Midterm Practice

midterm material

- Likelihood
  - Identify proportionality constants that can be excluded
- Cromwell's Rule
- Sufficient Statistics
- Data Generating Process
- Bias, Variance, Mean Squared Error
- Mixture Model
- Conjugate Prior
  - Pseudo-counts interpretations of conjugate priors
- Improper Priors
- Posterior Predictive Distribution
  - Integral definition involving likelihood and posterior (or prior)

Practice Problems

1. We are interested in the parameter λ of a Poisson(λ) distribution. We have a prior distribution for λ with density

$$p(\lambda) = \begin{cases} 0 & (\lambda < 0) \\ k_0 \lambda^3 e^{-\lambda} & (\lambda \geq 0) \end{cases}$$

   (a) Find the value of κ_0

   (b) Find the prior mean and prior standard eviation of λ.
   We now observe $y_1, ..., y_n$ which are independent observations from the Poisson(λ).

   (c) Write the likelihood

   (d) Write the posterior density of λ

   (e) Write the posterior mean of λ

2. In a fruit packaging factory apples are examined to see whether they are blemished. A sample of n apples is examined and, given the value of a parameter θ, representing the proportion of aples which are blemished, we regard y, the number of blemished apples in the sample, as an obervation from the binomial(n, θ) distribution. The chosen prior density for θ is

$$p(\theta) = \begin{cases} k_0 \left(20\theta(1-\theta)^3 + 1\right) & (0 \leq \theta \leq 1) \\ 0 & (\text{ otherwise }) \end{cases}$$

   We observe $n = 10$ apples and $y = 4$.

   (a) Find the likelihood function.

(b) Find the posterior density of θ.
(c) Find the posterior mean of θ.

3. In a medical experiment, patients with a chronic condition are asked to say which of two treatments, A, B, they prefer. (You may assume for the purpose of this question that every patient will express a preference one way or the other). Let the population proportion who prefer A be θ. We observe a sample of n patients. Given θ, the n responses are independent and the probability that a particular patient prefers A is θ. Our prior distribution for θ is a beta(a, a) distribution with a standard deviation of 0.25.

   (a) Find the value of a.
   (b) True or false: the 95% *prior* quantile interval is the same as the 95% prior HPD interval?
   (c) We observe n = 30 patients of whom 21 prefer treatment A. Find the posterior distribution of θ.
   (d) True or false: the 95% *posterior* quantile interval is the same as the 95% prior HPD interval?

4. An improper prior distribution (select all that are true):

   (a) can't be used for valid Bayesian inference
   (b) can only be used if the posterior distribution is integrable
   (c) is another name for the uniform prior distribution
   (d) integrates to infinity