

# Homework 1

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## Setup

```
library(tidyverse)
library(usethis)
# usethis::use_github()
set.seed(42)
```

## Functions

- **generate\_sample**
  - generates 1 sample

```
generate_sample <- function(N, y0 = 0, alpha = 0.8) {
  accumulate(rep(NA, N - 1), ~ alpha * .x + rnorm(1, 0, 1), .init = y0)
}
```

- **draw\_samples**
  - draws [times] samples of sample sizes [N\_values]

```
draw_samples <- function(N_values, times, y0 = 0, alpha = 0.8) {
  map(N_values, ~ rerun(times, generate_sample(N = .x, y0, alpha)))
}
```

- **estimate\_alpha**
  - estimates the alpha
  - easier for next steps to use dbl instead of list (list would be just map)

```
estimate_alpha <- function(samples) {
  map_dbl(samples, ~ ar(.x, order.max = 1, aic = FALSE, method = "ols")$ar)
}
```

- **compute\_statistics**
  - returns bias, variance and mse

```
compute_statistics <- function(estimates, true_alpha = 0.8) {
  bias <- mean(estimates) - true_alpha
  variance <- var(estimates)
  mse <- variance + bias^2

  stats <- tibble(bias, variance, mse)
```

```

return(stats)
}

```

- **plot\_statistics**

- plots bias, variance, and mse against sample size

```

plot_statistics <- function(stats) {
  stats %>%
    ggplot(aes(x = as.numeric(N), y = value)) +
    geom_point() +
    geom_line(linetype = "dotted") +
    facet_wrap(~ factor(stat, levels = c("bias", "variance", "mse")), scales = "free_y", nrow = 3) +
    theme_minimal(base_size = 15) +
    theme(
      panel.spacing = unit(1, "lines"),
      strip.text = element_text(face = "bold")
    ) +
    labs(x = "Sample Size (N)", y = NULL)
}

```

## 3.1 Generate observations DGP

```

true_alpha <- 0.8
sample_sizes <- c(10, 50, 100, 500, 1000)
samples <- draw_samples(sample_sizes, alpha = true_alpha, times = 5000)
alphas <- map(samples, estimate_alpha) %>% set_names(sample_sizes)

```

## 3.2 Compute and Plot Bias, Variance and MSE

i)

```

stats <- map(alphas, ~ compute_statistics(.x)) %>%
  bind_rows(.id = "N") %>%
  pivot_longer(-N, names_to = "stat", values_to = "value")

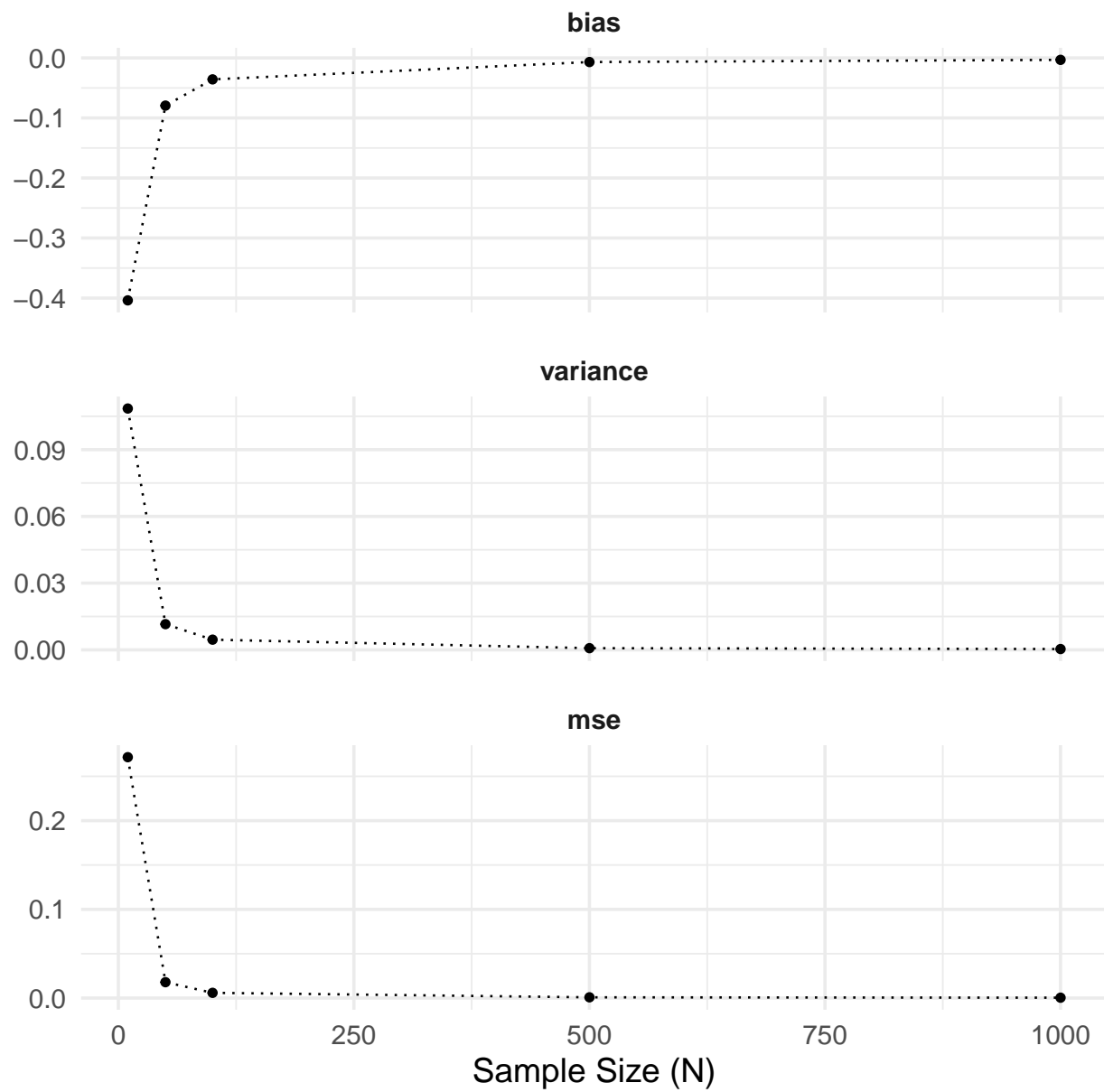
```

ii)

```

plot_statistics(stats)

```



### 3.3 Interpretation

The MSE, variance, and bias converge in probability to 0 or in other words the estimator converges in probability to the true parameter value. Hence the OLS estimator is a consistent estimator for  $\alpha$ .