

# **PSTAT 115: Bayesian Data Analysis**

**Professor Laura Baracaldo**

# Class Resources

## Required Textbook

- Bayes Rules: <https://www.bayesrulesbook.com/>

## Course Pages

- Nectir for course related questions and discussion:  
<https://app.nectir.io/us/ucsb/group/PSTAT115-Fall23>
- Gradescope: <https://www.gradescope.com/courses/640308>

# Grades

- 35% - expect approximately 4 homeworks
- 20% - Midterm (Nov 1st)
- 15% - Quizzes
- 30% - Final exam (Dec 6th)

# Homework

- There will be approximately 4 homeworks (35% of your grade total)
- You will typically have 2 weeks to complete the homework
- Homework turned in within 24 hrs after the deadline without prior approval will receive a 20% deduction
- Homework will not be accepted more than 24 hrs late.

# Homework submission format

- All code must be written to be reproducible in Rmarkdown
- All derivations can be done in any format of your choosing (latex, written by hand) but must be legible and *must be integrated into your Rmarkdown pdf*.
- Ask a TA *early* if you have problems regarding submissions.

# Software and Deliverables

## Software

- R (R studio)

## Homeworks submission format

- Electronic submission via Gradescope
- R markdown code
- Generated PDF file with eventual handwritten notes

# Labs and Quizzes

- There will be a handful of quizzes throughout the quarter.
- The quizzes will be on Canvas.
- You will have 10 minutes to take the quiz any time within a window of 12 hours.
- The quizzes will be given on lecture days
- There are no makeups, but the lowest quiz grade will be dropped from your final score.
- Quizzes will be multiple choice and will test your comprehension of the basic concepts.

# Class Policies

- All questions should be posted on nectir, *not by email* (unless they are personal or grade-related)



# RStudio Cloud Service

- Log on to [pstat115.lsit.ucsb.edu](https://pstat115.lsit.ucsb.edu)
  - Cloud based rstudio service
  - Log in with your UCSB NetID
- Use [Jupyter Hub](#) to sync new material (BOOKMARK THIS)
- Make sure you can write and compile an [R markdown](#) (Rmd) document online
- Text formatting is minimal but [syntax](#) is simple

# Markdown and mathematical formulas

The text inserted between two \$ signs will be interpreted as a Latex instruction, e.g.  $x$

Code	Rendered math
<code>\$x\$</code>	$x$
<code>\$\$\theta\$</code>	$\theta$
<code>\$x_i^2\$</code>	$x_i^2$
<code>\$\$\frac{1}{n}\sum_{i=1}^n x_i\$</code>	$\frac{1}{n} \sum_{i=1}^n x_i$
<code>\$\$\frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^2\$</code>	$\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$

Rmarkdown and Latex resources:

- [Introduction to RMarkdown](#)
- [Latex cheat sheet](#)
- [Introduction to Latex](#)

# Other R resources

- Cheatsheets: <https://www.rstudio.com/resources/cheatsheets/>
- *An Introduction to R* - Venables and Smith  
<http://cran.r-project.org/doc/manuals/R-intro.pdf>
- *Using R for Introductory Statistics* - John Verzani  
<http://cran.r-project.org/doc/contrib/Verzani-SimpleR.pdf>
- *R Markdown reference* - <https://www.rstudio.com/wp-content/uploads/2015/03/rmarkdown-reference.pdf>
- Probability cheatsheet in resources folder of cloud environment

# **What is Bayesian statistics?**

**What is the version of statistics  
you already know?**

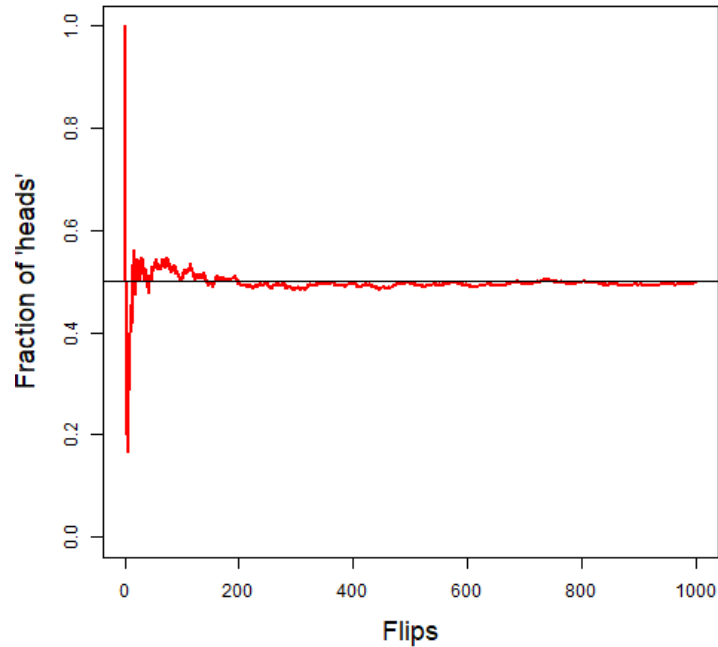
# Frequentist statistics

What you learned in PSTAT 120B

- Associated with the *frequentist* interpretation of probability
  - For any given event, only one of two possibilities may hold: it occurs or it does not.
  - The *frequency* of an event (in repeated experiments) is the *probability* of the event

# Frequentist probability

The probability of a coin landing on heads is 50%



The long run fraction of heads is 50%

# Frequentist statistics

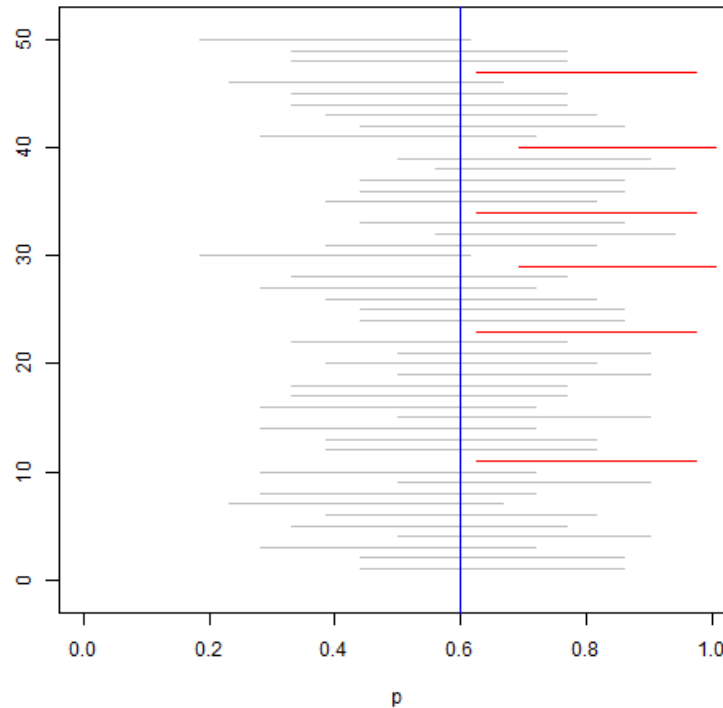
What you learned in PSTAT 120B

- Null Hypothesis Significant Testing (NHST) and Confidence Intervals
  - Frequentist uncertainty premised on imaginary resampling of data
  - Example: If the null model is true, and I re-run the experiment many times, how often will I reject?



# Confidence intervals

I have a 95% confidence interval for a parameter  $\theta$ . What does this mean?



We expect  $0.05 \times 50 = 2.5$  of the intervals to *not* cover the true parameter,  $p = 0.6$ , on average

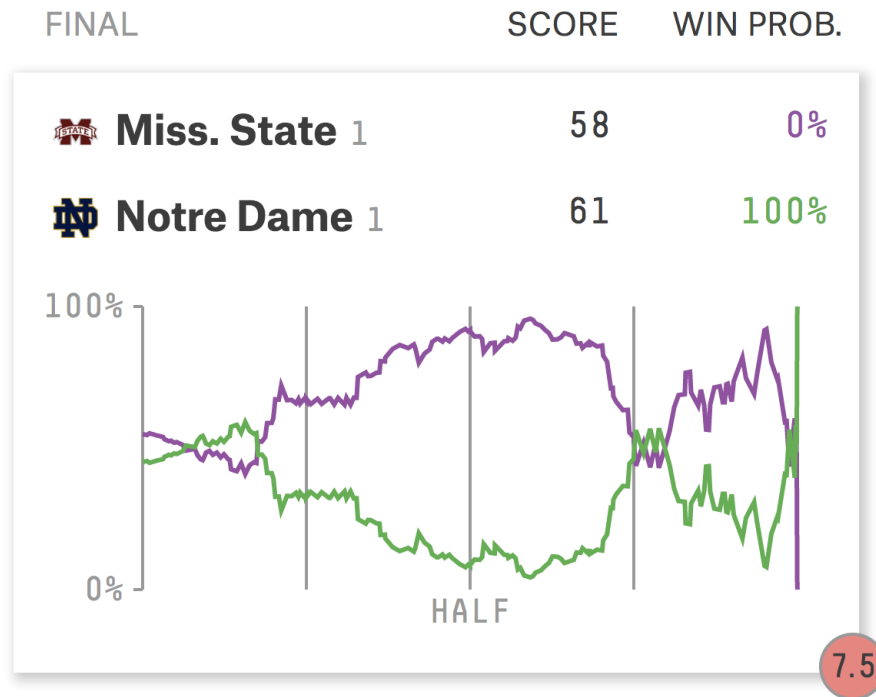
# Hypothesis Testing



$H_0$  : "All swans are white" vs  $H_A$  : "not all swans are white".

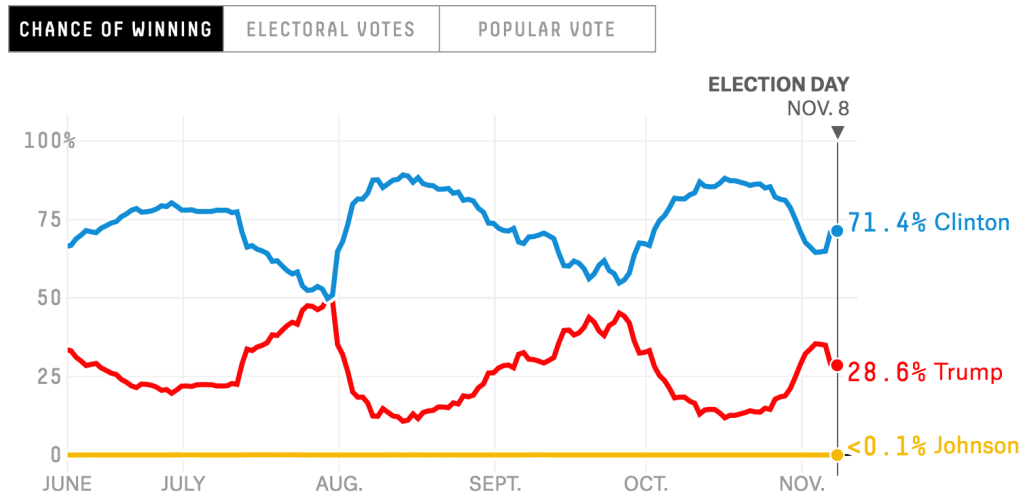
- What is the probability of rejecting the Null? -If we observe at least one black swan, we reject: Depends on how rare black swans are.
- A more complex null hypothesis: At least 50% of swans are black.

# Win probability



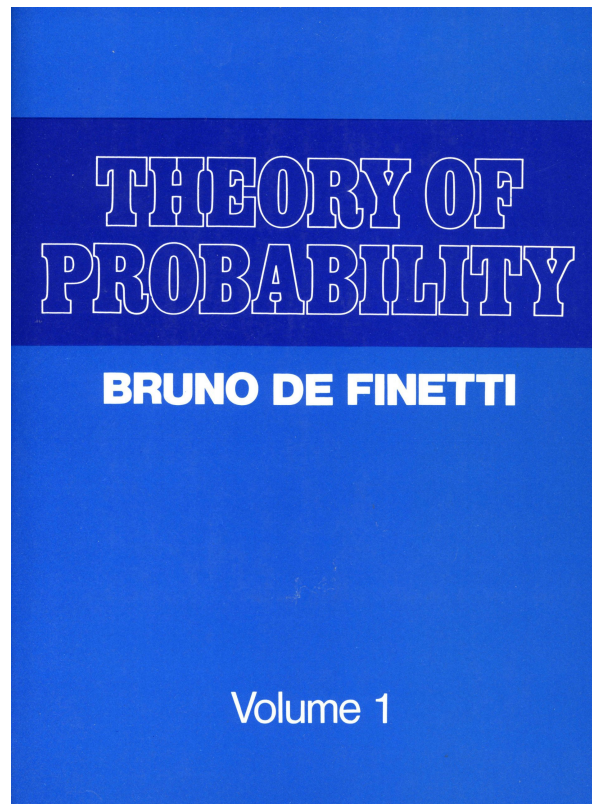
source: fivethirtyeight.com

# Win probability



source: fivethirtyeight.com

# Bayesian probability



Bruno de Finetti began his book on probability with:  
"PROBABILITY DOES NOT EXIST"

# Bayesian probability

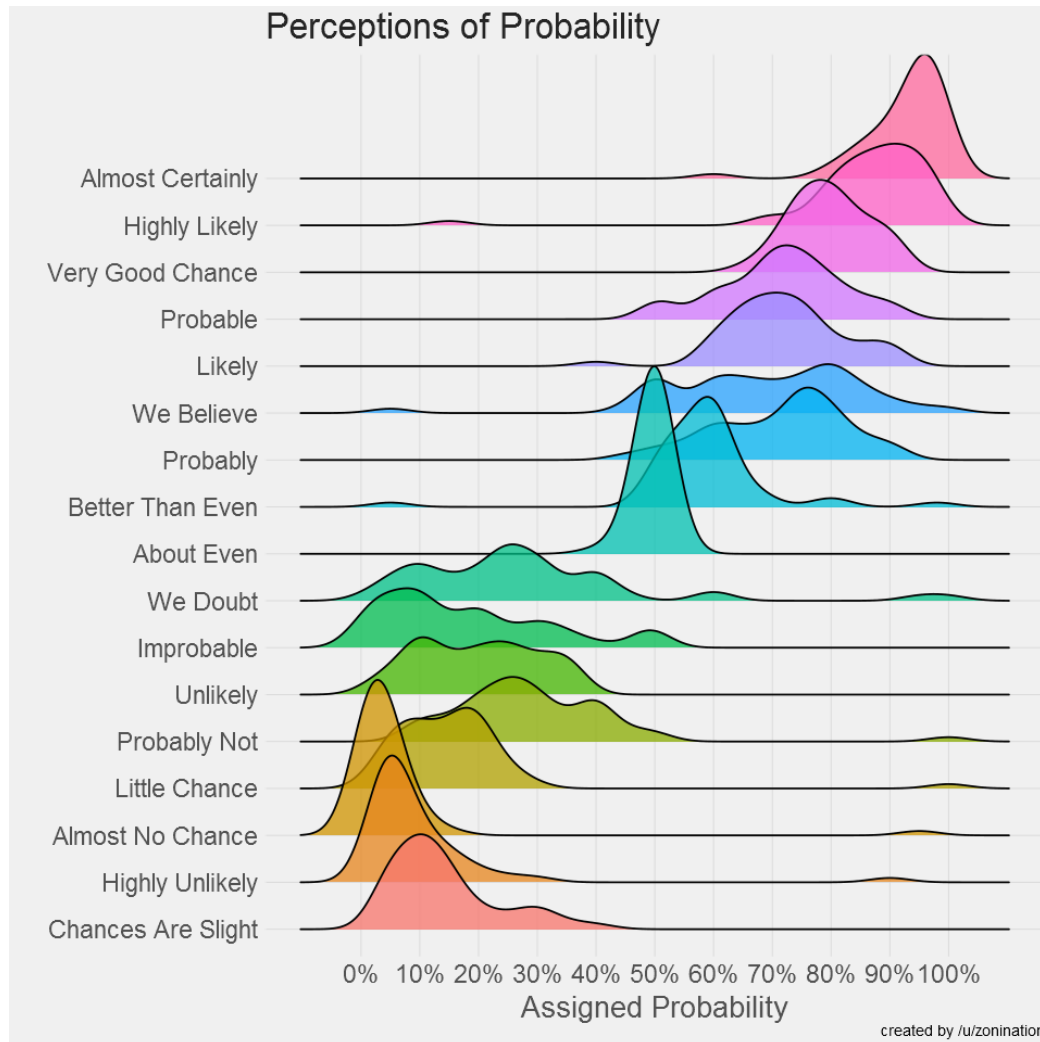
- de Finetti is arguing that probability is about *belief*
  - Probability doesn't exist in an *objective* sense
  - "The coin is fair" means *I believe* that its equally likely to be heads or tails.
  - "Hillary Clinton has a 71% chance to win" reflects a belief, since the election happens only once
- Rarely, if ever, get *true* replications to estimate frequentist probabilities
- Bayesian idea: focus statistical practice around belief about parameters

# Bayesian probability

"The terms *certain* and *probable* describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain. All propositions are true or false, but the knowledge we have of them depends on our circumstances

--- John M Keynes

# Perceptions of Probability



source: <https://github.com/zonination/perceptions>



# Why Bayesian statistics?

- Classic statistical toolbox may not be appropriate for all settings.
  - Inflexible and fragile
  - e.g. what if the assumptions of the test don't hold?
- Bayesian statistics provides a procedure for building our own tests / tools.
  - Design, build and refine procedures for you own models.
- A variety of powerful tools for inference with computer simulation
- Philosophy of science: quantifying degrees of belief often a more useful perspective than falsification

# Setup

- The *sample space*  $\mathcal{Y}$  is the set of all possible datasets.
  - $Y$  is a random variable with support in  $\mathcal{Y}$
  - We observe one dataset  $y$  from which we hope to learn about the world.
- The *parameter space*  $\Theta$  is the set of all possible parameter values  $\theta$
- $\theta$  encodes the population characteristics that we want to learn about!

# Three steps of Bayesian data analysis

1. Construct a plausible probability model governed by parameters  $\theta$ 
  - This includes specifying your belief about  $\theta$  before seeing data (*the prior*)
2. Condition on the observed data and compute *the posterior* distribution for  $\theta$
3. Evaluate the model fit, revise and extend. Then repeat.

# Bayesian Inference in a Nutshell

1. The *prior distribution*  $p(\theta)$  describes our belief about the true population characteristics, for each value of  $\theta \in \Theta$ .
2. Our *sampling model*  $p(y \mid \theta)$  describes our belief about what data we are likely to observe if  $\theta$  is true.
3. Once we actually observe data,  $y$ , we update our beliefs about  $\theta$  by computing *the posterior distribution*  $p(\theta \mid y)$ . We do this with Bayes' rule!

**Key difference:  $\theta$  is random!**

# Bayes' Rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- $P(A \mid B)$  is the conditional probability of A given B
- $P(B \mid A)$  is the conditional probability of B given A
- $P(A)$  and  $P(B)$  are called the marginal probability of A and B (unconditional)

# Bayes' Rule for Bayesian Statistics

$$P(\theta \mid y) = \frac{P(y \mid \theta)P(\theta)}{P(y)}$$

- $P(\theta \mid y)$  is the posterior distribution
- $P(y \mid \theta)$  is the likelihood
- $P(\theta)$  is the prior distribution
- $P(y) = \int_{\Theta} p(y \mid \tilde{\theta})p(\tilde{\theta})d\tilde{\theta}$  is the model evidence

# Bayes' Rule for Bayesian Statistics

$$P(\theta | y) = \frac{P(y | \theta)P(\theta)}{P(y)} \propto P(y | \theta)P(\theta)$$

- Start with a subjective belief (prior)
- Update it with evidence from data (likelihood)
- Summarize what you learn (posterior)

# Example: Estimating COVID Infection Rates

- We need to estimate the prevalence of a COVID in Isla Vista
- Get a small random sample of 20 individuals to check for infection

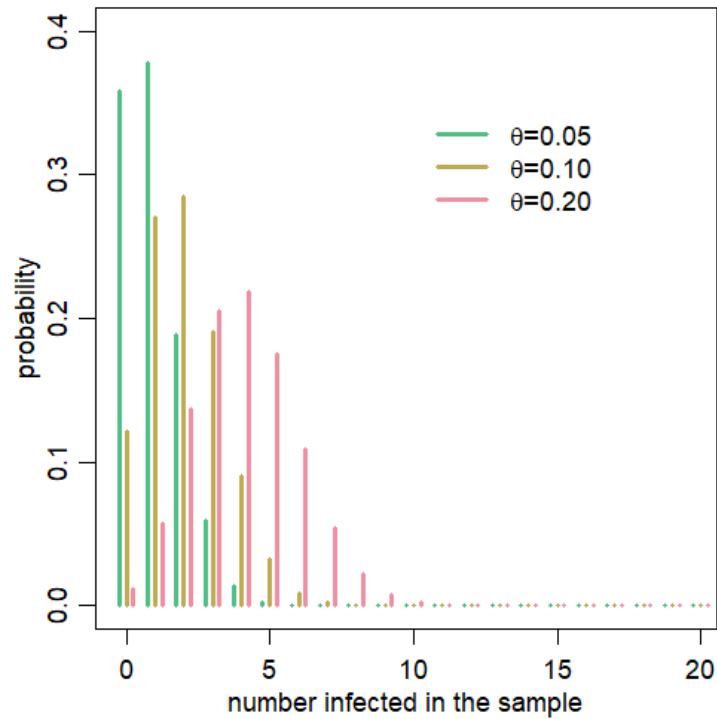




# Example: Estimating Infection Rates

- $\theta$  represents the population fraction of infected
- $Y$  is a random variable reflecting the number of infected in the sample
- $\Theta = [0, 1]$     $\mathcal{Y} = \{0, 1, \dots, 20\}$
- Sampling model:  $Y \sim \text{Binom}(20, \theta)$

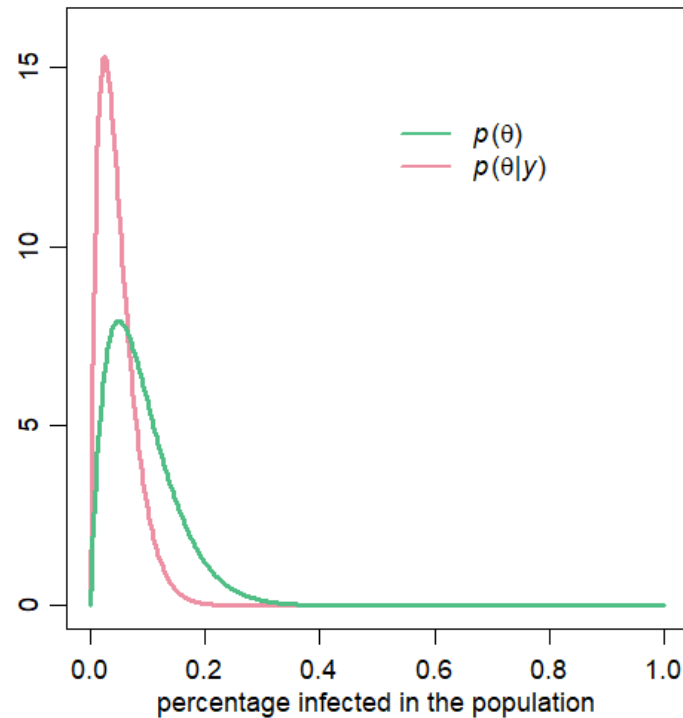
# Example: Estimating Infection Rates



# Example: Estimating Infection Rates

- Assume *a priori* that the population rate is low
  - The infection rate in comparable cities ranges from about 0.05 to 0.20
- Assume we observe  $Y = 0$  infected in our sample
- What is our estimate of the true population fraction of infected individuals?

# Example: Estimating Infection Rates



# Tentative syllabus

- One parameter models (binomial, poisson, and normal)
- Monte Carlo methods (i.e. simulation-based inference)
- Markov chain Monte Carlo (MCMC)
- Hierarchical modeling
- An introduction to probabilistic programming

# Assignment

- Check Nectir, bookmark important links
- Start reviewing probability cheat sheet!
- Read chapters 1 and 2 of Bayes Rules