Evolution de la population d'Homo Néanderthal

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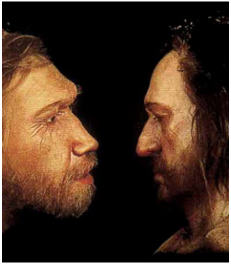


Introduction

Contexte biologique

? ? ? peut-être pas besoin de slide... le slide avec le titre intro peut suffire

Présentation des modèles étudiés

Cadre général

Modèle général

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x), \quad t \in \mathbb{R}, x \in \mathbb{R} \tag{1}$$

u(t,x) : Densité de population $\in [0,1]$

d : Constante de diffusion

Croissance Logistique

Croissance Logistique

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x)$$

$$f(u(t,x)) = \alpha u(1 - \frac{u}{K})$$

K : Capacité de transport

α : Taux de croissance maximum

Modèle Allee

Modèle Allee

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x)$$

$$f(u(t,x)) = ku(1-u)(u-A)$$

k : Taux de croissance normalisé constant

A : Densité critique

$$k = \frac{4}{(1-A)^2}$$

Système de Lotka-Volterra

Modèle de compétition

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = f(u,v) + d_1 \Delta u \\ \frac{\partial v(t,x)}{\partial t} = g(u,v) + d_2 \Delta v \end{cases}$$

$$f(u,v) = \alpha_1 u \left(1 - \frac{u}{K_1} - \gamma_1 \frac{v}{K_1}\right),\ g(u,v) = \alpha_2 v \left(1 - \frac{v}{K_2} - \gamma_2 \frac{u}{K_2}\right)$$

u(t,x) : Densité de population des Hommes Modernes

v(t,x) : Densité de population des Hommes de Néanderthal

K_1 et K_2 : Capacités d'accueil du milieu

γ_1 et γ_2 : Coefficients de compétition

α_1 et α_2 : Taux de croissance

Analyse mathématiques

Croissance logistique

- Équilibres : $u = 0$ *et* $u = K$
- MONOSTABLE
- Vitesse minimale $c_0 = 2\sqrt{\alpha}$

Effet Allee

- Équilibres : $u = 0, u = A$ et $u = 1$
- BISTABLE
- 2 scénarios selon la valeur de A :
  - $A < 0.5$: $c > 0 \rightarrow$ "1 envahit 0"
  - $A > 0.5$: $c < 0 \rightarrow$ "0 envahit 1"

Système de Lotka Volterra

? ? ?

Simulations numériques

Propagation 1D

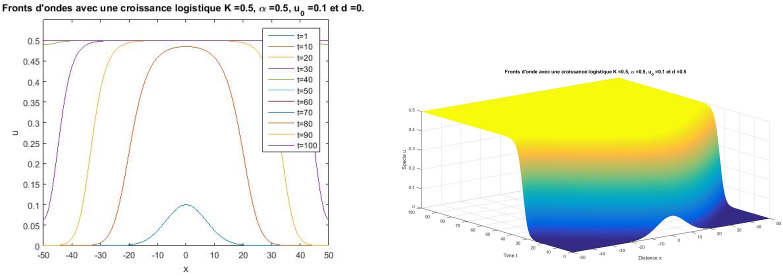


FIGURE – Propagation du front d'onde en 1D avec une petite perturbation initiale

Propagation 2D

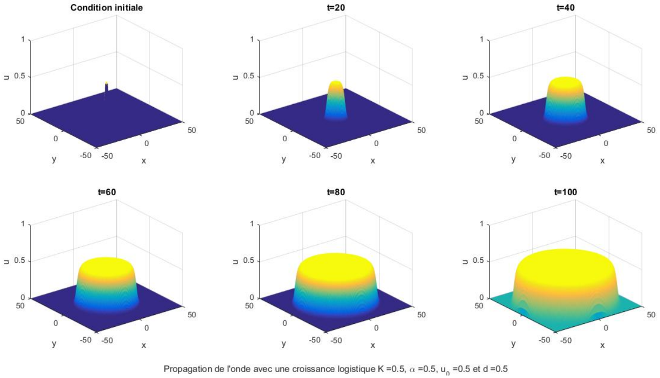


FIGURE – Propagation du front d'onde en 2D

Propagation 2D vu de haut

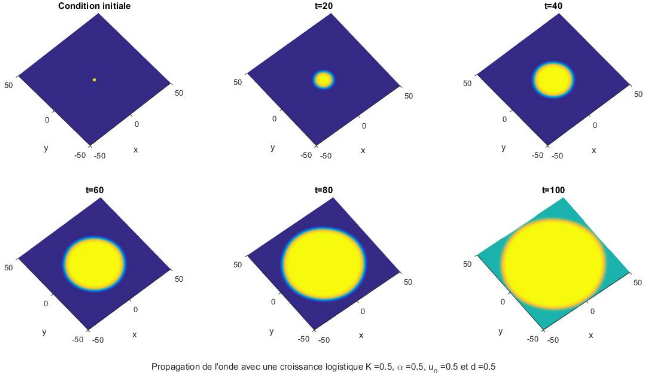


FIGURE – Propagation du front d'onde en 2D

Effet d’une barrière géographique franchissable

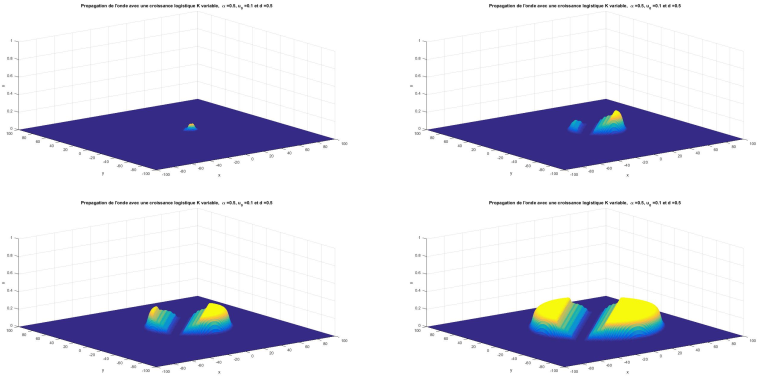


FIGURE – Diffusion 2D de la population face à une montagne

Effet d'une barrière géographique franchissable
Vue de haut

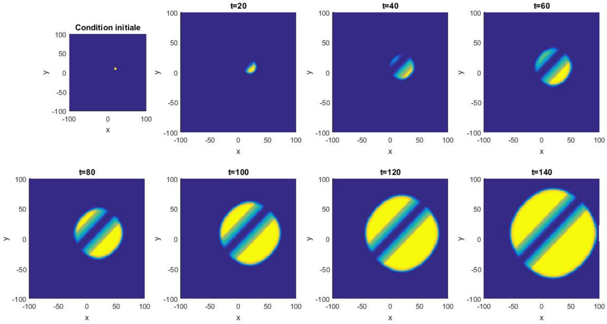


FIGURE – Diffusion 2D de la population face à une montagne

Effet d'une barrière géographique infranchissable

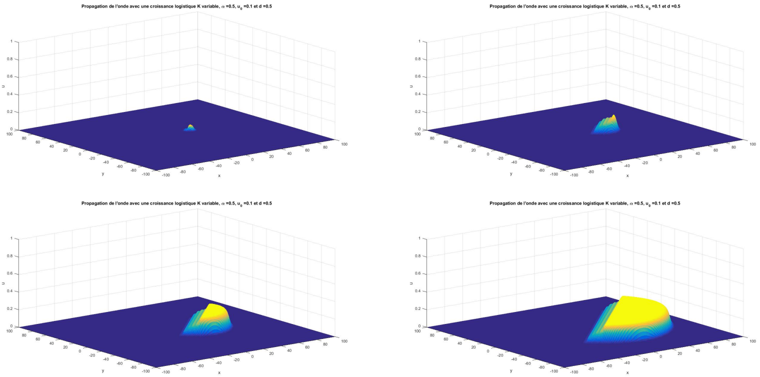


FIGURE – Diffusion 2D

Effet d'une barrière géographique infranchissable

Vue de haut

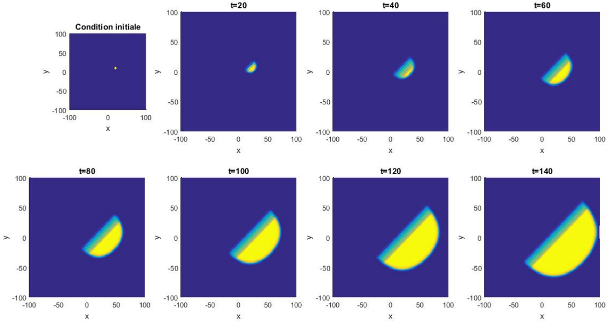
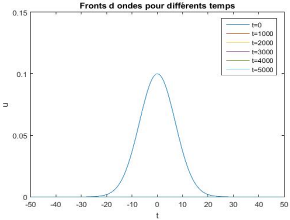
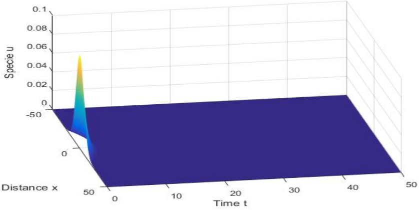
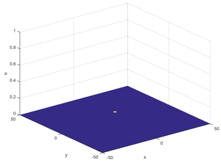


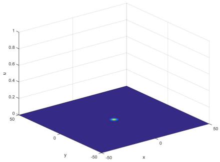
FIGURE – Diffusion 2D

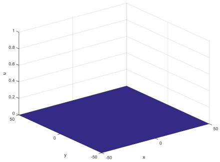
$d = 0.5, A = 0.25(k = 64), u_0 = 0.1$



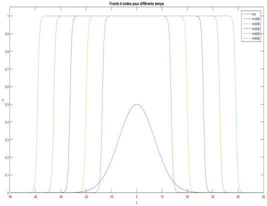


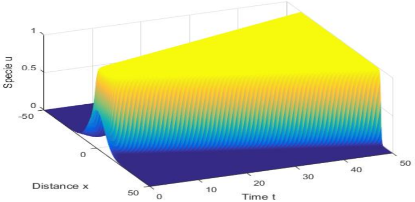


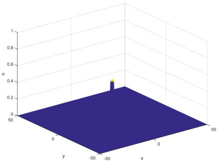


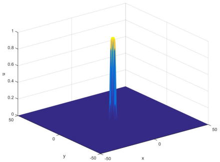


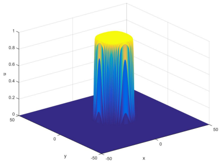
$d = 0.5, A = 0.25(k = 64), u_0 = 0.5$



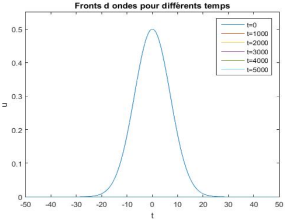


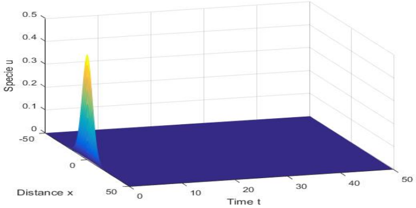


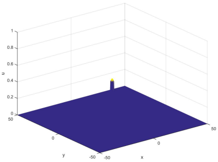


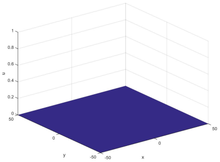


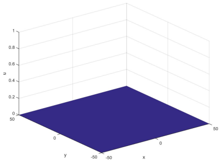
$d = 0.5, A = 0.75 (k = 7.1), u_0 = 0.5$



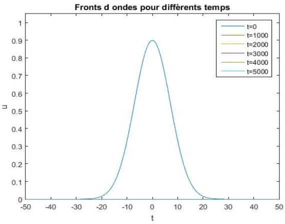


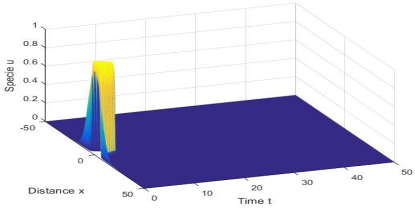


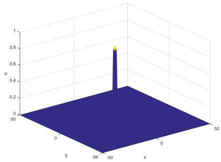


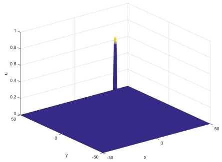


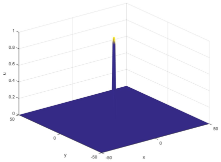
$d = 0.5, A = 0.75(k = 7.1), u_0 = 0.9$



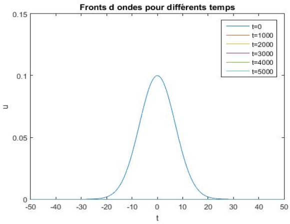


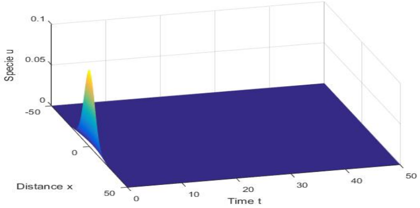


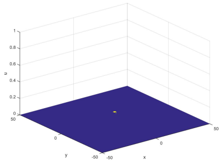


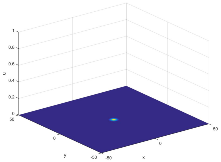


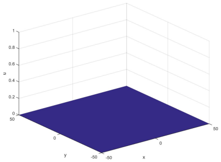
$d = 0.5, A = 0.5(k = 16), u_0 = 0.1$



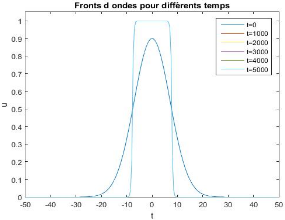


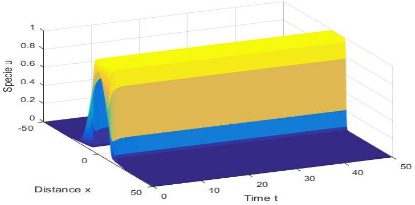


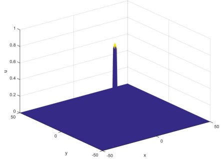


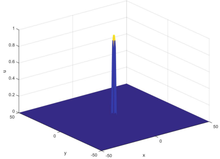


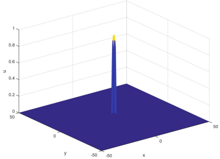
$d = 0.5, A = 0.5(k = 16), u_0 = 0.9$











Front d'ondes

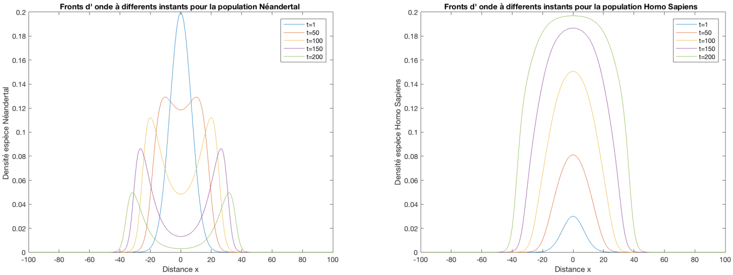


FIGURE – Evolution des fronts d'onde au cours du temps

Diffusion 1D

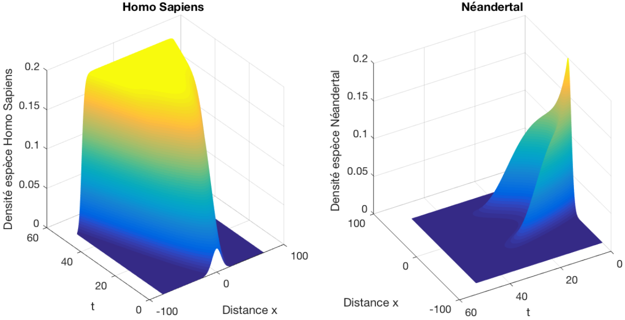


FIGURE – Diffusion 1D

Conclusion