

COMP4130 Linear and Discrete Optimization

Lecture 4: Post-optimality Analysis

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Post-optimality Analysis

- **The Role of Post-optimality Analysis**
- **Post-optimality Analysis Tools**
- **Good Practices in Developing Spreadsheet Models**

Additional Reading:

- Chapter 4.9 'Post-optimality Analysis', *Introduction to Operations Research*.
- Chapter 21 'The Art of Modeling with Spreadsheets', *Introduction to Operations Research*.

Post-optimality Analysis

Re-optimization:

- Post-optimality analysis (also called what-if analysis) is performed after an optimal solution has been found.
- It examines how the solution changes when model parameters are modified, without repeatedly solving the problem from scratch.
- Through re-optimization, alternative models and solutions can be explored by adjusting parameters.
- When applied effectively, re-optimization often requires fewer iterations to reach a new solution.

Post-optimality Analysis

Sensitivity Analysis:

- Sensitivity analysis is a common type of post-optimality (what-if) analysis.
- It interprets the optimal solution and evaluates the effect of parameter changes on the solution and objective value.
- Applicable primarily to linear programming (LP) models.
- For integer programming (IP/BIP) models: analysis is based on their LP relaxation.
- For mixed-integer programming (MIP) models: analysis can be conducted on the LP relaxation or on LP sub-problems after fixing integer variables to optimal values.

Post-optimality Analysis

Key Questions Answered:

- What is the slack (if any) of resources in the optimal solution?
- Which resources are **critical** to the optimization problem?
- How do changes in resource availability affect the optimal solution and objective value?
- What is the acceptable cost of increasing resource availability?
- What is the effect of modifying the **objective function parameters**?
- How do changes in decision variable values affect feasibility?
- Are there **alternative optimal solutions** of equal quality?
- How can the current optimal solution be improved?

Post-optimality Analysis Tools

Most optimization solvers, including Excel Solver and LP-Solve, provide built-in post-optimality analysis reports.

- **Excel Solver:** Answer Report, Sensitivity Report, Limits Report.
 - The layout and labelling of the spreadsheet model affect how results are displayed.
- **LP-Solve:** Constraints Report, Sensitivity Report.
 - To generate complete sensitivity results, all decision variables must appear in every constraint, preferably in standard form (e.g., $0X_1 + 10X_2 \leq 800$).

Other powerful solvers such as **CPLEX**, **GUROBI**, **LINGO**, **GAMS**, **XPRESS** also provide extensive post-optimality analysis tools, including support for re-optimization.

Post-optimality Analysis: Example

WENBU Product-mix Optimization Problem

WENBU is a food processing plant producing bacon and bread. The mill can grind at most 800 units of flour per day, and each kilogram of bread requires 10 units of flour. Bacon requires pork, with up to 1500 units supplied daily; each kilogram of bacon consumes 25 units. Labour is limited to two part-time employees working 5 hours each per day (600 minutes total). Producing a kilogram of bacon uses 5 minutes of labour, while a kilogram of bread uses 7 minutes. Each kilogram of bacon yields a profit of \$25 and each kilogram of bread yields \$10. The company wishes to determine the daily production quantities of bacon and bread that maximize profit, assuming fractional production is possible.

Post-optimality Analysis: Example

WENBU Product-mix Optimization Problem

1. Identify parameters (numerical data).
2. Define decision variables (competing activities).
3. State the objective function (maximize or minimize Z).
4. Formulate functional and non-negativity constraints.
5. Define integrality constraints (IP).

Post-optimality Analysis: Example

1. Parameters

- Flour capacity: 800 units/day; bread uses 10 units/kg.
- Pork supply: 1500 units/day; bacon uses 25 units/kg.
- Labour: 600 minutes/day; bacon uses 5 min/kg, bread 7 min/kg.
- Profits: bacon \$25/kg, bread \$10/kg.

2. Decision variables

X_1 = kg of bacon produced per day, X_2 = kg of bread produced per day

3. Objective function: $\max Z = 25X_1 + 10X_2$

4. Functional Constraints

$$10X_2 \leq 800 \quad (\text{flour})$$

$$25X_1 \leq 1500 \quad (\text{pork})$$

$$5X_1 + 7X_2 \leq 600 \quad (\text{labour})$$

5. Non-negativity

$$X_1 \geq 0, \quad X_2 \geq 0$$

Post-optimality Analysis: Example

The LP model can be formulated as:

$$\text{Maximize: } Z = 25X_1 + 10X_2$$

$$\text{Subject to: } 10X_2 \leq 800 \quad (1)$$

$$25X_1 \leq 1500 \quad (2)$$

$$5X_1 + 7X_2 \leq 600 \quad (3)$$

$$X_1, X_2 \geq 0 \quad (4)$$

Post-optimality Analysis: Excel

Formulating WENBU Product-mix Optimization Problem:

	A	B	C	D	E	F	G	H	I	J
1	Wenbu Company Production Problem									
2										
3	Requirements									
4		Labour	Flour	Pork		Profit/Unit	Produced	Profit per Product		
5	Bacon	5	0	25		25	60	1500		
6	Bread	7	10	0		10	42.85714	428.5714286		
7										
8	Used	600	428.57	1500			Total Profit	1928.571429		
9		<=	<=	<=						
10	Available	600	800	1500						
11	Solver Results									
12	<p>Solver found a solution. All Constraints and optimality conditions are satisfied.</p> <p><input checked="" type="radio"/> Keep Solver Solution <input type="radio"/> Restore Original Values</p> <p><input type="checkbox"/> Return to Solver Parameters Dialog <input type="checkbox"/> Outline Reports</p>									
13										
14										
15	<p>Reports</p> <p><input checked="" type="radio"/> Answer <input type="radio"/> Sensitivity <input type="radio"/> Limits</p>									
16										
17										
18										
19										
20										
21	<p><input type="button"/> OK <input type="button"/> Cancel <input type="button"/> Save Scenario...</p>									
22										
23										
24										
25										
26										
27	<p>Creates the type of report that you specify, and places each report on a separate sheet in the workbook</p>									
--										

Post-optimality Analysis: Excel

Excel Answer Report:

A	B	C	D	E	F	G	H
1	Microsoft Excel 16.0 Answer Report						
2	Worksheet: [Wenbu.xlsx]Sheet1						
3	Report Created: 9/6/2025 9:53:29 PM						
4	Result: Solver found a solution. All Constraints and optimality conditions are satisfied.						
5	Solver Engine						
6	Engine: Simplex LP						
7	Solution Time: 0.016 Seconds.						
8	Iterations: 2 Subproblems: 0						
9	Solver Options						
10	Max Time Unlimited, Iterations Unlimited, Precision 0.000001						
11	Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative						
12							
13							
14	Objective Cell (Max)						
15	Cell	Name	Original Value	Final Value			
16	\$I\$8	Total Profit	Profit per Product	0	1928.571429		
17							
18							
19	Variable Cells						
20	Cell	Name	Original Value	Final Value	Integer		
21	\$H\$5	Bacon Produced	0	60	Contin		
22	\$H\$6	Bread Produced	0	42.85714286	Contin		
23							
24							
25	Constraints						
26	Cell	Name	Cell Value	Formula	Status	Slack	
27	SC\$8	Used Labour	600	\$C\$8<= \$C\$10	Binding	0	
28	SD\$8	Used Flour	428.5714286	\$D\$8<= \$D\$10	Not Binding	371.4285714	
29	SE\$8	Used Pork	1500	\$E\$8<= \$E\$10	Binding	0	

Post-optimality Analysis: Excel

Interpreting Excel Answer Report:

- Summarizes the **optimal solution** found by Solver.
- Shows the final value of the **objective function** (maximum profit).
- Lists the values of the **decision variables** at the optimum.
- Reports the **status of each constraint** (binding or not binding) and any slack.
- Helps decision-makers identify which resources are fully utilized and which have spare capacity.

Post-optimality Analysis: Excel

Interpreting Excel Answer Report: Objective Cell

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$I\$8	Total Profit	Profit per Product	1928.571429

- The final value of the objective cell confirms the **maximum profit**.

$$Z = 1928.57$$

- Solver verified that all constraints and optimality conditions are satisfied.
- The solution is feasible and optimal under the given model.

Post-optimality Analysis: Excel

Interpreting Excel Answer Report: Variable Cells

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$H\$5	Bacon Produced		60	60 Contin
\$H\$6	Bread Produced	42.85714286	42.85714286	Contin

Decision Variables (Production Plan):

- The Solver report shows the decision variables (i.e., quantities of bacon and bread that maximize profit).
 - $X_1 = 60$ (Bacon produced per day)
 - $X_2 \approx 42.86$ (Bread produced per day)
- Both decision variables are continuous (not restricted to integers).

Post-optimality Analysis: Excel

Interpreting Excel Answer Report: Constraints

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$C\$8	Used Labour	600	$\$C\$8 \leq \$C\10	Binding	0
\$D\$8	Used Flour	428.5714286	$\$D\$8 \leq \$D\10	Not Binding	371.4285714
\$E\$8	Used Pork	1500	$\$E\$8 \leq \$E\10	Binding	0

Constraint Status:

- **Labour:** $600 = 600$ (Binding, no slack).
- **Flour:** $428.57 \leq 800$ (Not binding, slack = 371.43).
- **Pork:** $1500 = 1500$ (Binding, no slack).

Managerial Insight:

- Labour and pork are **critical resources** fully utilized at the optimum.
- Flour is underutilized — increasing its availability does not increase profit.
- Improvement requires relaxing the **binding constraints**.

Post-optimality Analysis: Excel

Excel Sensitivity Report:

	A	B	C	D	E	F	G	H
1	Microsoft Excel 16.0 Sensitivity Report							
2	Worksheet: [Wenbu.xlsx]Sheet1							
3	Report Created: 9/6/2025 9:53:30 PM							
4								
5								
6	Variable Cells							
7		Final Value		Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease	
9	\$H\$5	Bacon Produced	60	0	25	1E+30	17.85714286	
10	\$H\$6	Bread Produced	42.85714286	0	10	25	10	
11								
12	Constraints							
13		Final Value		Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease	
15	\$C\$8	Used Labour	600	1.428571429	600	260	300	
16	\$D\$8	Used Flour	428.5714286	0	800	1E+30	371.4285714	
17	\$E\$8	Used Pork	1500	0.714285714	1500	1500	1300	

Post-optimality Analysis: Excel

Reduced Cost

- If the Final Value of a decision variable is **positive** in the optimal solution, its reduced cost = 0.
- If the Final Value of a decision variable is **0** in the optimal solution, the reduced cost shows:
 - the loss in profit if it is forced into production, or
 - how much its profit coefficient must improve before it can enter the solution.
- If the Final Value of a variable is 0 and its reduced cost = 0 \Rightarrow multiple optimal solutions exist.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Variable Cells

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$5	Bacon Produced	60	0	25	1E+30	17.85714286
\$H\$6	Bread Produced	42.85714286	0	10	25	10

Bacon (X_1):

- Current profit per unit = 25.
- Since bacon (X_1) is included in the solution (positive value), its reduced cost = 0.
- Allowable increase = $\infty \Rightarrow$ even if the profit rises well above 25, the same solution remains optimal.
- Allowable decrease = 17.86 \Rightarrow the profit can fall to 7.14 before the solution changes.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Variable Cells

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$5	Bacon Produced	60	0	25	1E+30	17.85714286
\$H\$6	Bread Produced	42.85714286	0	10	25	10

Bread (X_2):

- Current profit per unit = 10.
- Since bread (X_2) is included in the solution (positive value), its reduced cost = 0.
- Allowable increase = 25 \Rightarrow the profit can rise to 35 before the solution changes.
- Allowable decrease = 10 \Rightarrow the profit can fall to 0 before the solution changes.

Post-optimality Analysis: Excel

Shadow Price

- If a constraint is **not binding** ($\text{slack} > 0$), its shadow price is 0.
- If a constraint is **binding**, the shadow price shows how much the objective value changes if the right-hand side (RHS) increases by 1 unit.
- Valid only within the allowable increase/decrease range for that constraint.
- Shadow price measures the “value” of one more unit of a scarce resource.

$$\Delta Z = (\text{Shadow Price}) \times (\Delta \text{RHS})$$

- A positive ΔRHS (more resources) **increases profit**, while a negative one **reduces profit**.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$8	Used Labour	600	1.428571429	600	260	300
\$D\$8	Used Flour	428.5714286	0	800	1E+30	371.4285714
\$E\$8	Used Pork	1500	0.714285714	1500	1500	1300

Labour Constraint:

- Shadow price = 1.4286
 - Each additional minute of labour increases profit by **\$1.4286**.
 - Indicates labour is a **binding and valuable resource**.
- Allowable range:
 - Can increase by up to 260 minutes or decrease by up to 300 minutes
 - Beyond this range, the optimal solution (basis) will change.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$8	Used Labour	600	1.428571429	600	260	300
\$D\$8	Used Flour	428.5714286	0	800	1E+30	371.4285714
\$E\$8	Used Pork	1500	0.714285714	1500	1500	1300

Flour Constraint:

- Shadow price = 0
 - Indicates flour is **not fully used** (slack exists).
 - Extra flour does not improve profit unless labour or pork availability changes.
- Allowable decrease = 371.43
 - Flour can be reduced to 428.57 units before it becomes binding.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$8	Used Labour	600	1.428571429	600	260	300
\$D\$8	Used Flour	428.5714286	0	800	1E+30	371.4285714
\$E\$8	Used Pork	1500	0.714285714	1500	1500	1300

Pork Constraint:

- Shadow price = 0.7143
 - Each additional unit of pork increases profit by about **\$0.7143**.
 - Confirms pork is a **binding resource**, though less valuable than labour.
- Allowable range:
 - Pork supply can increase by up to 1500 units or decrease by up to 1300 units
 - Outside this range, the solution will shift to a new corner point.

Post-optimality Analysis: Excel

Interpreting Excel Sensitivity Report: Managerial Insights

- **Resources:**
 - Labour is most valuable (\$1.43/min).
 - Pork also valuable (\$0.71/unit).
 - Flour is surplus, not currently constraining.
- **Strategic decision:** Invest in expanding labour or pork supply for the greatest return.

Post-optimality Analysis: Excel

Excel Limits Report:

	A	B	C	D	E	F	G	H	I	J
1	Microsoft Excel 16.0 Limits Report									
2	Worksheet: [Wenbu.xlsx]Sheet1									
3	Report Created: 9/6/2025 9:53:30 PM									
4										
5										
6	Objective									
7	Cell	Name	Value							
8	\$I\$8	Total Profit	Profit per Product	1928.571429						
9										
10										
11	Variable									
12	Cell	Name	Value	Lower Limit	Objective Result		Upper Limit	Objective Result		
13	\$H\$5	Bacon Produced	60	0	428.5714286		60	1928.571429		
14	\$H\$6	Bread Produced	42.85714286	0	1500		42.85714286	1928.571429		

Post-optimality Analysis: Excel

Interpreting Excel Limits Report:

- Shows the range (lower and upper limits) for each decision variable.
- Indicates how the objective function value changes when a variable is pushed to its minimum or maximum, holding the other variable fixed.
- Helps decision-makers see the **flexibility** of each variable within the model.

Post-optimality Analysis: Excel

Interpreting Excel Limits Report:

Objective		
Cell	Name	Value
\$I\$8	Total Profit Profit per Product	1928.571429

Variable		
Cell	Name	Value
\$H\$5	Bacon Produced	60
\$H\$6	Bread Produced	42.85714286

Lower Limit	Objective Result	Upper Limit	Objective Result
0	428.5714286	60	1928.571429
0	1500	42.85714286	1928.571429

Bacon Produced (X_1):

- Current optimal value: $X_1 = 60$.
- If bacon production is forced down to 0, the model produces only bread, giving profit \$428.57.
- If bacon stays at 60 (upper bound), we keep the maximum profit of \$1928.57.
- Suggests bacon is a critical driver of profit.

Post-optimality Analysis: Excel

Interpreting Excel Limits Report:

Objective		
Cell	Name	Value
\$I\$8	Total Profit Profit per Product	1928.571429

Variable		
Cell	Name	Value
\$H\$5	Bacon Produced	60
\$H\$6	Bread Produced	42.85714286

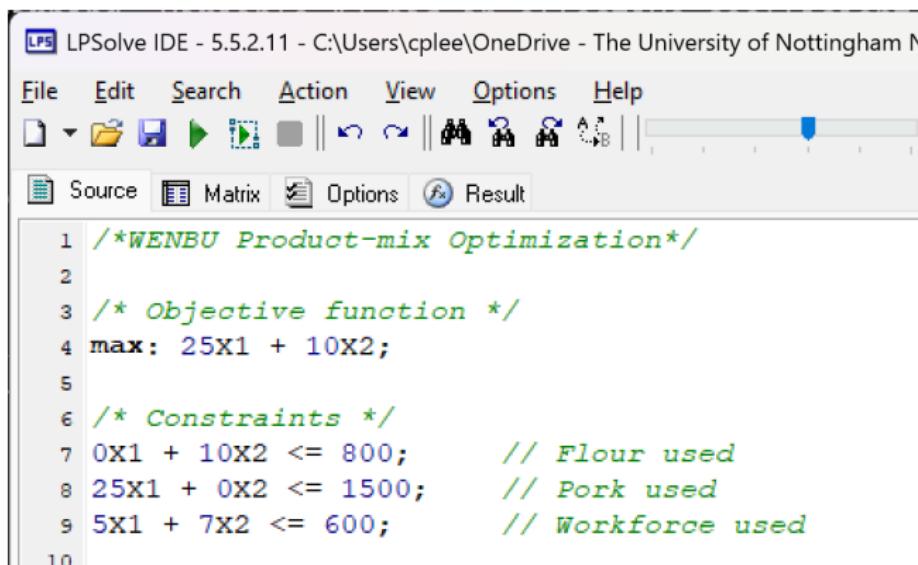
Lower Limit	Objective Result	Upper Limit	Objective Result
0	428.5714286	60	1928.571429
0	1500	42.85714286	1928.571429

Bread Produced (X_2):

- Current optimal value: $X_2 \approx 42.86$.
- If bread production is reduced to 0, only bacon is produced, yielding profit \$1500.
- At the upper limit of 42.86, profit reaches the maximum of \$1928.57.
- Bread adds profit, but its contribution is less than bacon.

Post-optimality Analysis: LP-Solve

Formulating WENBU Product-mix Optimization Problem:

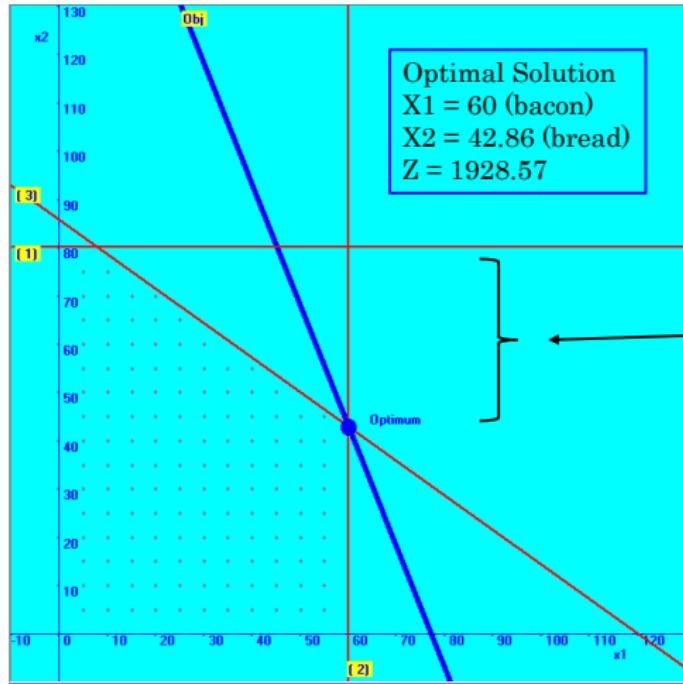


The screenshot shows the LPSolve IDE interface. The title bar reads "LPSolve IDE - 5.5.2.11 - C:\Users\cplee\OneDrive - The University of Nottingham". The menu bar includes File, Edit, Search, Action, View, Options, and Help. Below the menu is a toolbar with various icons. The tabs at the top of the main window are Source, Matrix, Options, and Result, with Source being the active tab. The code area contains the following text:

```
1 /*WENBU Product-mix Optimization*/
2
3 /* Objective function */
4 max: 25X1 + 10X2;
5
6 /* Constraints */
7 0X1 + 10X2 <= 800;           // Flour used
8 25X1 + 0X2 <= 1500;           // Pork used
9 5X1 + 7X2 <= 600;            // Workforce used
10
```

Note: For the post-optimality (sensitivity) report to be complete in LP-Solve, all decision variables must appear in every constraint, preferably written in standard form (e.g., $0X_1 + 10X_2 \leq 800$).

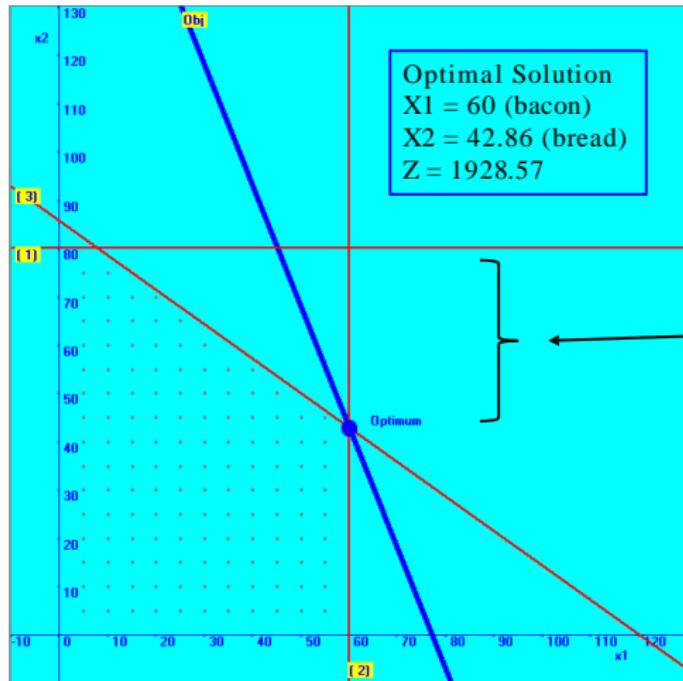
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 && (1) \\ & \text{Subject to: } 10X_2 \leq 800 && (2) \\ & \quad 25X_1 \leq 1500 && (3) \\ & \quad 5X_1 + 7X_2 \leq 600 && (3) \\ & \quad X_1, X_2 \geq 0 && (4) \end{aligned}$$

- What is the 'slack' (if any) in the resources?
- Which are the critical resources for optimality?

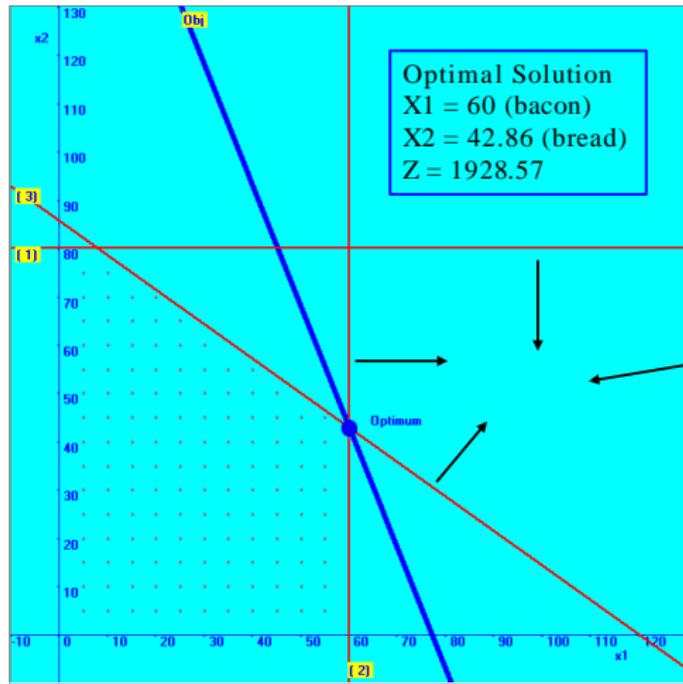
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 \\ & \text{Subject to: } 10X_2 \leq 800 \quad (1) \\ & \quad 25X_1 \leq 1500 \quad (2) \\ & \quad 5X_1 + 7X_2 \leq 600 \quad (3) \\ & \quad X_1, X_2 \geq 0 \quad (4) \end{aligned}$$

- What is the ‘slack’ (if any) in the resources?
The distance between the optimal solution to the line of constraint (1).
- Which are the critical resources for optimality?
The resources of constraint (2) and (3): pork and labor time.

Post-optimality Analysis: Graphical Method



$$\text{Maximize: } Z = 25X_1 + 10X_2 \quad (1)$$

$$\text{Subject to: } 10X_2 \leq 800 \quad (2)$$

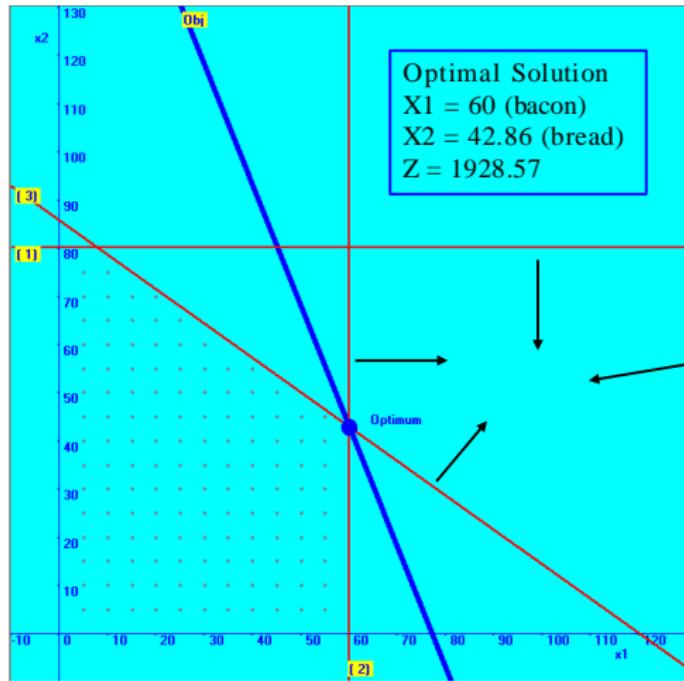
$$25X_1 \leq 1500 \quad (3)$$

$$5X_1 + 7X_2 \leq 600 \quad (4)$$

$$X_1, X_2 \geq 0 \quad (4)$$

- How do changes in the availability of resources affect the optimal solution?
- What is the additional profit with the increased resources?

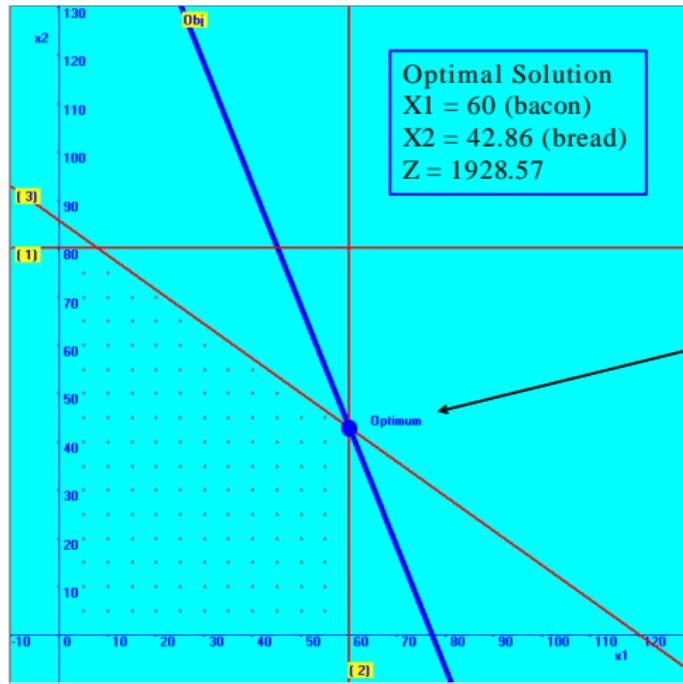
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 && (1) \\ & \text{Subject to: } 10X_2 \leq 800 && (2) \\ & \quad 25X_1 \leq 1500 && (3) \\ & \quad 5X_1 + 7X_2 \leq 600 && (4) \\ & \quad X_1, X_2 \geq 0 \end{aligned}$$

- How do changes in the availability of resources affect the optimal solution?
The increase of pork and labor time will improve the optimal solution.
- What is the additional profit with the increased resources?
Affected by the slopes of the lines of binding constraints.

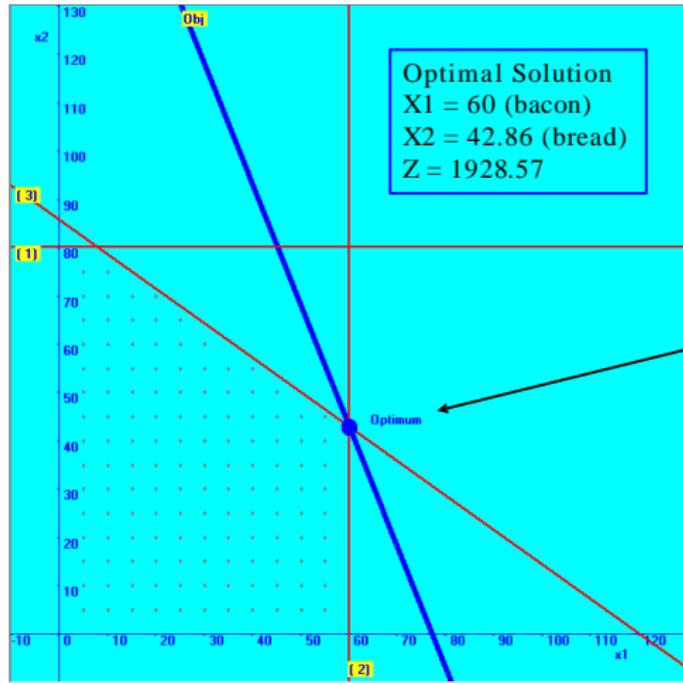
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 && (1) \\ & \text{Subject to: } 10X_2 \leq 800 && (2) \\ & \quad 25X_1 \leq 1500 && (3) \\ & \quad 5X_1 + 7X_2 \leq 600 && (3) \\ & \quad X_1, X_2 \geq 0 && (4) \end{aligned}$$

- Are there alternative optimal solutions?
- How can the current optimal solution be improved?

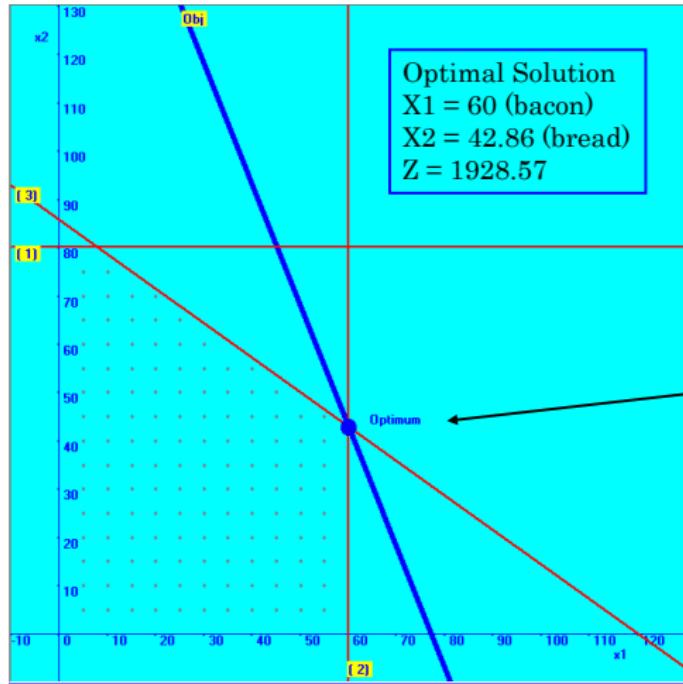
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 \\ & \text{Subject to: } 10X_2 \leq 800 \quad (1) \\ & \quad 25X_1 \leq 1500 \quad (2) \\ & \quad 5X_1 + 7X_2 \leq 600 \quad (3) \\ & \quad X_1, X_2 \geq 0 \quad (4) \end{aligned}$$

- Are there alternative optimal solutions?
No. We have only one intersection between the line of objective function and feasible region.
- How can the current optimal solution be improved?
Provide more binding resources.

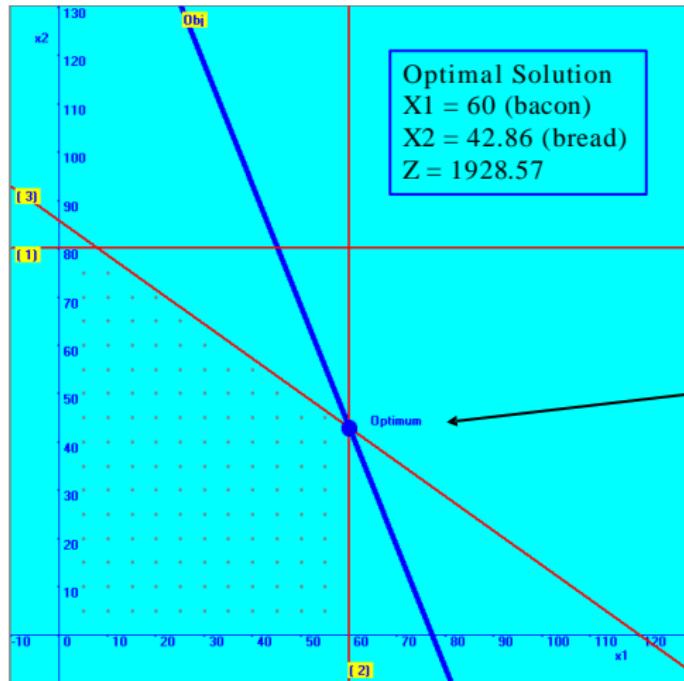
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 && (1) \\ & \text{Subject to: } 10X_2 \leq 800 && (2) \\ & \quad 25X_1 \leq 1500 && (2) \\ & \quad 5X_1 + 7X_2 \leq 600 && (3) \\ & \quad X_1, X_2 \geq 0 && (4) \end{aligned}$$

- How much can the profits per product change without changing the optimal solution?
- What changes in the profits per product would result in multiple optimal solutions?

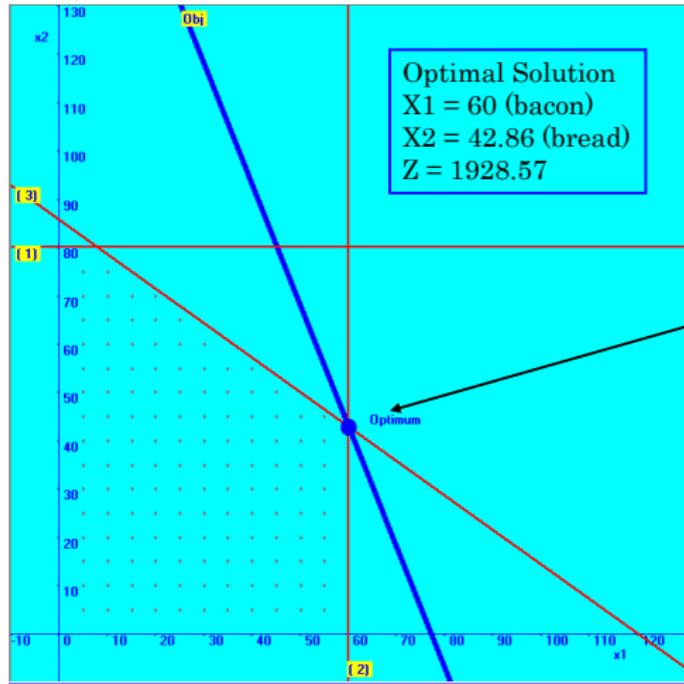
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 \\ & \text{Subject to: } 10X_2 \leq 800 \quad (1) \\ & \quad 25X_1 \leq 1500 \quad (2) \\ & \quad 5X_1 + 7X_2 \leq 600 \quad (3) \\ & \quad X_1, X_2 \geq 0 \quad (4) \end{aligned}$$

- How much can the profits per product change without changing the optimal solution?
Slightly adjust the slope of the line of objective function.
- What changes in the profits per product would result in multiple optimal solutions?
Make the line of objective function parallel to the lines of binding constraints.

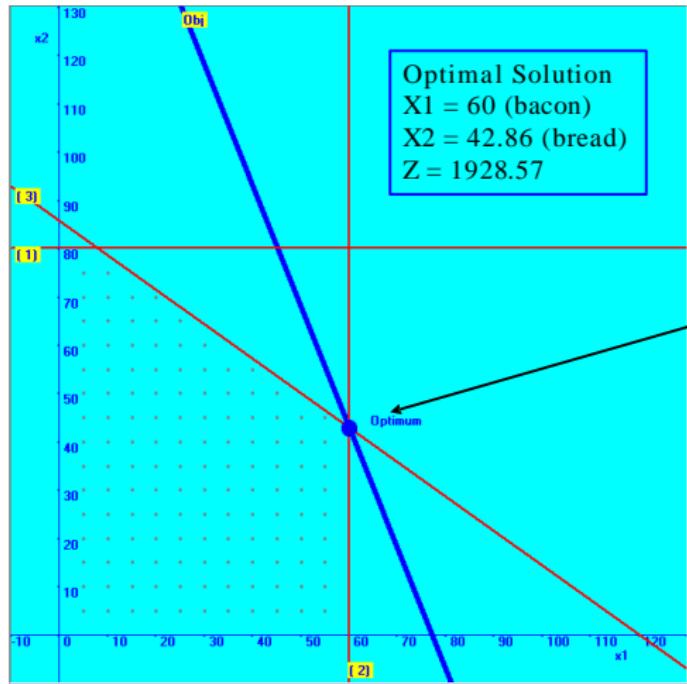
Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 && (1) \\ & \text{Subject to: } 10X_2 \leq 800 && (2) \\ & \quad 25X_1 \leq 1500 && (2) \\ & \quad 5X_1 + 7X_2 \leq 600 && (3) \\ & \quad X_1, X_2 \geq 0 && (4) \end{aligned}$$

- How much can the values of the decision variables change while maintaining optimality or feasibility?

Post-optimality Analysis: Graphical Method



$$\begin{aligned} & \text{Maximize: } Z = 25X_1 + 10X_2 \\ & \text{Subject to: } 10X_2 \leq 800 \quad (1) \\ & \quad 25X_1 \leq 1500 \quad (2) \\ & \quad 5X_1 + 7X_2 \leq 600 \quad (3) \\ & \quad X_1, X_2 \geq 0 \quad (4) \end{aligned}$$

- How much can the values of the decision variables change while maintaining optimality or feasibility?
No change can maintain optimality.
Slight change within the feasible region maintains feasibility.

Good Practice in Developing Spreadsheet Models

Spreadsheets offer an **intuitive way for developing and visualize models of optimization problems** (instead of the algebraic form).

The **formal algebraic model is always essential** in this module.

Like algebraic models, spreadsheet models also contain:

- **Data:** cells with fixed given values
- **Decision variables:** the changing cells
- **Constraints:** output cells and solver parameters
- **Objective function:** special output cell called the target cell

Parameters in the **solver dialog box** complete the model.

Good Practice in Developing Spreadsheet Models

The overall process for modeling using spreadsheets:

1. Plan/Design the model
2. Build the model
3. Test and revise the model
4. Analyze model and results

Applying good planning and principles in spreadsheet modeling will help to produce models that are easy to understand, easy to debug and easy to modify.

It is not advisable to develop a spreadsheet model without proper planning as this might lead to a model that is poorly organized and difficult to interpret and expand.

Good Practice in Developing Spreadsheet Models

Unlike formal algebraic models, spreadsheets offer high flexibility in modeling, but may lead to less accurate, less robust, or non-linear formulations if not carefully designed.

The Excel Solver:

- Developed by Frontline Systems Inc.
- Applies the simplex method for LP models and branch-and-bound for IP models.
- Uses the Generalized Reduced Gradient (GRG2) method for nonlinear optimization.
- Sometimes referred to as a 'What-if Analysis Tool'.

Good Practice in Developing Spreadsheet Models

Follow these **spreadsheet modeling principles** whenever possible:

- First, enter all the data available
- Enter each piece of data only once in the model
- The model structure should conform the data if possible
- Organize and clearly identify the data and calculations, for clarity and easier application of formulas and visualization of the solution
- Always aim for a good layout as this can facilitate the modeling process and post-optimality analysis

Good Practice in Developing Spreadsheet Models

Follow these **spreadsheet modeling principles** whenever possible:

- Use good formatting (labels, colors, shading, borders, etc.) to make the model easier to interpret
- Separate data from formulas, avoid numbers directly in a formula or in the solver parameter settings
- Avoid complicated formulas, this helps to ensure the model is linear
- Use the various tools in the formulas menu for debugging spreadsheet models
- Show clearly the full optimization model including the equality and inequality signs of constraints
- Be very careful, it is tedious to debug a spreadsheet model!!

Guidelines for Designing Spreadsheet Models

- **Sketch the Spreadsheet:** careful planning tends to result in less time spent correcting mistakes (carpenters say: “measure twice, cut once”).
- **Organize the Spreadsheet into Sections:** grouping similar or related items into sections can help to organize the information and improve the spreadsheet design.
- **Start Small:** if the problem is considerably large, isolate one part of the problem or one module of the spreadsheet, then design, build and test that one part.
- **Isolate Input Parameters:** place numerical values of parameters in a single location and definitely not directly in formulas, for easy identification and modification.

Guidelines for Designing Spreadsheet Models

- **Design for Use:** try to anticipate what questions the user will want to address, include auxiliary outputs and graphs to enhance usability.
- **Keep it Simple:** avoid excessively long formulas, decompose complex calculations and display intermediate results in separate cells to enhance readability.
- **Design for Communication:** logical design, layout, organization and visual cues (labels and notes) all help to understand the spreadsheet by users not familiar with the model and over time.
- **Document Important Data and Formulas:** use practical ways like cell comments, brief notes and explanations within the spreadsheet itself, perhaps include influence diagrams as well.