COMP4130 Linear and Discrete Optimization
Workshop 4: Post-optimality Analysis on Product-Mix Optimization

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Purpose: To formulate and solve LP product-mix problems using both Excel Solver and LP-Solve, and to interpret the post-optimality analysis reports generated by these tools.

LIGHTS PRODUCTION Problem

The **LIGHTS PRODUCTION** problem was introduced and solved in Workshop 3. You may already have developed your own Excel model with a layout that produces the correct optimal solution. A reference spreadsheet model is shown below. Once the model has been solved, review the three post-optimality analysis reports generated by Excel Solver and complete the tasks that follow.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**LIGHTS PRODUCTION Problem**													
2														
3			Units Produced											
4			Regular Time	Overtime	Total Produced		Total Demand	Initial Demand	Material Cost Per Unit	Material Cost	Production Cost	Selling Price	Revenue	Net Profit
5	Department 1	Table Lamp	67200	0	67200	<=	67200	60000	66	4435200	1075200	120	8064000	2543600
6		Floor Lamp	20000	0	20000	<=	20000	20000	85	1700000	320000	150	3000000	980000
7	Department 2	Ceiling Fixture	50800	24000	74800	<=	100000	100000	50	3740000	969600	100	7480000	2770400
8		Chandelier	39200	0	39200	<=	39200	35000	80	3136000	470400	160	6272000	2657600
9														
10			Department 1										Total Profit	8951600
11			Regular Time	Overtime			Additional Demand	Edvertising Effect	Advertising Expenditure		Maximum Advertising			
12		Units to Produce Dept 1	87200	0		Table Lamp	7200	0.12	10000	<=	10000			
13			<=	<=		Floor Lamp	0	0.1	0	<=	10000			
14		Production Capacity Dept 1	100000	25000		Ceiling Fixture	0	0.08	0	<=	10000			
15		Production Cost Per Unit Dept 1	16	18		Chandelier	4200	0.15	8000	<=	10000			
16		Production Cost Dept 1	1395200	0										
17							Total Advertising Expenditure		18000					
18			Department 2						<=					
19			Regular Time	Overtime				Advertising Budget	18000					
20		Units to Produce Dept 2	90000	24000										
21			<=	<=										
22		Production Capacity Dept 2	90000	24000										
23		Production Cost Per Unit Dept 2	12	15										
24		Production Cost Dept 2	1080000	360000										
25														

Task 1. Answer Report

Answer the following questions using the **Answer Report** provided below.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$N$10	Total Profit Net Profit	8951600	8951600

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$5	Table Lamp Regular Time	67200	67200	Contin
$D$5	Table Lamp Overtime	0	0	Contin
$C$6	Floor Lamp Regular Time	20000	20000	Contin
$D$6	Floor Lamp Overtime	0	0	Contin
$C$7	Ceiling Fixture Regular Time	50800	50800	Contin
$D$7	Ceiling Fixture Overtime	24000	24000	Contin
$C$8	Chandelier Regular Time	39200	39200	Contin
$D$8	Chandelier Overtime	0	0	Contin
$I$12	Table Lamp Advertising Expenditure	10000	10000	Contin
$I$13	Floor Lamp Advertising Expenditure	0	0	Contin
$I$14	Ceiling Fixture Advertising Expenditure	0	0	Contin
$I$15	Chandelier Advertising Expenditure	8000	8000	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$12	Units to Produce Dept 1 Regular Time	87200	$C$12<=$C$14	Not Binding	12800
$D$12	Units to Produce Dept 1 Overtime	0	$D$12<=$D$14	Not Binding	25000
$C$20	Units to Produce Dept 2 Regular Time	90000	$C$20<=$C$22	Binding	0
$D$20	Units to Produce Dept 2 Overtime	24000	$D$20<=$D$22	Binding	0
$E$5	Table Lamp Total Produced	67200	$E$5<=$G$5	Binding	0
$E$6	Floor Lamp Total Produced	20000	$E$6<=$G$6	Binding	0
$E$7	Ceiling Fixture Total Produced	74800	$E$7<=$G$7	Not Binding	25200
$E$8	Chandelier Total Produced	39200	$E$8<=$G$8	Binding	0
$I$17	Total Advertising Expenditure Advertising Expenditure	18000	$I$17<=$I$19	Binding	0
$I$12	Table Lamp Advertising Expenditure	10000	$I$12<=$K$12	Binding	0
$I$13	Floor Lamp Advertising Expenditure	0	$I$13<=$K$13	Not Binding	10000
$I$14	Ceiling Fixture Advertising Expenditure	0	$I$14<=$K$14	Not Binding	10000
$I$15	Chandelier Advertising Expenditure	8000	$I$15<=$K$15	Not Binding	2000

a. According to the Answer Report, what does the optimal solution tell us about the company's maximum achievable profit? 8951600

b. Which products and activities are actually carried out in the optimal plan?

c. Which resources are fully utilized, and which still have unused capacity?

d. If Dept 1 Regular Time capacity were reduced from 100,000 to 95,000, would the solution and profit change?

e. If Ceiling Fixture Total Produced were required to be at most 60,000 units, would the solution change?

Task 2. Sensitivity Report

Answer the following questions using the **Sensitivity Report** shown below.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$5	Table Lamp Regular Time	67200	0	38	1E+30	2
$D$5	Table Lamp Overtime	0	-2	36	2	1E+30
$C$6	Floor Lamp Regular Time	20000	0	49	29.75	2
$D$6	Floor Lamp Overtime	0	-2	47	2	1E+30
$C$7	Ceiling Fixture Regular Time	50800	0	38	0	22.1142857
$D$7	Ceiling Fixture Overtime	24000	0	35	1E+30	0
$C$8	Chandelier Regular Time	39200	0	68	22.1142857	0
$D$8	Chandelier Overtime	0	0	65	0	1E+30
$I$12	Table Lamp Advertising Expenditure	10000	11.61	-1	1E+30	11.61
$I$13	Floor Lamp Advertising Expenditure	0	-5.95	-1	5.95	1E+30
$I$14	Ceiling Fixture Advertising Expenditure	0	-15.75	-1	15.75	1E+30
$I$15	Chandelier Advertising Expenditure	8000	0	-1	11.61	5.95

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$12	Units to Produce Dept 1 Regular Time	87200	0	100000	1E+30	12800
$D$12	Units to Produce Dept 1 Overtime	0	0	25000	1E+30	25000
$C$20	Units to Produce Dept 2 Regular Time	90000	38	90000	25200	50800
$D$20	Units to Produce Dept 2 Overtime	24000	35	24000	25200	24000
$E$5	Table Lamp Total Produced	67200	38	0	12800	67200
$E$6	Floor Lamp Total Produced	20000	49	0	12800	20000
$E$7	Ceiling Fixture Total Produced	74800	0	0	1E+30	25200
$E$8	Chandelier Total Produced	39200	30	0	50800	25200
$I$17	Total Advertising Expenditure Advertising Expenditure	18000	14.75	18000	2000	8000

a. Which decision variables are part of the optimal solution?

b. How can you use the Reduced Cost to interpret whether an unused variable could become profitable in the future?

c. If the company produces 1 unit of *Floor Lamp Overtime* (instead of 0), what would be the change in the objective function?

d. If the company spends \$10 on Ceiling Fixture Advertising (instead of \$0), what would be the change in the objective function?

e. According to the Sensitivity Report, which resources (constraints) are the most valuable to the company, and how can you tell?

f. If the production capacity of Dept 2 Overtime is increased from $24,000$ to $24,010$, what would be the change in the objective function?

g. If the Total Advertising Budget is reduced from $18,000$ to $17,900$, what would be the change in the objective function?

h. How can you detect from the Sensitivity Report if multiple optimal solutions exist?

i. Suppose a new policy requires that at least one unit of *Chandelier Overtime* must be produced. What would happen to the optimal objective function value?

Task 3. Limits Report

Answer the following questions using the **Limits Report** shown below.

Cell	Objective Name	Value
$N$10	Total Profit Net Profit	8951600

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$5	Table Lamp Regular Time	67200	0	6398000	67200	8951600
$D$5	Table Lamp Overtime	0	0	8951600	-1.16415E-08	8951600
$C$6	Floor Lamp Regular Time	20000	0	7971600	20000	8951600
$D$6	Floor Lamp Overtime	0	0	8951600	0	8951600
$C$7	Ceiling Fixture Regular Time	50800	0	7021200	50800	8951600
$D$7	Ceiling Fixture Overtime	24000	0	8111600	24000	8951600
$C$8	Chandelier Regular Time	39200	0	6286000	39200	8951600
$D$8	Chandelier Overtime	0	0	8951600	0	8951600
$I$12	Table Lamp Advertising Expenditure	10000	10000	8951600	10000	8951600
$I$13	Floor Lamp Advertising Expenditure	0	0	8951600	0	8951600
$I$14	Ceiling Fixture Advertising Expenditure	0	0	8951600	0	8951600
$I$15	Chandelier Advertising Expenditure	8000	8000	8951600	8000	8951600

a. What information does the Limits Report provide that is not shown in the Answer or Sensitivity Reports?

b. Which decision variables are critical drivers of profit, according to the Limits Report?

FARMLAND Problem

Develop the optimization model in Excel to solve the following product-mix optimization problem. In addition to formulating the model correctly, aim to design the spreadsheet carefully by following the **good practice guidelines** provided in Lecture 4. These will help you produce a model that is easy to understand, debug, and modify. No questions will be asked about the post-optimality reports produced by the Excel solver for this step.

Problem description: A farmer owns 1,000 acres of more or less homogeneous farmland. His options are to breed cattle or cultivate wheat, corn, or tomatoes. It takes 4 acres to support one head of cattle. Annually, 12,000 hours of labor are available which can be used in any way needed during the year. The table provides information regarding the profit, yield, and labor needs for the four activities.

It is required that at least 20% of the farmland that is cultivated must be used for cattle breeding. At most 30% of the farmland can be used for growing tomatoes. The ratio between the amount of farmland assigned to growing wheat and the farmland left uncultivated should not exceed 2 to 1. The problem is to decide the most profitable way to use the farmland. *Note: a non-linearity arises in the ratio constraint that requires reformulation.*

	Cattle	Wheat	Corn	Tomatoes
Profit	£1,600 per head	£5 per bushel	£6 per bushel	50 pence per pound
Yield per acre	$\frac{1}{4}$ head per acre	50 bushels	80 bushels	1,000 pounds
Annual labor requirement	40 hrs per head	10 hrs per acre	12 hrs per acre	25 hrs per acre