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Instructions: Update this file (or recreate a similar one, e.g. in Word) to prepare your answers to the questions. Feel free to add text, equations and figures as needed. Hand-written notes, e.g. for the development of equations, can also be included e.g. as pictures (from your cell phone or from a scanner). ***This lab is not graded. However, the lab exercises are meant as a way to familiarise with dynamical systems and to study them using Python to prepare you for the final project.*** *This file does not need to be submitted and is provided for your own benefit. The graded exercises will have a similar format.*

The file `lab#.py` *is provided to run all exercises in Python. Each* `exercise#.py` *can be run to run an exercise individually. The list of exercises and their dependencies are shown in Figure 1. When a file is run, message logs will be printed to indicate information such as what is currently being run and and what is left to be implemented. All warning messages are only present to guide you in the implementation, and can be deleted whenever the corresponding code has been implemented correctly.*

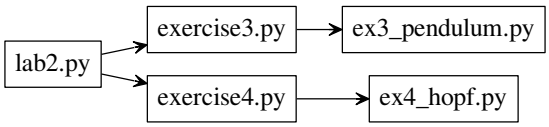

Figure 1: Exercise files dependencies. In this lab, you will be modifying `exercise3.py`, `ex3_pendulum.py`, `exercise4.py` *and* `ex4_hopf.py`.

Question 3: Pendulum with friction

3.a Find the fixed points of the pendulum with friction (i.e. damping), and analyze their stability using a local linearization (briefly describe the calculation steps).

$$\ddot{\theta} = -\frac{g}{L}\sin\theta - d\dot{\theta} \tag{1}$$

where θ is the angle, g the gravity constant, L the length of the pendulum and d is the damping coefficient.

3.b Numerically solve the differential equations of the pendulum with different initial conditions. Show several time evolutions and phase portraits with different initial conditions that illustrate several aspects of the interesting behavior of the pendulum. See `exercise3.py` and `ex3_pendulum.py` for help with implementation.

3.c Investigate and describe how the behavior of the pendulum changes if friction is zero (d=0). Show a new phase portrait.

3.d Does the pendulum without friction (d=0) produce stable limit cycles? Discuss, and try to support your statement with some numerical simulations (show figures) and/or analytical arguments.

3.e Investigate how the behavior of the pendulum changes if the viscous friction term is replaced with a dry (Coulomb) friction term. Unlike viscous friction, dry friction does not depend on speed, only the direction of movement. What are the main differences between the two types of pendulum? (discuss and show some examples). And is there anything notable about the numerical integration of the pendulum with dry friction? If yes, what and why?

$$\ddot{\theta} = -\frac{g}{L}\sin\theta - d \cdot sign(\dot{\theta}) \tag{2}$$

Question 4: Coupled Hopf oscillators

4.a Implement a single Hopf oscillator and illustrate its various behaviors using time evolution and phase plane figures. See `exercise4.py` and `ex4_hopf.py` for help with implementation.

4.b Implement a system of two coupled Hopf oscillators and illustrate its behavior using figures showing time evolutions of states and of angles. You are free to propose your own coupling. Investigate phase locked behavior, and more generally how coupling weights and different parameters such as intrinsic frequency affect the global behavior.