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(ALIROB)

3D Cellular Automata And Pentominos

Simulations and configurations

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Contents

1 Brief summary

1.1 Cellular Automaton

For more information, please consult [1]–[6].

A cellular automaton is a discrete dynamical system that consists of a set of simple units (cells) that change their states at each time step, depending of the states of their neighbors (there are no actions at a distance), according to a local update rule; the cells evolve in parallel at discrete time steps and they all use the same update rule.

They are also known as **CA** and have proof been quite useful, both as general models of complexity and as specific representations of non-linear dynamics in a great variety of fields. Despite functioning in a non-traditionally way, the **CA** can solve algorithmic problems or compute functions, all of this with the right suitable rule, of course.

Cellular automata come in a variety of shapes; the type of grid (for example, square, triangular or hexagonal cells in a two-dimensional **CA**) is one of it's most fundamental properties. The neighborhood over which cells affect one another is also very important; two common neighborhoods in the case of a two-dimensional **CA** on a square grid are the Moore and the von Newmann neighborhoods.

The **elementary cellular automata** are the simplest type of cellular automaton, being a binary, nearest-neighbor, one-dimensional automaton; these **CA** have been thoroughly studied. There are 256 **ECA**.

The best known cellular automaton is the *Game of life*, discovered by J. H. Conway in 1970; this is a binary automaton with a Moore neighborhood. Over the years, it has been compiled a library of patterns with various behaviours:

- **Still life**. A fixed point pattern; the update rule keeps each cell unchanged.

- **Oscillator**. A temporally periodic pattern; the update rule may change the pattern, but after some steps, the original pattern reapears in the same location with the same orientation.
- **Spaceship**. A pattern that reappears, after some steps, although not necessarily in the same location.
- **Gun**. A pattern that, as an oscillator, periodically returns back to the initial state and emits spaceships.
- **Glider gun**. A pattern that emmits gliders.

1.1.1 The Diffusion Rule

Conway's *Game of Life* inspired the cientific community and soon they started to experiment with variations of the game's rules to find new and interesing behaviours, for example, *high life*, *life 43* and *long life*; later they started experimenting with new neighborhoods, different shapes in the grids and the numer of dimensions in the universe. One rule stands out of the rest: the diffusion rule; mainly because it's behaviour is chaotic and with it, there has been discovered a great deal of gliders, oscillators, glider guns and puffer trains. The rule is defined as follow:

- Any living cell with less than seven neighbours dies out of loneliness.
- Any living cell with more than seven neighbours dies suffocated.
- Any living cell with seven neighbours lives onto the next generation.
- Any death cell with two living neighbours will be born in the next generation.

1.2 Pentominos

For more information, please consult [7]–[11].

A polyomino is a finite collection of orthogonally connected cells; or, in another workds, a collection of equally-sized squares that form a connected piece, meaning that each square can reach any other in it by going through adjacent squares; the order of a polyomino is the number of squares used to make it, so a fifth order polyomino (made of five squares), is called pentomino. Both terms, polyomino and pentomino were first used by S. Golomb in 1953, in a talk to the Harvard Mathematics Club and a year later in an article. There are twelve distinct pentominoes (see Figure 1) and two naming conventions, in this paper the Conway convention (using letters *O* through *Z*) will be used; although the resemblance to the letters with this labeling scheme seems a little more strained with the other scheme, specially when using the *O* instead of *I*, it has the advantage that uses 12 consecutive letters; also, the Conway scheme tends to be used when discussing topics related to cellular automata.

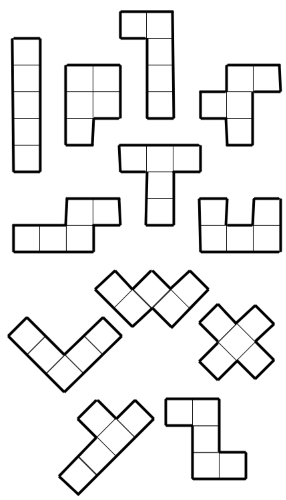

Figure 1: Conway pentominoes from O through Z (left to right).

Conway used polyominoes, specially pentominoes, in his initial investigations of Life and other cellular automata: he tracked the histories of small polyominoes as a way to explore the behaviour of different cellular automata when the calculations had to be done by hand; the studies of the polyominoes led to important discoveries in Life.

1.2.1 R-Pentomino

Among the other pentominoes, the **R-pentomino** outstands; mainly because of two reasons:

- It is a methuselah[1], meaning is a pattern that takes a large number of generations to stabilize and, at some point during it's evolution, becomes much larger than it's initial configuration.
- The first glider ever observed is the glider that releases in the 69th generation.

Since all the others pentominoes stabilize in ten generations or less, this is, by far, the most active pentomino: it takes 1103 generations and by then it's population is of 116.

[1] M. Gardner defined methuselahs as patterns of less than ten cells that take more than fifty generations to stabilize[12].

2 Experiment Results

The experiment consisted in evolving in a three dimensional universe, with cubes as units and an adaptation of the diffusion rule, each Conway pentomino and observing it's behaviour while watching out for configurations such as still lifes, oscillators gliders and specially, glider guns.

2.1 The Settings

The simulations were done in Ready, a cross-platform implementation of various reaction-difussion systems designed to explore both, continuous and discrete cellular automata on grids and arbitrary meshes [13]. Alas, precisely because it can be used with continuous and discrete universes, Ready is not optimized for experiments of this kind, provoquing quite slow evolutions.

Each experiment started with a pentomino in the center of a 256×256 empty cube; the simulation ended when there were no more alive cells or when the cells filled the space and started to interfere one another. Since Ready does not permit to move the view when the simulation is running, each pentomino scenario was run, at least, three times in order to obtain a front, lateral and in angle view of the evolution. The complete evolutions can be observed in A.

2.2 Pentomino By Pentomino

For each pentomino entry there will be an isometric figure to ilustrate the initial scenario. The pentominoes are alphabetically ordered and will be included only some moments of the evolution; ergo, the appendix contains the complete evolution of each pentomino.

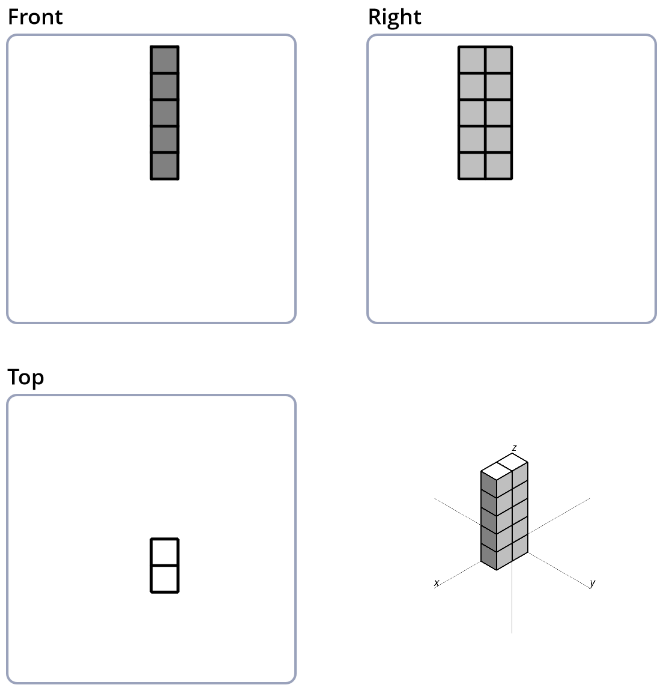

Figure 2: Isometric of the O-pentomino.

2.2.1 O-Pentomino

This is, as seen in figure 2, the simplest pentomino in the set. As seen in the figure 3, in the 9^{th} generation, it splits into two identical oscillators (see figure 32), sadly, each one obstructs the way of the other and by the 21^{th} generation, all the cells are dead.

2.2.2 P-Pentomino

This pentomino (see figure 4) is quite interesting; three configurations can bee seen whithin less than 47 generations. The first one to appear is a puffer train (see figure 33), latter, the first (and one of the simplest) glider appears (see figure 28); finally, another glider appears (see figure 29).

Figure 7 shows the state of the evolution of the p-pentomino in the 42^{th} generation;

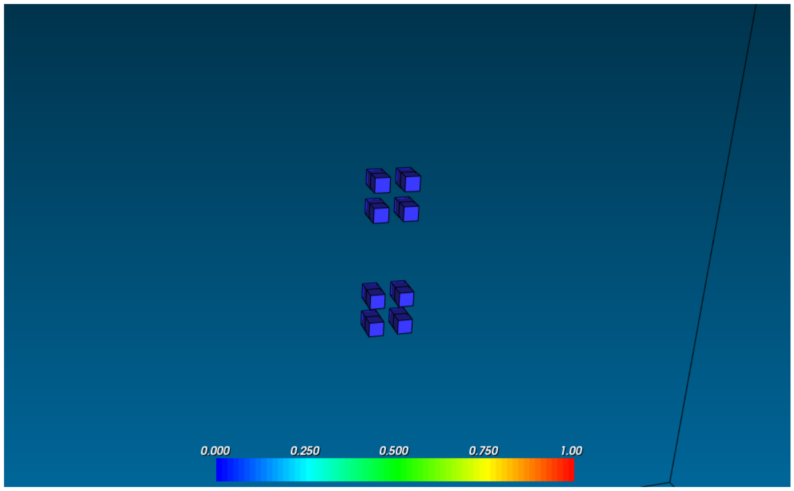

Figure 3: Evolution of the O-Pentomino, 9^{th} generation.

the puffer can be seen in the north of the evolution; while the gliders are located in the south of the population.

The simulation of this pentomino stopped at 210 generations because the space was overcrowded.

2.2.3 Q-Pentomino

This pentomino (see figure 8) shows in ten generations, two structures, one is a puffer train (see figure 33), and a *modified* glider: the glider-1 (see figure 28) with a satellite (see figure 30). The evolution of the 10^{th} generation with both structures can be seen in figure 9.

As shown in 10, in the 50^{th} generation, a mirroed glider-4 (see figure 31) emerges, both in the north and south of the population; the figures 11 and 12 show the gliders, zoomed, in the 50^{th} and 52^{th} generation, respectively.

The simulation of this pentomino stopped at 109 generations because the space was overcrowded.

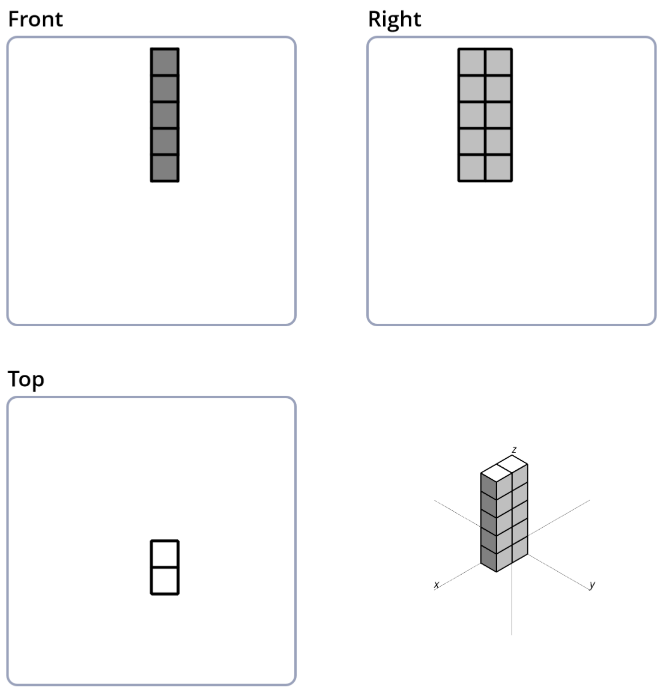

Figure 4: Isometric of the P-pentomino.

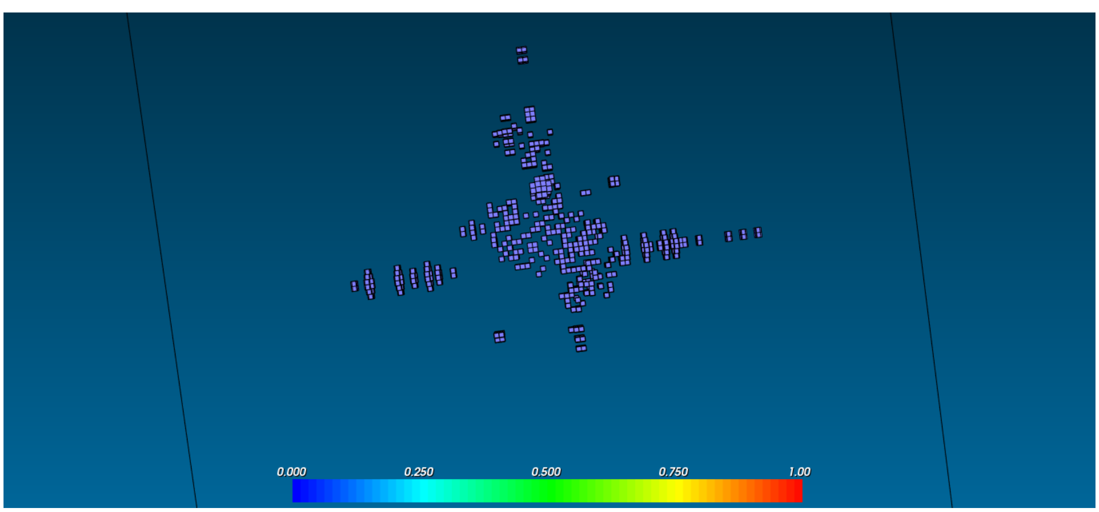

Figure 5: Evolution of the P-Pentomino, 42^{th} generation.

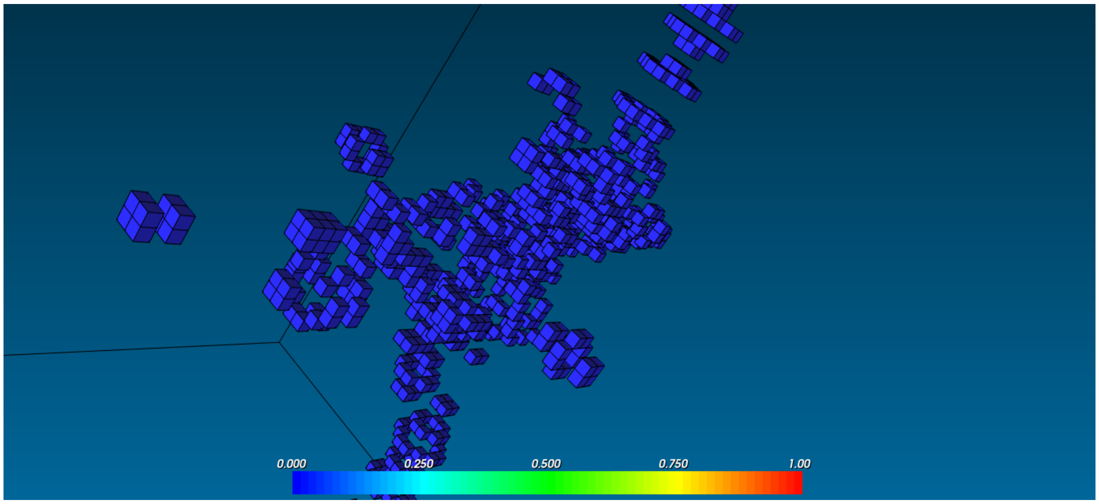

Figure 6: Evolution of the P-Pentomino, zoomed puffer train, 42^{th} generation.

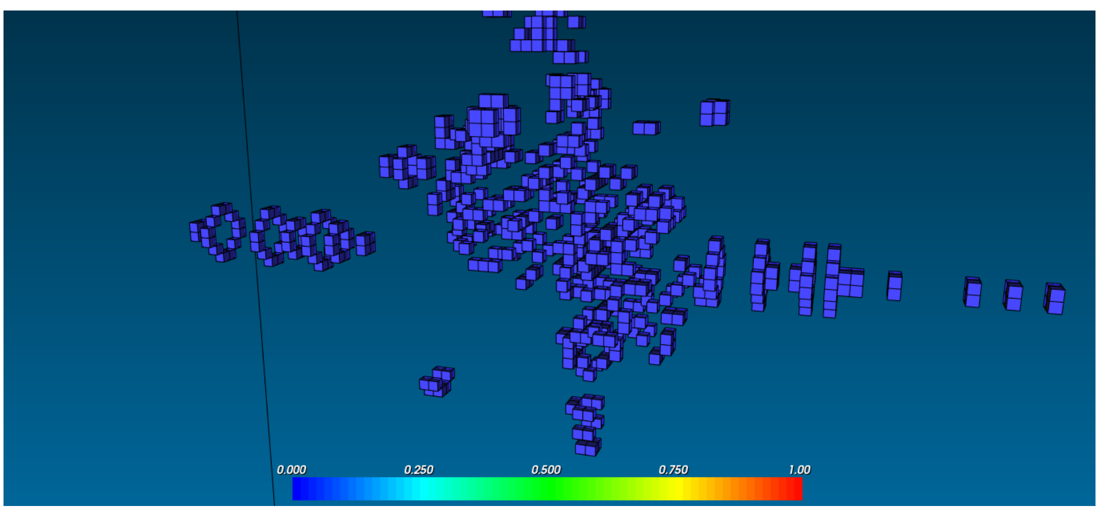

Figure 7: Evolution of the P-Pentomino, zoomed gliders, 42^{th} generation.

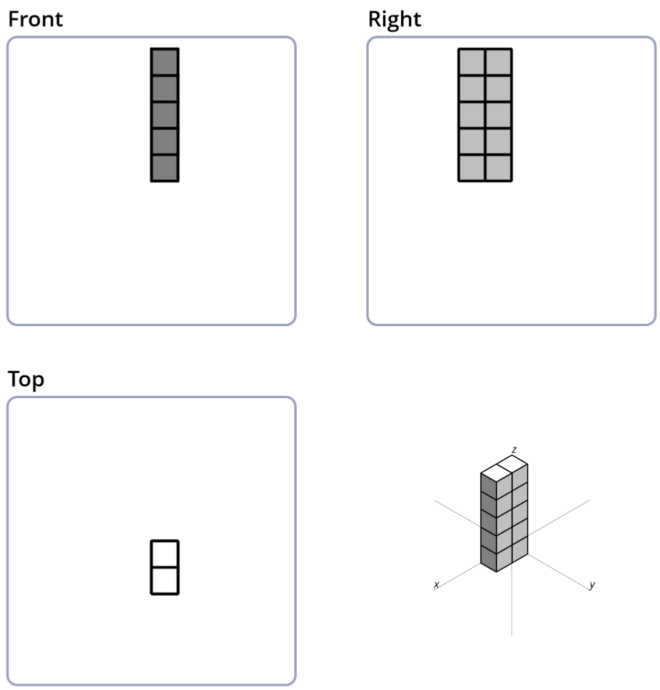

Figure 8: Isometric of the Q-pentomino.

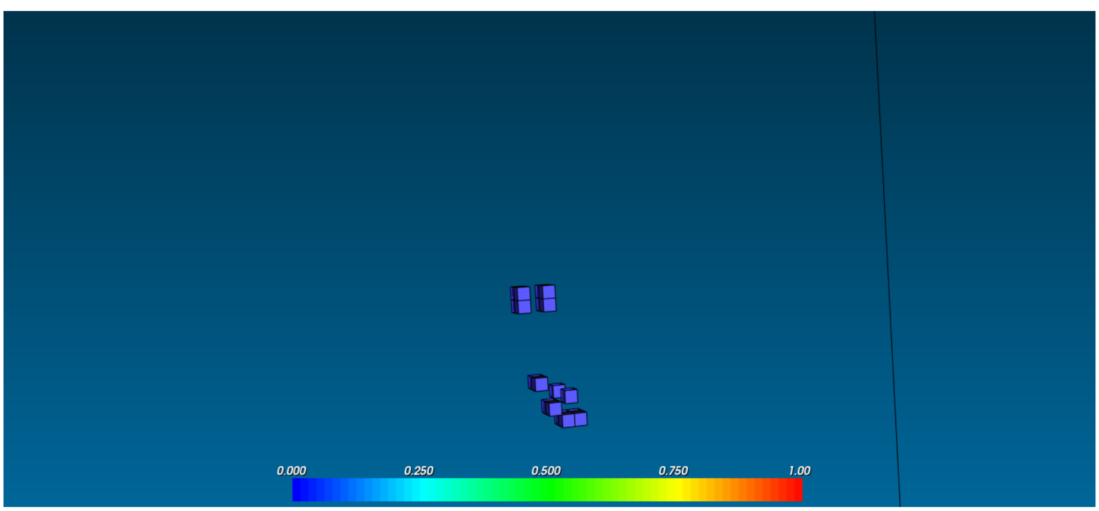

Figure 9: Evolution of the Q-Pentomino, 10^{th} generation.

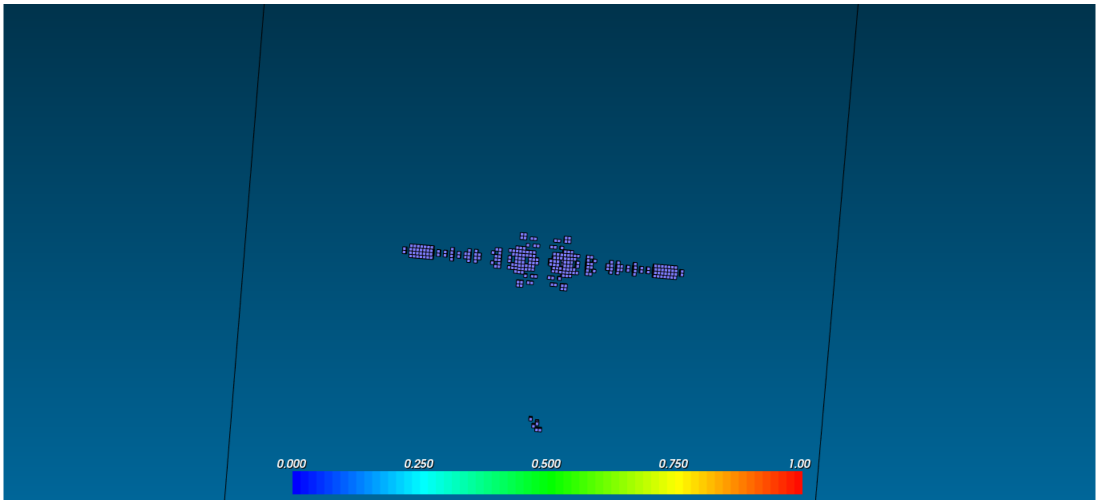

Figure 10: Evolution of the Q-Pentomino, 50^{th} generation.

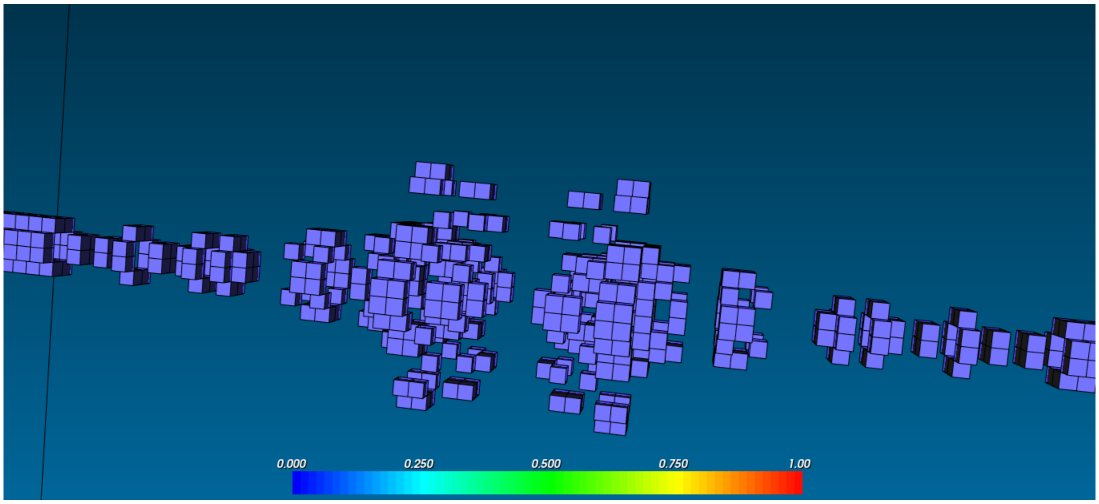

Figure 11: Evolution of the Q-Pentomino, zoomed gliders, 50^{th} generation.

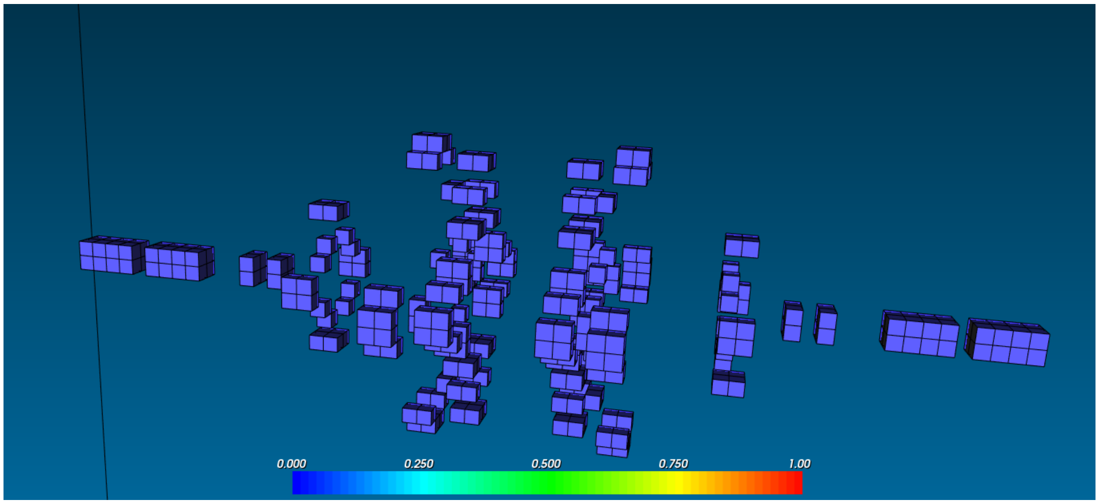

Figure 12: Evolution of the Q-Pentomino, zoomed gliders, 52^{th} generation.

2.2.4 R-Pentomino

This pentomino (see figure 13), just like it's equivalent in two dimensions, expands very quickly in a short period of time, this can be seen in the figure 14, where a comparative between the evolution achieved in the same generation for different pentominos.

The puffer train (see figure 33) appears here, too; as the glider-1 (see figure 28) and glider-4 (see figure 31); the gliders can be seen in the 81^{th} generation, in the north-east, nort-west, south-east and south-west areas of the figure 15; in the figure 16, a pair of glider-1 can be seen easily in the left of the figure; the glider-4 is shown in the upper-left of figure 17.

2.2.5 S-Pentomino

This pentomino (see figure 18) shows, quite fast, two puffer trains; first, the small one seen in previous pentominoes; later, however, a brand new clean puffer train (see figure 34) emerges; four of these puffers can be seen in the figure 19

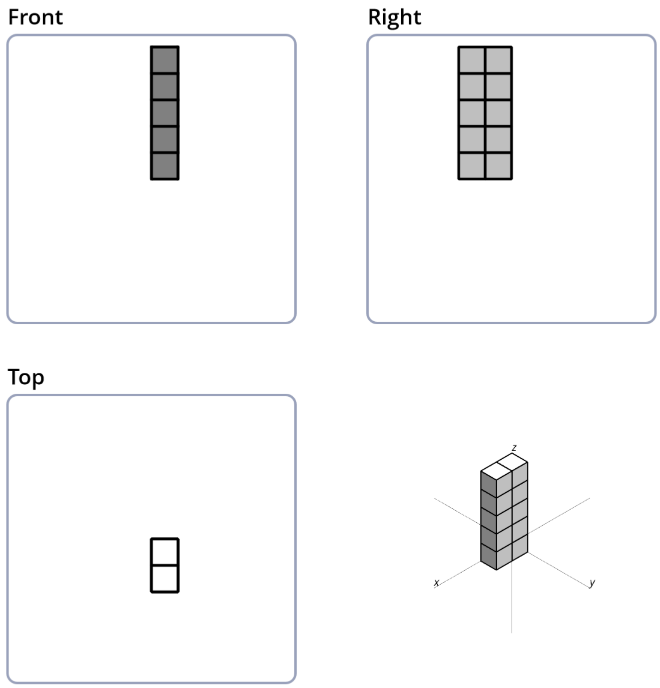

Figure 13: Isometric of the R-pentomino.

2.2.6 T-Pentomino

This pentomino (see figure 20),

2.2.7 U-Pentomino

This pentomino (see figure 21),

2.2.8 V-Pentomino

This pentomino (see figure 22),

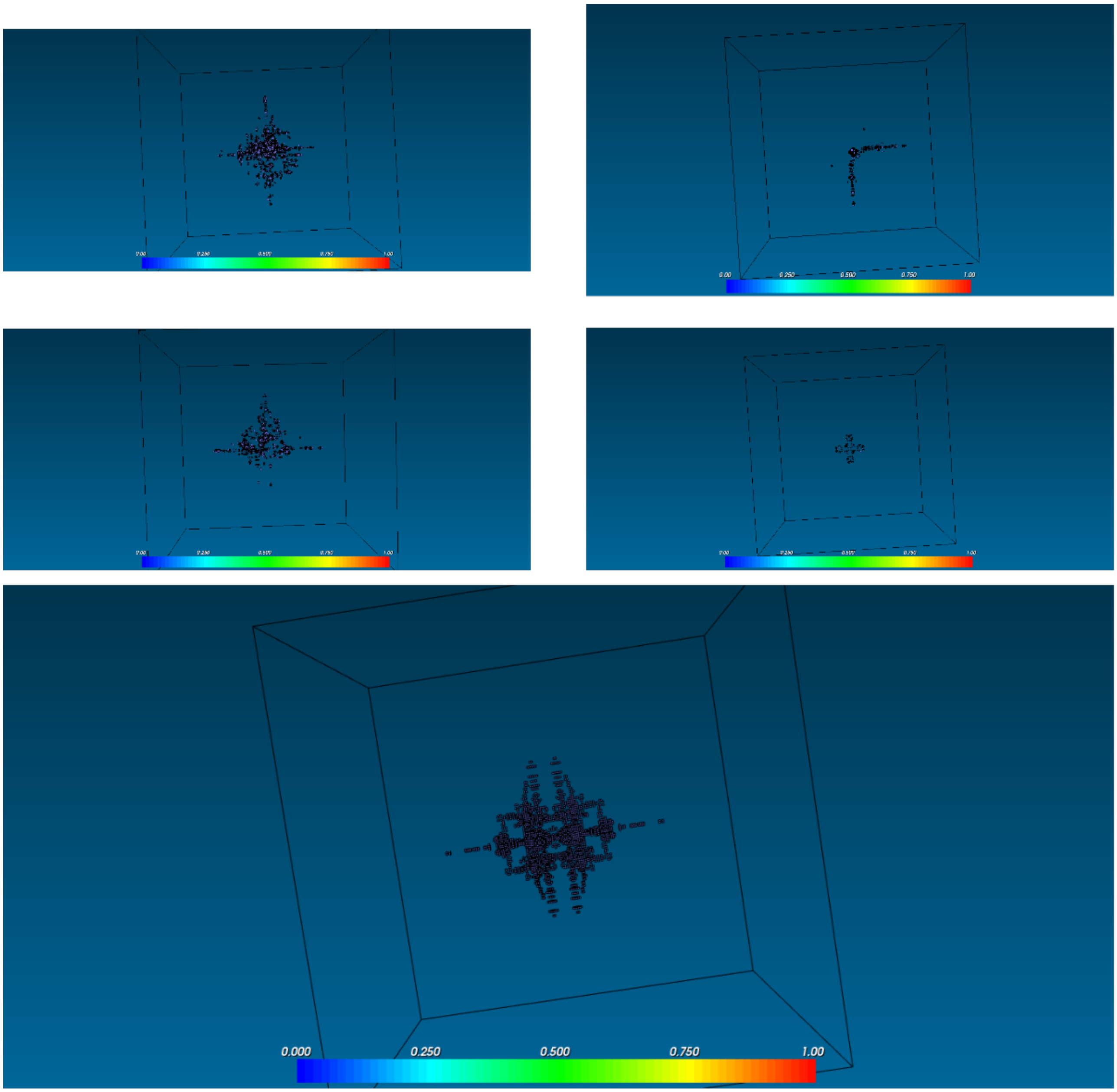

Figure 14: Evolution in the 52^{th} generation for the Y, W, P, X and R pentominoes respectively.

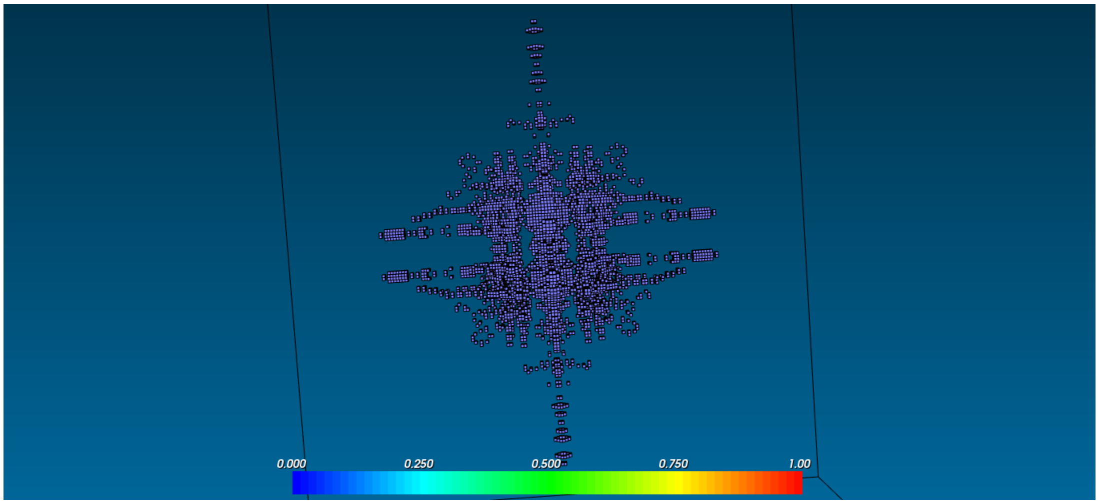

Figure 15: Evolution of the Q-Pentomino, 81^{th} generation.

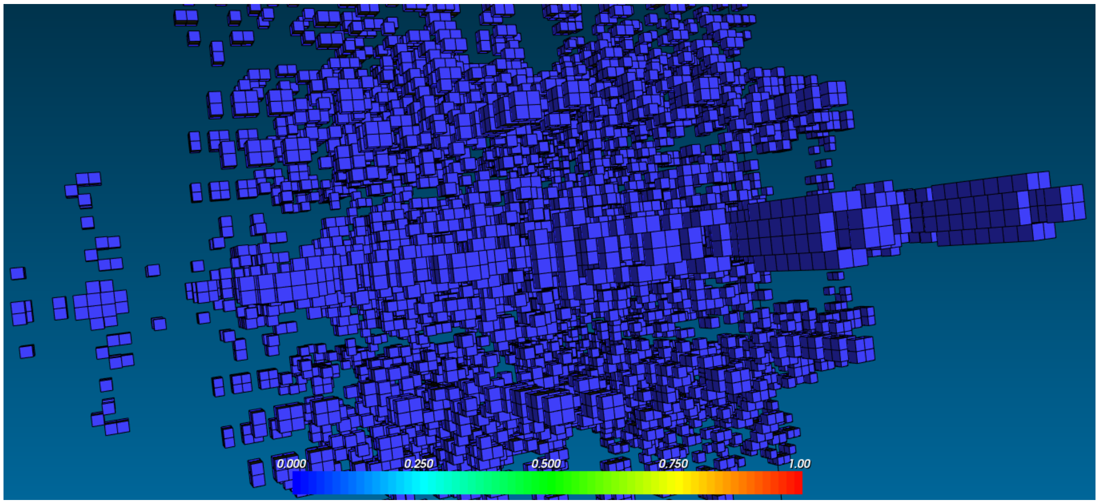

Figure 16: Evolution of the Q-Pentomino, zoomed glider-1, 81^{th} generation.

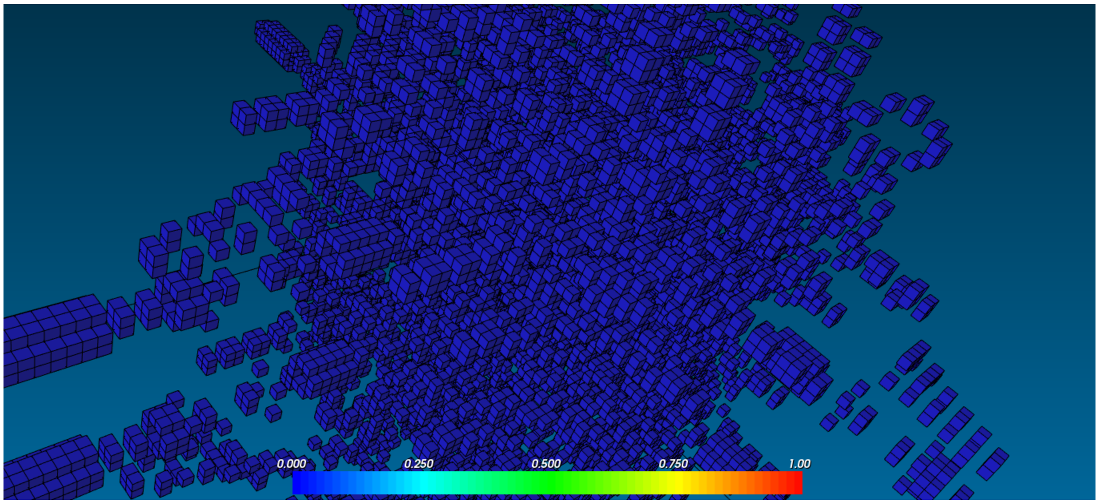

Figure 17: Evolution of the Q-Pentomino, zoomed glider-4, 81^{th} generation.

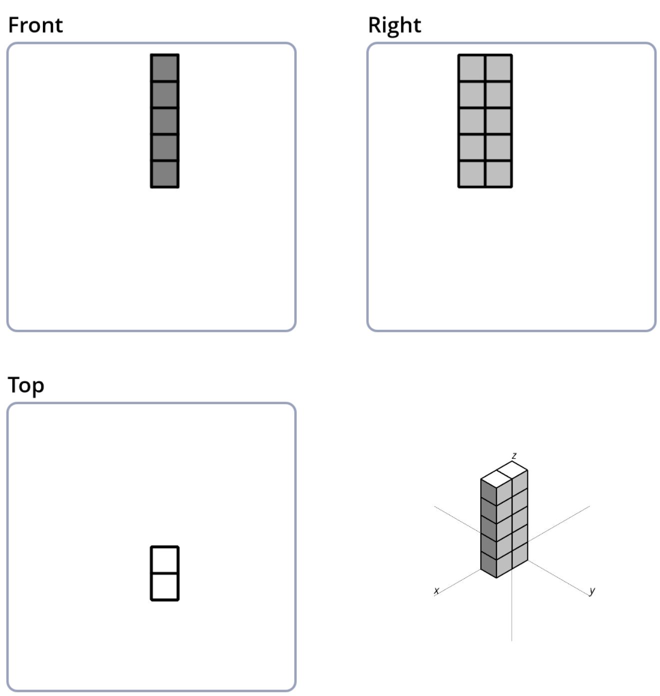

Figure 18: Isometric of the S-pentomino.

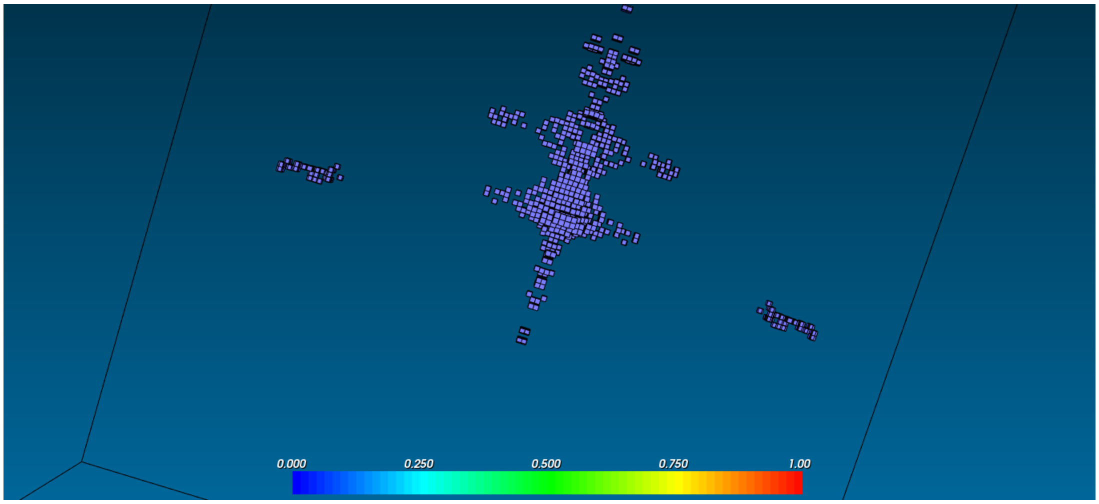

Figure 19: Evolution of the S-Pentomino, 56^{th} generation.

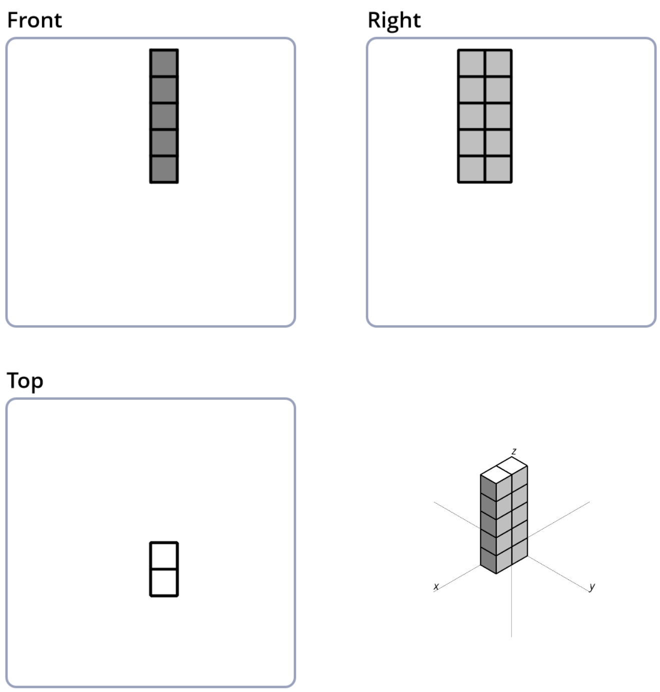

Figure 20: Isometric of the T-pentomino.

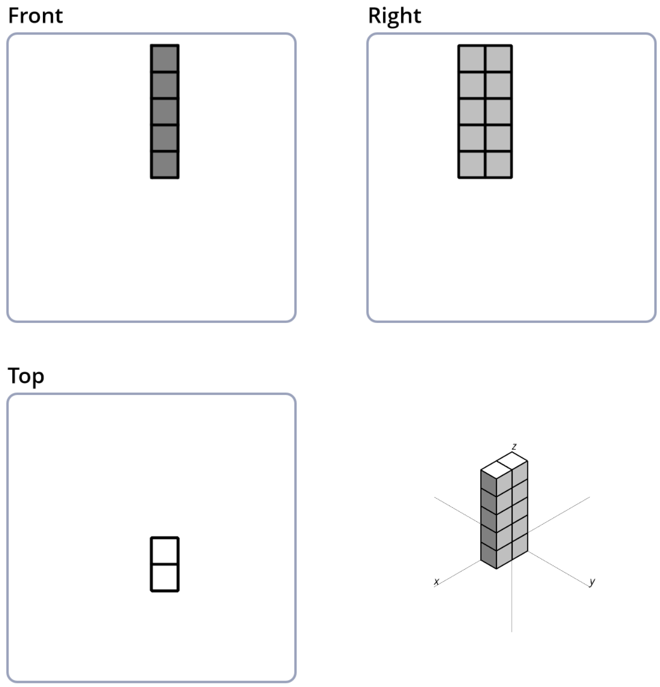

Figure 21: Isometric of the U-pentomino.

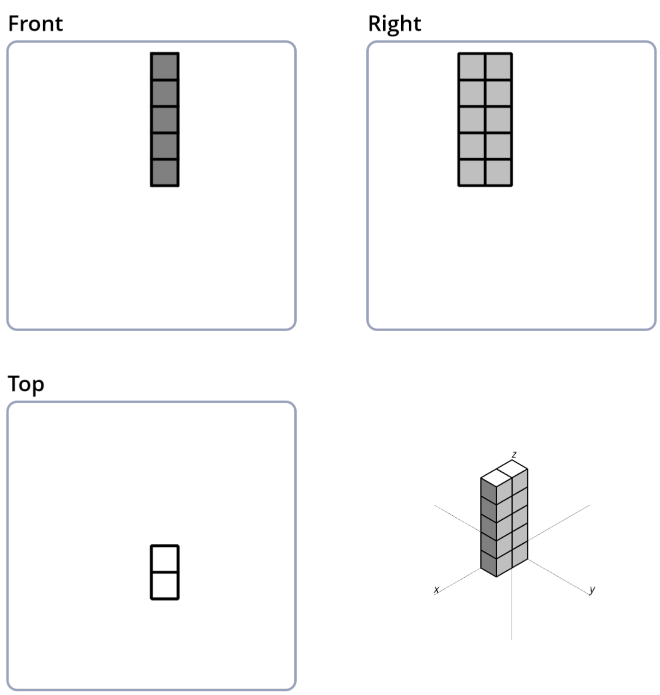

Figure 22: Isometric of the V-pentomino.

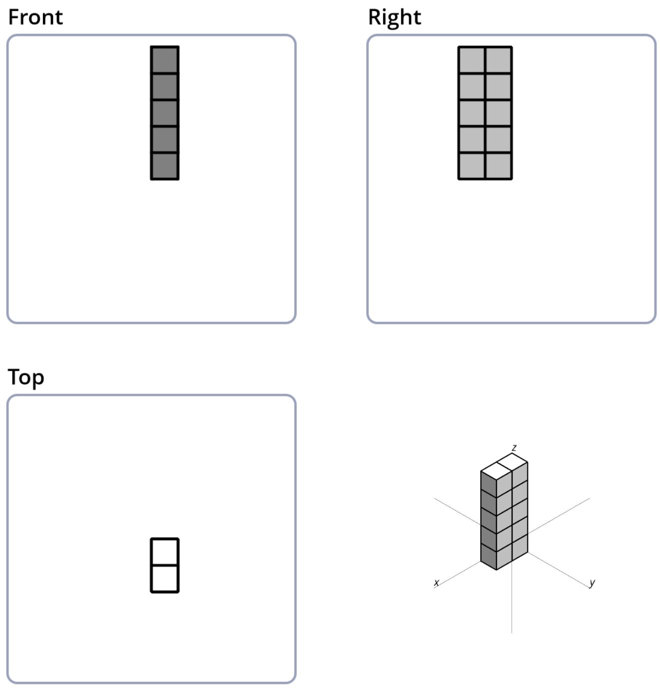

Figure 23: Isometric of the W-pentomino.

2.2.9 W-Pentomino

This pentomino (see figure 23),

2.2.10 X-Pentomino

This pentomino (see figure 24),

2.2.11 Y-Pentomino

This pentomino (see figure 25),

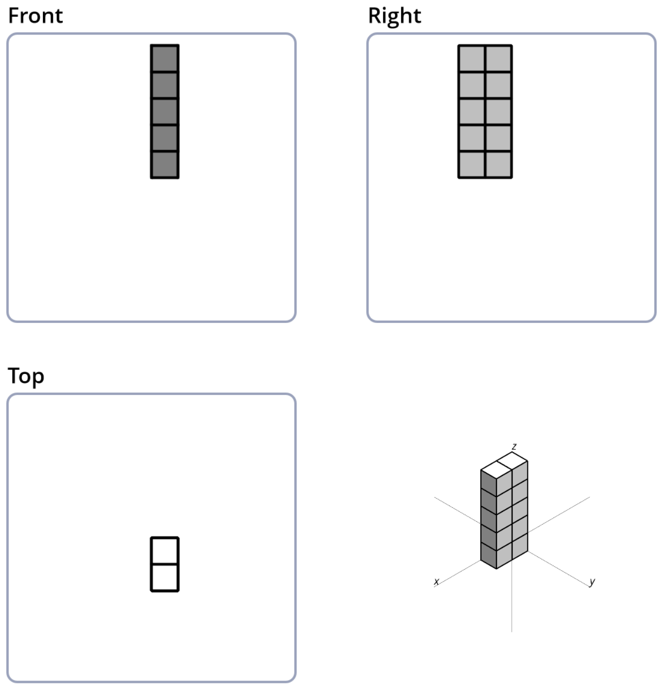

Figure 24: Isometric of the X-pentomino.

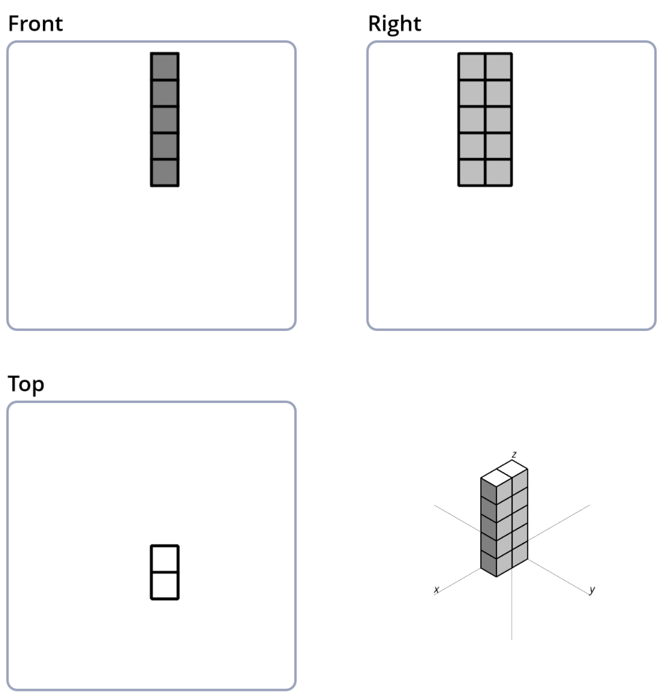

Figure 25: Isometric of the Y-pentomino.

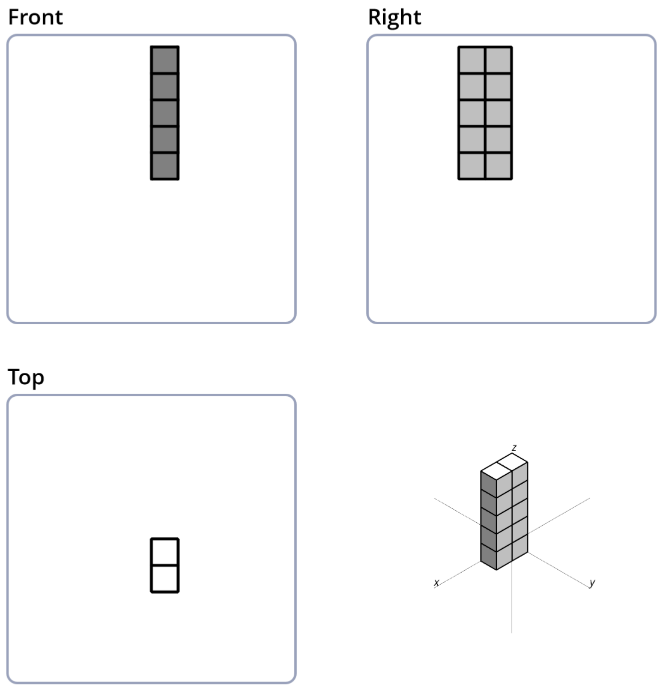

Figure 26: Isometric of the Z-pentomino.

2.2.12 Z-Pentomino

This pentomino (see figure 26),

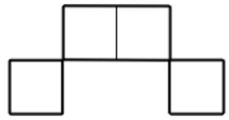

Figure 27: G1 glider in the Diffusion Rule.

2.3 Configurations found

Below are presented the configurations that were found while observing the evolutions of the pentominos; it is important to remark that, due to the fast paced population growth in some pentominoes, it is possible that some configurations remain hidden. The configurations are listed in the order that were found.

2.3.1 Gliders

For more information, please consult [5].

The gliders, as explained in section 1.1, are mobile particles traveling in the space. There are two types of gliders: primary and compound; the first ones cannot be decomposed into smaller mobile localizations, whereas a compound glider is made of at least two primary gliders. Some properties of the gliders are listed below:

- Volume
- Translation
- Period
- Speed
- Weight

One of the primary gliders in the Diffusion Rule is shown in figure 27.

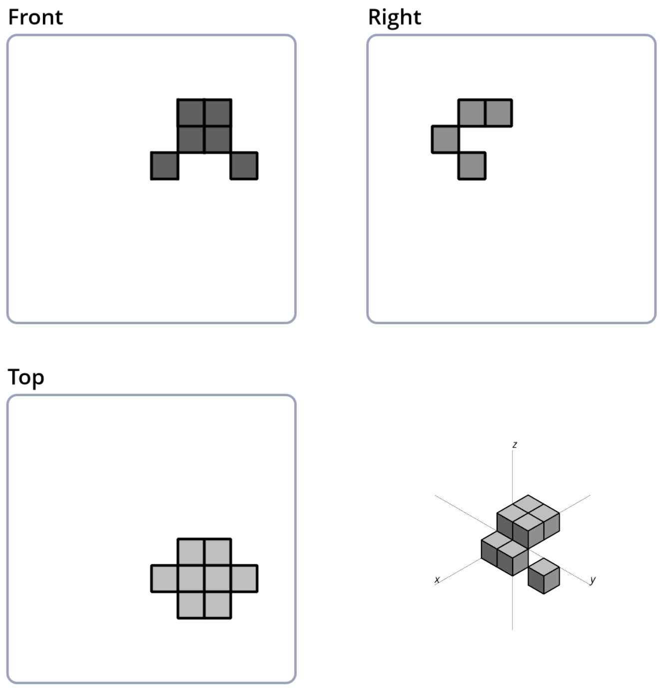

Figure 28: Isometric of glider-1.

Glider-1

Glider-2

Glider-3

Glider-4 This is, probably, the most simple glider; the equivalent of 27 in three dimensions.

2.3.2 Oscillators

Oscillator-1

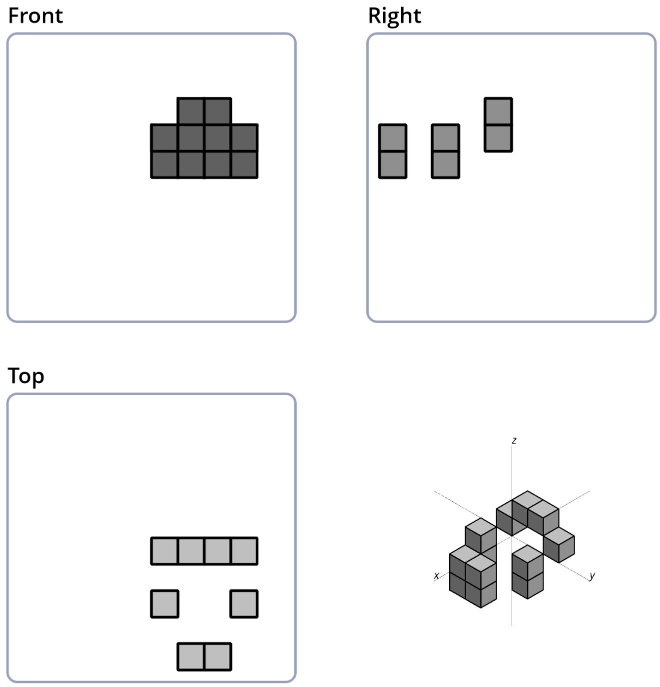

Figure 29: Isometric of glider-2.

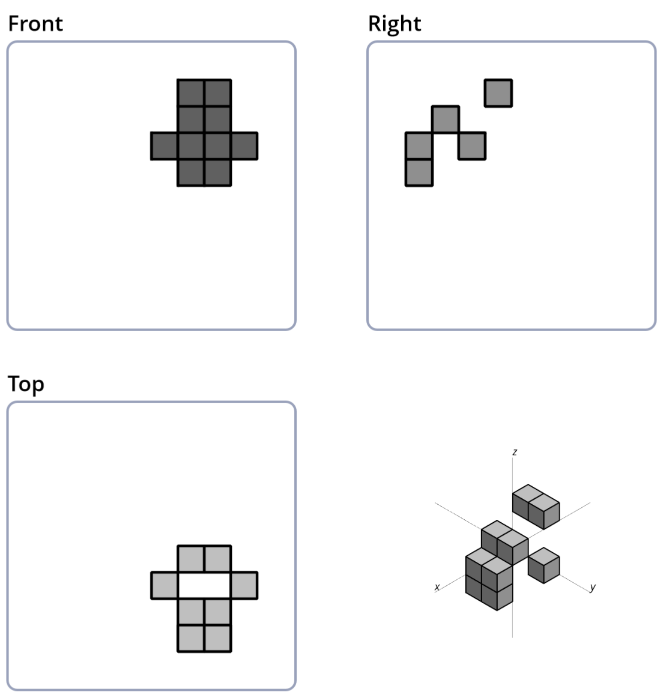

Figure 30: Isometric of glider-3.

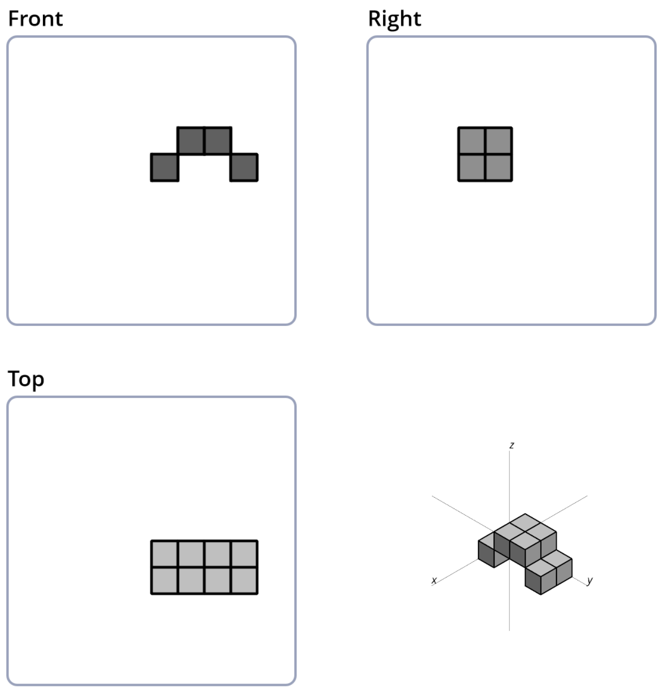

Figure 31: Isometric of glider-4.

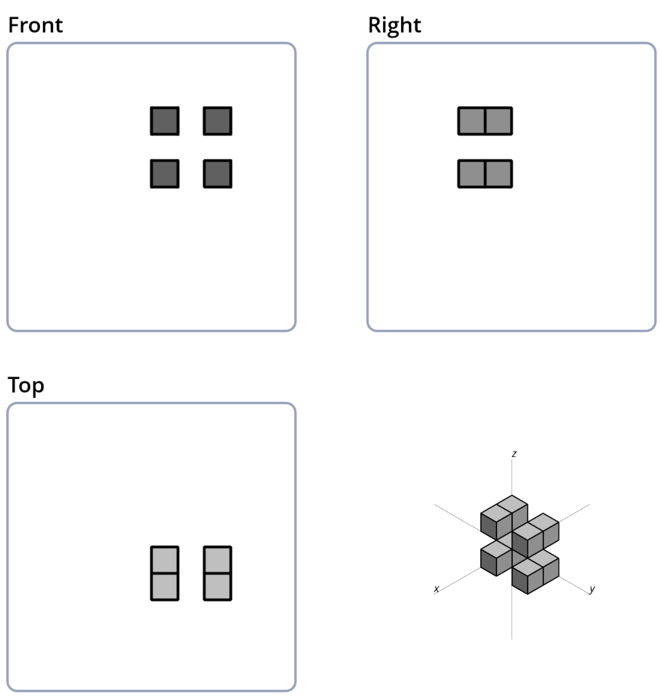

Figure 32: Isometric of oscillator-1.

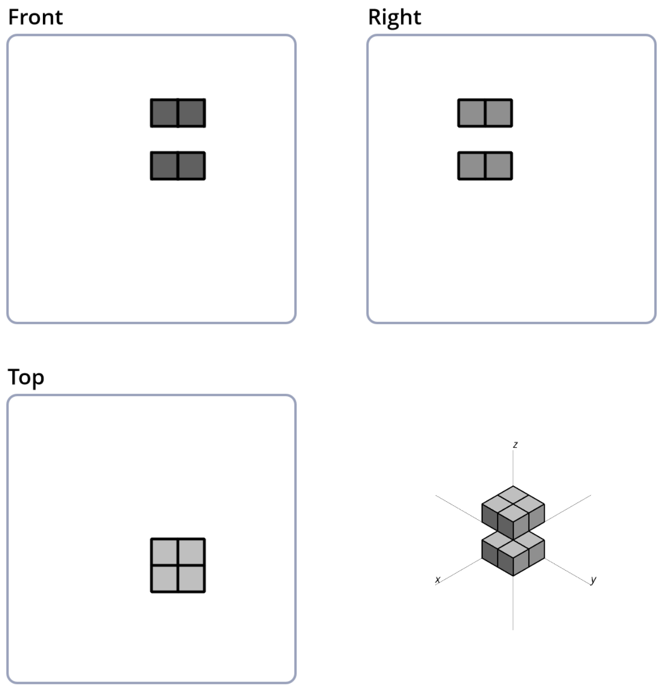

Figure 33: Isometric of puffer-1.

2.3.3 Puffer trains

Puffer Train-1

Puffer Train-2

3 Conclusion

Talk about how long it took to view the simulations, even when the processor was HQ Talk about the configurations found (yayy), why it is important to find a glider gun (then, they would have ”cómputo universal”).

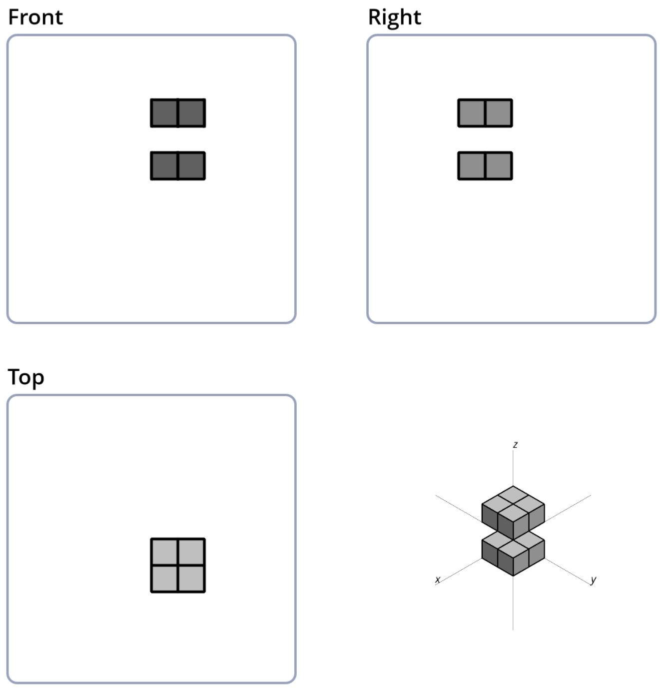

Figure 34: Isometric of puffer-2.

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Appendices

A Full Pentomino Evolution

Below are the full evolutions of each pentomino.

A.1 Pentomino P

A.2 Pentomino Q

A.3 Pentomino R

A.4 Pentomino S

A.5 Pentomino T

A.6 Pentomino U

A.7 Pentomino V

A.8 Pentomino W

A.9 Pentomino X

A.10 Pentomino Y

A.11 Pentomino Z