Evolution de la population d'Homo Néanderthal

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INSA Lyon - Bioinformatique et Modélisation

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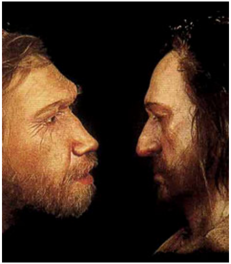

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Introduction

Contexte biologique

? ? ? peut-être pas besoin de slide... le slide avec le titre intro peut suffire

Cadre général

Cadre général

Modèle général

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x), \quad t \in \mathbb{R}, x \in \mathbb{R} \tag{1}$$

u(t,x) : Densité de population $\in [0,1]$

d : Constante de diffusion

Croissance logistique

Présentation du modèle

Croissance Logistique

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x)$$

$$f(u(t,x)) = \alpha u(1 - \frac{u}{K})$$

K : Capacité de transport

α : Taux de croissance maximum

Analyse mathématique

- Équilibres : $u = 0$ *et* $u = K$
- MONOSTABLE
- Vitesse minimale $c_0 = 2\sqrt{\alpha}$

Propagation 1D

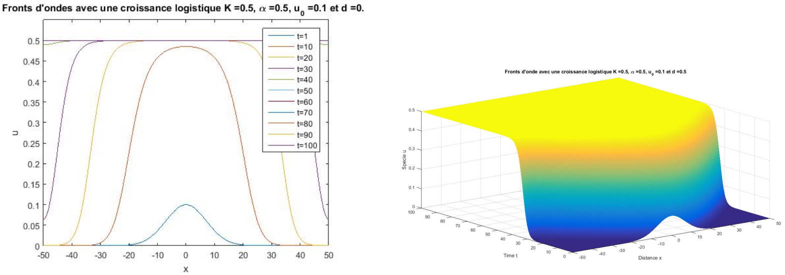
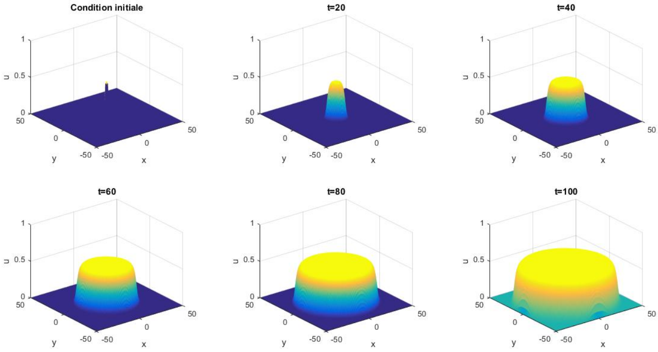


FIGURE: Propagation du front d'onde en 1D avec une petite perturbation initiale

Propagation 2D



Propagation de l'onde avec une croissance logistique K =0.5, α =0.5, u_0 =0.5 et d =0.5

FIGURE: Propagation du front d'onde en 2D

Propagation 2D vu de haut

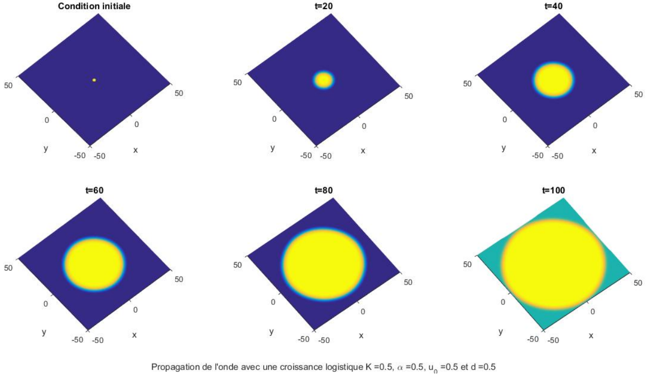


FIGURE: Propagation du front d'onde en 2D

Effet d’une barrière géographique franchissable

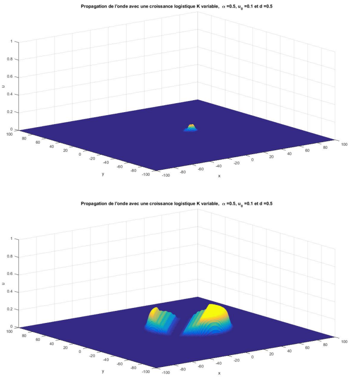


FIGURE: Diffusion 2D de la population face à une montagne

Effet d'une barrière géographique franchissable

Vue de haut

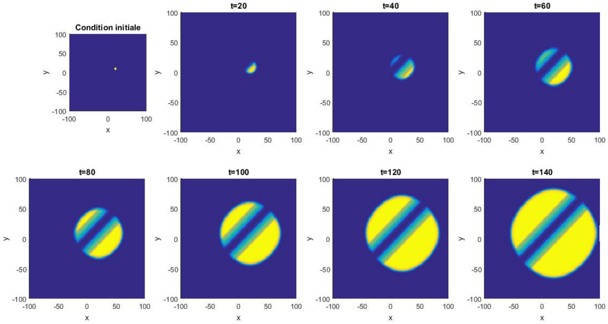


FIGURE: Diffusion 2D de la population face à une montagne

Effet d'une barrière géographique infranchissable

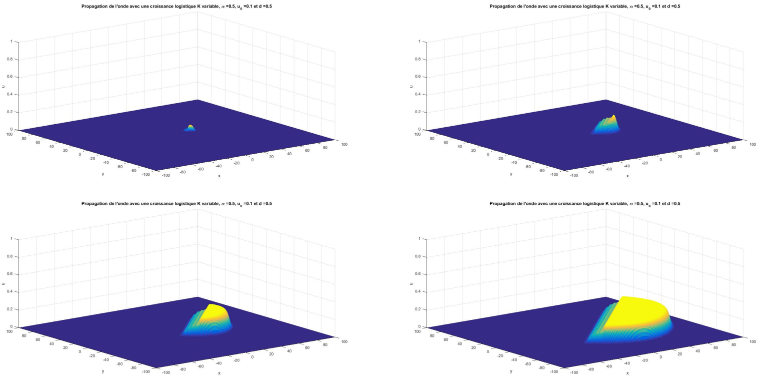


FIGURE: Diffusion 2D

Effet d'une barrière géographique infranchissable

Vue de haut

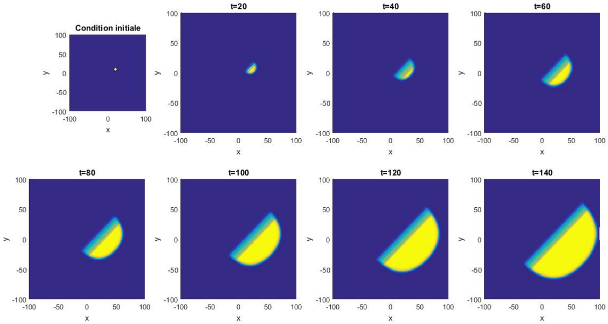


FIGURE: Diffusion 2D

Effet Allee

Présentation du modèle

Modèle Allee

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x)$$

$$f(u(t,x)) = ku(1-u)(u-A)$$

k : Taux de croissance normalisé constant

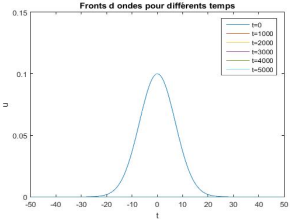
A : Densité critique

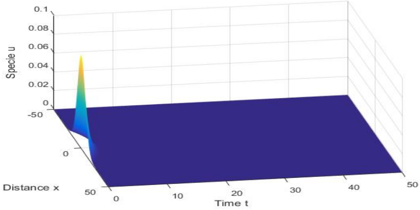
$$k = \frac{4}{(1-A)^2}$$

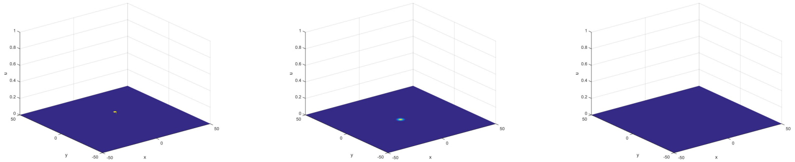
Analyse mathématique

- Équilibres : $u = 0, u = A$ et $u = 1$
- BISTABLE
- 2 scénarios selon la valeur de A :
  - $A < 0.5$: $c > 0 \rightarrow$ "1 envahit 0"
  - $A > 0.5$: $c < 0 \rightarrow$ "0 envahit 1"

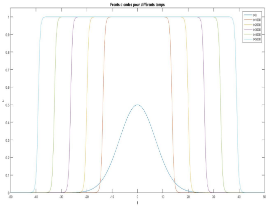
$d = 0.5, A = 0.25(k = 64), u_0 = 0.1$

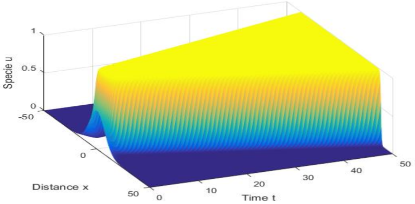


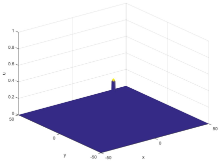


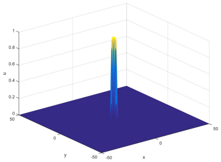


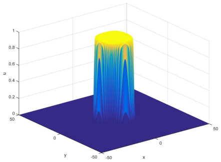
$d = 0.5, A = 0.25(k = 64), u_0 = 0.5$



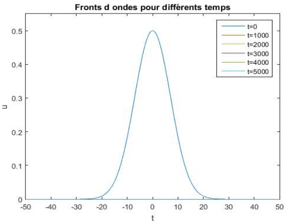


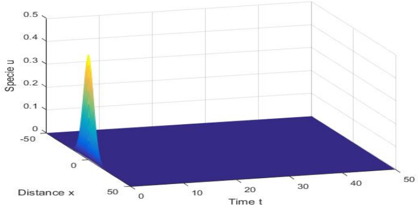


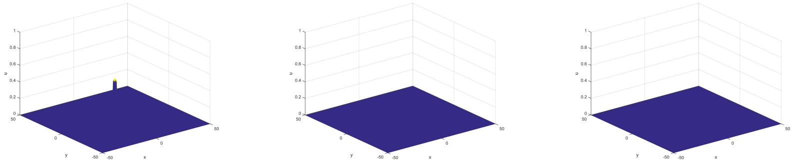




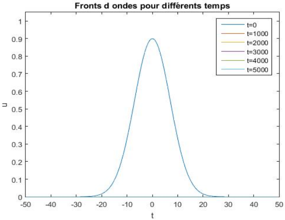
$d = 0.5, A = 0.75(k = 7.1), u_0 = 0.5$

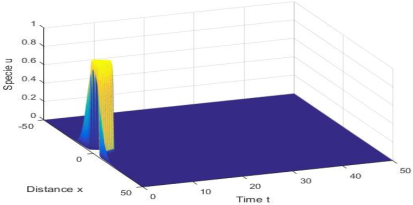


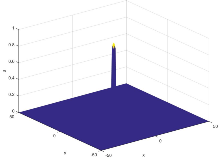


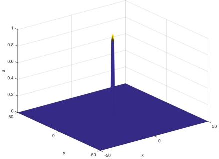


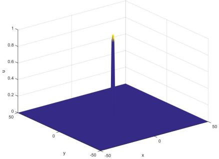
$d = 0.5, A = 0.75 (k = 7.1), u_0 = 0.9$



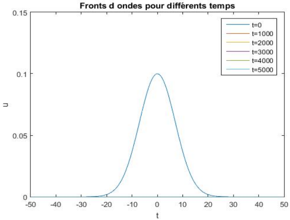


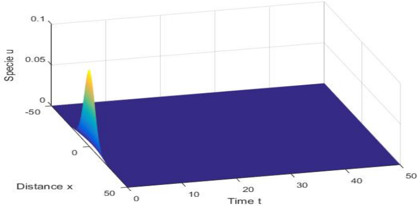


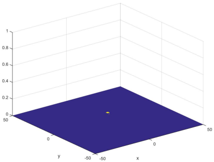


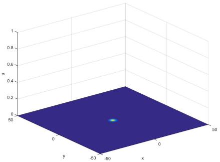


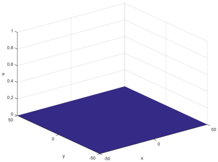
$d = 0.5, A = 0.5(k = 16), u_0 = 0.1$



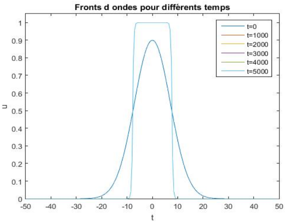


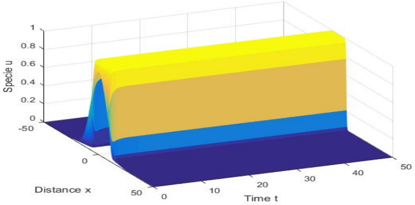


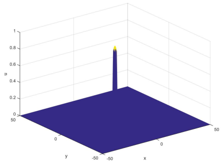


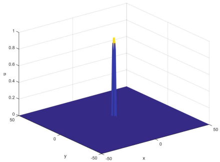


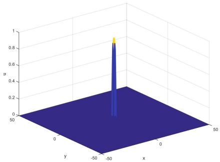
$d = 0.5, A = 0.5 (k = 16), u_0 = 0.9$











Système de Lotka-Volterra

Présentation du modèle

Modèle de compétition

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = f(u,v) + d_1 \Delta u \\ \frac{\partial v(t,x)}{\partial t} = g(u,v) + d_2 \Delta v \end{cases}$$

$$f(u,v) = \alpha_1 u \left(1 - \frac{u}{K_1} - \gamma_1 \frac{v}{K_1}\right), \; g(u,v) = \alpha_2 v \left(1 - \frac{v}{K_2} - \gamma_2 \frac{u}{K_2}\right)$$

u(t,x) : Densité de population des Hommes Modernes

v(t,x) : Densité de population des Hommes de Néanderthal

K_1 et K_2 : Capacités d'accueil du milieu

γ_1 et γ_2 : Coefficients de compétition

α_1 et α_2 : Taux de croissance

Analyse mathématique

- On prend $\gamma_1 < \frac{K_1}{K_2}$ et $\gamma_2 > \frac{K_2}{K_1}$
- Trois équilibres $(0,0)$, $(0,K_2)$, $(K_1,0)$.
- En posant $\alpha = \alpha_1 = \alpha_2$, $K = K_1 = K_2$, $D = d_1 = d_2$ et $\gamma_1 + \gamma_2 = 2$: existence de fronts d'ondes reliant 0 et K.
- $c_{min} = 2\sqrt{\alpha D(1 - \gamma_1)}$

Front d'ondes

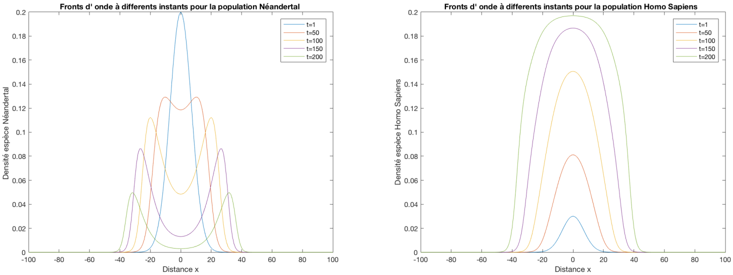


FIGURE: Evolution des fronts d'onde au cours du temps

Diffusion 1D

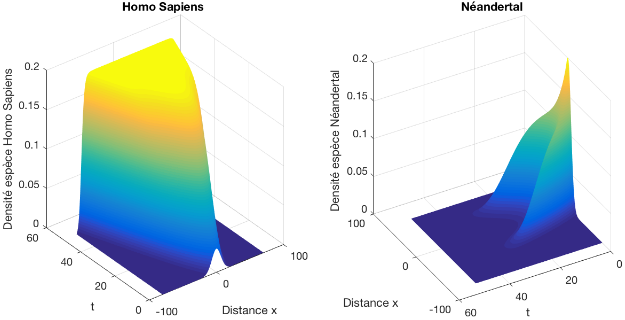


FIGURE: Diffusion 1D

Conclusion