Modelling M/EEG data with Bayesian nonparametric multilevel models

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6 Abstract

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Time-resolved electrophysoiological measurements such as those offered by magneto- or electro-encephalography (M/EEG) provide a unique window onto neural activity underlying cognitive process and how they unfold over time. Typically, we are interested in testing whether such measures differ across conditions and/or groups. The conventional approach consists in conducting mass-univariate statistics followed by some form of multiplicity correction (e.g., FDR, FWER) or cluster-based inference. However, these cluster-based methods have an important downside: they shift the focus of inference from the timepoint to the cluster level, thus preventing any conclusion to be made about the onset and offset of effects (e.g., differences across conditions). Here, we introduce a novel model-based approach for analysing one-dimensional M/EEG timeseries such as ERPs or decoding timecourses and their differences across conditions. This approach relies on Bayesian nonparametric multilevel modelling (multilevel generalised additive models or Gaussian processes), which outputs posterior probabililities of the effect being above chance at every timestep/voxel, while naturally taking into account the temporal dependencies and between-subject variability present in such data.

Keywords: EEG, MEG, generalised additive models, gaussian processes, mixed-effects models, Bayesian statistics, brms

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Modelling M/EEG data with Bayesian nonparametric multilevel models

Introduction (in progress)

Here are some useful references to be discussed (Combrisson & Jerbi, 2015; Ehinger & Dimigen, 2019; Frossard & Renaud, 2021, 2022; Gramfort, 2013; Hayasaka, 2003; Luck & Gaspelin, 2017; Maris & Oostenveld, 2007; E. J. Pedersen et al., 2019; C. R. Pernet et al., 2015; Riutort-Mayol et al., 2023; Rousselet, 2025)... See also (Maris, 2011)... and (Rosenblatt et al., 2018) (history of cluster-based approaches and using a data split?)... Cluster failure (Eklund et al., 2016)...

In the following, we consider two approaches to modelling the non-linear timecourse of M/EEG ERPs or decoding performance: i) generalised additive models (GAMs) with thin-plate smoothing splines (Wood, 2003; Wood, 2004) and ii) Gaussian processes (GPs) with a smooth covariance kernel (Rasmussen & Williams, 2005) and low-rank approximation (Riutort-Mayol et al., 2023).

13 Previous modelling work

Disentangling overlapping processes (Skukies et al., 2024; Skukies & Ehinger, 2021)...
Using Bayes factors (Teichmann, 2022)... Weighting single trials (C. Pernet, 2022)...

See https://www.let.rug.nl/nerbonne/teach/rema-stats-meth-seminar/presentations/Wieling-2014-GAMs.pdf and https://www.let.rug.nl/wieling/Statistics/GAM-EEG/lab/, https://www.let.rug.nl/wieling/Statistics/GAM-EEG/GAM-EEG.pdf (Abugaber et al., 2023; Meulman et al., 2015)...

Recent example of GLM for EEG (Fischer & Ullsperger, 2013; Wüllhorst et al., 2025)... "Similar approaches have been successfully applied to EEG time- (Rousselet et al., 2008) and frequency-domain (Cohen and Cavanagh, 2011) data and allow the simultaneous investigation of multiple independent variables while preserving the high temporal resolution of the EEG."... See also (Hauk et al., 2006; Rousselet et al., 2008)... Example of two-stage regression analysis (individual-level then group-level, 10.1101/2024.04.10.588861, Dunagan et al., 2024)...

Generalised additive models

In generalised additive models (GAMs), the functional relationship between predictors and response variable is decomposed into a sum of low-dimensional non-parametric functions. A typical GAM has the following form:

$$y_i \sim \text{EF}(\mu_i, \phi)$$

$$g(\mu_i) = A_i + \mathbf{X}_i \gamma + \sum_{i=1}^{J} f_j(x_{ij})$$

where $y_i \sim \text{EF}(\mu_i, \phi)$ denotes that the observations y_i are distributed as some member of the exponential family of distributions (e.g., Gaussian, Gamma, Beta, Poisson) with mean μ_i and scale parameter ϕ ; $g(\cdot)$ is the link function, A is an offset, \mathbf{X}_i is the ith row of a parametric model matrix, γ is a vector of parameters for the parametric terms, f_j is a smooth function of covariate x_j . The smooth functions f_j are represented in the model via penalised splines basis expansions of the covariates, that are a weighted sum of basis functions:

$$f_{j}\left(x_{ij}\right) = \sum_{k=1}^{K} \beta_{jk} b_{jk}\left(x_{ij}\right)$$

where β_{jk} is the weight (coefficient) associated with the kth basis function b_{jk} () evaluated at the covariate value x_{ij} for the jth smooth function f_j . Splines' coefficients are penalised...

Gaussian process regression

A Gaussian process (GP) is a stochastic process that defines the distribution over a collection of random variables indexed by a continuous variable, that is $\{f(t): t \in \mathcal{T}\}$ for some index set \mathcal{T} (Rasmussen & Williams, 2005; Riutort-Mayol et al., 2023). Whereas Bayesian linear regression outputs a distribution over the parameters of some predefind parametric model, the GP approach, in contrast, is a non-parametric approach, in that it finds a distribution over the possible functions that are consistent with the observed data. However, note that nonparametric does not mean there aren't parameters, it means that there are infinitely many parameters.

From brms documentation: A GP is a stochastic process, which describes the relation between one or more predictors $x = (x_1, ..., x_d)$ and a response f(x), where d is the number of predictors. A GP is the generalization of the multivariate normal distribution to an infinite number of dimensions. Thus, it can be interpreted as a prior over functions. The values of f() at any finite set of locations are jointly multivariate normal, with a covariance matrix defined by the covariance kernel $k_p(x_i, x_j)$, where p is the vector of parameters of the GP:

$$\left(f\left(x_{1}\right), \dots f\left(x_{n}\right) \sim \text{MVN}\left(0, \left(k_{p}\left(x_{i}, x_{j}\right)\right)_{i, j=1}^{n}\right)\right)$$

The smoothness and general behaviour of the function f depends only on the choice of covariance kernel, which ensures that values that are close together in the input space will be mapped to similar output values...

From this perspective, f is a realisation of an infinite dimensional normal distribution:

$$f \sim \text{Normal}(0, C(\lambda))$$

where C is a covariance kernel with hyperparameters λ that defines the covariance between two function values $f(t_1)$ and $f(t_2)$ for two time points t_1 and t_2 (Rasmussen & Williams, 2005). Similar to the different choices of the basis function for splines, different choices of the covariance kernel lead to different GPs. In this article, we consider the squared-exponential (a.k.a. radial basis function) kernel, which computes the squared distance between points and converts it into a measure of similarity. It is defined as:

$$C(\lambda) := C(t_1, t_2, \sigma, \gamma) := \sigma^2 \exp\left(-\frac{||t_1 - t_2||^2}{2\gamma^2}\right)$$

with hyperparameters $\lambda = (\sigma, \gamma)$, expressing the overall scale of GP and the lengthscale, respectively (Rasmussen & Williams, 2005). The advantages of this kernel are that it is computationally efficient and (infinitely) smooth making it a reasonable choice for the purposes of the present article. Here again, λ hyperparameters are estimated from the data, along with all other model parameters.

Taken from https://michael-franke.github.io/Bayesian-Regression/practice-sheets/10c-Gaussian-processes.html: For a given vector \mathbf{x} , we can use the kernel to construct finite multi-variate normal distribution associated with it like so:

$$\mathbf{x} \mapsto_{GP} \text{MVNormal}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}))$$

where m is a function that specifies the mean for the distribution associated with \mathbf{x} . This mapping is essentially the Gaussian process: a systematic association of vectors of arbitrary length with a suitable multi-variate normal distribution.

See also https://jtimonen.github.io/lgpr-usage/articles/math.html...

Low-rank approximate Gaussian processes are of main interest in machine learning and statistics due to the high computational demands of exact Gaussian process models (Riutort-Mayol et al., 2023)...

Objectives

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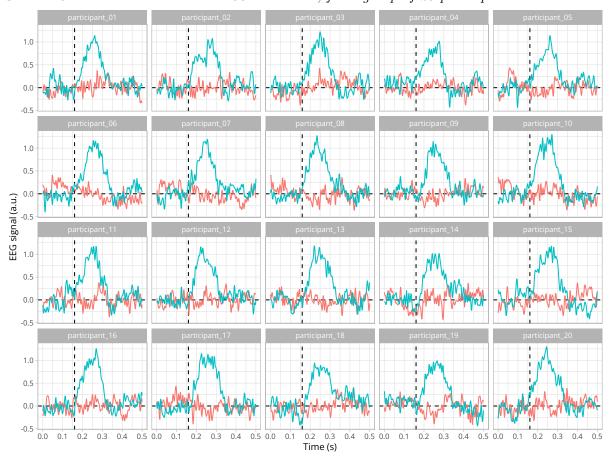
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Methods 79

M/EEG data simulation 80

Following the approach used by Sassenhagen & Draschkow (2019) and Rousselet (2025), we simulated EEG data stemming from two conditions, one with noise only, and the other with noise + signal. As in previous studies, the noise was generated by superimposing 50 sinusoids at different frequencies, following an EEG-like spectrum (see details and code in Yeung et al., 2004). As in Rousselet (2025), the signal was generated from truncated Gaussian with an objective onset at 160 ms, a peak at 250 ms, and an offset at 342 ms. We simulated this signal for 250 timesteps between 0 and 0.5s, akin to a 500 Hz sampling rate. We simulated such data for a group of 20 participants with 50 trials per participant and condition (Figure 1).

Figure 1 Some ERPs in two conditions with 50 trials each, for a group of 20 participants.



We computed the average of the ERP difference (Figure 2)...

Model fitting 90

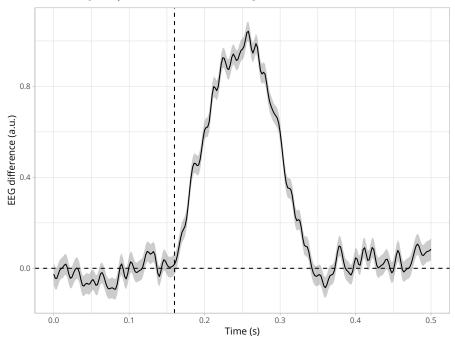
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We then fitted a GAM and GP regression model using the brms package (Bürkner, 2017, 2018; Nalborczyk et al., 2019)... 92

Figure 2

Group-level average difference between conditions (mean +/- standard error of the mean). The 'true' (population value of the) onset is indicated by the vertical dashed line.



```
# averaging across participants
ppt_df <- raw_df %>%
    group_by(participant, condition, time) %>%
    summarise(eeg = mean(eeg) ) %>%
    # mutate(eeg = as.numeric(scale(x = eeg, scale = FALSE) ) ) %>%
    ungroup()
# defining a contrast for condition
contrasts(ppt_df$condition) <- c(-0.5, 0.5)
# fitting the GAM
gam <- brm(</pre>
    # cubic regression splines with k-1 basis functions
    eeg ~ condition + s(time, bs = "cr", k = 10, by = condition),
    data = ppt_df,
    family = gaussian(),
    iter = 5000,
    chains = 4,
    cores = 4,
    file = "models/gam.rds"
```

Now we fit the GP...

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```
# computing eeg_diff and averaging across participants
# group_df <- raw_df %>%
# group_by(participant, time) %>%
```

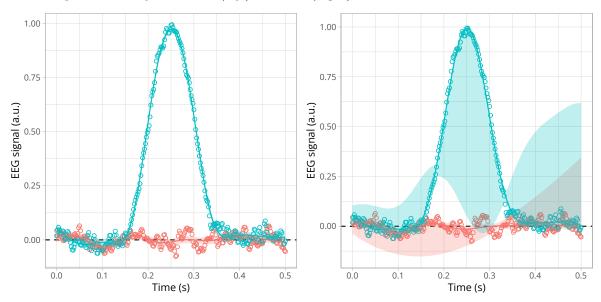
```
pivot_wider(names_from = condition, values_from = eeg, values_fn = mean) %>%
      mutate(eeg_diff = cond2 - cond1) %>%
#
      summarise(eeg_diff = mean(eeg_diff) ) %>%
#
      ungroup()
# see https://betanalpha.github.io/assets/case_studies/gp_part3/part3.html
# and https://betanalpha.github.io/assets/case_studies/gaussian_processes.html
gp_priors <- c(</pre>
    set_prior("normal(0, 1)", class = "Intercept"),
    set_prior("normal(0, 1)", class = "b"),
    set_prior("exponential(1)", class = "sigma"),
    set_prior("exponential(1)", class = "sdgp")
gp_model <- brm(</pre>
    # k refers to the number of basis functions for
    # computing Hilbert-space approximate GPs
    # if k = NA (default), exact GPs are computed
    # eeg ~ gp(time, by = condition),
    eeg ~ condition + gp(time, by = condition, k = 20, cov = "exp_quad"),
   data = ppt_df,
   family = gaussian(),
    # prior = gp_priors,
    control = list(adapt_delta = 0.99, max_treedepth = 15),
    backend = "cmdstanr",
    iter = 2000,
    chains = 4,
    cores = 4,
    file = "models/gp.rds"
```

And we plot the posterior predictions...

```
# plotting the posterior predictions
plot_post_preds(model = gam) + plot_post_preds(model = gp_model)
```

Figure 3

Posterior predictions of the GAM (left) and GP (right) models.



Posterior probability of difference above 0

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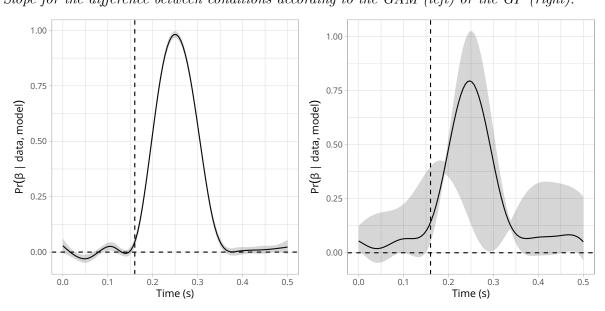
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We can retrieve the posterior probability of the slope for condition at each timestep.

Figure 4

Slope for the difference between conditions according to the GAM (left) or the GP (right).



We can also compute the posterior probability of the slope for condition being above 0 (Figure 5).

We can also express this as the ratio of posterior probabilities (i.e., p/(1-p)) and visualise the timecourse of this ratio superimposed with the conventional BF thresholds (Figure 6). Also check this paper (https://link.springer.com/article/10.3758/s13428-021-01613-6? fromPaywallRec=false)?

Figure 5

Posterior probability of the EEG signal being above 0+eps according to the GAM (left) and the GP (right).

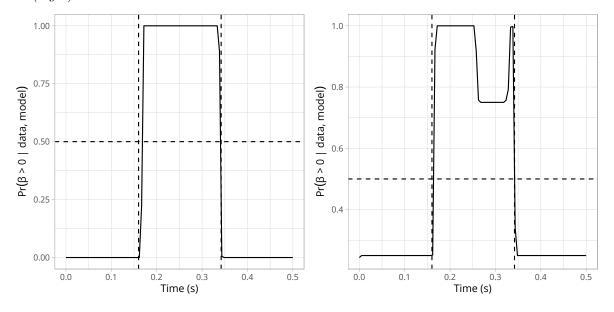


Figure 6

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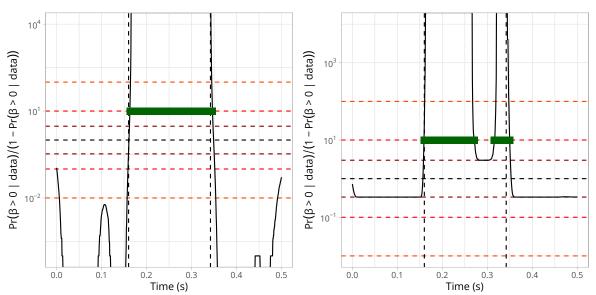
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Ratio of posterior probability according to the GAM (left) and the GP (right). Timesteps above threshold (>10) are highlighted in green.



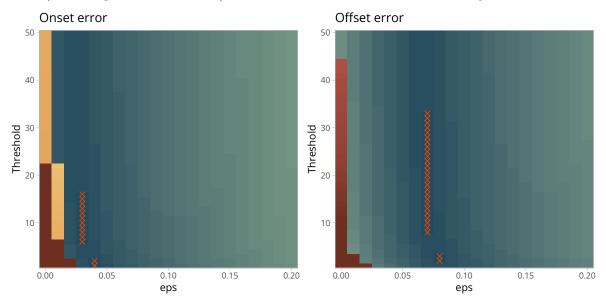
103 Error properties of the proposed approach

Below we are plotting the error function (where the error is computed as error := $|\hat{\theta} - \theta|$) for both the onset and offset values of the ERP difference, according to various eps and threshold values. Remember that the signal is generated from a truncated Gaussian defining an objective onset at 160 ms, a maximum at 250 ms, and an offset at 342 ms. Figure 7 shows that the GP model can exactly recover the true onset and offset values, given some reasonable choice of eps and threshold values.

110	eps	${\tt threshold}$	${\tt estimated_onset}$	${\tt estimated_offset}$	$\verb error_onset $	error_offset		
111	1 0.03	6	0.16	0.351	0	0.009		
112	2 0.03	7	0.16	0.351	0	0.009		
113	3 0.03	8	0.16	0.351	0	0.009		
114	4 0.03	9	0.16	0.351	0	0.009		
115	5 0.03	10	0.16	0.351	0	0.009		
116	6 0.03	11	0.16	0.351	0	0.009		
117	eps	threshold	${\tt estimated_onset}$	${\tt estimated_offset}$	$\verb error_onset $	error_offset		
118	1 0.07	8	0.166	0.342	0.006	0		
119	2 0.07	9	0.166	0.342	0.006	0		
120	3 0.07	10	0.166	0.342	0.006	0		
121	4 0.07	11	0.166	0.342	0.006	0		
122	5 0.07	12	0.166	0.342	0.006	0		
123	6 0.07	13	0.166	0.342	0.006	0		
124	[1] eps	5	threshold	estimated_or	nset estimat	ted_peak		
125	[5] estimated_offset error_onset error_peak error_offset							
126	<pre><0 rows> (or 0-length row.names)</pre>							

Figure 7

Error function of onset (left) and offset (right) estimation according to various eps and threshold values (according to the GP model). Minimum error values are indicated by red crosses.



Using summary statistics of ERPs

Next we fit a hierarchical GAM using summary statistics of ERPs (mean and SD) at the participant level (similar to what is done in meta-analysis).

```
# averaging across participants
summary_df <- raw_df %>%
   summarise(
        eeg_mean = mean(eeg),
        eeg_sd = sd(eeg),
        .by = c(participant, condition, time)
# defining a contrast for condition
contrasts(summary_df$condition) <- c(-0.5, 0.5)</pre>
# fitting the GAM
meta_gam <- brm(</pre>
    # using by-participant SD of ERPs across trials
    eeg_mean | se(eeg_sd) ~
        condition + s(time, bs = "cr", k = 10, by = condition) +
        (1 | participant),
    data = summary_df,
    family = gaussian(),
    iter = 5000,
    chains = 4,
    cores = 4,
    file = "models/meta_gam.rds"
```

```
# fitting the GP
meta_gp <- brm(
    # using by-participant SD of ERPs across trials
    eeg_mean | se(eeg_sd) ~
        condition + gp(time, k = 20, by = condition) +
        (1 | participant),
    data = summary_df,
    family = gaussian(),
    control = list(adapt_delta = 0.99),
    iter = 5000,
    chains = 4,
    cores = 4,
    file = "models/meta_gp.rds"
    )</pre>
```

```
# plotting the posterior predictions
meta_gam_preds <- plot(
    conditional_effects(x = meta_gam, effect = "time:condition"),
    points = FALSE, theme = theme_light(), plot = FALSE
    )[[1]] +
    labs(x = "Time (s)", y = "EEG signal (a.u.)")
meta_gp_preds <- plot(</pre>
```

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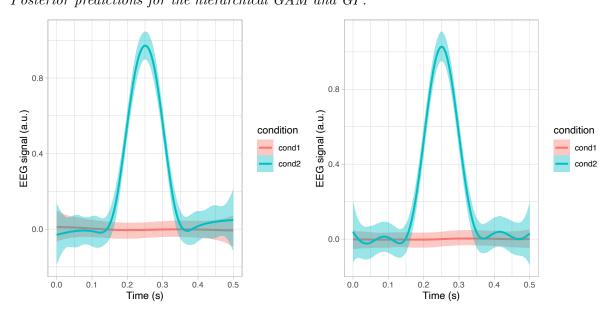
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```
conditional_effects(x = meta_gp, effect = "time:condition"),
  points = FALSE, theme = theme_light(), plot = FALSE
  )[[1]] +
  labs(x = "Time (s)", y = "EEG signal (a.u.)")

# combining the two plots
meta_gam_preds + meta_gp_preds
```

Figure 8

Posterior predictions for the hierarchical GAM and GP.



Application to time-resolved decoding results (accuracy over time)

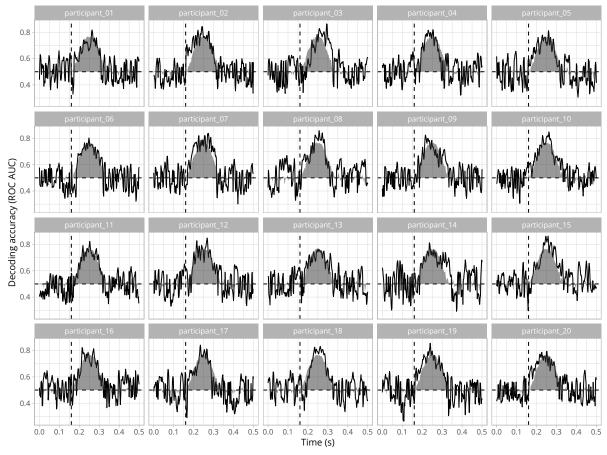
We conducted time-resolved multivariate pattern analysis (MVPA), also known as decoding. As a result, we have a timecourse of decoding accuracies (e.g., ROC AUC), bounded between 0 and 1, per participant (Figure 9)...

Now, we want to test whether the group-level average decoding accuracy is above chance (i.e., 0.5) at each timestep. We use a similar GAM/GP as previously, but we replace the Normal likelihood function by a Beta one to account for the bounded nature of AUC values (between 0 and 1).

```
# fitting the GAM
decoding_gam <- brm(
    auc ~ s(time, bs = "cr", k = 10),
    data = decoding_data,
    family = Beta(),
    iter = 5000,
    chains = 4,
    cores = 4,
    file = "models/decoding_gam.rds"
    )</pre>
```

Figure 9

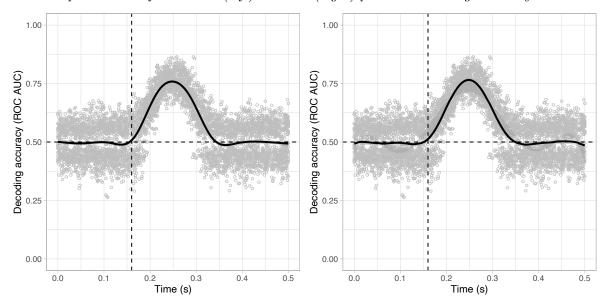
Exemplary average timecourse of binary decoding accuracy (ROC AUC) for each participant. Group-level average decoding accuracy is depicted as a grey background density in each panel.



```
# fitting the GP
decoding_gp <- brm(
    auc ~ gp(time, k = 20),
    data = decoding_data,
    family = Beta(),
    control = list(adapt_delta = 0.99),
    iter = 5000,
    chains = 4,
    cores = 4,
    file = "models/decoding_gp.rds"
    )</pre>
```

Figure 10

Posterior predictions of the GAM (left) and GP (right) fitted on decoding accuracy over time.



Next, we plot the posterior probability of decoding accuracy being above chance level (plus some epsilon) (Figure 11).

Figure 11

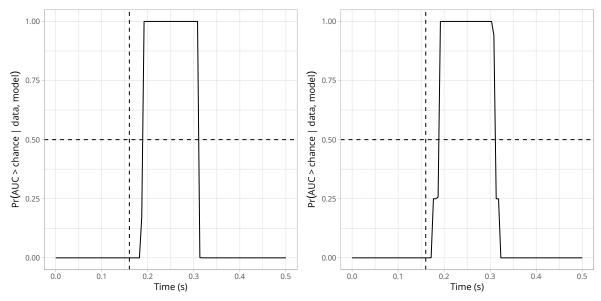
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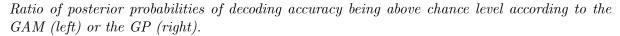
Posterior probability of decoding accuracy being above chance level according to the GAM (left) or the GP (right).

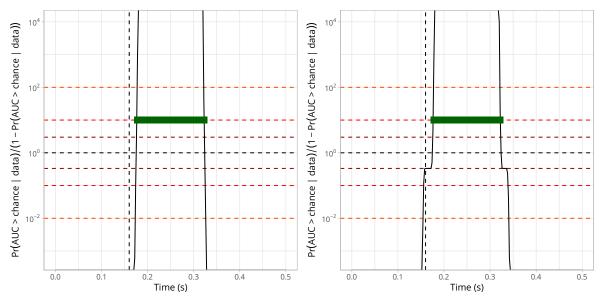


Application to actual M/EEG data

Assessing the reliability of the proposed approach (in comparison to other methods) using some sort of split-half reliability (Rosenblatt et al., 2018)?

Figure 12





143 Comparing the identified onsets/offsets to other approaches

From Rousselet (2024): Five methods for multiple comparison correction were considered: two FDR methods; two cluster-based methods; and the maximum statistics. The two FDR methods were BH95 (Benjamini & Hochberg, 1995) and BY01 (Benjamini & Yekutieli, 2001), which were applied to the permutation p-values using the p.adjust function in R. The first cluster-based inference was implemented using a cluster-sum statistic of squared t-values...

Comparing performance of the GAM and GP models with:

- uncorrected univariate tests
- univariate tests + Bonferroni/Holm
- univariate tests + BH95 (Benjamini-Hochberg, 1995) aka FDR
- univariate tests + BY01 (Benjamini-Yekutieli, 2001)
- cluster-based permutation test (cluster sum)
- cluster-based permutation test (cluster depth)
- threshold-free cluster enhancement
- change point

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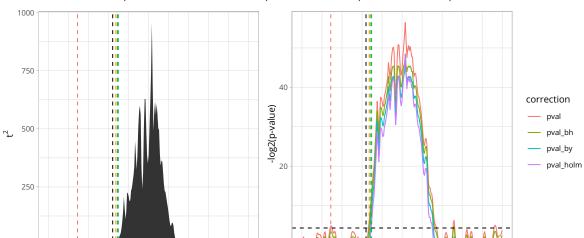
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- We used the R package changepoint v 2.2.4 (Killick et al., 2022a)...
- S-values (Greenland, 2019)...

Figure 13

Timecourse of squared t-values and s-values (continuous measure of evidence given by -log2(p-value)) with true onset and onset identified using the raw (uncorrected) p-values or the corrected p-values (BH, BY, Holm).



True onset: 0.16s, Uncorrected onset: 0.072s, BH onset: 0.168s, BY onset: 0.172s, Holm onset: 0.174s

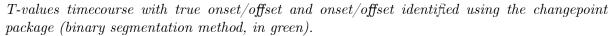
Figure 14

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0.0

0.1



0.0

0.1

0.3

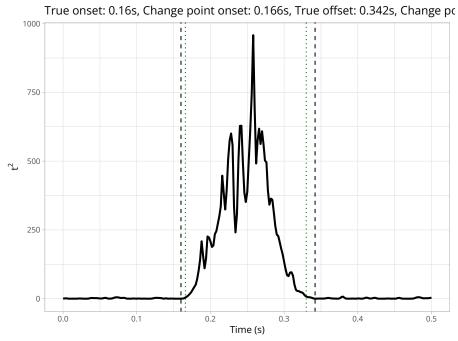
Time (s)

0.5

0.4

Time (s)

0.5

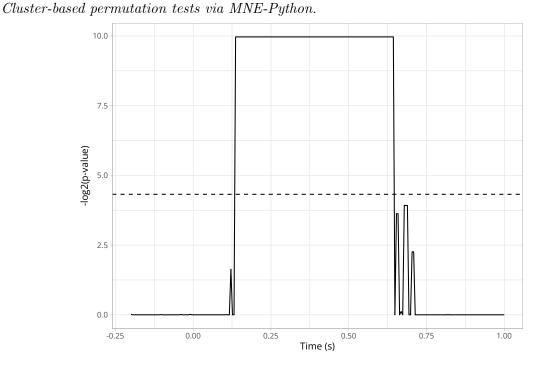


Now cluster-based permutations using MNE-Python (Gramfort, 2013) via the R package reticulate v 1.35.0 (Ushey et al., 2024)...

```
# importing the decoding scores
p_values_df <- readRDS(file = "results/mne_permutation_decoding_scores.rds")

# plotting the s-values (-log2(p-values))
p_values_df %>%
    mutate(sval = -log2(pval) ) %>%
    ggplot(aes(x = time, y = sval) ) +
    geom_line(linewidth = 0.5) +
    geom_hline(yintercept = -log2(0.05), linetype = 2) +
    labs(
        x = "Time (s)",
        y = "-log2(p-value)"
        )
```

Figure 15



162 Results

163 ...

Discussion 164 165 Summary of the proposed approach 166 167 Increasing potential usage 168 Prepare a wrapper R package and show how to call it in Python and integrate it with MNE-Python (Gramfort, 2013) pipelines... 170 Limitations and future directions 171 Extending the approach to spatiotemporal data (i.e., time + sensors)... 172 Conclusions 173 174

Packages

We used R version 4.2.3 (R Core Team, 2023) and the following R packages: brms v. 2.22.0 (Bürkner, 2017, 2018, 2021), changepoint v. 2.2.4 (Killick et al., 2022b; Killick & Eckley, 2014), grateful v. 0.2.10 (Rodriguez-Sanchez & Jackson, 2023), knitr v. 1.45 (Xie, 2014, 2015, 2023), MetBrewer v. 0.2.0 (Mills, 2022), pakret v. 0.2.2 (Gallou, 2024), patchwork v. 1.2.0 (T. L. Pedersen, 2024), rmarkdown v. 2.29 (Allaire et al., 2024; Xie et al., 2018, 2020), scales v. 1.3.0 (Wickham et al., 2023), scico v. 1.5.0 (T. L. Pedersen & Crameri, 2023), tidybayes v. 3.0.6 (Kay, 2023), tidyverse v. 2.0.0 (Wickham et al., 2019).

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Appendix A

Application to 2D time-resolved decoding results (cross-temporal generalisation)

Assume we have M/EEG data and we have conducted cross-temporal generalisation analyses (King & Dehaene, 2014). As a result, we have a 2D matrix where each element contains the decoding accuracy (e.g., ROC AUC) of a classifier trained at timestep training_i and tested at timestep testing_i (Figure A1).

Figure A1

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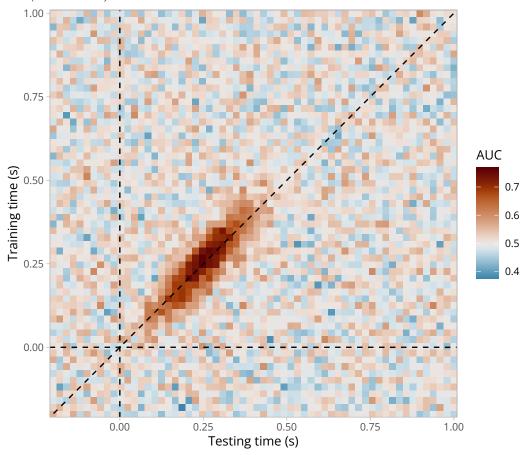
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Exemplary (simulated) group-level average cross-temporal generalisation matrix of decoding performance (ROC AUC).



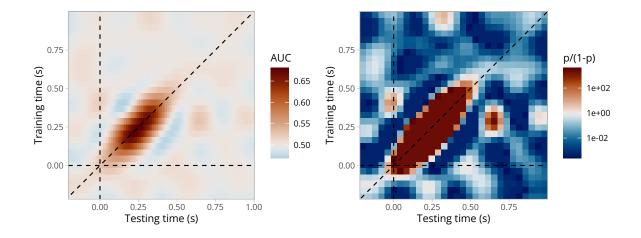
Now, we want to test whether and when decoding performance is above chance level (0.5 for a binary decoding task). These two models are computationally heavier to fit (more observations and 2D smooth functions)...

```
# fitting a GAM with two temporal dimensions
timegen_gam <- brm(
    # 2D thin-plate spline (tp)
    # auc ~ s(train_time, test_time, bs = "tp", k = 10),
    auc ~ t2(train_time, test_time, bs = "tp", k = 10),
    data = timegen_data,
    family = Beta(),
    iter = 2000,
    chains = 4,
    cores = 4,
    file = "models/timegen_gam_t2.rds"</pre>
```

```
# fitting a GP with two temporal dimensions
# timegen_gp <- brm(</pre>
      auc ~ gp(train_time, test_time, k = 20),
#
#
      data = timegen_data,
#
      family = Beta(),
      control = list(adapt_delta = 0.95),
#
#
      iter = 2000,
#
      chains = 4,
      cores = 4,
#
      file = "models/timegen_gp.rds"
```

Figure A2

Posterior probability of decoding accuracy being above chance level (2D GAM).



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Appendix B

Mathematical formulation of the bivariate GAM

To model cross-temporal generalisation matrices of decoding performance (ROC AUC), we extended the initial (decoding) GAM to take into account the bivariate temporal distribution of AUC values, thus producing naturally smoothed estimates (timecourses) of AUC values and posterior probabilities. This model can be written as follows:

$$AUC_i \sim Beta(\mu_i, \phi)$$

 $g(\mu_i) = f(train_i, test_i)$

where we assume that AUC values come from a Beta distribution with two parameters μ and ϕ . We can think of f (train_i, test_i) as a surface (a smooth function of two variables) that we can model using a 2-dimensional splines. Let $\mathbf{s}_i = (\text{train}_i, \text{test}_i)$ be some pair of training and testing samples, and let $\mathbf{k}_m = (\text{train}_m, \text{test}_m)$ denote the m^{th} knot in the domain of train_i and test_i. We can then express the smooth function as:

$$f(\text{train}_i, \text{test}_i) = \alpha + \sum_{m=1}^{M} \beta_m b_m \left(\tilde{s}_i, \tilde{k}_m\right)$$

Note that $b_m(,)$ is a basis function that maps $R \times R \to R$. A popular bivariate basis function uses *thin-plate splines*, which extend to $\mathbf{s}_i \in \mathbb{R}^d$ and ∂l_g penalties. These splines are designed to interpolate and approximate smooth surfaces over two dimensions (hence the "bivariate" term). For d=2 dimensions and l=2 (smoothness penalty involving second order derivative):

$$f(\tilde{s}_i) = \alpha + \beta_1 x_i + \beta_2 z_i + \sum_{m=1}^{M} \beta_{2+m} b_m \left(\tilde{s}_i, \tilde{k}_m\right)$$

using the the radial basis function given by:

$$b_m\left(\tilde{s}_i, \tilde{k}_m\right) = \left\|\tilde{s}_i - \tilde{k}_m\right\|^2 \log \left\|\tilde{s}_i - \tilde{k}_m\right\|$$

where $\|\mathbf{s}_i - \mathbf{k}_m\|$ is the Euclidean distance between the covariate \mathbf{s}_i and the knot location \mathbf{k}_m .

Appendix C

Threshold-free cluster enhancement

Cluster-based permutation approaches require defining a cluster-forming threshold (e.g., a tor f-value) as the initial step of the algorithm. As different cluster-forming thresholds lead to clusters with different spatial or temporal extent, this threshold modulates the sensitivity of the subsequent permutation test. The threshold-free cluster enhancement method (TFCE) was introduced by Smith & Nichols (2009) to overcome this arbitrary threshold.

In brief, the TFCE method works as follows. Instead of picking an arbitrary cluster-forming threshold (e.g., t=2), we try all (or many) possible thresholds in a given range and check whether a given timestep/voxel belongs to a significant cluster under any of the set of thresholds... Then, instead of using cluster mass, we use a weighted average between the cluster extend (e, how broad is the cluster, that is, how many connected samples it contains) and the cluster height (h, how high is the cluster, that is, how large is the test statistic) according to the formula:

$$TFCE = \int_{h} e(h)^{E} h^{H} dh$$

Where... the parameters E and H are set a priori and control the influence of the extend and height on the TFCE. Then, p-value for timestep/voxel i is computed by comparing it TFCE with the null distribution of TFCE values. For each permuted signal, we keep the maximal value over the whole signal for the null distribution of the TFCE.... But see Sassenhagen & Draschkow (2019)...

Appendix D

Using the R package and integration with MNE-Python

Explain how to use the R package and to integrate it with MNE epochs...

to-do adding some code here...