COMP4130 Linear and Discrete Optimization
Workshop 2: Fundamentals of Linear Programming

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Purpose: Apply the graphical method to two-variable LP models and develop algebraic and spreadsheet models for small optimization problems.

Step 1

Solve the following LP model in LP-Solve. Then, apply the graphical method to solve the same model. Ensure that you clearly identify the following on the graph: constraints, feasible region, objective function, corner-point feasible (CPF) solutions, the optimal CPF solution, binding constraints, and non-binding constraints. It is recommended that you draw the graph yourself for practice. The ZWEIG MEDIA online toolbox is suggested for the graphical method. You may also use Desmos or GeoGebra.

$$
\begin{aligned}
\text{Maximize}: \quad & Z = 3X_1 + 4X_2 \\
\text{Subject to}: \quad & X_1 + 2X_2 \geq 15 && (1) \\
& -2X_1 + X_2 \leq 5 && (2) \\
& X_1 + X_2 \leq 20 && (3) \\
& 2X_1 - 5X_2 \leq 25 && (4) \\
& X_1 \geq 0 && (5) \\
& X_2 \geq 0 && (6)
\end{aligned}
$$

Step 2

Using the graphical method, explain how the optimal solution changes when the following modifications are made to the above LP model. Please note that each case below is applied to the original model independently of the others.

Case 1: Remove constraint (1).
Case 2: Change the objective function to maximize $Z = 1.5X_1 + 2X_2$ and remove constraint (4).

Case 3: Change the objective function to minimize $Z = 1.5X_1 + 3X_2$ and add the constraint $X_1 \geq 12$.

Step 3

A paper recycling company collects paper from different sources and produces several types of paper for various markets. The three types of paper collected are: white paper, mixed paper, and newsprint. The company produces three products: office paper, catalog paper, and tan stock. The yield for each process is given in the table. For example, if 10 tons of white paper are processed into office paper, the output is 8.5 tons.

	Office Paper	Catalog Paper	Tan Stock
White Paper	85%	90%	95%
Mixed Paper	60%	60%	80%
Newsprint	—	75%	60%

The monthly collection is currently 300 tons of white paper, 600 tons of mixed paper, and 400 tons of newsprint. In the current market, the company can sell no more than: 150 tons of office paper at £25/ton, 750 tons of catalog paper at £20/ton, and 550 tons of tan stock at £18/ton. The company wishes to determine the monthly production plan that maximizes total sales.

1. Use the spreadsheet `PaperRecycling` provided to implement and solve the corresponding optimization model, and determine the optimal solution.

2. Formulate the corresponding algebraic optimization model and implement it in LP-Solve.

Step 4

In Step 3, the PAPER RECYCLING problem was formulated using nine decision variables, one for each combination of collected paper type and produced paper type.

In this step, consider whether the problem could instead be represented using only three decision variables, either one for each type of paper produced (office paper, catalog paper, and tan stock), or one for each type of paper collected (white paper, mixed paper, and newsprint).

1. If such a three-variable formulation is feasible, write the corresponding algebraic model and solve it in LP-Solve. Express all algebraic constraints in standard form, with all decision variables on the left-hand side and the scalar constant on the right-hand side. Compare the optimal solutions obtained with those from the nine-variable model in Step 3.

2. If it is not feasible to represent the problem with only three decision variables, justify clearly why this simplification cannot capture the problem structure.

Step 5

Write the mathematical programming model for the following optimization problem.

"A BUFFET company must determine the minimum-cost mix of food items for an event, subject to nutritional and composition constraints. There are $N = 20$ food items. Items 1–10 are liquids and items 11–20 are solids. The amount chosen of each item may be fractional.

For each item i, the following parameters are given per unit: fat F_i, energy E_i, protein P_i, sugar S_i, and cost C_i. The mix must satisfy:

- the total energy is at least EL and at most EH;
- exactly 40% of the total protein comes from liquids;
- the sugar from liquids is at most twice the sugar from solids;
- the total amount of liquids is at least half the total amount of solids."