

# Common Distributions

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For more details, see Chapter 4 of <https://www.openintro.org/book/os/>, or on Wikipedia.

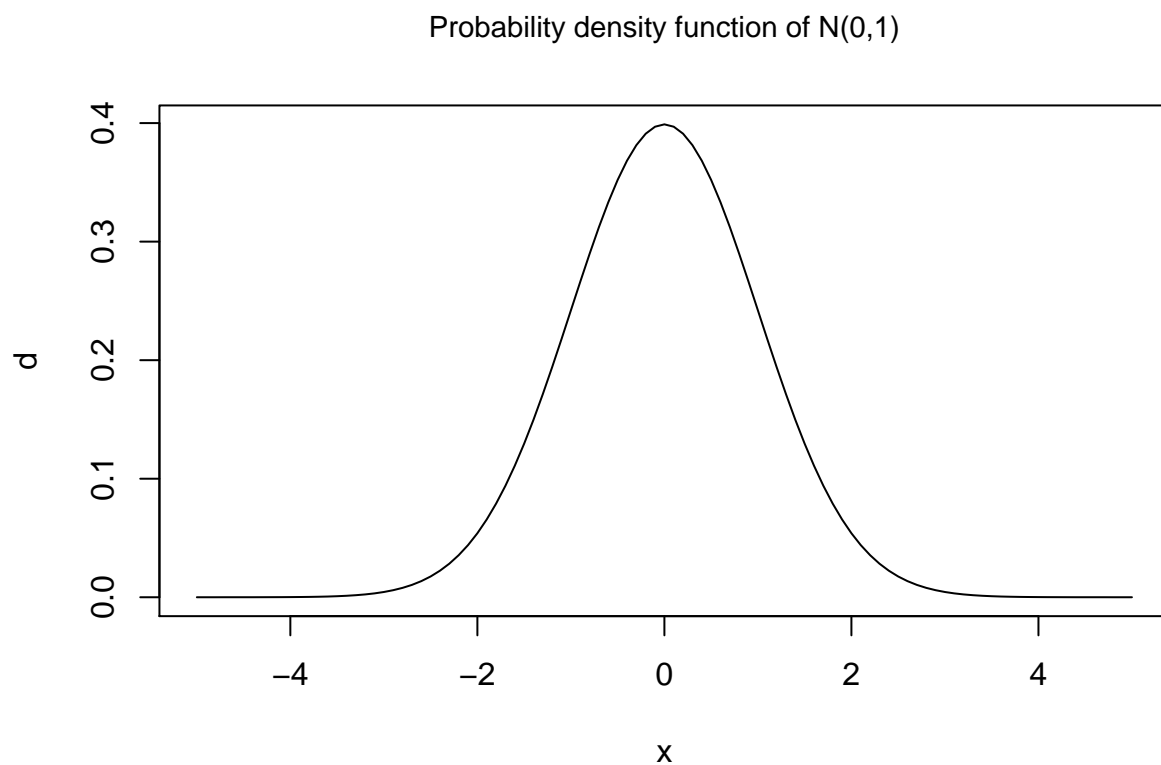
**Normal Distribution :**  $N(\mu, \sigma^2)$

Lots of real-life distributions are nearly normal, because of the Central Limit Theorem. The density plot of a normal distribution is symmetric and bell-shaped.

```
dnorm(0)
```

```
## [1] 0.3989423
```

```
x <- seq(-5, 5, by=0.1)
d <- dnorm(x, mean=0, sd=1)
plot(x, d, type="l", font.main=1, cex.main=0.9, main="Probability density function of N(0,1)")
```



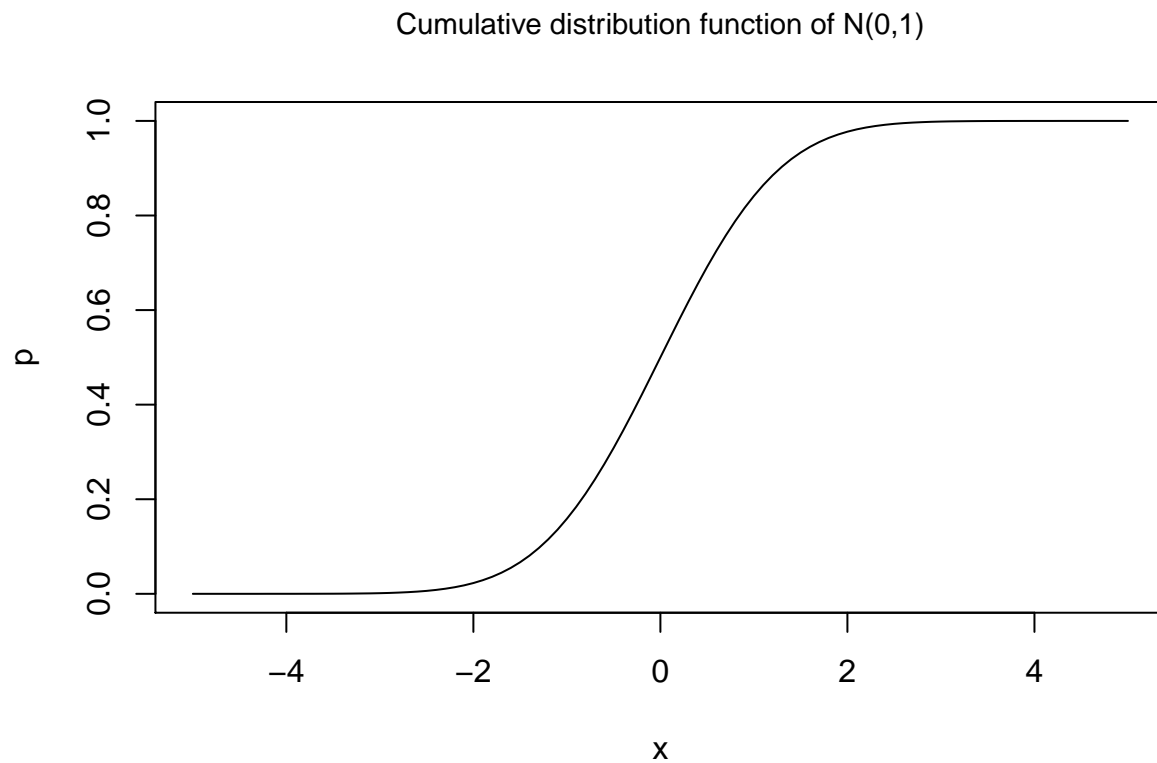
```
pnorm(0)
```

```
## [1] 0.5
```

```
x <- seq(-5, 5, by=0.1)
```

```
p <- pnorm(x)
```

```
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of N(0,1)")
```



```
qnorm(0.5)
```

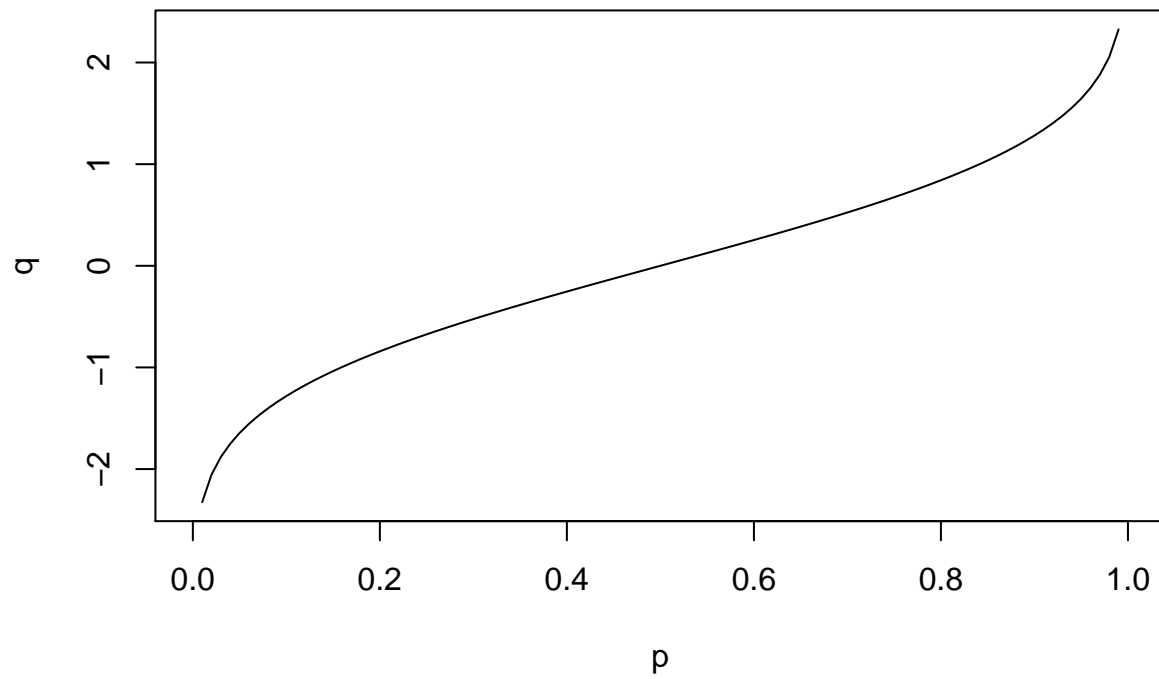
```
## [1] 0
```

```
p <- seq(0, 1, by=0.01)
```

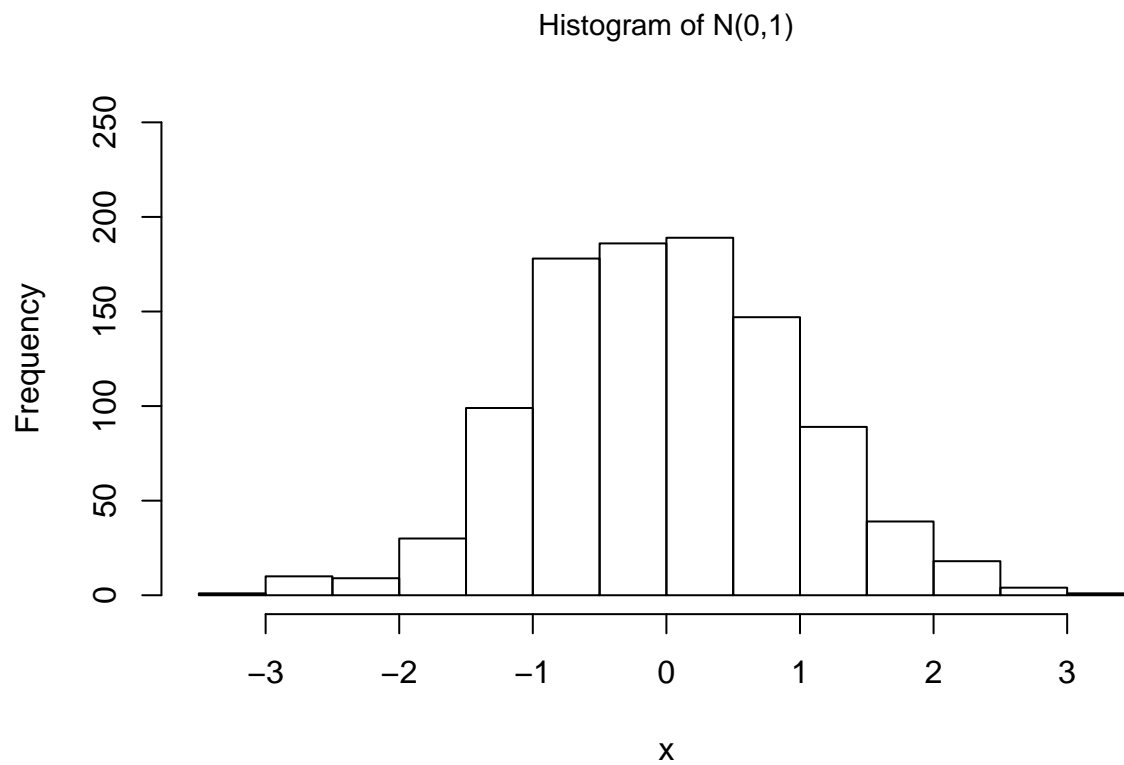
```
q <- qnorm(p)
```

```
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of N(0,1)")
```

Quantile function of  $N(0,1)$



```
x <- rnorm(1000)
hist(x, font.main=1, cex.main=0.9, main="Histogram of N(0,1)", ylim = c(0, 250))
```

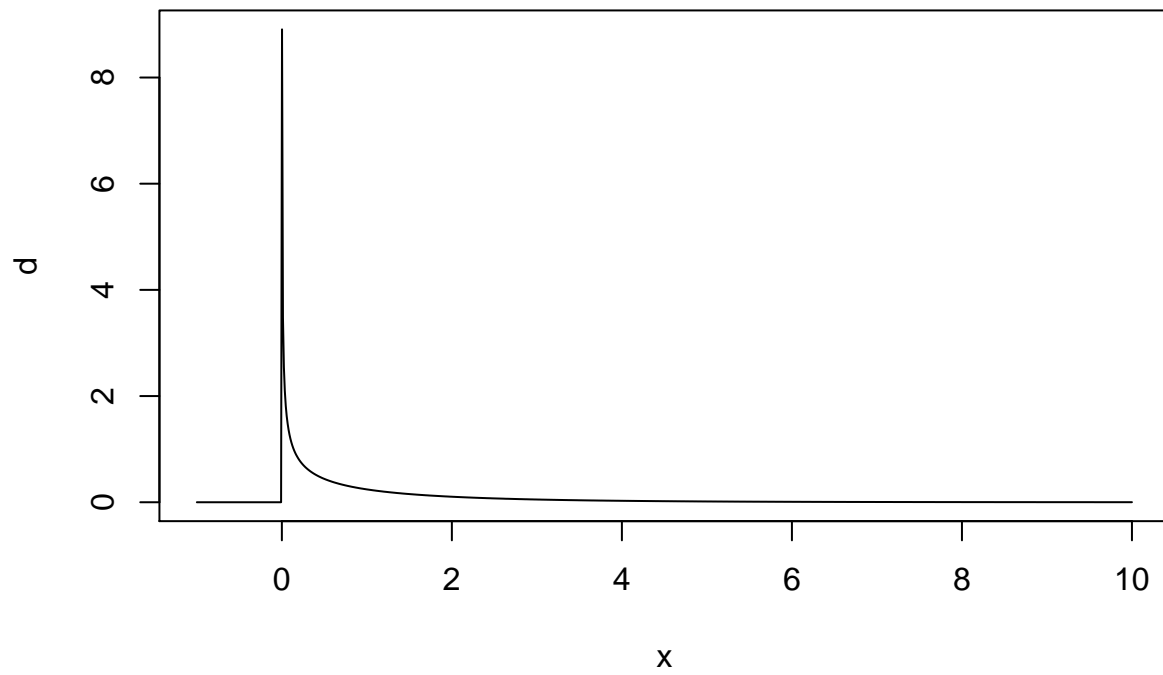


**Chi-squared Distribution:**  $\chi_k^2$

Sum of square of  $k$  independent  $N(0,1)$ .

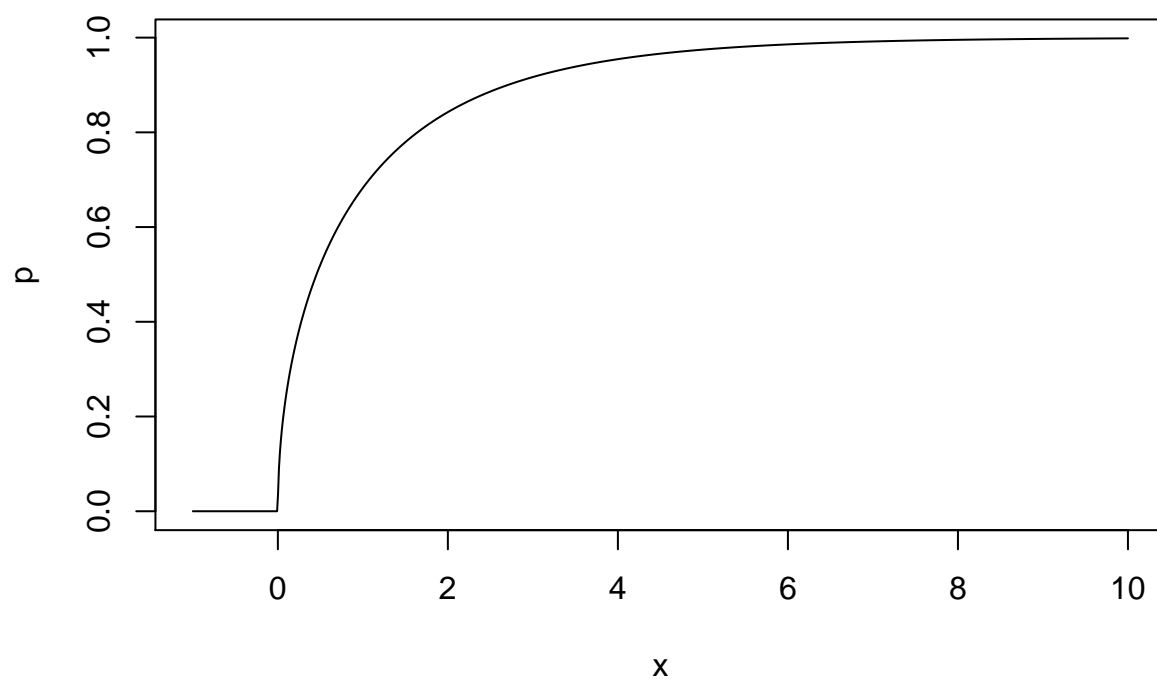
```
x <- seq(-1, 10, length.out=1000)
d <- dchisq(x, df=1)
plot(x, d, type="l", font.main=1, cex.main=0.9, main="Probability density function of Chi-squared(1)")
```

Probability density function of Chi-squared(1)



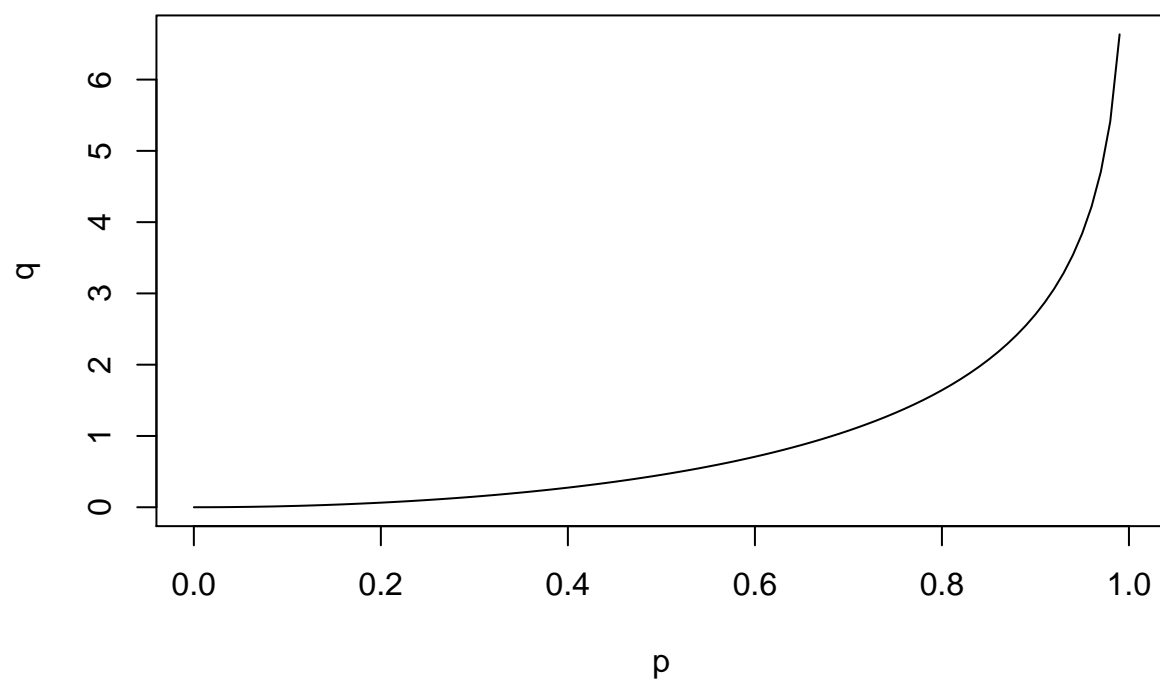
```
x <- seq(-1, 10, length.out=1000)
p <- pchisq(x, df=1)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Chi-squared(1)
```

Cumulative distribution function of Chi-squared(1)

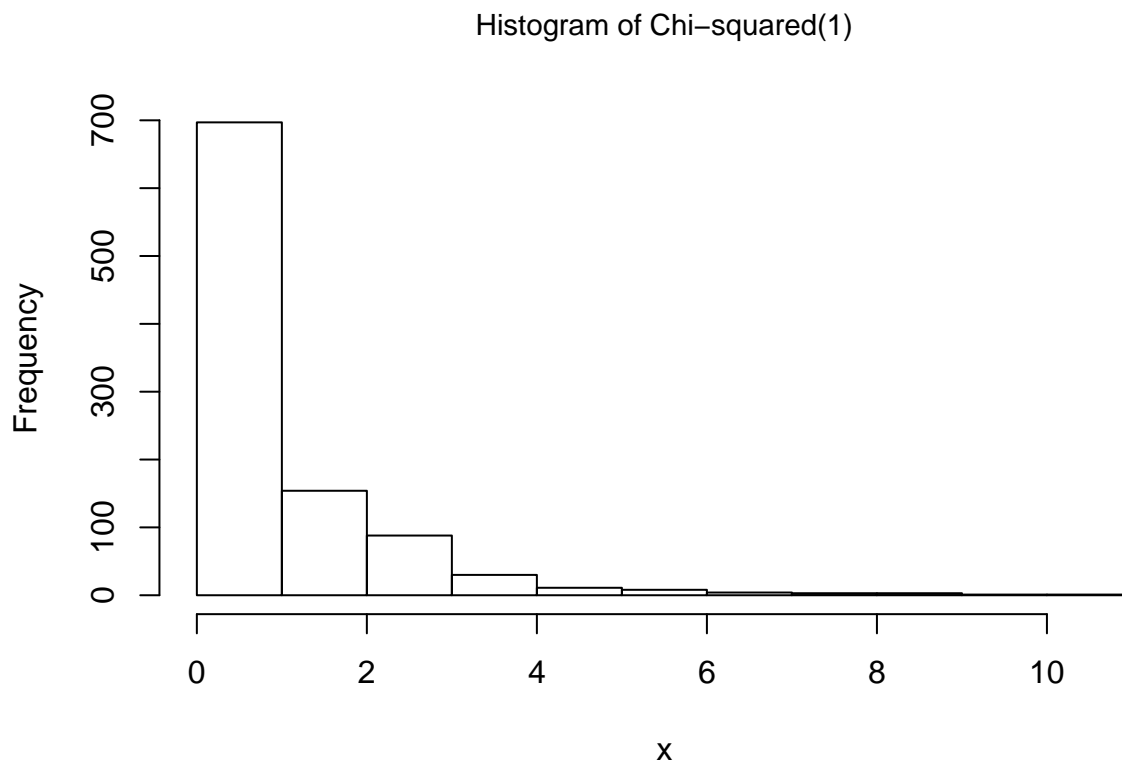


```
p <- seq(0, 1, by=0.01)
q <- qchisq(p, df=1)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Chi-squared(1)")
```

Quantile function of Chi-squared(1)



```
x <- rchisq(1000, df=1)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Chi-squared(1)")
```



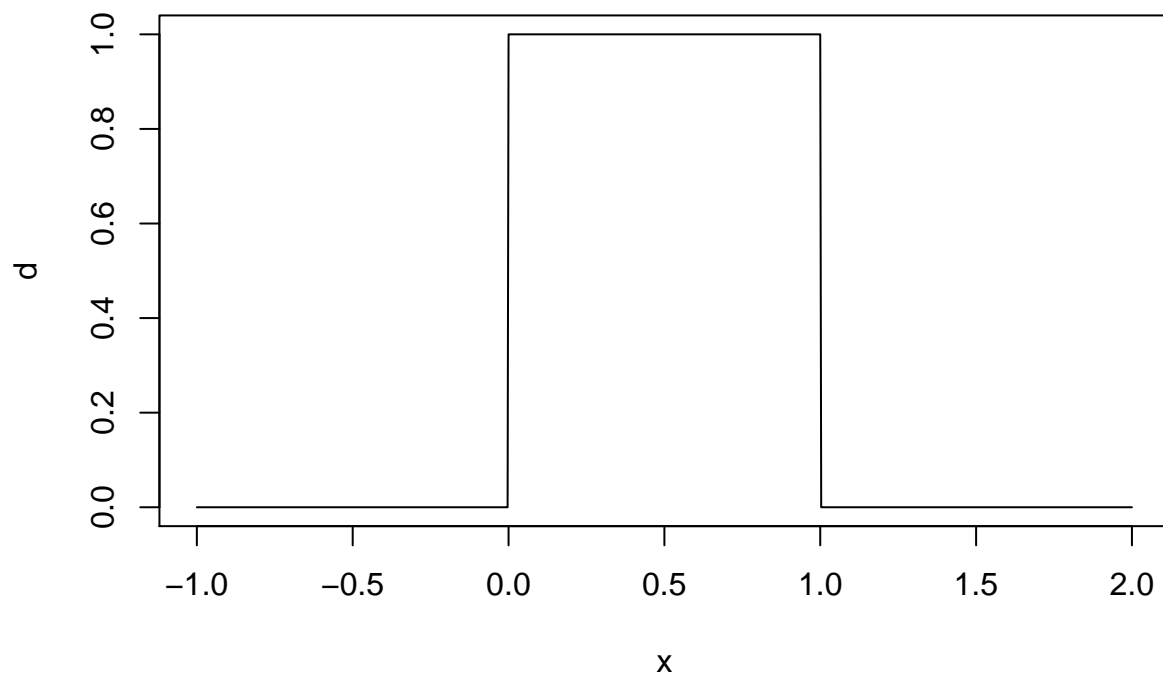
**Uniform Distribution (continous):**  $\text{Unif}(a, b)$

Density is a constant on the support  $(a, b)$ .

```
x <- seq(-1, 2, length.out=1000)
d <- dunif(x, min=0, max=1)
plot(x, d, type="l", font.main=1, cex.main=0.9, main="Probability density function of Unif(0,1)")
```

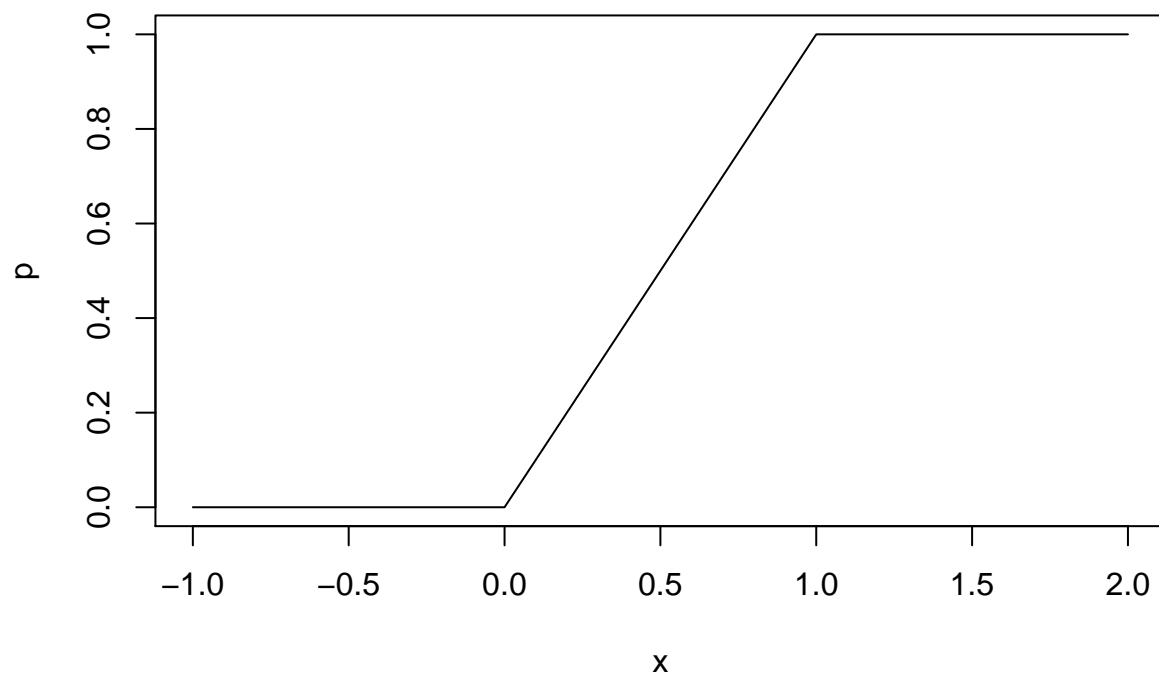


Probability density function of Unif(0,1)



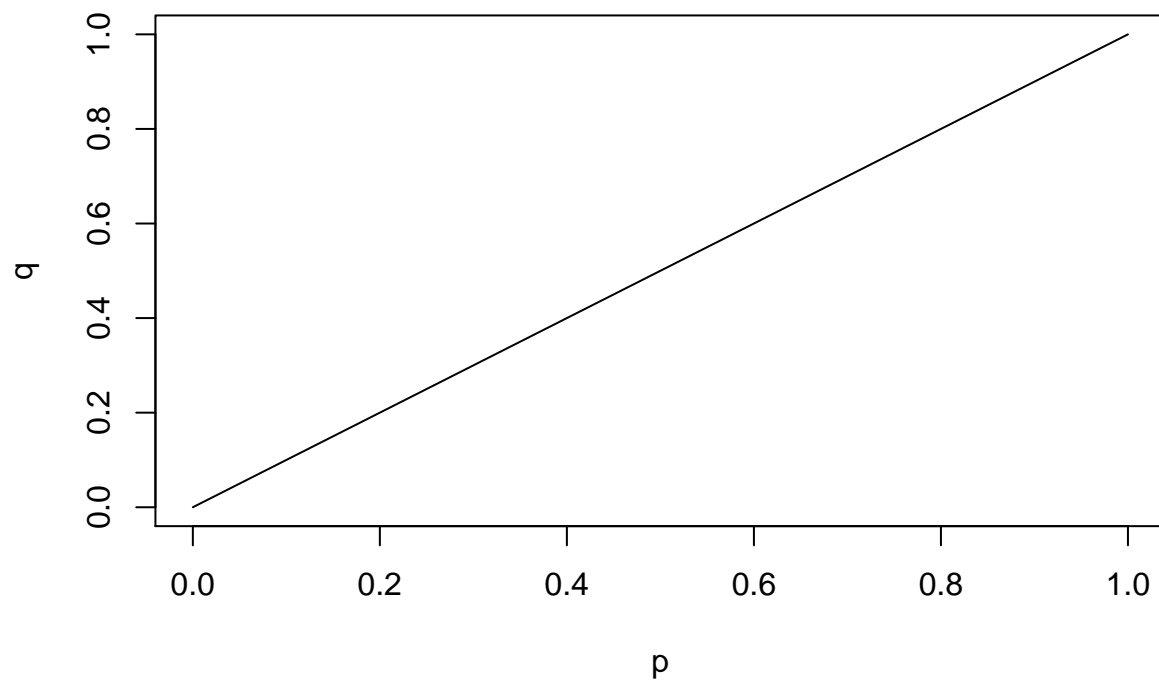
```
x <- seq(-1, 2, length.out=1000)
p <- punif(x)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Unif(0,1)")
```

Cumulative distribution function of Unif(0,1)

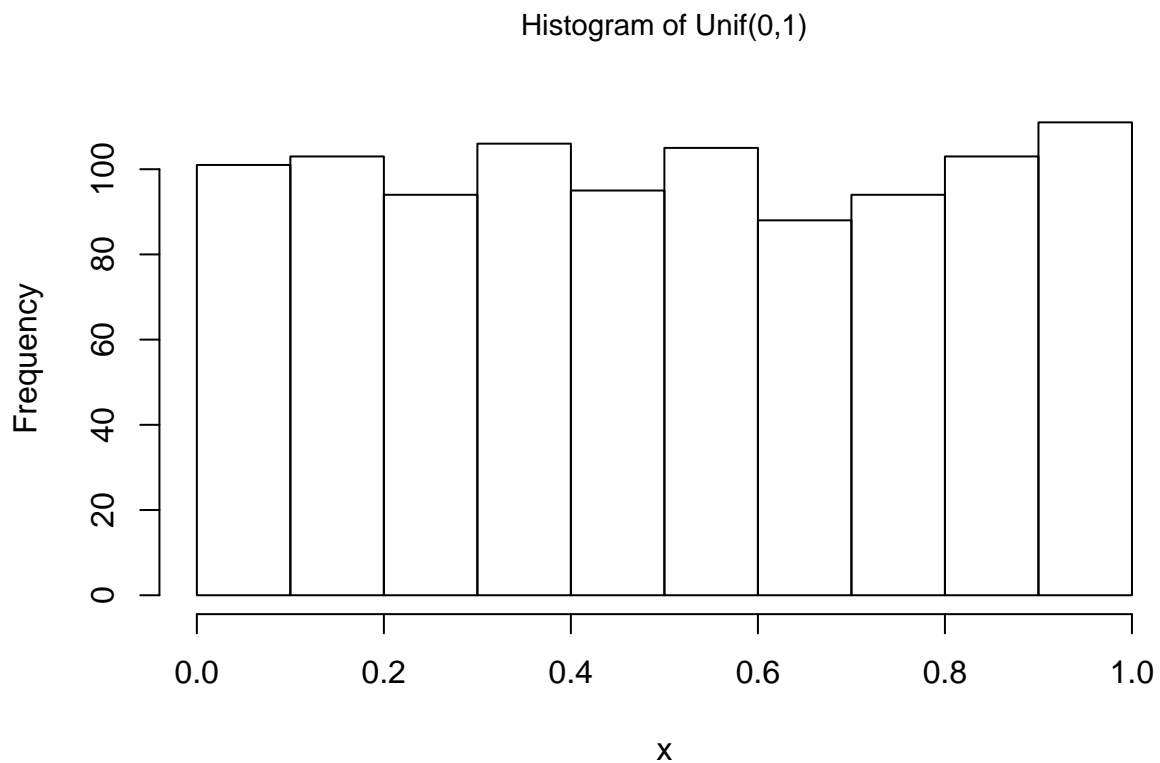


```
p <- seq(0, 1, by=0.01)
q <- qunif(p)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Unif(0,1)")
```

Quantile function of Unif(0,1)



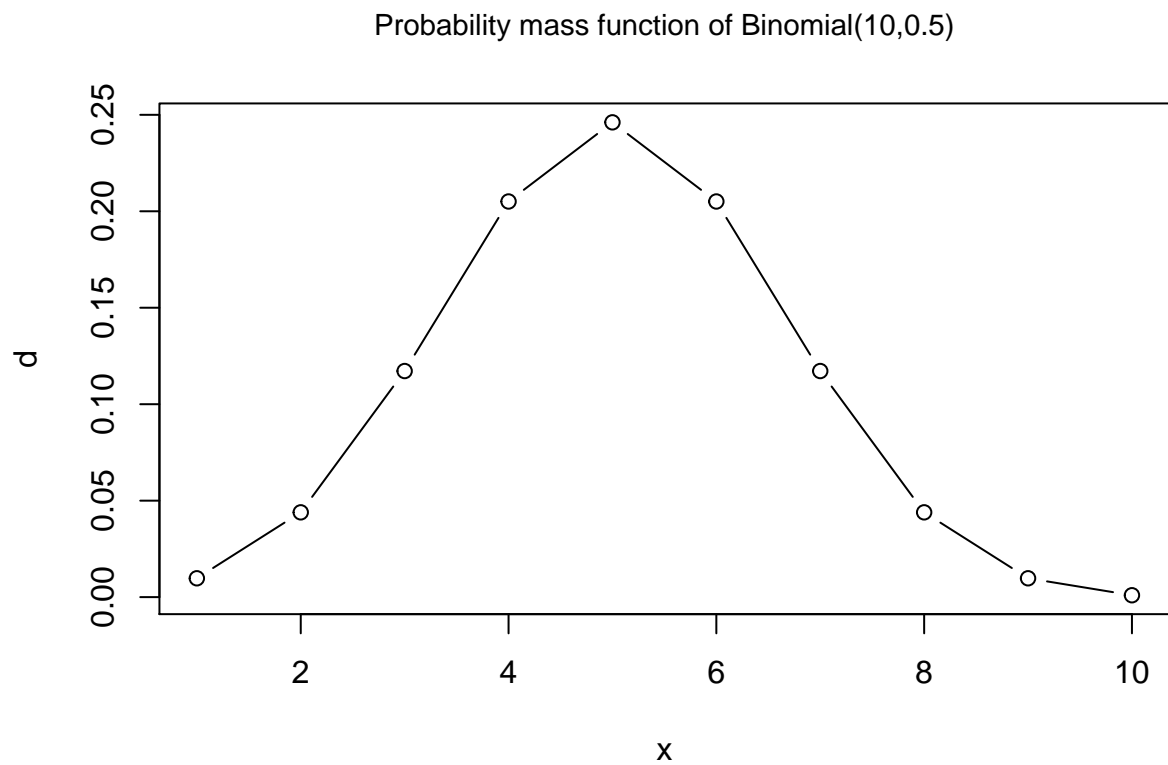
```
x <- runif(1000)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Unif(0,1)")
```



### Binomial Distribution: $\text{Binomial}(n, p)$

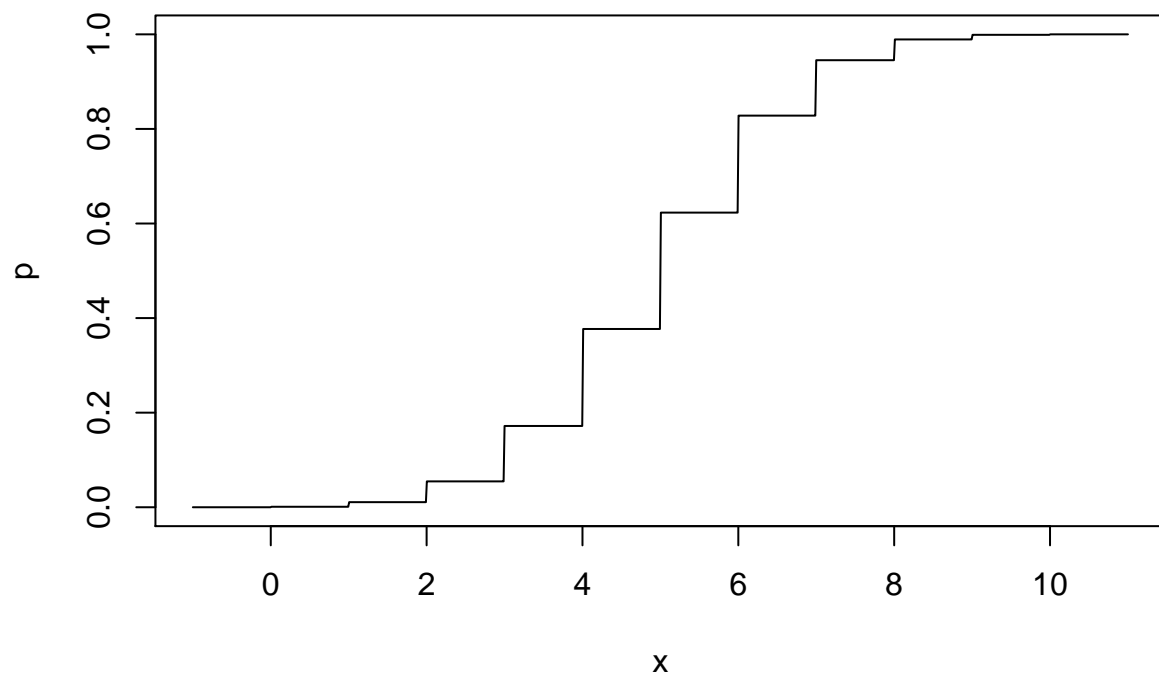
Number of heads in  $n$  flips,  $p$  is the probability of landing a head.  $\text{Bernoulli}(p)$  is equivalent to  $\text{Binomial}(n, p)$ .

```
x <- seq(1, 10, by=1)
d <- dbinom(x, size=10, prob=0.5)
plot(x, d, type="b", font.main=1, cex.main=0.9, main="Probability mass function of Binomial(10,0.5)")
```



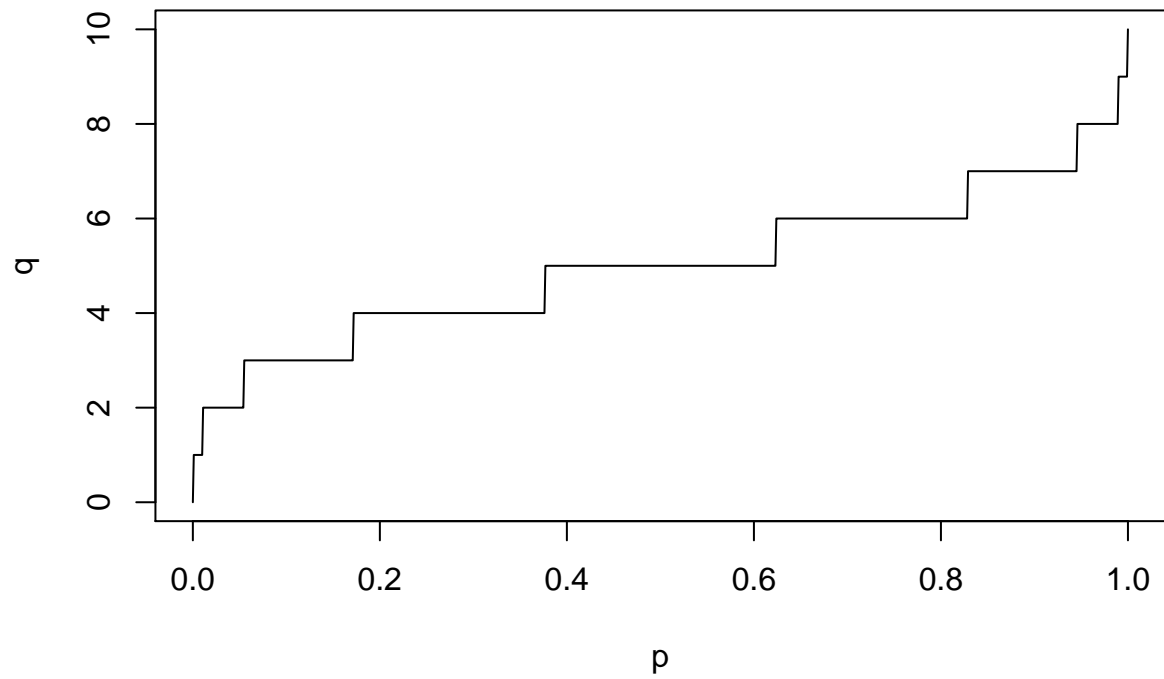
```
x <- seq(-1, 11, length.out=1000)
p <- pbinom(x, size=10, prob=0.5)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Binomial(10,0.5)")
```

Cumulative distribution function of Binomial(10,0.5)

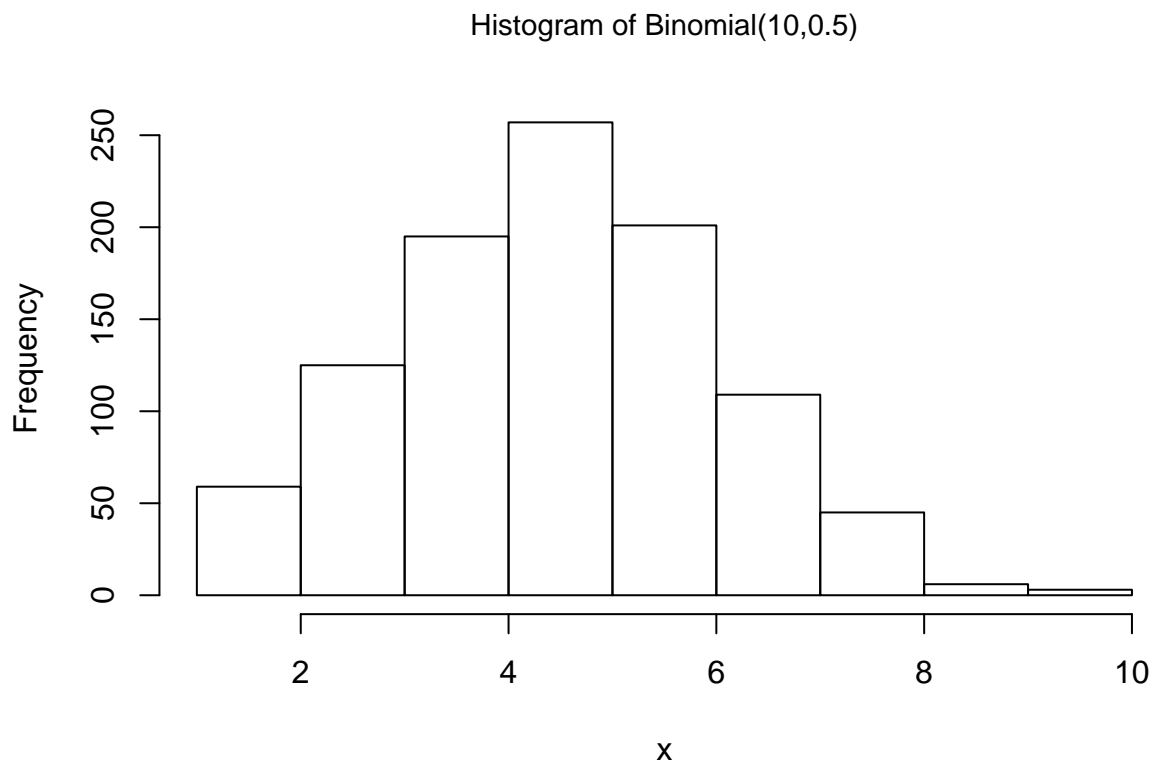


```
p <- seq(0, 1, by=0.001)
q <- qbinom(p, size=10, prob=0.5)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Binomial(10,0.5)")
```

Quantile function of Binomial(10,0.5)



```
x <- rbinom(1000, size=10, prob=0.5)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Binomial(10,0.5)")
```



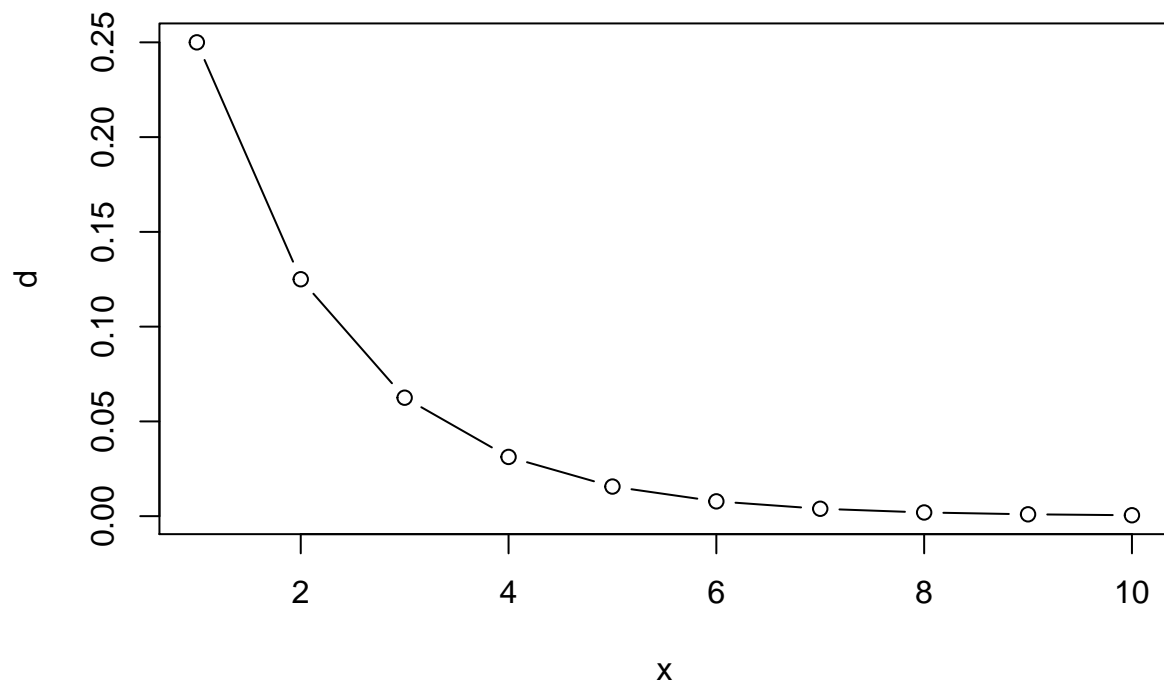
### Geometric Distribution: $\text{Geometric}(p)$

Number of flips until the first head.

```
x <- seq(1, 10, by=1)
d <- dgeom(x, prob=0.5)
plot(x, d, type="b", font.main=1, cex.main=0.9, main="Probability mass function of Geometric(0.5)")
```

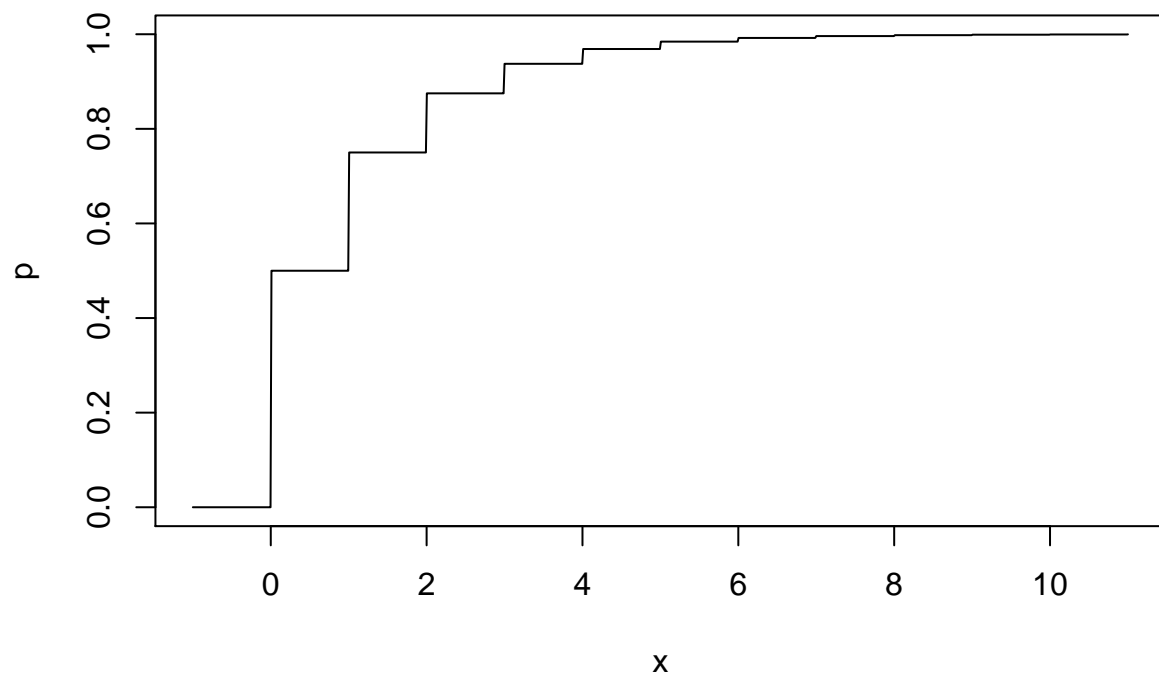


Probability mass function of Geometric(0.5)



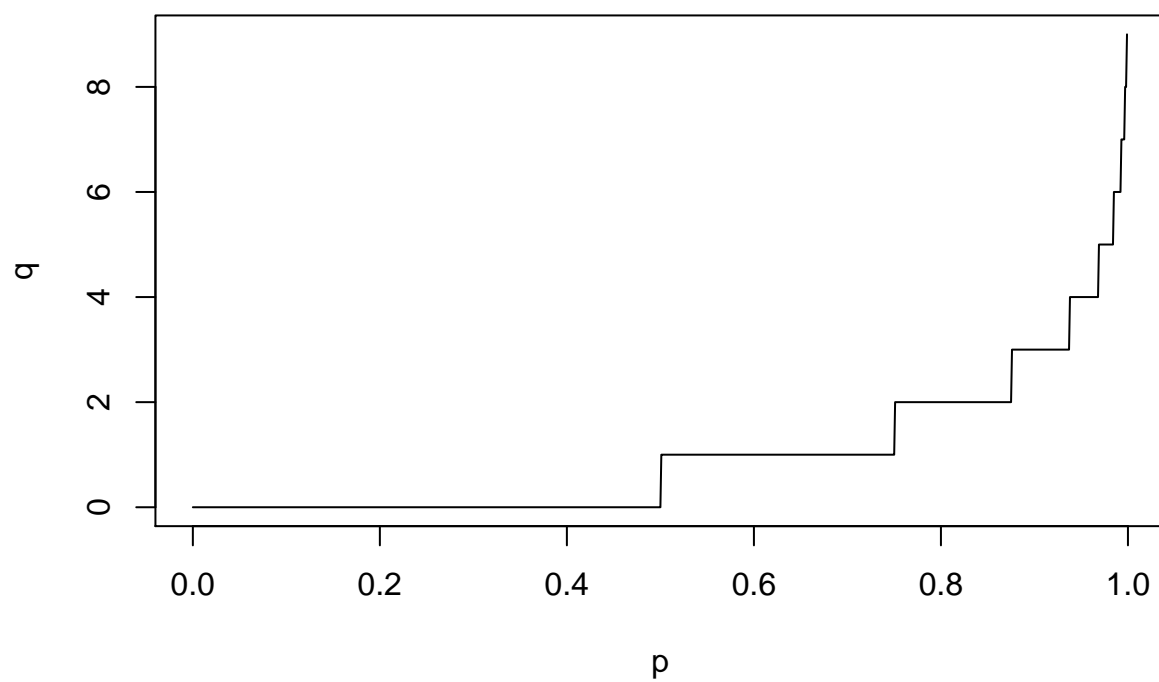
```
x <- seq(-1, 11, length.out=1000)
p <- pgeom(x, prob=0.5)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Geometric(0.5)
```

Cumulative distribution function of Geometric(0.5)



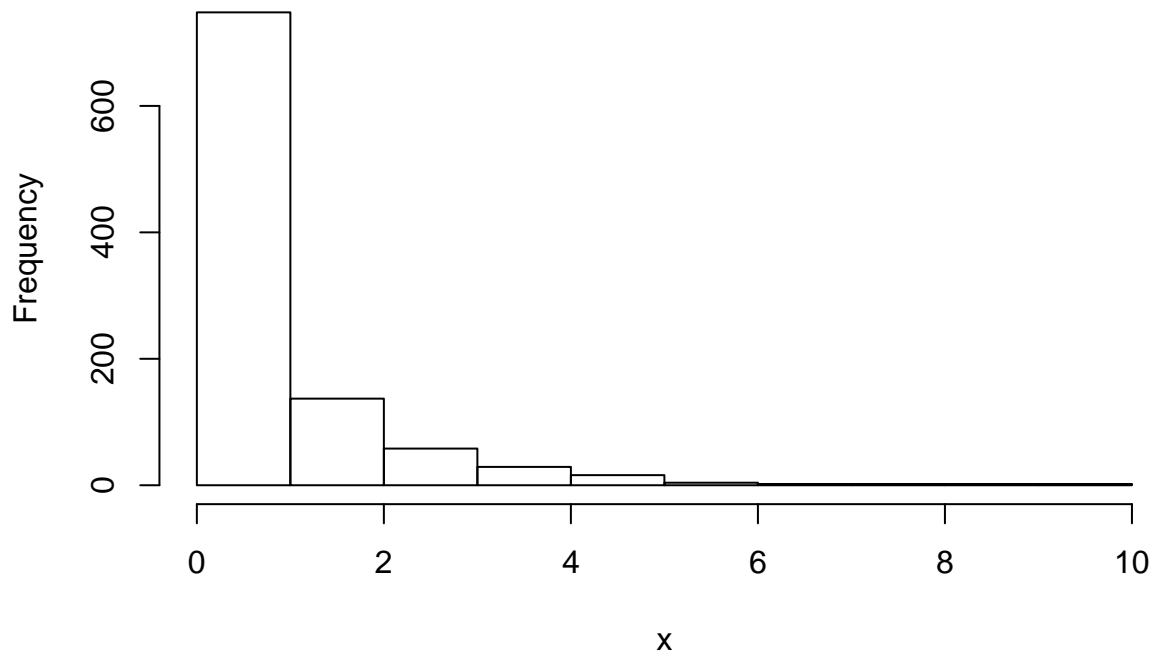
```
p <- seq(0, 1, by=0.001)
q <- qgeom(p, prob=0.5)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Geometric(0.5)")
```

Quantile function of Geometric(0.5)



```
x <- rgeom(1000, prob=0.5)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Geometric(0.5)")
```

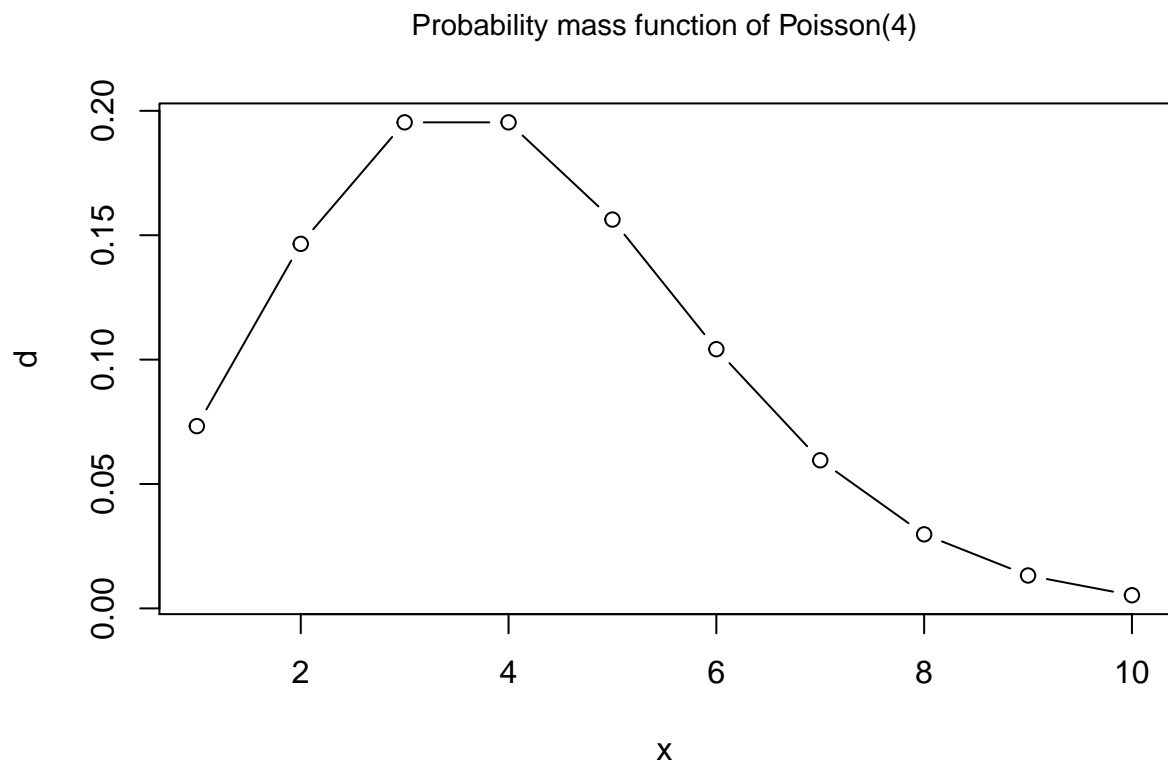
Histogram of Geometric(0.5)



### Poisson Distribution: $\text{Poisson}(\lambda)$

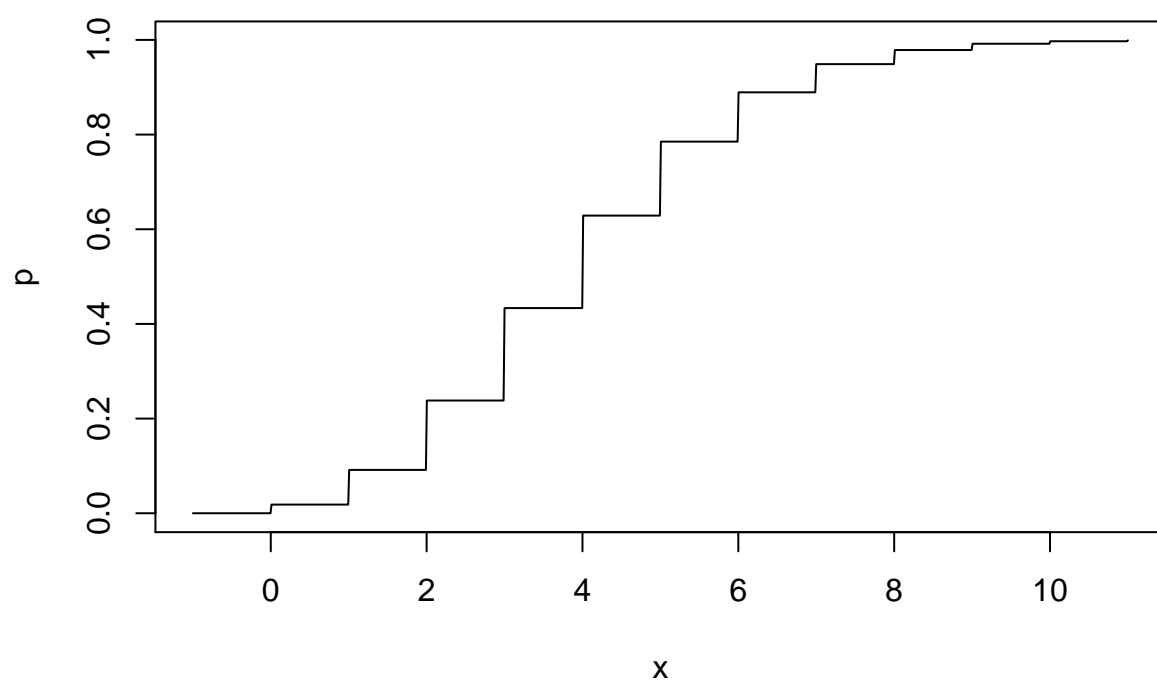
Assume a bus comes at a constant rate.  $\text{Poisson}(\lambda)$  denotes the number of buses appears in 1 hour, if the average (expectation) is  $\lambda$ .

```
x <- seq(1, 10, by=1)
d <- dpois(x, lambda=4)
plot(x, d, type="b", font.main=1, cex.main=0.9, main="Probability mass function of Poisson(4)")
```



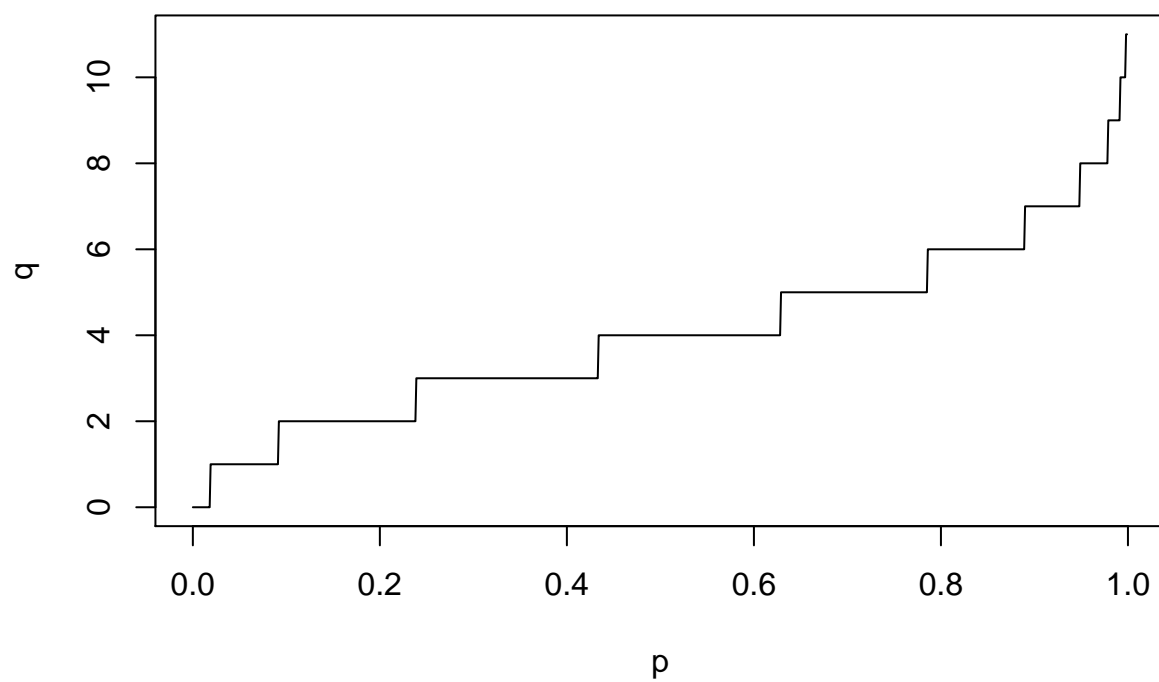
```
x <- seq(-1, 11, length.out=1000)
p <- ppois(x, lambda=4)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Poisson(4)")
```

Cumulative distribution function of Poisson(4)

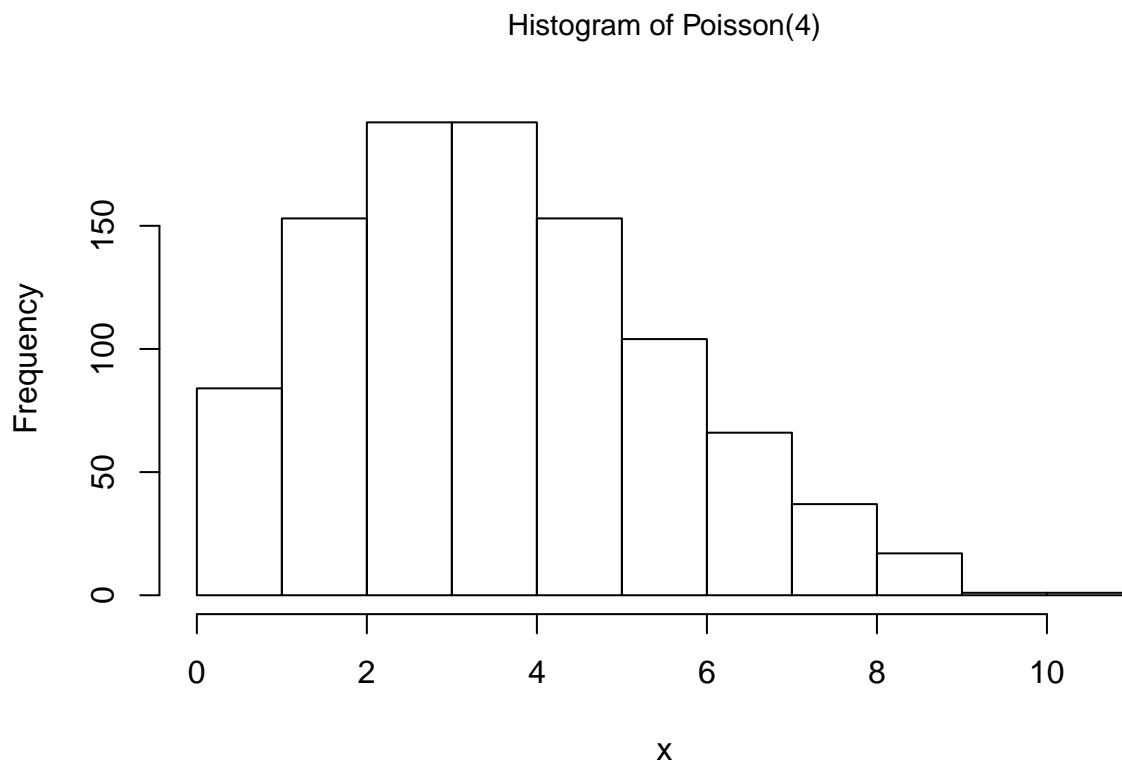


```
p <- seq(0, 1, by=0.001)
q <- qpois(p, lambda=4)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Poisson(4)")
```

Quantile function of Poisson(4)



```
x <- rpois(1000, lambda=4)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Poisson(4)")
```



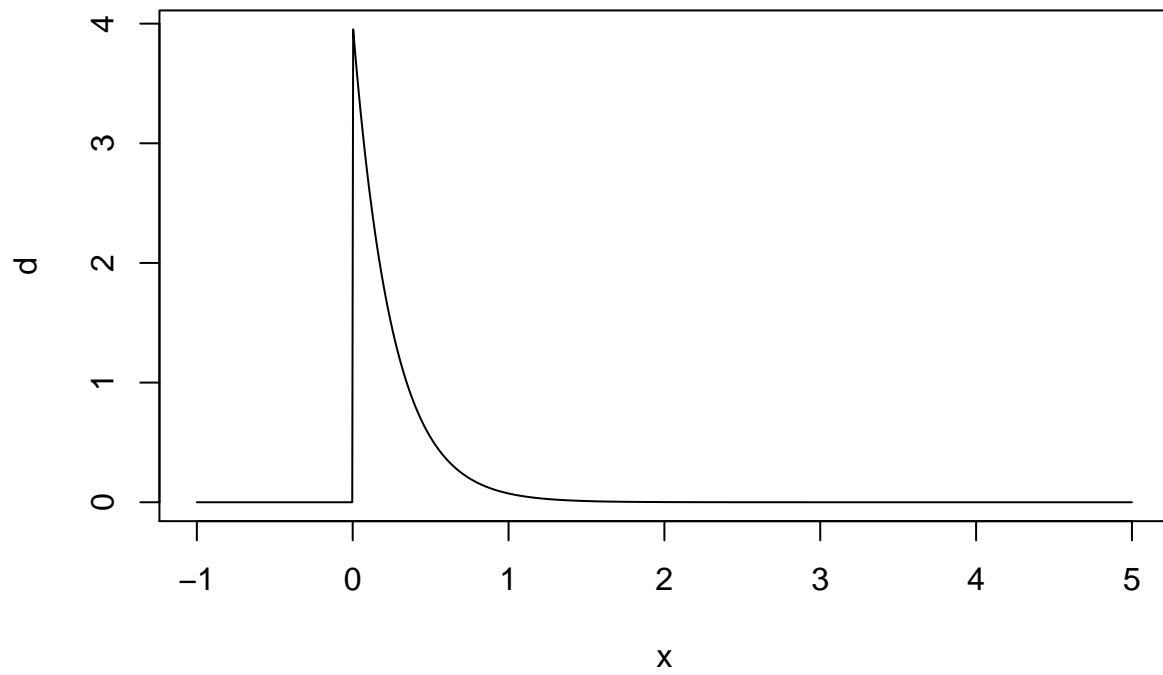
**Exponential Distribution:  $\text{Exponential}(\lambda)$**

Time until the next bus show up.

```
x <- seq(-1, 5, length.out=1000)
d <- dexp(x, rate=4)
plot(x, d, type="l", font.main=1, cex.main=0.9, main="Probability density function of Exponential(4)")
```

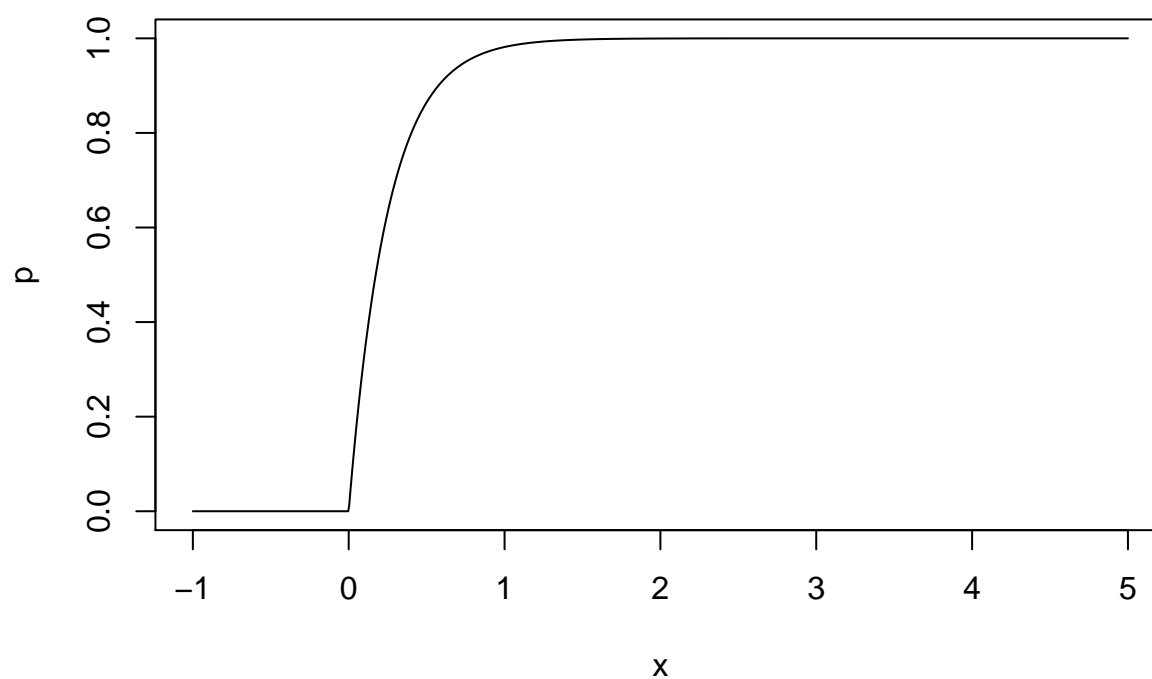


Probability density function of Exponential(4)



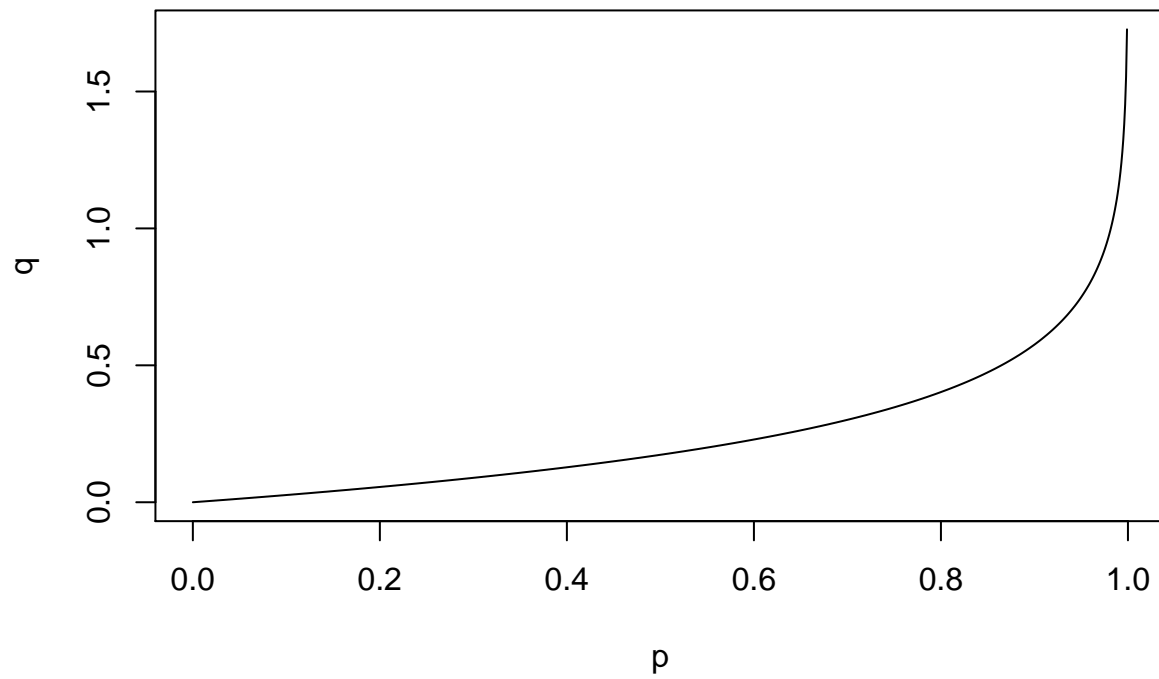
```
x <- seq(-1, 5, length.out=1000)
p <- pexp(x, rate=4)
plot(x, p, type="l", font.main=1, cex.main=0.9, main="Cumulative distribution function of Exponential(4)
```

Cumulative distribution function of Exponential(4)



```
p <- seq(0, 1, by=0.001)
q <- qexp(p, rate=4)
plot(p, q, type="l", font.main=1, cex.main=0.9, main="Quantile function of Exponential(4)")
```

Quantile function of Exponential(4)



```
x <- rexp(1000, rate=4)
hist(x, font.main=1, cex.main=0.9, main="Histogram of Exponential(4)")
```

Histogram of Exponential(4)

