REPORT

Problem 1

Part 1

To find the lowest-cost dedicated bond portfolio that covers that stream of liabilities given while allow cash to be carried over at the forward rates, the below linear programming model has been formulated.

x_i: number of bonds

z_i: cash carry over

$f_{i,j}$: forward rate between time period i and j

$f_{i,j} = (1 + s_j)^j/(1 + s_i)^i$-1

$f_{1,2} = 0.02, f_{2,3} = 0.03, f_{3,4} = 0.04, f_{4,5} = 0.05, f_{5,6} = 0.06$

Formulate the model:

$$\min_x 108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$$

Such that:

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + 1.02z_1 - z_2 \geq 200$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + 1.03z_2 - z_3 \geq 800$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + 1.04z_3 - z_4 \geq 400$$

$$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + 1.05\, z_4 - z_5 \geq 700$$

$$110x_1 + 107x_2 + 108x_3 + 1.06\, z_5 \geq 900$$

$$x_1, x_2, x_3, \dots, x_{13} \geq 0$$

$$z_1, z_2, \dots, z_5 \geq 0$$

Optimal portfolio:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 8.1818 \\ 0 \\ 0 \\ 0 \\ 5.7774 \\ 2.6202 \\ 0 \\ 0 \\ 6.1298 \\ 0 \\ 0.1180 \\ 0 \\ 3.1180 \end{bmatrix} \qquad z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Lowest cost:

2639.9694

MATLAB Code & Output

Problem 1

Part 1

```
% spot rate
s_1 = 0.01;
s_2 = 0.015;
s_3 = 0.02;
s_4 = 0.025;
s_5 = 0.03;
s_6 = 0.035;
% forward rate
f_12 = (1+s_2)^2/(1+s_1)-1;
f_23 = (1+s_3)^3/(1+s_2)^2-1;
f_34 = (1+s_4)^4/(1+s_3)^3-1;
f_45 = (1+s_5)^5/(1+s_4)^4-1;
f_56 = (1+s_6)^6/(1+s_5)^5-1;
f_12
```

f_12 = 0.0200

```
f_23
```

f_23 = 0.0301

```
f_34
```

f_34 = 0.0401

```
f_45
```

f_45 = 0.0502

```
f_56
```

f_56 = 0.0604

```
% input arguments
c = [108,94,99,92.7,96.6,95.9,92.9,110,...
    104,101,107,102,95.2,0,0,0,0,0]';
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1+f_12,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1+f_23,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1+f_34,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1+f_45,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1+f_56];
b= -[500,200,800,400,700,900];
Aeq=[];
beq =[];
lb =[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
ub = [inf;inf;inf;inf;inf;inf;inf;inf;...
    inf;inf;inf;inf;inf;inf;inf;inf;inf;inf];
% linprog output
[x_part1, fval_part1] = linprog(c,A,b,Aeq,beq,lb,ub);
```

Optimal solution found.

```
% optimal solution
x_part1 = round(x_part1,4);
x_part1
```

	1
1	8.1818
2	0
3	0
4	0
5	5.7774
6	2.6202
7	0
8	0
9	6.1298
10	0
11	0.118
12	0
13	3.118
14	0
15	0
16	0
17	0
18	0

```
% objective function value
fval_part1 = round(fval_part1,6);
fval_part1
```

fval_part1 = 2639.969419

Part 2

To find the lowest-cost dedicated bond portfolio that covers that stream of liabilities given, where at most 50% of the bond portfolio's value can be in bond rated B, while allow cash to be carried over at the forward rates, the below linear programming model has been formulated.

x_i: number of bonds

z_i: cash carry over

$f_{i,j}$: forward rate between time period i and j

$f_{i,j} = (1 + s_j)^j/(1 + s_i)^i$-1

$f_{1,2} = 0.02, f_{2,3} = 0.03, f_{3,4} = 0.04, f_{4,5} = 0.05, f_{5,6} = 0.06$

Formulate the model:

$$\min_x 108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$$

Such that:

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + 1.02z_1 - z_2 \geq 200$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + 1.03z_2 - z_3 \geq 800$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + 1.04z_3 - z_4 \geq 400$$

$$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + 1.05\, z_4 - z_5 \geq 700$$

$$110x_1 + 107x_2 + 108x_3 + 1.06\, z_5 \geq 900$$

$$x_1, x_2, x_3, \dots, x_{13} \geq 0$$

$$z_1, z_2, \dots, z_5 \geq 0$$

$$(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6)/(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}) \leq 50\%$$

Optimal portfolio:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 8.4112 \\ 0 \\ 0 \\ 5.5027 \\ 0 \\ 3.3565 \\ 0 \\ 6.3502 \\ 0 \\ 0.3184 \\ 0 \\ 3.3184 \end{bmatrix} \qquad z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 49.8338 \\ 0 \end{bmatrix}$$

Lowest cost:

2644.4248

Compare with optimal bond portfolio from Part 1:

Portion of money spent on A rating bonds in Part 1:

$(6.1298 \times 104 + 0.1180 \times 107 + 3.1180 \times 95.2)/2639.9694 = 35.87\%$

Portion of money spent on A rating bonds in Part 2:

$$(3.3565 \times 92.9 + 6.3502 \times 104 + 0.3184 \times 107 + 3.3184 \times 95.2)/2644.4248 \ = \ 50.00\%$$

Portion of money spent on A rating bonds is higher in Part 2's portfolio than in Part 1's portfolio.

Cost of portfolio from Part 1 is 2639.9694 which is less than that of portfolio from Part 2 which is 2644.4248.

Although the cost of portfolio from Part 1 is a little lower than that from Part 2, the portion of money spent on A rating bonds in Part 2 is much higher than that in Part 1, because higher rating bonds are usually more expensive than lower rating bonds.

MATLAB Code & Output

Problem 1

Part 2

```
% input arguments
c = [108,94,99,92.7,96.6,95.9,92.9,110,...
    104,101,107,102,95.2,0,0,0,0,0]';
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1+f_12,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1+f_23,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1+f_34,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1+f_45,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1+f_56;
      -108*0.5,-94*0.5,-99*0.5,-92.7*0.5,-96.6*0.5,...
      -95.9*0.5,92.9*0.5,110*0.5,104*0.5,101*0.5,...
      107*0.5,102*0.5,95.2*0.5,0,0,0,0,0];
b= -[500,200,800,400,700,900,0];
Aeq=[];
beq =[];
lb =[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
ub = [inf;inf;inf;inf;inf;inf;inf;inf;...
    inf;inf;inf;inf;inf;inf;inf;inf;inf;inf];
% linprog output
[x_part2, fval_part2] = linprog(c,A,b,Aeq,beq,lb,ub);
```

Optimal solution found.

```
% optimal solution
x_part2 = round(x_part2,4);
x_part2
```

	1
1	0
2	8.4112
3	0
4	0
5	5.5027
6	0
7	3.3565
8	0
9	6.3502
10	0
11	0.3184
12	0
13	3.3184
14	0
15	0
16	0
17	49.8338
18	0

```
% objective function value
fval_part2 = round(fval_part2,6);
fval_part2
```

fval_part2 = 2644.424804

Part 3

To find the lowest-cost dedicated bond portfolio that covers that stream of liabilities given, where at most 25% of the bond portfolio's value can be in bond rated B, while allow cash to be carried over at the forward rates, the below linear programming model has been formulated.

x_i: number of bonds

z_i: cash carry over

$f_{i,j}$: forward rate between time period i and j

$f_{i,j} = (1 + s_j)^j/(1 + s_i)^i$-1

$f_{1,2} = 0.02, f_{2,3} = 0.03, f_{3,4} = 0.04, f_{4,5} = 0.05, f_{5,6} = 0.06$

Formulate the model:

$$\min_x 108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$$

Such that:

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + 1.02z_1 - z_2 \geq 200$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + 1.03z_2 - z_3 \geq 800$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + 1.04z_3 - z_4 \geq 400$$

$$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + 1.05\, z_4 - z_5 \geq 700$$

$$110x_1 + 107x_2 + 108x_3 + 1.06\, z_5 \geq 900$$

$$x_1, x_2, x_3, \dots, x_{13} \geq 0$$

$$z_1, z_2, \dots, z_5 \geq 0$$

$$(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6)/(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}) \leq 25\%$$

Optimal portfolio:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 7.1267 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10.4054 \\ 0 \\ 6.4638 \\ 0 \\ 0.4216 \\ 0 \\ 3.4216 \end{bmatrix} \qquad z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 742.4528 \\ 129.6111 \end{bmatrix}$$

Lowest cost:

2679.6516

Compare with the optimal portfolio from Part 1 and Part 2:

Portion of money spent on A rating bonds in Part 1:

$(6.1298 \times 104 + 0.1180 \times 107 + 3.1180 \times 95.2)/2639.9694 = 35.87\%$

Portion of money spent on A rating bonds in Part 2:

$$(3.3565 \times 92.9 + 6.3502 \times 104 + 0.3184 \times 107 + 3.3184 \times 95.2)/2644.4248 \ = \ 50.00\%$$

Portion of money spent on A rating bonds in Part 3:

$$(10.4054 \times 92.9 + 6.4638 \times 104 + 0.4216 \times 107 + 3.4216 \times 95.2)/2679.6516 \ = \ 75.00\%$$

Portion of money spent on A rating bonds is higher in Part 3's portfolio than in Part 2 and Part 1's portfolio.

Cost of portfolio from Part 1 is less than that of portfolio from Part 2 and is less than that of portfolio from Part 3.

Although the cost of portfolio from Part 1 is lower than that from Part 2 and Part 3, the portion of money spent on A rating bonds in Part 3 is much higher than that in Part 2 and Part 1, because higher rating bonds are usually more expensive than lower rating bonds.

Rank according to cost:

Part 3: 2679.6516 (costs the most)

Part 2: 2644.4248

Part 1: 2639.9694 (costs the least)

MATLAB Code & Output

Part 3

```
% input arguments
c = [108,94,99,92.7,96.6,95.9,92.9,110,...
    104,101,107,102,95.2,0,0,0,0,0]';
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1+f_12,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1+f_23,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1+f_34,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1+f_45,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1+f_56;
      -0.75*108,-0.75*94,-0.75*99,-0.75*92.7,...
      -0.75*96.6,-0.75*95.9,0.25*92.9,0.25*110,...
      0.25*104,0.25*101,0.25*107,0.25*102,0.25*95.2,0,0,0,0,0];
b= -[500,200,800,400,700,900,0];
Aeq=[];
beq =[];
lb =[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
ub = [inf;inf;inf;inf;inf;inf;inf;inf;inf;...
    inf;inf;inf;inf;inf;inf;inf;inf;inf];
% linprog output
[x_part3, fval_part3] = linprog(c,A,b,Aeq,beq,lb,ub);
```

Optimal solution found.

```
% optimal solution
x_part3 = round(x_part3,4);
x_part3
```

	1
1	0
2	7.1267
3	0
4	0
5	0
6	0
7	10.4054
8	0
9	6.4638
10	0
11	0.4216
12	0
13	3.4216
14	0
15	0
16	0
17	742.4528
18	129.6111

```
% objective function value
fval_part3 = round(fval_part3,6);
fval_part3
```

fval_part3 = 2679.651552

Problem 2

Part 1

a)

Below is the arithmetic mean, **expected return** (geometric mean) and **standard deviation** of SPY, GOVT and EEMV:

	SPY	GOVT	EEMV
arithmatic mean	0.01210	0.00190	0.00435
expected return (geometeric mean)	0.01127	0.00184	0.00370
standerd deviation	0.04097	0.01143	0.03627

Below is the **covariance matrix**:

	SPY	GOVT	EEMV
SPY	0.00168	-0.00011	0.00101
GOVT	-0.00011	0.00013	-0.00003
EEMV	0.00101	-0.00003	0.00132

b)

Below is the table of expected return goal R, and the corresponding optimal weights of the assets as well as the portfolio variance:

Expected return goal R	Optimal weight of SPY	Optimal weight of GOVT	Optimal weight of EEMV	Portfolio variance
0.2	0.1105	0.8772	0.0123	0.00010152
0.3	0.1228	0.8735	0.0037	0.00010166
0.4	0.2452	0.8367	-0.0819	0.00011928
0.5	0.3682	0.7997	-0.168	0.00016652
0.6	0.4913	0.7628	-0.254	0.00024336
0.7	0.6143	0.7258	-0.3401	0.0003498
0.8	0.7373	0.6888	-0.4261	0.00048586
0.9	0.8603	0.6519	-0.5122	0.0006515
1.0	0.9833	0.6149	-0.5982	0.00084676
1.1	1.1063	0.5779	-0.6842	0.00107162
1.2	1.2293	0.5409	-0.7703	0.00132608

For the mean-variance optimization model, Weight of SPY: w_1, Weight of GOVT: w_2, Weight of EEMV: w_3

R: expected return goal

Formulate the model:

$$\min_{w} 0.00168w_1^2 + 0.00013w_2^2 + 0.00132w_3^2 - 0.00022w_1w_2 + 0.00202w_1w_3 - 0.00006w_2w_3$$

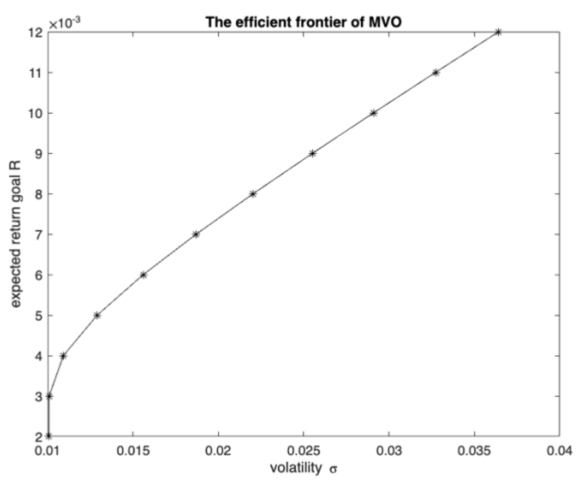
Such that:

$$0.01127w_1 + 0.00184w_2 + 0.0037w_3 \geq R$$

$$w_1 + w_2 + w_3 = 1$$

Assume short selling is allowed.

Plot the efficient frontier:



MATLAB Code & Output

Problem 2

Part 1

b)

```
format long g
n=3;
% expected returns of assets
mu=[0.01127,0.00184,0.00370];
% covariance matrix
Q=[.00168,-.00011,.00101;
   -.00011,.00013,-.00003;
   .00101,-.00003,.00132];
```

```
% expected return goals range from 0.2% to 1.2%
goal_R=(0.2:.1:1.2)/100;
c=zeros(n,1);
x=zeros(11,3);
fval=zeros(11,1);
std_devi=zeros(11,1);
Aeq=[ones(1,n)];
beq=1;
for a=1:length(goal_R)
    A=-mu;
    b=-goal_R(a);
    [x(a,:), fval(a,1)] = quadprog(Q, c, A,b, Aeq,beq,[],[]);
    % standard deviation = (x'*Q*x)^.5
    std_devi(a,1)=(x(a,:)*Q*x(a,:)')^.5;
    x = round(x,4);
    fval = round(fval,8);
end
% optimal solution
x
```

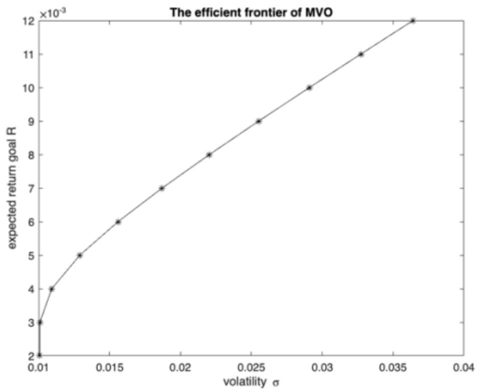
	1	2	3
1	0.1105	0.8772	0.0123
2	0.1228	0.8735	0.0037
3	0.2452	0.8367	-0.0819
4	0.3682	0.7997	-0.168
5	0.4913	0.7628	-0.254
6	0.6143	0.7258	-0.3401
7	0.7373	0.6888	-0.4261
8	0.8603	0.6519	-0.5122
9	0.9833	0.6149	-0.5982
10	1.1063	0.5779	-0.6842
11	1.2293	0.5409	-0.7703

```
% variance
var = fval*2;
var
```

	1
1	0.00010152
2	0.00010166
3	0.00011928
4	0.00016652
5	0.00024336
6	0.0003498
7	0.00048586
8	0.0006515
9	0.00084676
10	0.00107162
11	0.00132608

```
% plot efficient frontier
```

```
plot(std_devi, goal_R, '-k*')
xlabel('volatility \sigma')
ylabel('expected return goal R')
title('The efficient frontier of MVO')
```



c)

	SPY	GOVT	EEMV
Monthly return of Feb 2022	-0.02952	-0.00847	-0.00112

Covariance matrix in a):

	SPY	GOVT	EEMV
SPY	0.00168	-0.00011	0.00101
GOVT	-0.00011	0.00013	-0.00003
EEMV	0.00101	-0.00003	0.00132

Minimum variance portfolio: (the portfolio with the smallest variance in b)

$w_1 = 0.1105, w_2 = 0.8772, w_3 = 0.0123$

Return: $0.1105 \times (-0.02952) + 0.8772 \times (-0.00847) + 0.0123 \times (-0.00112) = -0.0107$

Risk: $0.00168 \times 0.1105^2 + 0.00013 \times 0.8772^2 + 0.00132 \times 0.0123^2 - 0.00022 \times 0.1105 \times 0.8772 + 0.00202 \times 0.1105 \times 0.0123 - 0.00006 \times 0.8772 \times 0.0123 = 0.00010$

Equal weighted portfolio:

$w_1 = 1/3, w_2 = 1/3, w_3 = 1/3$

Return: $(1/3) \times (-0.02952) + (1/3) \times (-0.00847) + (1/3) \times (-0.00112) = -0.0130$

Risk: $0.00168 \times 1/3^2 + 0.00013 \times 1/3^2 + 0.00132 \times 1/3^2 - 0.00022 \times 1/3^2 + 0.00202 \times 1/3^2 - 0.00006 \times 1/3^2 = 0.00054$

Portfolio has 70% in SPY, 20% in GOVT, 10% in EEMV:

$w_1 = 70\%, w_2 = 20\%, w_3 = 10\%$

Return: $70\% \times (-0.02952) + 20\% \times (-0.00847) + 10\% \times (-0.00112) = -0.0225$

Risk: $0.00168 \times 70\%^2 + 0.00013 \times 20\%^2 + 0.00132 \times 10\%^2 - 0.00022 \times 70\% \times 20\% + 0.00202 \times 70\% \times 10\% - 0.00006 \times 20\% \times 10\% = 0.00095$

Rank the three portfolios in terms of return: (from highest to lowest)

Minimum variance portfolio: -0.0107 (highest)

Equal weighted portfolio: -0.0130

Portfolio has 70% in SPY, 20% in GOVT, 10% in EEMV: -0.0225 (lowest)

Explain relative performance of portfolios in terms of risk and return:

Among these three portfolios, the minimum variance portfolio has the highest expected return and the lowest risk which performs the best, while the portfolio with 70% in SPY, 20% in GOVT and 10% in EEMV has the lowest expected return and the highest risk which performs the worst.

Part 2

Below is the arithmetic mean, expected return (geometric mean) and standard deviation of SPY, GOVT, EEMV, CME, BR, CBOE, ICE and ACN:

	SPY	GOVT	EEMV	CME	BR	CBOE	ICE	ACN
arithmatic mean	0.01210	0.00190	0.00435	0.01650	0.01879	0.01145	0.01418	0.01892
expected return (geometeric mean)	0.01127	0.00184	0.00370	0.01502	0.01709	0.00930	0.01267	0.01716
standerd deviation	0.04097	0.01143	0.03627	0.05515	0.05937	0.06516	0.05589	0.05988

Below is the covariance matrix:

	SPY	GOVT	EEMV	CME	BR	CBOE	ICE	ACN
SPY	0.00168	-0.00011	0.00101	0.00084	0.00151	0.00095	0.00130	0.00187
GOVT	-0.00011	0.00013	-0.00003	-0.00010	0.00003	-0.00001	-0.00008	-0.00007
EEMV	0.00101	-0.00003	0.00132	0.00022	0.00082	0.00033	0.00036	0.00082
CME	0.00084	-0.00010	0.00022	0.00304	0.00108	0.00195	0.00186	0.00103
BR	0.00151	0.00003	0.00082	0.00108	0.00352	0.00105	0.00143	0.00220
CBOE	0.00095	-0.00001	0.00033	0.00195	0.00105	0.00425	0.00167	0.00127
ICE	0.00130	-0.00008	0.00036	0.00186	0.00143	0.00167	0.00312	0.00183
ACN	0.00187	-0.00007	0.00082	0.00103	0.00220	0.00127	0.00183	0.00359

Below is the table of expected return goal R, and the corresponding optimal weights of the assets as well as the portfolio variance:

R	SPY	GOVT	EEMV	CME	BR	CBOE	ICE	ACN	Portfolio variance
0.2	0.1603	0.8723	0.0049	0.0507	-0.0485	-0.0169	-0.0034	-0.0194	9.09×10^{-5}
0.4	0.1929	0.8298	-0.0425	0.0866	-0.0224	-0.0274	-0.0189	0.0019	0.0001035
0.6	0.2485	0.7572	-0.1234	0.148	0.0223	-0.0454	-0.0454	0.0383	0.00018332
0.8	0.3041	0.6845	-0.2043	0.2094	0.0669	-0.0634	-0.0719	0.0747	0.0003367
1.0	0.3597	0.6119	-0.2852	0.2707	0.1116	-0.0814	-0.0983	0.111	0.00056358
1.2	0.4153	0.5392	-0.3663	0.3321	0.1562	-0.0994	-0.1248	0.1474	0.00086402
1.4	0.4709	0.4666	-0.447	0.3935	0.2009	-0.1174	-0.1512	0.1838	0.00123798
1.6	0.5265	0.394	-0.5279	0.4548	0.2455	-0.1353	-0.1777	0.2202	0.00168548
1.8	0.5821	0.3213	-0.6088	0.5162	0.2902	-0.1533	-0.2042	0.2565	0.00220652
2.0	0.6377	0.2487	-0.6897	0.5776	0.3348	-0.1713	-0.2306	0.2929	0.00280108

For the mean-variance optimization model,

Weight of SPY: w_1, Weight of GOVT: w_2, Weight of EEMV: w_3, Weight of CME: w_4, Weight of BR: w_5, Weight of CBOE: w_6, Weight of ICE: w_7, Weight of ACN: w_8

R: expected return goal

Formulate the model:

$$\begin{aligned}\min_{w}\ & 0.00168w_1^2 + 0.00013w_2^2 + 0.00132w_3^2 + 0.00304w_4^2 + 0.00352w_5^2 + 0.00425w_6^2 \\ & + 0.00312w_7^2 + 0.00359w_8^2 + 2\times(-0.00011)w_1w_2 + 2\times 0.00101w_1w_3 \\ & + 2\times 0.00084w_1w_4 + 2\times 0.00151w_1w_5 + 2\times 0.00095w_1w_6 \\ & + 2\times 0.00130w_1w_7 + 2\times 0.00187w_1w_8 + 2\times(-0.00003)w_2w_3 \\ & + 2\times(-0.00010)w_2w_4 + 2\times 0.00003w_2w_5 + 2\times(-0.00001)w_2w_6 \\ & + 2\times(-0.00008)w_2w_7 + 2\times(-0.00007)w_2w_8 + 2\times 0.00022w_3w_4 \\ & + 2\times 0.00082w_3w_5 + 2\times 0.00033w_3w_6 + 2\times 0.00036w_3w_7 \\ & + 2\times 0.00082w_3w_8 + 2\times 0.00108w_4w_5 + 2\times 0.00195w_4w_6 \\ & + 2\times 0.00186w_4w_7 + 2\times 0.00103w_4w_8 + 2\times 0.00105w_5w_6 \\ & + 2\times 0.00143w_5w_7 + 2\times 0.00220w_5w_8 + 2\times 0.00167w_6w_7 \\ & + 2\times 0.00127w_6w_8 + 2\times 0.00183w_7w_8\end{aligned}$$

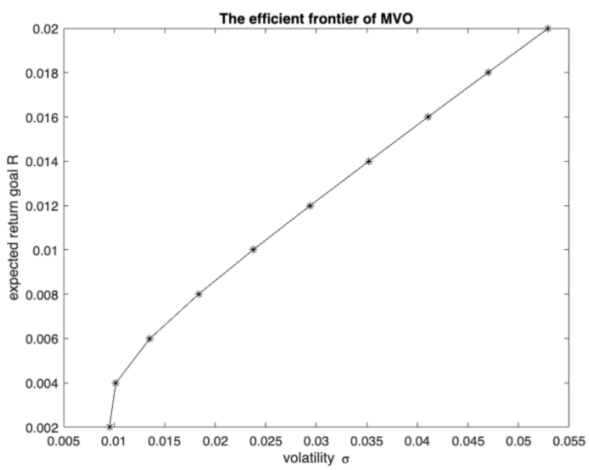
Such that:

$$\begin{aligned}& 0.01127w_1 + 0.00184w_2 + 0.0037w_3 + 0.01502w_4 + 0.01709w_5 + 0.0093w_6 \\ & + 0.01267w_7 + 0.01716w_8 \geq R\end{aligned}$$

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 = 1$$

Assume short selling is allowed.

Plot the efficient frontier:



MATLAB Code & Output

Part 2

```
n=8;
% expected returns of assets
mu=[0.01127,0.00184,0.00370,0.01502,...
    0.01709,0.00930,0.01267,0.01716];
% covariance matrix
```

```
Q=[0.00168,-0.00011,0.00101,0.00084,...
    0.00151,0.00095,0.00130,0.00187;
   -0.00011,0.00013,-0.00003,-0.00010,...
   0.00003,-0.00001,-0.00008,-0.00007;
   0.00101,-0.00003,0.00132,0.00022,...
   0.00082,0.00033,0.00036,0.00082;
   0.00084,-0.00010,0.00022,0.00304,...
   0.00108,0.00195,0.00186,0.00103;
   0.00151,0.00003,0.00082,0.00108,...
   0.00352,0.00105,0.00143,0.00220;
   0.00095,-0.00001,0.00033,0.00195,...
   0.00105,0.00425,0.00167,0.00127;
   0.00130,-0.00008,0.00036,0.00186,...
   0.00143,0.00167,0.00312,0.00183;
   0.00187,-0.00007,0.00082,0.00103,...
   0.00220,0.00127,0.00183,0.00359];
% expected return goals range from 0.2% to 2.0%
goal_R=(0.2:0.2:2.0)/100;
c=zeros(n,1);
x=zeros(10,8);
fval=zeros(10,1);
std_devi=zeros(10,1);
Aeq=[ones(1,n)];
beq=1;
for a=1:length(goal_R)
    A=-mu;
    b=-goal_R(a);
    [x(a,:), fval(a,1)] = quadprog(Q, c, A,b, Aeq,beq,[],[]);
    % standard deviation = (x'*Q*x)^.5
    std_devi(a,1) = (x(a,:)*Q*x(a,:)')^.5;
    x = round(x,4);
    fval = round(fval,8);
end
% optimal solution
x
```

	1	2	3	4	5	6	7	8
1	0.1603	0.8723	0.0049	0.0507	-0.0485	-0.0169	-0.0034	-0.0194
2	0.1929	0.8298	-0.0425	0.0866	-0.0224	-0.0274	-0.0189	0.0019
3	0.2485	0.7572	-0.1234	0.148	0.0223	-0.0454	-0.0454	0.0383
4	0.3041	0.6845	-0.2043	0.2094	0.0669	-0.0634	-0.0719	0.0747
5	0.3597	0.6119	-0.2852	0.2707	0.1116	-0.0814	-0.0983	0.111
6	0.4153	0.5392	-0.3661	0.3321	0.1562	-0.0994	-0.1248	0.1474
7	0.4709	0.4666	-0.447	0.3935	0.2009	-0.1174	-0.1512	0.1838
8	0.5265	0.394	-0.5279	0.4548	0.2455	-0.1353	-0.1777	0.2202
9	0.5821	0.3213	-0.6088	0.5162	0.2902	-0.1533	-0.2042	0.2565
10	0.6377	0.2487	-0.6897	0.5776	0.3348	-0.1713	-0.2306	0.2929

```
% variance
var = fval*2;
var
```

	1
1	9.09e-05
2	0.0001035
3	0.00018332
4	0.0003367
5	0.00056358
6	0.00086402
7	0.00123798
8	0.00168548
9	0.00220652
10	0.00280108

```
% plot efficient frontier
plot(std_devi, goal_R, '-k*')
xlabel('volatility \sigma')
ylabel('expected return goal R')
title('The efficient frontier of MVO')
```