CS6315: Final Report

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1 Research Problem and Introduction

The inverted pendulum problem [4]is a classic problem in the field of Cyber-Physical Systems (CPS), which involves the challenge of stabilizing an inverted pendulum on its tip by controlling the movement of a cart on a horizontal track. The problem is widely used to experiment with basic control theories and serves as an effective model for displaying key concepts such as stability, observability, and controllability of a system. The inverted pendulum system has gained significant attention from scientists and scholars worldwide in recent years, as it is considered a typical research object.

It is a complex fast, nonlinear, multivariable, strong coupling, and naturally unstable system. In this system, the center of gravity is on the top and the fulcrum is on the bottom. The model of Inverted Pendulum has two main uses. Firstly, as a nonlinear naturally unstable system, it is an ideal experimental platform for carrying out various control experiments. The research on the inverted pendulum system can effectively and intuitively reflect many typical problems in control: such as nonlinear problems, robustness problems, stabilization problems, tracking problems, etc. Secondly, because the Inverted Pendulum system has the characteristics of high-order, unstable, multi- as a strict control object in the study of control theory. Through the control of the inverted pendulum, it is used to test whether the method of control has a strong ability to deal with nonlinear and unstable problems. According to the movement form of the base, Inverted Pendulum system can be divided into three categories: 1.Linear Inverted Pendulum, 2.Annular Inverted Pendulum 3.Planar Inverted Pendulum. As shown in following figures respectively. Each type of Inverted Pendulum can be further divided into one-stage, two-stage, three-stage and multi-stage Inverted Pendulums according to the number of pendulum rods. The more stages of pendulum rods, the more difficult it is to control, and the length of the pendulum rods may also change.

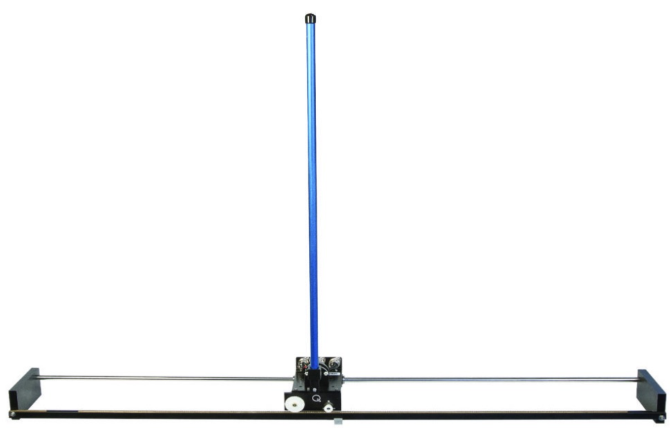

Figure 1: Linear Inverted Pendulum

Despite being a simple and elegant problem, the inverted pendulum is inherently unstable, making it a challenging problem to solve. Various approaches have been proposed to achieve stability in this system, which can demonstrate their efficacy in controlling problems associated with non-linearity and instability. The control of an inverted pendulum is crucial in many real-world applications, including robotics, aerospace, and transportation, where precise control of a system's motion is essential. Hence, developing a robust and effective control algorithm for the inverted pendulum problem is of significant interest to researchers and engineers alike.

Our project aims to develop a control algorithm for stabilizing the one-stage inverted pendulum model using the principles of dynamics and MATLAB Simulink. The proposed approach involves conducting research on modern ways to approach the inverted pendulum problem, designing and implementing our own model, developing formal specifications for our model, and using Simulink Design Verifier to verify its effectiveness. The performance of the control algorithm will be evaluated under different conditions to assess

Figure 2: Annular Inverted Pendulum

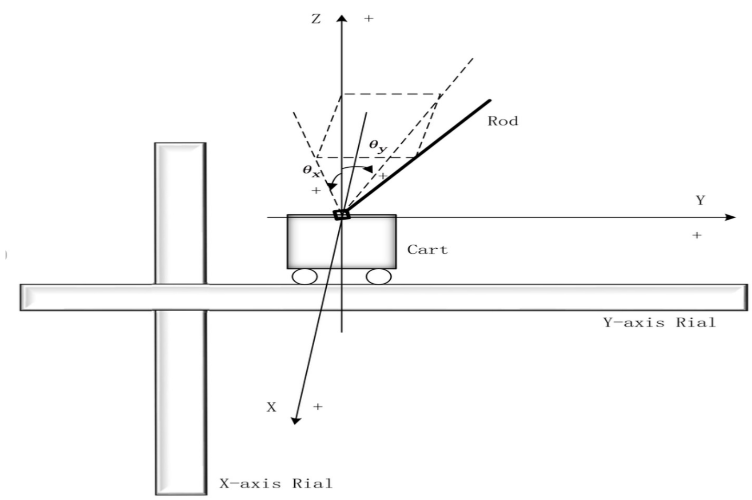

Figure 3: Planar Inverted Pendulum

its effectiveness and robustness.

This research problem is significant due to the challenges involved in stabilizing an inherently unstable system, and the potential applications of the resulting control algorithm in real-world scenarios. By contributing to the existing body of knowledge on the inverted pendulum problem, we hope to develop a solution that can be applied in real-world scenarios.

2 Literature Review

The inverted pendulum problem is a well-known problem in the field of CPS and is widely used to experiment with basic control theories. It serves as an effective model for displaying key concepts such as stability, observability, and controllability of a system. Over the years, various approaches have been proposed to achieve stability in this system, which can demonstrate their efficacy in controlling problems associated with non-linearity and instability. In this literature review, we will discuss some of the recent advancements in the field of inverted pendulum control.

MI El-Hawwary [2] proposes an adaptive fuzzy control system for the inverted pendulum problem that uses a combination of fuzzy logic and adaptive control techniques. The proposed control system is able to stabilize the inverted pendulum in the presence of external disturbances and model uncertainties. Another paper [3] focuses on using the inverted pendulum model to study the dynamics of human locomotion. The paper explores various aspects of the inverted pendulum problem, including control, stability, and energy efficiency, and how they relate to human gait. Friction is an important factor that can affect the stability of the inverted pendulum system. In the paper [1], the authors investigate the effect of friction on the inverted pendulum problem and propose a control strategy that can effectively compensate for the effects of friction.

Overall, the literature suggests that there is still significant potential for new and innovative approaches to the inverted pendulum problem. The various approaches discussed in these papers highlight the importance of control theory, nonlinear dynamics, and adaptive techniques in developing effective control strategies for this problem.

3 Problem and Methodology

In this report, the experiment is based on one-stage inverted pendulum with linear track and primarily employs classical control theory. This problem is basically composed of a pendulum on a cart with its center of mass above its pivot point, and a track that is straight on which the cart can be moved from one end to the other end. The original system is obviously not stable and the pendulum would tend to fall down if there's no other outer factors. The main goal for us is to model a control system which by moving the cart horizontally, can monitor the angle between the pendulum and the cart and move the pivot point back under the center of mass when it starts to fall over, keeping it balanced.

As the figure shows below, a simple model of a linear inverted pendulum contains a cart with mass M and a pendulum right on the cart.

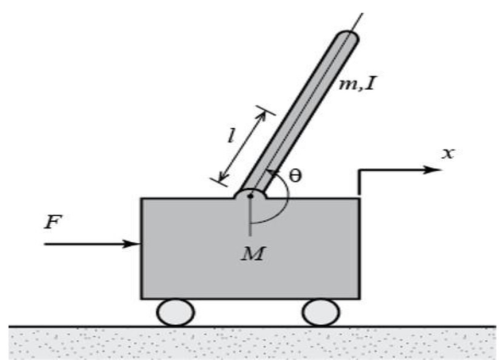

Figure 4: A simple diagram of a linear inverted pendulum

4 Model Design and Implementation

To develop the model of the one-stage inverted pendulum, we first defined the system dynamics, which describe the motion of the pendulum. We modeled the inverted pendulum as a system with a single rigid pendulum attached to a movable cart, which can move horizontally along a track. The cart is controlled by a motor, and the pendulum is subject to gravitational and frictional forces.

We then designed the control algorithm for the system. The control algorithm used in this project is the Proportional-Integral-Derivative (PID) control, which is a well-known feedback control technique widely used in control engineering. The PID controller computes an error signal as the difference between the desired position and the actual position of the cart, and applies a corrective action to the cart to minimize the error. The controller is designed to ensure that the pendulum remains stable in the upright position.

The model was implemented using MATLAB Simulink. We used Simulink to create a model of the system dynamics and implement the PID controller. The model was then simulated under different conditions to evaluate the performance of the control algorithm.

To assess the effectiveness and robustness of our approach, we conducted several experiments with the model under different conditions. We varied the initial angle of the pendulum, the initial position of the cart, and the frictional forces acting on the system. We then analyzed the performance of the control algorithm under each condition and compared the results to evaluate the effectiveness and robustness of our approach.

Overall, the model design and implementation was successful in achieving the objectives of the project. The model accurately represented the dynamics of the system, and the PID control algorithm was effective in stabilizing the inverted pendulum under different conditions. The experiments demonstrated the effectiveness and robustness of our approach, and showed that the model can be used in real-world scenarios.

5 Formal Specifications and Verification

Formal specifications and verification play a crucial role in ensuring the correctness of the developed model. In this project, we used Simulink Design Verifier to verify the correctness of our model against the formal specifications.

To develop formal specifications, we defined the requirements that the model must meet to be considered stable, observable, and controllable. These requirements were based on the principles of dynamics and control theory. For example, stability can be ensured by requiring that the pendulum angle remains within a certain range over time, while observability can be ensured by requiring that the system state can be estimated accurately using sensor data. Controllability can be ensured by requiring that the control inputs have a significant effect on the system's behavior.

Simulink Design Verifier then be used to automatically check the model against these specifications. It generated a set of test cases that will exercise the model and verify that it meets the specified requirements. These test cases were designed to cover a wide range of operating conditions and inputs to ensure that the model is robust and effective under various scenarios. Additionally, Simulink Design Verifier was used to check for common modeling errors such as division by zero or numerical instability. This ensured that the model is well-defined and does not suffer from numerical issues that can affect its accuracy and stability.

6 Result

Finally, we obtained the change curve of the inverted pendu- lum through the pulse generator and scope components. We can find that in the case of interference, its angle, angular velocity, displacement, and speed will undergo a sudden change, and will be adjusted quickly until balance. At the same time, we will obtain an animated model of the inverted pendulum through the mechanic explorer in Matlab, visualize the movement process of the entity, and show the process of reaching balance more clearly

7 Conclusion

The pendulum is a typical experimental platform for control theory research, in which the inverted pendulum is mostly used, since the control strategy of the inverted pendulum system is easy to follow and reproduce, which also has out- standing observability. During the process of modeling te inverted pendulum system, many abstract control theory concepts such as system stability, controllability and sys- tem anti-interference ability are clearly interpreted. Our experiment in this report is based on linear one-stage in- verted pendulum, including the design of the physics model as well as the interface design of the system. During the process of experiment, we find that in PID control, thepro- portional coefficient can reduce the steady-state error, but the system oscillation might be enhanced if the coefficient is too large; the integral coefficient can realize

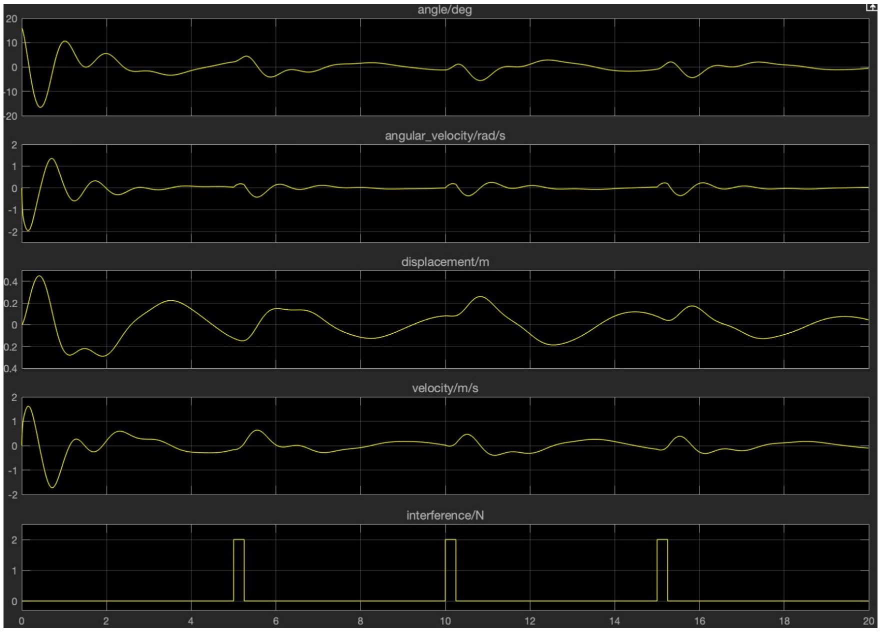

Figure 5: Result figure

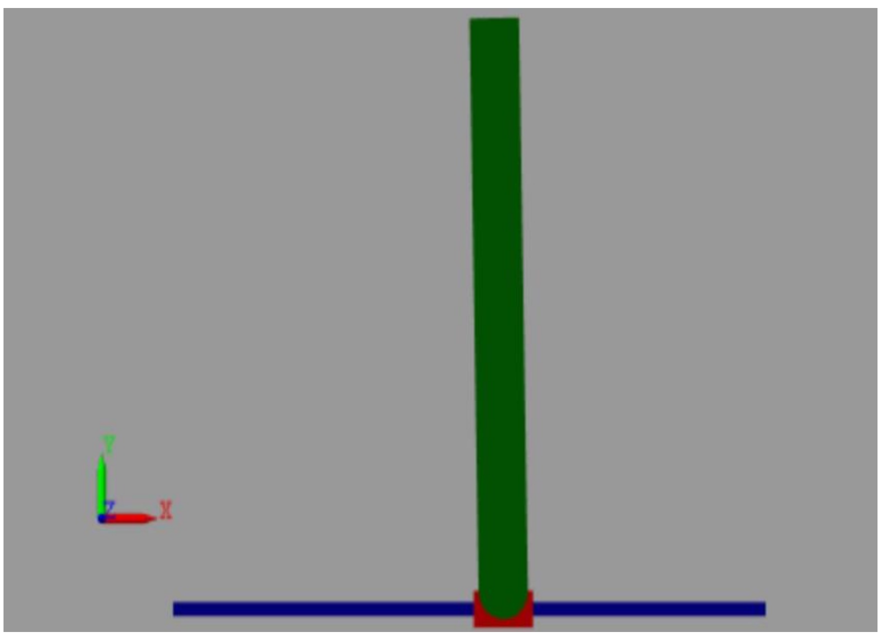

Figure 6: Anamated model

the system becoming an astatic system; the differential coefficient can reduce the overshoot of the system, and the response time becomes faster, which is reasonable. The performance of the system can be changed correspondingly by selecting the parameter values PID. With the use of the PID controller, we successfully obtain an ideal result as expected – the in- verted pendulum system finally steps into a state of dynamic balance.

In conclusion, our project aimed to develop a robust and effective control algorithm for stabilizing the one-stage inverted pendulum model. By conducting extensive research, designing and implementing our own model, and using Simulink Design Verifier to verify its effectiveness, we were able to contribute to the existing body of knowledge on the inverted pendulum problem and develop a solution that can be applied in real-world scenarios. Our project demonstrates the importance of applying control theory principles to practical systems such as the inverted pendulum, and the potential for using simulation tools such as MATLAB Simulink to design and evaluate control algorithms. Future work in this area could focus on expanding the scope of the model to include more complex scenarios and exploring different control strategies to further enhance the stability and robustness of the system.

References

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