

Exam 2  
Math 4753  
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Question 1:

$$\hat{\lambda}_{\text{mom}} = \frac{1}{n} \sum_{i=1}^n y_i = \bar{y} = \frac{1+2+3}{3} = 2$$

Question 2:

1. Joint  $P(y_1, y_2, y_3, \dots, y_n) = P(y_1) P(y_2) P(y_3) \dots P(y_n)$

$$\Rightarrow L(\lambda) = \frac{e^{-\lambda n} \lambda^{\sum_{i=1}^n y_i}}{y_1! \dots y_n!}$$

$$\Rightarrow \ell(\lambda) = -\lambda n + \sum_{i=1}^n y_i \ln(\lambda) - \ln(y_1! \dots y_n!)$$

$$\ell'(\lambda) = -n + \sum_{i=1}^n y_i / \lambda = 0$$

$$\Rightarrow \sum_{i=1}^n y_i = n\lambda \quad \Rightarrow \hat{\lambda}_{\text{MLE}} = \frac{\sum y_i}{n} = \bar{y}$$

$$\Rightarrow \hat{\lambda}_{\text{MLE}} = \frac{1+2+3}{3} = 2$$

Question 3:

$$E(\hat{\lambda}_{mle}) = E(\bar{y}) = \frac{1}{n} E(\sum y_i) = \frac{1}{n} \cdot n E(y) = \lambda \quad \text{as } E(y_i) = \lambda$$

Question 4:

$$V(\hat{\lambda}_{mle}) = V(\bar{y}) = \frac{1}{n^2} V(\sum y_i) = \frac{n}{n^2} V(y_i) = \frac{1}{n} V(y) = \frac{\lambda}{n} \quad \text{as } V(y) = \lambda$$

Question 5: we use B because we know  $\sigma^2$  not  $s^2$

Question 6:

$$L: \text{mean}(y) - qt(1 - \alpha/2, \text{length}(y) - 1) \cdot \text{sd}(y) / \sqrt{\text{length}(n)}$$

$$\text{where } \alpha = 0.2 \quad \text{and } y = c(3, 4, 5) \\ = 2.9113$$

Question 7: they are matched pairs because we are taking two measurements from the same sample

Question 8:

```
S = read.csv("shallow.csv")  
t.test(S$Actual, S$Predict, Paired=TRUE, conf.level=0.99) $ Conf
```

Question 9:

$$\text{Cov}(S\$Actual, S\$Predict) / (\text{sd}(S\$Actual) * \text{sd}(S\$Predict)) \\ \text{because } \rho = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$

Question 10:

```
S = read.csv("mow-mow.csv")
```

```
Var.test(S$y1, S$y2, conf.level=0.8) $ Conf
```

Question 11:

```
t.test(S$y1, S$y2, conf.level=0.92, var.equal=FALSE) $ Conf
```

Question 11

$$V(L) = V(y_1 - 2y_2 + 3y_3) = V(y_1) + V(-2y_2) + V(3y_3)$$

$$= V(y_1) + (-2)^2 V(y_2) + 3^2 V(y_3) = 2 + 4 \cdot 2 + 9 \cdot 2 = 28$$