Calculating Accuracy, Weighted Precision, Weighted Recall, and Weighted F1-Score for Multi-class Classification

Let M be a $n \times n$ confusion matrix for n classes ($n \geq 3$) where each entry is M_{ij} for $i = 0, 1, ..., n-1$ and $j = 0, 1, ..., n-1$. The rows of M represent true classes while the columns represent predicted classes. Let y_true be a list of N true classes where $y\_true = \{y_0, y_1, ..., y_{N-1}\}$, $y_k \in \{0, 1, ..., n-1\}$, and $k \in \{0, 1, ..., N-1\}$. Let y_pred be a list of N corresponding predicted classes. Therefore, M_{ij} is the number of pairs (i, j) where $i = y\_true_k$ and $j = y\_pred_k$ for all k where $k \in \{0, 1, ..., N-1\}$. One vs. all multi-class setting is used. For all classes, one class is chosen to represent the positive instance and the rest of the other classes together represent the negative instance. Let $support_i$ be the number of elements in class i in y_true, so $support_i = \sum_{j=0}^{n-1} M_{ij}$. Class i has a weight (w_i) where $w_i = \frac{support_i}{N}$. Let tp be the number of true positives, fp be the number of false positives, and fn be the number of false negatives. For class i, $tp_i = M_{ii}$, $fp_i = \sum_{j=0}^{n-1} M_{ji} - M_{ii}$, and $fn_i = \sum_{j=0}^{n-1} M_{ij} - M_{ii}$.

1. **Accuracy**

   Accuracy $a = \frac{1}{N} \sum_{i=0}^{n-1} tp_i = \frac{1}{N} \sum_{i=0}^{n-1} M_{ii}$.

2. **Weighted Precision**

   For class i, precision $p_i = \frac{tp_i}{tp_i + fp_i}$.

   Thus, $p_i = \frac{M_{ii}}{M_{ii} + \sum_{j=0}^{n-1} M_{ji} - M_{ii}} = \frac{M_{ii}}{\sum_{j=0}^{n-1} M_{ji}}$.

   $p_{weighted} = \sum_{i=0}^{n-1} w_i p_i = \frac{1}{N} \sum_{i=0}^{n-1} \frac{support_i M_{ii}}{\sum_{j=0}^{n-1} M_{ji}}$.

3. **Weighted Recall**

   For class i, recall $r_i = \frac{tp_i}{tp_i + fn_i}$.

   Thus, $r_i = \frac{M_{ii}}{M_{ii} + \sum_{j=0}^{n-1} M_{ij} - M_{ii}} = \frac{M_{ii}}{\sum_{j=0}^{n-1} M_{ij}} = \frac{M_{ii}}{support_i}$.

   $r_{weighted} = \sum_{i=0}^{n-1} w_i r_i = \frac{1}{N} \sum_{i=0}^{n-1} M_{ii}$.

   Note that the weighted recall is also the same as accuracy.

4. **Weighted F1-score**

F1-score is the harmonic mean of precision and recall.

For class i, F1-score $f_i = \frac{2p_i r_i}{p_i + r_i}$.

Thus, $f_i = \frac{2\left(\frac{M_{ii}}{\sum_{j=0}^{n-1} M_{ji}}\right)\left(\frac{M_{ii}}{support_i}\right)}{\frac{M_{ii}}{\sum_{j=0}^{n-1} M_{ji}} + \frac{M_{ii}}{support_i}} = \frac{2M_{ii}}{support_i + \sum_{j=0}^{n-1} M_{ji}}$.

$f_{weighted} = \sum_{i=0}^{n-1} w_i f_i = \frac{2}{N} \sum_{i=0}^{n-1} \frac{support_i M_{ii}}{support_i + \sum_{j=0}^{n-1} M_{ji}}$.

5. **Simple Example**

   A simple example showing a 3-class ('bird', 'cat', 'dog') classification problem is given below.

   y_true = ['cat', 'dog', 'bird', 'bird', 'cat', 'dog', 'dog', 'bird', 'bird', 'cat', 'dog', 'cat', 'cat', 'cat', 'cat', 'dog', 'dog', 'dog', 'dog', 'dog', 'bird', 'bird', 'bird', 'dog', 'dog', 'cat', 'bird', 'dog'].

   y_pred = ['bird', 'dog', 'bird', 'cat', 'cat', 'bird', 'dog', 'bird', 'bird', 'dog', 'dog', 'cat', 'cat', 'cat', 'cat', 'dog', 'dog', 'dog', 'dog', 'dog', 'bird', 'bird', 'bird', 'cat', 'cat', 'dog', 'dog', 'bird'].

   These 3 classes ('bird', 'cat', 'dog') are mapped to 0, 1, and 2, respectively.

   y_true = [1, 2, 0, 0, 1, 2, 2, 0, 0, 1, 2, 1, 1, 1, 1, 2, 2, 2, 2, 2, 0, 0, 0, 2, 2, 1, 0, 2].

   y_pred = [0, 1, 0, 1, 1, 0, 2, 0, 0, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 2, 0, 0, 0, 1, 1, 2, 2, 0].

   $N = 28$.

   The corresponding 3×3 confusion matrix is shown below.

   $$M = \begin{bmatrix} 6 & 1 & 1 \\ 1 & 5 & 2 \\ 2 & 3 & 7 \end{bmatrix}$$

   (a) **Accuracy**
   $a = \frac{1}{28}(6 + 5 + 7) = \frac{9}{14} \approx 0.6428571428571429$.

   (b) **Weighted Precision**
   $p_{weighted} = \frac{1}{28}\left(\frac{8(6)}{9} + \frac{8(5)}{9} + \frac{12(7)}{10}\right) = \frac{409}{630} \approx 0.6492063492063492$.

   (c) **Weighted Recall**
   $r_{weighted} = \frac{1}{28}(6 + 5 + 7) = \frac{9}{14} \approx 0.6428571428571429$.

   (d) **Weighted F1-Score**
   $f_{weighted} = \frac{2}{28}\left(\frac{8(6)}{8+9} + \frac{8(5)}{8+9} + \frac{12(7)}{12+10}\right) = \frac{841}{1309} \approx 0.6424751718869367$.