Instituto Politécnico Nacional

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Artificial Life Robotics Laboratory

(ALIROB)

3D Cellular Automata And Pentominos

Simulations and configurations

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Contents

1 Brief summary

1.1 Cellular Automaton

For more information, please consult [1]–[6].

A cellular automaton is a discrete dynamical system that consists of a set of simple units (cells) that change their states at each time step, depending on the states of their neighbors (there are no actions at a distance), according to a local update rule; the cells evolve in parallel at discrete time steps and they all use the same update rule.

Also known as **CA**, the cellular automata have proof been quite useful, both as general models of complexity and as specific representations of non-linear dynamics in a great variety of fields. Despite functioning in a non-traditionally way, the **CA** can solve algorithmic problems or compute functions, all of this with the right suitable rule, of course.

Cellular automata come in a variety of shapes; the type of grid (for example, square, triangular or hexagonal cells in a two-dimensional **CA**) is one of it's most fundamental properties. The neighborhood over which cells affect one another is also very important; two common neighborhoods in the case of a two-dimensional **CA** on a square grid are the Moore and the von Newmann neighborhoods.

The **elementary cellular automata** are the simplest type of cellular automaton, being a binary, nearest-neighbor, one-dimensional automaton; these **CA** have been thoroughly studied. There are 256 **ECA**.

The best known cellular automaton is the *Game of life*, discovered by J. H. Conway in 1970; this is a binary automaton with a Moore neighborhood. Over the years, it has been compiled a library of patterns with various behaviors:

- **Still life**. A fixed point pattern; the update rule keeps each cell unchanged.
- **Oscillator**. A temporally periodic pattern; the update rule may change the pattern, but after some steps, the original pattern reappears in the same location with the same orientation.
- **Spaceship**. A pattern that reappears, after some steps, although not necessarily in the same location.
- **Gun**. A pattern that, as an oscillator, periodically returns back to the initial state and emits spaceships.
- **Glider gun**. A pattern that emits gliders.

1.1.1 The Diffusion Rule

Conway's *Game of Life* inspired the scientific community and soon they started to experiment with variations of the game's rules to find new and interesting behaviors, for example, *high life*, *life 43* and *long life*; later they started experimenting with new neighborhoods, different shapes in the grids and the number of dimensions in the universe. One rule stands out of the rest: the diffusion rule; mainly because it's behavior is chaotic and with it, there has been discovered plenty of gliders, oscillators, glider guns, and puffer trains. The rule is defined as follow:

- Any living cell with less than seven neighbors dies out of loneliness.
- Any living cell with more than seven neighbors dies suffocated.
- Any living cell with seven neighbors lives onto the next generation.
- Any death cell with two living neighbors will become alive in the next generation.

1.2 Pentominos

For more information, please consult [7]–[11].

A polyomino is a finite collection of orthogonally connected cells; or, in another words, a collection of equally-sized squares that form a connected piece, meaning that each square can reach any other in it by going through adjacent squares; the order of a polyomino is the number of squares used to make it, so a fifth order polyomino (made of five squares), is called pentomino. Both terms, polyomino and pentomino, were first used by S. Golomb in 1953, in a talk to the Harvard Mathematics Club and a year later in an article. There are twelve distinct pentominoes (see Figure 1) and two naming conventions, in this paper the Conway convention (using letters *O* through *Z*) will be used; although the resemblance to the letters with this labeling scheme seems a little more strained with the other scheme, especially when using the *O* instead of *I*, it has the advantage that uses 12 consecutive letters; also, the Conway scheme tends to be used when discussing topics related to cellular automata.

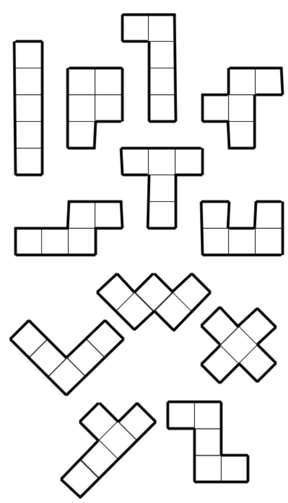

Figure 1: Conway pentominoes from O through Z (left to right).

Conway used polyominoes, specially pentominoes, in his initial investigations of Life and other cellular automata: he tracked the histories of small polyominoes as a way to explore the behavior of different cellular automata when the calculations had to be done by hand; the studies of the polyominoes led to important discoveries in Life.

1.2.1 R-Pentomino

Among the other pentominoes, the **R-pentomino** outstands; mainly because of two reasons:

- It is a methuselah[1], meaning that is a pattern that takes a large number of generations to stabilize and, at some point during its evolution, becomes much larger than it's initial configuration.
- The first glider ever observed is the glider that releases in the 69th generation.

Since all the others pentominoes stabilize in ten generations or less, this is, by far, the most active pentomino: it takes 1103 generations and by then it's population is of 116.

[1] M. Gardner defined methuselahs as patterns of less than ten cells that take more than fifty generations to stabilize[12].

2 Experiment Results

The experiment consisted in evolving in a three-dimensional universe, with cubes as units and an adaptation of the diffusion rule, each Conway pentomino and observing its behavior while watching out for configurations such as still lifes, oscillators gliders and especially, glider guns.

2.1 The Settings

The simulations were done in Ready, a cross-platform implementation of various reaction-diffusion systems designed to explore both, continuous and discrete cellular automata on grids and arbitrary meshes [13]. Alas, precisely because it can be used with continuous and discrete universes, Ready is not optimized for experiments of this kind, provoquing quite slow evolutions.

Each experiment started with a pentomino in the center of a 256×256 empty cube; the simulation ended when there were no more alive cells or when the cells filled the space and started to interfere one another. Since Ready does not permit to move the view when the simulation is running, each pentomino scenario was run, at least, three times in order to obtain a front, lateral and in an angle view of the evolution.

2.2 Pentomino By Pentomino

For each pentomino entry, there will be an isometric figure to illustrate the initial scenario. The pentominoes are alphabetically ordered and will be included only some moments of the evolution.

2.2.1 O-Pentomino

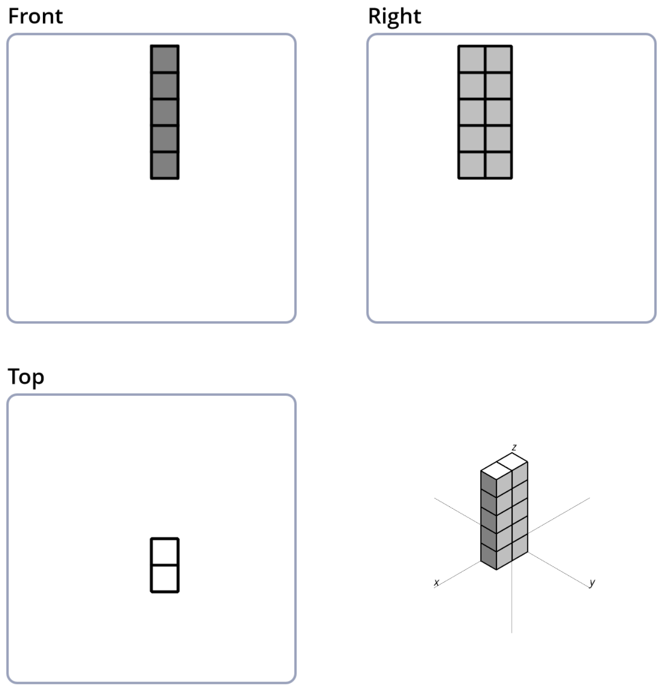

Figure 2: Isometric of the O-pentomino.

This is, as seen in figure 2, the simplest pentomino in the set. As observed in figure 3, in the 9^{th} generation, it splits into two identical oscillators (see figure 49), sadly, each one obstructs the way of the other and by the 21^{st} generation, all the cells are dead.

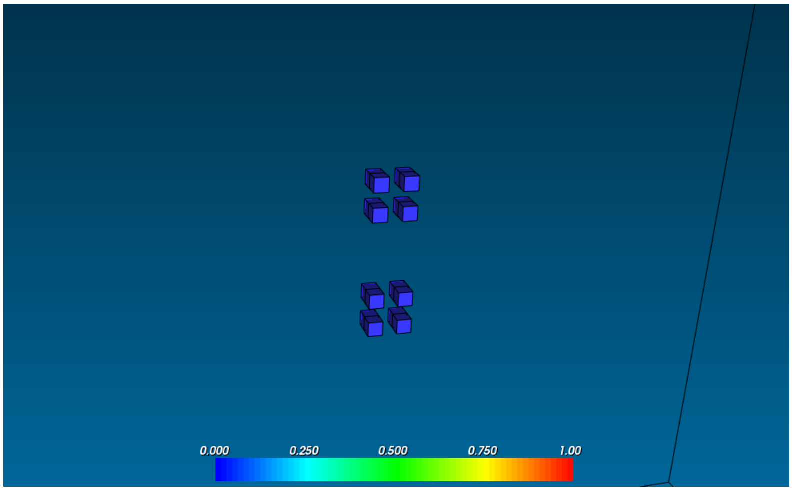

Figure 3: Evolution of the O-Pentomino, 9^{th} generation.

2.2.2 P-Pentomino

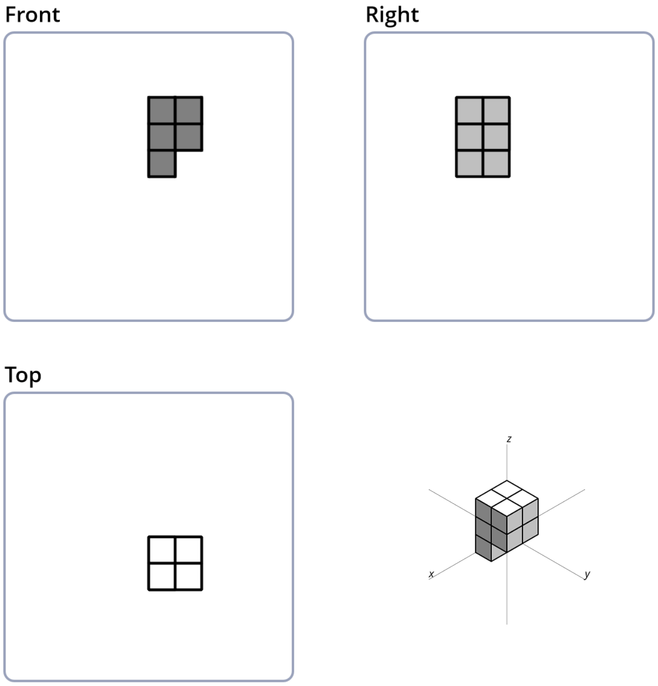

Figure 4: Isometric of the P-pentomino.

This pentomino (see figure 4) is quite interesting; three configurations can be seen within less than 47 generations. The first one to appear is a replicator (see figure 50), latter, the first (and one of the simplest) glider appears (see figure 44); finally, another glider appears (see figure 45).

Figure 5 shows the state of the evolution of the p-pentomino in the 42^{th} generation; the replicator can be seen in the north of the evolution; while the gliders are located in the south of the population; in 6 the replicator can be seen in the left of the figure and in 7, the gliders can be seen in the south.

The simulation of this pentomino stopped at 210 generations because space was overcrowded.

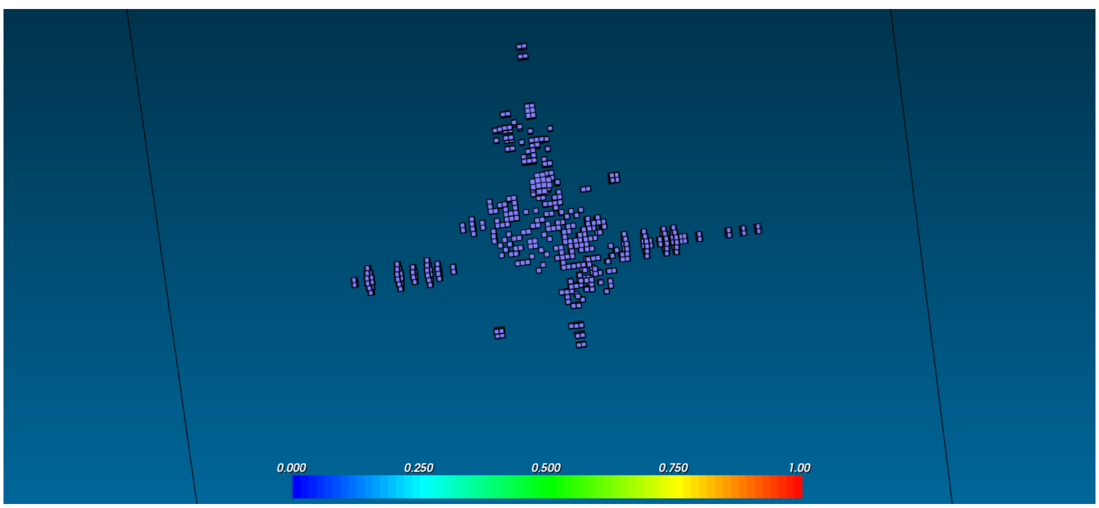

Figure 5: Evolution of the P-Pentomino, 42^{nd} generation.

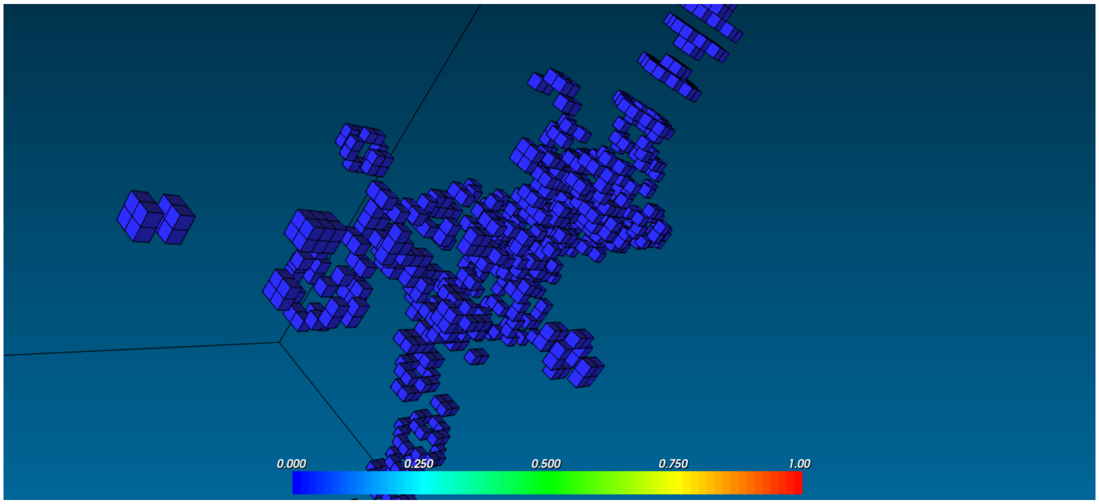

Figure 6: Evolution of the P-Pentomino, zoomed replicator, 42^{nd} generation.

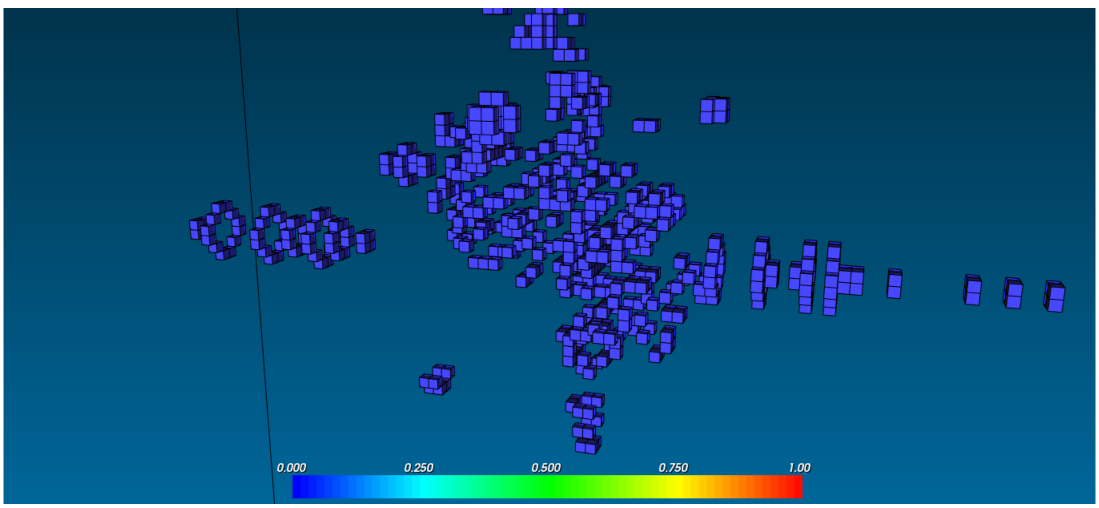

Figure 7: Evolution of the P-Pentomino, zoomed gliders, 42^{nd} generation.

2.2.3 Q-Pentomino

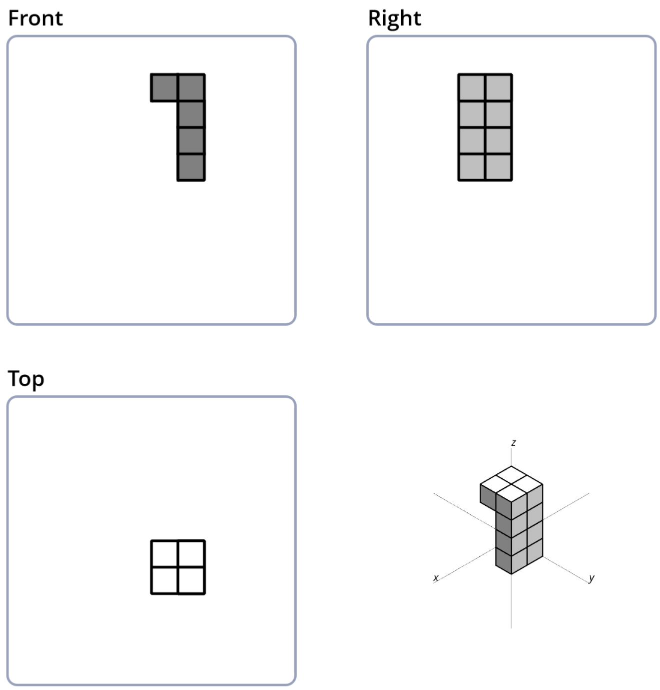

Figure 8: Isometric of the Q-pentomino.

This pentomino (see figure 8) shows in ten generations, two structures, one is the replicator-1 (see figure 50), and a *modified* glider: the glider-1 (see figure 44) with a satellite (see figure 46). The evolution of the 10^{th} generation with both structures can be seen in figure 9.

As shown in 10, in the 50^{th} generation, a mirrored glider-4 (see figure 47) emerges, both in the north and south of the population; the figures 11 and 12 show the gliders, zoomed, in the 50^{th} and 52^{nd} generation, respectively.

The simulation of this pentomino stopped at 109 generations because space was overcrowded.

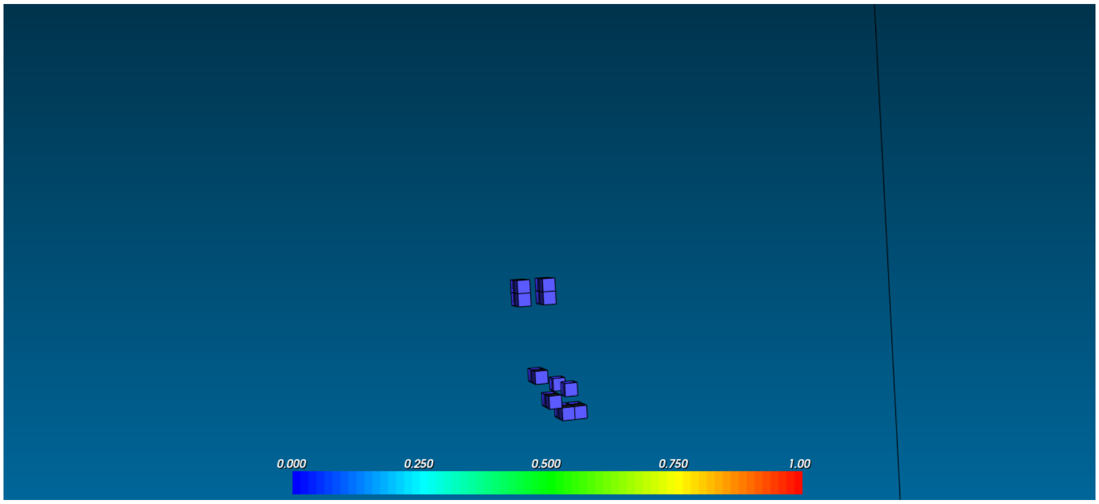

Figure 9: Evolution of the Q-Pentomino, 10^{th} generation.

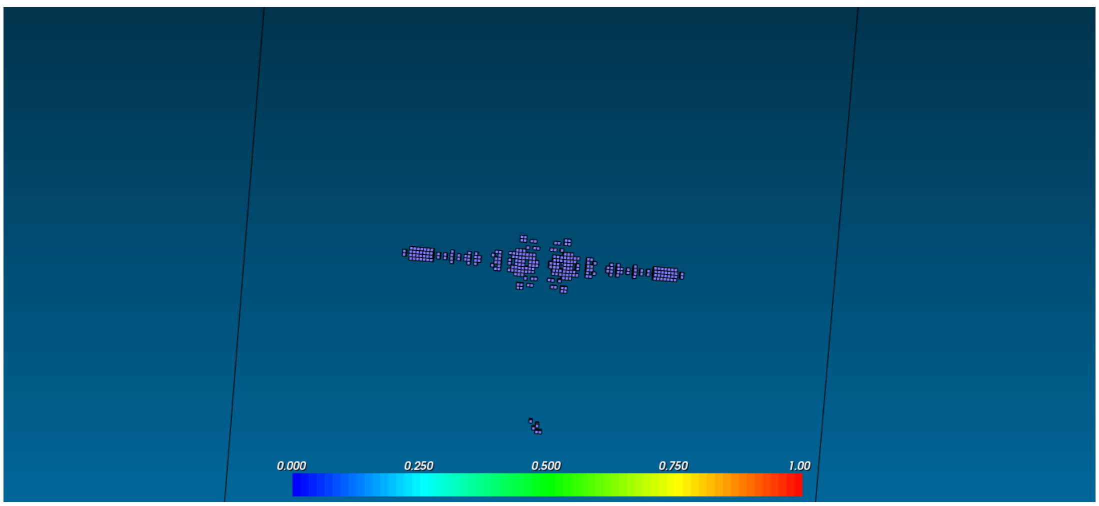

Figure 10: Evolution of the Q-Pentomino, 50^{th} generation.

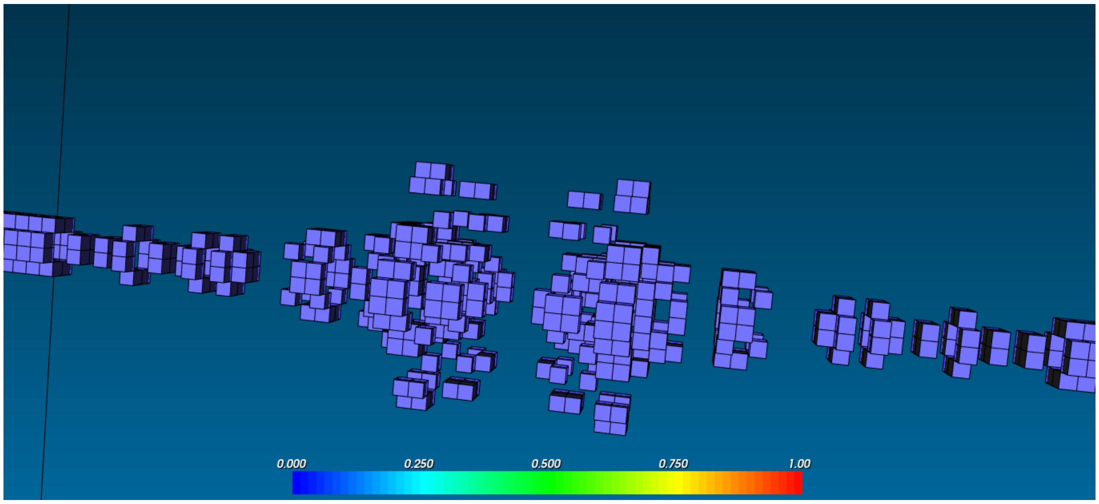

Figure 11: Evolution of the Q-Pentomino, zoomed gliders, 50^{th} generation.

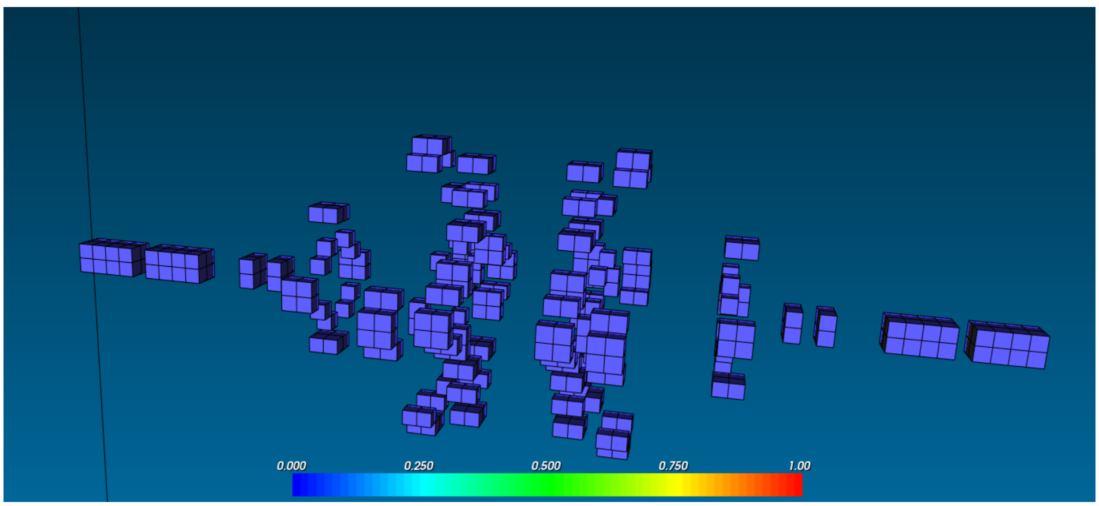

Figure 12: Evolution of the Q-Pentomino, zoomed gliders, 52^{nd} generation.

2.2.4 R-Pentomino

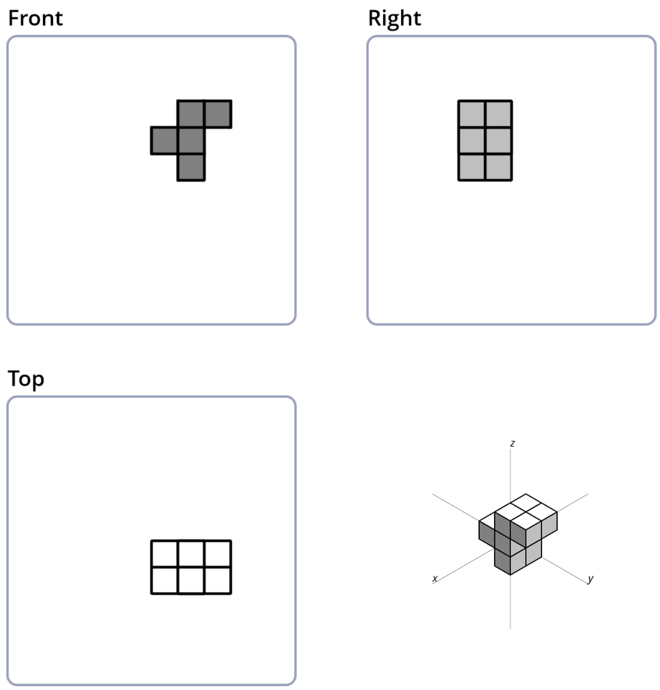

Figure 13: Isometric of the R-pentomino.

This pentomino (see figure 13), just like it's equivalent in two dimensions, expands very quickly in a short period of time, this can be observed in figure 14, where a comparison between the evolution achieved in the same generation for different pentominoes.

The replicator (see figure 50) appears here, too; as the glider-1 (see figure 44) and glider-4 (see figure 47); the gliders can be seen in the 81^{st} generation, in the north-east, north-west, south-east and south-west areas of figure 15; in figure 16, a pair of glider-1 can be seen easily in the left of the figure; the glider-4 is shown in the upper-left of figure 17.

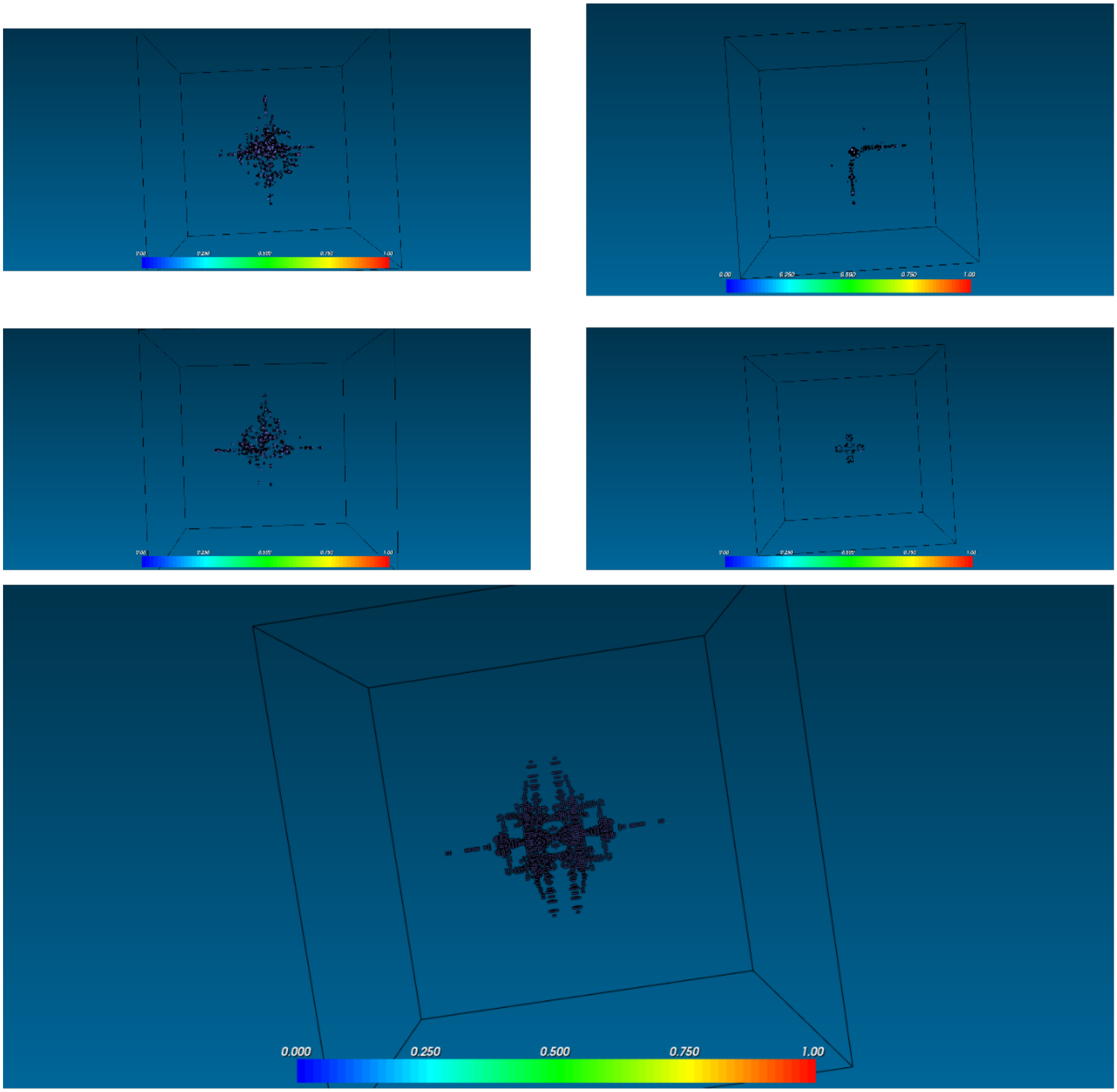

Figure 14: Evolution in the 52^{th} generation for the Y, W, P, X and R pentominoes, from left to right and top to bottom.

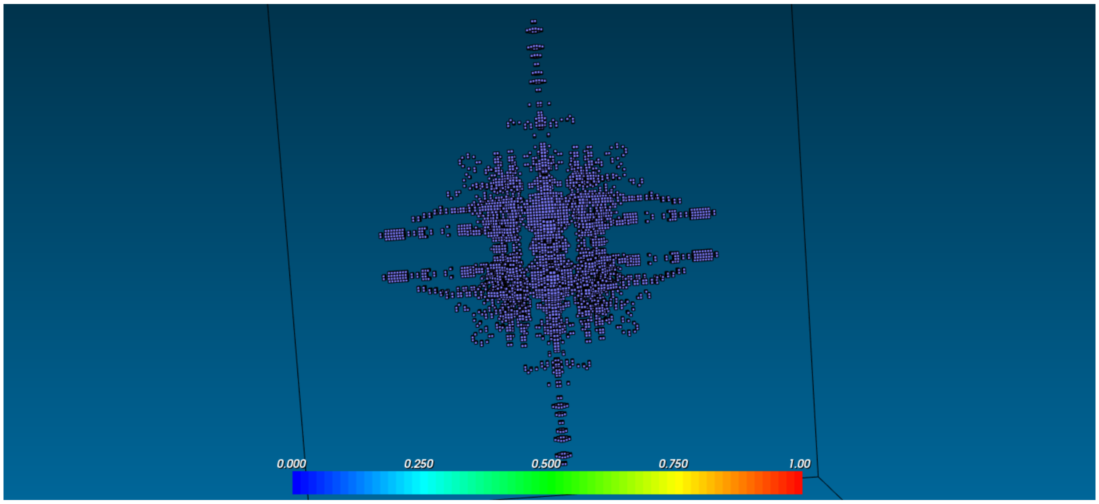

Figure 15: Evolution of the R-Pentomino, 81^{st} generation.

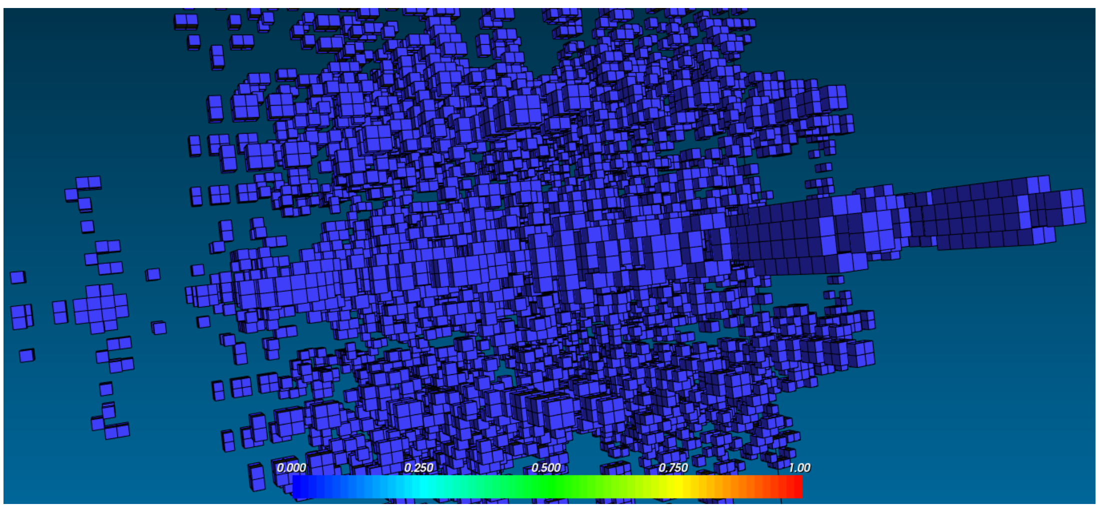

Figure 16: Evolution of the R-Pentomino, zoomed glider-1, 81^{st} generation.

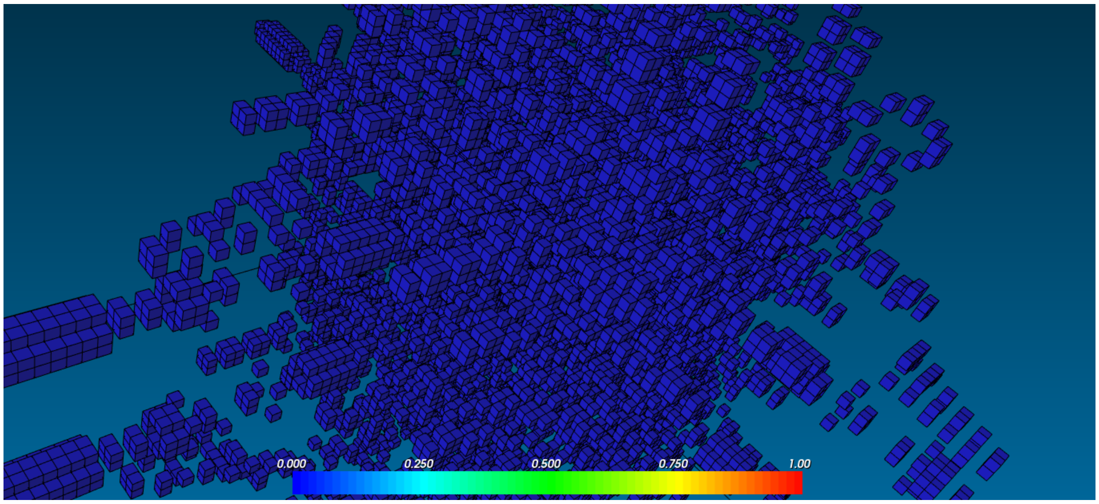

Figure 17: Evolution of the R-Pentomino, zoomed glider-4, 81^{st} generation.

2.2.5 S-Pentomino

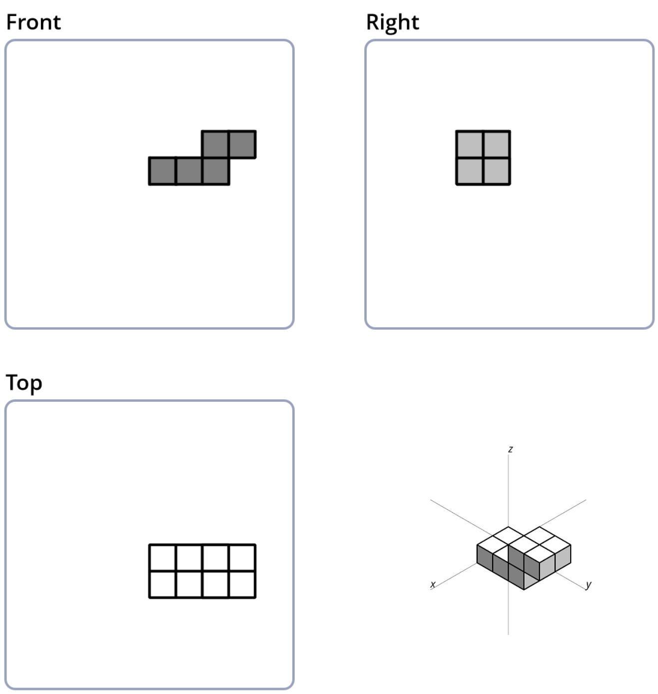

Figure 18: Isometric of the S-pentomino.

This pentomino (see figure 18) shows, quite fast, the replicator-1, seen in previous pentominoes first; later a clean puffer (see figure 52) emerges; four of these puffers can be observed in figure 19. This was one of the most active pentominoes; and, although the simulation had to be stopped after 131 generations. Interestingly, the growth in the population seemed more ordered, symmetrical and configuration driven: seems easier to find individual configurations in the s-pentomino than in the others. Two pairs of puffer-2 can be seen again, a pair moving towards the east, and the other towards west; in figures 20 and 21.

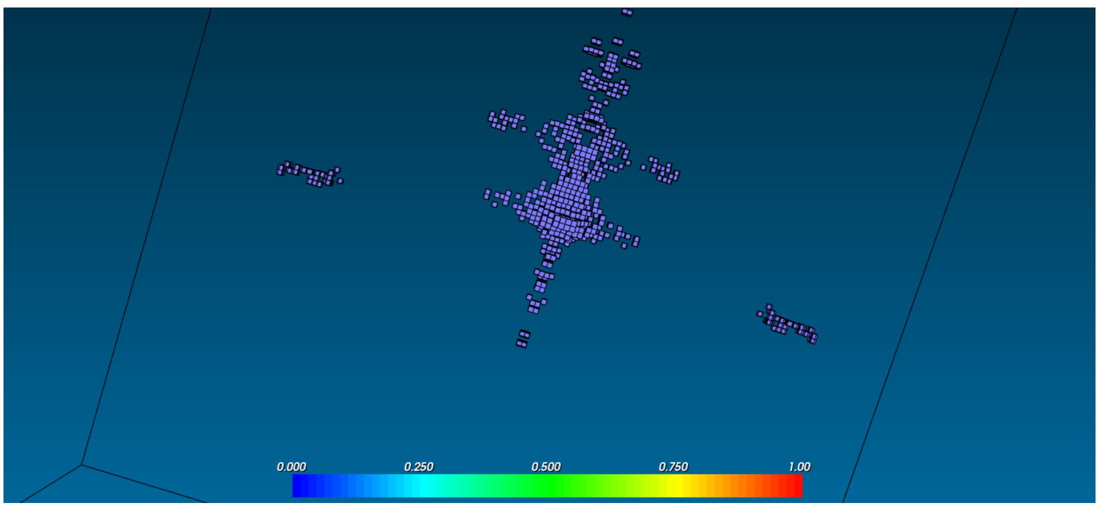

Figure 19: Evolution of the S-Pentomino, 56^{th} generation.

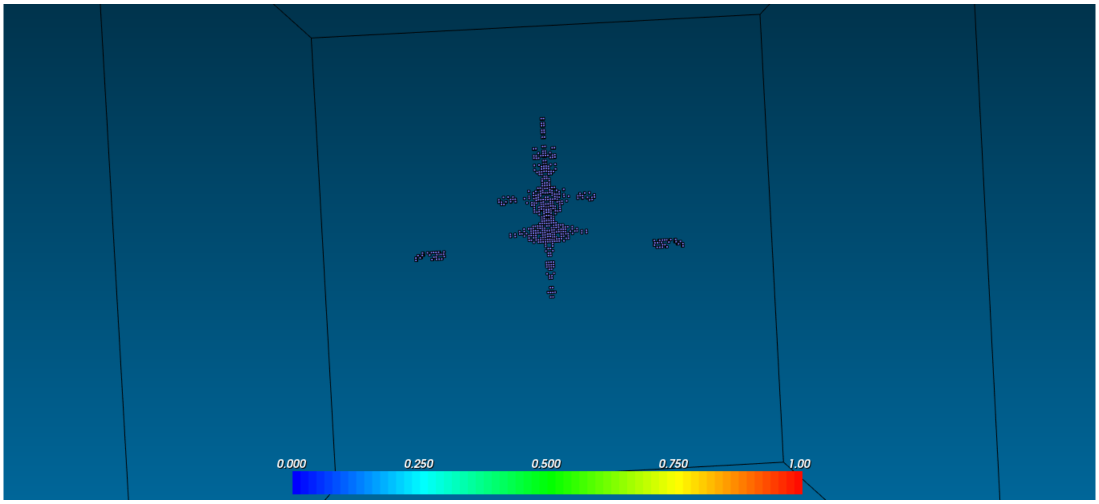

Figure 20: Evolution of the S-Pentomino, 57^{th} generation.

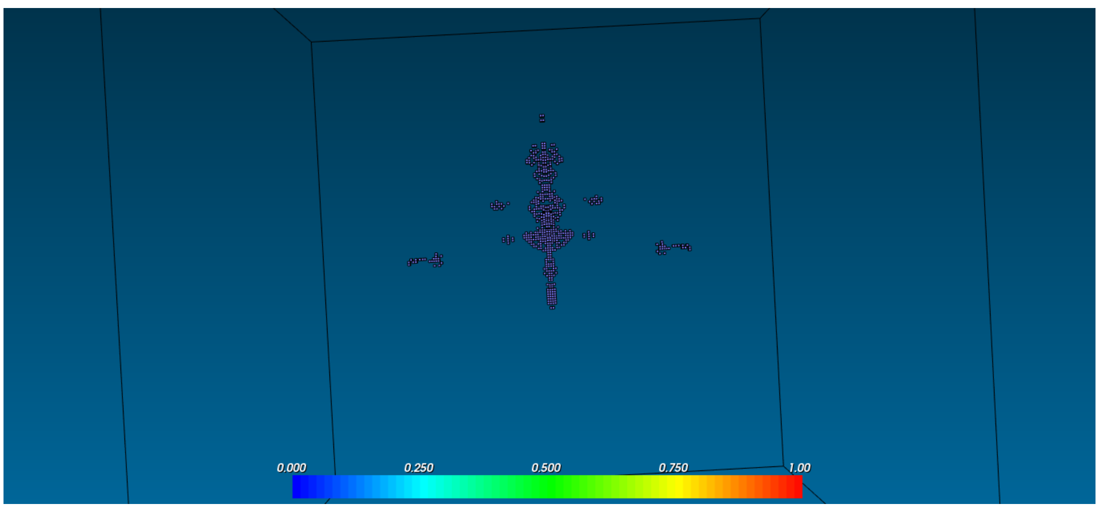

Figure 21: Evolution of the S-Pentomino, 60^{th} generation.

2.2.6 T-Pentomino

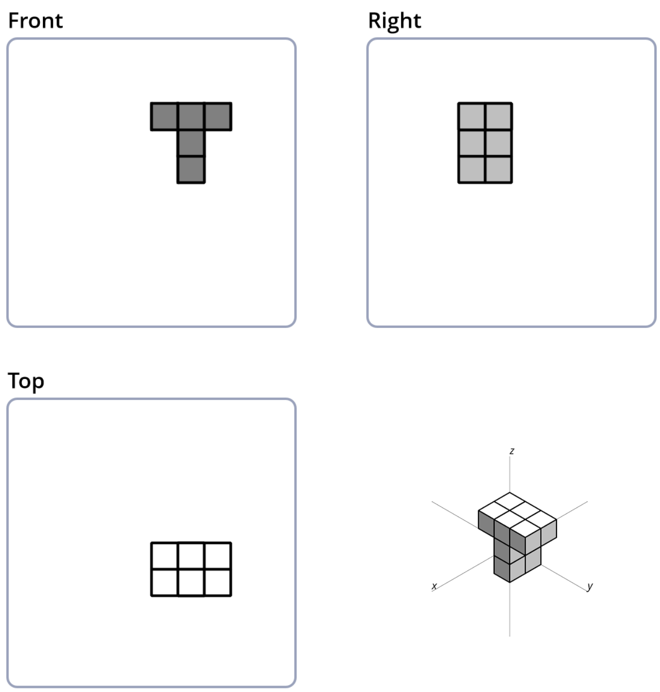

Figure 22: Isometric of the T-pentomino.

This pentomino (see figure 22) behaves rather differently than it's equivalents in other rules (and dimensions); instead of evolving in a replicator, spaceship, pulsar or such, dies within five generations.

2.2.7 U-Pentomino

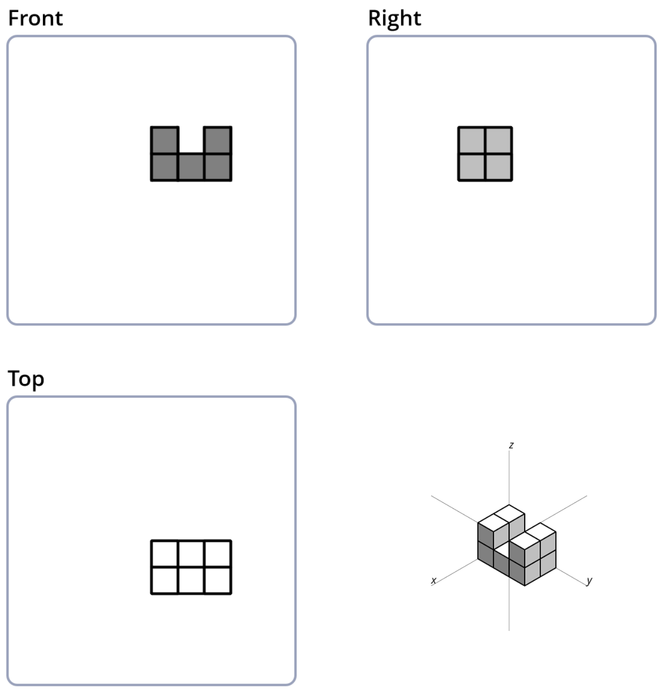

Figure 23: Isometric of the U-pentomino.

This pentomino (see figure 23),shows the puffer-1 (see figure 51) in less than 7 generations; unlike the puffer-2 (see figure 52), this puffer is not clean at all and pollutes very quickly. Interestingly enough, the population at its early stages appears to be formed by configurations that look almost like gliders and puffers found previously, but each fails a few cells. The glider-1 (see figure 44) and the replicator-1 (see figure 50) appear heavily through the evolution of the pentomino. A clean new puffer, the puffer-3 (see figure 53) and a glider, the glider-5 (see figure 48, appear in this pentomino; both, the new puffer and glider can be observed in figure 24: the glider-5 in the north and the puffer-3 in the north-east and east of the figure.

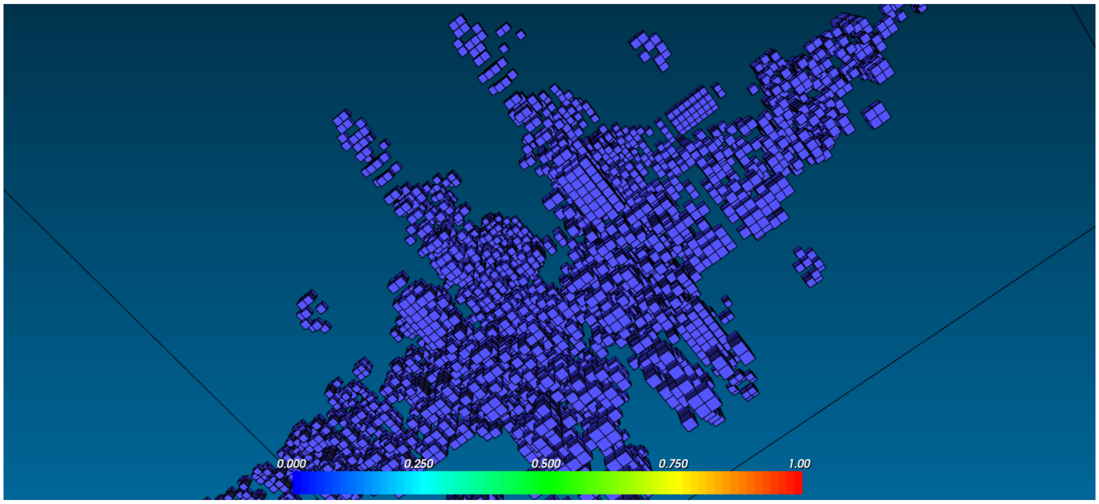

Figure 24: Evolution of the U-Pentomino, 70^{th} generation.

2.2.8 V-Pentomino

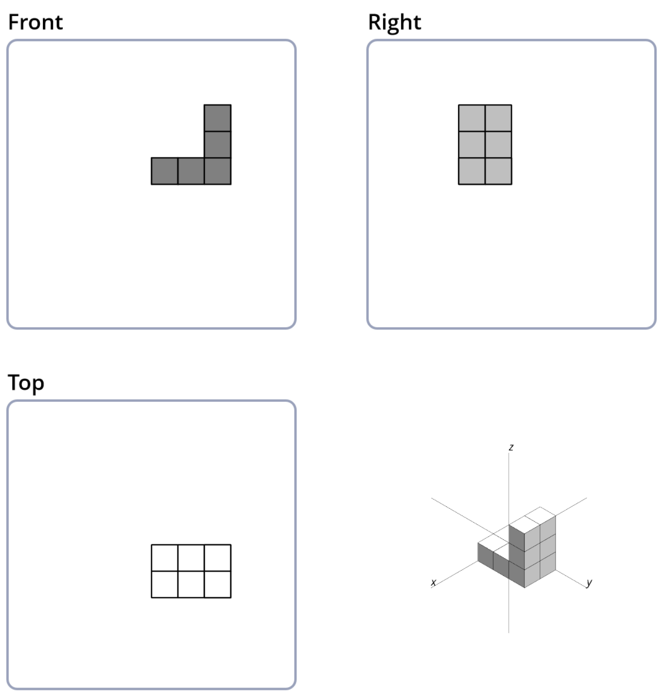

Figure 25: Isometric of the V-pentomino.

In this pentomino (see figure 25), a pair of replicator-1 (see figure 50) appear in the early stages of its evolution. Although the replicator-1 starts by replicating itself in the z-axis, soon it starts to replicate perpendicularly to itself, as observed in figure 26.

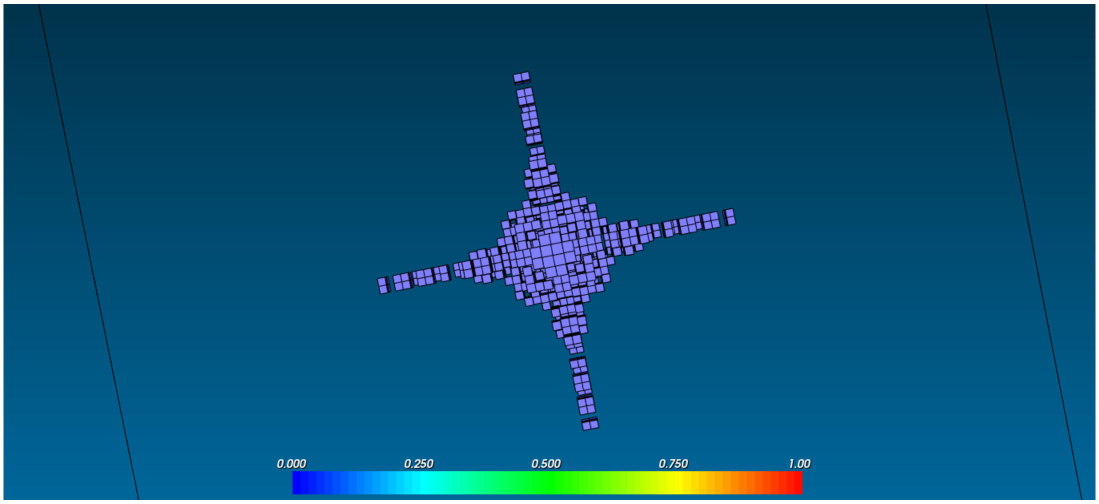

Figure 26: Evolution of the V-Pentomino, 48^{th} generation.

2.2.9 W-Pentomino

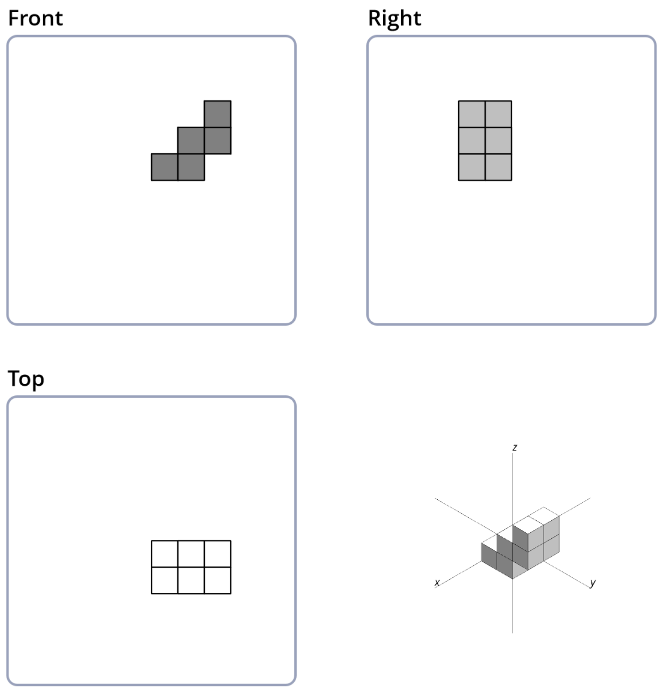

Figure 27: Isometric of the W-pentomino.

This pentomino (see figure 27), unlike its predecessor, the V-pentomino, *folds* since the beginning and the two initial replicator-1 start to replicate perpendicularly since the beginning (see figure 28).

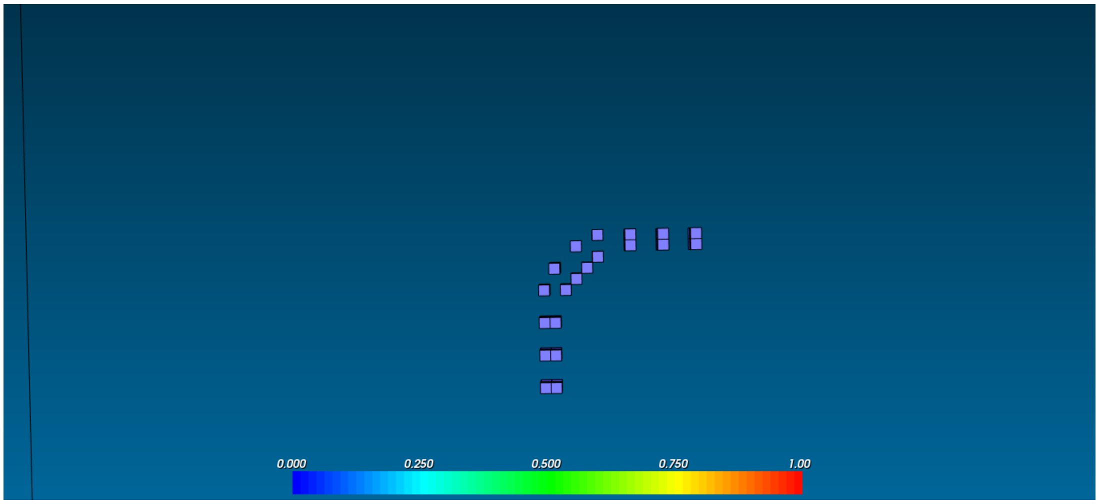

Figure 28: Evolution of the W-Pentomino, 12^{th} generation.

2.2.10 X-Pentomino

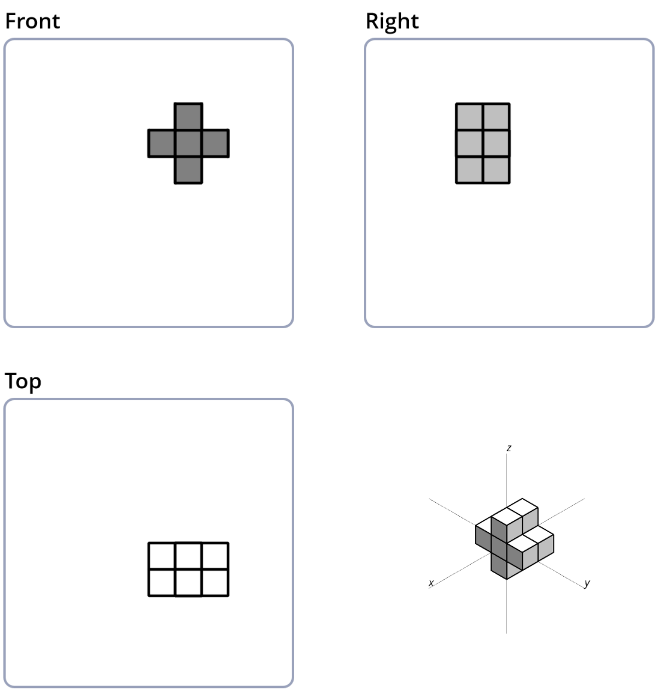

Figure 29: Isometric of the X-pentomino.

This pentomino's (see figure 29) first evolution's stages appear to be related to the oscillator-1 (see figure 49 for the oscillator and figures 30, 31, 32, 33, 34 and 35 for the pentomino's evolution). Then, as shown in figure 36, it evolves into a set of replicator-1 (see figure 50).

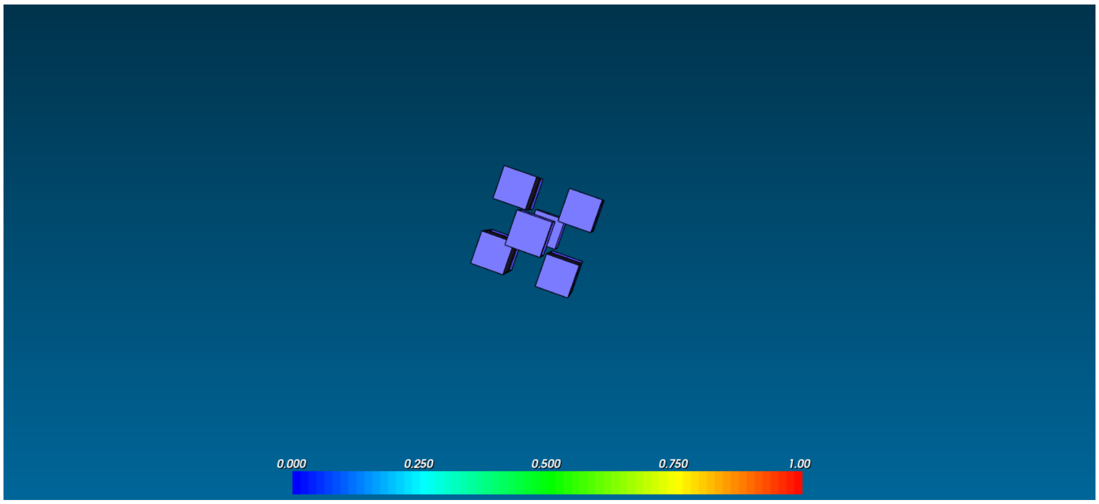

Figure 30: Evolution of the X-Pentomino, 2^{nd} generation.

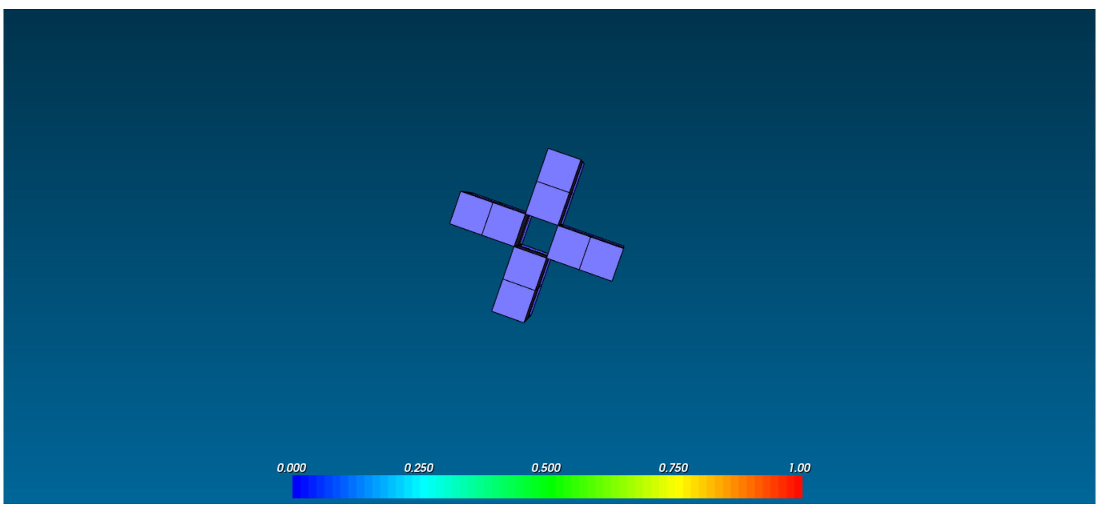

Figure 31: Evolution of the X-Pentomino, 3^{th} generation.

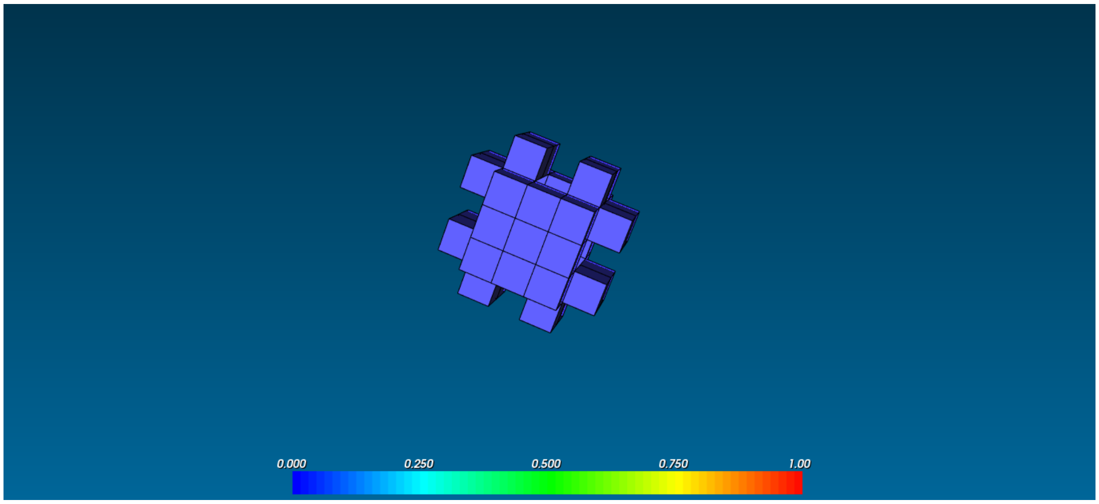

Figure 32: Evolution of the X-Pentomino, 4^{th} generation.

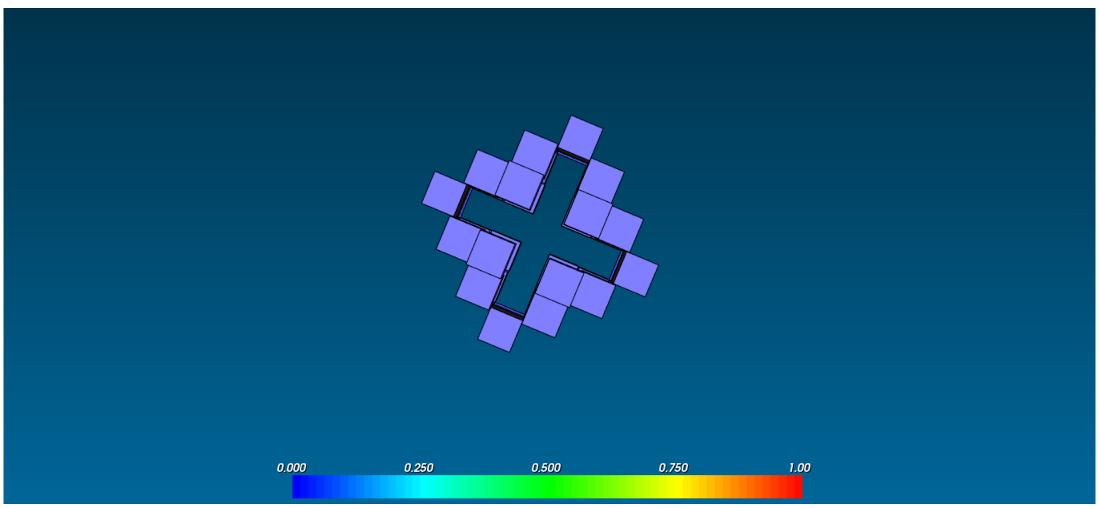

Figure 33: Evolution of the X-Pentomino, 5^{th} generation.

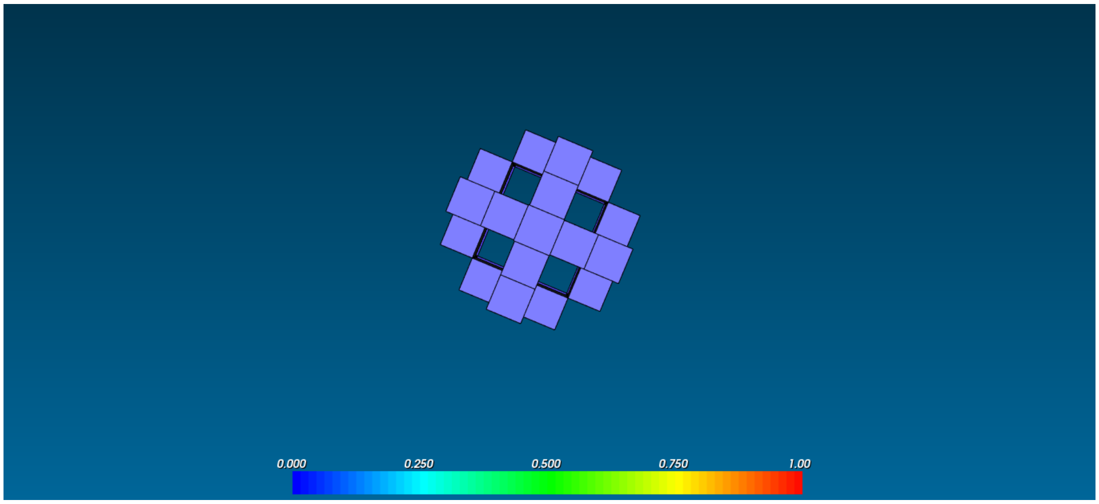

Figure 34: Evolution of the X-Pentomino, 6^{th} generation.

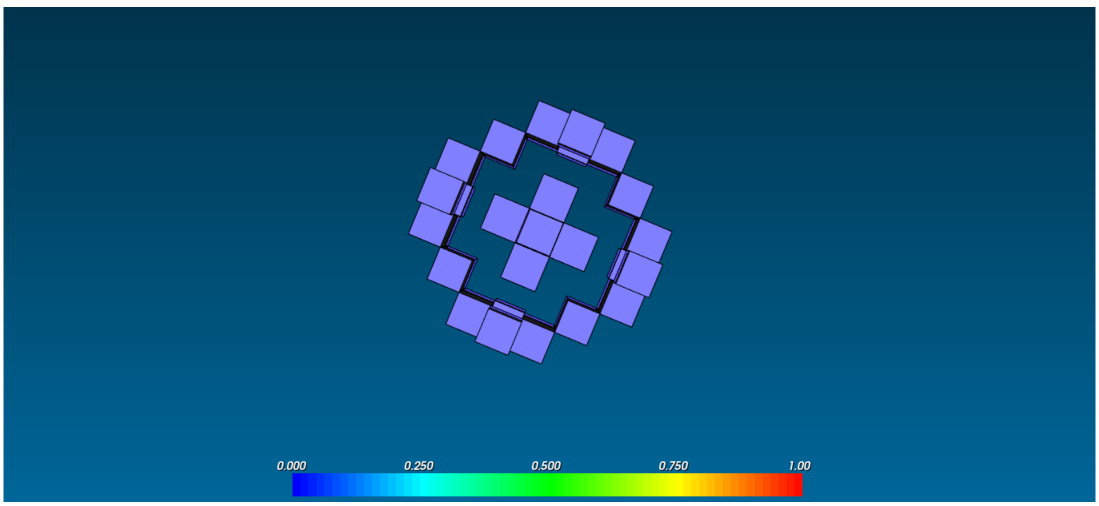

Figure 35: Evolution of the X-Pentomino, 7^{th} generation.

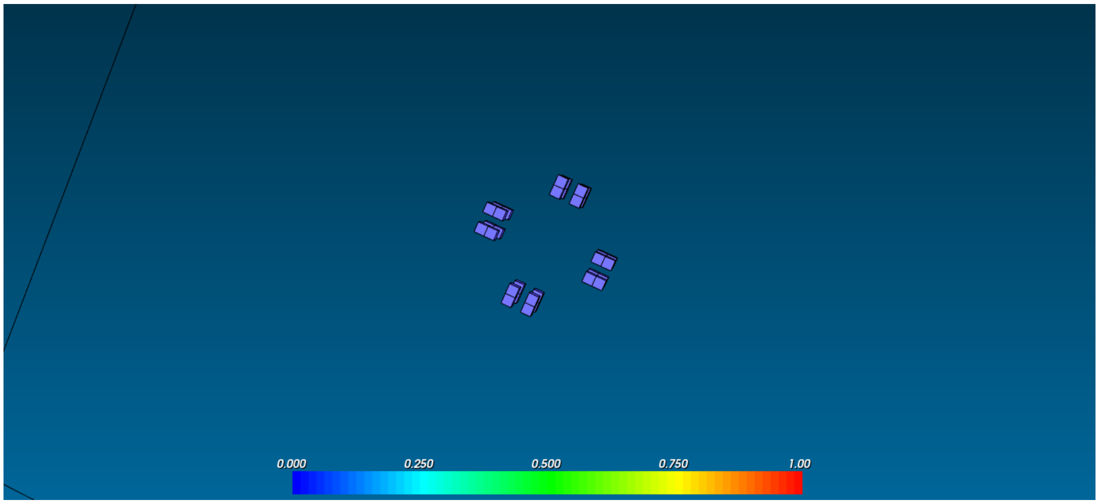

Figure 36: Evolution of the X-Pentomino, 15^{th} generation.

2.2.11 Y-Pentomino

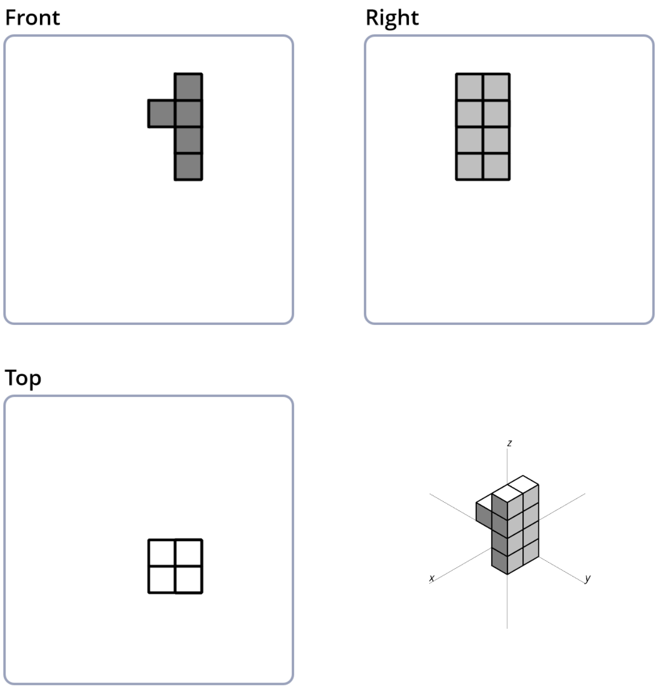

Figure 37: Isometric of the Y-pentomino.

This pentomino (see figure 37) grows quickly due to all the replicator-1 (see figure 50) present almost since the beginning of the evolution; a puffer-1 (see figure 51) before the 20^{th} generation (see figure 38).

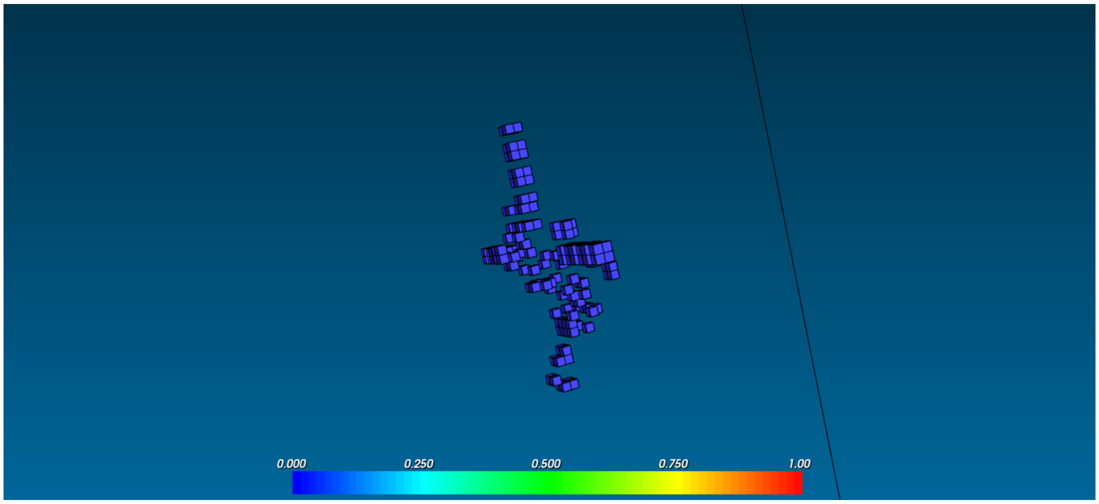

Figure 38: Evolution of the Y-Pentomino, 16^{th} generation.

2.2.12 Z-Pentomino

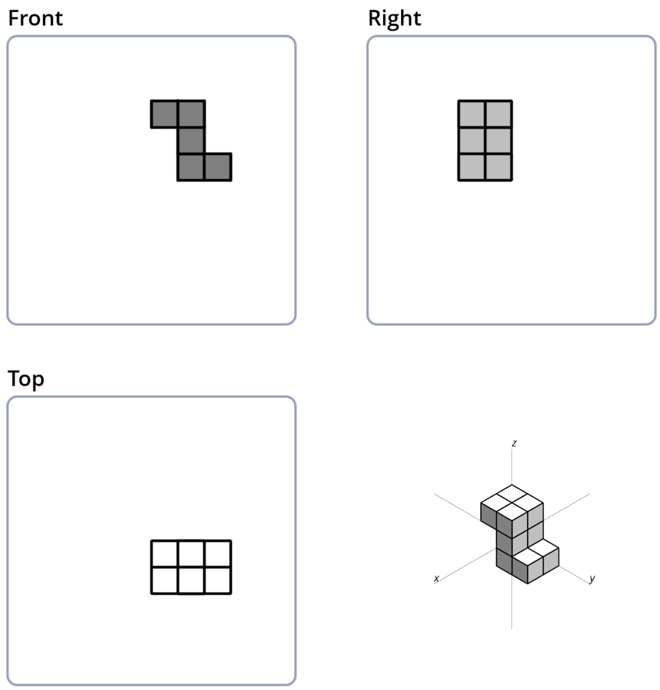

Figure 39: Isometric of the Z-pentomino.

In this pentomino (see figure 39), right from the beginning, as shown in figure 40, a pair of puffer-1 emerge (see figure 51), one in the left and another in the right of the population. The replicator-1 appears in this evolution as well, as can be observed in figures 41 and 41.

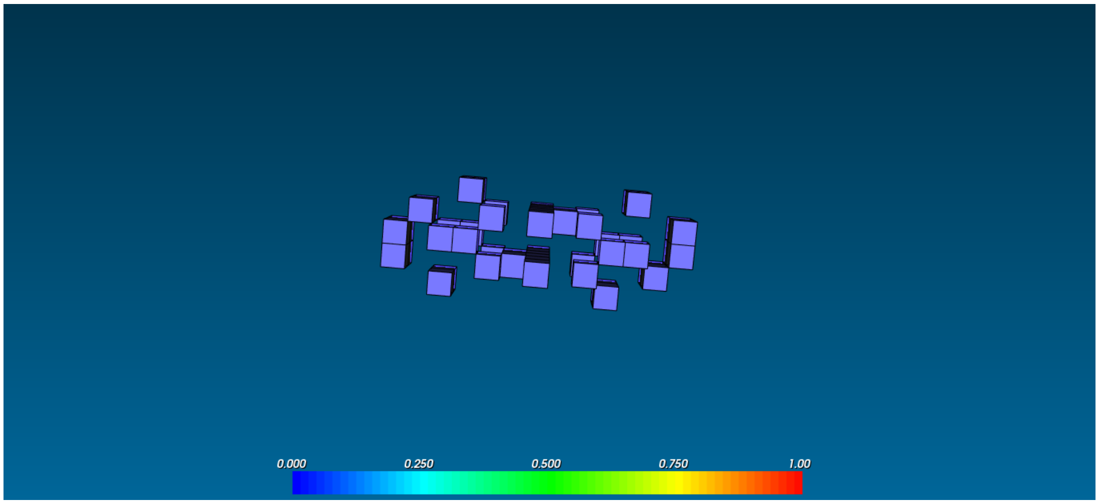

Figure 40: Evolution of the Z-Pentomino, 5^{th} generation.

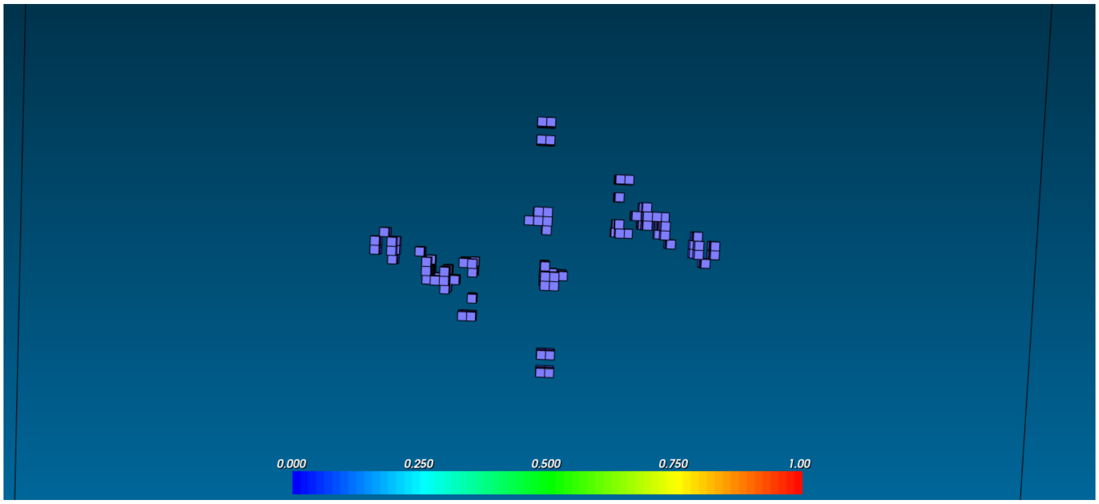

Figure 41: Evolution of the Z-Pentomino, 18^{th} generation.

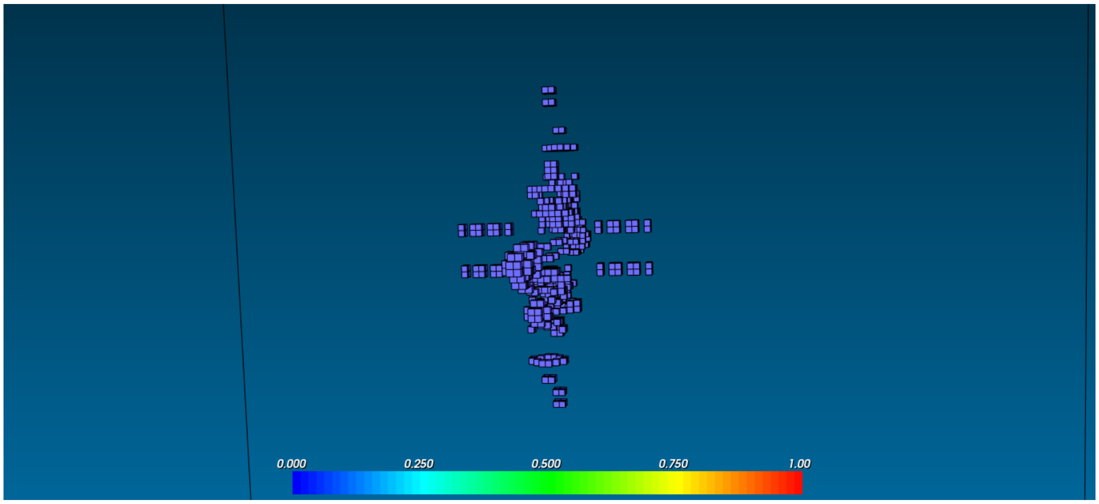

Figure 42: Evolution of the Z-Pentomino, 30^{th} generation.

2.3 Configurations found

Below are presented the configurations that were found while observing the evolutions of the pentominoes; it is important to remark that, due to the fast-paced population growth in some pentominoes, it is possible that some configurations remain hidden. The configurations are listed in the order that they were found.

2.3.1 Gliders

For more information, please consult [5].

The gliders, as explained in section 1.1, are mobile particles traveling in the space. There are two types of gliders: primary and compound; the first ones cannot be decomposed into smaller mobile localizations, whereas a compound glider is made of at least two primary gliders. Some properties of the gliders are listed below:

- Volume
- Translation
- Period
- Speed
- Weight

One of the primary gliders in the Diffusion Rule is shown in figure 43.

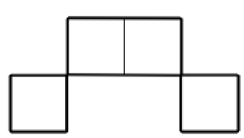

Figure 43: G1 glider in the Diffusion Rule.

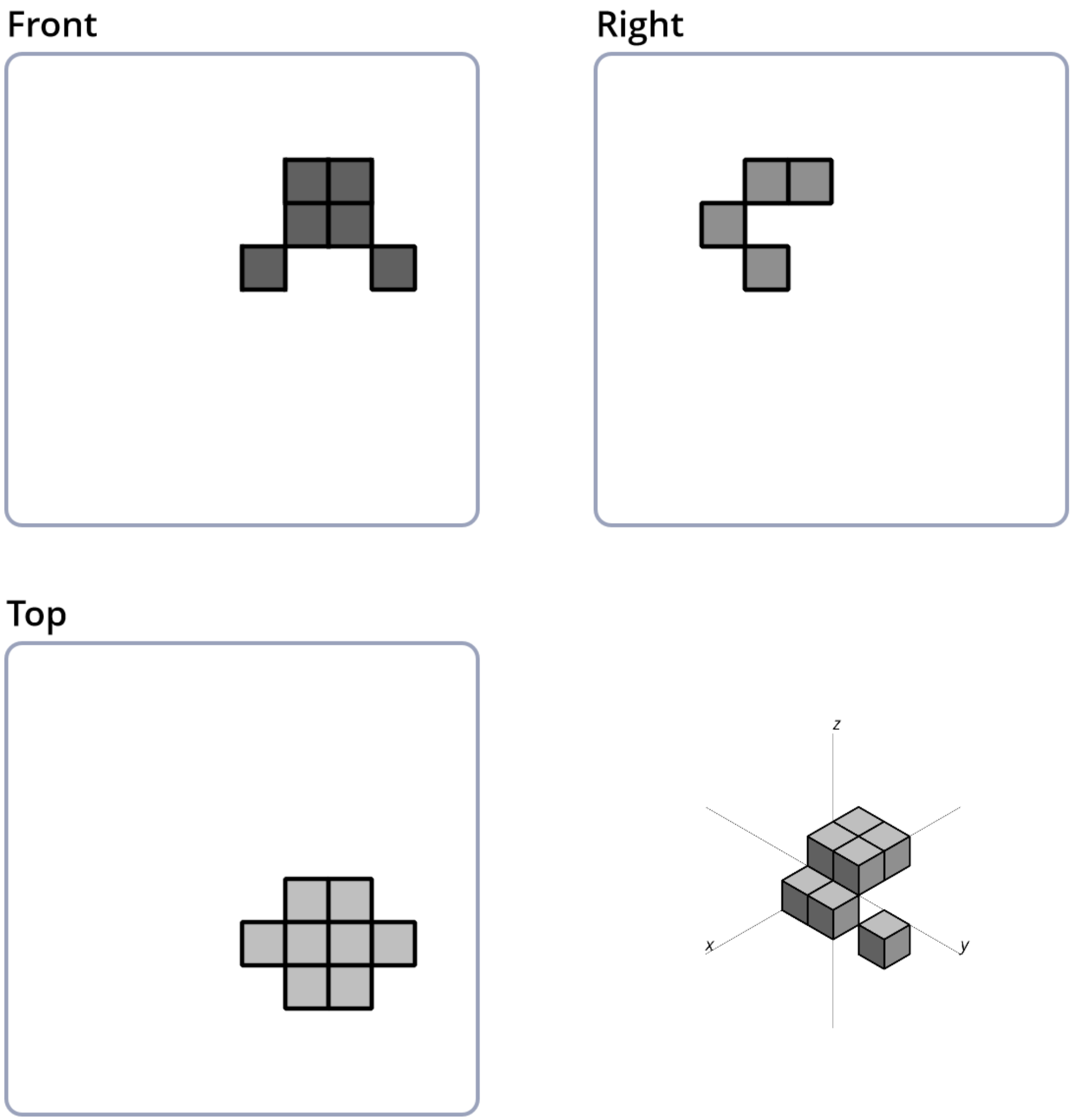

Figure 44: Isometric of glider-1.

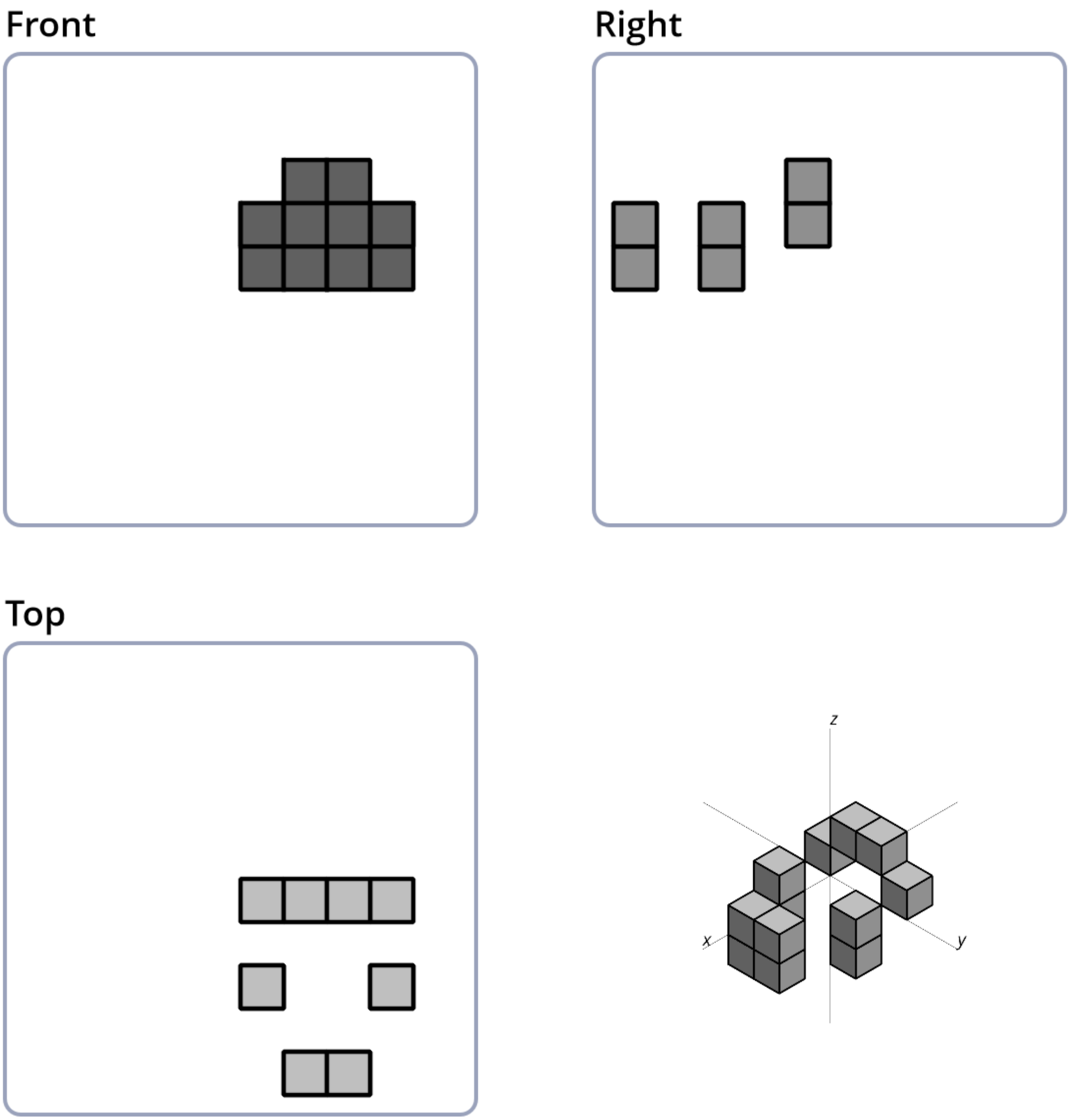

Figure 45: Isometric of glider-2.

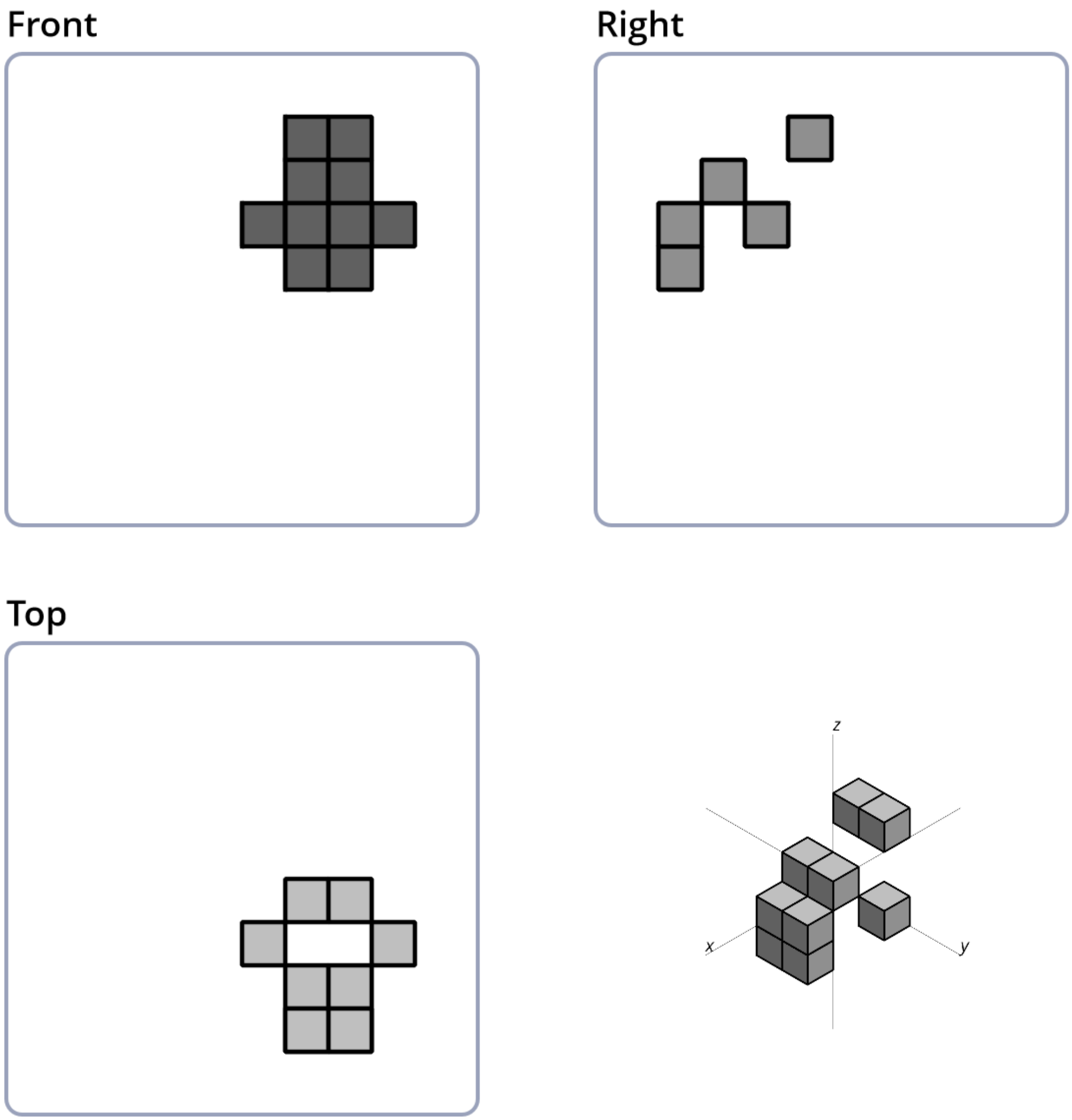

Figure 46: Isometric of glider-3.

The glider-4 (see figure 47) probably is the most simple glider; the equivalent of G1 (see figure 43) in three dimensions.

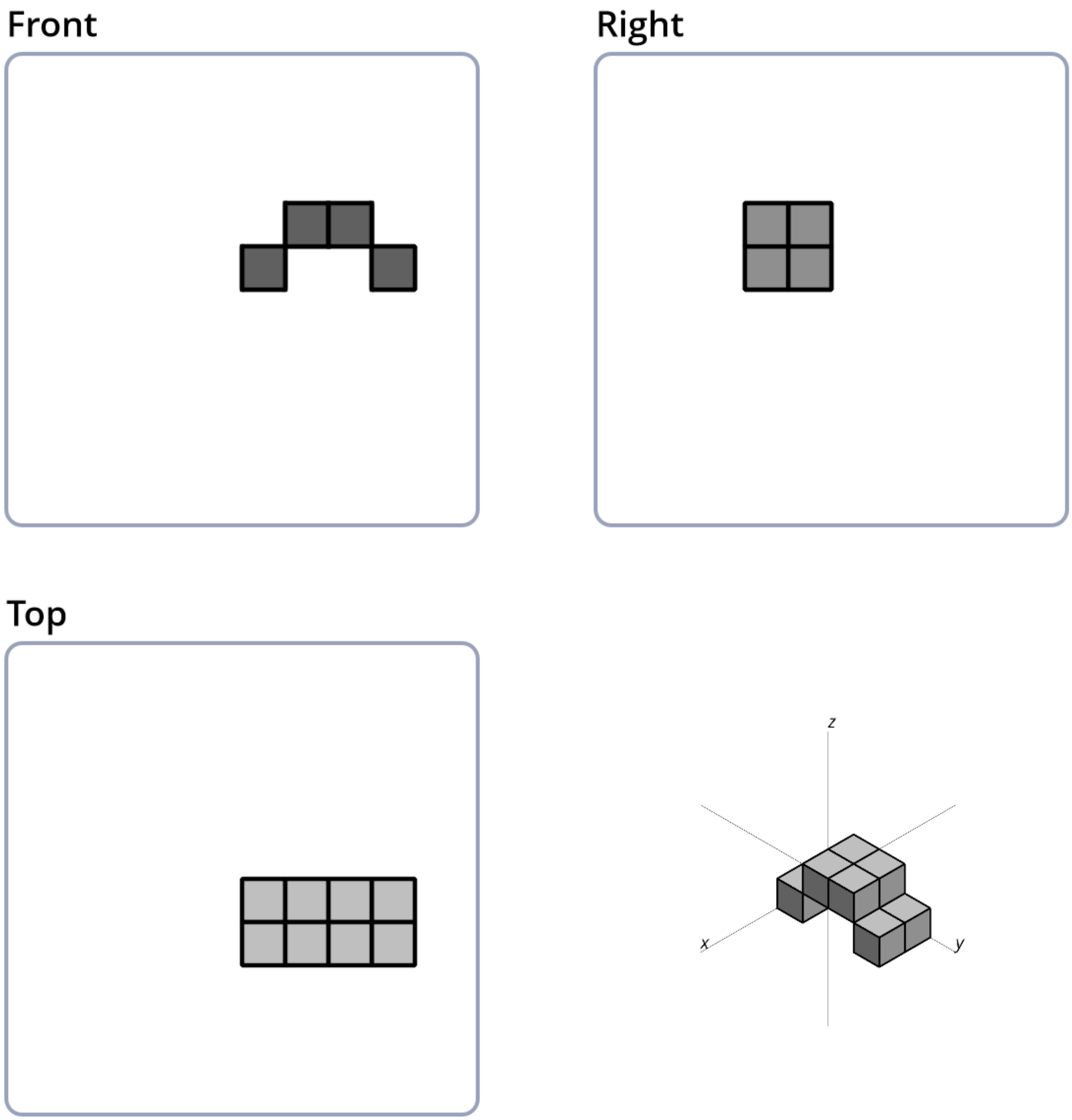

Figure 47: Isometric of glider-4.

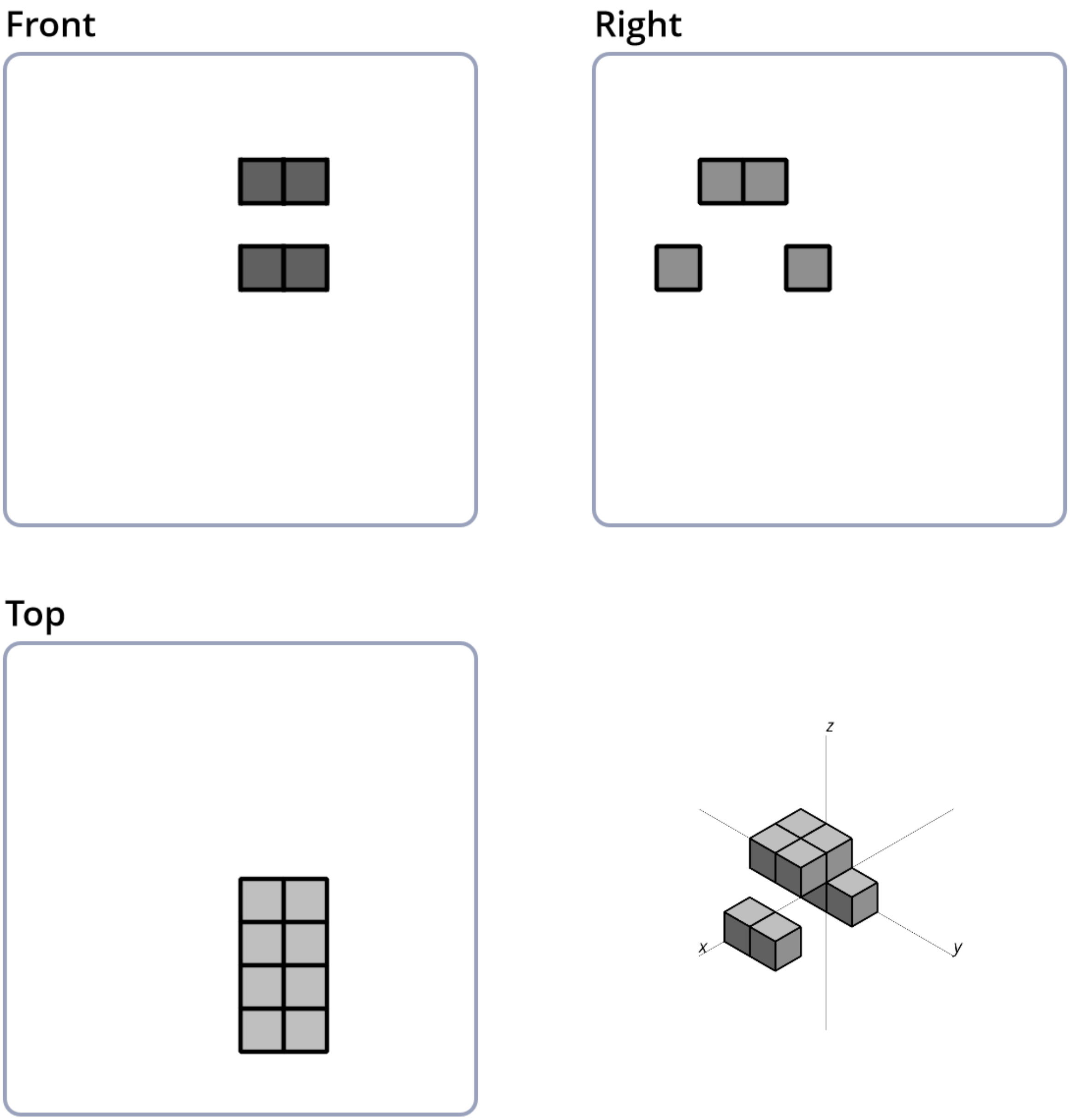

Figure 48: Isometric of glider-5.

2.3.2 Oscillators

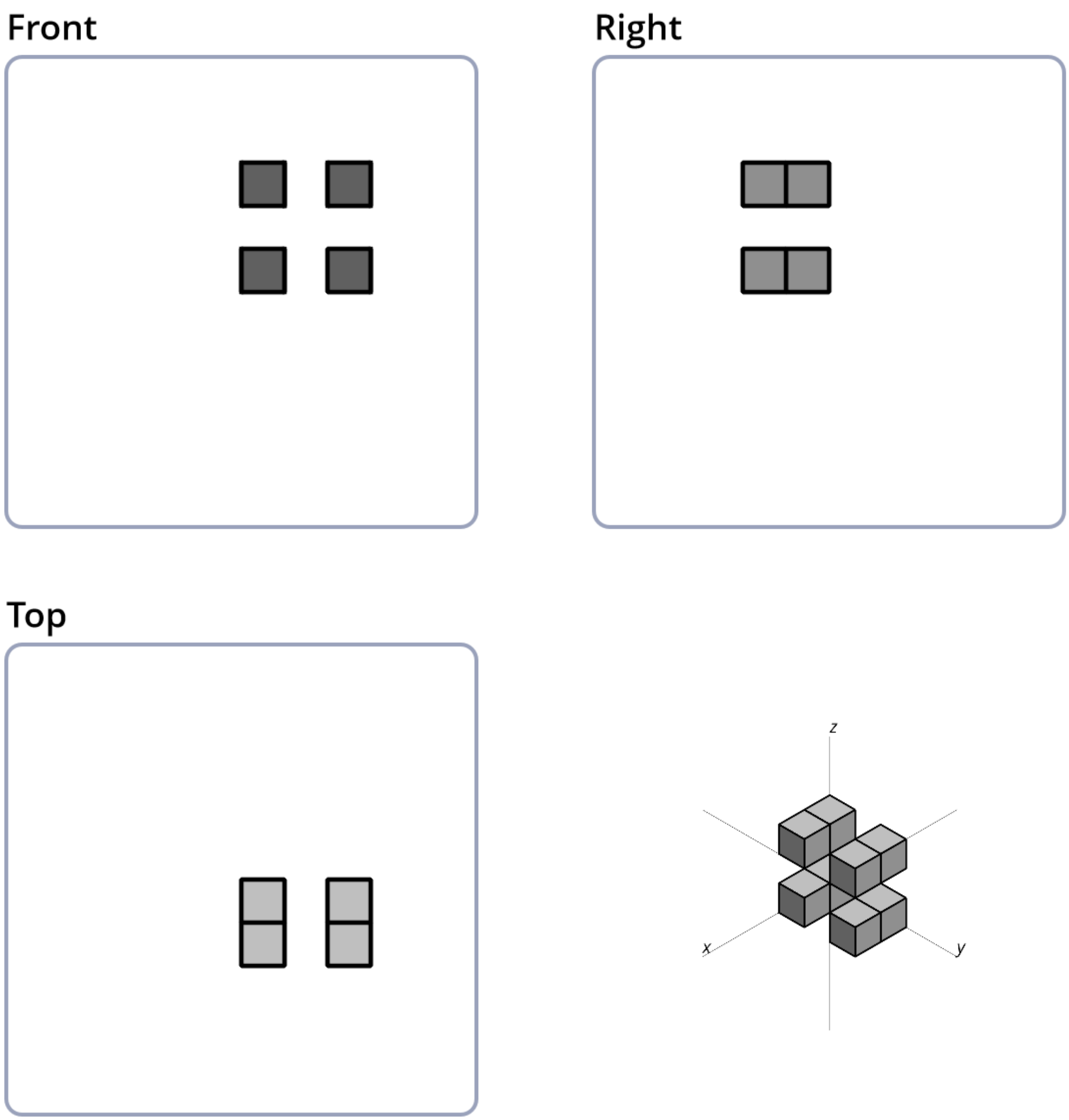

Figure 49: Isometric of oscillator-1.

2.3.3 Replicator

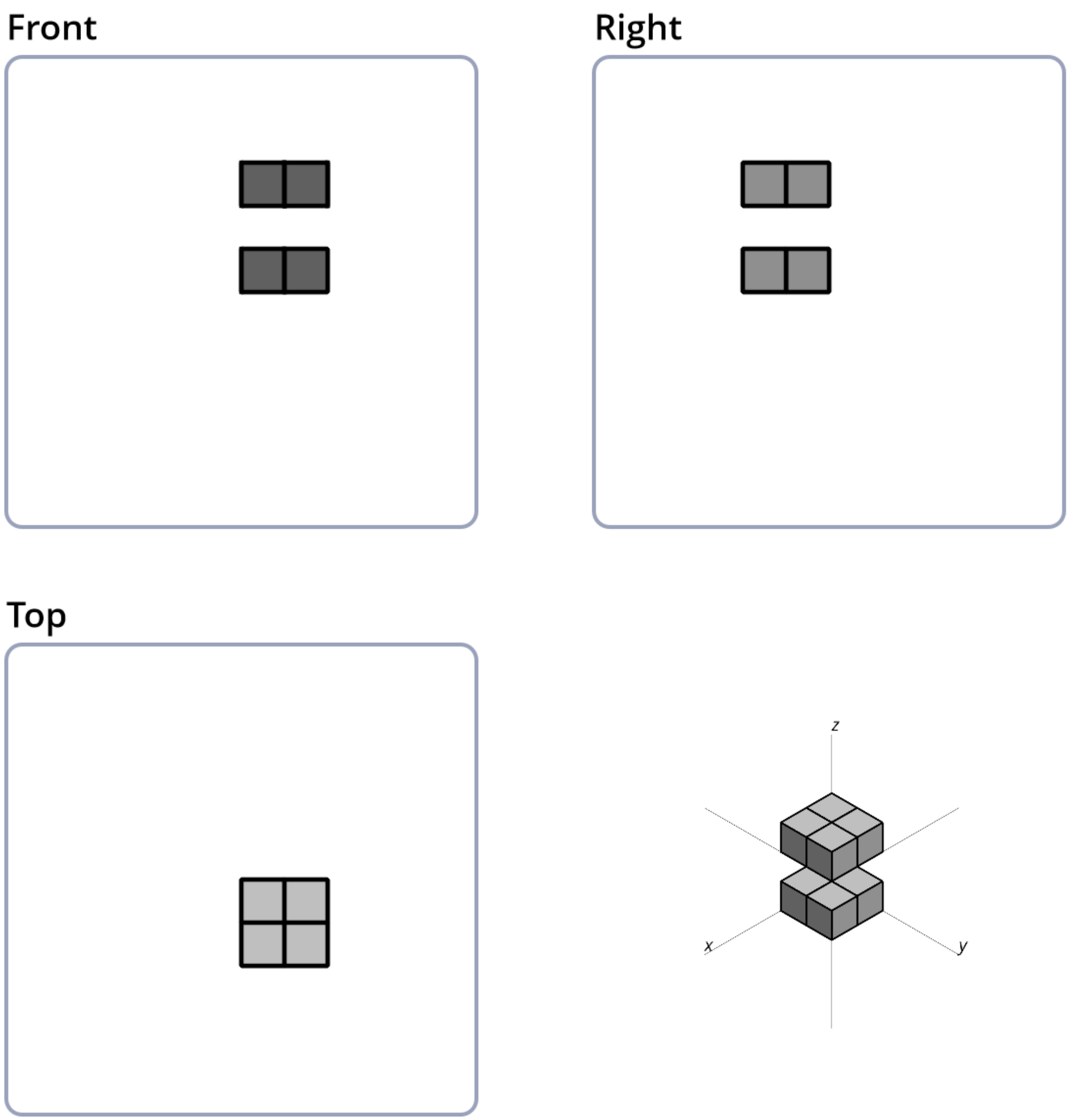

Figure 50: Isometric of replicator-1.

2.3.4 Puffer trains

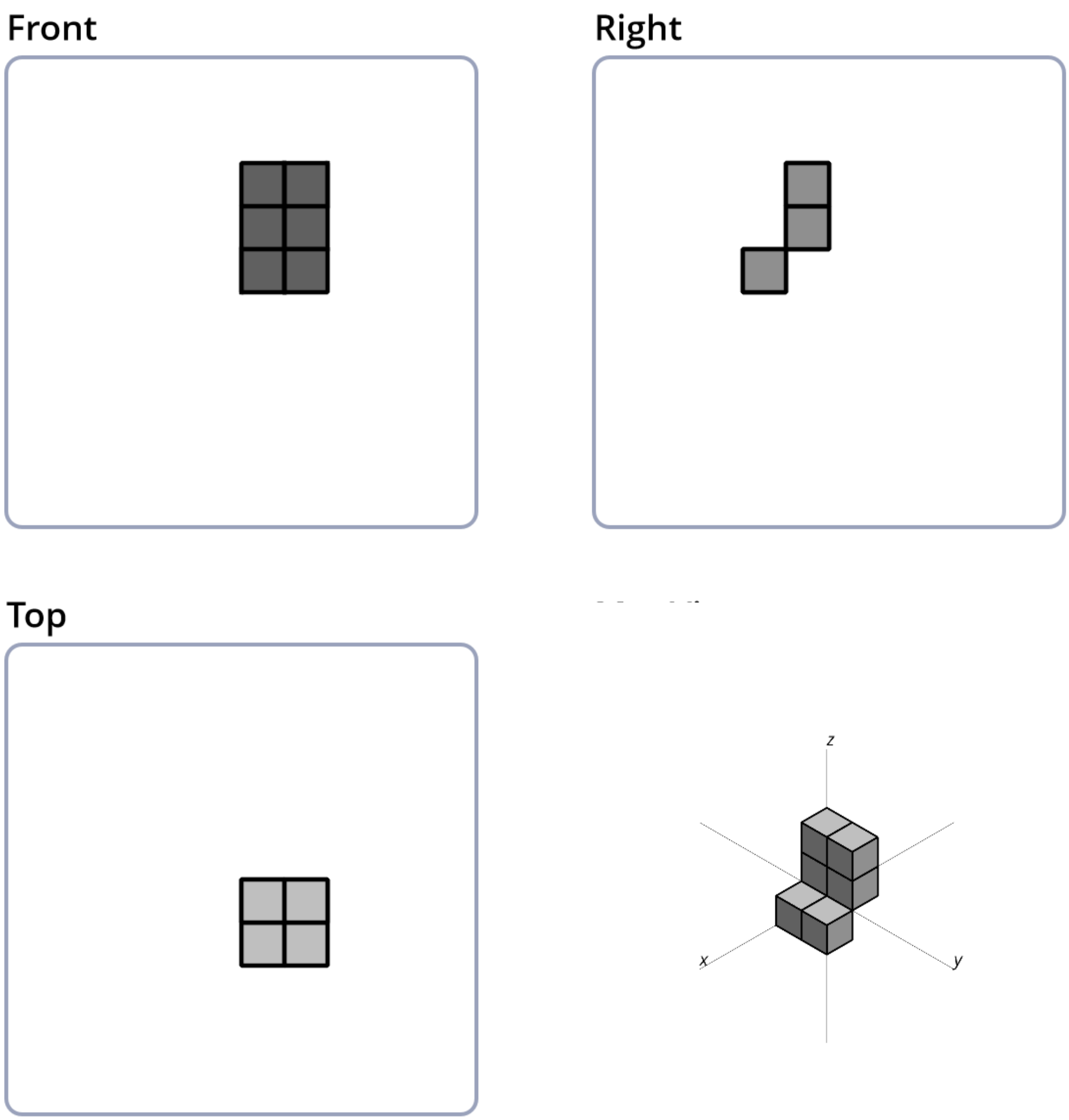

Figure 51: Isometric of puffer-1.

When two of this puffer-2 (see figure 52) frontally collide, a glider-4 (see figure 47) emerges.

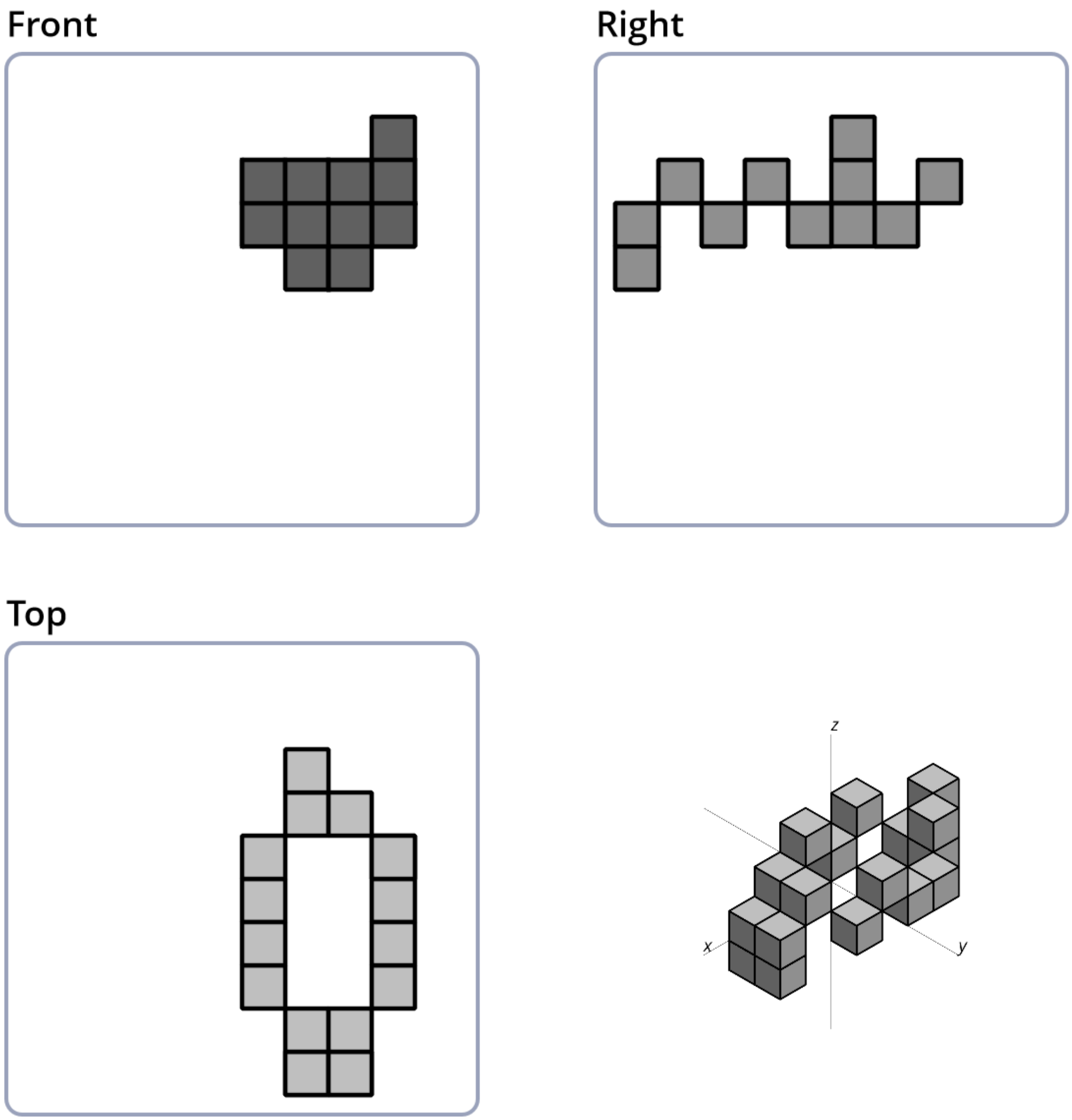

Figure 52: Isometric of puffer-2.

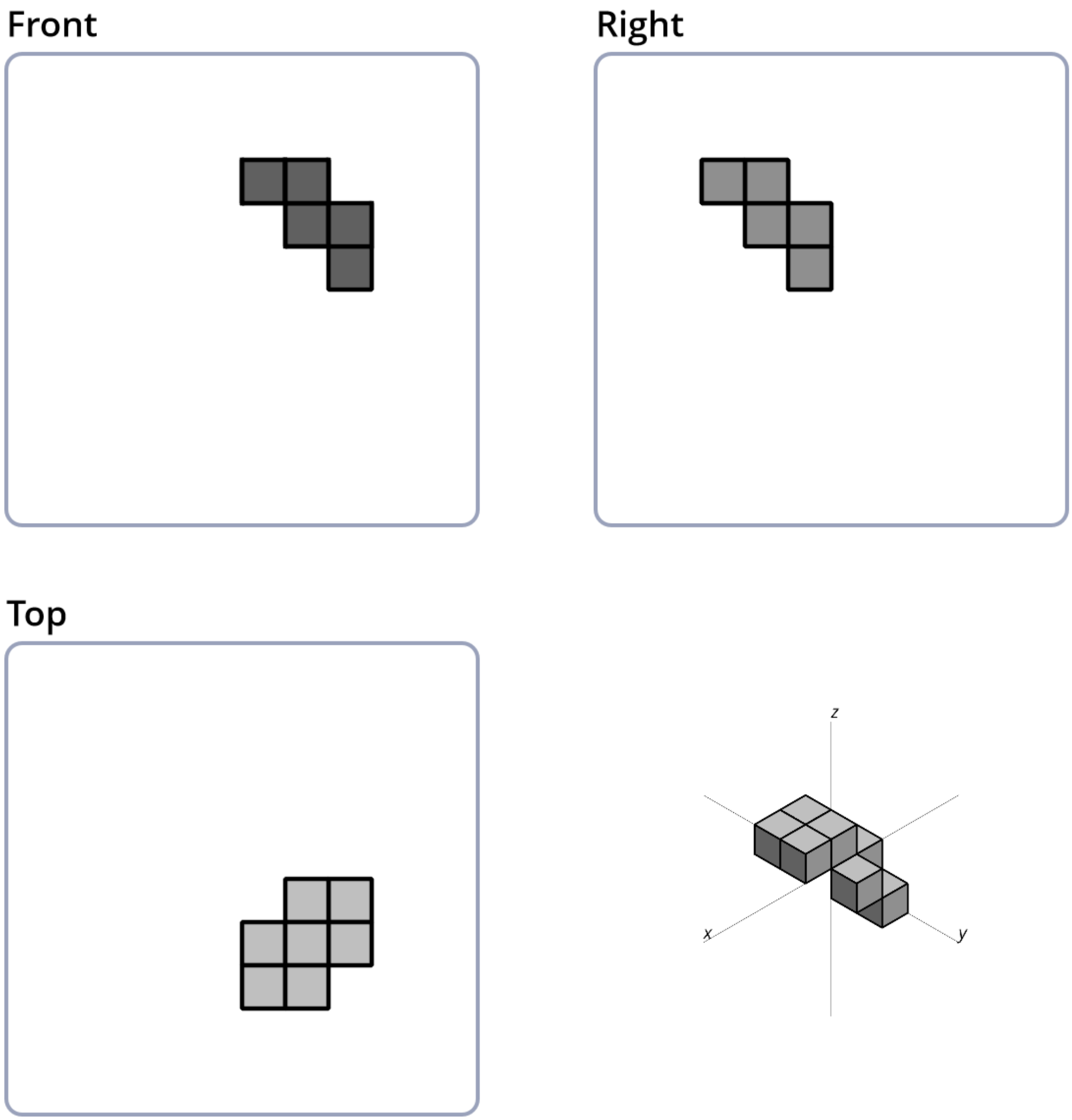

Figure 53: Isometric of puffer-3.

3 Conclusion

Talk about how long it took to view the simulations, even when the processor was HQ Talk about the configurations found (yayy), why it is important to find a glider gun (then, they would have ”cómputo universal”).

Even though the main objective was not achieved (a glider gun was not found), these experiments leaded to some interesting results (see section 2.3); for example, a replicator structure (see figure 50); the replicator-1, although could not be probed in this set of experiments, this configuration appears to be somehow important in the process of generating gliders; this supposition is made by the observation of the evolution of the s-pentomino population; at least four gliders were created and the pattern that appeared to create them kept replicating in the population; but the more the population grew, the more the population got in its way.

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Appendices

A Full Pentomino Evolution

Below are the full evolutions of each pentomino.

A.1 Pentomino P

A.2 Pentomino Q

A.3 Pentomino R

A.4 Pentomino S

A.5 Pentomino T

A.6 Pentomino U

A.7 Pentomino V

A.8 Pentomino W

A.9 Pentomino X

A.10 Pentomino Y

A.11 Pentomino Z