Evolution de la population d'*Homo Néanderthalensis*

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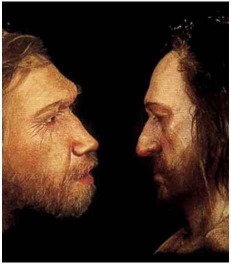

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La disparition mystérieuse de Néandertal

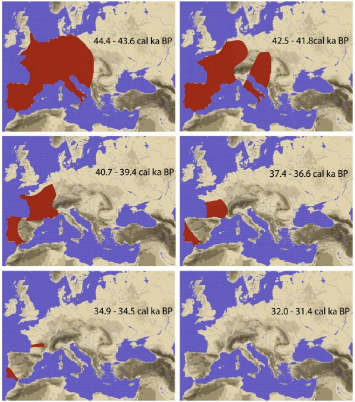

FIGURE – Régions occupées par Néandertal à différentes époques en Europe

FIGURE – La grotte de Bruniquel, plus ancienne construction humaine

Cadre général

Modèle général

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x), \tag{1}$$

$$t \in \mathbb{R}, \quad x \in \mathbb{R}^n, \quad n = 1 \text{ ou } n = 2,$$

u(t,x) : Densité de population à la position x et au temps t.

d : Constante de diffusion.

f : Terme de réaction.

Plan

Présentation du modèle

Croissance Logistique

$$\frac{\partial u(t,x)}{\partial t} = \alpha u(t,x)\left(1 - \frac{u(t,x)}{K}\right) + d\Delta u(t,x),$$

K : Capacité du milieu liée à des facteurs locaux environnementaux ($K > 0$).

α : Taux de croissance intrinsèque de la population ($\alpha > 0$).

Analyse mathématique

Analyse de la partie réaction

0 équilibre instable et K équilibre stable

Analyse de l'équation de réaction-diffusion

L'équation de réaction diffusion est de type **KPP monostable.**

- Pour tout $c > 2\sqrt{\alpha}$ il existe un unique front d'onde U(z) monotone se propageant à la vitesse c tel que $lim_{z \to +\infty} U(z) = 0$ et $lim_{z \to -\infty} U(z) = K$.
- Effet *"hair trigger"* : Si on prend une condition initiale positive sur un intervalle borné et nul en dehors de l'intervalle, alors la solution converge vers K à la vitesse $2\sqrt{\alpha}$.

Propagation d’un front d’onde en 1D

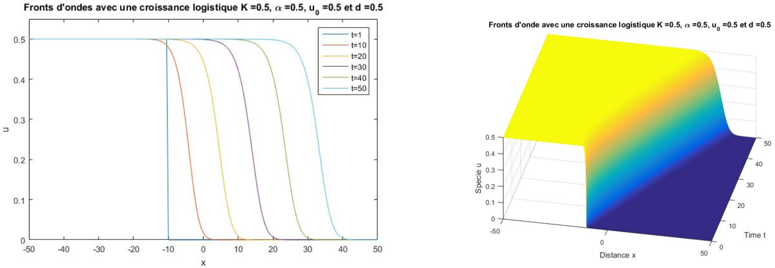

FIGURE – Propagation du front d’onde en 1D (`KPP1D.avi`)

Effet "hair trigger"

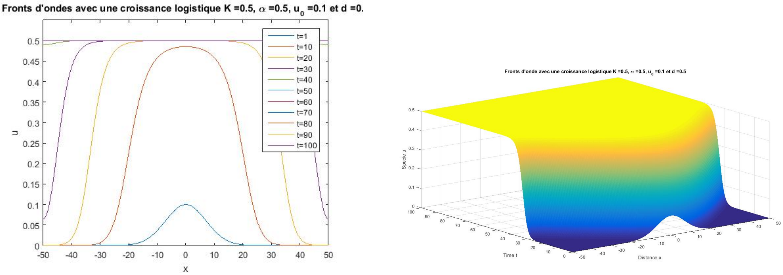

FIGURE – Propagation du front d'onde en 1D avec une petite perturbation initiale à support compact.

Effet d’une barrière géographique franchissable

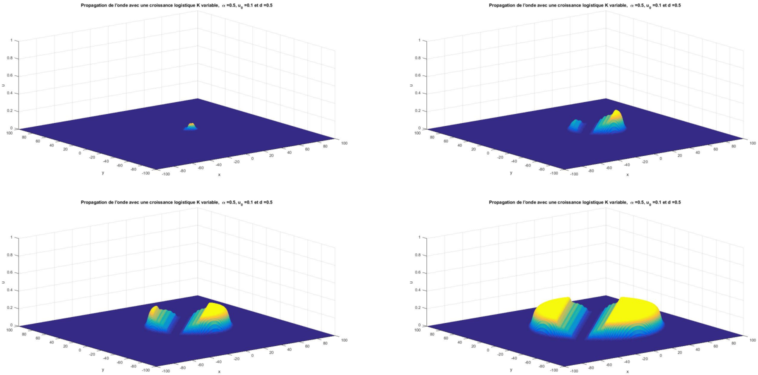

FIGURE – Diffusion 2D de la population face à une barrière géographique franchissable : `KPPFranchissable.avi`

Effet d’une barrière géographique infranchissable

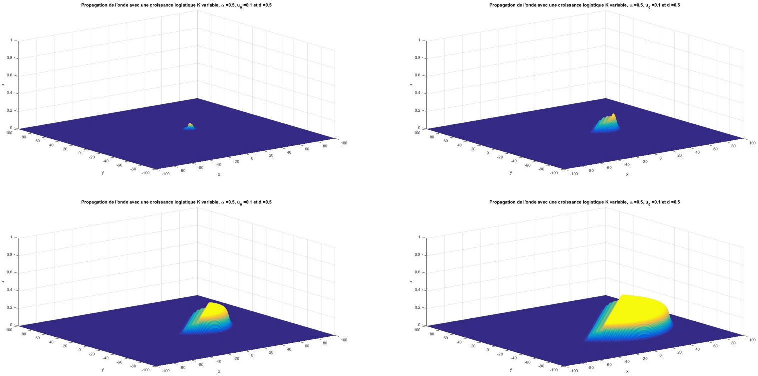

FIGURE – Diffusion 2D de la population face à une barrière géographique infranchissable : `KPPNonFranchissable.avi`

Plan

Présentation du modèle

Modèle Allee

$$\frac{\partial u(t,x)}{\partial t} = f(u(t,x)) + d\Delta u(t,x)$$

$$f(u(t,x)) = ku(1-u)(u-A)$$

k : Taux de croissance normalisé constant

A : Densité critique

$$k = \frac{4}{(1-A)^2}$$

Analyse mathématique

- Équilibres : $u = 0, u = A$ et $u = 1$
- BISTABLE
- 2 scénarios selon la valeur de A :
  - $A < 0.5$: $c > 0 \rightarrow$ "1 envahit 0"
  - $A > 0.5$: $c < 0 \rightarrow$ "0 envahit 1"

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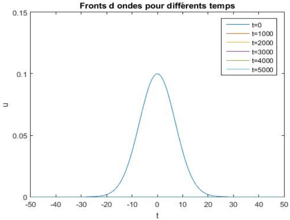
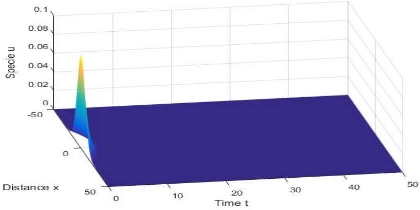
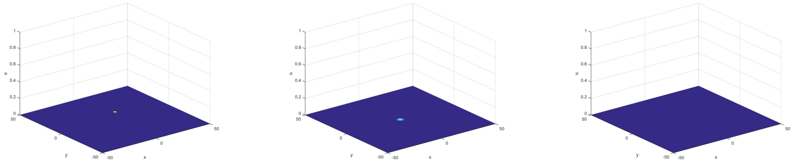
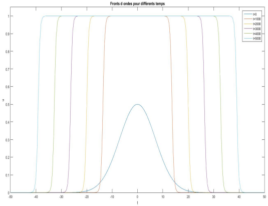
$d = 0.5, A = 0.25 (k = 64), u_0 = 0.1$

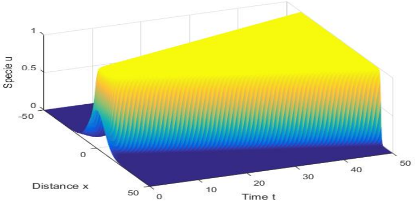


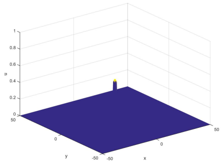


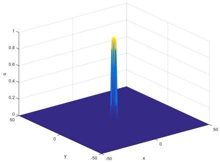


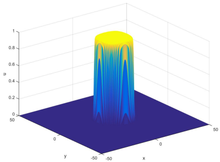
$d = 0.5, A = 0.25(k = 64), u_0 = 0.5$



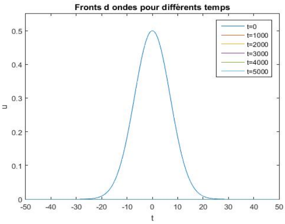


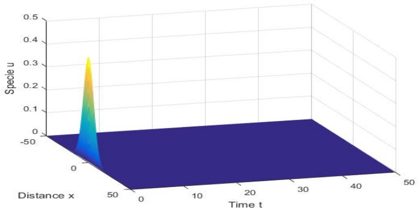


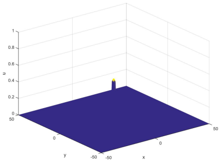


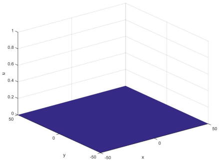


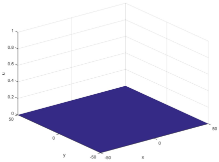
$d = 0.5, A = 0.75(k = 7.1), u_0 = 0.5$



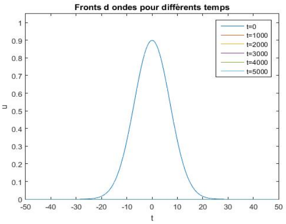


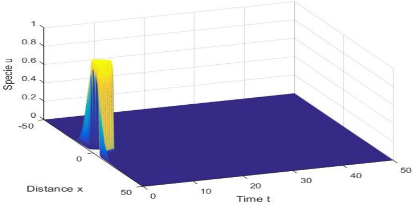


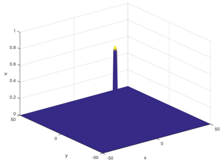


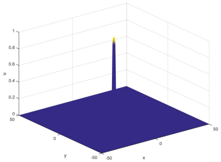


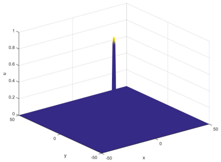
$d = 0.5, A = 0.75(k = 7.1), u_0 = 0.9$



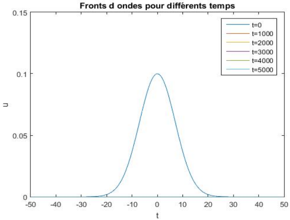


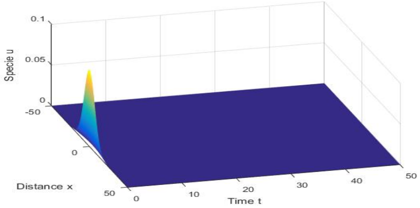


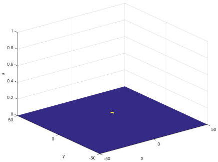


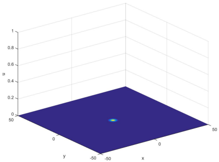


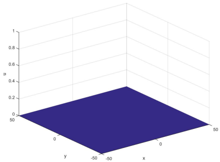
$d = 0.5, A = 0.5(k = 16), u_0 = 0.1$



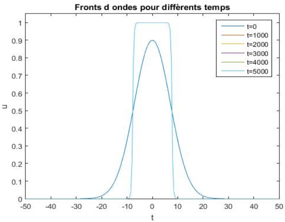


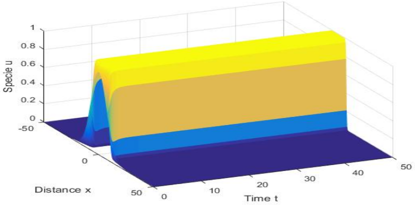


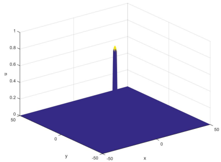


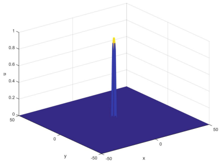


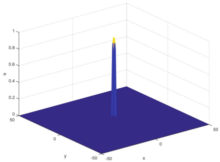
$d = 0.5, A = 0.5(k = 16), u_0 = 0.9$











Plan

Présentation du modèle

Modèle de compétition

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = f(u,v) + d_1 \Delta u \\ \frac{\partial v(t,x)}{\partial t} = g(u,v) + d_2 \Delta v \end{cases}$$

$$f(u,v) = \alpha_1 u \left(1 - \frac{u}{K_1} - \gamma_1 \frac{v}{K_1}\right), \; g(u,v) = \alpha_2 v \left(1 - \frac{v}{K_2} - \gamma_2 \frac{u}{K_2}\right)$$

u(t,x) : Densité de population des Hommes Modernes

v(t,x) : Densité de population des Hommes de Néanderthal

K_1 et K_2 : Capacités d'accueil du milieu

γ_1 et γ_2 : Coefficients de compétition

α_1 et α_2 : Taux de croissance

Analyse mathématique

? ? ?

Front d'ondes

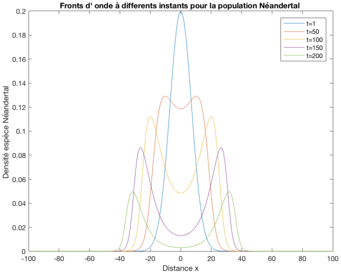


FIGURE – Evolution des fronts d'onde au cours du temps (`Competition.avi`)

Diffusion 1D

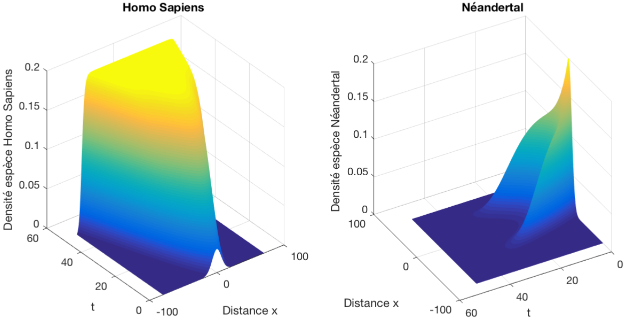


FIGURE – Diffusion 1D

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