This is a function to estimate the distance from a CY with a defining polynomial f, to the discriminant locus.
The method is to find the discrete solutions x_i of grad f=0 in each patch.
Then find the minimal normalized distance between one of these points and the manifold f=0, by constrained minimization of d(x,y)^2.
The first routine uses the Euclidean distance in the patch,
and the second one uses the distance in P^n, which is cos^{-1} |z_1 \cdot \bar z_2|/|z_1|/|z_2|.
The second one is better motivated but somewhat slower.

Note that we have to run this once for each independent patch. Often symmetry will relate the results in different patches in which case we need not run it on all the patches.
For the function f, symmetry relates all patches.
For f2, symmetry relates (z1,z2,z5) and (z3,z4), so we need two runs.
Should the result depend on patch or not??

In these runs, NMinimize is searching complex values of the coordinates.
Complex parameters might work but they are too slow.

In[206]:=
```
Zsubs = {z1 → x1 + I y1, z2 → x2 + I y2, z3 → x3 + I y3, z4 → x4 + I y4, z5 → x5 + I y5}
```

Out[206]=
```
{z1 → x1 + ⅈ y1, z2 → x2 + ⅈ y2, z3 → x3 + ⅈ y3, z4 → x4 + ⅈ y4, z5 → x5 + ⅈ y5}
```

In[208]:=
```
XYsubs[vars_] := Variables[Apply[Times, vars] /. Zsubs]
```

In[218]:=
```
EstimateDistanceInCPN[ f_, patch_, vars_] := Module[ {eqs, gradzero},
  eqs = Map[# == 0 &, Grad[f /. patch → 1 , vars ]];
  gradzero = DeleteDuplicates[Solve[eqs, vars]];
  dmin[sol_] := Norm[Map[(# /. sol) - # &, vars]];
  reim = ReIm[f /. patch → 1];
  MinimalBy[Map[NMinimize[{dmin[#], 0 ==  reim[[1]], 0 == reim[[2]]} /. Zsubs,
      XYsubs[vars]] &, gradzero], #[[1]] &]
 ]
```

In[227]:=
```
EstimateTrueDistanceInCPN[ f_, patch_, vars_] := Module[ {eqs, gradzero},
  eqs = Map[# == 0 &, Grad[f /. patch → 1 , vars ]];
  gradzero = DeleteDuplicates[Solve[eqs, vars]];
  dmin[sol_] := Abs[1 + Total[Map[# Conjugate[# /. sol] &, vars]]^2 /
     (1 + Norm[vars]^2) / (1 + Norm[Map[(# /. sol) &, vars]]^2);
  reim = ReIm[f /. patch → 1];
  res =
   MaximalBy[Map[NMaximize[{dmin[#], 0 ==  reim[[1]], 0 == reim[[2]] } /. Zsubs,
       XYsubs[vars]] &, gradzero], #[[1]] &];
  Map[{ArcCos[#[[1]]] / Pi, #[[2]]} &, res]
 ]
```

The conifold is psi=-5 in these conventions.

In[212]:=
```
f = z1^5 + z2^5 + z3^5 + z4^5 + z5^5 + psi z1 z2 z3 z4 z5
```

Out[212]=
```
z1^5 + z2^5 + z3^5 + z4^5 + psi z1 z2 z3 z4 z5 + z5^5
```

```
z1^5 + z2^5 + z3^5 + z4^5 + psi z1 z2 z3 z4 z5 + z5^5
```

In[219]:=
```
EstimateDistanceInCPN[f /. psi → 0, z1, {z2, z3, z4, z5}]
```

Out[219]=
```
{{1. + 0. i,
  {x2 → -6.69067 × 10^-9, y2 → -5.34941 × 10^-9, x3 → -7.2824 × 10^-9, y3 → -7.80349 × 10^-9,
   x4 → -5.43845 × 10^-9, y4 → -2.08305 × 10^-9, x5 → 0.809017, y5 → -0.587785}}}
```

In[220]:=
```
EstimateDistanceInCPN[f /. psi → -1, z1, {z2, z3, z4, z5}]
```

Out[220]=
```
{{0.845117, {x2 → 0.0600428, y2 → -0.195952, x3 → 0.80746, y3 → -0.587842,
   x4 → 0.066602, y4 → 0.193821, x5 → 0.0666019, y5 → 0.193821}}}
```

In[221]:=
```
EstimateDistanceInCPN[f /. psi → -2, z1, {z2, z3, z4, z5}]
```

Out[221]=
```
{{0.694836, {x2 → -0.361387, y2 → -0.23309, x3 → 0.155358, y3 → 0.400991,
   x4 → 0.782842, y4 → -0.589975, x5 → -0.333356, y5 → 0.271667}}}
```

In[222]:=
```
EstimateDistanceInCPN[f /. psi → -3, z1, {z2, z3, z4, z5}]
```

Out[222]=
```
{{0.517283, {x2 → 0.278849, y2 → 0.598793, x3 → 0.792343, y3 → 0.449458,
   x4 → -0.577566, y4 → -0.320533, x5 → 0.278849, y5 → 0.598793}}}
```

In[223]:=
```
EstimateDistanceInCPN[f /. psi → -4, z1, {z2, z3, z4, z5}]
```

Out[223]=
```
{{0.316066, {x2 → 0.848861, y2 → 0.217256, x3 → -0.662494, y3 → 0.481322,
   x4 → 0.468957, y4 → 0.740175, x5 → -0.662494, y5 → 0.481323}}}
```

In[224]:=
```
EstimateDistanceInCPN[f /. psi → -4.5, z1, {z2, z3, z4, z5}]
```

Out[224]=
```
{{0.210096, {x2 → -0.830711, y2 → 0.421781, x3 → -0.734027, y3 → -0.534293,
   x4 → 0.279694, y4 → -0.863735, x5 → 0.424037, y5 → 0.829532}}}
```

In[225]:=
```
EstimateDistanceInCPN[f /. psi → -4.9, z1, {z2, z3, z4, z5}]
```

Out[225]=
```
{{0.0902418 + 0. i, {x2 → -0.758187, y2 → 0.629592, x3 → -0.794325,
   y3 → 0.576321, x4 → -0.793576, y4 → 0.577352, x5 → 0.364484, y5 → -0.915634}}}
```

In[226]:=
```
EstimateDistanceInCPN[f /. psi → -6, z1, {z2, z3, z4, z5}]
```

Out[226]=
```
{{0.248543, {x2 → 0.442569, y2 → 1.36209, x3 → -0.929406, y3 → -0.675255,
   x4 → 1.1488, y4 → 1.71445 × 10^-6, x5 → 0.354997, y5 → 1.09258}}}
```

In[239]:=
```
EstimateDistanceInCPN[f /. psi → 0.5, z1, {z2, z3, z4, z5}]
```

Out[239]=
```
{{0.900093, {x2 → -0.0992015, y2 → 5.83743 × 10^-9, x3 → -0.030655, y3 → 0.0943463,
   x4 → -0.030655, y4 → 0.0943463, x5 → 0.809091, y5 → -0.587839}}}
```

In[240]:= **EstimateDistanceInCPN[f /. psi → 4, z1, {z2, z3, z4, z5}]**

Out[240]= $\{\{0.388713, \{x2 \to -0.749087, y2 \to 6.45915 \times 10^{-7}, x3 \to -0.231483, y3 \to -0.712424, x4 \to 0.95349, y4 \to 0.69275, x5 \to -0.231483, y5 \to -0.712424\}\}\}$

In[241]:= **EstimateDistanceInCPN[f /. psi → 6, z1, {z2, z3, z4, z5}]**

Out[241]= {{0.367686, {x2 → -0.350997, y2 → 1.08024, x3 → 1.25437, y3 → -0.911353, x4 → -0.350996, y4 → -1.08026, x5 → 0.918905, y5 → 0.667625}}}

In[139]:= **EstimateDistanceInCPN[f /. psi → 0.1 I, z1, {z2, z3, z4, z5}]**

Out[139]= $Aborted

In[228]:= **EstimateTrueDistanceInCPN[f /. psi → 0, z1, {z2, z3, z4, z5}]**

Out[228]= $\{\{0.333333, \{x2 \to 0.809017, y2 \to -0.587785, x3 \to -1.53187 \times 10^{-8}, y3 \to -1.42181 \times 10^{-8}, x4 \to 1.13199 \times 10^{-9}, y4 \to 1.83833 \times 10^{-9}, x5 \to -1.9778 \times 10^{-9}, y5 \to 1.02064 \times 10^{-8}\}\}\}$

In[229]:= **EstimateTrueDistanceInCPN[f /. psi → -1, z1, {z2, z3, z4, z5}]**

Out[229]= {{0.256394, {x2 → -0.293533, y2 → -0.183278, x3 → 0.345201, y3 → -0.02426, x4 → -0.265017, y4 → 0.222531, x5 → 0.809843, y5 → -0.582017}}}

In[230]:= **EstimateTrueDistanceInCPN[f /. psi → -2, z1, {z2, z3, z4, z5}]**

Out[230]= {{0.175661, {x2 → -0.504162, y2 → 0.302145, x3 → 0.773544, y3 → -0.616999, x4 → 0.585472, y4 → 0.0518976, x5 → 0.230281, y5 → -0.540779}}}

In[231]:= **EstimateTrueDistanceInCPN[f /. psi → -3, z1, {z2, z3, z4, z5}]**

Out[231]= {{0.113723, {x2 → -0.178552, y2 → 0.969644, x3 → -0.645955, y3 → 0.394783, x4 → -0.645955, y4 → 0.394783, x5 → 0.754637, y5 → 0.0602926}}}

In[232]:= **EstimateTrueDistanceInCPN[f /. psi → -4, z1, {z2, z3, z4, z5}]**

Out[232]= {{0.0662463, {x2 → -0.0392173, y2 → 0.98823, x3 → 0.319352, y3 → -0.829885, x4 → -0.690502, y4 → -0.560293, x5 → -0.746192, y5 → 0.483632}}}

In[233]:= **EstimateTrueDistanceInCPN[f /. psi → -4.5, z1, {z2, z3, z4, z5}]**

Out[233]= {{0.0432114, {x2 → 0.323483, y2 → 0.889801, x3 → 0.52969, y3 → 0.840393, x4 → 0.323478, y4 → 0.889803, x5 → -0.746293, y5 → 0.58261}}}

In[234]:= **EstimateTrueDistanceInCPN[f /. psi → -4.9, z1, {z2, z3, z4, z5}]**

Out[234]= {{0.0182487, {x2 → -0.808697, y2 → -0.570492, x3 → 0.992489, y3 → -0.10931, x4 → 0.318923, y4 → 0.936879, x5 → 0.318923, y5 → 0.936879}}}

In[235]:= **EstimateTrueDistanceInCPN[f /. psi → -5.5, z1, {z2, z3, z4, z5}]**

Out[235]= {{0.0247182, {x2 → -1.03767, y2 → -0.75391, x3 → -1.03767, y3 → -0.75391, x4 → -1.03767, y4 → 0.75391, x5 → -1.03767, y5 → 0.75391}}}

In[236]:=
```
EstimateTrueDistanceInCPN[f /. psi → -6, z1, {z2, z3, z4, z5}]
```

Out[236]=
$$\{\{0.0274821, \{x2 \to 1.44221, y2 \to -3.55397 \times 10^{-6}, x3 \to 1.44222, y3 \to 1.47179 \times 10^{-9}, x4 \to 1.44222, y4 \to 8.12741 \times 10^{-7}, x5 \to 1.44221, y5 \to 2.73253 \times 10^{-6}\}\}\}$$

In[243]:=
```
EstimateTrueDistanceInCPN[f /. psi → -10, z1, {z2, z3, z4, z5}]
```

Out[243]=
$$\{\{0.0211915, \{x2 \to 2.49353, y2 \to 9.30276 \times 10^{-11}, x3 \to 0.770544, y3 \to 2.37149, x4 \to 0.770544, y4 \to 2.37149, x5 \to -2.01731, y5 \to -1.46566\}\}, \{0.0211915, \{x2 \to 0.770544, y2 \to -2.37149, x3 \to -2.01731, y3 \to -1.46566, x4 \to -2.01731, y4 \to 1.46566, x5 \to 0.770544, y5 \to 2.37149\}\}, \{0.0211915, \{x2 \to 0.770544, y2 \to -2.37149, x3 \to -2.01731, y3 \to 1.46566, x4 \to -2.01731, y4 \to -1.46566, x5 \to 0.770544, y5 \to 2.37149\}\}, \{0.0211915, \{x2 \to 0.770544, y2 \to -2.37149, x3 \to 0.770544, y3 \to 2.37149, x4 \to -2.01731, y4 \to -1.46566, x5 \to -2.01731, y5 \to 1.46566\}\}\}$$

In[237]:=
```
EstimateTrueDistanceInCPN[f /. psi → 4, z1, {z2, z3, z4, z5}]
```

Out[237]=
$$\{\{0.0659886, \{x2 \to -1.08758, y2 \to -8.04811 \times 10^{-11}, x3 \to 0.879872, y3 \to -0.639264, x4 \to -0.336081, y4 \to 1.03435, x5 \to -0.377528, y5 \to 1.16191\}\}\}$$

In[238]:=
```
EstimateTrueDistanceInCPN[f /. psi → 6, z1, {z2, z3, z4, z5}]
```

Out[238]=
$$\{\{0.036718, \{x2 \to -0.477119, y2 \to -1.46842, x3 \to -1.54399, y3 \to -6.91824 \times 10^{-9}, x4 \to -1.54399, y4 \to 7.40304 \times 10^{-10}, x5 \to -0.477119, y5 \to 1.46842\}\}\}$$

In[242]:=
```
f2 = f + phi (z3 z4^4 + z3^2 z4^3 + z3^3 z4^2 + z3^4 z4)
```

Out[242]=
$$z1^5 + z2^5 + z3^5 + z4^5 + \text{phi}\left(z3^4\, z4 + z3^3\, z4^2 + z3^2\, z4^3 + z3\, z4^4\right) + \text{psi}\, z1\, z2\, z3\, z4\, z5 + z5^5$$

In[262]:=
```
Reduce[Append[ Map[# == 0 &, Grad[f2 /. psi → 0.5 , {z1, z2, z3, z4, z5} ]],
  (f2 == 0 /. psi → 0.5) && z1 == 1] , {z1, z2, z3, z4, z5}]
```

Out[262]=
$$\begin{aligned}
&\Big((\text{phi} == -0.5 - 0.00158114\, i \;||\; \text{phi} == -0.5 + 0.00158114\, i) \;\&\&\; z1 == 1. \;\&\&\\
&\quad (z2 == 1. \;||\; z2 == -0.809017 - 0.587785\, i \;||\; z2 == 0.309017 + 0.951057\, i \;||\\
&\qquad z2 == 0.309017 - 0.951057\, i \;||\; z2 == -0.809017 + 0.587785\, i) \;\&\&\\
&\quad \Big(z3 == 10.\, (-1. - 2.\, \text{phi})^{1/5} \;||\; z3 == (-8.09017 - 5.87785\, i)\, (-1. - 2.\, \text{phi})^{1/5} \;||\\
&\qquad z3 == (3.09017 + 9.51057\, i)\, (-1. - 2.\, \text{phi})^{1/5} \;||\\
&\qquad z3 == (3.09017 - 9.51057\, i)\, (-1. - 2.\, \text{phi})^{1/5} \;||\\
&\qquad z3 == (-8.09017 + 5.87785\, i)\, (-1. - 2.\, \text{phi})^{1/5}\Big) \;\&\&\\
&\quad z4 == -7.54119 \times 10^{-56}\, z3\, \Big(1.227 \times 10^{59} + 1.03285 \times 10^{55}\, \text{phi} - 2.45424 \times 10^{59}\, \text{phi}^2 +\\
&\qquad 3.33776 \times 10^{59}\, \text{phi}^3 - 2.46406 \times 10^{59}\, \text{phi}^4 + 1.15762 \times 10^{59}\, \text{phi}^5 -\\
&\qquad 3.58123 \times 10^{58}\, \text{phi}^6 + 6.91115 \times 10^{57}\, \text{phi}^7 - 6.28287 \times 10^{56}\, \text{phi}^8 + 3.12992 \times 10^{54}\, z3^5 -\\
&\qquad 1.03023 \times 10^{54}\, \text{phi}\, z3^5 - 8.48053 \times 10^{53}\, \text{phi}^2\, z3^5 + 7.98986 \times 10^{53}\, \text{phi}^3\, z3^5 -\\
&\qquad 3.10131 \times 10^{53}\, \text{phi}^4\, z3^5 + 2.12426 \times 10^{52}\, \text{phi}^5\, z3^5 + 9.42731 \times 10^{51}\, \text{phi}^6\, z3^5\Big) \;\&\&\\
&\quad z5 == 6.03295 \times 10^{-57}\, z2^4\, z3^3\, \Big(7.83746 \times 10^{57} + 4.01461 \times 10^{56}\, \text{phi} - 1.56511 \times 10^{58}\, \text{phi}^2 +\\
&\qquad 2.05072 \times 10^{58}\, \text{phi}^3 - 1.46762 \times 10^{58}\, \text{phi}^4 + 6.68662 \times 10^{57}\, \text{phi}^5 -\\
&\qquad 2.00436 \times 10^{57}\, \text{phi}^6 + 3.73471 \times 10^{56}\, \text{phi}^7 - 3.26036 \times 10^{55}\, \text{phi}^8 + 1.15203 \times 10^{53}\, z3^5 -
\end{aligned}$$

$$1.45129\times10^{53}\,\text{phi}\,z3^5+1.09538\times10^{53}\,\text{phi}^2\,z3^5-5.31283\times10^{52}\,\text{phi}^3\,z3^5+$$
$$1.70647\times10^{52}\,\text{phi}^4\,z3^5-3.44875\times10^{51}\,\text{phi}^5\,z3^5+3.2733\times10^{50}\,\text{phi}^6\,z3^5\Big)\Big)\;||$$

$$\Big((\text{phi}==2.54367\;||\;\text{phi}==0.995883-1.99903\,i\;||\;\text{phi}==0.995883+1.99903\,i\;||$$
$$\text{phi}==1.00412-2.00098\,i\;||\;\text{phi}==1.00412+2.00098\,i\;||$$
$$\text{phi}==2.47816-0.038147\,i\;||\;\text{phi}==2.47816+0.038147\,i)\;\&\&\;z1==1.\;\&\&$$
$$(z2==1.\;||\;z2==-0.809017-0.587785\,i\;||\;z2==0.309017+0.951057\,i\;||$$
$$z2==0.309017-0.951057\,i\;||\;z2==-0.809017+0.587785\,i)\;\&\&$$
$$\Big(z3==\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-$$
$$399.965\,\text{phi}^5-177.782\,\text{phi}^6-1.37443\times10^{-13}\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-$$
$$1.41476\times10^{17}\,\text{phi}-1.16207\times10^{17}\,\text{phi}^2+1.09538\times10^{17}\,\text{phi}^3-$$
$$4.25218\times10^{16}\,\text{phi}^4+2.91005\times10^{15}\,\text{phi}^5+1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||$$
$$z3==(-0.809017-0.587785\,i)\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-$$
$$15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-399.965\,\text{phi}^5-177.782\,\text{phi}^6-1.37443\times10^{-13}$$
$$\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-1.41476\times10^{17}\,\text{phi}-1.16207\times10^{17}\,\text{phi}^2+$$
$$1.09538\times10^{17}\,\text{phi}^3-4.25218\times10^{16}\,\text{phi}^4+2.91005\times10^{15}\,\text{phi}^5+$$
$$1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||\;z3==(0.309017+0.951057\,i)$$
$$\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-$$
$$399.965\,\text{phi}^5-177.782\,\text{phi}^6-1.37443\times10^{-13}\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-$$
$$1.41476\times10^{17}\,\text{phi}-1.16207\times10^{17}\,\text{phi}^2+1.09538\times10^{17}\,\text{phi}^3-$$
$$4.25218\times10^{16}\,\text{phi}^4+2.91005\times10^{15}\,\text{phi}^5+1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||$$
$$z3==(0.309017-0.951057\,i)\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-$$
$$15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-399.965\,\text{phi}^5-177.782\,\text{phi}^6-$$
$$1.37443\times10^{-13}\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-1.41476\times10^{17}\,\text{phi}-$$
$$1.16207\times10^{17}\,\text{phi}^2+1.09538\times10^{17}\,\text{phi}^3-4.25218\times10^{16}\,\text{phi}^4+$$
$$2.91005\times10^{15}\,\text{phi}^5+1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||$$
$$z3==(-0.809017+0.587785\,i)\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-$$
$$15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-399.965\,\text{phi}^5-177.782\,\text{phi}^6-$$
$$1.37443\times10^{-13}\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-1.41476\times10^{17}\,\text{phi}-$$
$$1.16207\times10^{17}\,\text{phi}^2+1.09538\times10^{17}\,\text{phi}^3-4.25218\times10^{16}\,\text{phi}^4+$$
$$2.91005\times10^{15}\,\text{phi}^5+1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||$$
$$z3==\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-15\,055.3\,\text{phi}^3+5844.32\,\text{phi}^4-$$
$$399.965\,\text{phi}^5-177.782\,\text{phi}^6+1.37443\times10^{-13}\sqrt{\Big(5.29365\times10^{30}+\Big(4.29467\times10^{17}-$$
$$1.41476\times10^{17}\,\text{phi}-1.16207\times10^{17}\,\text{phi}^2+1.09538\times10^{17}\,\text{phi}^3-$$
$$4.25218\times10^{16}\,\text{phi}^4+2.91005\times10^{15}\,\text{phi}^5+1.2935\times10^{15}\,\text{phi}^6\Big)^2\Big)}\Big)^{1/5}\;||$$
$$z3==(-0.809017-0.587785\,i)\Big(-59\,027.2+19\,444.9\,\text{phi}+15\,971.8\,\text{phi}^2-$$

$$15\,055.3\,\text{phi}^3 + 5844.32\,\text{phi}^4 - 399.965\,\text{phi}^5 - 177.782\,\text{phi}^6 +$$
$$1.37443\times10^{-13}\sqrt{\left(5.29365\times10^{30} + \left(4.29467\times10^{17} - 1.41476\times10^{17}\,\text{phi} -\right.\right.}$$
$$1.16207\times10^{17}\,\text{phi}^2 + 1.09538\times10^{17}\,\text{phi}^3 - 4.25218\times10^{16}\,\text{phi}^4 +$$
$$\left.\left.2.91005\times10^{15}\,\text{phi}^5 + 1.2935\times10^{15}\,\text{phi}^6\right)^2\right)^{1/5}\ ||$$

$$z3 == (0.309017 + 0.951057\,i)\left(-59\,027.2 + 19\,444.9\,\text{phi} + 15\,971.8\,\text{phi}^2 -\right.$$
$$15\,055.3\,\text{phi}^3 + 5844.32\,\text{phi}^4 - 399.965\,\text{phi}^5 - 177.782\,\text{phi}^6 +$$
$$1.37443\times10^{-13}\sqrt{\left(5.29365\times10^{30} + \left(4.29467\times10^{17} - 1.41476\times10^{17}\,\text{phi} -\right.\right.}$$
$$1.16207\times10^{17}\,\text{phi}^2 + 1.09538\times10^{17}\,\text{phi}^3 - 4.25218\times10^{16}\,\text{phi}^4 +$$
$$\left.\left.2.91005\times10^{15}\,\text{phi}^5 + 1.2935\times10^{15}\,\text{phi}^6\right)^2\right)^{1/5}\ ||$$

$$z3 == (0.309017 - 0.951057\,i)\left(-59\,027.2 + 19\,444.9\,\text{phi} + 15\,971.8\,\text{phi}^2 -\right.$$
$$15\,055.3\,\text{phi}^3 + 5844.32\,\text{phi}^4 - 399.965\,\text{phi}^5 - 177.782\,\text{phi}^6 +$$
$$1.37443\times10^{-13}\sqrt{\left(5.29365\times10^{30} + \left(4.29467\times10^{17} - 1.41476\times10^{17}\,\text{phi} -\right.\right.}$$
$$1.16207\times10^{17}\,\text{phi}^2 + 1.09538\times10^{17}\,\text{phi}^3 - 4.25218\times10^{16}\,\text{phi}^4 +$$
$$\left.\left.2.91005\times10^{15}\,\text{phi}^5 + 1.2935\times10^{15}\,\text{phi}^6\right)^2\right)^{1/5}\ ||$$

$$z3 == (-0.809017 + 0.587785\,i)\left(-59\,027.2 + 19\,444.9\,\text{phi} + 15\,971.8\,\text{phi}^2 -\right.$$
$$15\,055.3\,\text{phi}^3 + 5844.32\,\text{phi}^4 - 399.965\,\text{phi}^5 - 177.782\,\text{phi}^6 +$$
$$1.37443\times10^{-13}\sqrt{\left(5.29365\times10^{30} + \left(4.29467\times10^{17} - 1.41476\times10^{17}\,\text{phi} -\right.\right.}$$
$$1.16207\times10^{17}\,\text{phi}^2 + 1.09538\times10^{17}\,\text{phi}^3 - 4.25218\times10^{16}\,\text{phi}^4 +$$
$$\left.\left.\left.2.91005\times10^{15}\,\text{phi}^5 + 1.2935\times10^{15}\,\text{phi}^6\right)^2\right)^{1/5}\right)\ \&\&$$

$$z4 == -5.49755\times10^{-29}\,z3\left(-2.24833\times10^{28} + 1.59651\times10^{28}\,\text{phi} + 1.16331\times10^{27}\,\text{phi}^2 -\right.$$
$$1.096\times10^{27}\,\text{phi}^3 + 4.25418\times10^{26}\,\text{phi}^4 - 2.91392\times10^{25}\,\text{phi}^5 -$$
$$1.29318\times10^{25}\,\text{phi}^6 + 4.29343\times10^{27}\,z3^5 -$$
$$1.41321\times10^{27}\,\text{phi}\,z3^5 - 1.16331\times10^{27}\,\text{phi}^2\,z3^5 +$$
$$1.096\times10^{27}\,\text{phi}^3\,z3^5 - 4.25418\times10^{26}\,\text{phi}^4\,z3^5 +$$
$$\left.2.91392\times10^{25}\,\text{phi}^5\,z3^5 + 1.29318\times10^{25}\,\text{phi}^6\,z3^5\right)\ \&\&$$

$$z5 == 4.39804\times10^{-30}\,z2^4\,z3^3\left(9.96308\times10^{30} - 1.53441\times10^{31}\,\text{phi} +\right.$$
$$1.18957\times10^{31}\,\text{phi}^2 - 5.8015\times10^{30}\,\text{phi}^3 +$$
$$1.79503\times10^{30}\,\text{phi}^4 - 3.44687\times10^{29}\,\text{phi}^5 +$$
$$3.37513\times10^{28}\,\text{phi}^6 + 1.58028\times10^{26}\,z3^5 -$$
$$1.99079\times10^{26}\,\text{phi}\,z3^5 + 1.50258\times10^{26}\,\text{phi}^2\,z3^5 -$$
$$7.2878\times10^{25}\,\text{phi}^3\,z3^5 + 2.34082\times10^{25}\,\text{phi}^4\,z3^5 -$$
$$\left.4.73077\times10^{24}\,\text{phi}^5\,z3^5 + 4.4901\times10^{23}\,\text{phi}^6\,z3^5\right)\Big)$$

```
EstimateDistanceInCPN[f2 /. {psi → 0.5, phi → 1}, z1, {z2, z3, z4, z5}]
```

Out[178]= {{0.901534 + 0. i, {z2 → −0.999766, z3 → 0.149416, z4 → −0.125699, z5 → 0.131492}},
{0.901534 + 0. i, {z2 → −0.999766, z3 → 0.149416, z4 → −0.125699, z5 → 0.131492}}}

```
EstimateDistanceInCPN[f2 /. {psi → 0.5, phi → 1}, z4, {z1, z2, z3, z5}]
```

Out[181]= $\left\{\left\{0.584779 + 0.\,i, \left\{z1 \to -6.43016 \times 10^{-9}, z2 \to -6.61161 \times 10^{-9}, z3 \to -1., z5 \to -9.1173 \times 10^{-9}\right\}\right\}, \left\{0.584779 + 0.\,i, \left\{z1 \to -6.43016 \times 10^{-9}, z2 \to -6.61161 \times 10^{-9}, z3 \to -1., z5 \to -9.1173 \times 10^{-9}\right\}\right\}\right\}$

In[179]:=
```
EstimateDistanceInCPN[f2 /. {psi → 0.5, phi → 2}, z1, {z2, z3, z4, z5}]
```

Out[179]= {{0.835949 + 0. i, {z2 → -0.99723, z3 → 0.369146, z4 → -0.335788, z5 → 0.245961}}, {0.835949 + 0. i, {z2 → -0.99723, z3 → 0.369146, z4 → -0.335788, z5 → 0.245961}}}

In[182]:=
```
EstimateDistanceInCPN[f2 /. {psi → 0.5, phi → 2}, z4, {z1, z2, z3, z5}]
```

Out[182]= $\left\{\left\{0.357282 + 0.\,i, \left\{z1 \to -7.27525 \times 10^{-9}, z2 \to -7.18925 \times 10^{-9}, z3 \to -1., z5 \to -7.02754 \times 10^{-9}\right\}\right\}, \left\{0.357282 + 0.\,i, \left\{z1 \to -7.27525 \times 10^{-9}, z2 \to -7.18925 \times 10^{-9}, z3 \to -1., z5 \to -7.02754 \times 10^{-9}\right\}\right\}\right\}$

In[244]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 0}, z1, {z2, z3, z4, z5}]
```

Out[244]= {{0.291029, {x2 → 0.145636, y2 → 0.105811, x3 → -0.0556279, y3 → 0.171205, x4 → -0.0556279, y4 → -0.171205, x5 → 0.809397, y5 → -0.588061}}}

In[245]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 0}, z4, {z1, z2, z3, z5}]
```

Out[245]= {{0.291029, {x1 → 0.145636, y1 → 0.105811, x2 → -0.0556279, y2 → 0.171205, x3 → -0.0556279, y3 → -0.171205, x5 → 0.809397, y5 → -0.588061}}}

In[246]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 1}, z1, {z2, z3, z4, z5}]
```

Out[246]= {{0.274846, {x2 → -0.0685271, y2 → -0.217561, x3 → 0.00945118, y3 → -0.14496, x4 → 0.809252, y4 → 0.615961, x5 → -0.22809, y5 → -0.00205777}}}

In[247]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 1}, z4, {z1, z2, z3, z5}]
```

Out[247]= {{0.100177, {x1 → 0.291374, y1 → -0.119705, x2 → 0.291374, y2 → -0.119705, x3 → 0.500529, y3 → 0.866945, x5 → 0.203886, y5 → 0.240122}}}

In[248]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2}, z1, {z2, z3, z4, z5}]
```

Out[248]= {{0.202251, {x2 → 0.297598, y2 → -0.223127, x3 → 0.340795, y3 → 0.190015, x4 → -0.441519, y4 → -1.02765, x5 → -0.109612, y5 → 0.355438}}}

In[249]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2}, z4, {z1, z2, z3, z5}]
```

Out[249]= {{0.0608233, {x1 → -0.302668, y1 → -0.100814, x2 → 0.30412, y2 → -0.0963429, x3 → -0.812156, y3 → 0.590101, x5 → -0.18941, y5 → 0.256701}}}

In[250]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 3}, z1, {z2, z3, z4, z5}]
```

Out[250]= {{0.194815, {x2 → 0.101076, y2 → -0.311079, x3 → -0.330873, y3 → 1.01832, x4 → 0.0352418, y4 → -0.108466, x5 → -0.26462, y5 → -0.192257}}}

In[251]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 3}, z4, {z1, z2, z3, z5}]
```

Out[251]=
$$\{\{0.0538794, \{x1 \to 0.252789, y1 \to 0.183662, x2 \to -0.0965568, y2 \to 0.297171, x3 \to -0.993698, y3 \to -8.05418 \times 10^{-8}, x5 \to 0.252789, y5 \to 0.183662\}\}\}$$

In[263]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2.5}, z1, {z2, z3, z4, z5}]
```

Out[263]=
$$\{\{0.00234036, \{x2 \to -0.0540671, y2 \to -0.066603, x3 \to 0.427804, y3 \to 16.4797, x4 \to -0.427804, y4 \to -16.4797, x5 \to 0.0574509, y5 \to -0.0637082\}\}\}$$

In[264]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2.5}, z4, {z1, z2, z3, z5}]
```

Out[264]=
$$\{\{0. + 6.70788 \times 10^{-9}\, i, \{x1 \to -2.50986 \times 10^{-9}, y1 \to 5.76312 \times 10^{-10}, x2 \to 8.82588 \times 10^{-9}, y2 \to 7.42003 \times 10^{-10}, x3 \to -1., y3 \to -4.78238 \times 10^{-9}, x5 \to -1.34373 \times 10^{-9}, y5 \to -8.57799 \times 10^{-9}\}\}\}$$

In[276]:=
```
Reduce[Append[ Map[# == 0 &, Grad[f2 /. psi → 0.5 , {z1, z2, z3, z4, z5} ]],
  (f2 == 0 /. psi → 0.5) && z1 == 0 && z3 == 1] , {z1, z2, z3, z4, z5}]
```

Out[276]=
$$(phi == -0.5 \&\& z1 == 0 \&\& z2 == 0 \&\& z3 == 1. \&\& z4 == 1. \&\& z5 == 0) \,||\, \left((phi == 1. - 2.\, i \,||\, phi == 1. + 2.\, i \,||\, phi == 2.5) \&\& z1 == 0 \&\& z2 == 0 \&\& z3 == 1. \&\& \left(z4 == 0.05 \left(-5. - 1.\, phi - 2.\, phi^2 - 1. \sqrt{-400. + \left(5. + phi + 2.\, phi^2\right)^2}\right) \,||\, z4 == 0.05 \left(-5. - 1.\, phi - 2.\, phi^2 + \sqrt{-400. + \left(5. + phi + 2.\, phi^2\right)^2}\right)\right) \&\& z5 == 0\right)$$

In[277]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2.4}, z4, {z1, z2, z3, z5}]
```

Out[277]=
$$\{\{0.025252, \{x1 \to 0.140666, y1 \to -0.286745, x2 \to -0.229243, y2 \to -0.222391, x3 \to -0.975724, y3 \to -0.284747, x5 \to -0.229243, y5 \to -0.222391\}\}\}$$

In[278]:=
```
EstimateTrueDistanceInCPN[f2 /. {psi → 0.5, phi → 2.6}, z4, {z1, z2, z3, z5}]
```

Out[278]=
$$\{\{0.0177361, \{x1 \to -0.295191, y1 \to -0.214469, x2 \to 0.364876, y2 \to 1.82726 \times 10^{-11}, x3 \to -1.30517, y3 \to -2.5996 \times 10^{-10}, x5 \to -0.295191, y5 \to 0.214469\}\}\}$$