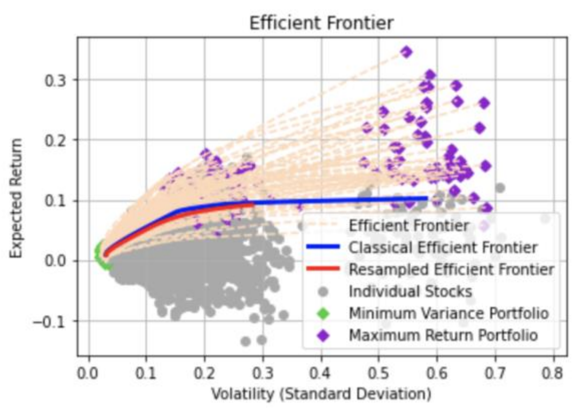
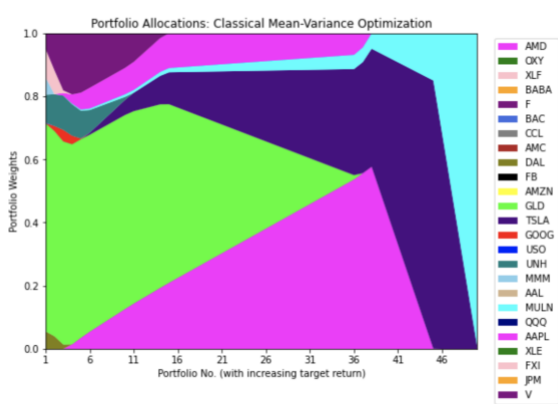
Report

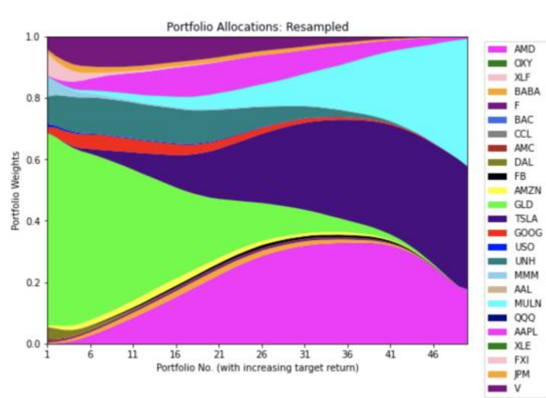
In this study, a data set of monthly closing prices for 25 assets from Jan 1st, 2018, to Dec 1st, 2020 (in-sample) and closing prices for the same assets in 2021 (out-of-sample) is provided. The data set has been loaded and cleaned, matrix of monthly returns, covariance matrix, means of the percent change matrix, minimum variance portfolio, maximum return portfolio, classical return portfolio, classical Markowitz efficient frontier and resampled efficient frontier of in-sample data have been calculated in Question 1. In Question 2, assuming $1000 total investment and a 0.2% monthly risk-free return, minimum variance, maximum return, Sharpe ratio portfolio weights on the classical and resampled efficient frontier of the out-of-sample data have been calculated, as well as the expected return of the 6 portfolios.

Question 3. Plot the classical and resampled efficient frontier.



Question 4. Plot the portfolio compositions for classical and resampled frontiers.





Question 5. Discuss your findings from comparing the performance of all 6 portfolios and from observing the graphs from question 3 & 4.

From the minimum variance portfolio on the classical efficient frontier, it shows that the weight is 2.82%, and the expected return on $1000 investment on the 2021 data is $28.22. Since the minimum variance portfolio is highly risk-averse, it would prefer low volatility and low variance, and at the same time, it will not experience relatively very high return or very huge loss. From the maximum return portfolio on the classical efficient frontier, it shows that the weight is -58.21%, and the expected return on $1000 investment on the 2021 data is -$582.10. It is not surprised to see the relatively high loss, because maximum return portfolio is the optimal portfolio for investors that are indifferent to risk and are looking for high return based on historical data. However, since the portfolio is based on historical data and cares a lot about high return, it is reasonable that a stock which generated the highest return in the past would suddenly experience huge loss which makes the maximum return portfolio experiences huge loss on out-of-sample data, due to the riskiness. From the Sharpe ratio portfolio on the classical efficient frontier, it shows that the weight is 27.96%, and the expected return on $1000 investment on the 2021 data is $279.57. Since when calculating Sharpe ratio, it uses return minus risk-free rate and then divide by standard deviation, by doing this, it takes into account both return and risk. If it has high return and high risk, it would not be an optimal Sharpe ratio portfolio. Therefore, Sharpe ratio portfolio would rather prefer higher return and lower risk, so the out-of-sample performance would have relatively low volatility and high return, which performs quite well on these set of data. On the resampled efficient frontier, the weight of minimum variance, maximum return and Sharpe ratio portfolios are 6.13%, 1.29% and 31.84% respectively, while their expected returns on $1000 invested in 2021 are $61.33, $12.95, and $318.44 respectively, which perform better than these portfolio on the classical efficient frontier. Because when using Monte Carlo resampling and bootstrapping method into mean-variance (MV) optimization, it would help generate more stable and useful result for investment, and since it is the average of many MV optimal portfolios, it is much safer (Michaud & Michaud, 2007). In addition, although resampling might not be optimal on in-sample data, it acts as a constraint and improves out-of-sample performances (Michaud & Michaud, 2007).

The plot of Question 3 shows the efficient frontiers, classical efficient frontier, resampled efficient frontier, individual stocks, minimum variance portfolio and maximum return portfolio. From the plot, we could see that the resampled efficient frontier is below the classical efficient frontier, but at the same time, it is short than the classical efficient frontier, nearly half of the length. It is because that resampling expects less return and lower risk on in-sample data, so it has a little bit lower expected return, but relatively much less risks.

There are two plots in Question 4. On both plots, they use different colors to represent different stocks, and also different weights combination of stocks for different target returns are shown on the plots. The classical MV optimization plot contains much fewer stocks as compared to the resampled plot. And at the same time, the resampled plot has much smoother line between transitions. In the classical MV optimization plot, the portfolio that has the largest target return invested fully into MULN, while in the resampled case, it splits among several stocks which might be less risky and more realistic for investors.

Reference:

Michaud, R. O., & Michaud, R. (2007). Estimation error and portfolio optimization: A. Resampling Solution. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2658657