

# Crosscheck of the ANSYS-COMSOL 2D FEM Implementations for Superconducting Accelerator Magnets

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Within ANSYS there is the capability for users to create their own element type by writing the code which defines the element's properties and the generation of its finite element matrices. After the compilation of a custom ANSYS executable, all other aspects of the software (such as geometry generation, meshing, solving, and post-processing) are compatible with the user element. Control over the generation of the matrices allows for customization of the mathematical formulation, material properties, and many other aspects of the element. This note documents a verification study which was completed to crosscheck results from user defined elements in ANSYS developed at Berkeley [1] with a similar 2D FEM implementation in COMSOL and STEAM developed at CERN [2, 3].

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## 1 Introduction and General Comments

This note documents a verification study for the 2D ANSYS user elements developed at LBNL. Results are compared between these elements and a similar implementation developed at CERN in COMSOL. Two separate Nb<sub>3</sub>Sn dipoles models were used as a basis for the comparison. First a series of tests were performed for a single layer geometry slowly building up checks of inductance, material property fits, IFCC, quench, electromagnetic-thermal coupling, and finally coupling to an external circuit. A second geometry with two layers was then used in a final series of tests which built up the full range of multiphysics effects. For these final tests, both a dump resistor extraction and CLIQ based protection are compared.

More information about the user elements and their use can be found in

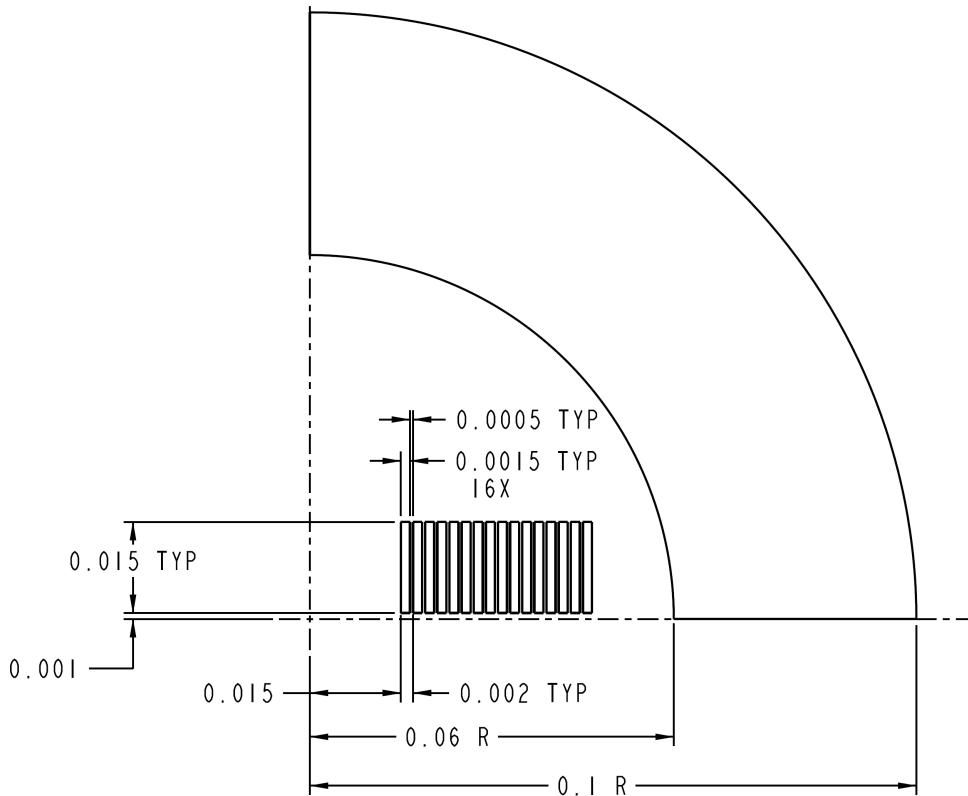
- [4]: general website
- [5]: documentation for USER101
- [6]: documentation for USER102
- [7]: check of equivalent magnetization IFCC in a uniformly changing background field for a single strand (comparison between USER102 results and analytic expectations)

## 2 A Single Layer Nb<sub>3</sub>Sn Dipole Model

### 2.1 Model and Geometry

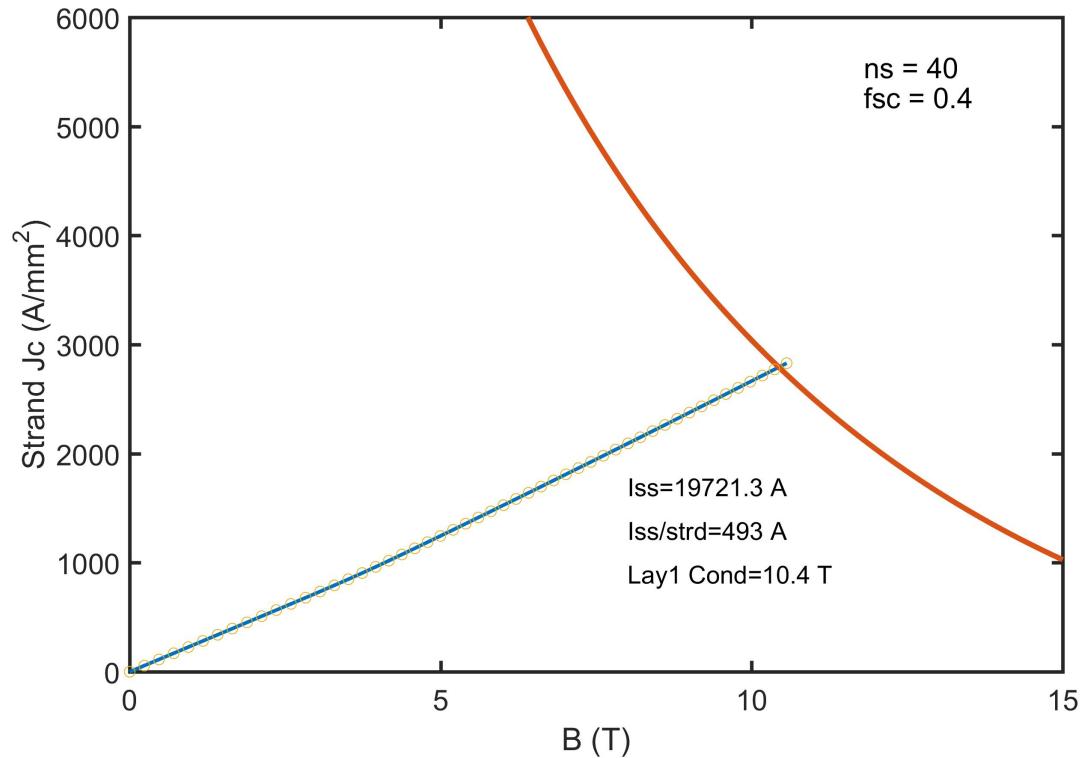
A dipole model was designed to allow for comparison of results in a field and current regime representative of realistic Nb<sub>3</sub>Sn accelerator magnets. This model consists of a single, block-like conductor layer surrounded by a cylindrical iron yoke. The figure below details the model dimensions (with units of meters), and a list of high level model parameters are given in the table. The cable and conductor properties are given in Appendices A and B. The non-linear BH curve for the iron yoke can be found in Appendix D. The outer, radial boundary conditions are assumed to be far enough from the coil and yoke to no longer have an impact on the results. There are also .stp and .dxf cad files (named “geo”) included. In many cases the compared results are averaged over the first half turn labeled “HT0”. This is the cable nearest to the aperture (farthest left of the 16).

Turns (per quadrant)	16	
Nb <sub>3</sub> Sn J <sub>c</sub> (4.5 K, 12 T)	2040	A/mm <sup>2</sup>
Short-sample current (4.5 K)	19.7	kA
Short-sample cond. field (4.5 K)	10.4	T
Lc: effective res. coil length	10.11	m
Li: effective ind. coil length	9.2	m



## 2.2 Short-Sample

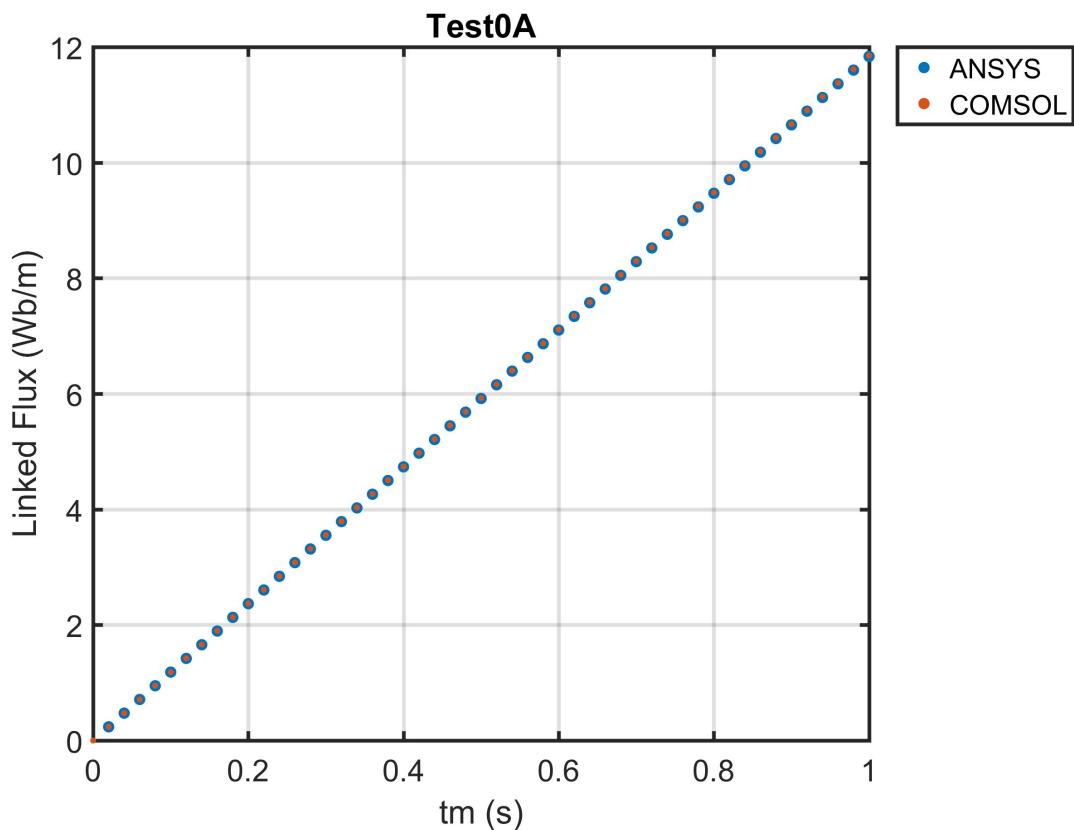
The strand and cable parameters in Appendix A, load line with iron (from Test0B), and  $J_c$  fit found in Appendix B were used to calculate the short-sample for the single layer model with iron. As seen in the figure below, the short-sample conductor field is 10.4 T at a current of 19.72 kA when operating at a temperature of 4.5 K.

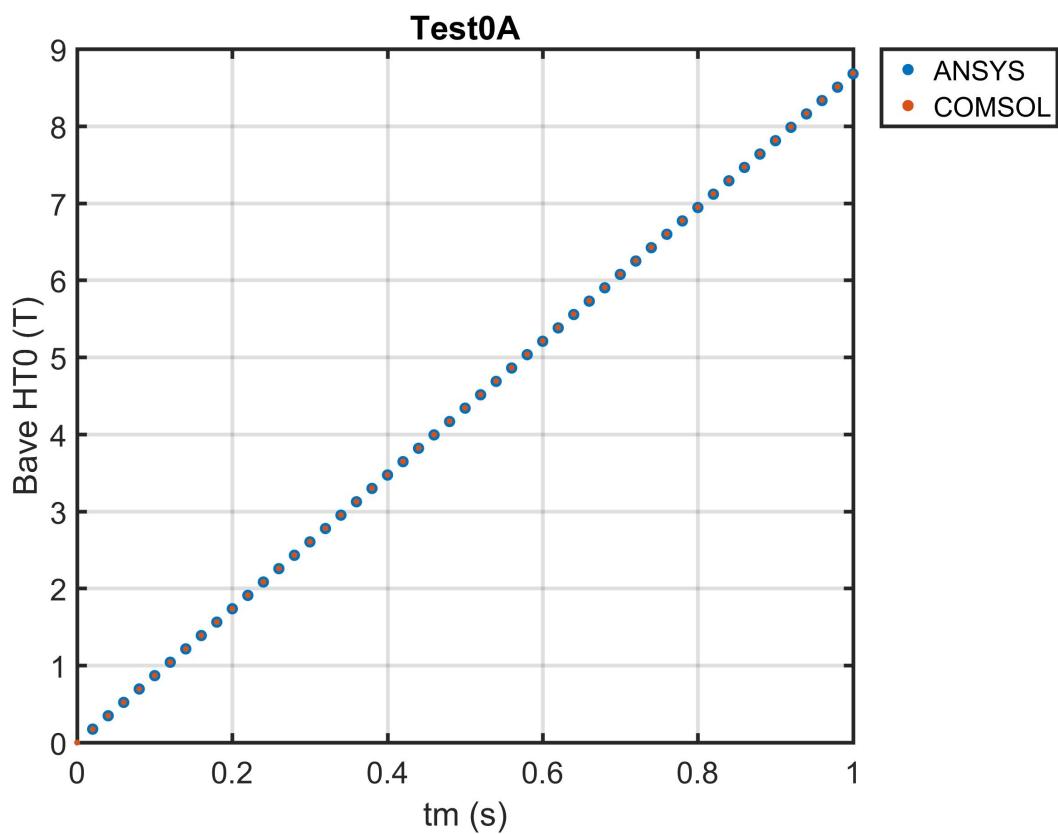
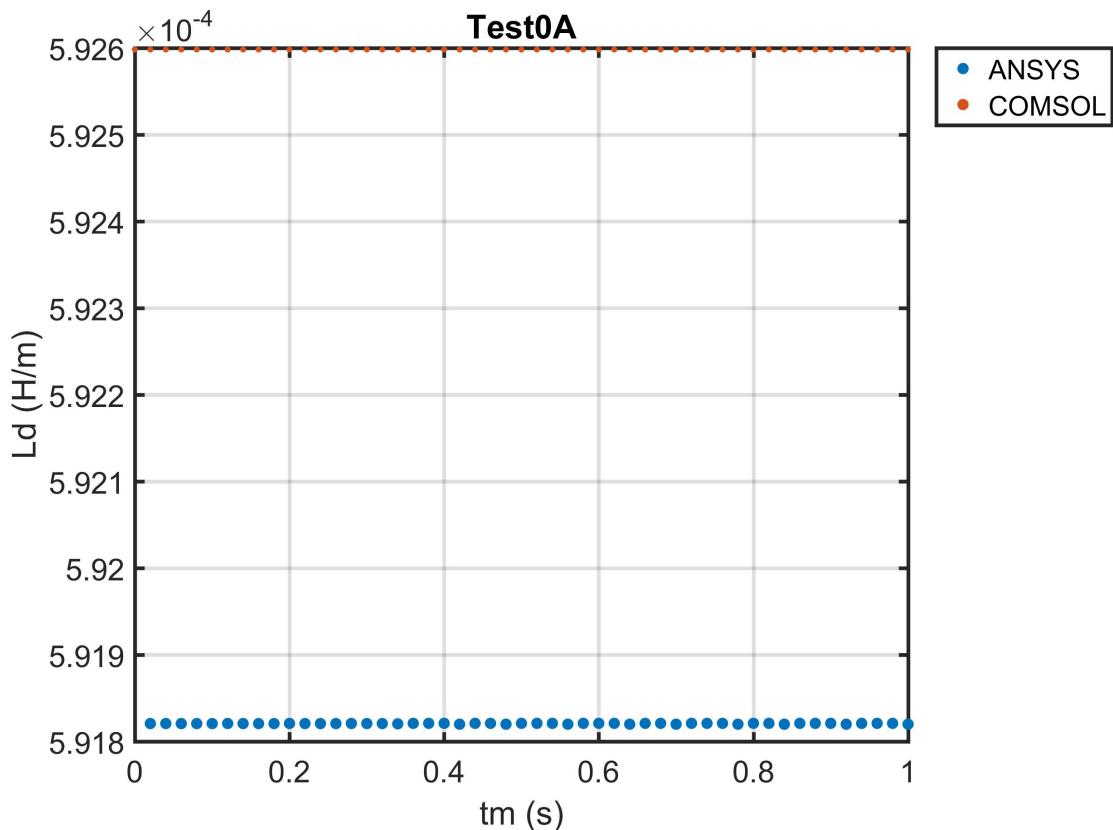


### 3 Single Layer Tests

#### 3.1 Test 0.A Differential Inductance (no iron yoke)

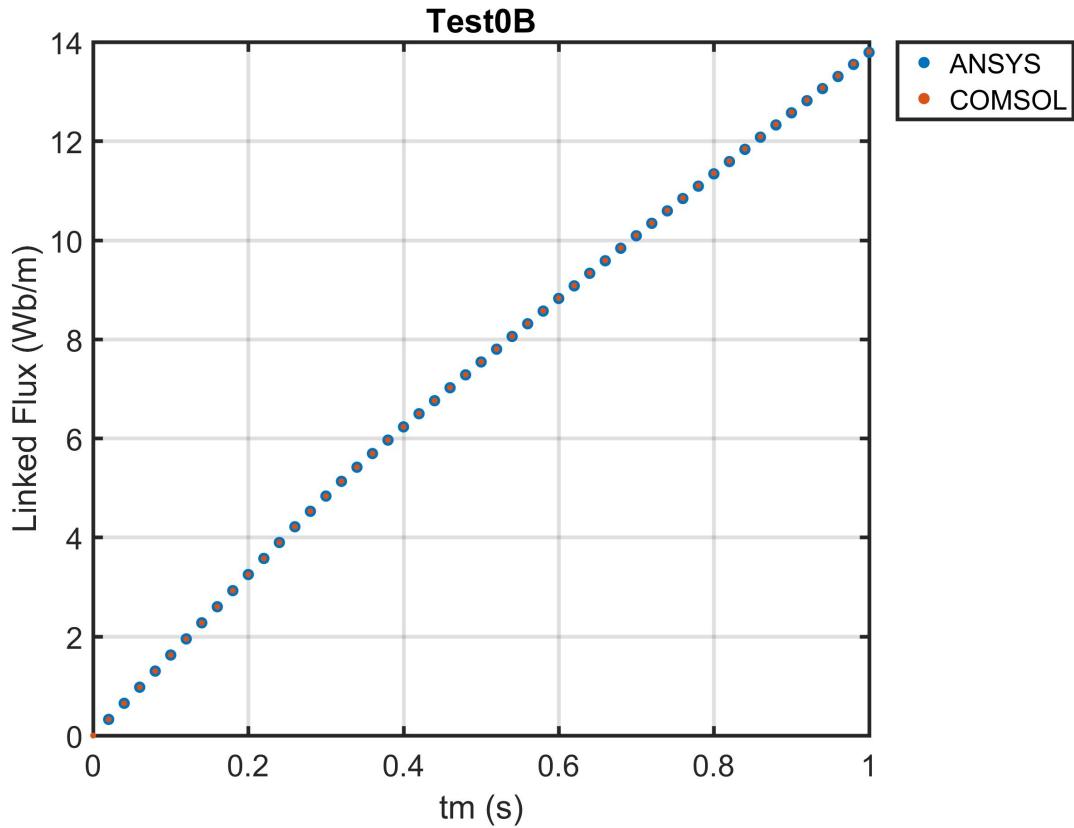
Active physics	EM
Iron yoke	NO
IFCC	NO
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	None
External loads	Linear ramp-up, $t=(0,1)$ [s], $\Delta t=20e-3$ [s], $i=(0,20e3)$ [A]
<b>Output</b>	Coil magnetic flux $\phi$ [Wb/m]
	Differential inductance as $L_d = \Delta\phi/\Delta i$ [H/m] (finite difference)
	$B_{aveHT0}$ [T]

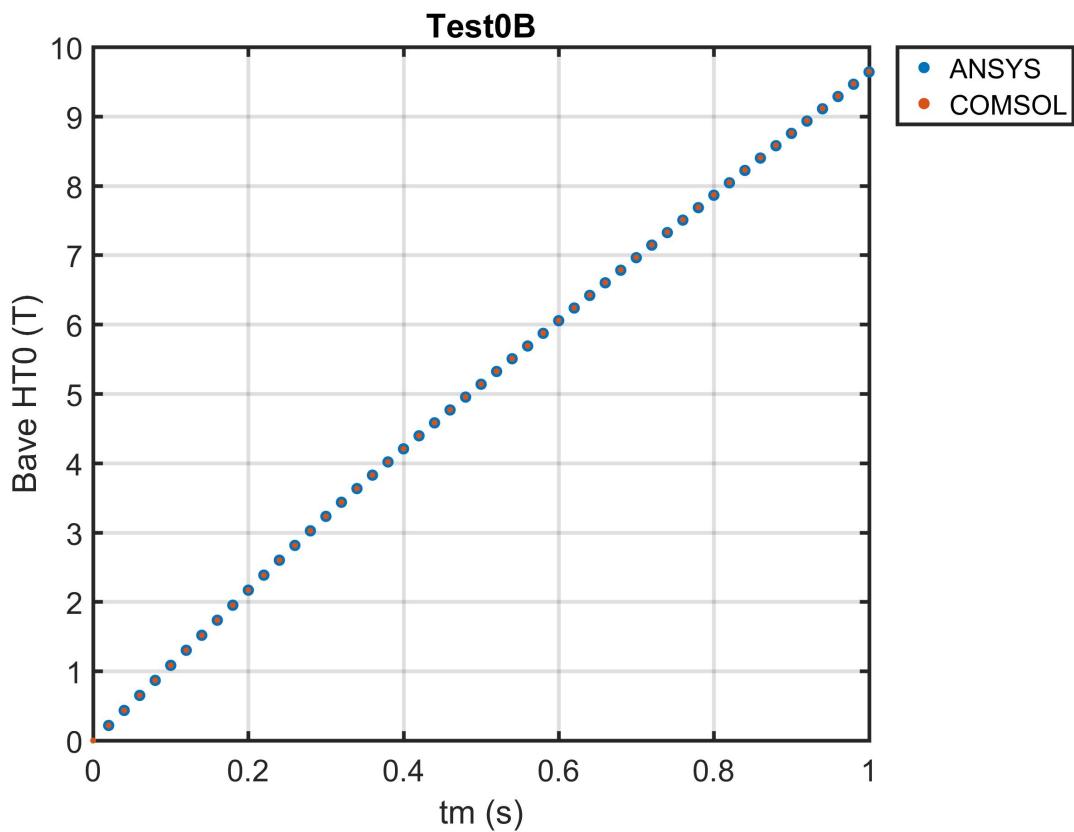
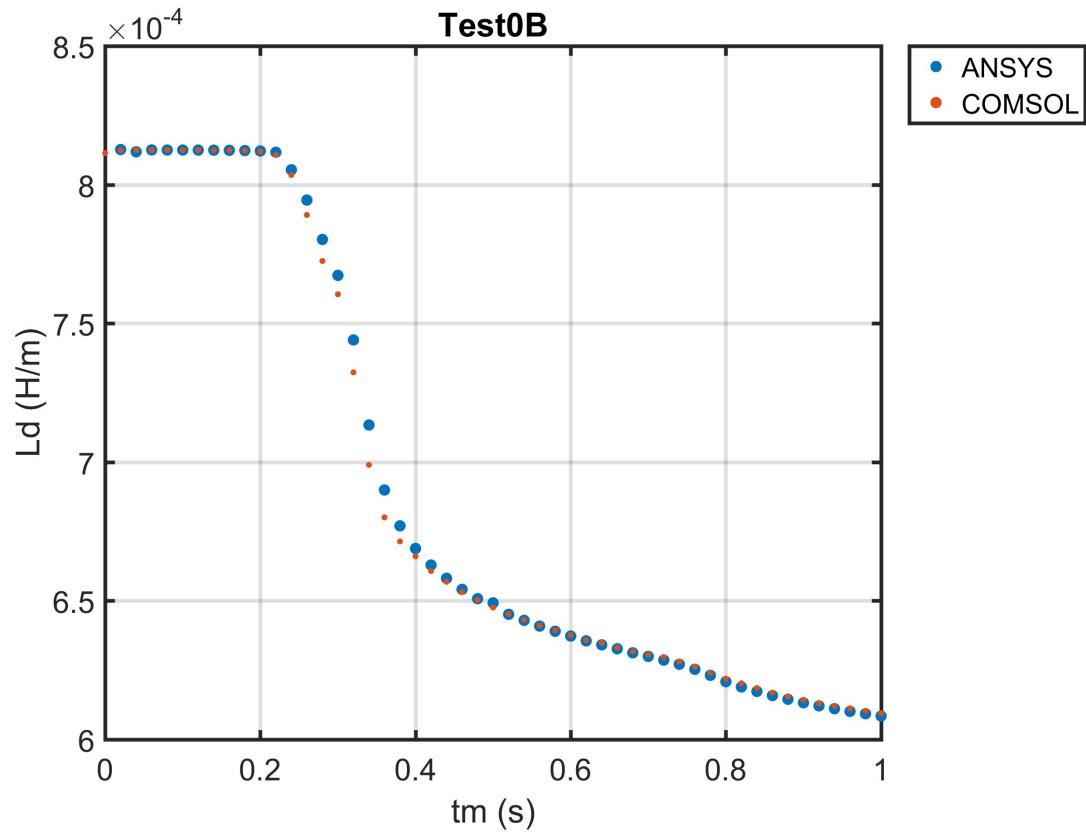




### 3.2 Test 0.B Differential Inductance (with iron yoke)

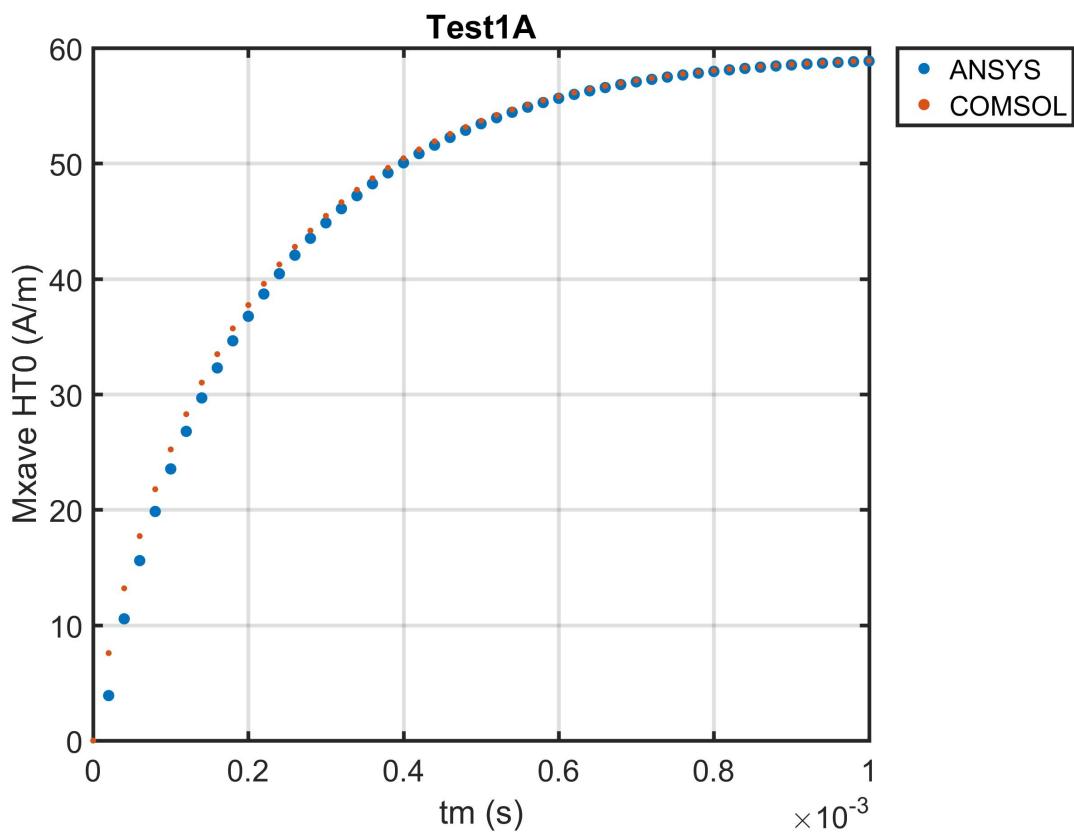
Active physics	EM
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	None
External loads	Linear ramp-up, $t=(0,1)$ [s], $\Delta t=20e-3$ [s], $i=(0,20)$ [kA]
<b>Output</b>	Coil magnetic flux $\phi$ [Wb/m]
	Differential inductance as $L_d = \Delta\phi/\Delta i$ [H/m] (finite difference)
	$B_{aveHT0}$ [T]

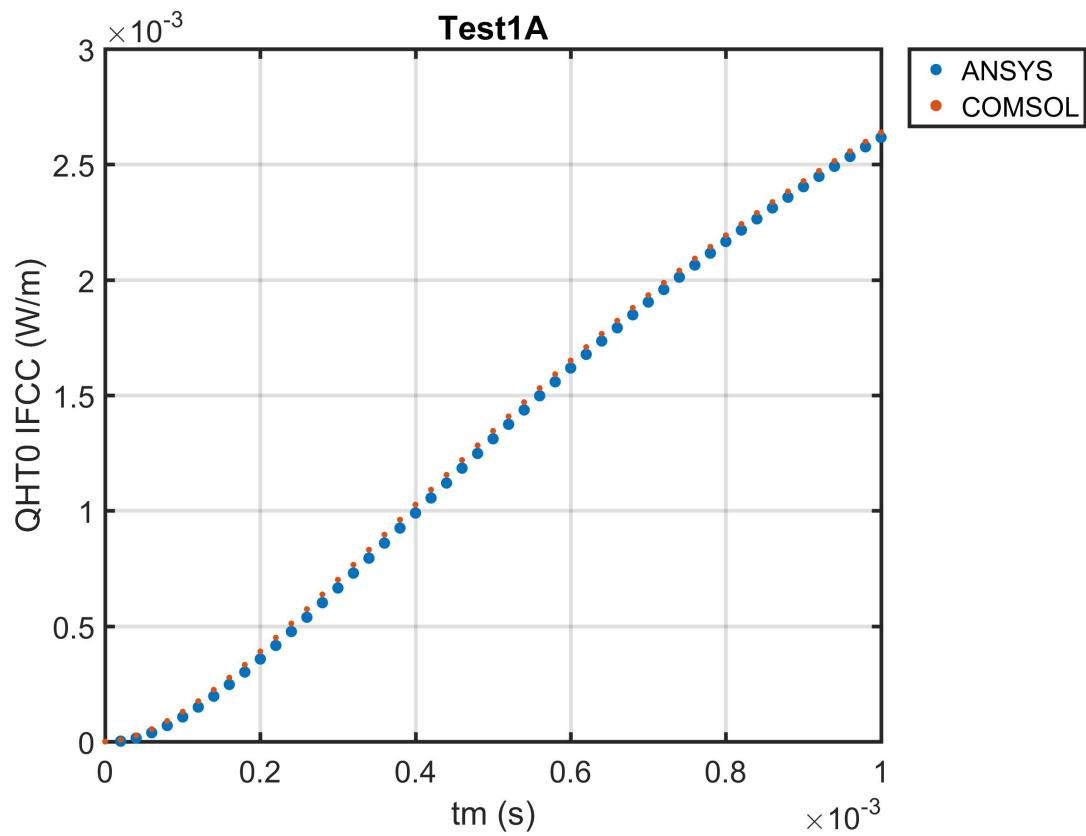
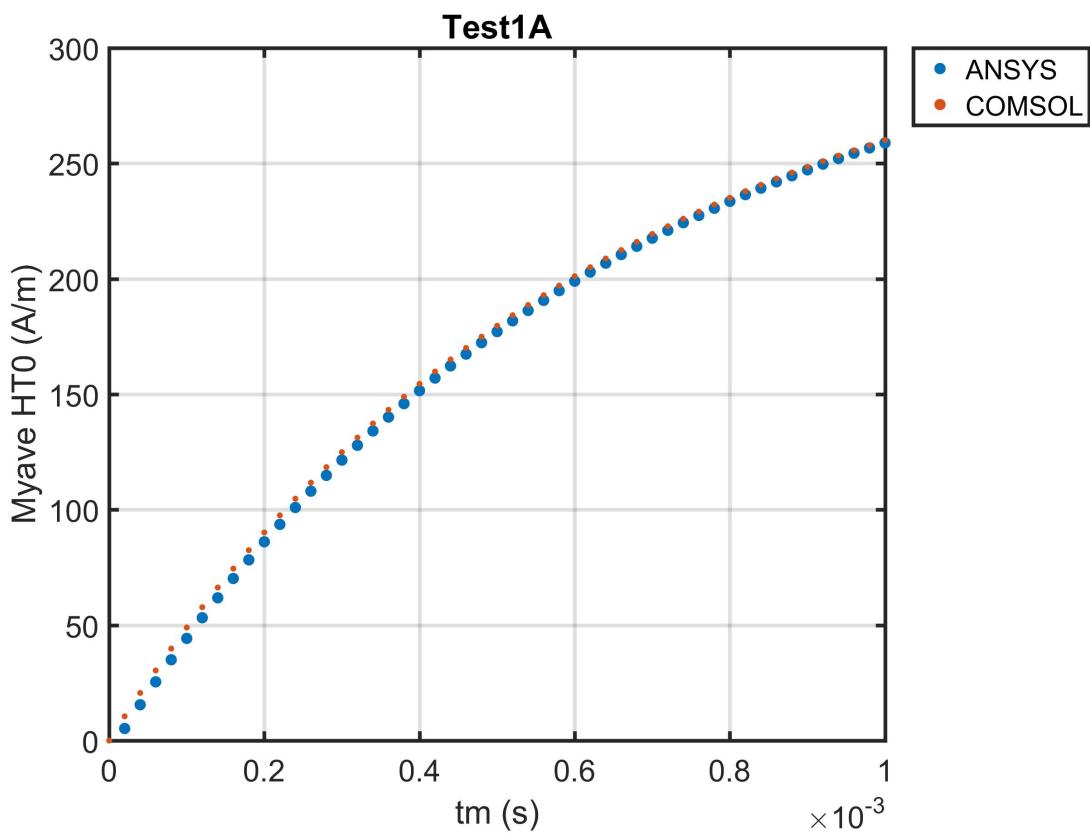


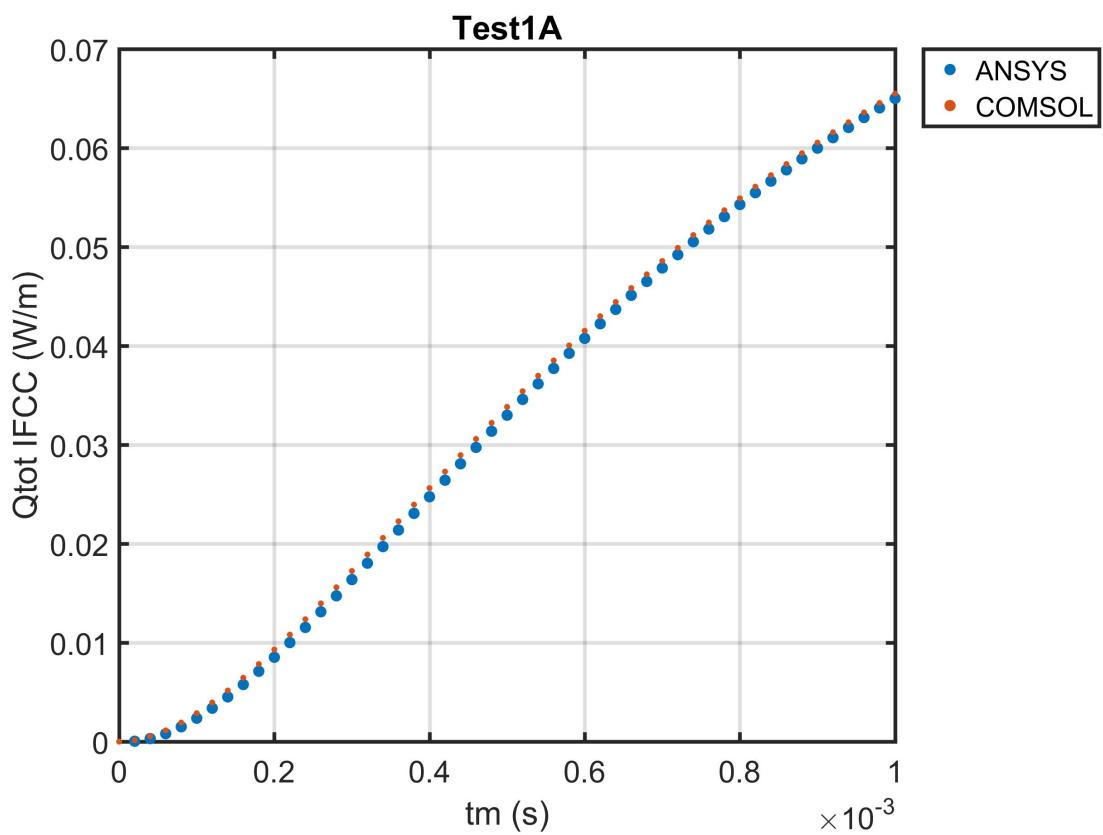


### 3.3 Test 1.A Interfilament Coupling Currents

Active physics	EM
Iron yoke	YES
IFCC	YES
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	None
External loads	Linear ramp-up, $t=(0,1e-3)$ [s], $\Delta t=20e-6$ [s], $i=(0,1)$ [A]
Fixed $\tau$ -IFCC	$1e-3$ [s]
<b>Output</b>	$M_x\text{-IFCC}_{aveHT0}$ [A/m]
	$M_y\text{-IFCC}_{aveHT0}$ [A/m]
	$Q\text{-IFCC}_{totHT0}$ [W]
	$Q\text{-IFCC}_{totcoil}$ [W]

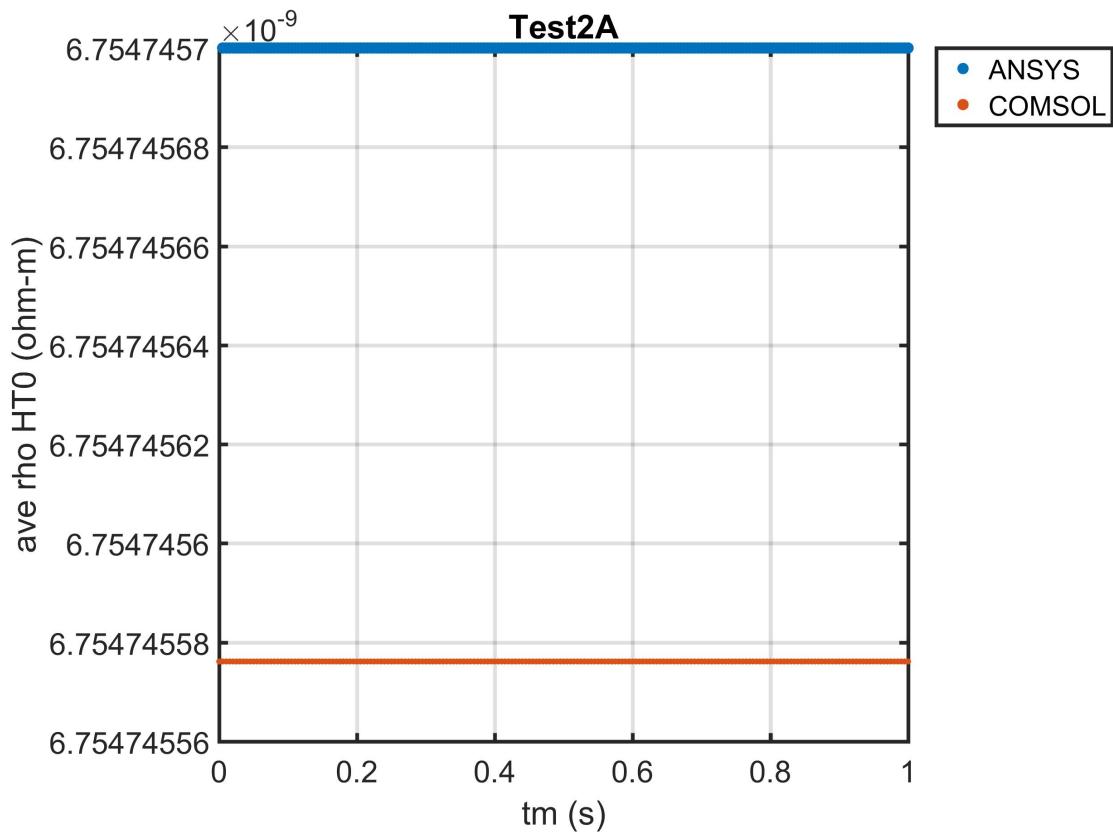


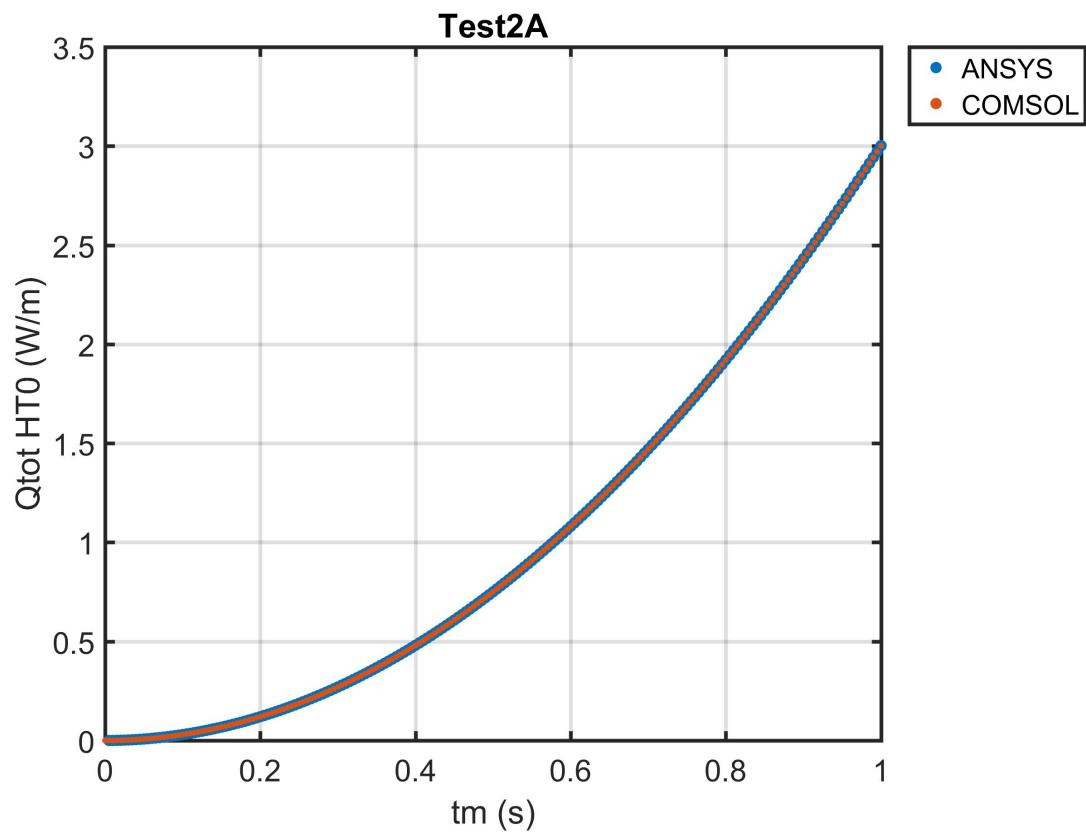
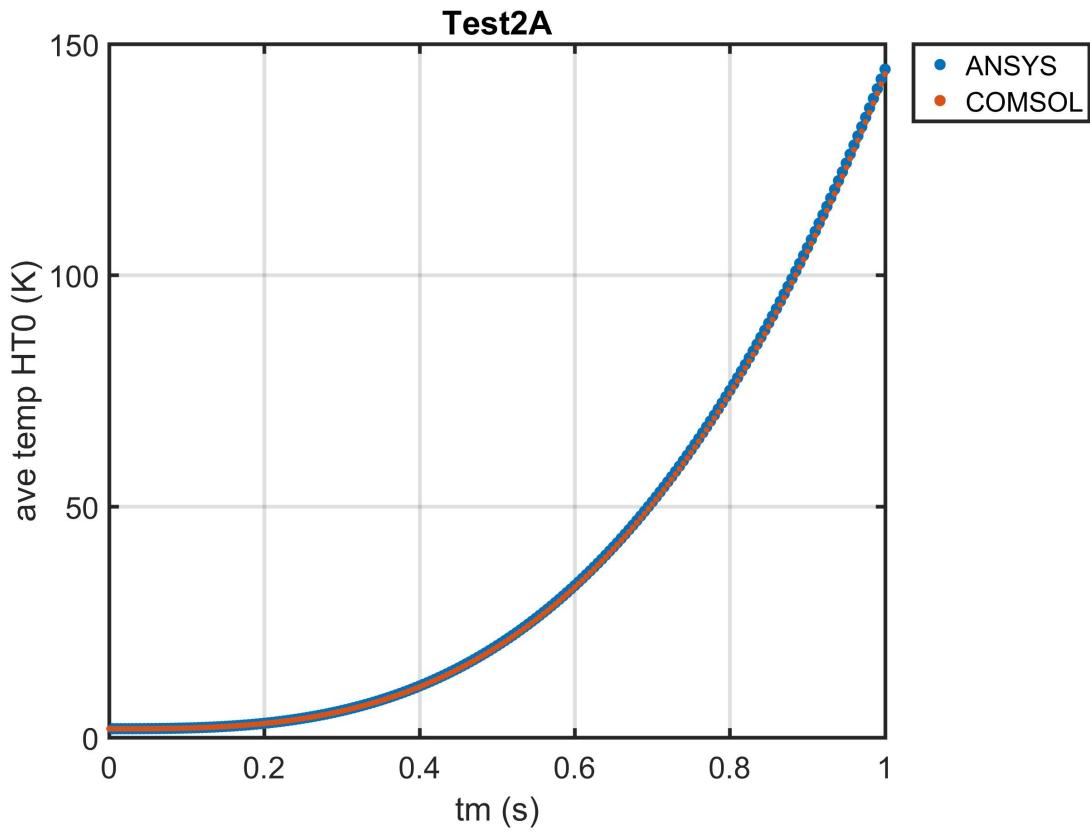


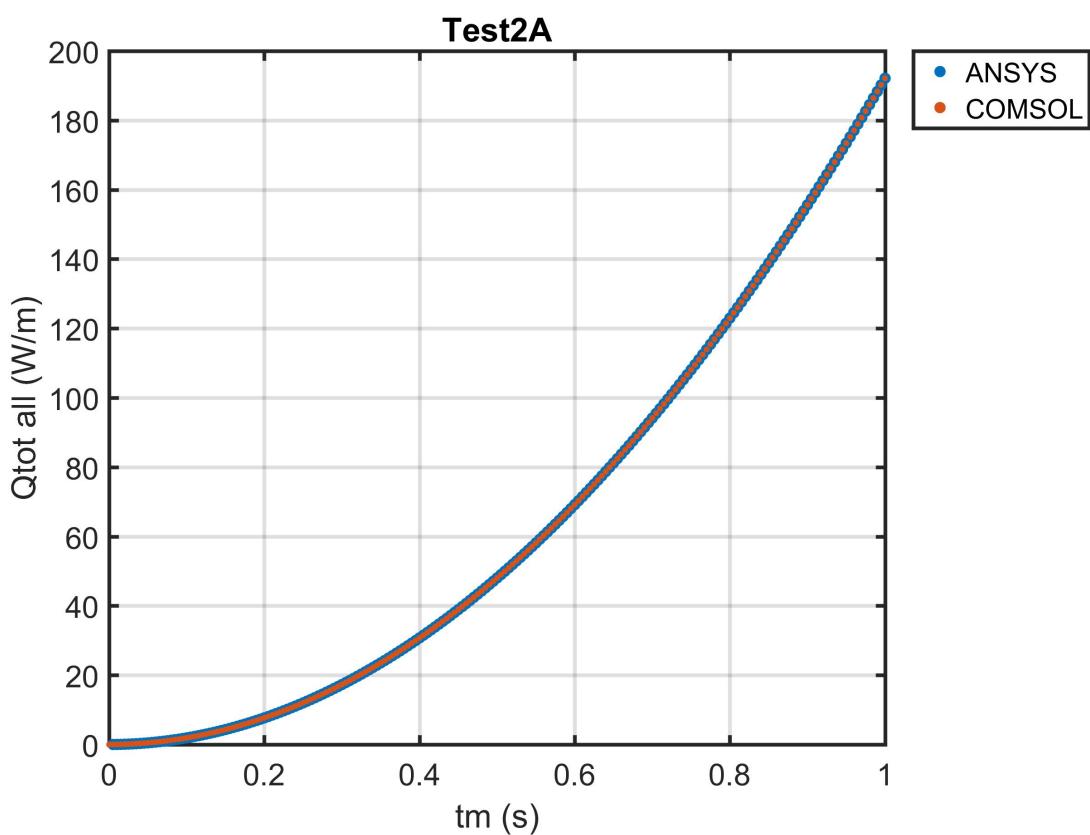


### 3.4 Test 2.A Ohmic Losses - Bare Cable

Active physics	EM, TH
Coil initial temp	1.9 [K]
Iron yoke	YES
IFCC	NO
Quench state	Force Quenched
Cable insulation material	None
Cable voids filling material	None
External loads	Constant, $t=(0,1)$ [s], $\Delta t=0.02$ [s], $i=(0,100)$ [A]
Material prop.	Fixed, see Appendix F
<b>Output</b>	$\rho_{aveHT0}$ [ $\Omega\text{m}$ ]
	$T_{aveHT0}$ [K]
	$Q_{totHT0}$ [W]
	$Q_{totcoil}$ [W]

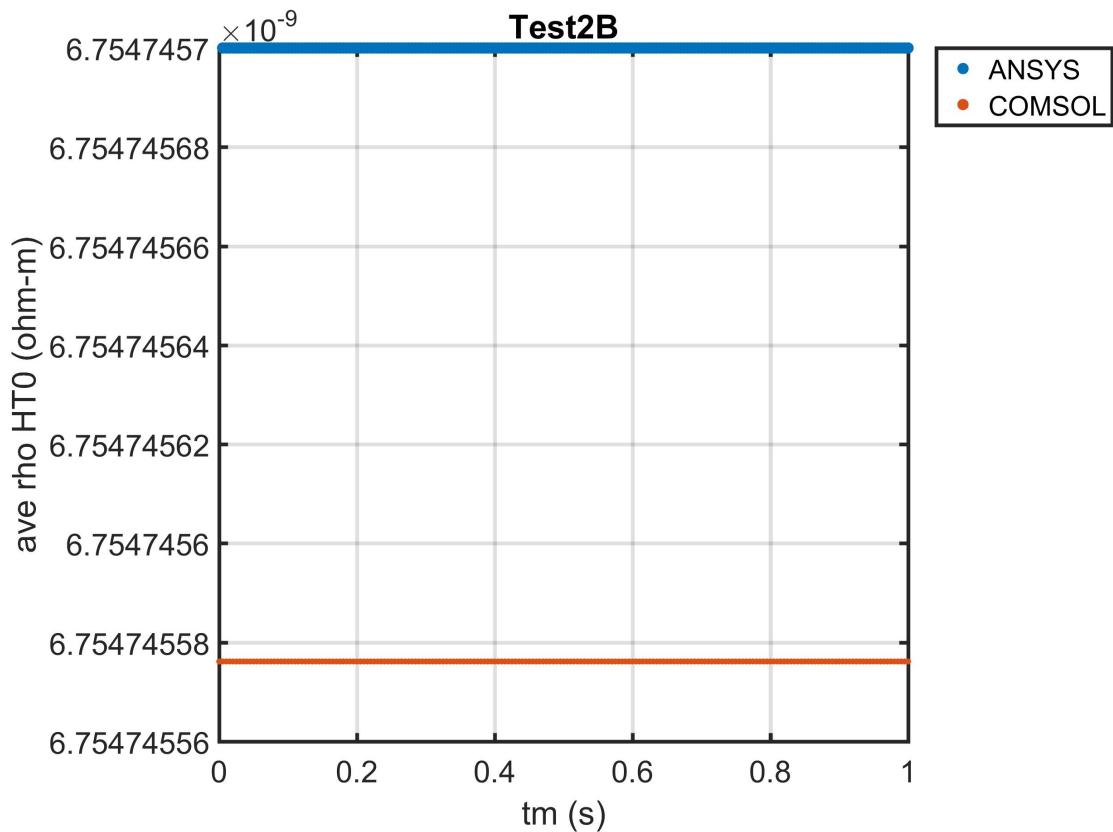


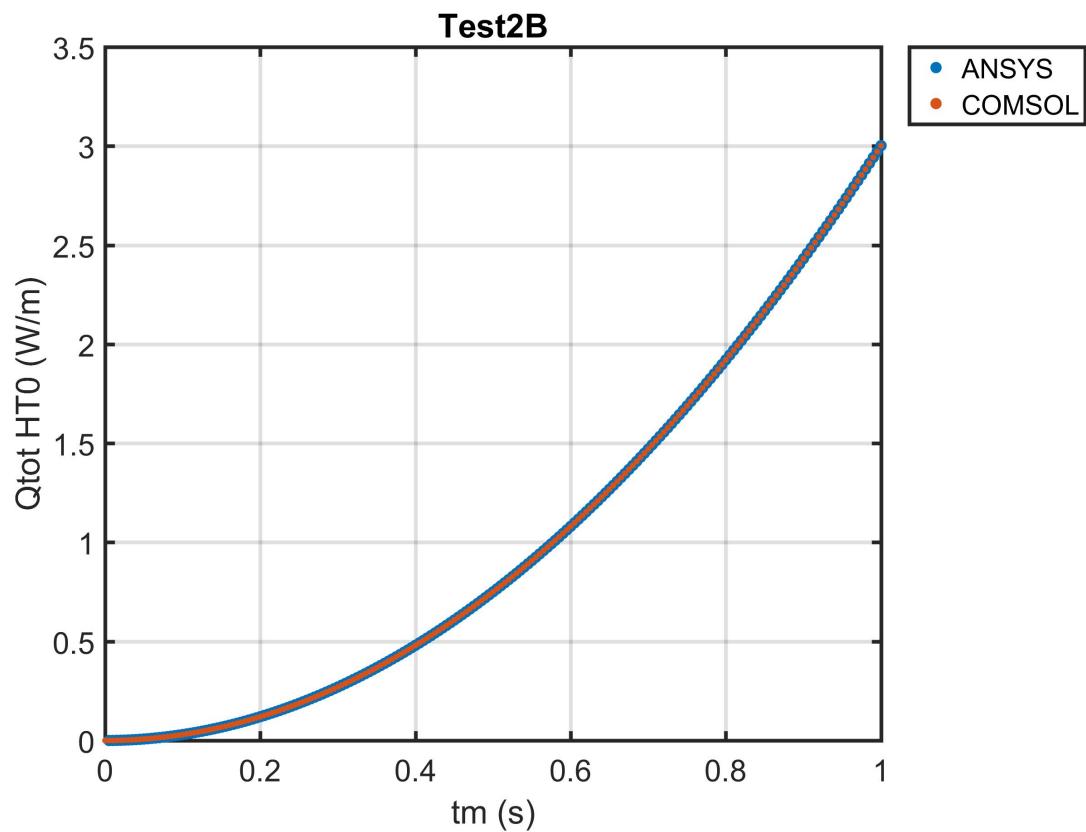
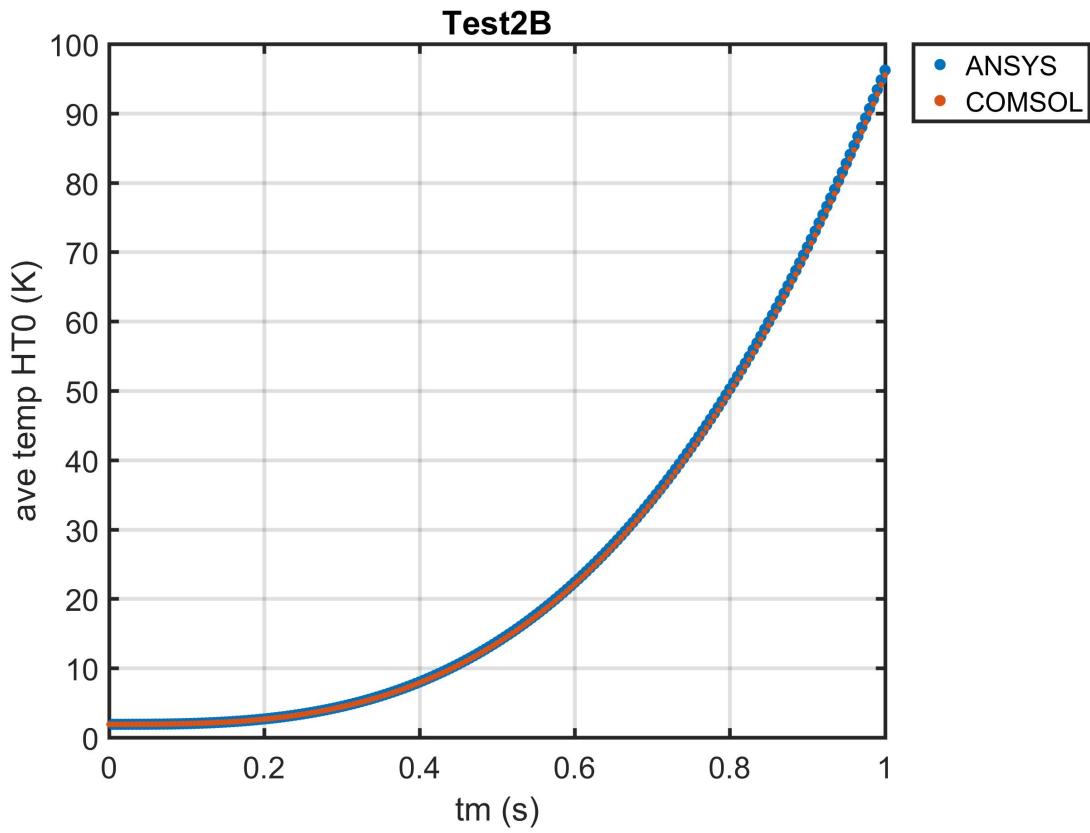


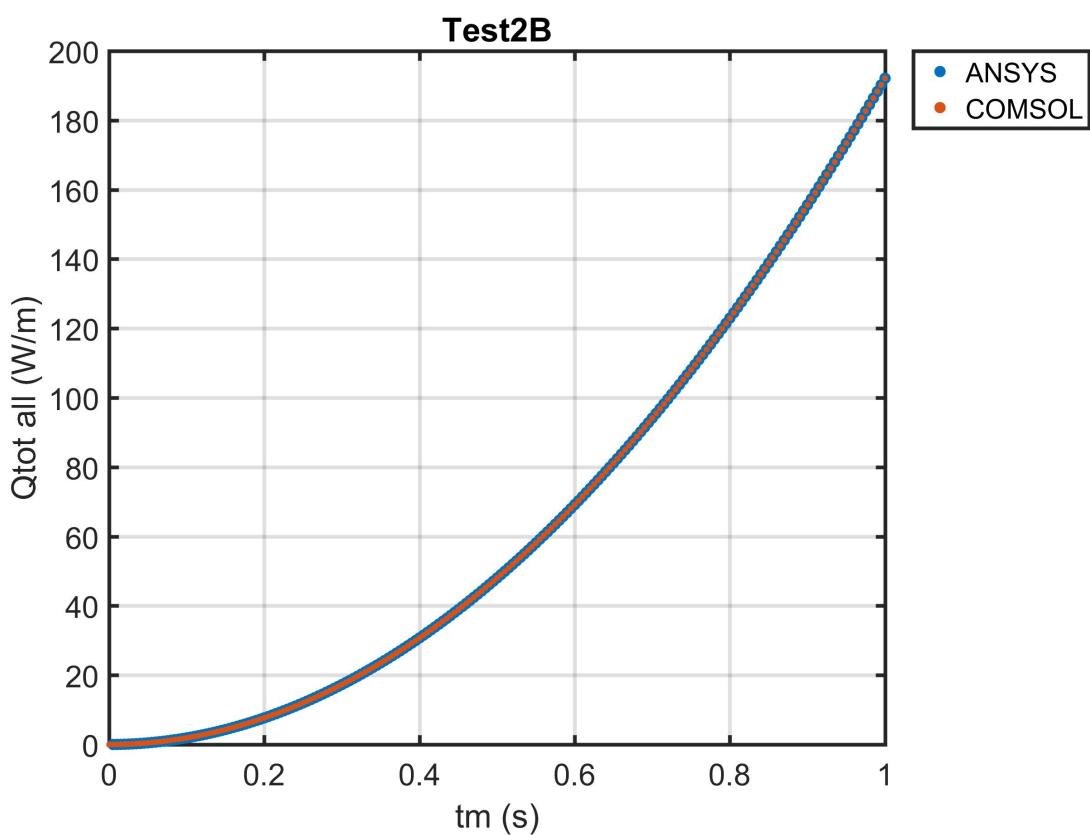


### 3.5 Test 2.B Ohmic Losses - Bare Cable with Filler

Active physics	EM, TH
Coil initial temp	1.9 [K]
Iron yoke	YES
IFCC	NO
Quench state	Force Quenched
Cable insulation material	None
Cable voids filling material	Glass Fiber
External loads	Constant, $t=(0,1)$ [s], $\Delta t=0.02$ [s], $i=(0,100)$ [A]
Material prop.	Fixed, see Appendix F
<b>Output</b>	$\rho_{aveHT0}$ [ $\Omega\text{m}$ ]
	$T_{aveHT0}$ [K]
	$Q_{totHT0}$ [W]
	$Q_{totcoil}$ [W]

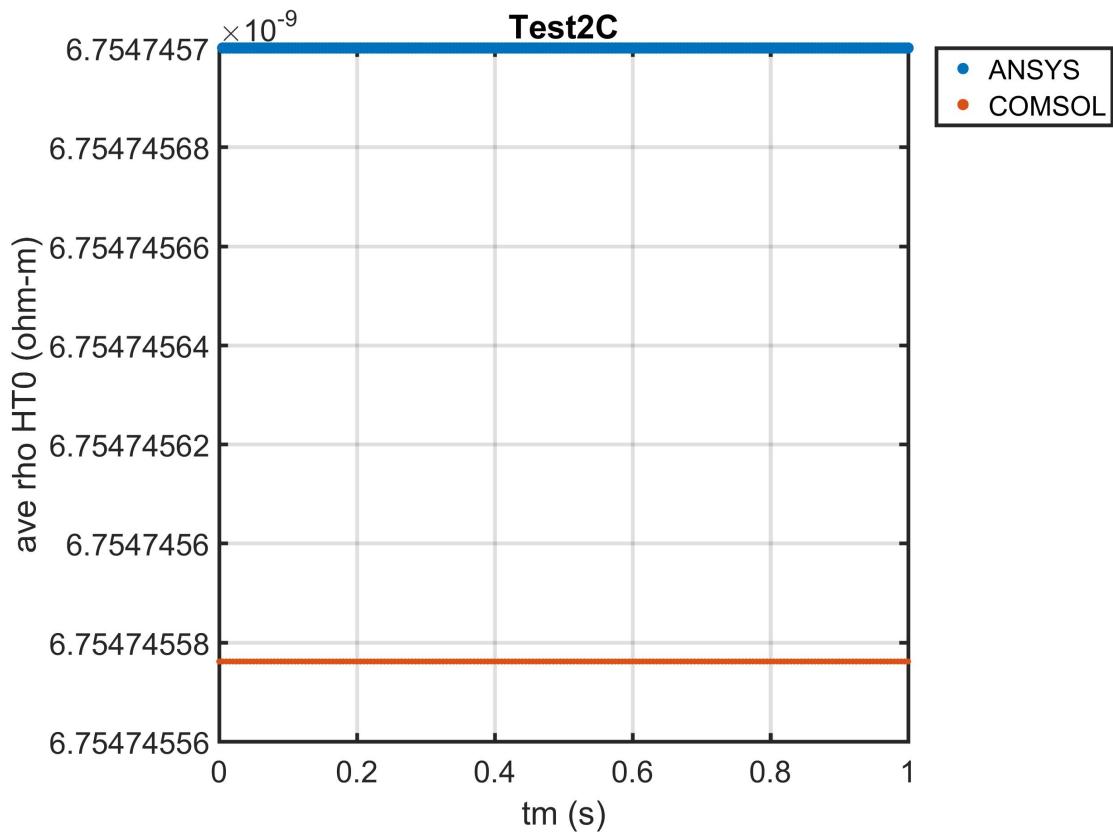


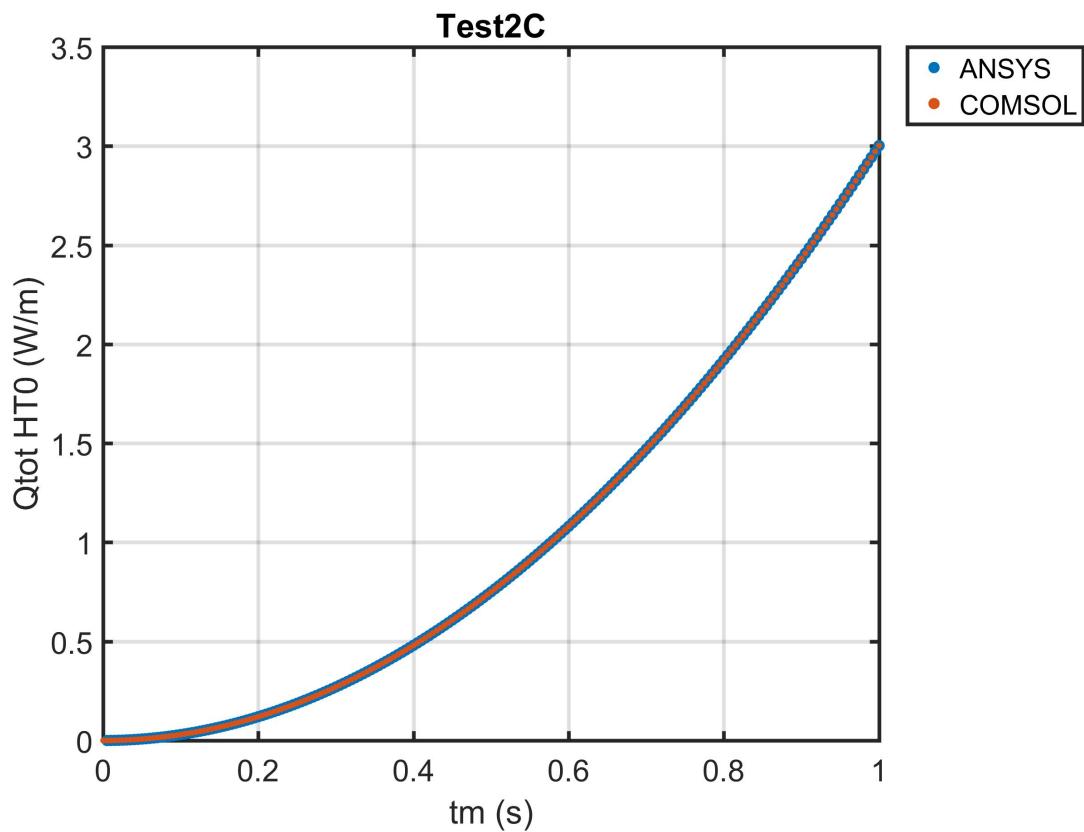
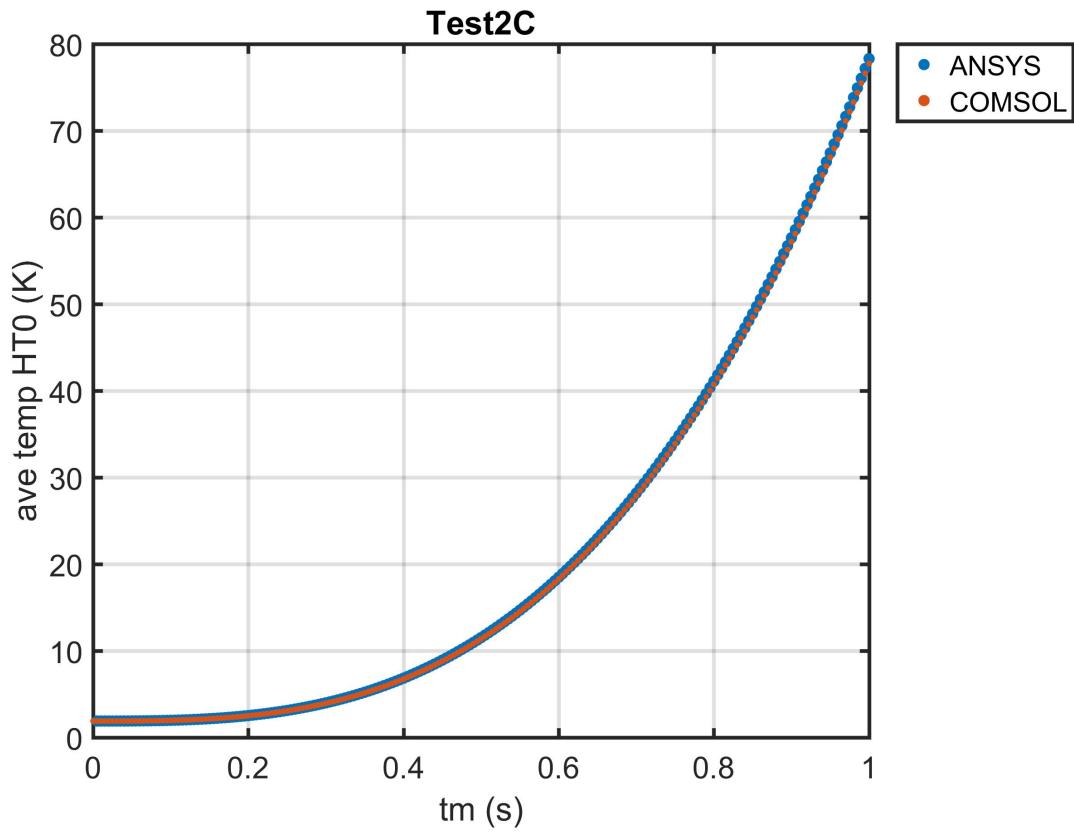


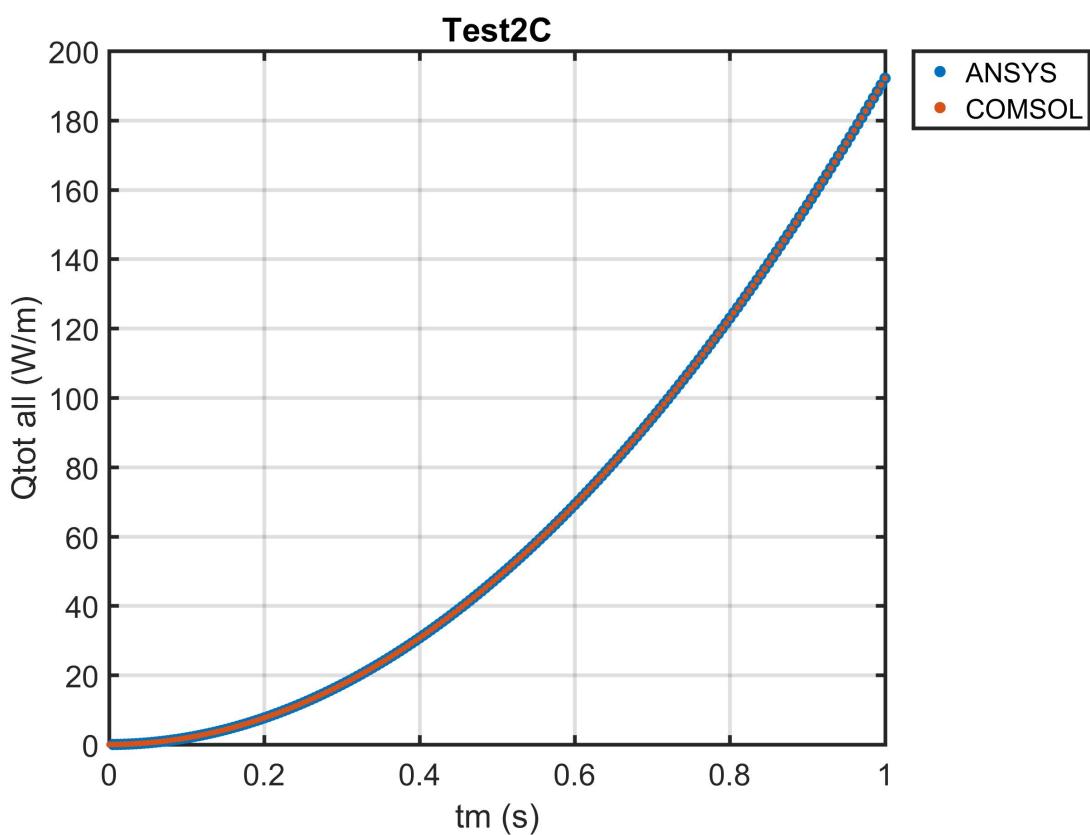


### 3.6 Test 2.C Ohmic Losses - Ins. Cable with Filler

Active physics	EM, TH
Coil initial temp	1.9 [K]
Iron yoke	YES
IFCC	NO
Quench state	Force Quenched
Cable insulation material	Glass Fiber
Cable voids filling material	Glass Fiber
External loads	Constant, $t=(0,1)$ [s], $\Delta t=0.02$ [s], $i=(0,100)$ [A]
Material prop.	Fixed, see Appendix F
<b>Output</b>	$\rho_{aveHT0}$ [ $\Omega\text{m}$ ]
	$T_{aveHT0}$ [K]
	$Q_{totHT0}$ [W]
	$Q_{totcoil}$ [W]



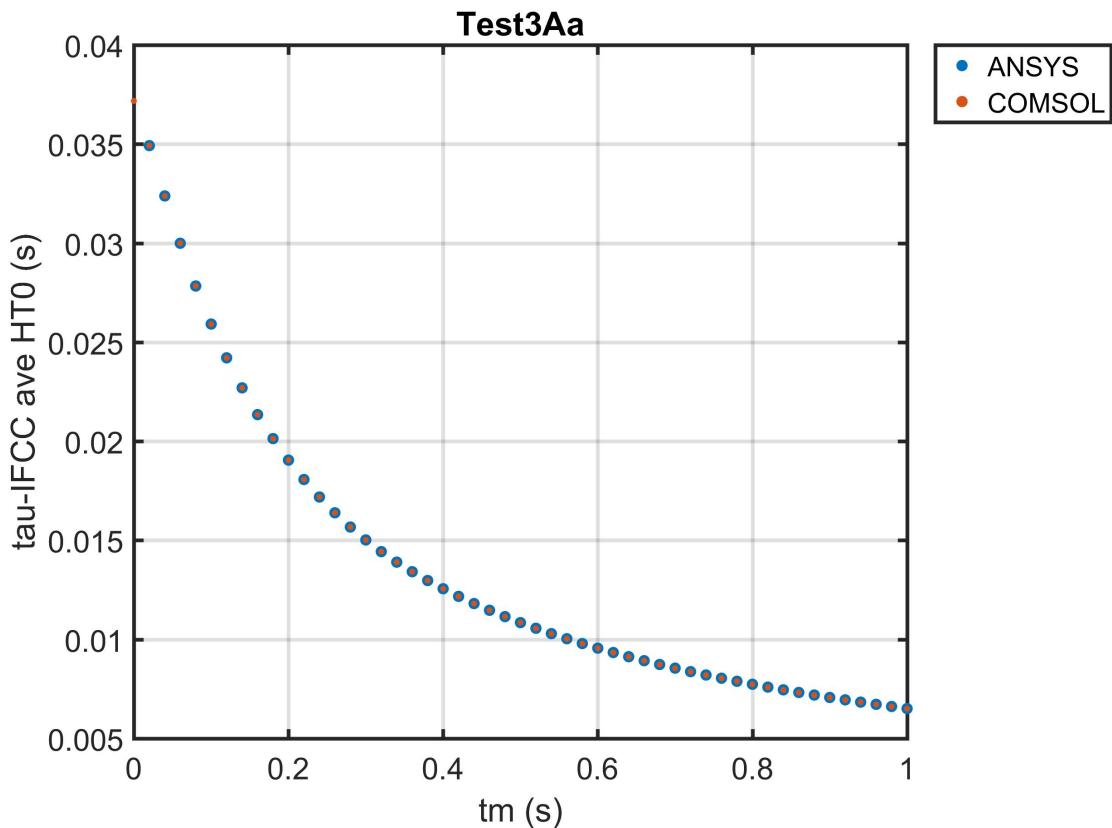


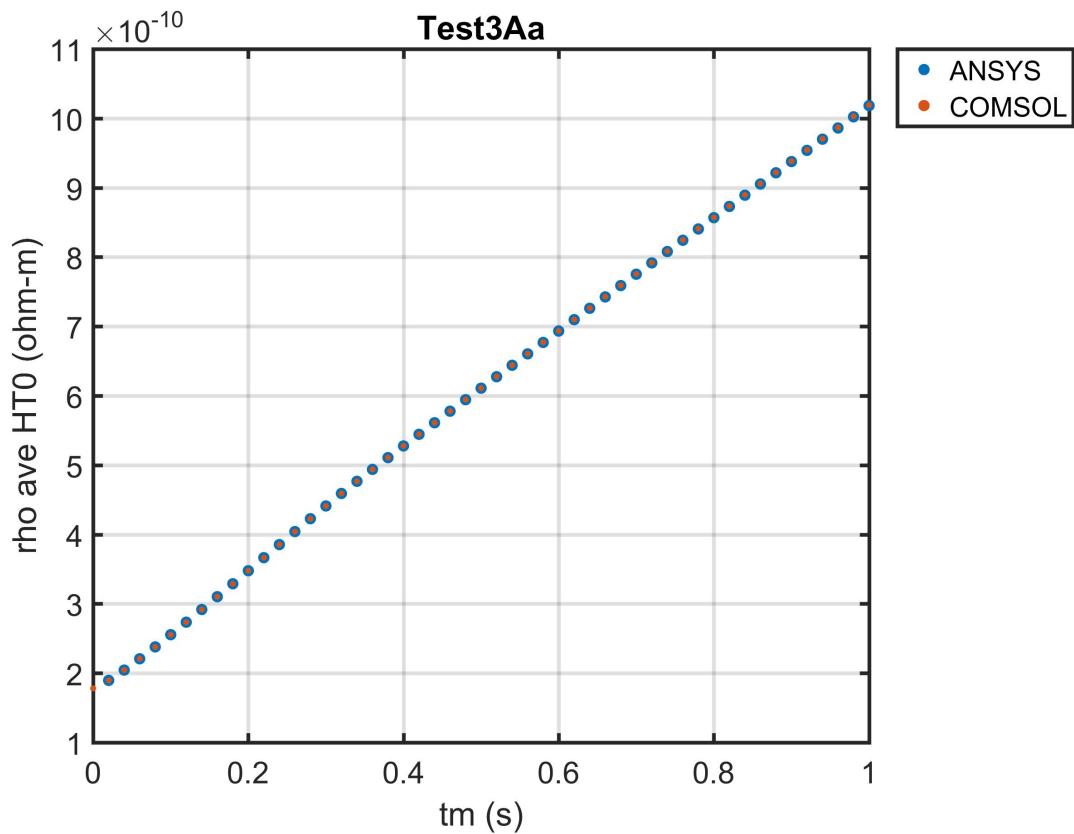
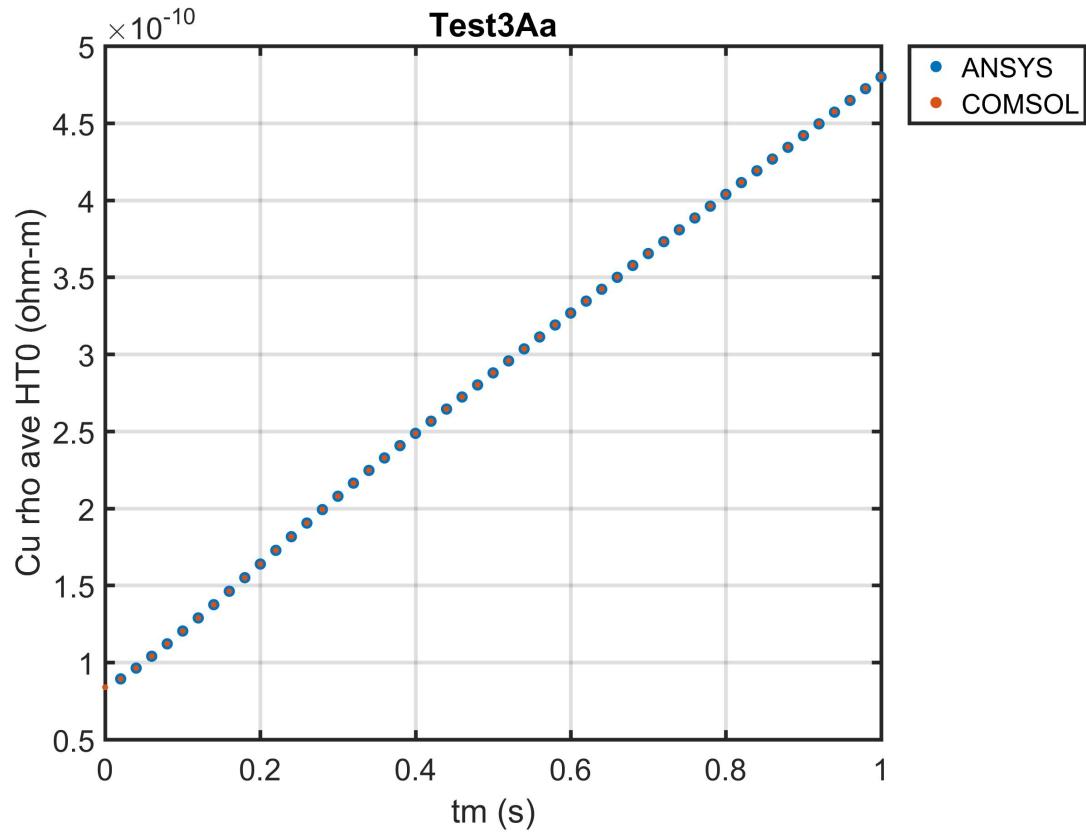


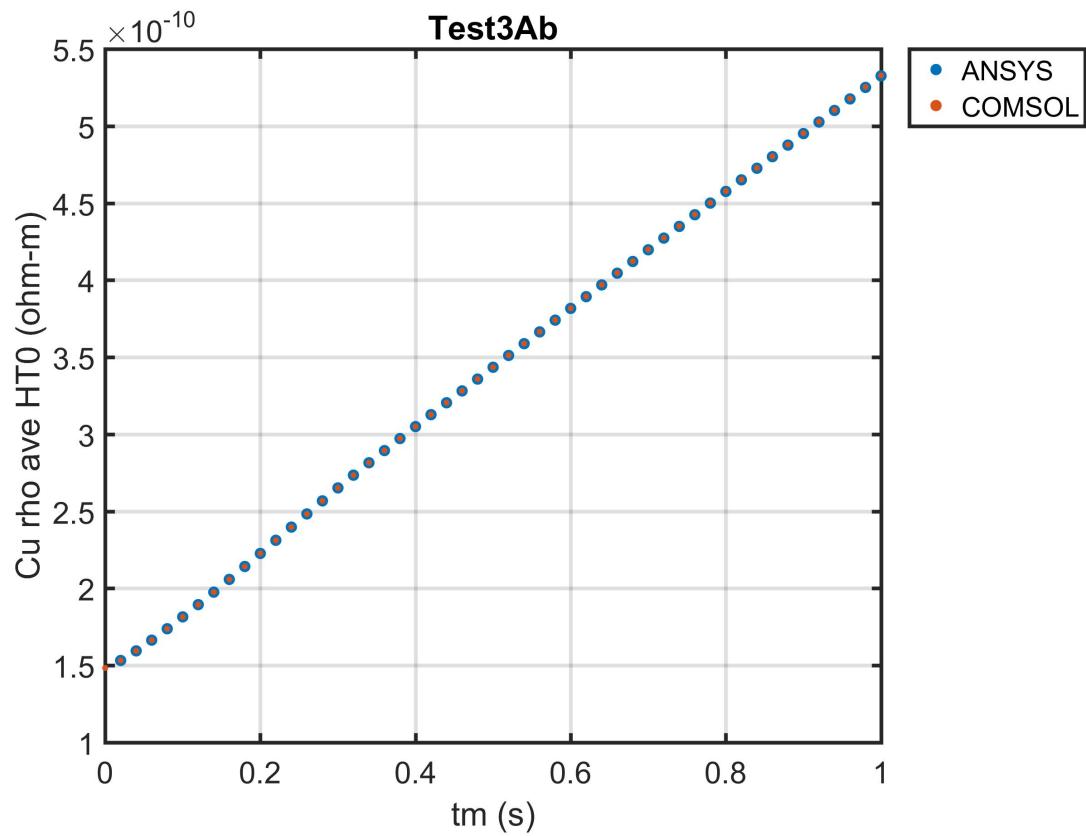
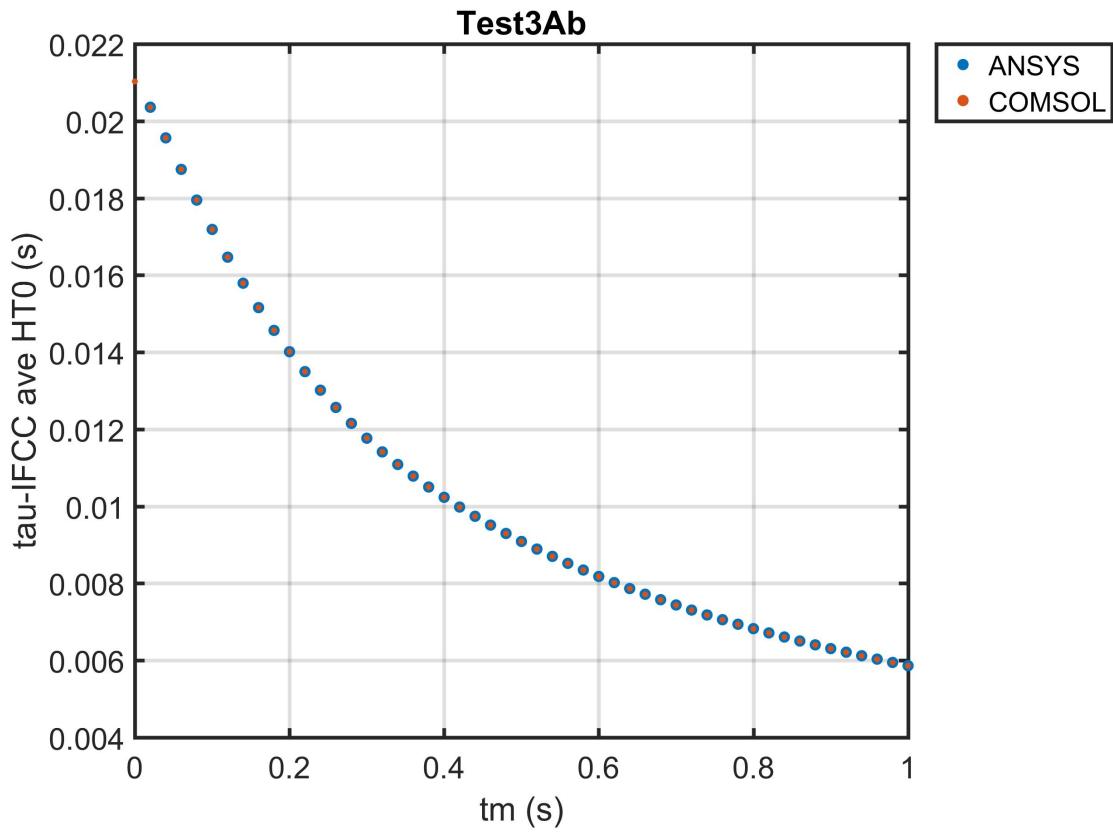
### 3.7 Test 3.A Resistivity and Tau IFCC (B and T dependence)

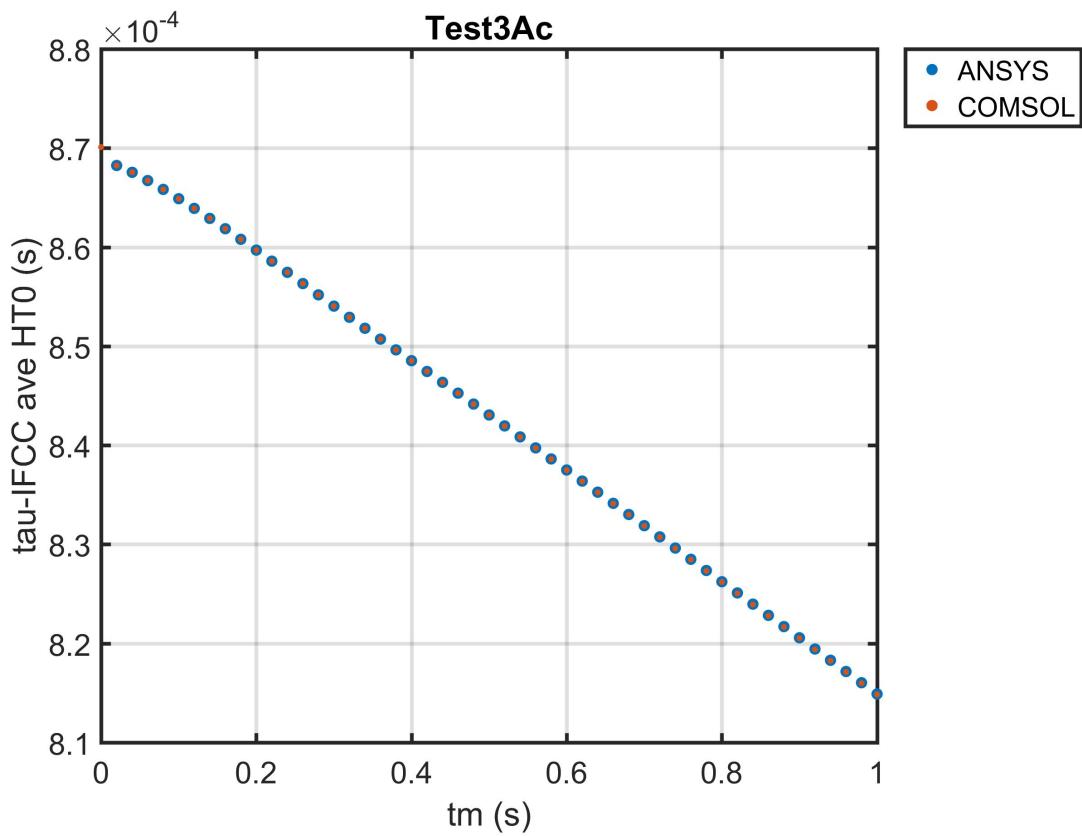
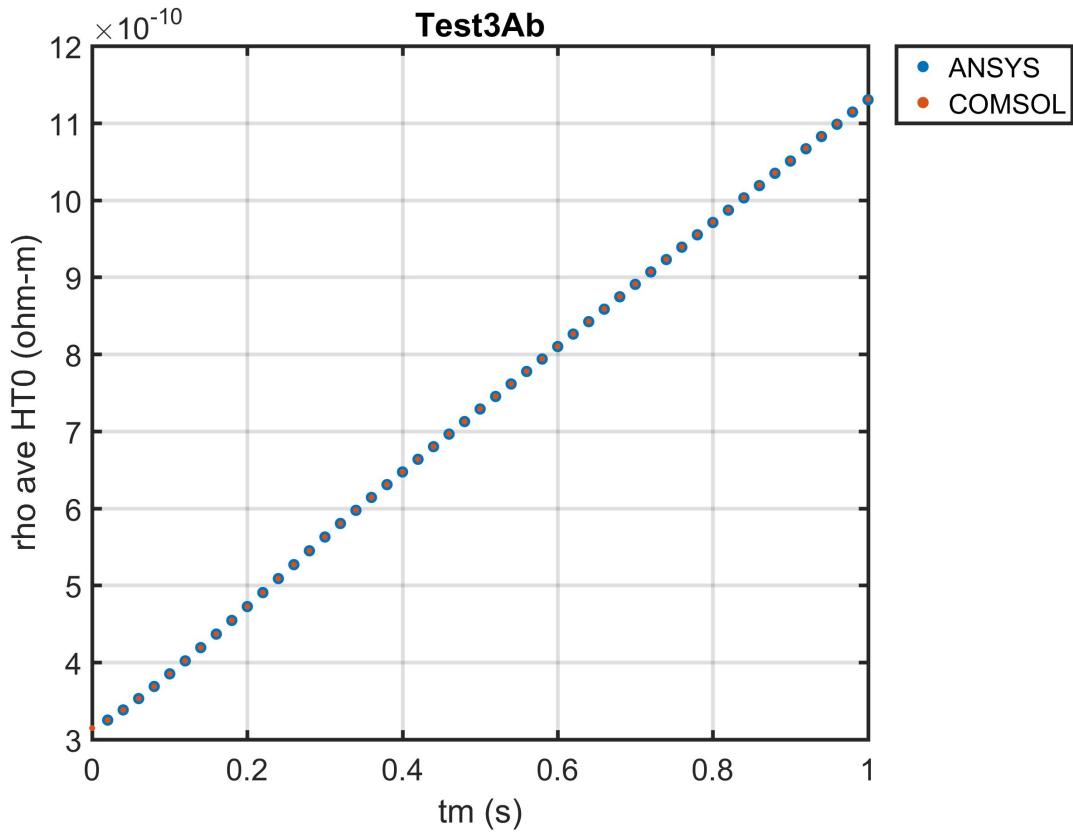
This test is added to check the temperature and field dependence of the resistivity of the copper using the NIST fit, as well as the calculation method for tau. The temperature is kept fixed for three different cases “a”, “b”, and “c” as shown in the table. In this case the effect of the IFCC currents (magnetization) is not included, only the material properties and tau are output.

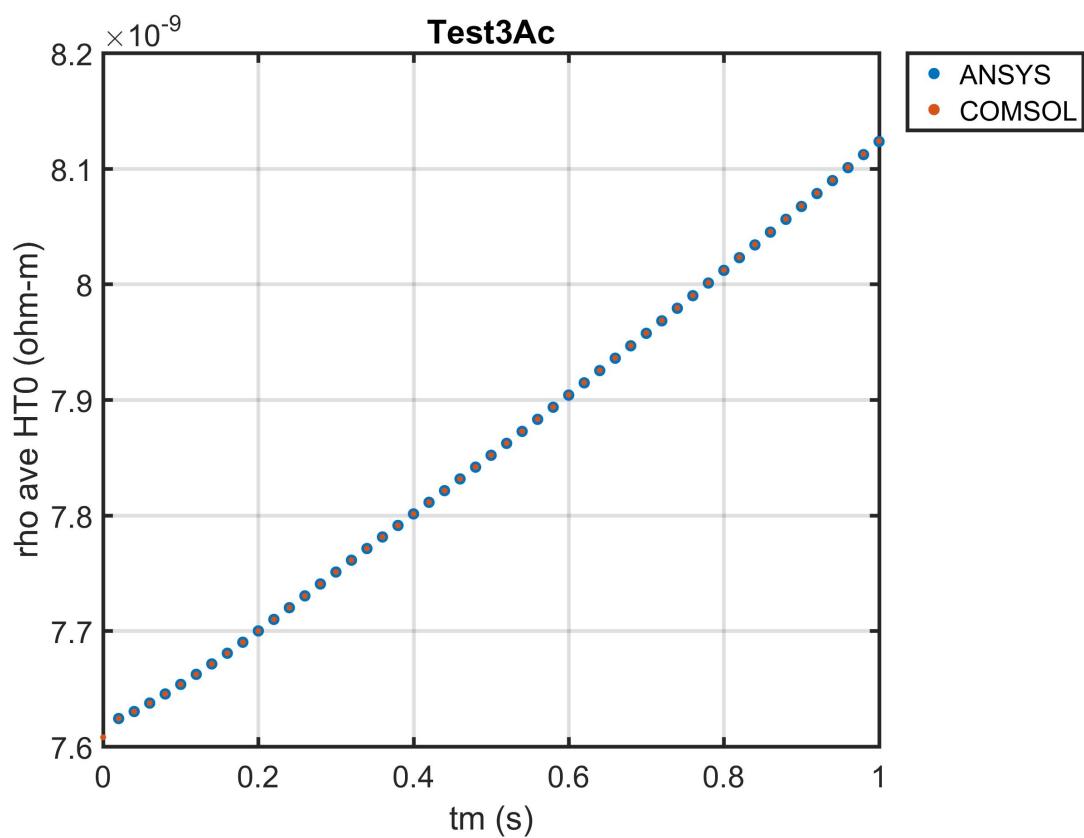
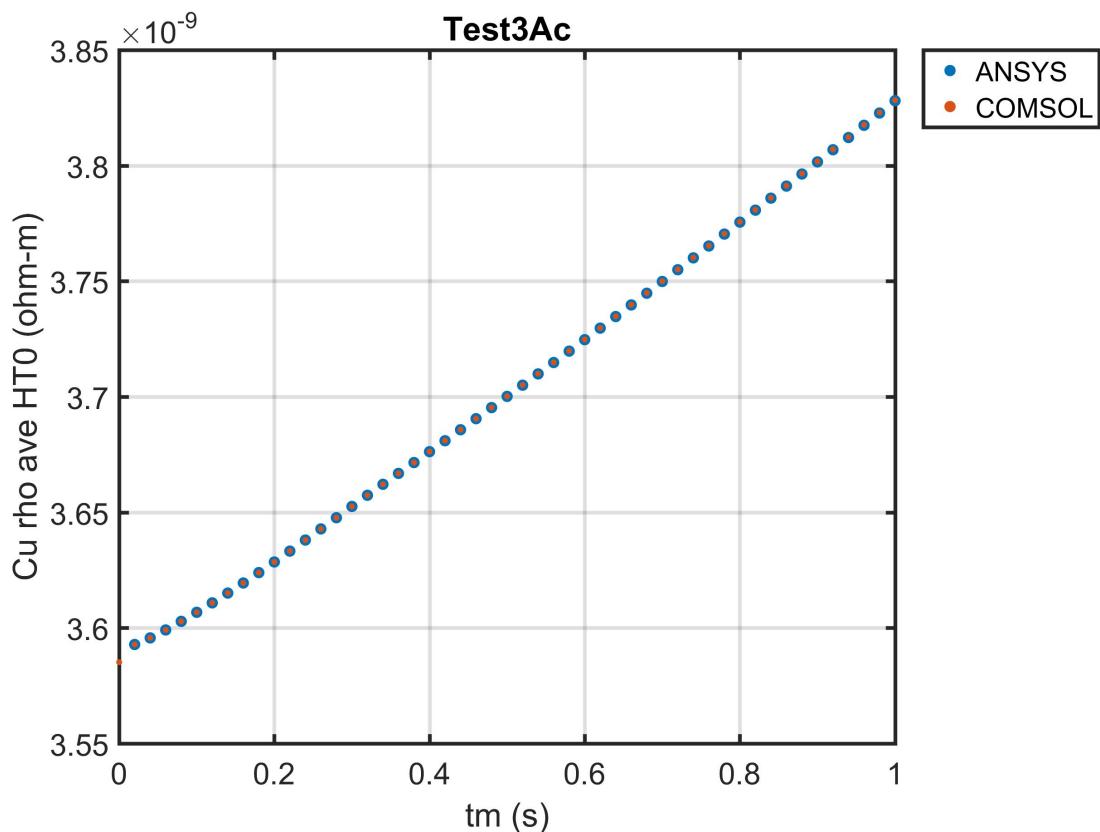
Active physics	EM
Coil initial temp	a=1.9 [K], b=30 [K], c=100 [K]
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	None
External loads	Linear ramp-up, t=(0,1) [s], $\Delta t=20e-3$ [s], i=(0,20) [kA]
$\rho_{cu}$ fit	NIST, see Appendix E
<b>Output</b>	Tau-IFCC <sub>aveHT0</sub> [s]
	$\rho_{cuHT0}$ [ $\Omega\text{m}$ ]
	$\rho_{aveHT0}$ [ $\Omega\text{m}$ ]









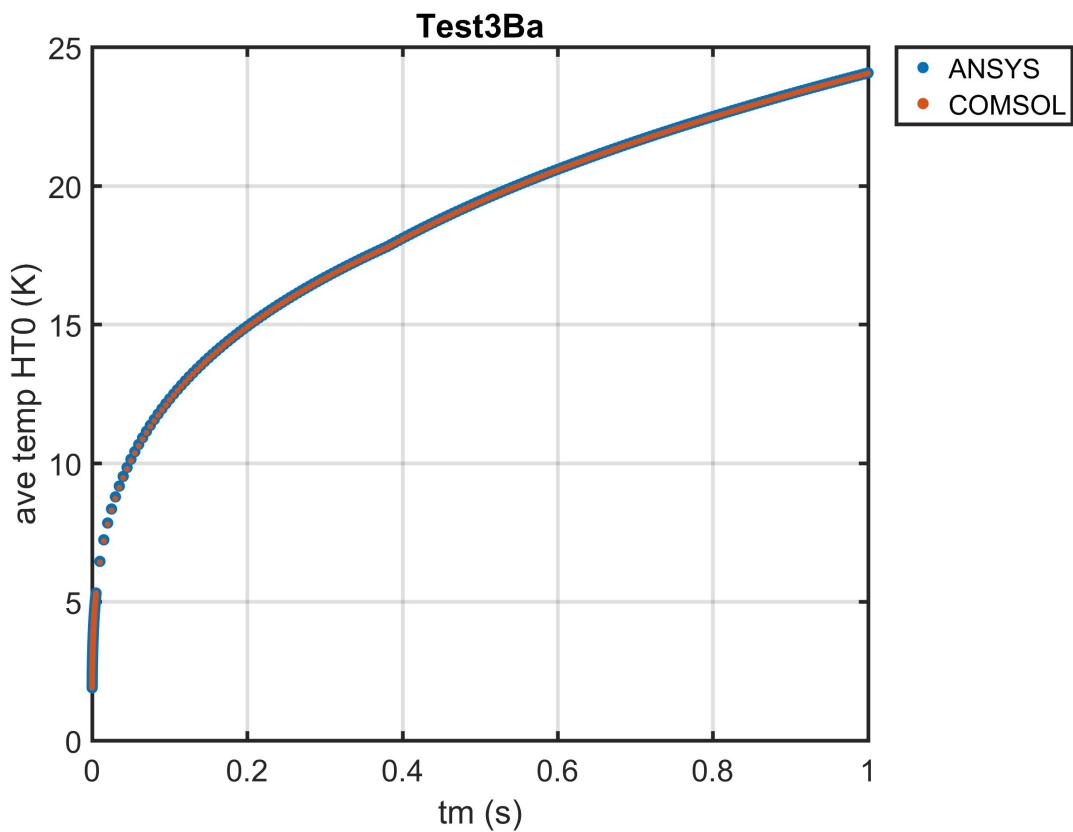
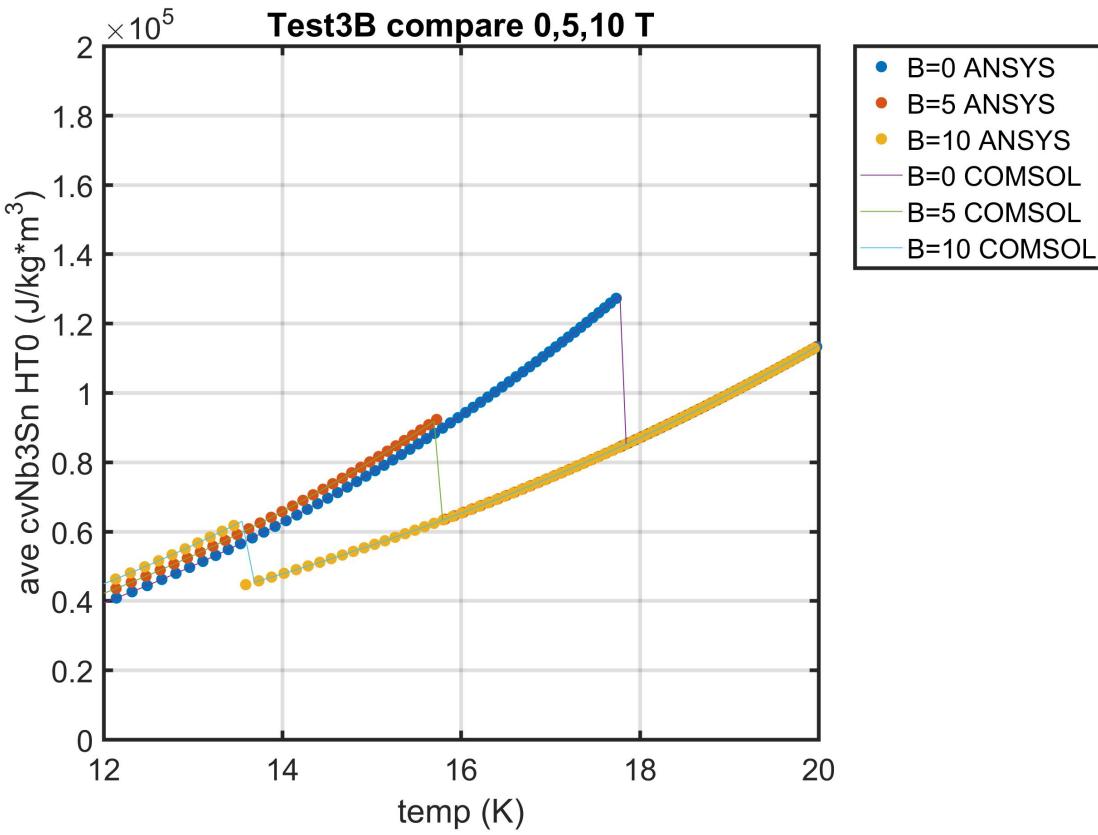


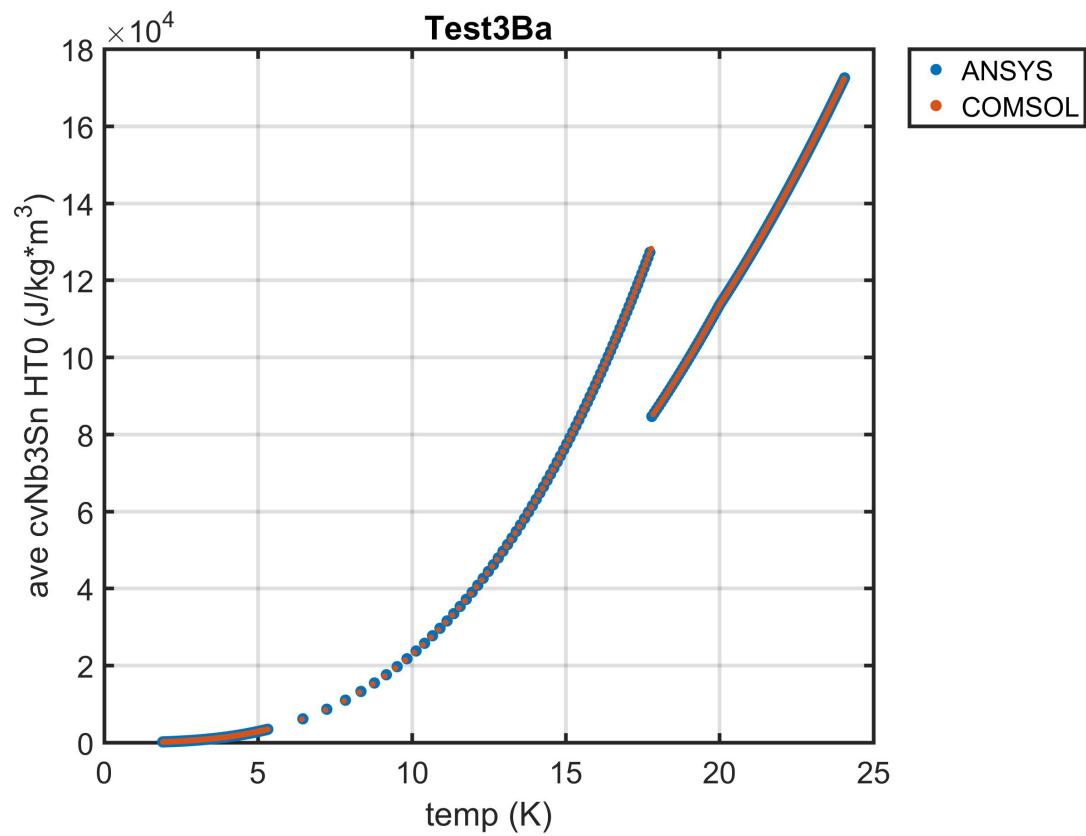
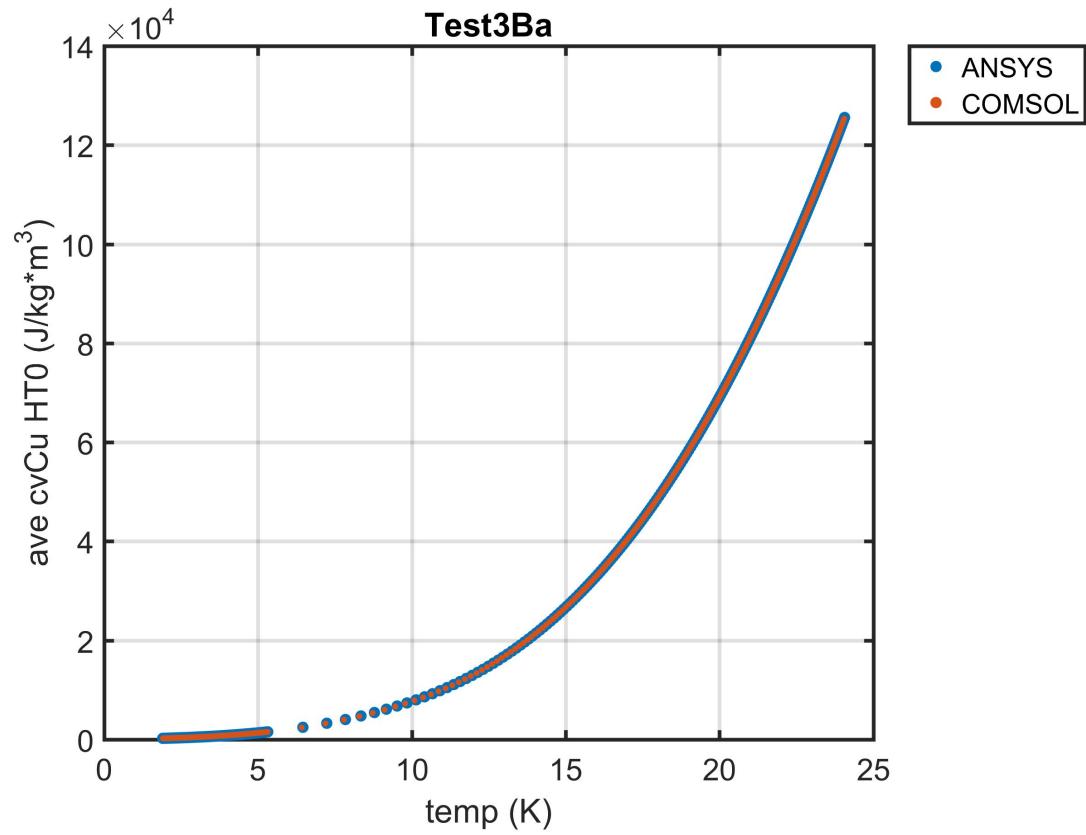
### 3.8 Test 3.B Heat Capacity (B and T dependence)

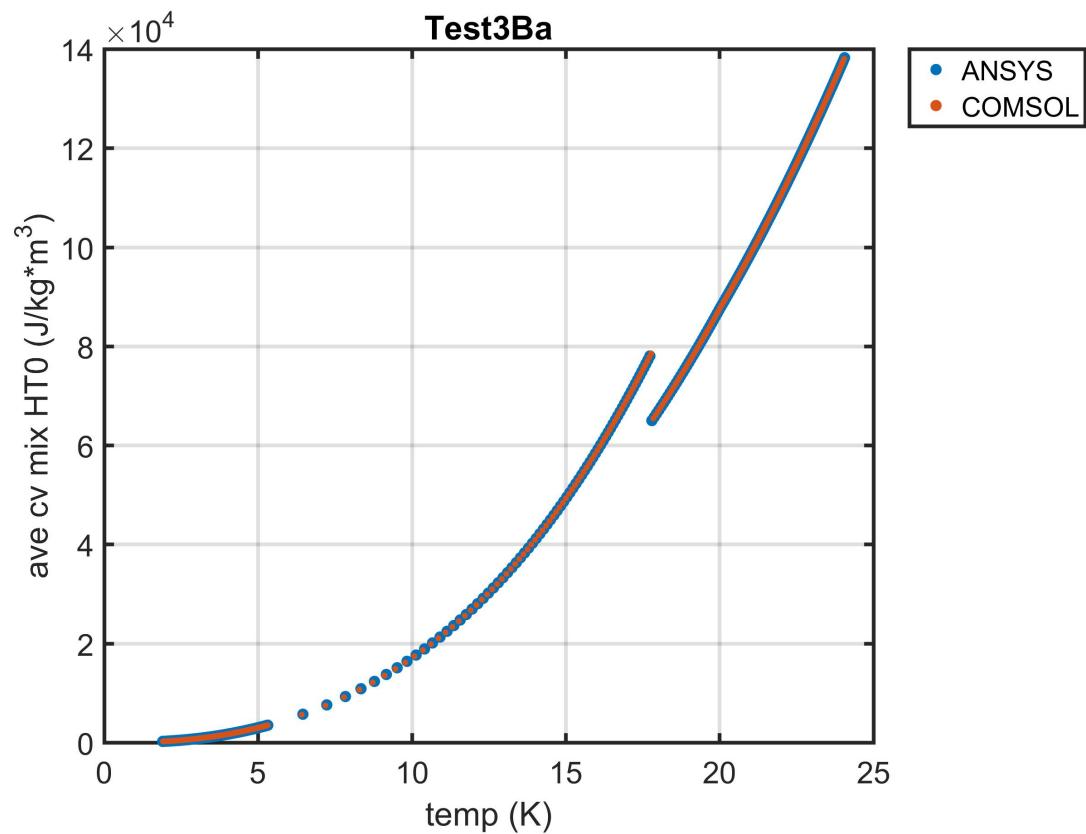
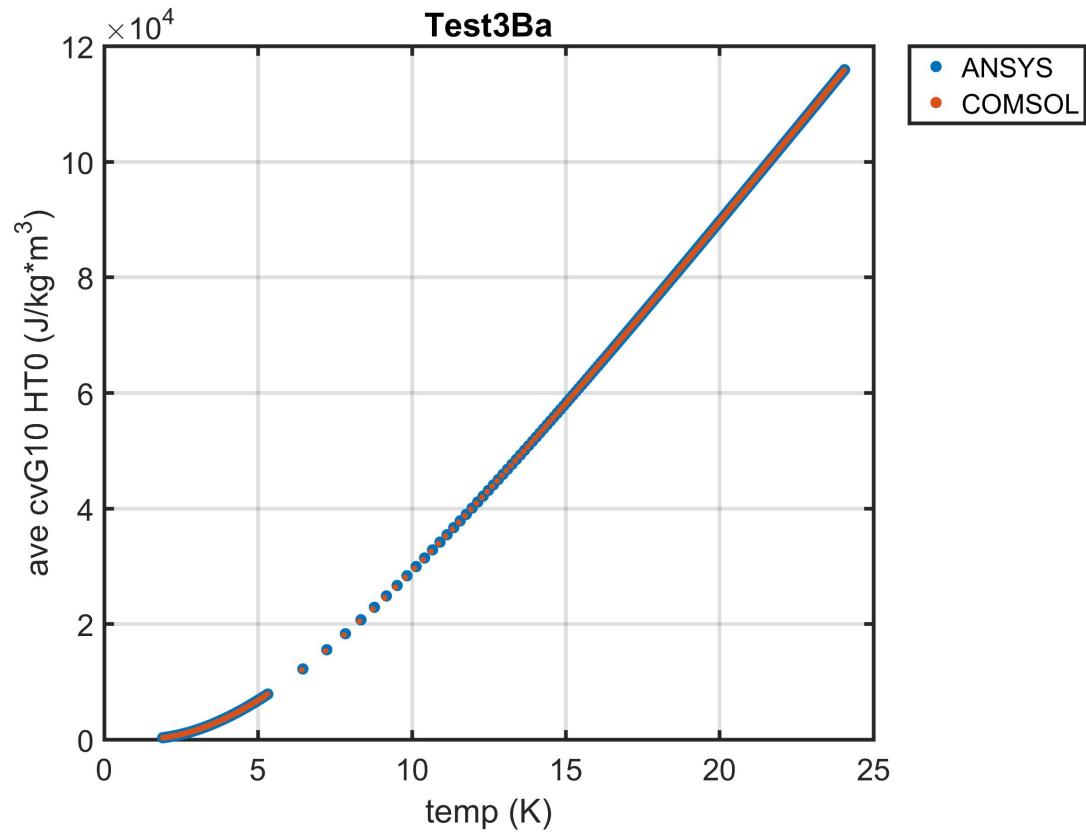
This test is designed to test the temperature dependence of the heat capacity fits and also the field dependence (quench) for Nb<sub>3</sub>Sn. A constant heat load is applied on the turns, which are then allowed to heat up. Thermal Conductivity should not play a role as the heat load is uniform, but is output as a check of the fits between codes. This test is run for three different background fields labeled a, b, and c.

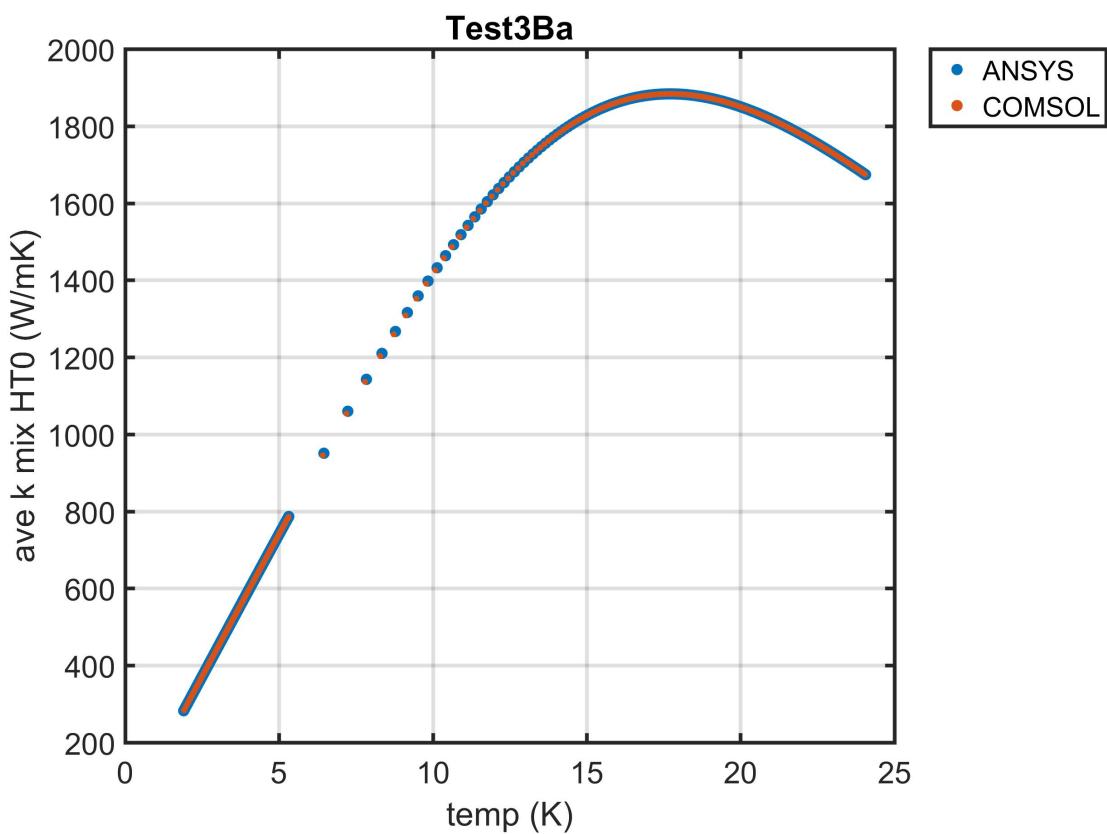
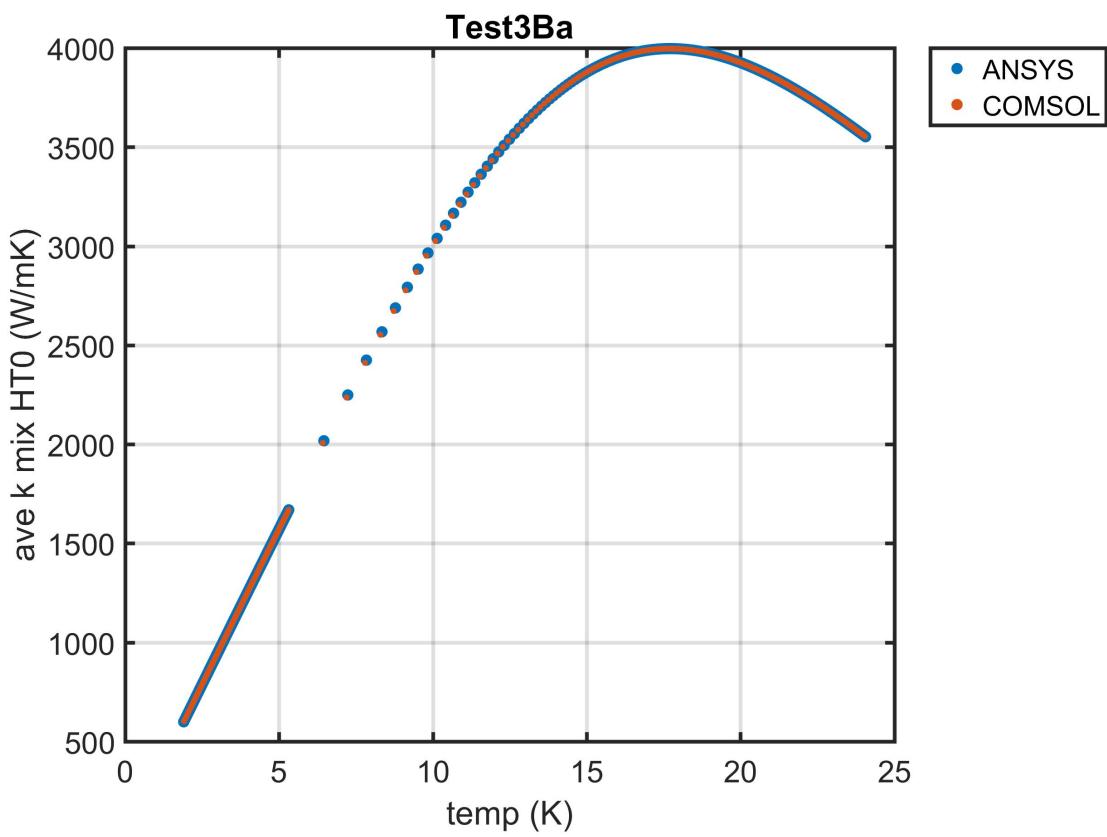
Active physics	TH
Coil initial temp	a=1.9 [K]
Input applied background field	a=0.0 [T], b=5 [T], c=10 [T]
Cable insulation material	None
Cable voids filling material	GLASS FIBER
External loads	Uniform heat load of 1e6 W/m <sup>3</sup> , t=(0,1) [s], Δt=suff. for conv.
C <sub>v</sub> -Cu fit	CUDI, see Appendix E
C <sub>v</sub> -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
C <sub>v</sub> -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
<b>Output</b>	T <sub>aveHT0</sub> [K]
	C <sub>v-cuHT0</sub> [J/(kg*m <sup>3</sup> )]
	C <sub>v-Nb3SnHT0</sub> [J/(kg*m <sup>3</sup> )]
	C <sub>v-G10HT0</sub> [J/(kg*m <sup>3</sup> )]
	C <sub>v-mixHT0</sub> [J/(kg*m <sup>3</sup> )]
	k <sub>cuHT0</sub> [W/(mK)]
	k <sub>mixHT0</sub> [W/(mK)]

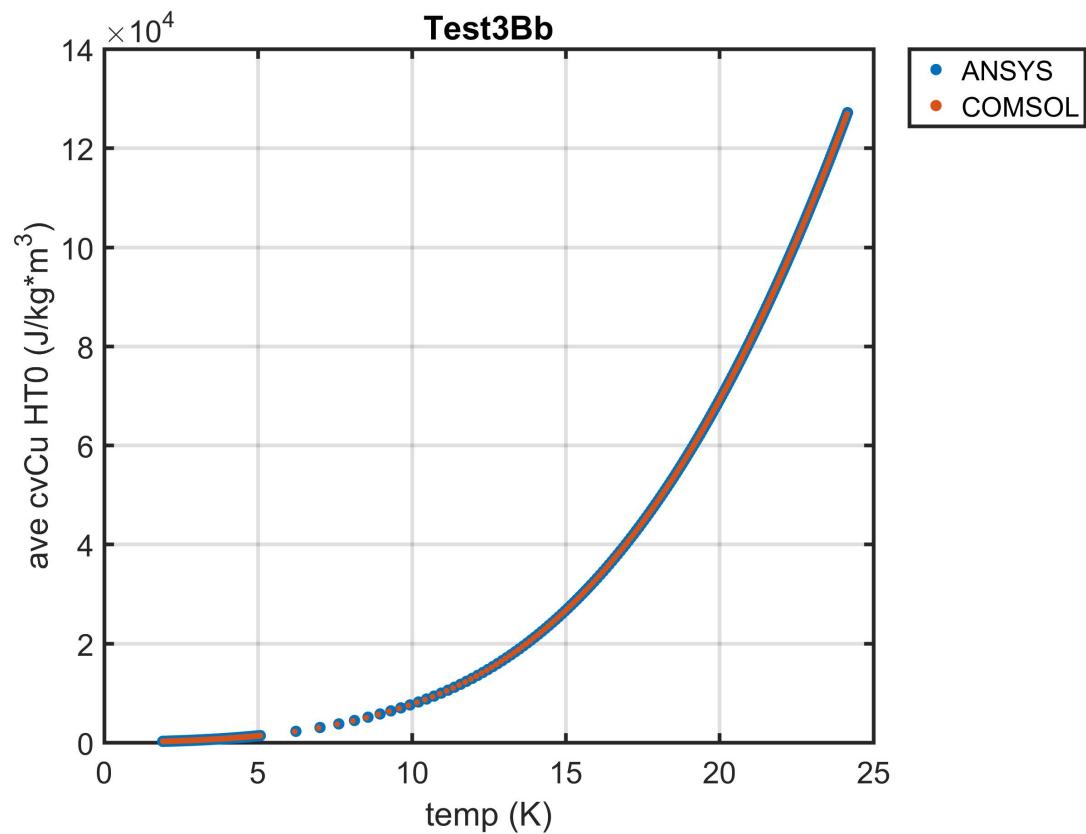
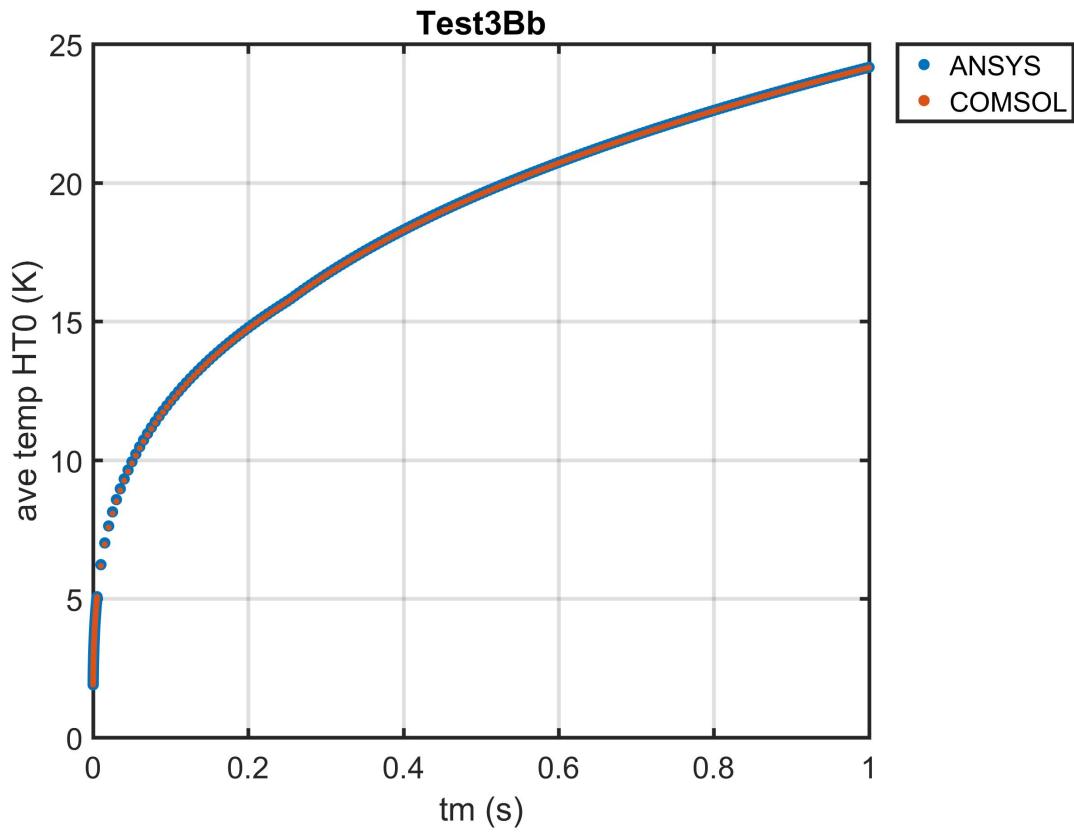
A plot of the Nb<sub>3</sub>Sn Cv vs. temp for each case is shown first which demonstrates the Cv jump at quench for the different magnetic field levels.

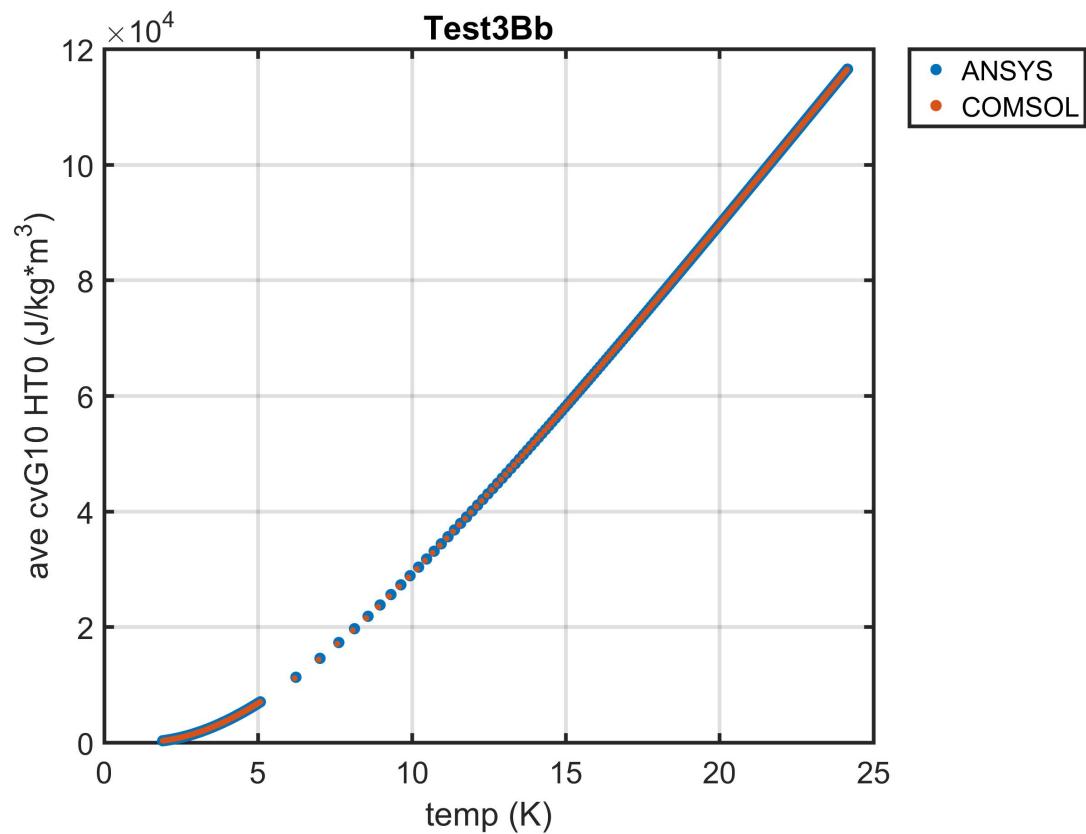
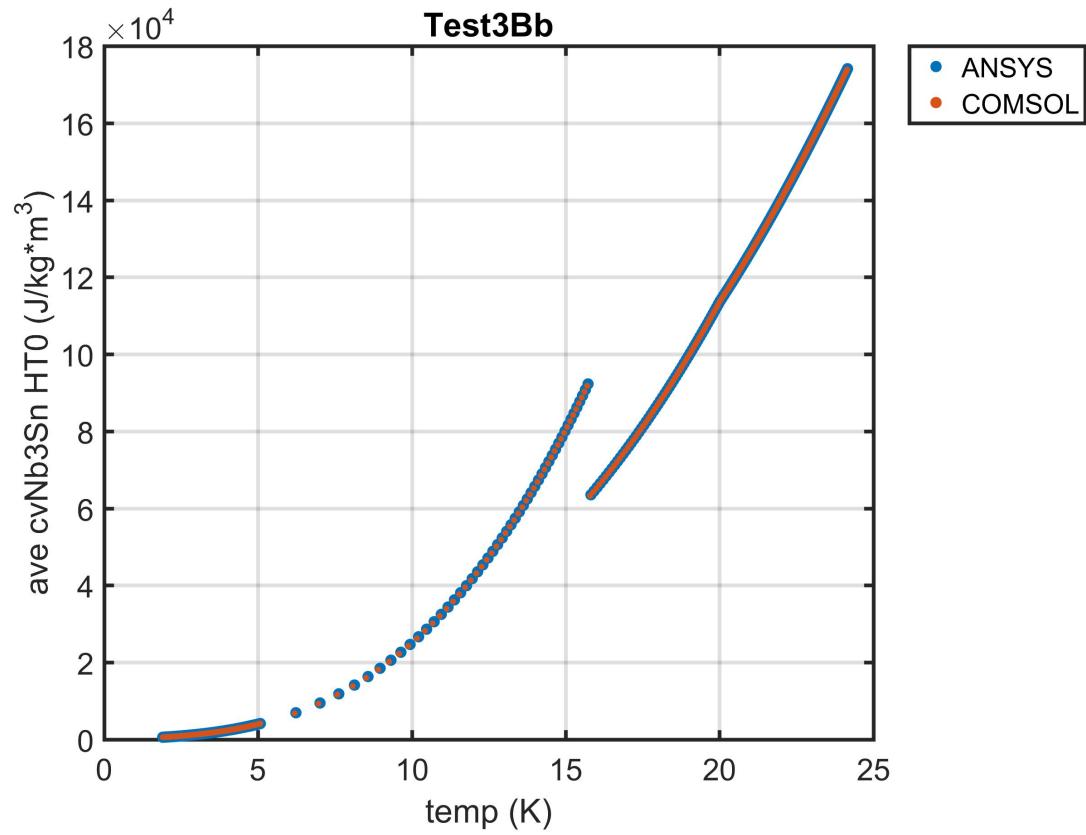


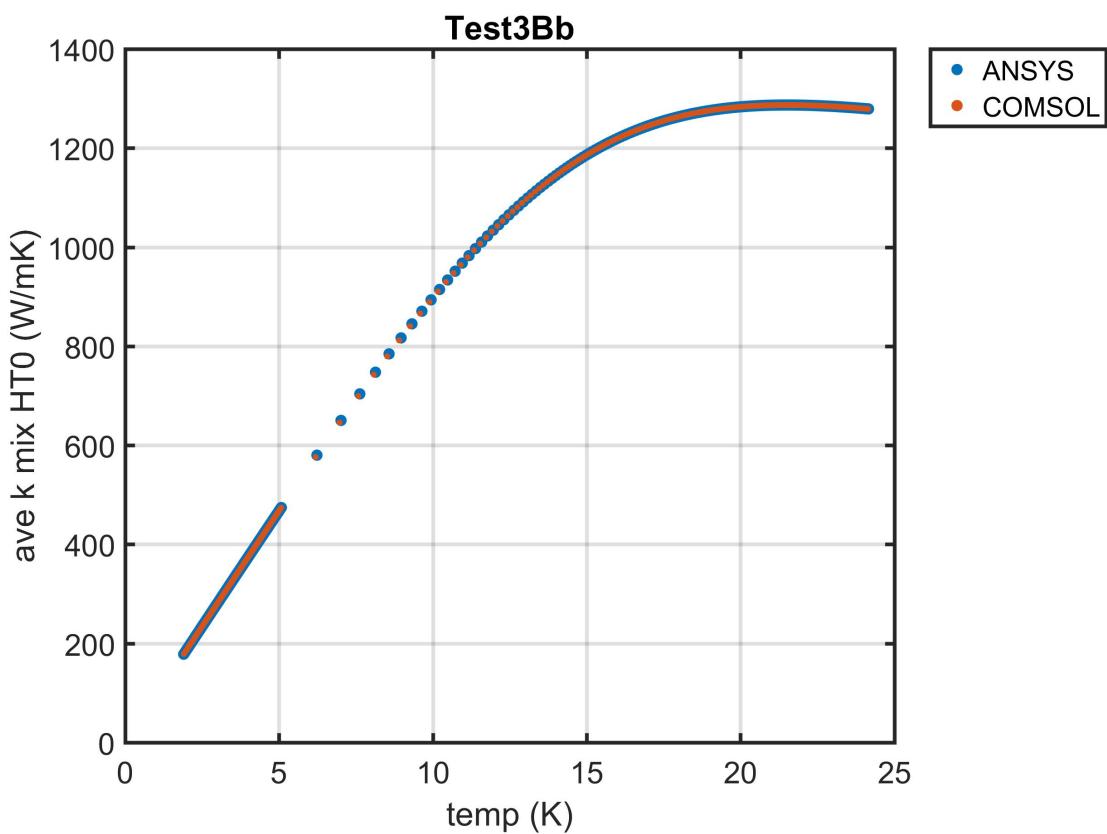
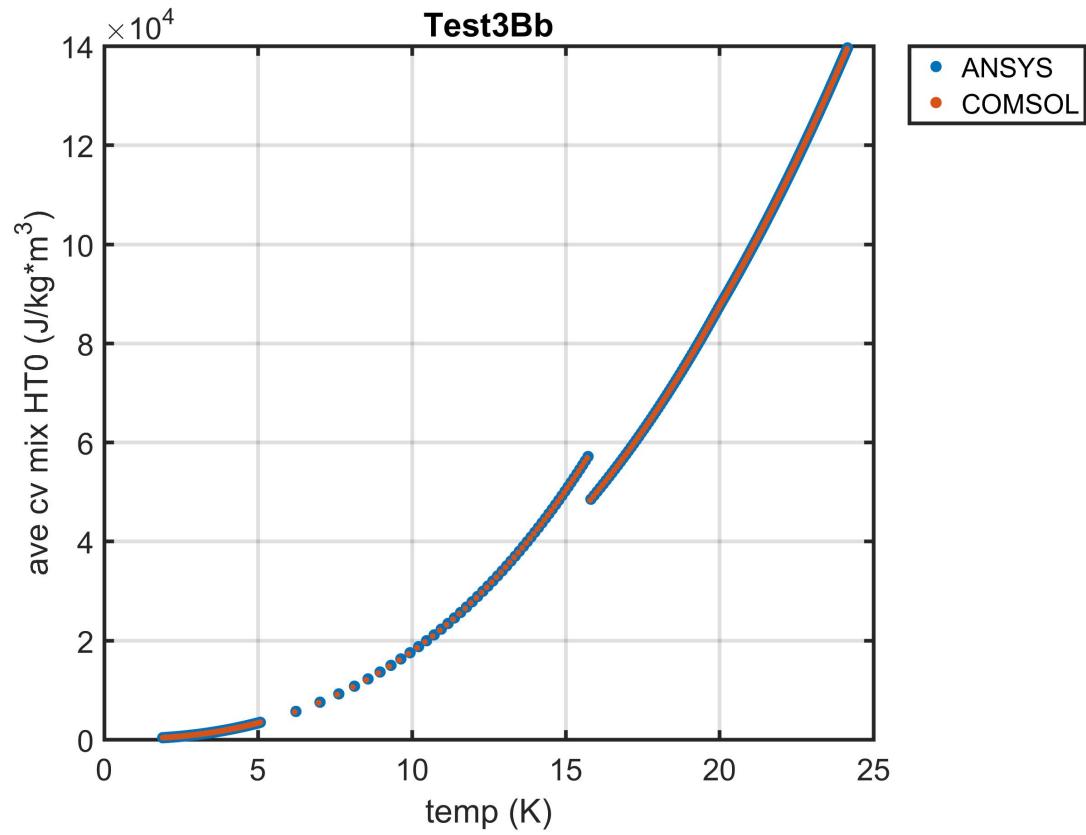


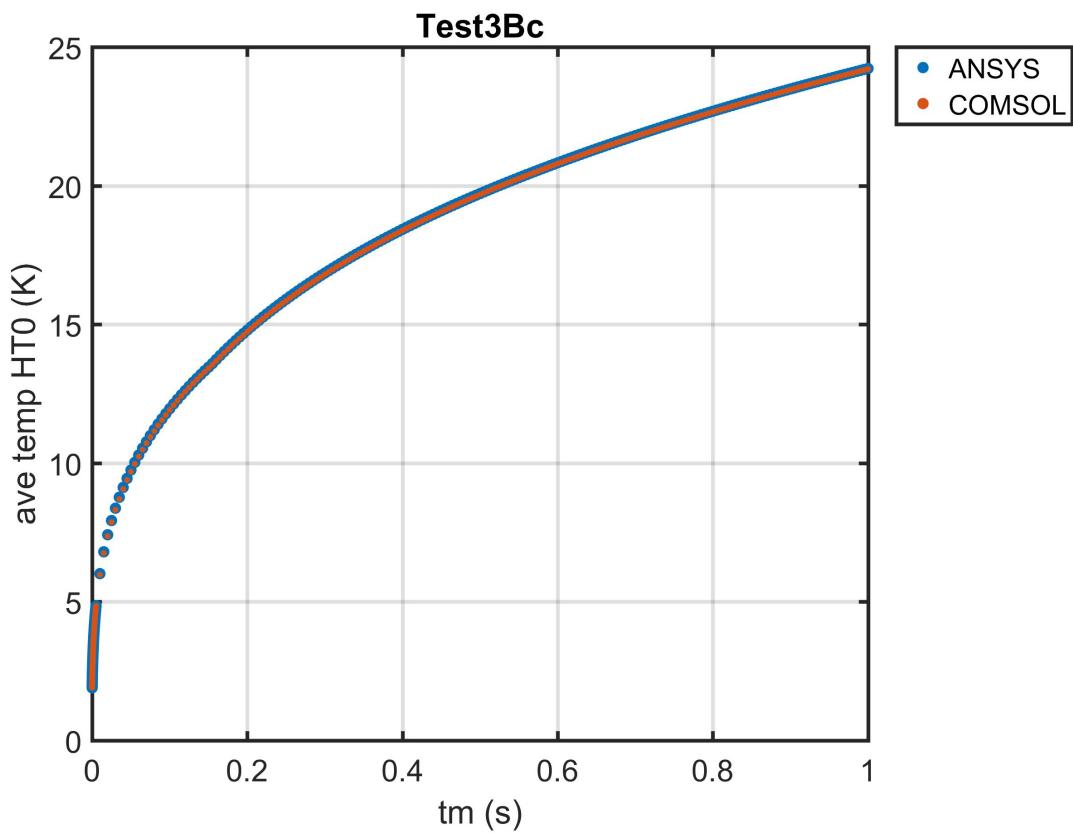
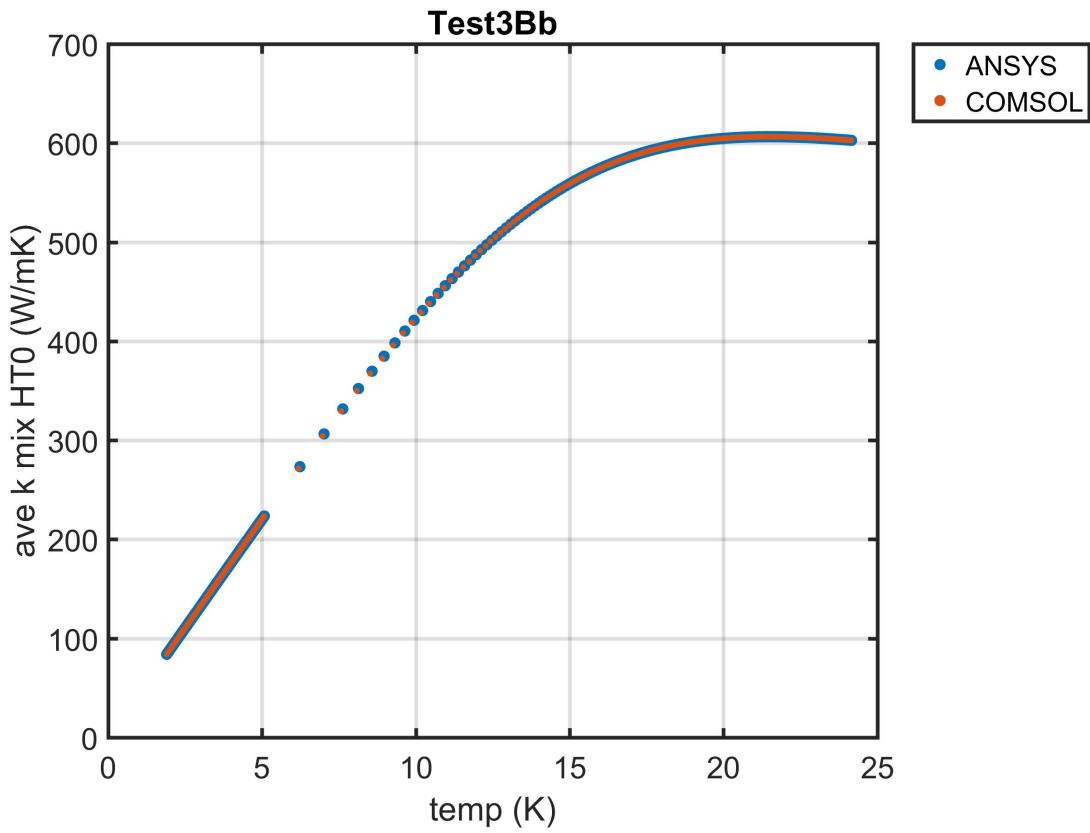


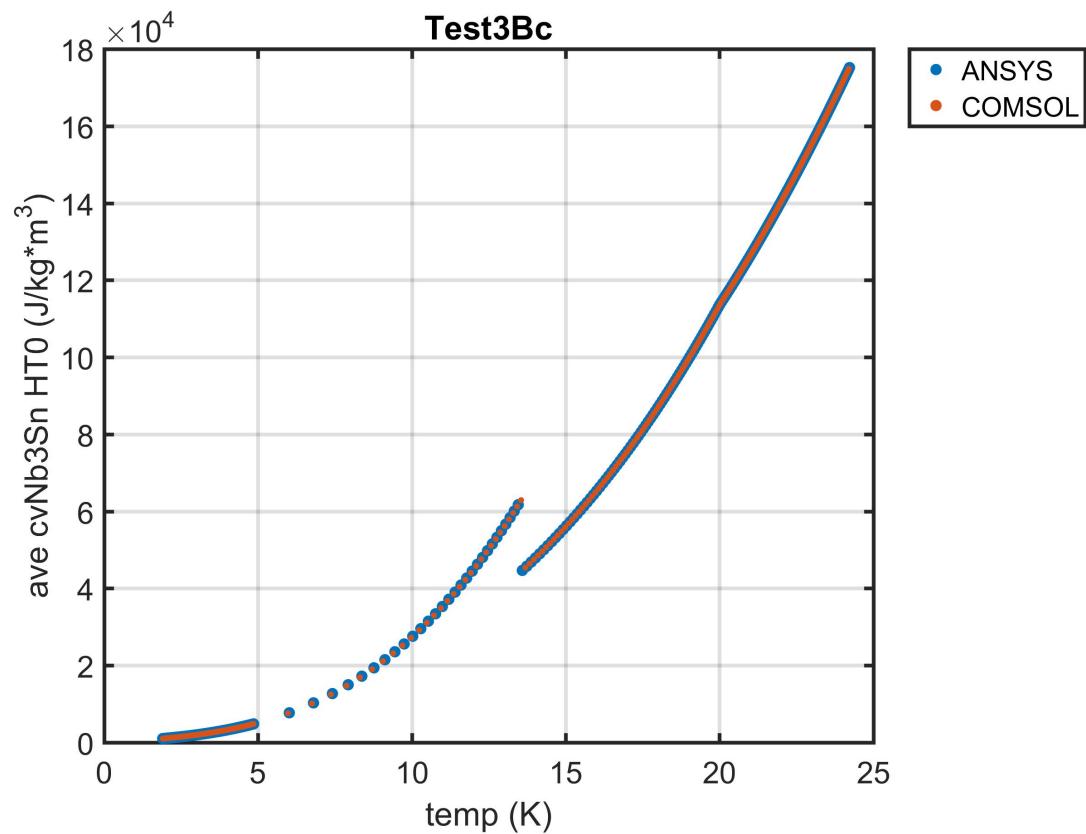
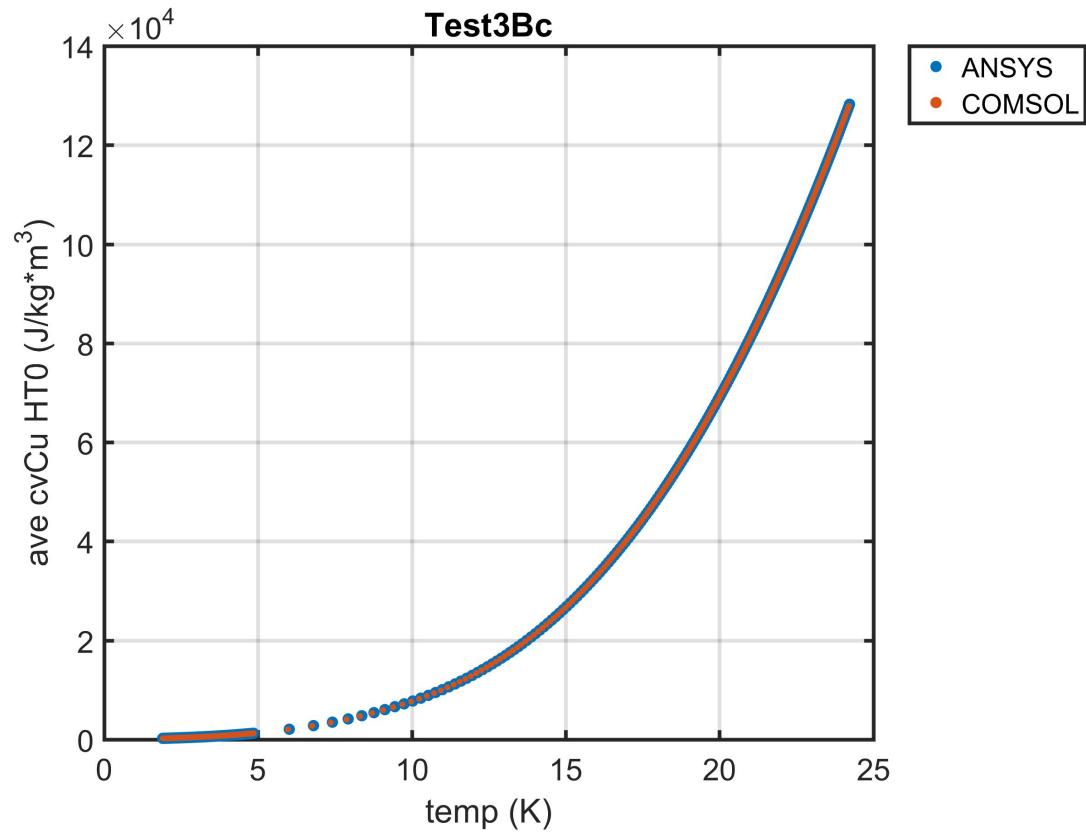


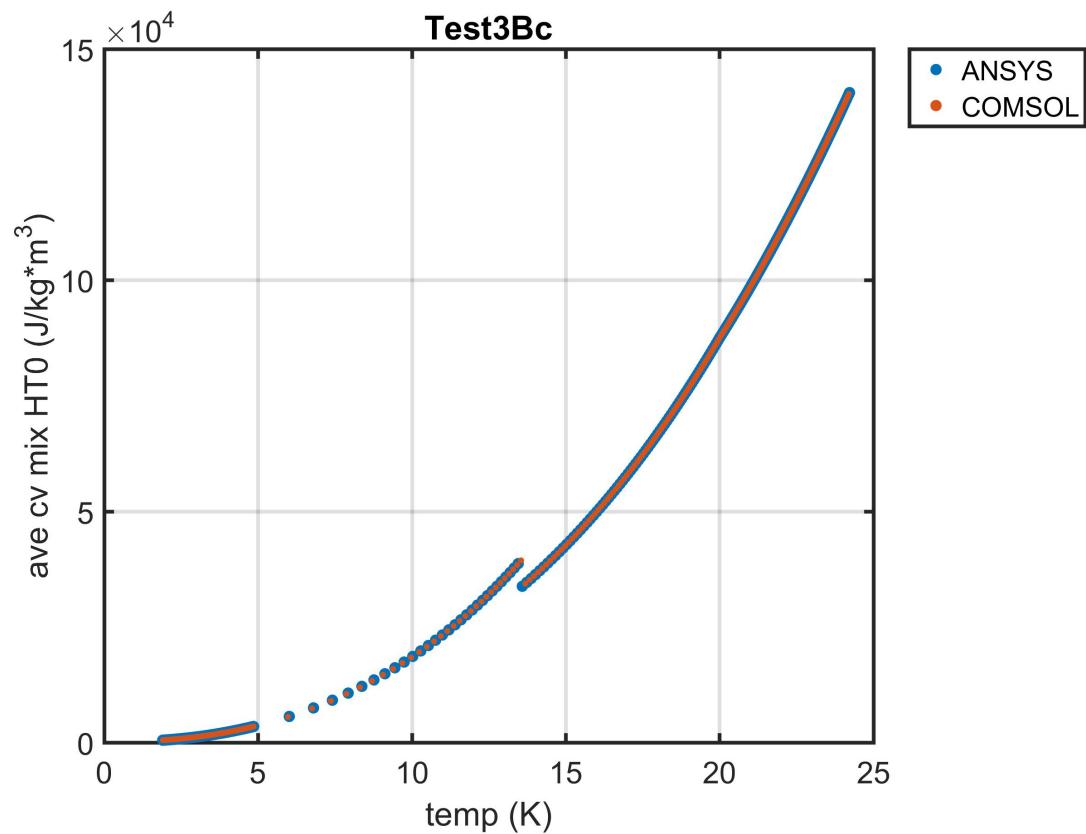
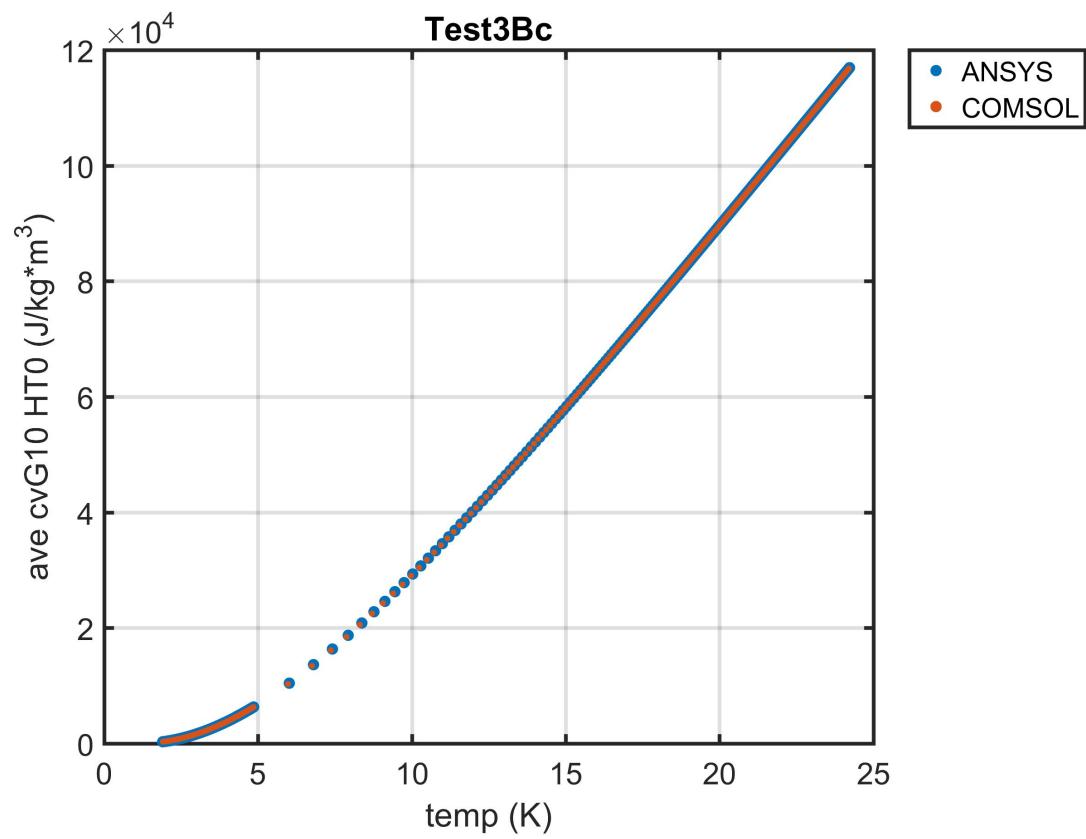


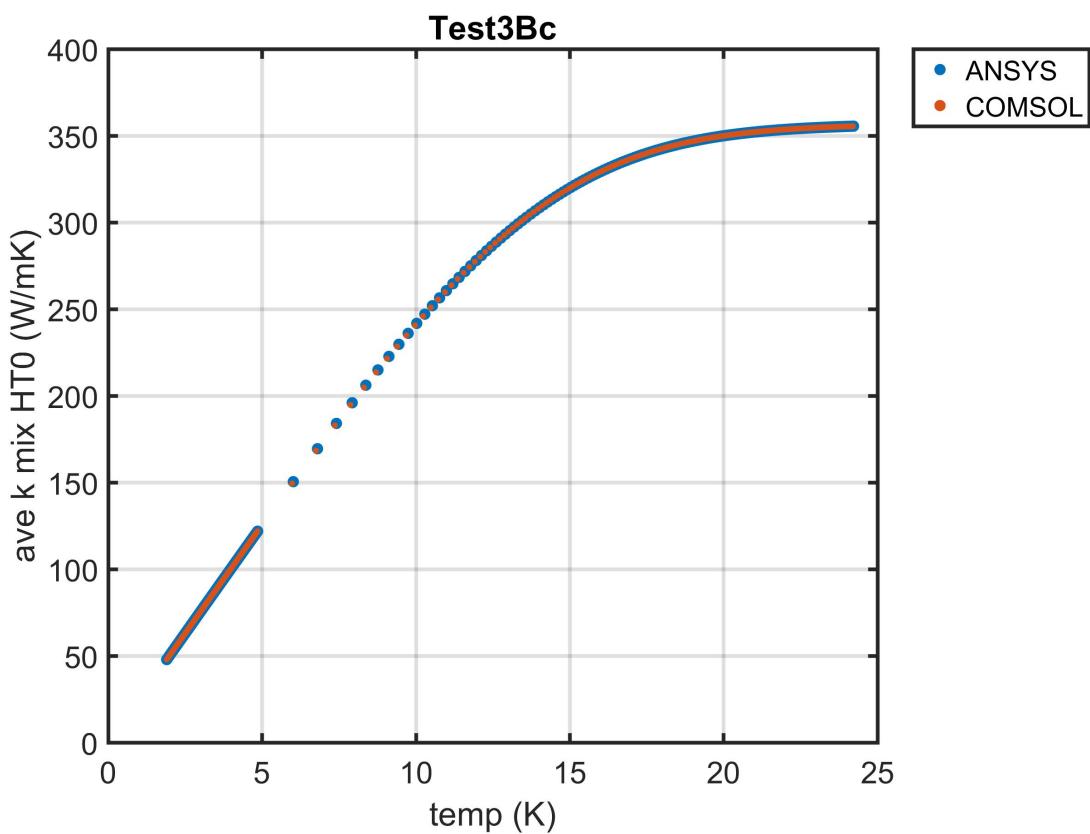
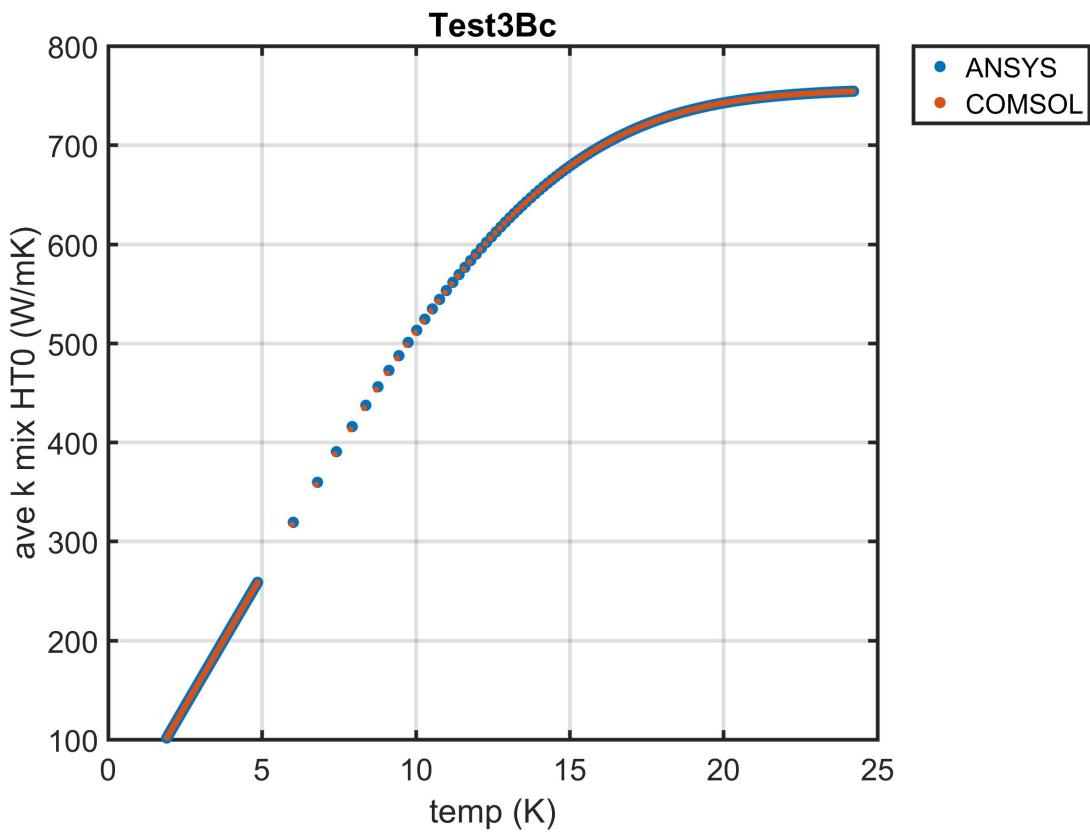






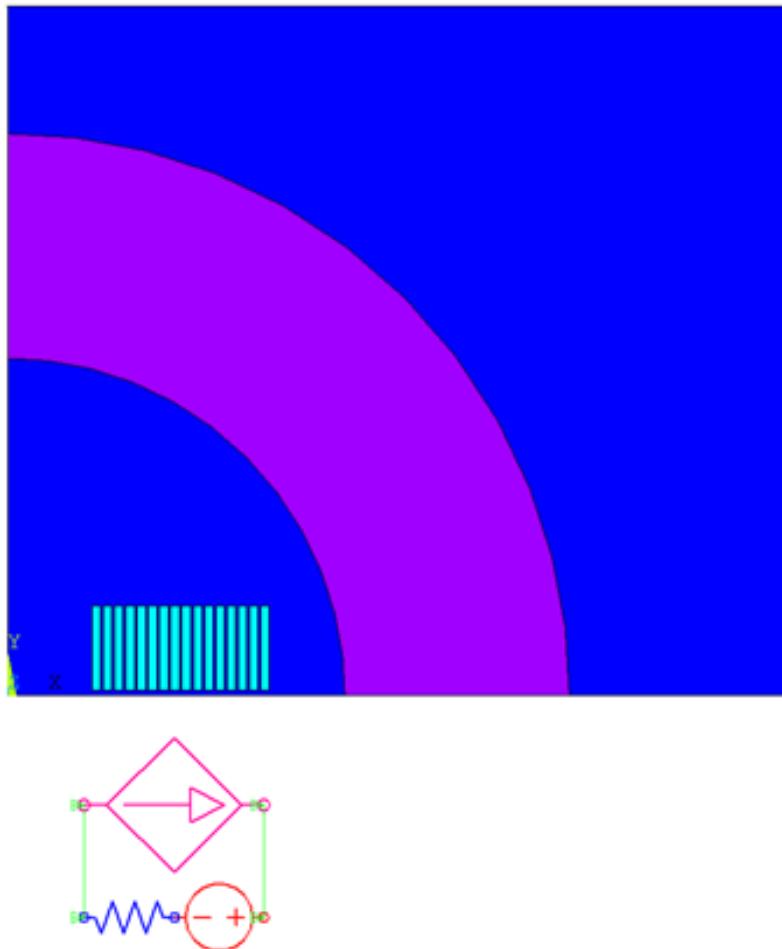






## 4 Single Layer Tests With Circuit Coupling

A series of comparison tests are added which build up the effects included for a single simulation including circuit coupling. The finite element region is assumed to be coupled to a simple dump resistor and power supply circuit. A voltage source is used to apply the desired current before extraction is triggered. At 5.0 ms the voltage source is ramped down to zero over 0.1 ms which acts like a switch to allow for the decay of the magnet in series with the dump resistor. This circuit is shown in the figure below. The parameters  $I_0$  and  $R_{\text{dump}}$  are used as the initial current in the circuit (steady state before 5.0 ms) and the dump resistance. Two length scaling factors are used.  $L_c$  is the coil length which can be scaled to match the resistive voltage between 2D and 3D models, and  $L_i$  is a similar factor but for matching the inductive voltage. These four parameters will be added to the table for each simulation. For the following comparisons, a current of nearly 90% of short sample and a dump resistor which keeps the magnet voltage from exceeding 500 V were chosen.

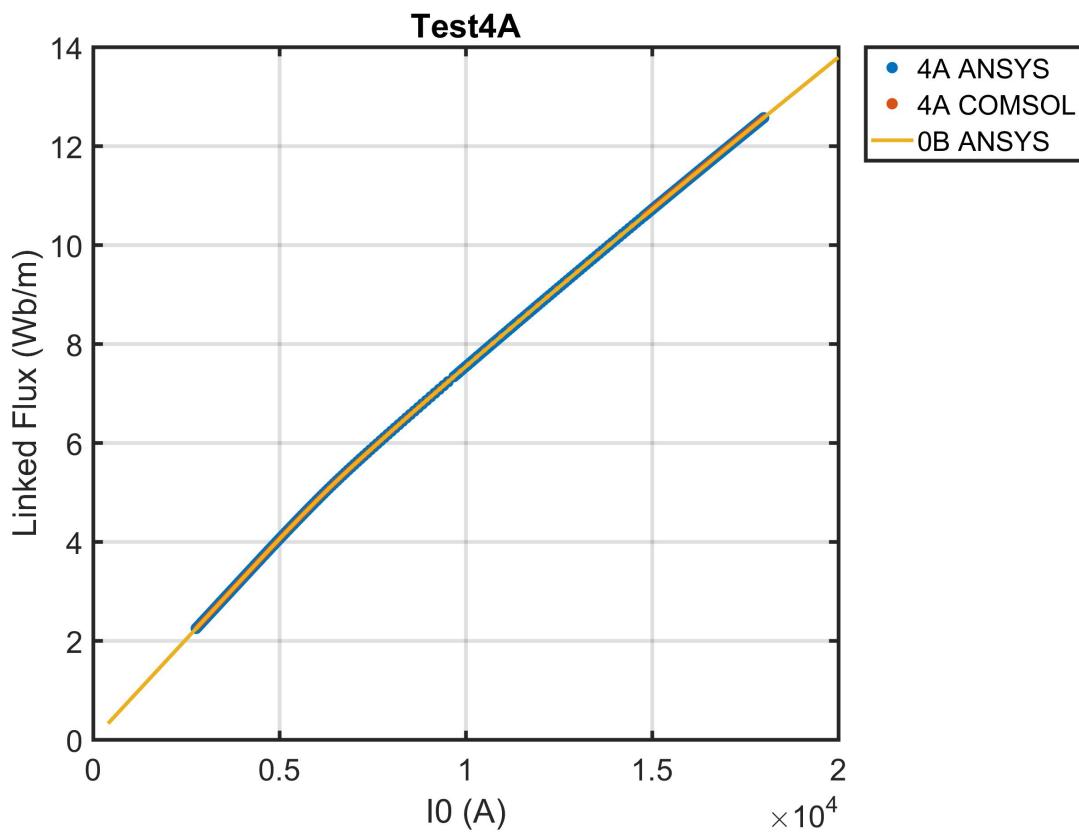
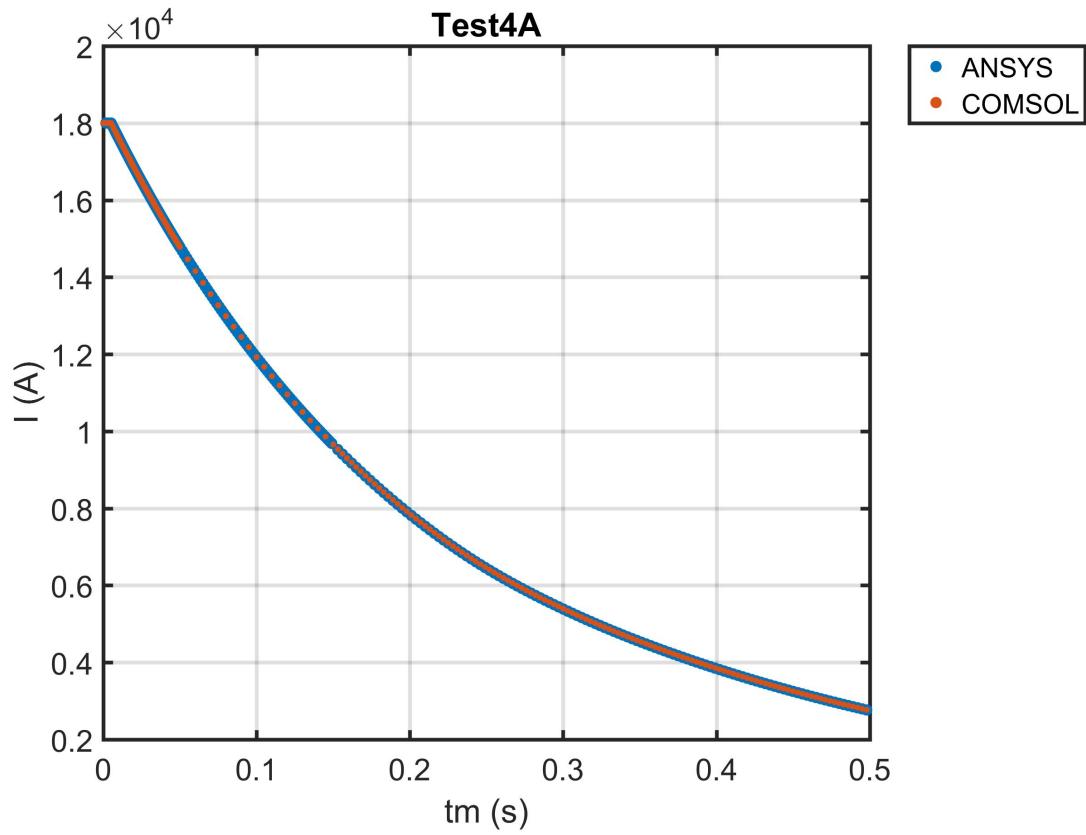


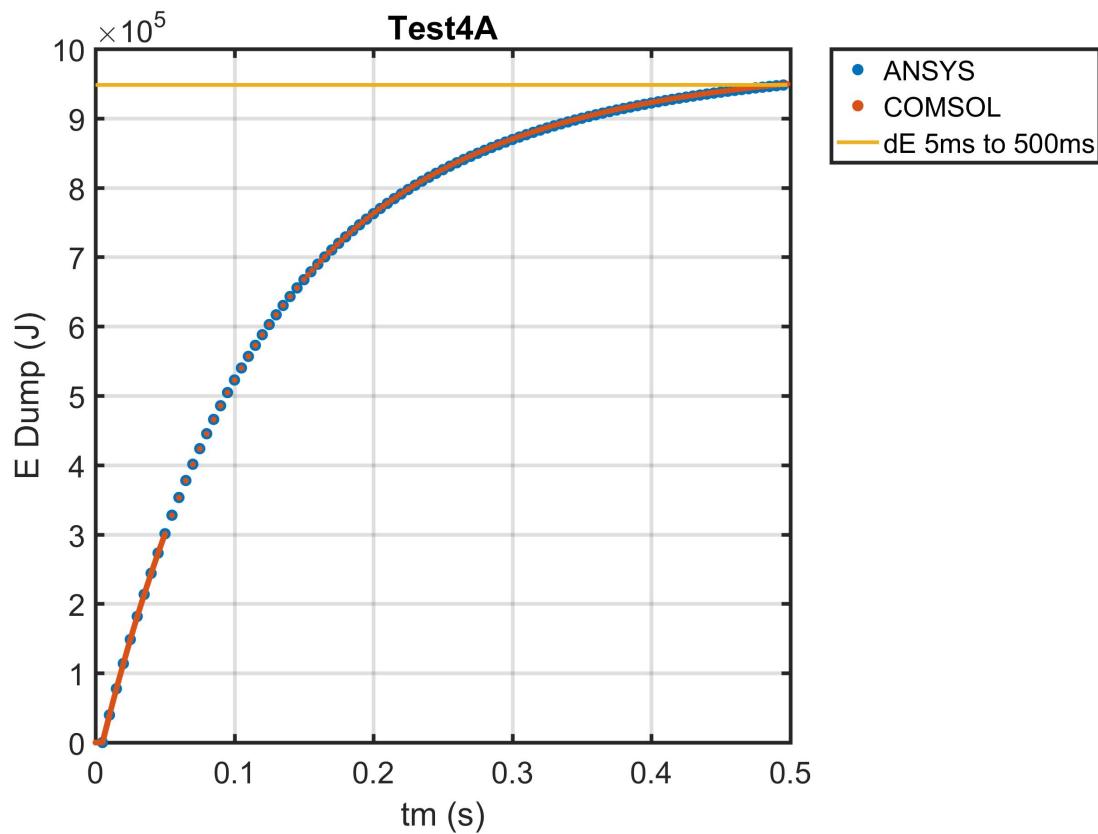
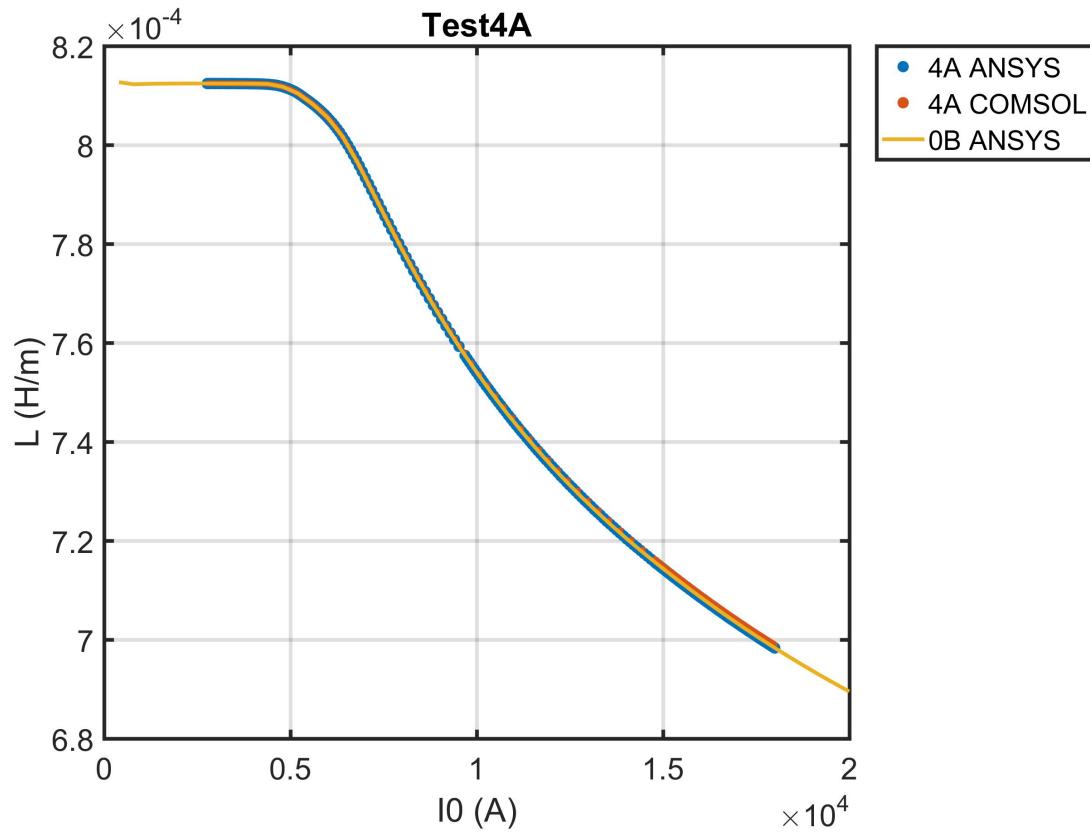
## 4.1 Test 4.A LR Decay With No Losses

This first simulation is intended to test the circuit coupling with the coil only contributing inductive voltage. Interfilament coupling currents and quench are turned off so there are no losses outside of the dump resistor. Since there are no heating terms, thermal coupling is not needed and the coil will stay superconducting and at the initial temperature of 4.5 K. I like to track energy, and for this case its a good check that 100% is dissipated in the dump. The length  $L_i$  is the effective magnet length used for inductive voltage. Note that both the linked flux and inductance can be checked against Test0B if plotted vs current.

Active physics	EM, CIRCU
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	18.0 kA
$R_{\text{dump}}$	25 m $\Omega$
$L_c$	N/A
$L_i$	9.20 m
<b>Output</b>	$I(t)$ [A]
	Coil magnetic flux $\phi$ [Wb/m]
	Flux inductance as $L = \phi/i$ [H/m] (finite difference)
	$E_{\text{dump}}(t)$ [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	976.77 kJ
Stored Energy at 500ms	28.61 kJ
$\Delta E$	948.16 kJ
$E_{\text{dump}}$	948.58 kJ (100% of $\Delta E$ )



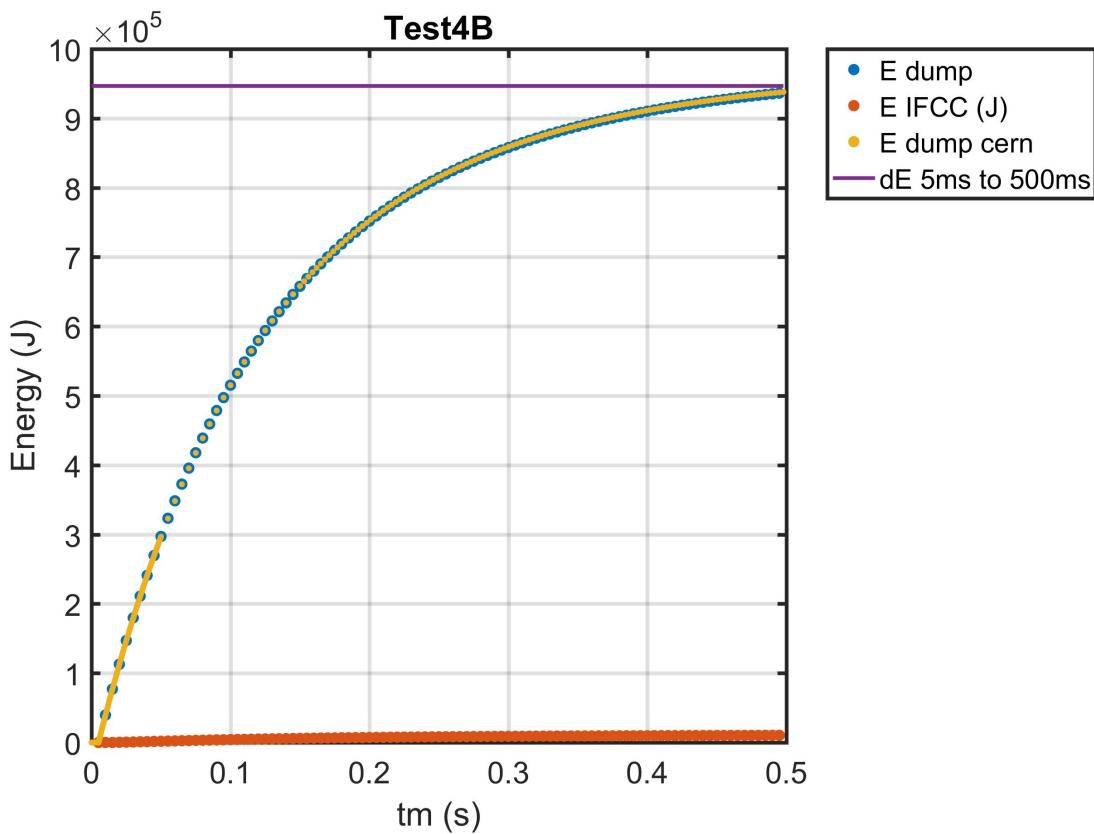
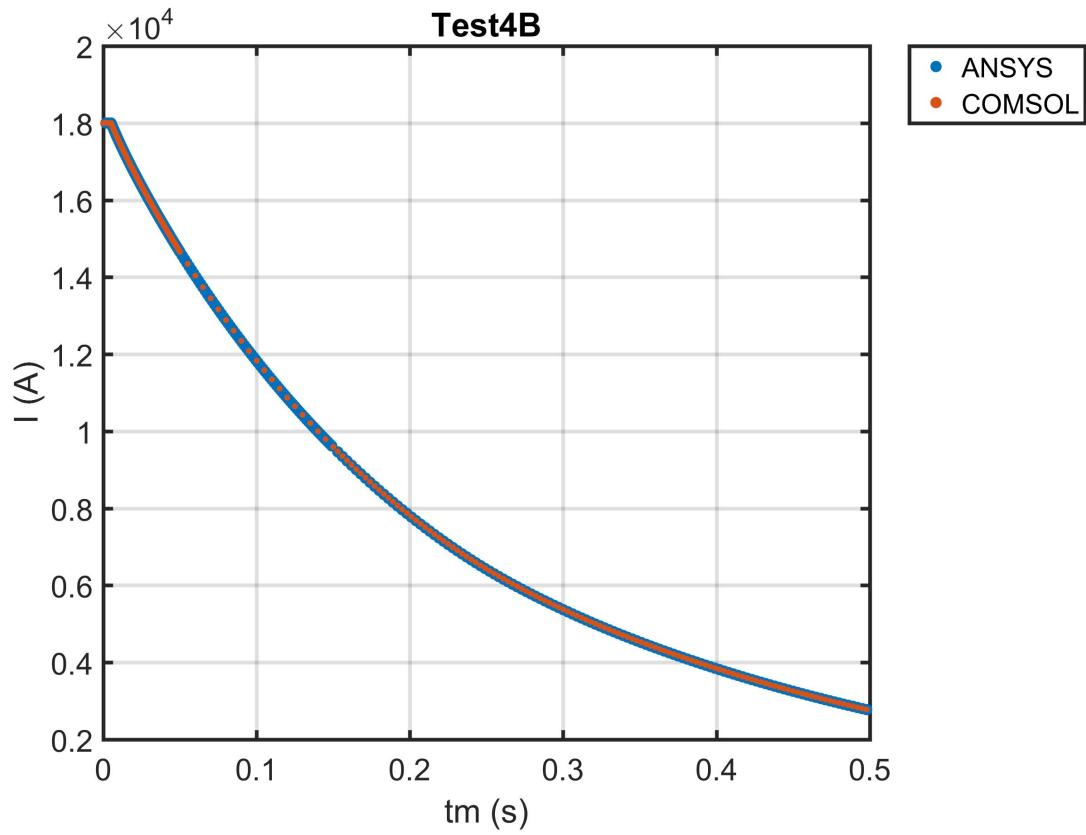


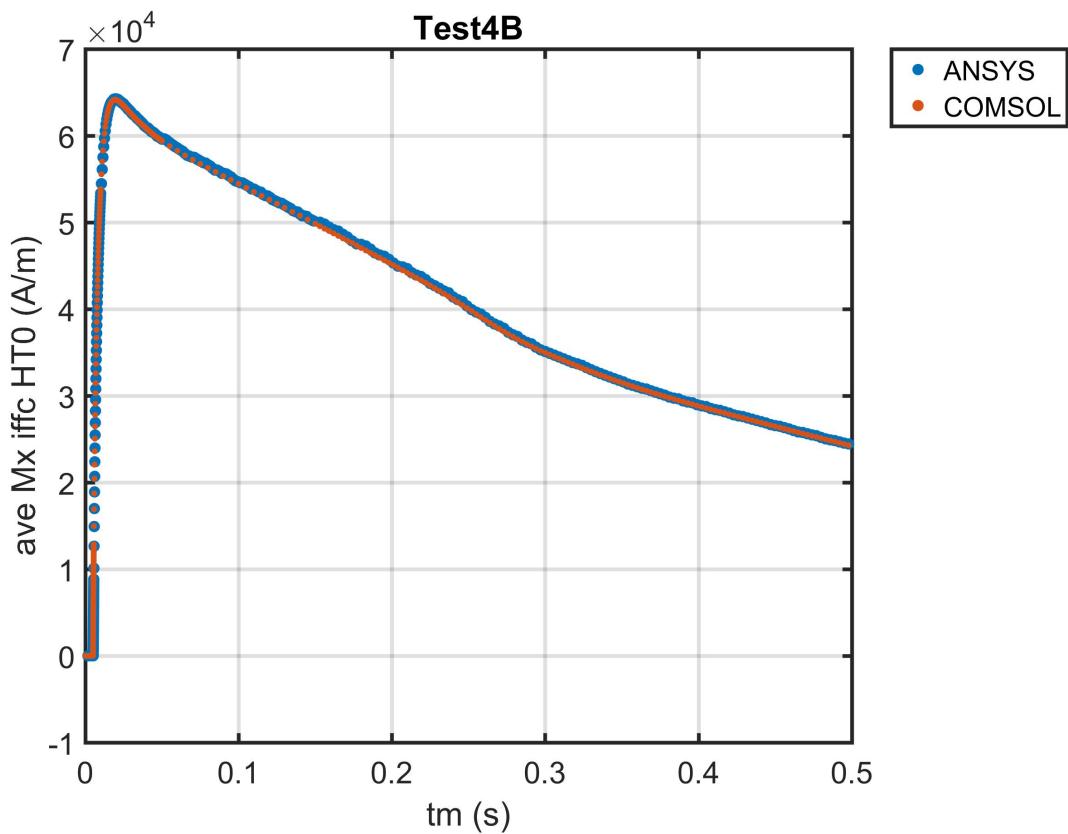
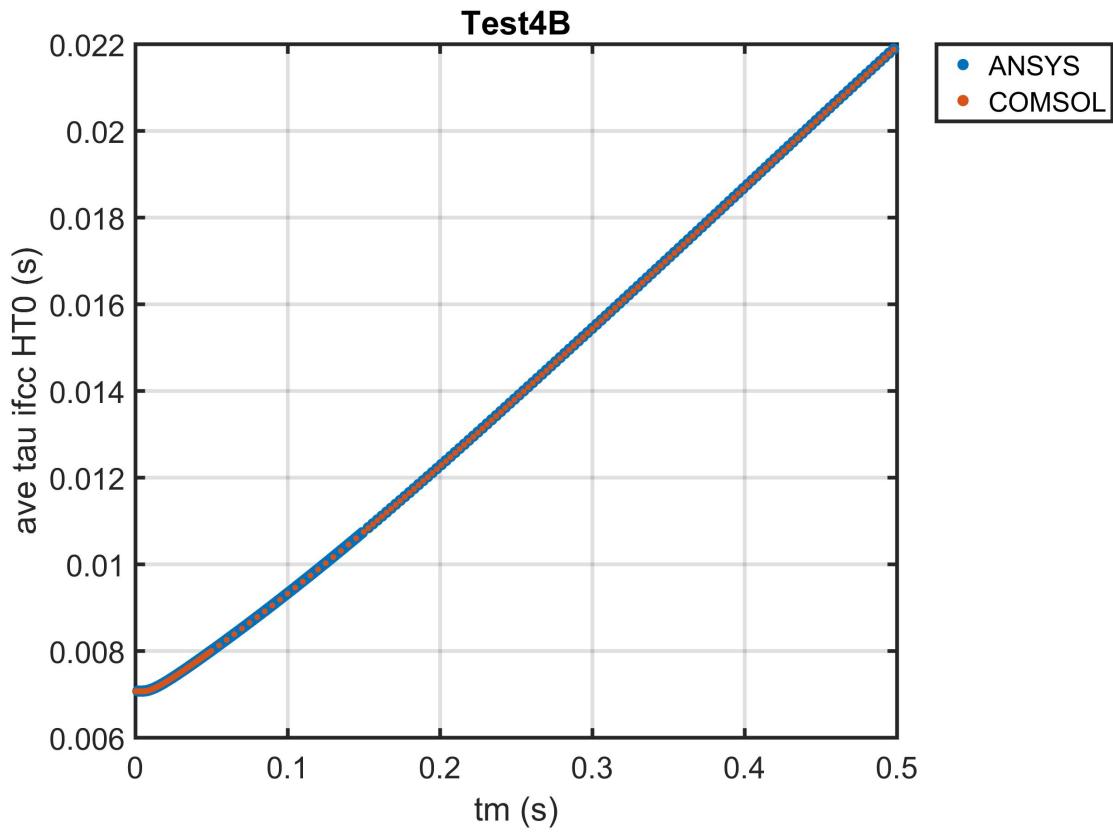
## 4.2 Test 4.B LR Decay With IFCC Losses

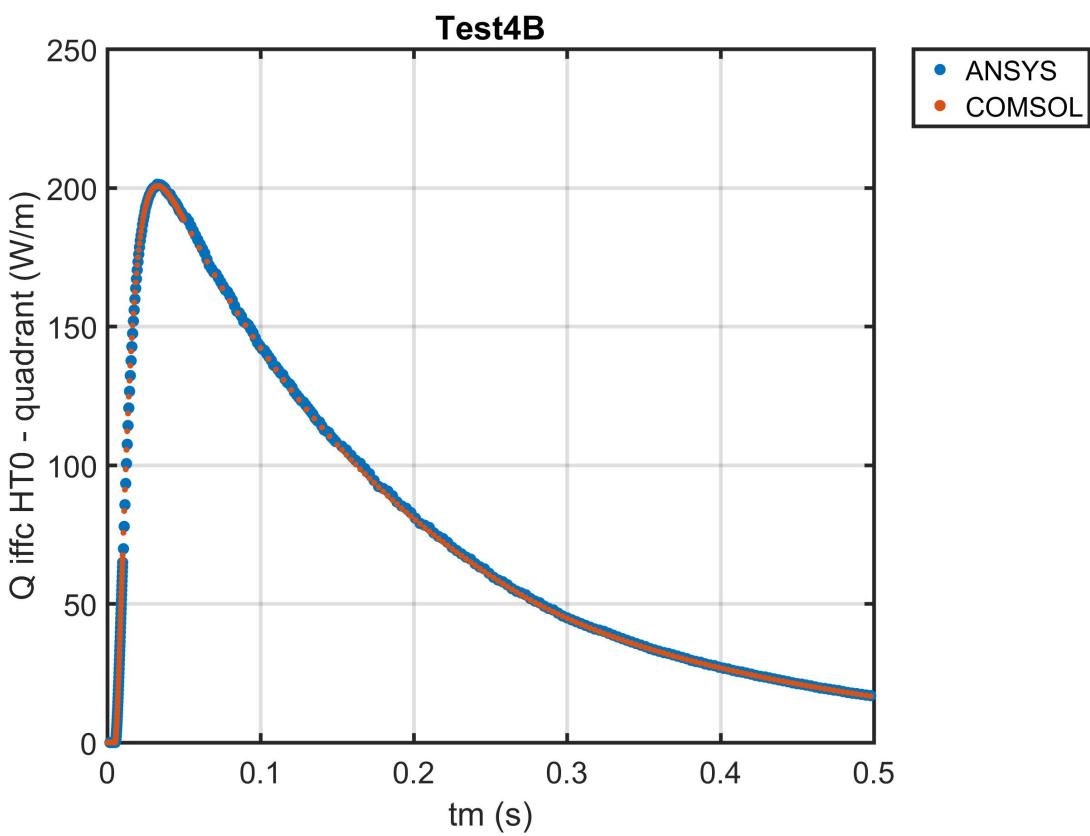
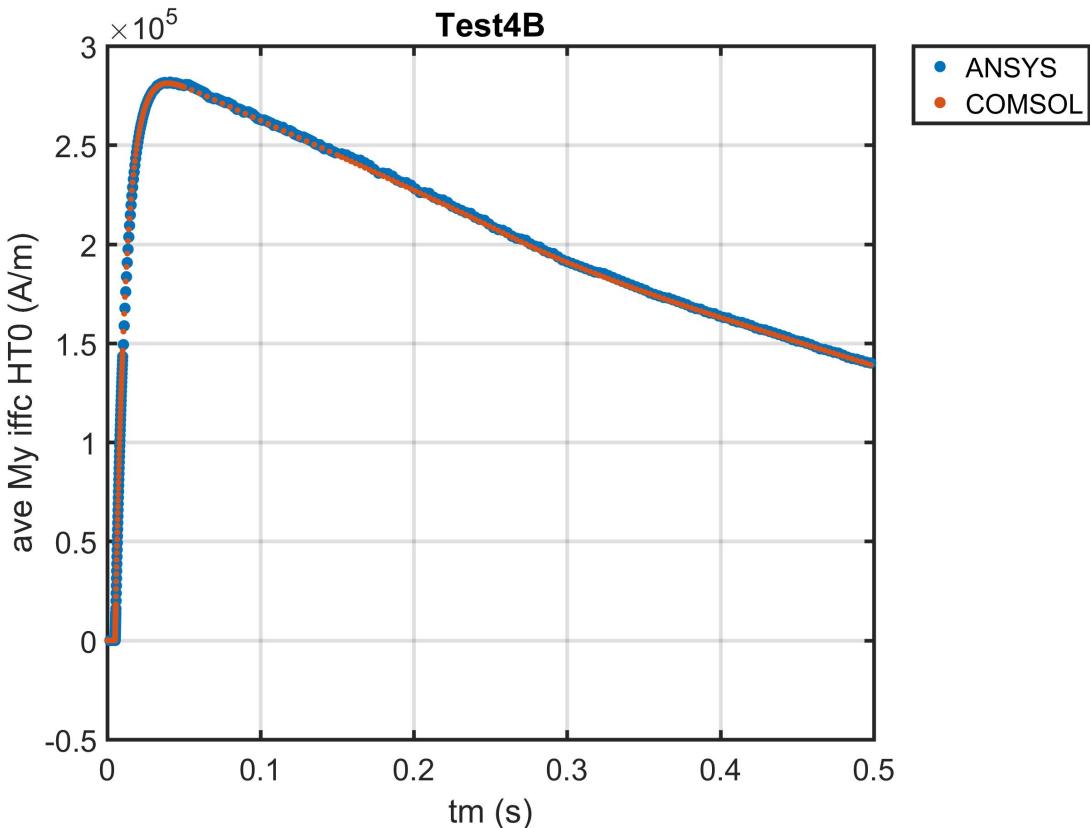
The previous simulation is repeated with inter-filament coupling current losses added, but kept at fixed temperature. This test checks the effect of IFCC on the current decay as some of the stored energy is now dissipated as IFCC loss.

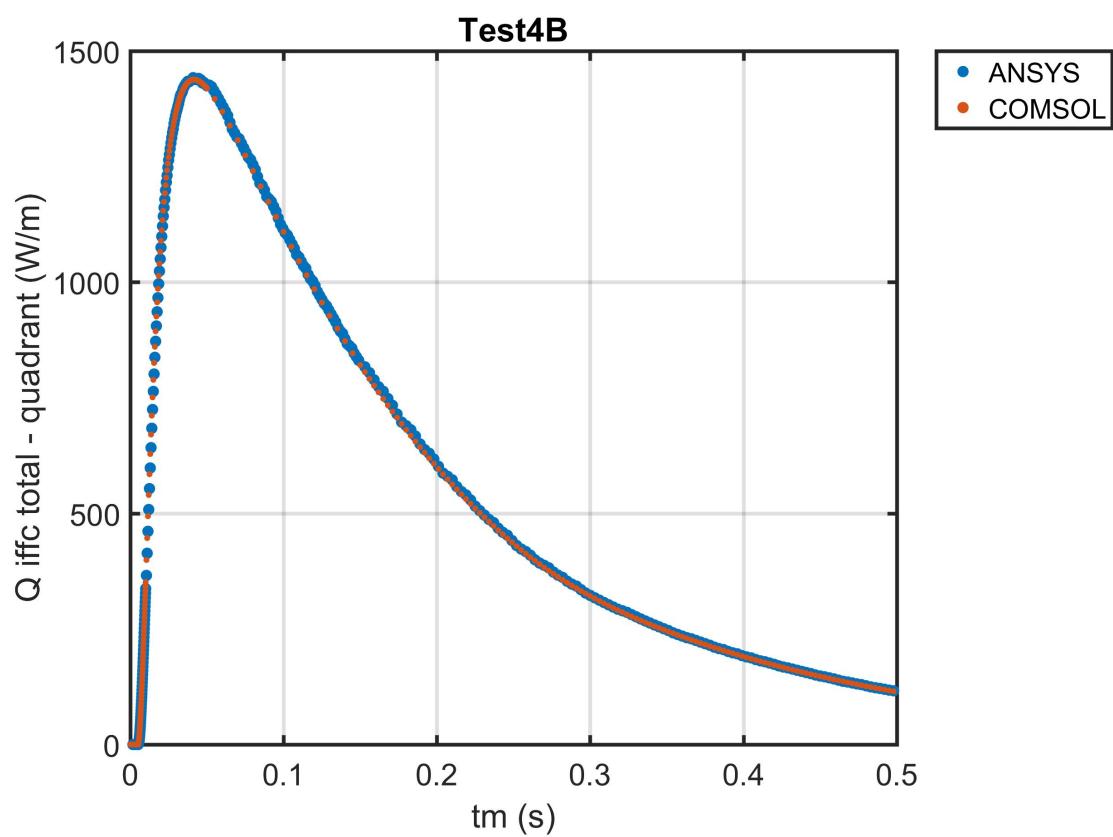
Active physics	EM, CIRCU
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Always SC
Time range for comparison	t=(0,500) [ms], Δt=suff. for conv.
I <sub>0</sub>	18.0 kA
Rdump	25 mΩ
L <sub>c</sub>	N/A
L <sub>i</sub>	9.20 m
ρ <sub>cu</sub> fit	NIST, see Appendix E
τ-IFCC	Internally calculated
<b>Output</b>	I(t) [A]
	E-dump(t) [J]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	976.77 kJ
Stored Energy at 500ms	30.05 kJ
ΔE	946.72 kJ
Edump	936.59 kJ (98.96% of ΔE)
Eifcc	10.37 kJ (1.10% of ΔE)







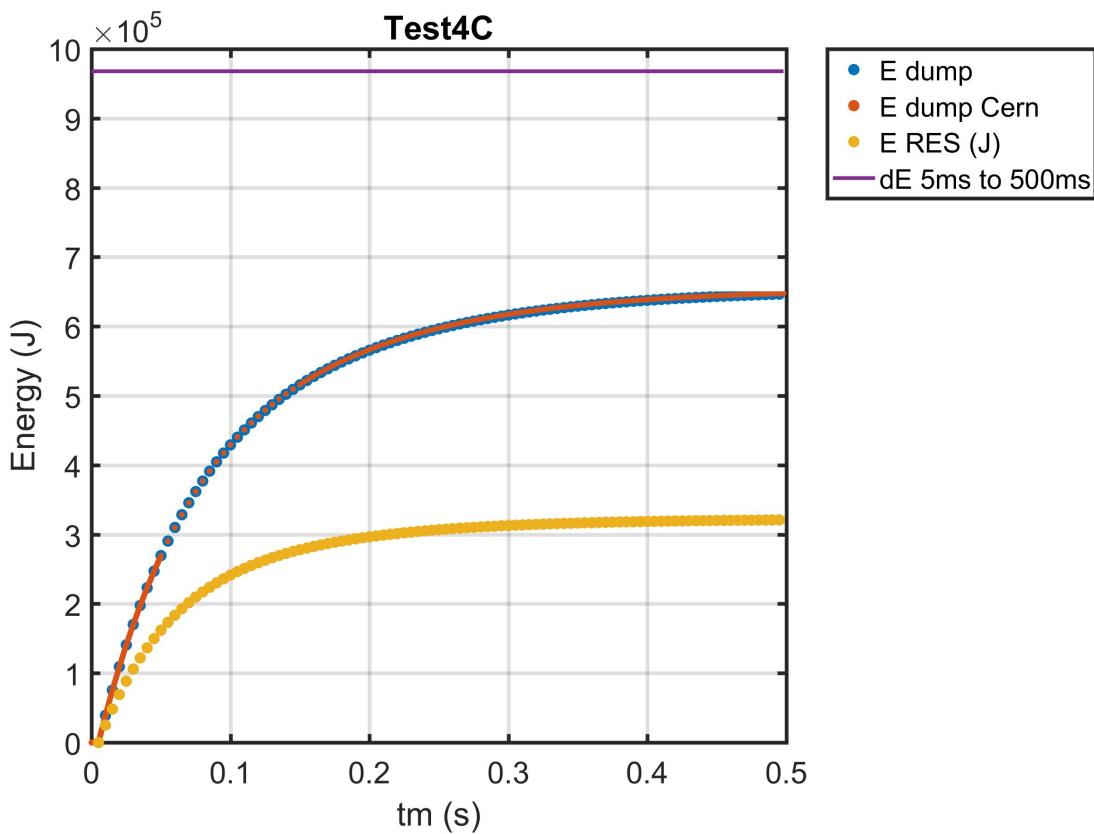
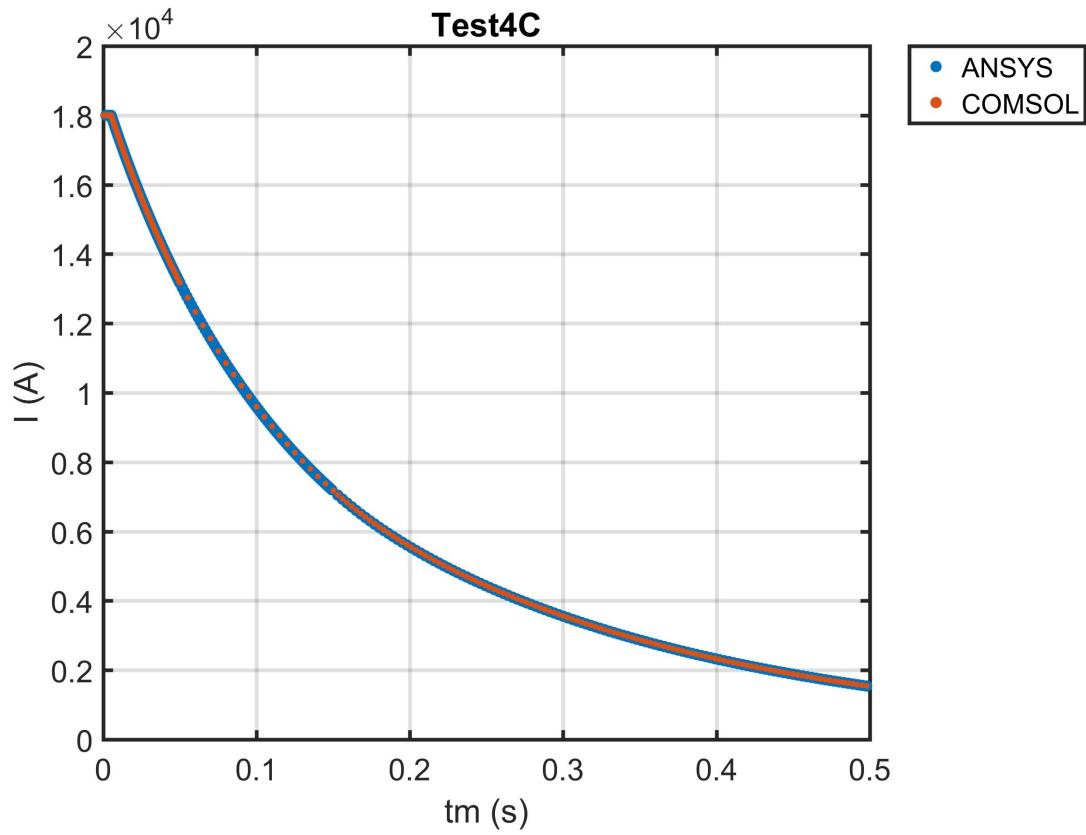


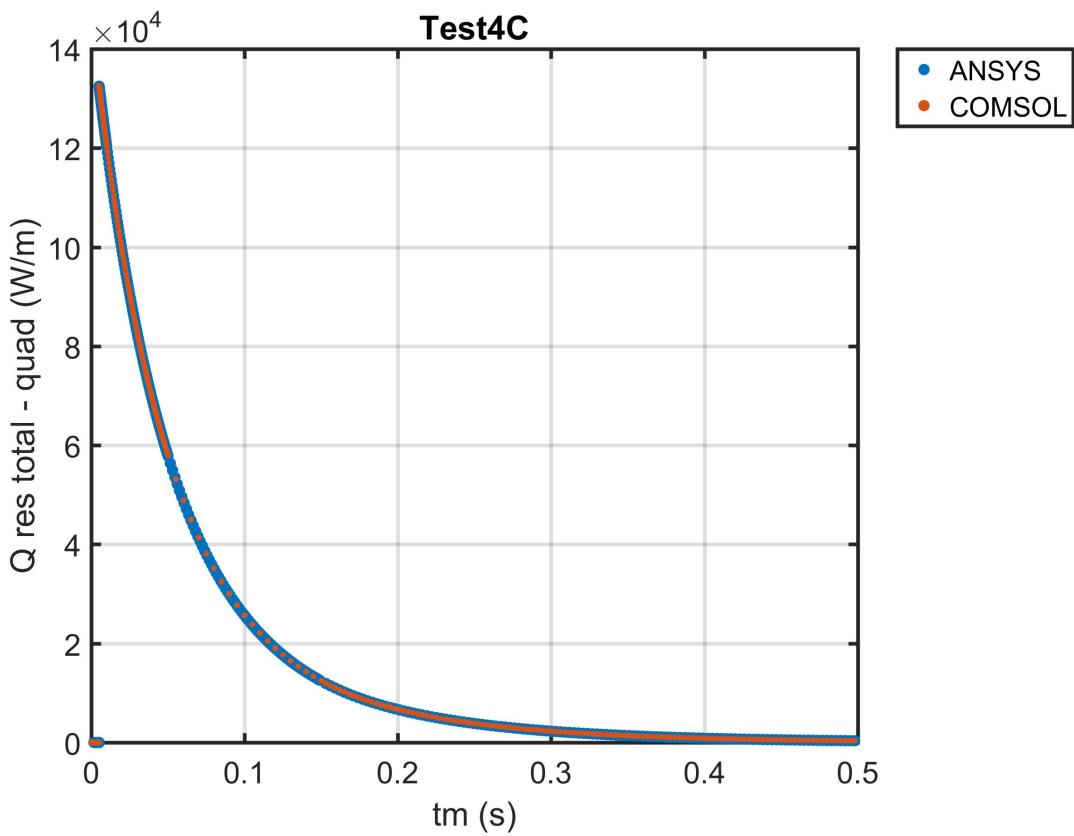
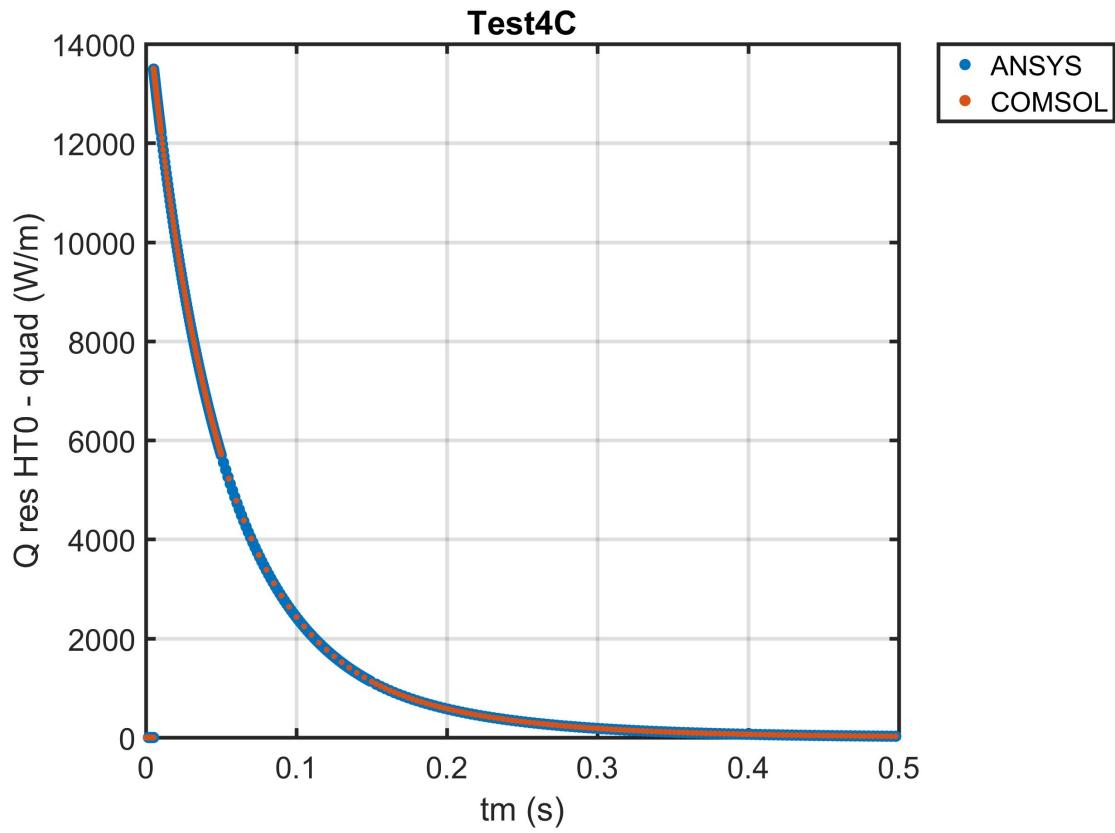
### 4.3 Test 4.C LR Decay With Coil Forced Quenched

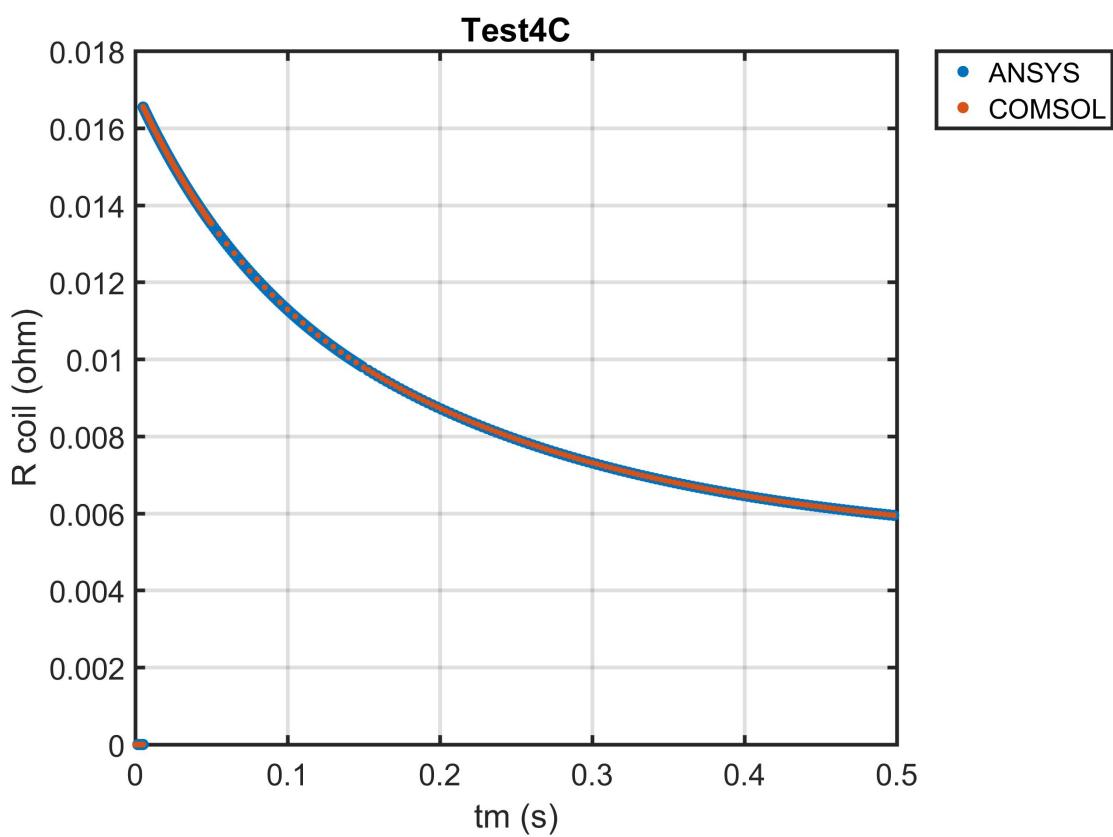
In this simulation IFCC is again turned off, but now the coil is forced to quench at a certain time. In this case we choose right when the supply voltage is dropped (5.1 ms). This adds the coil resistance to the decay seen in test 4A. Note an effective coil length for resistance is now included as  $L_c$ . As before, no heating is allowed. In this case the temperature is kept fixed and coil resistance only changes due to magneto resistivity.

Active physics	EM, CIRCU
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	NO
Quench state	Fully Quenched at 5.1 ms
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	18.0 kA
Rdump	25 mΩ
$L_c$	10.11 m
$L_i$	9.20 m
$\rho_{cu}$ fit	NIST, see Appendix E
<b>Output</b>	I(t) [A] E-dump(t) [J] Q-Ohm <sub>totHT0</sub> [W/m] (per quad) Q-Ohm <sub>totcoil</sub> [W/m] (per quad) R-coil(t) [Ω] E-Ohm(t) [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	976.77 kJ
Stored Energy at 500ms	8.89 kJ
$\Delta E$	967.88 kJ
Edump	646.63 kJ (66.81% of $\Delta E$ )
Eohm	320.97 kJ (33.16% of $\Delta E$ )





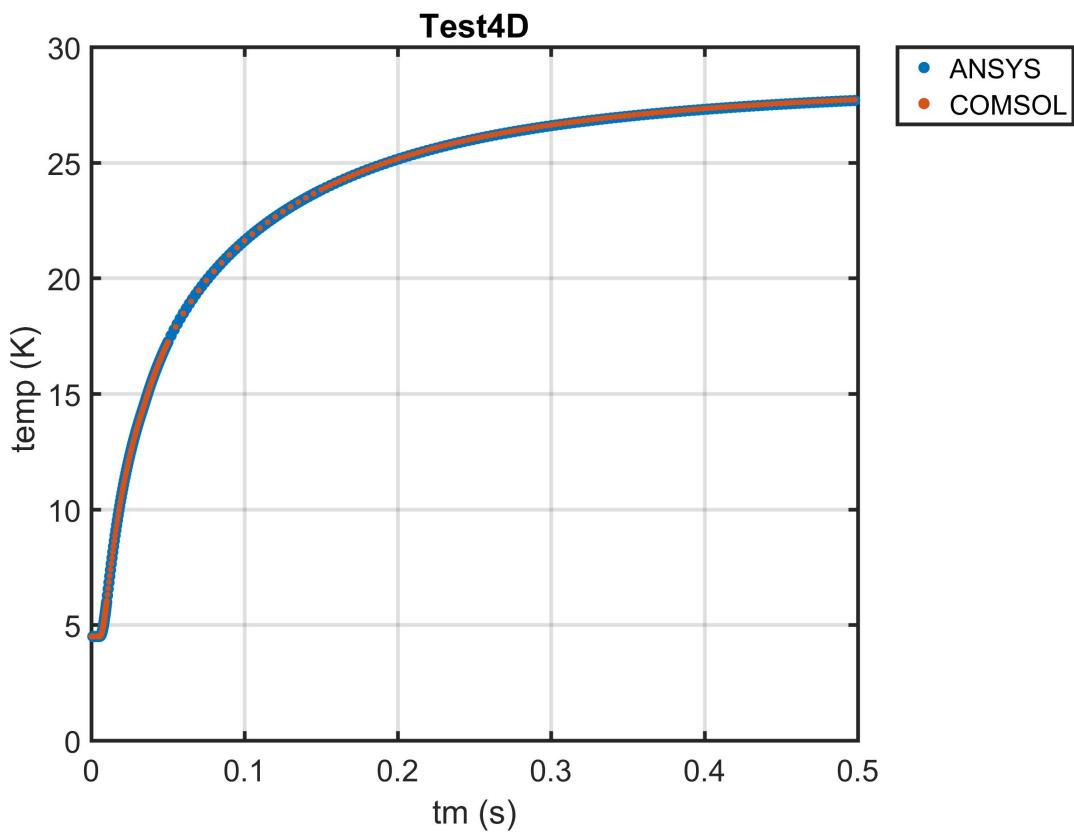
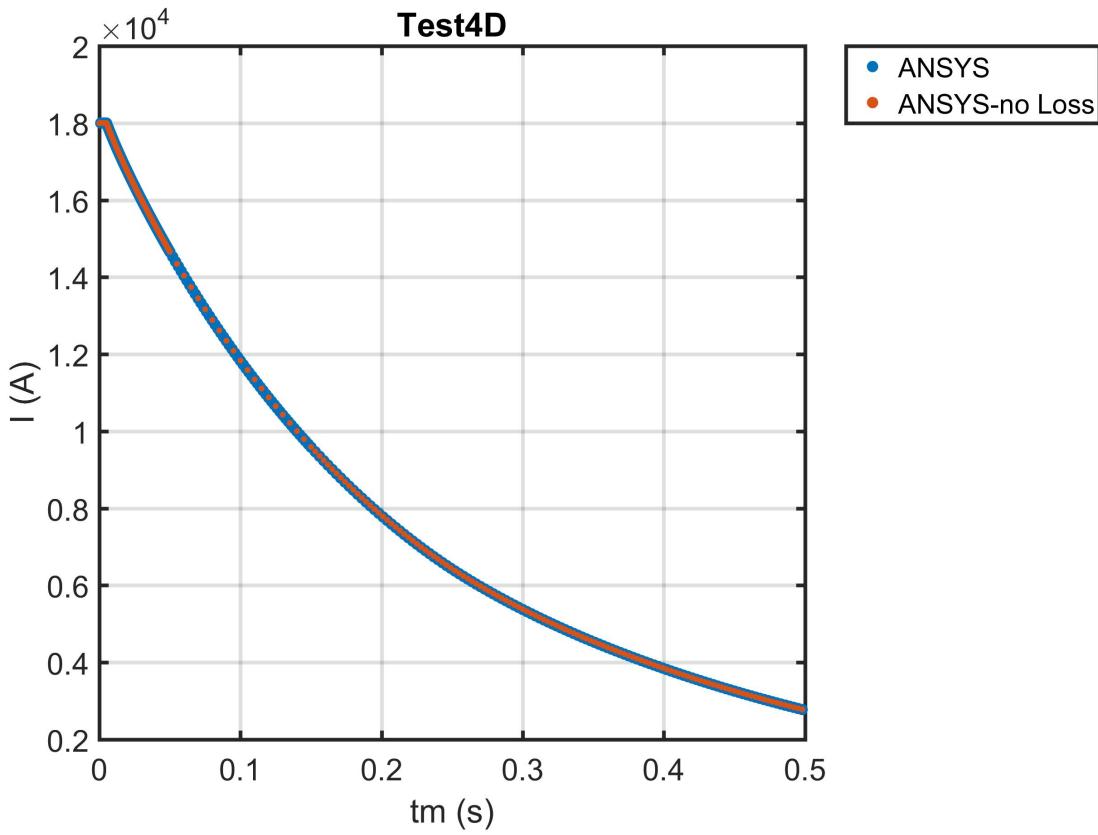


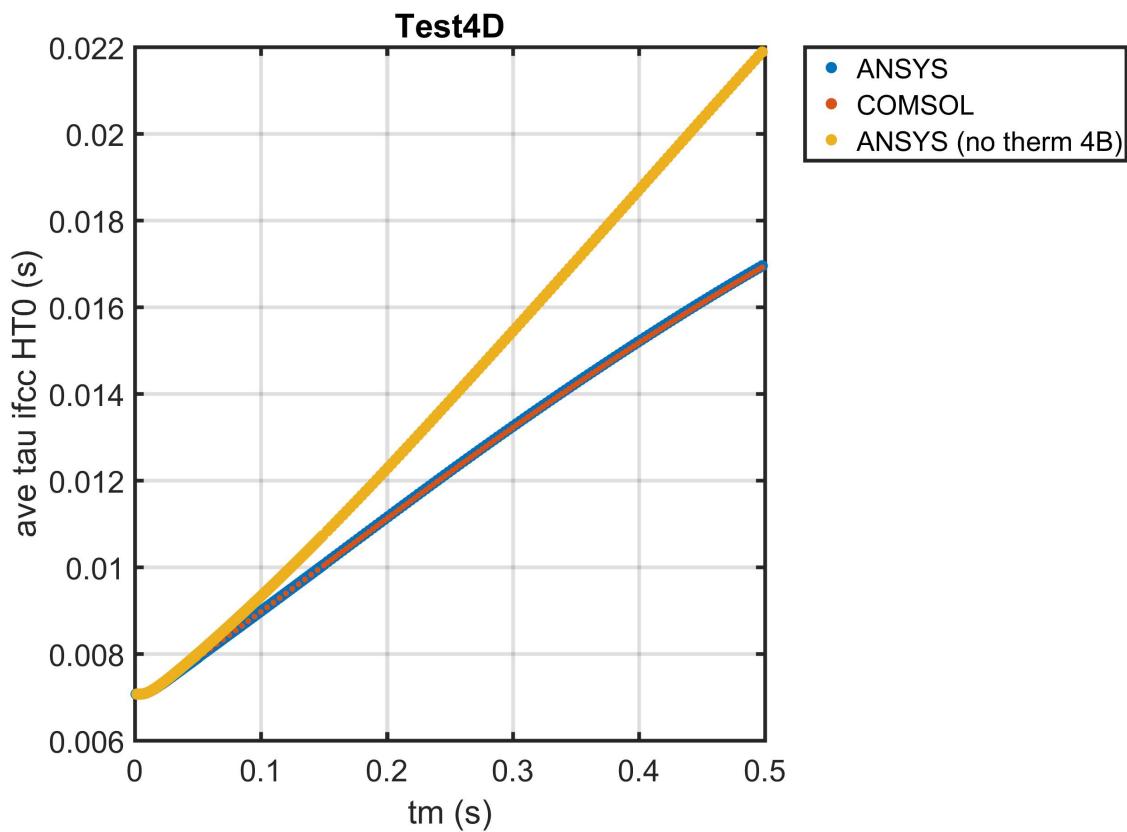
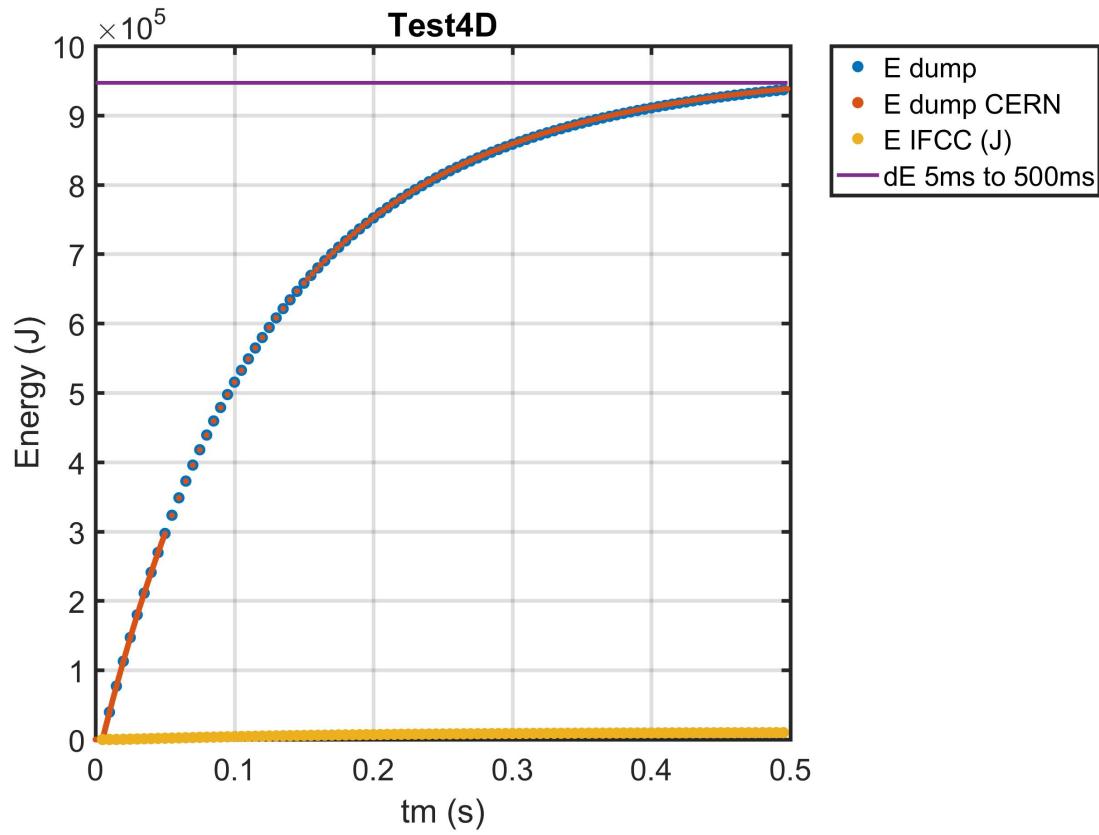
#### 4.4 Test 4.D LR Decay With IFCC Loss and Thermal

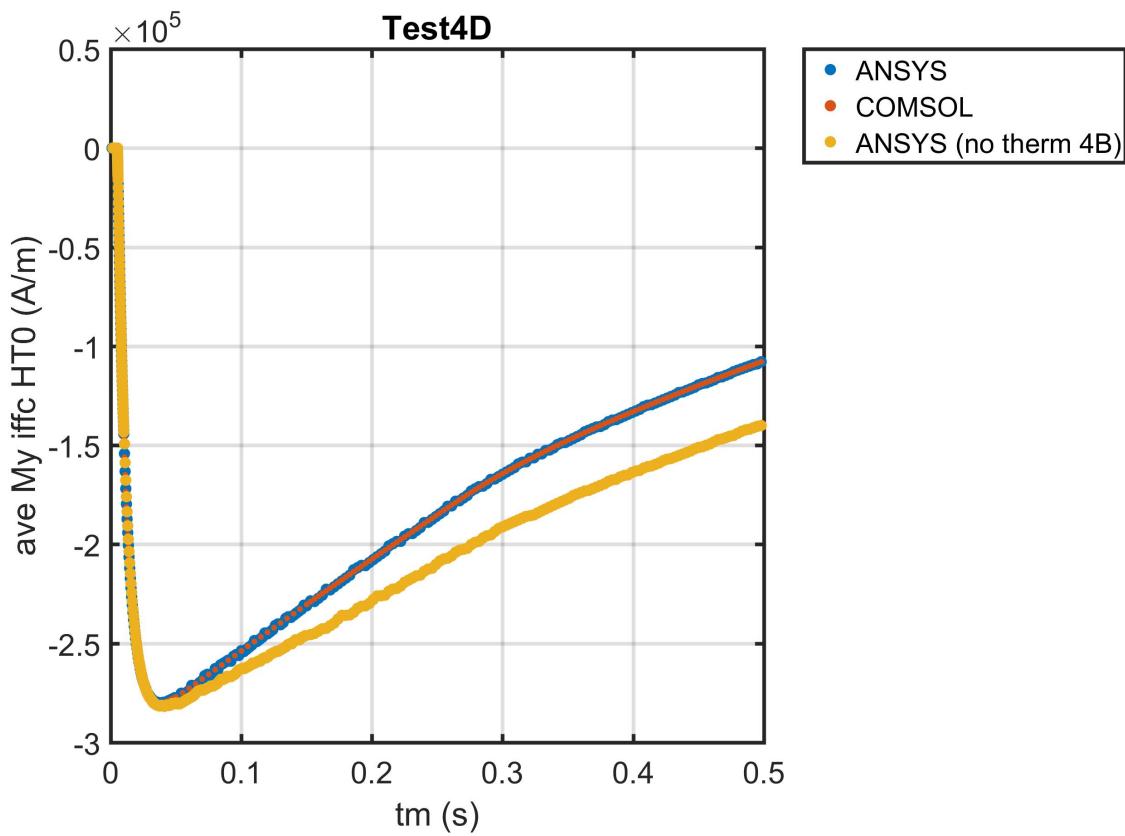
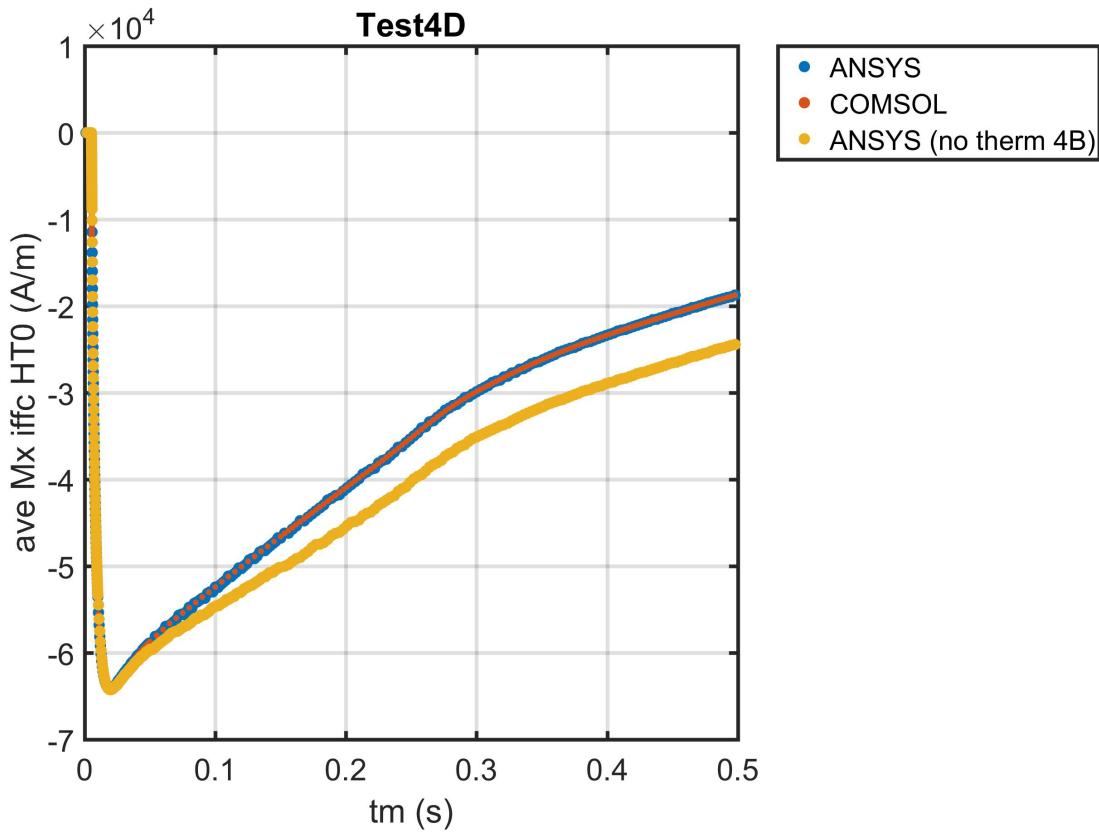
Test 4B is repeated with coupling to thermal. This allows the conductor to heat up due to the IFCC losses with the variation of material properties with temperature and field included. This is the first step towards a fully coupled case of both IFCC losses and quench.

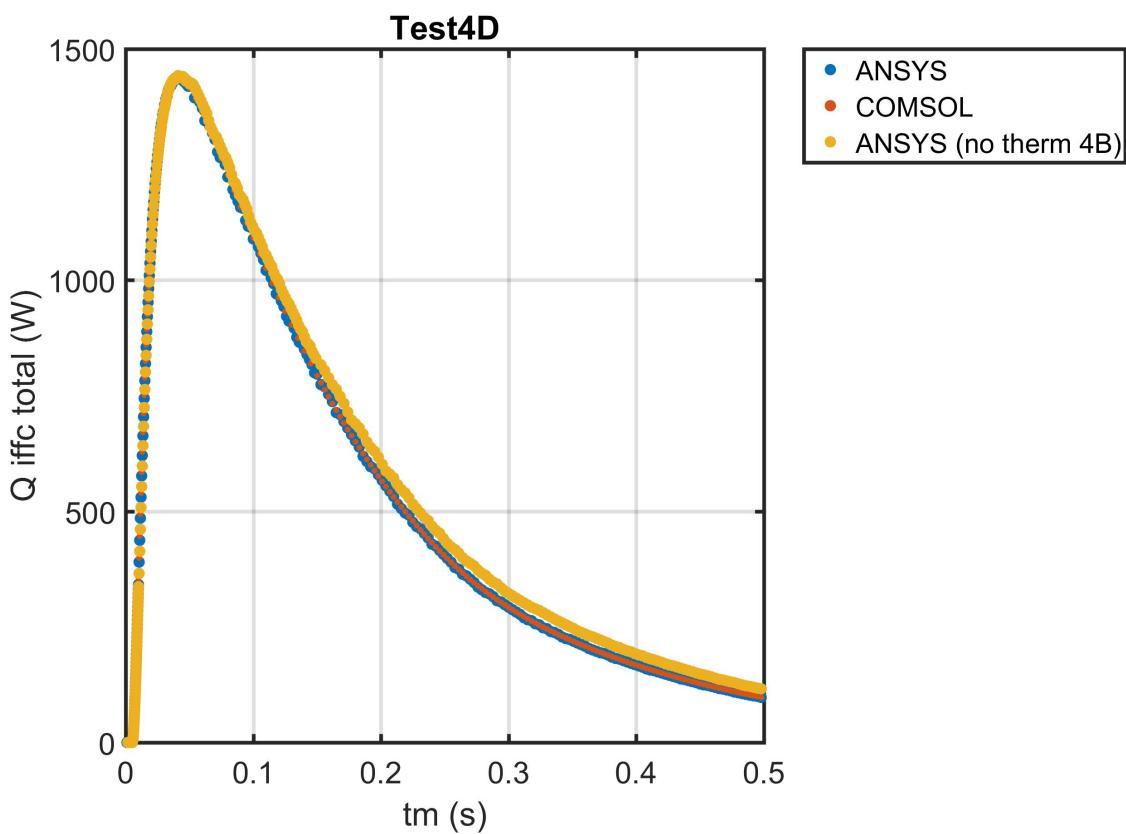
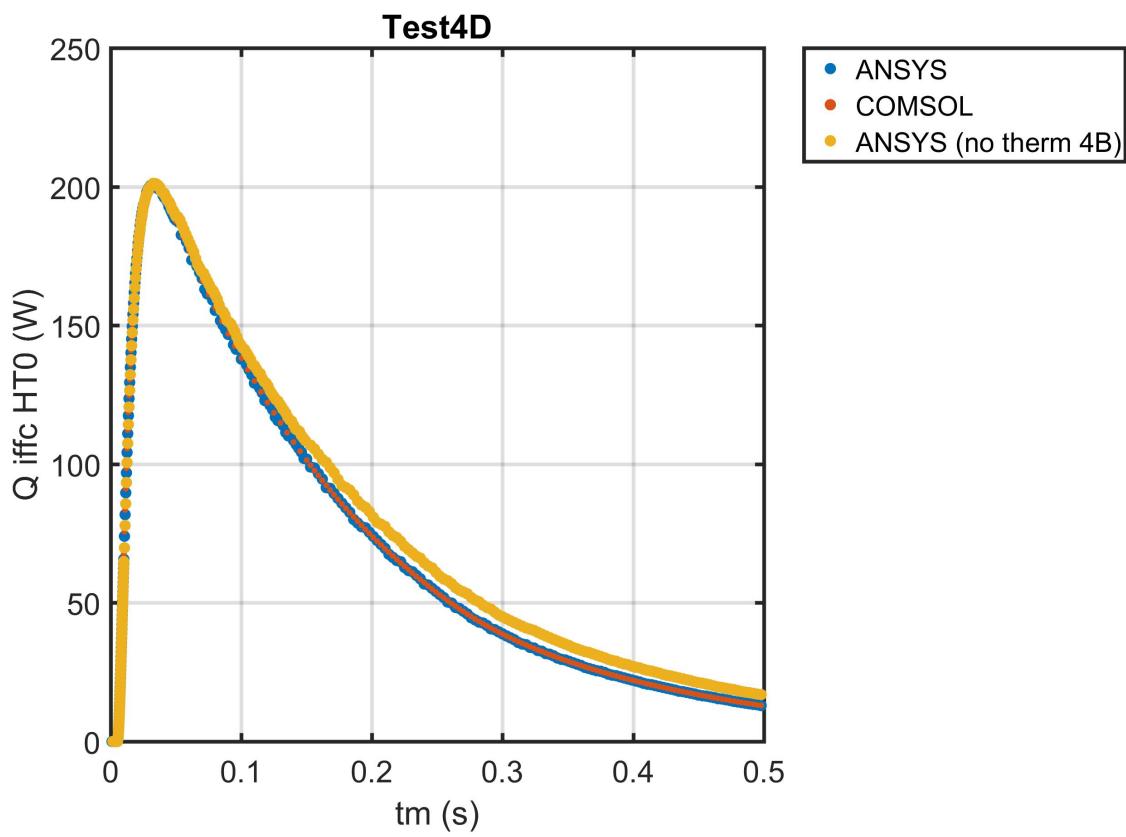
Active physics	EM, CIRCU, THERM
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	GLASS FIBER
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	18.0 kA
Rdump	25 mΩ
$L_c$	N/A
$L_i$	9.20 m
$\rho_{cu}$ fit	NIST, see Appendix E
$C_v$ -Cu fit	CUDI, see Appendix E
$C_v$ -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
$C_v$ -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
$\tau$ -IFCC	Internally calculated
<b>Output</b>	I(t) [A]
	$T_{aveHT0}$ [K]
	E-dump(t) [J]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	976.77 kJ
Stored Energy at 500ms	29.92 kJ
$\Delta E$	946.85 kJ
Edump	937.39 kJ (99.00% of $\Delta E$ )
Eifcc	9.91 kJ (1.05% of $\Delta E$ )







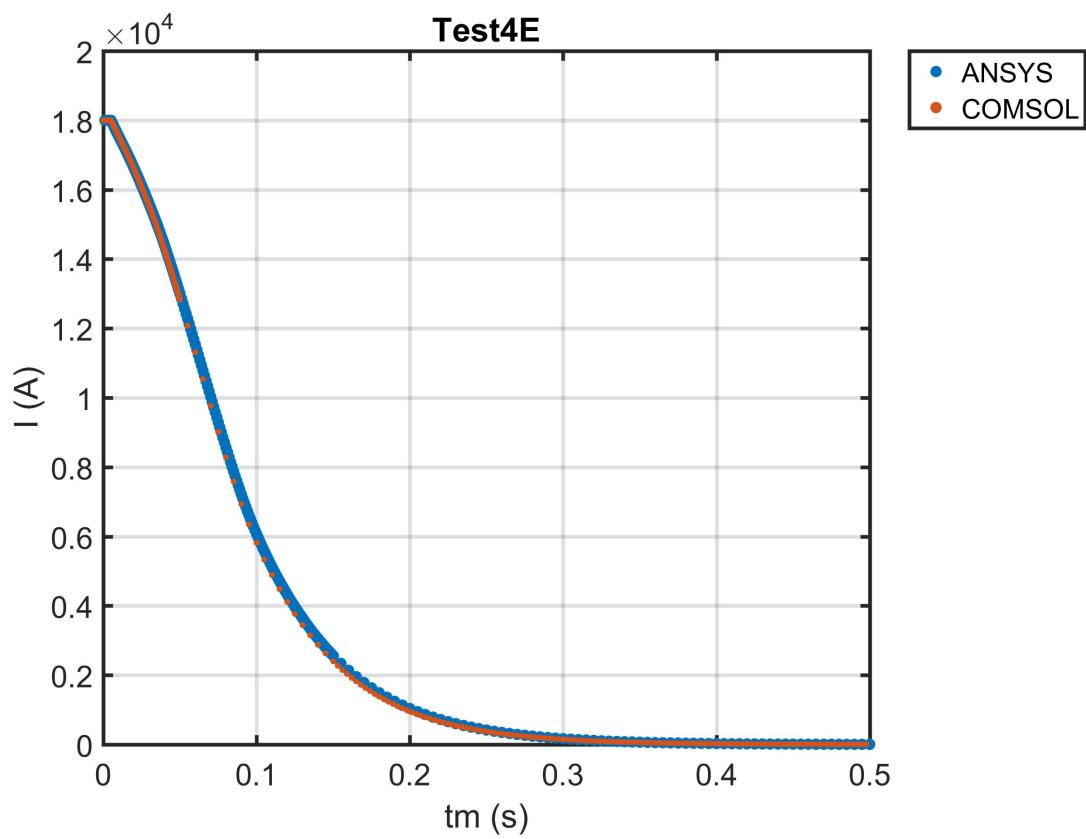


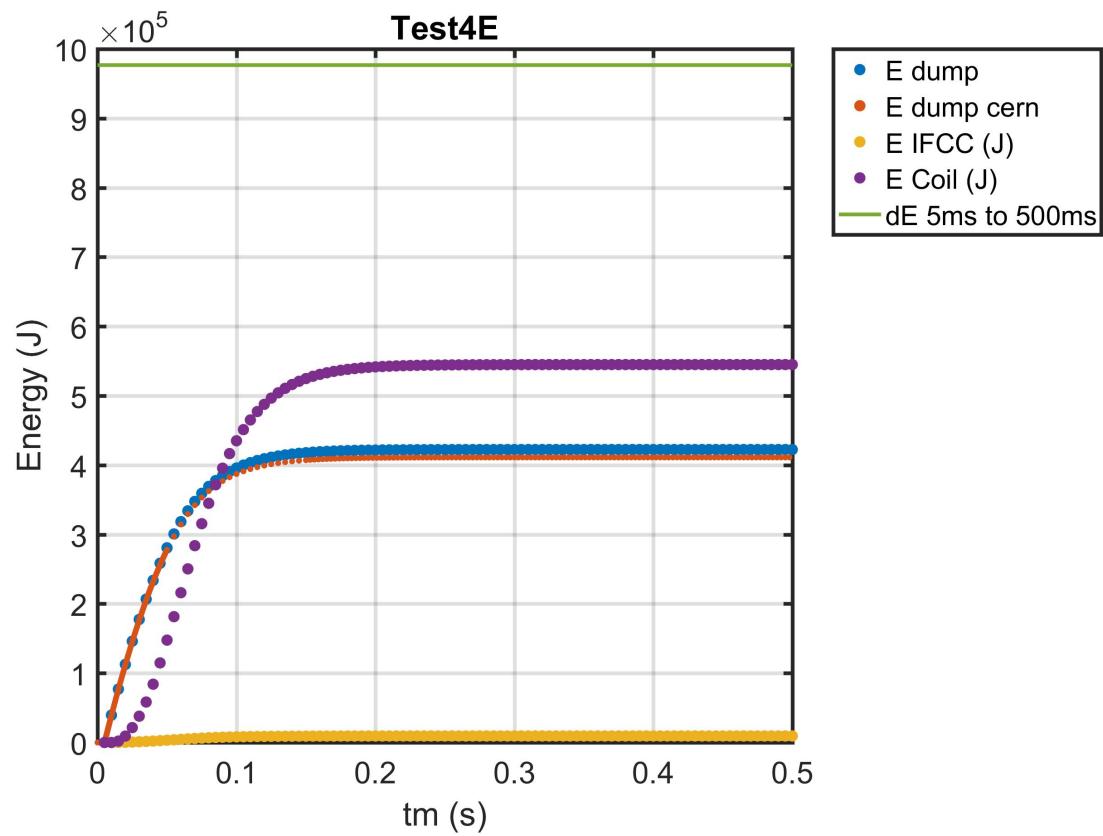
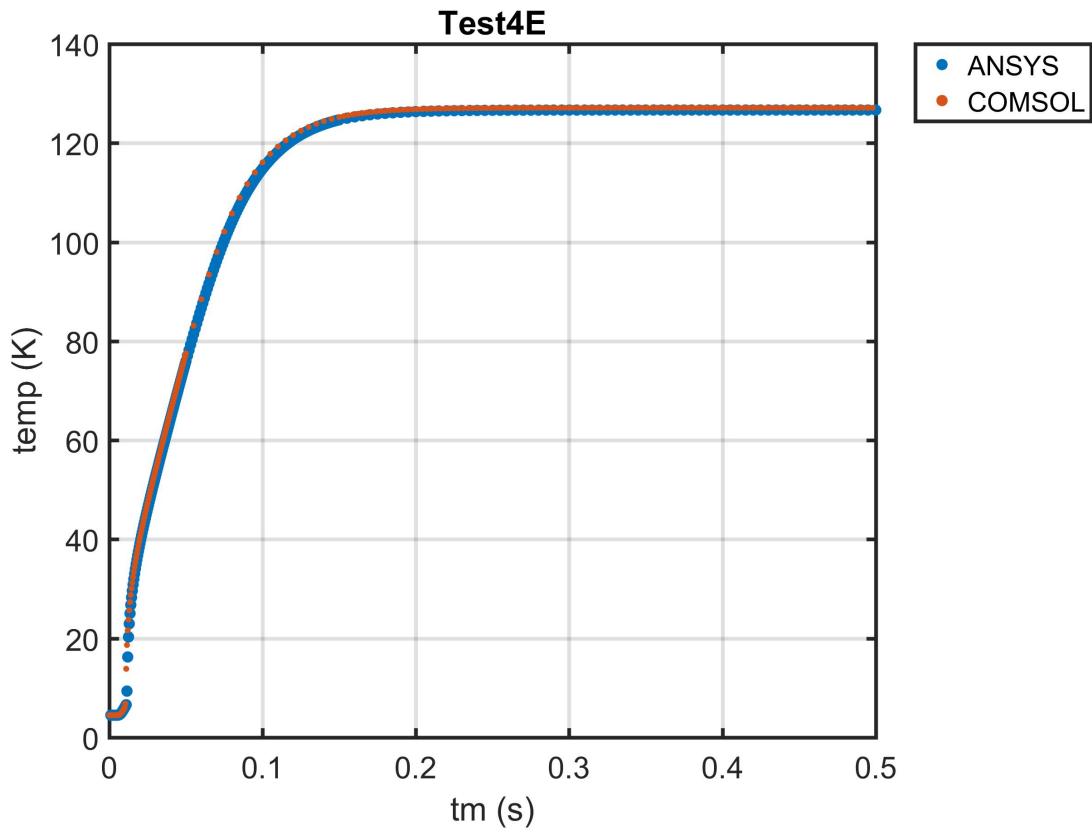
## 4.5 Test 4.E LR Decay With IFCC Induced Quench Back

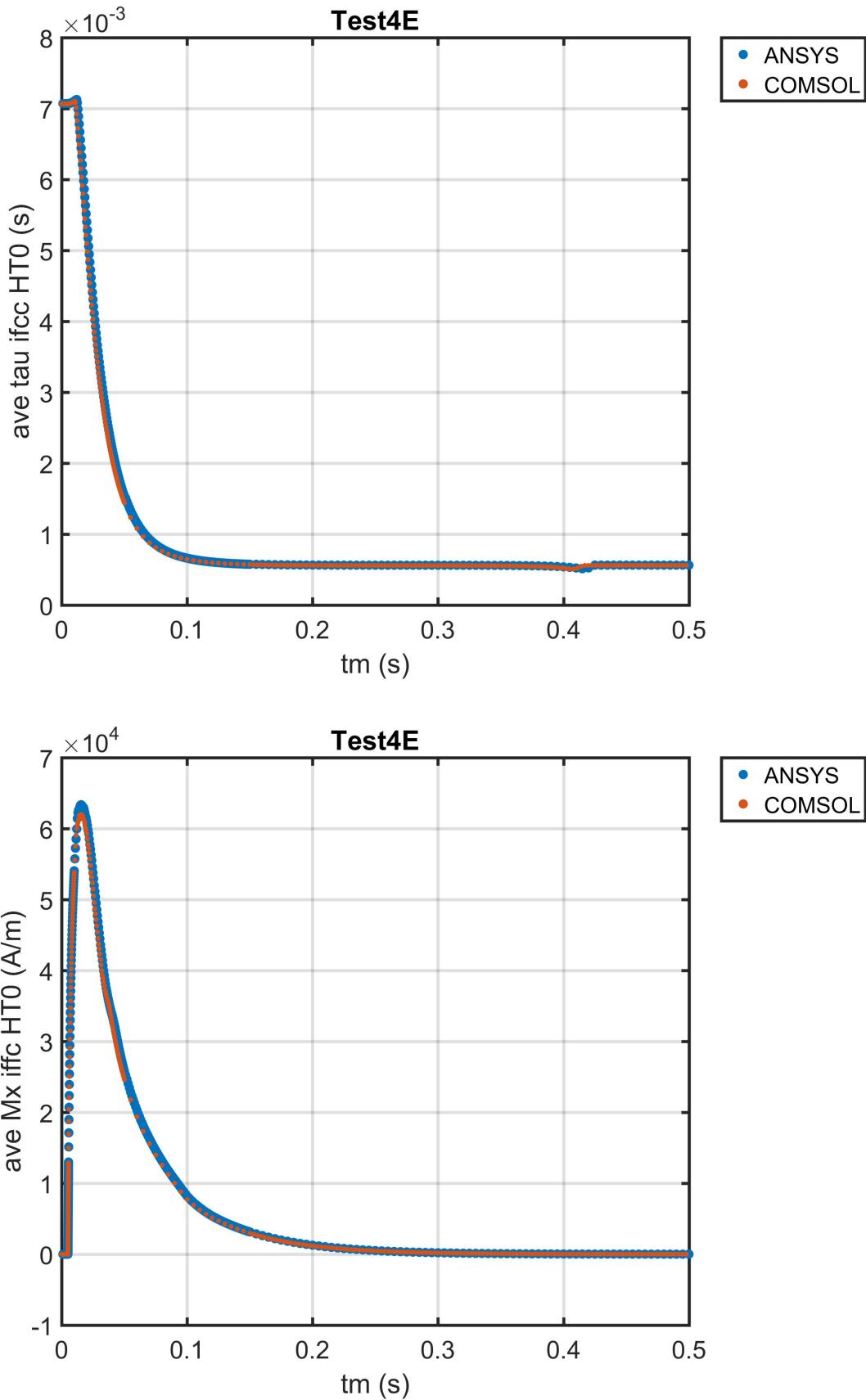
This is the first case where all the effects are combined for a quench back scenario. The high dB/dt due to the dump resistor extraction initiates IFCC losses which heats the coil to the point of quench. Then the coil resistance grows quickly and dominates the decay.

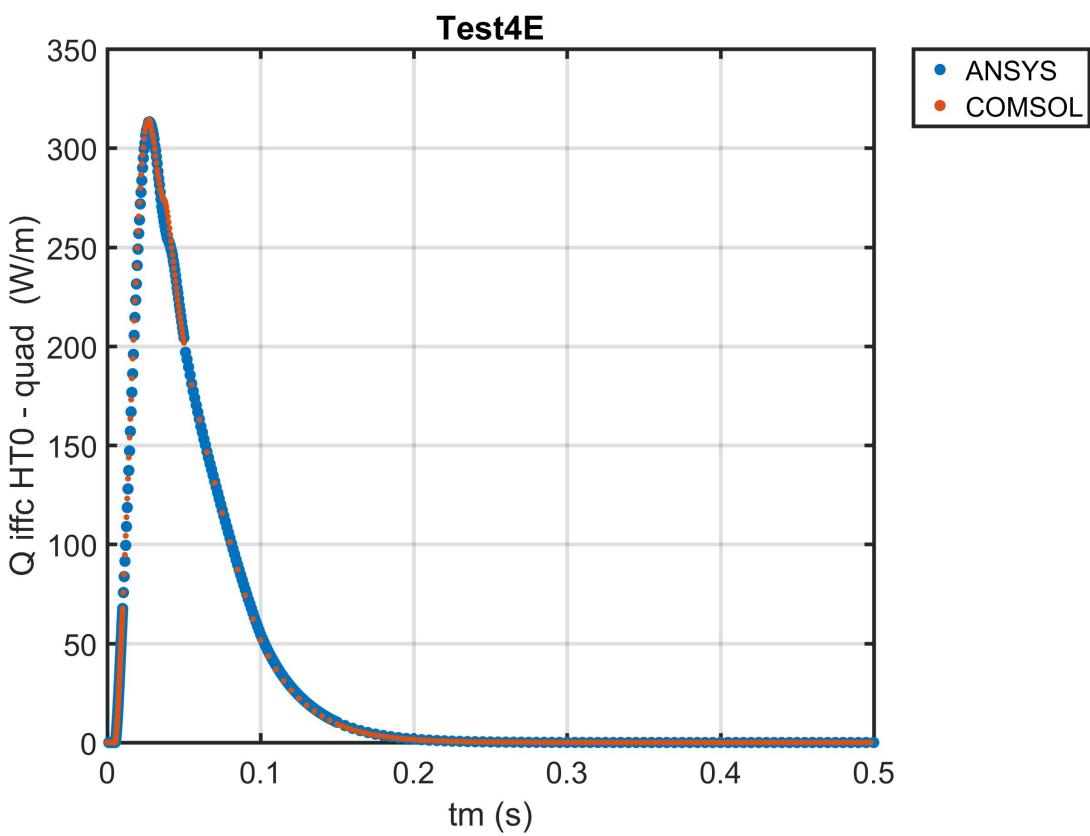
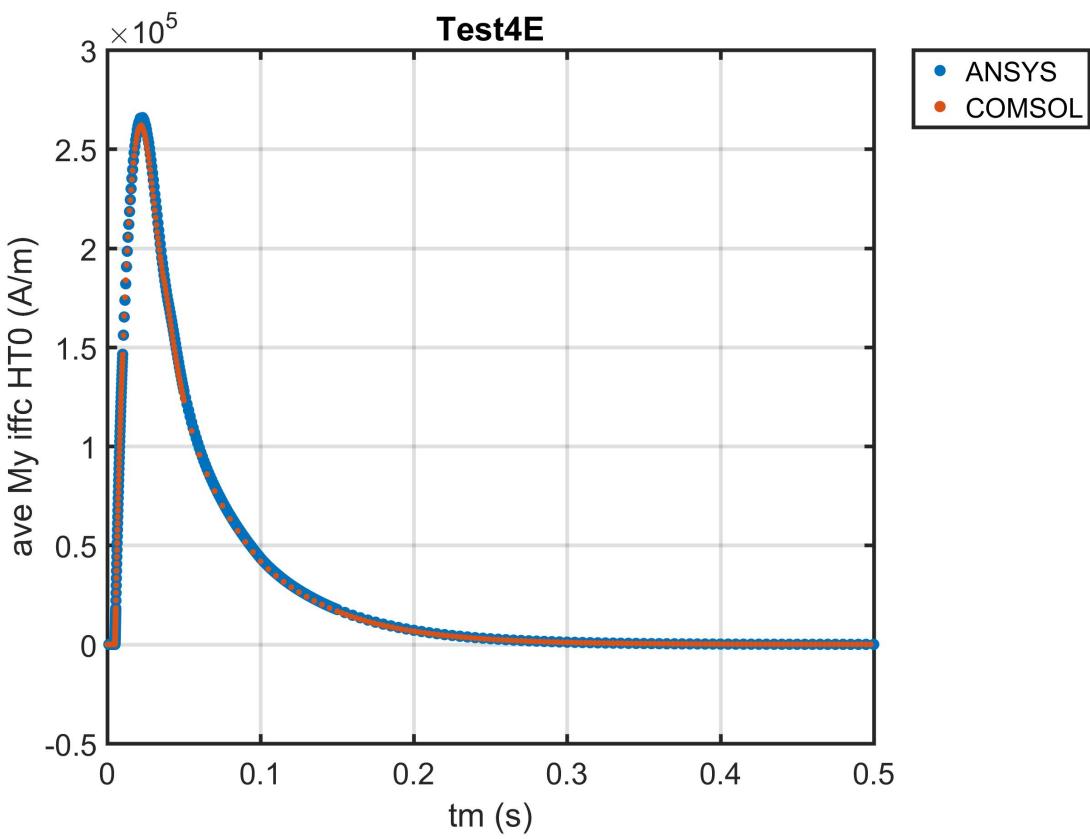
Active physics	EM, CIRCU, THERM
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Allow Quench with no Current Sharing
Cable insulation material	None
Cable voids filling material	GLASS FIBER
Time range for comparison	t=(0,500) [ms], Δt=suff. for conv.
I <sub>0</sub>	18.0 kA
R <sub>dump</sub>	25 mΩ
L <sub>c</sub>	10.11 m
L <sub>i</sub>	9.20 m
ρ <sub>cu</sub> fit	NIST, see Appendix E
C <sub>v</sub> -Cu fit	CUDI, see Appendix E
C <sub>v</sub> -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
C <sub>v</sub> -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
τ-IFCC	Internally calculated
<b>Output</b>	I(t) [A]
	T <sub>aveHT0</sub> [K]
	E-dump(t) [J]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]
	Q-Ohm <sub>totHT0</sub> [W/m] (per quad)
	Q-Ohm <sub>totcoil</sub> [W/m] (per quad)
	R-coil(t) [Ω]
	E-Ohm(t) [J]

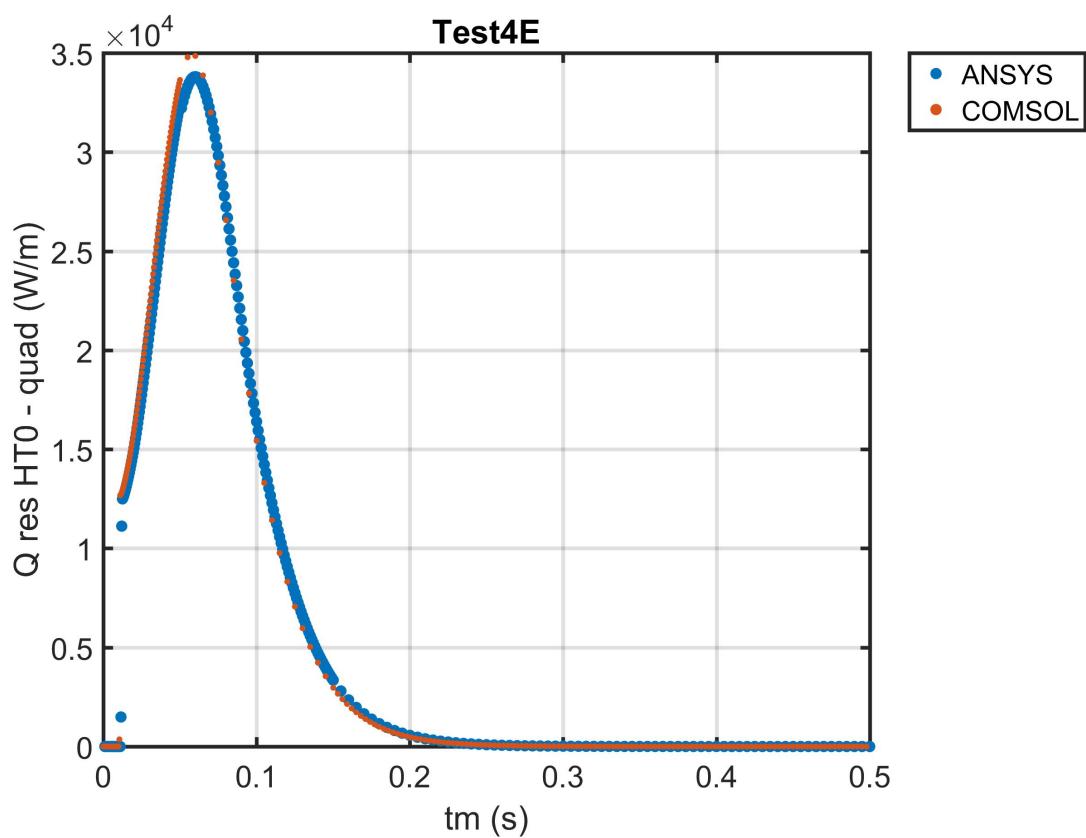
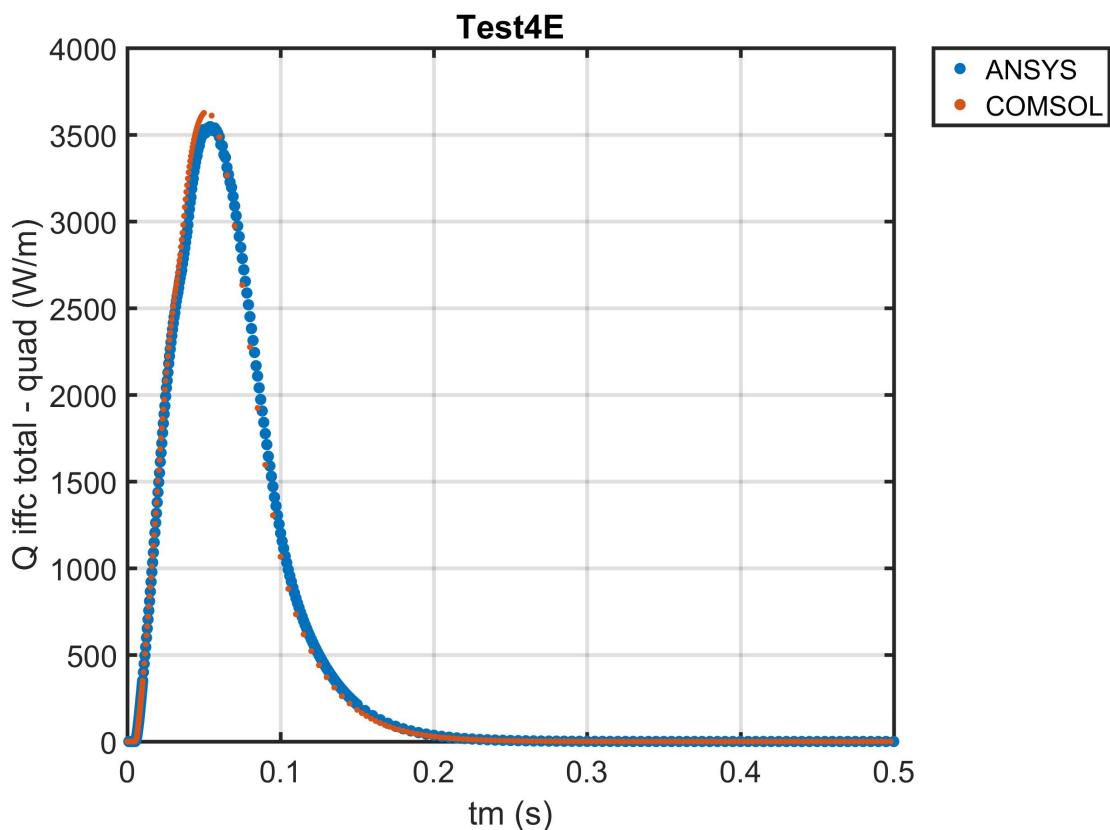
ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	976.77 kJ
Stored Energy at 500ms	0.00 kJ
$\Delta E$	976.77 kJ
Edump	422.51 kJ (43.26% of $\Delta E$ )
Eifcc	9.41 kJ (0.96% of $\Delta E$ )
Eohm	544.98 kJ (55.79% of $\Delta E$ )

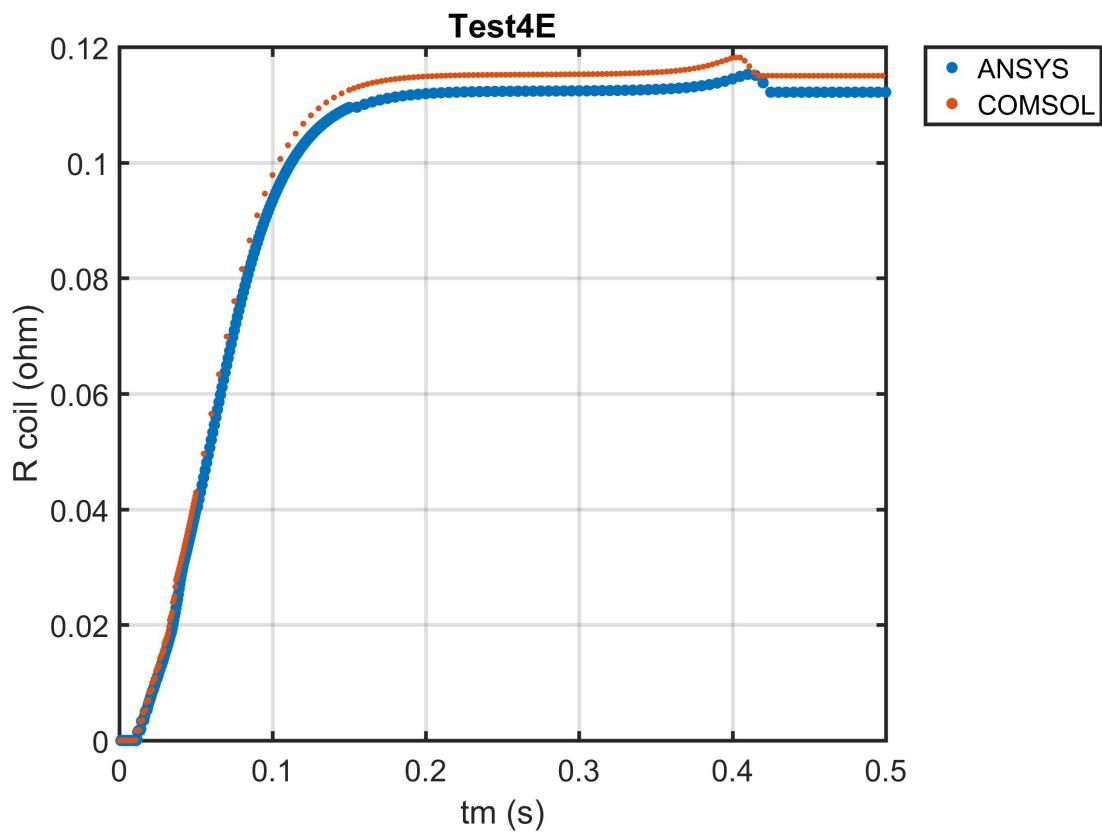
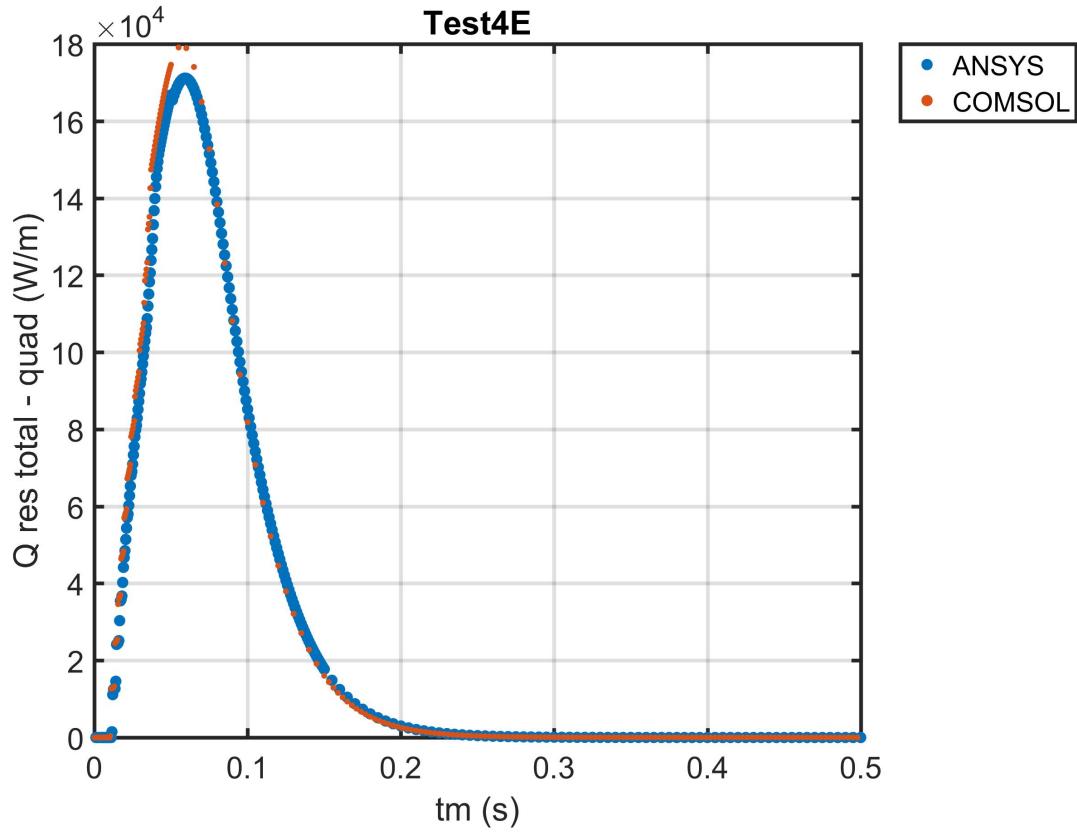










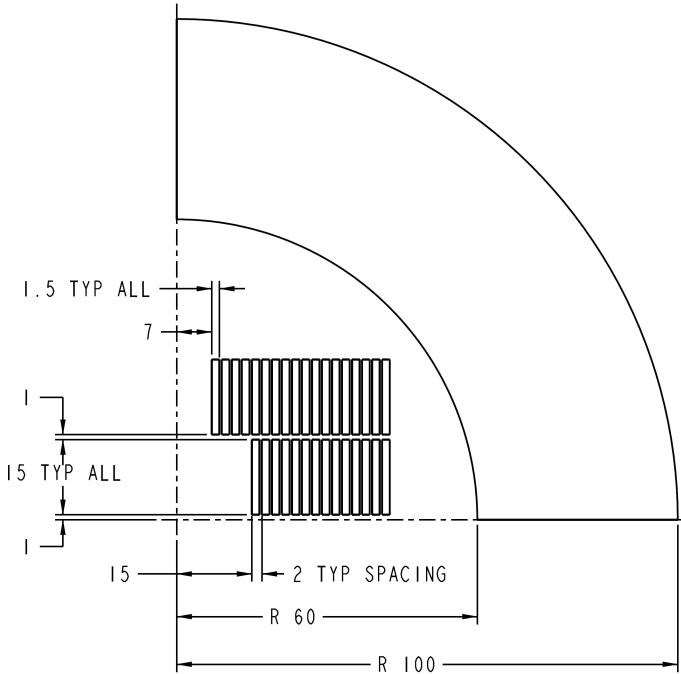


## 5 A Double Layer Nb<sub>3</sub>Sn Dipole Model

### 5.1 Model and Geometry

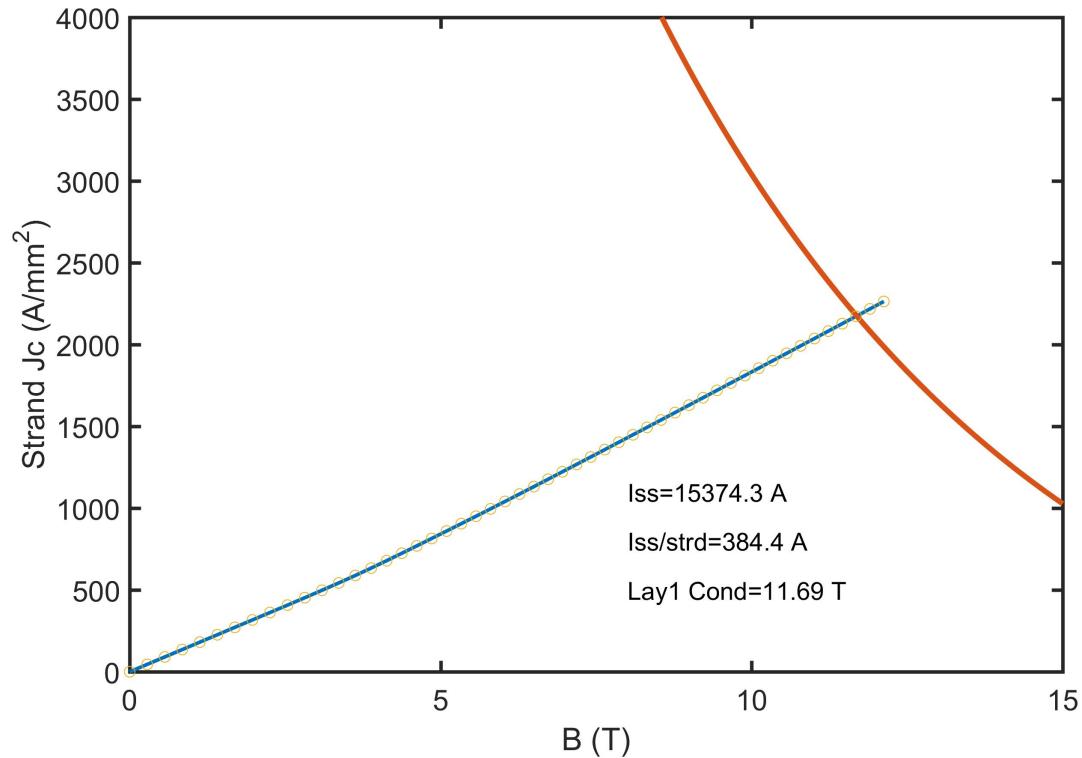
A dipole model with two layers (to allow for layer-layer CLIQ tests) was designed in a current and field regime representative of realistic Nb<sub>3</sub>Sn accelerator magnets. This model consists of two, block-like conductor layers surrounded by a cylindrical iron yoke. The inner layer contains 14 turns and the outer layer has 18 turns. The figure below details the model dimensions (with units of millimeters), and a list of high level model parameters are given in the table. The cable and conductor properties are given in Appendices A and B. The non-linear BH curve for the iron yoke can be found in Appendix D. The outer, radial boundary conditions are assumed to be far enough from the coil and yoke to no longer have an impact on the results. There are also .stp and .dxf CAD files (named “geo2”) included. In many cases the compared results are averaged over the first half turn labeled “HT0”. This is the cable nearest to the aperture in the inner layer (farthest left of the 14).

Inner layer turns (per quad.)	14	
Outer layer turns (per quad.)	18	
Nb <sub>3</sub> Sn $J_c$ (4.5 K, 12 T)	2040	A/mm <sup>2</sup>
Short-sample current (4.5 K)	15.4	kA
Short-sample cond. field (4.5 K)	11.7	T
Lc: effective res. coil length	10.11	m
Li: effective ind. coil length	9.2	m



## 5.2 Short-Sample

The strand and cable parameters in Appendix A, load line with iron (from Test5A), and  $J_c$  fit found in Appendix B were used to calculate the short-sample for the double layer model with iron. As seen in the figure below, the short-sample conductor field is 11.7 T at a current of 15.37 kA when operating at a temperature of 4.5 K.

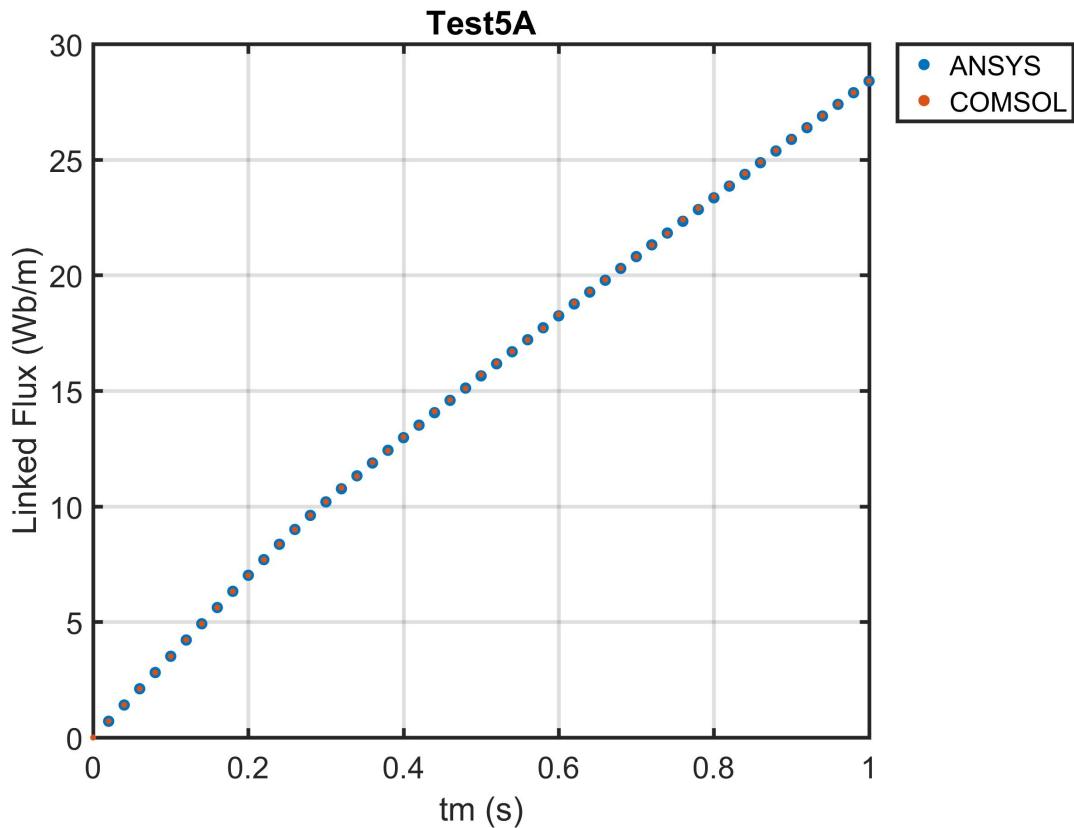


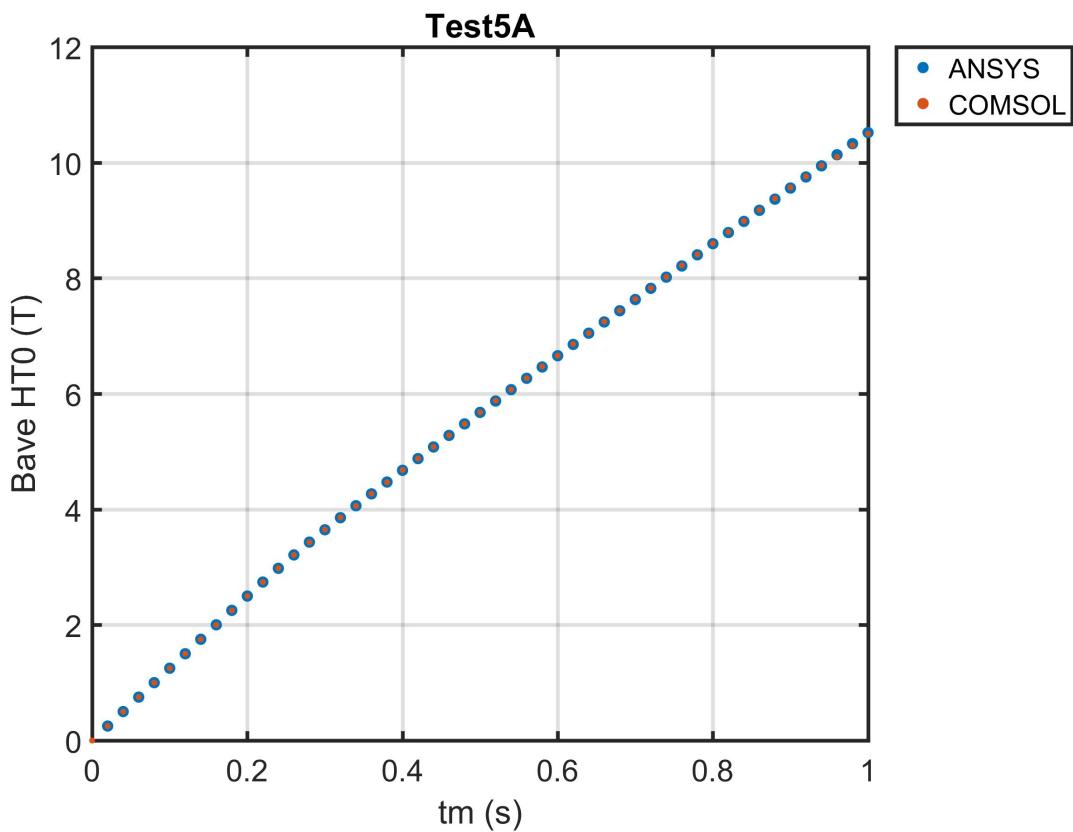
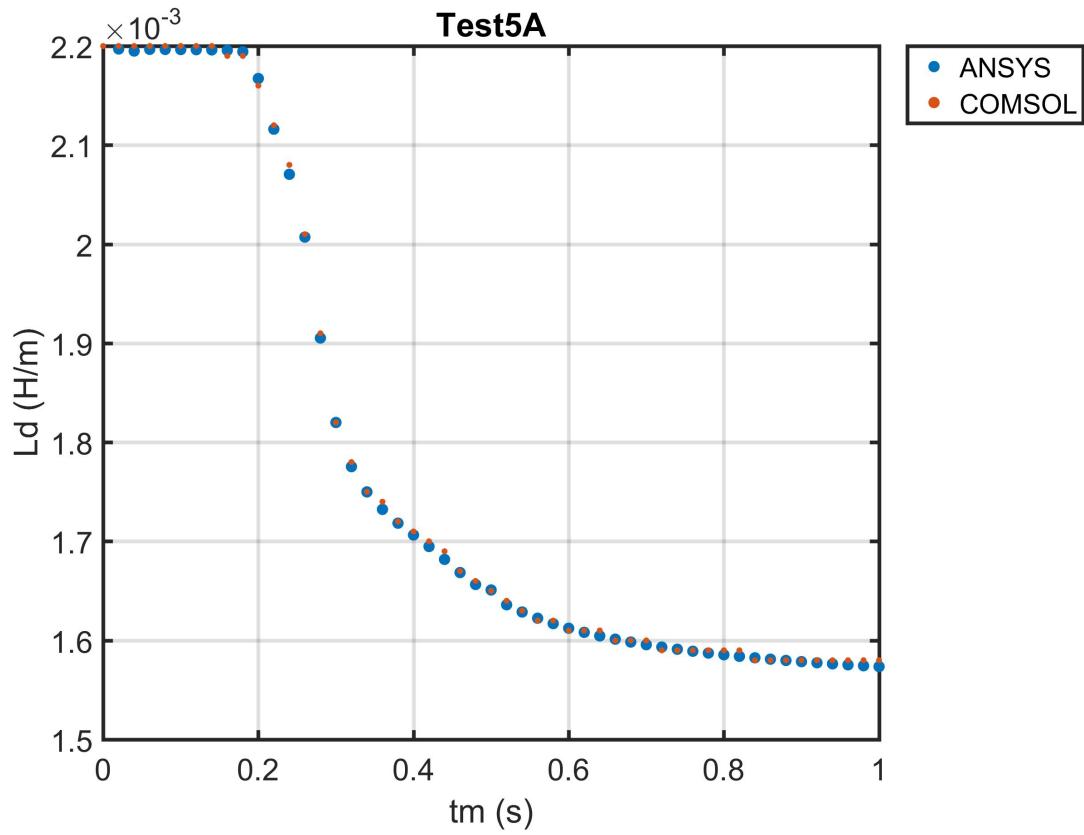
## 6 Double Layer LR Tests

A set of tests is completed for a two layer model simulating dump resistor based extraction and a CLIQ discharge. These tests form the basis of the comparison published in [1]. With respect to previous tests, a second conductor layer is now included with the details given in Section 5. An initial test of the inductance is performed, similar to 0B, and then repeat of 4A, 4D, and 4E with the new geometry. This builds up a dump resistor extraction case to the point where quench back from IFCC losses is included. A final series of tests, 6A and 6B, simulate a CLIQ unit attached in a layer-layer configuration.

## 6.1 Test 5.A Differential Inductance (with iron yoke)

Active physics	EM
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	None
External loads	Linear ramp-up, $t=(0,1)$ [s], $\Delta t=20e-3$ [s], $i=(0,16)$ [kA]
<b>Output</b>	Coil magnetic flux $\phi$ [Wb/m]
	Differential inductance as $L_d = \Delta\phi/\Delta i$ [H/m] (finite difference)
	$B_{aveHT0}$ [T]



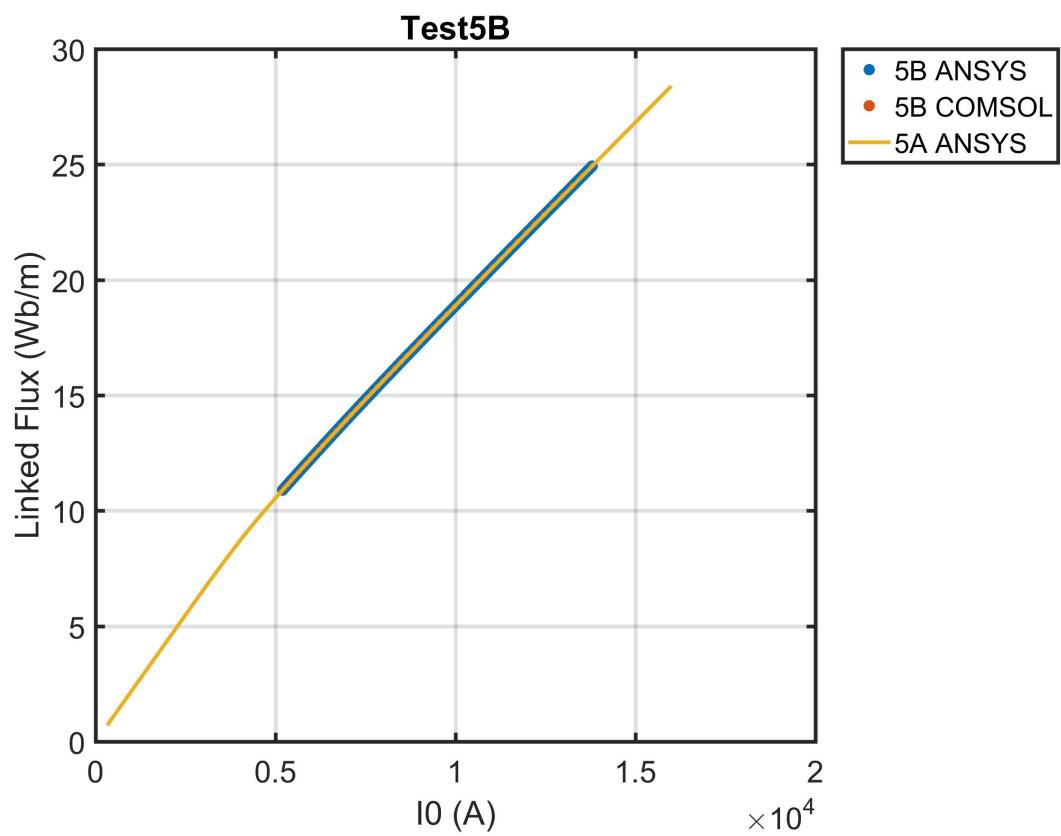
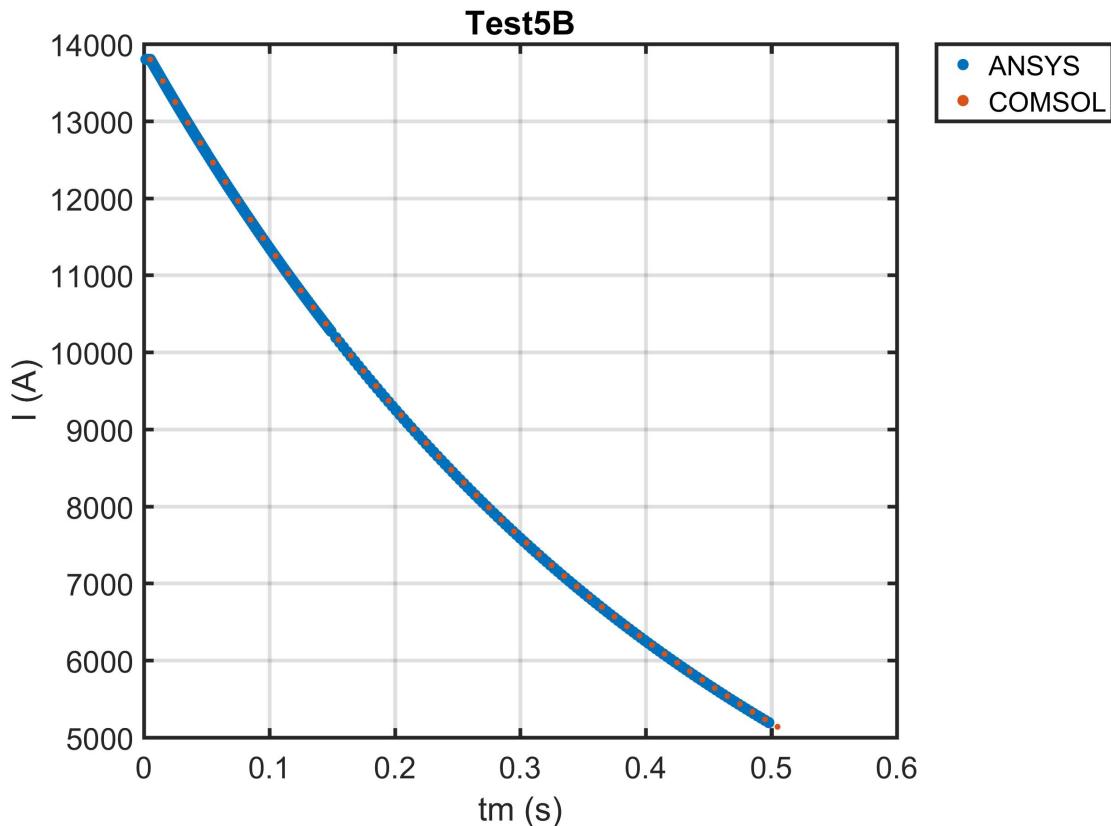


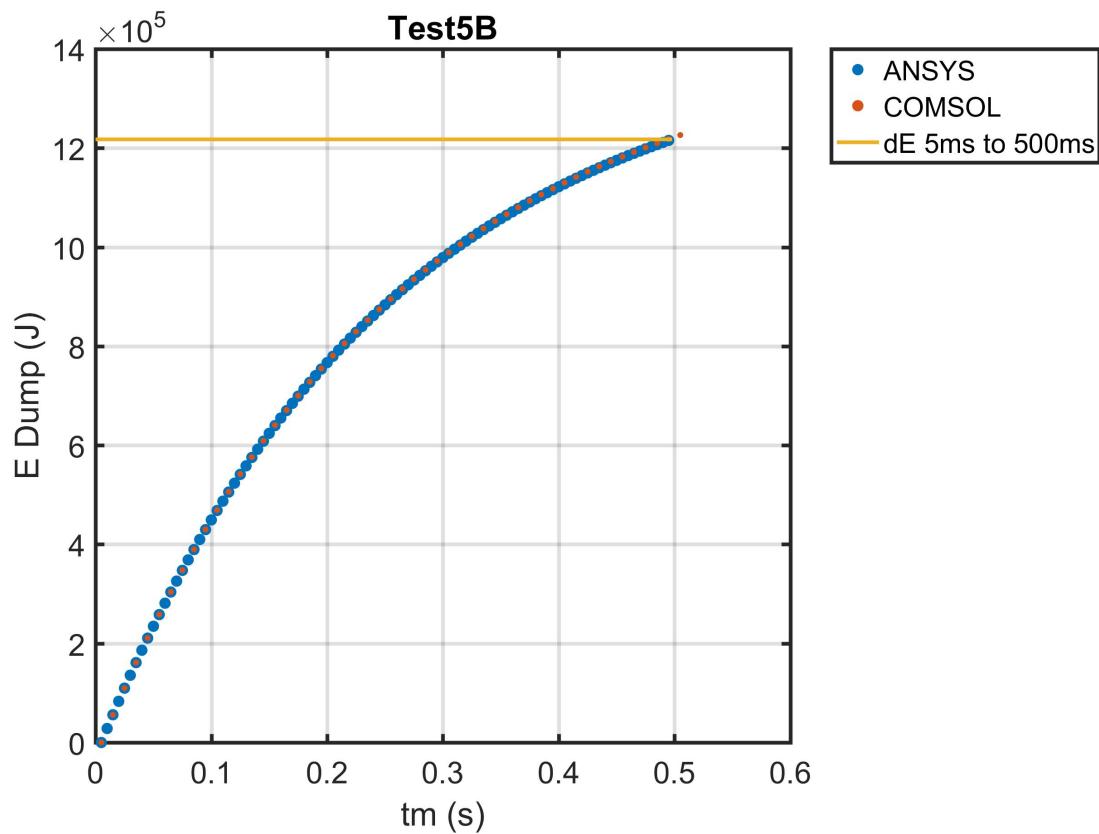
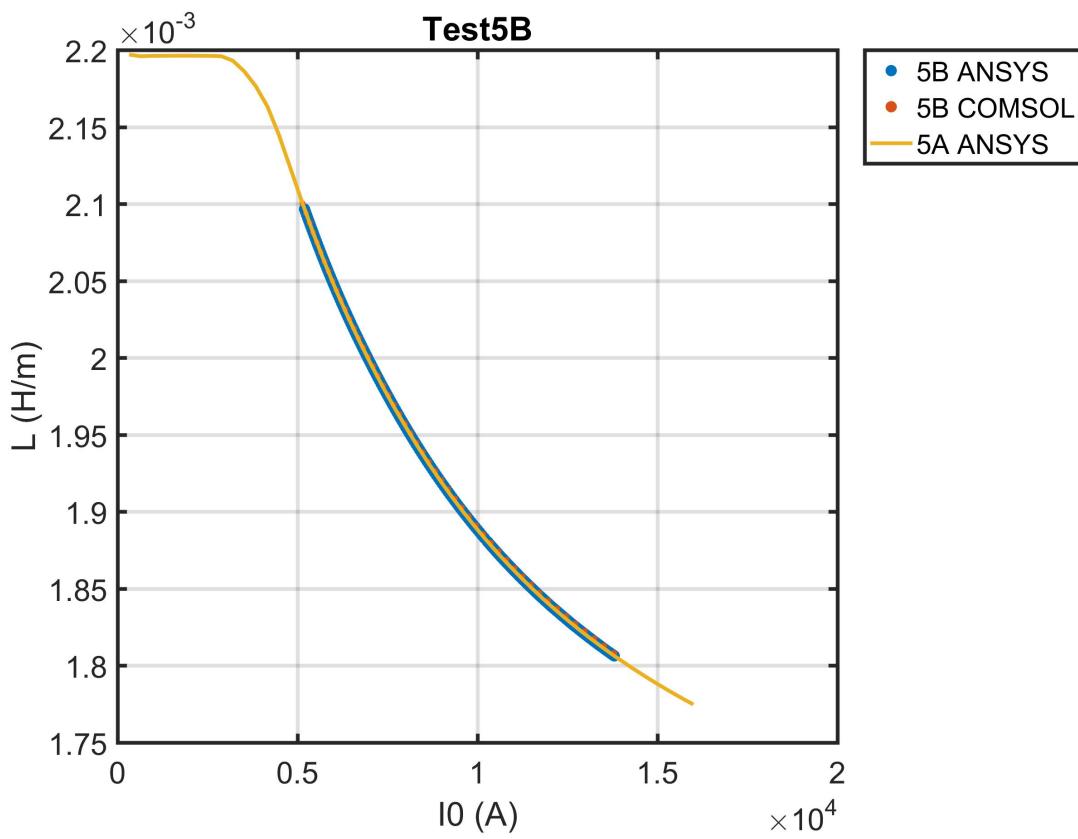
## 6.2 Test 5.B LR Decay With No Losses

Tests 4A, 4D, and 4E are repeated with the new geometry to build up a dump resistor extraction case. The initial current is set to 90% of the short-sample for this new geometry (13.8 kA) and the dump resistor is increased to keep a similar voltage maximum. This first simulation is intended to test the circuit coupling with the coil only contributing inductive voltage. Interfilament coupling currents and quench are turned off so there are no losses outside of the dump resistor. Since there are no heating terms, thermal coupling is not needed and the coil will stay superconducting and at the initial temperature of 4.5 K. I like to track energy, and for this case its a good check that 100% is dissipated in the dump. The length  $L_i$  is the effective magnet length used for inductive voltage. Note that both the linked flux and inductance can be checked against Test5A if plotted vs current.

Active physics	EM, CIRCU
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	13.8 kA
$R_{dump}$	30 m $\Omega$
$L_c$	N/A
$L_i$	9.20 m
<b>Output</b>	$I(t)$ [A]
	Coil magnetic flux $\phi$ [Wb/m]
	Flux inductance as $L = \phi/i$ [H/m] (finite difference)
	$E_{dump}(t)$ [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	1.469 MJ
Stored Energy at 500ms	0.251 MJ
$\Delta E$	1.218 kJ
$E_{dump}$	1.218 kJ (100% of $\Delta E$ )
$E_{dump-CERN}$	1.218 kJ



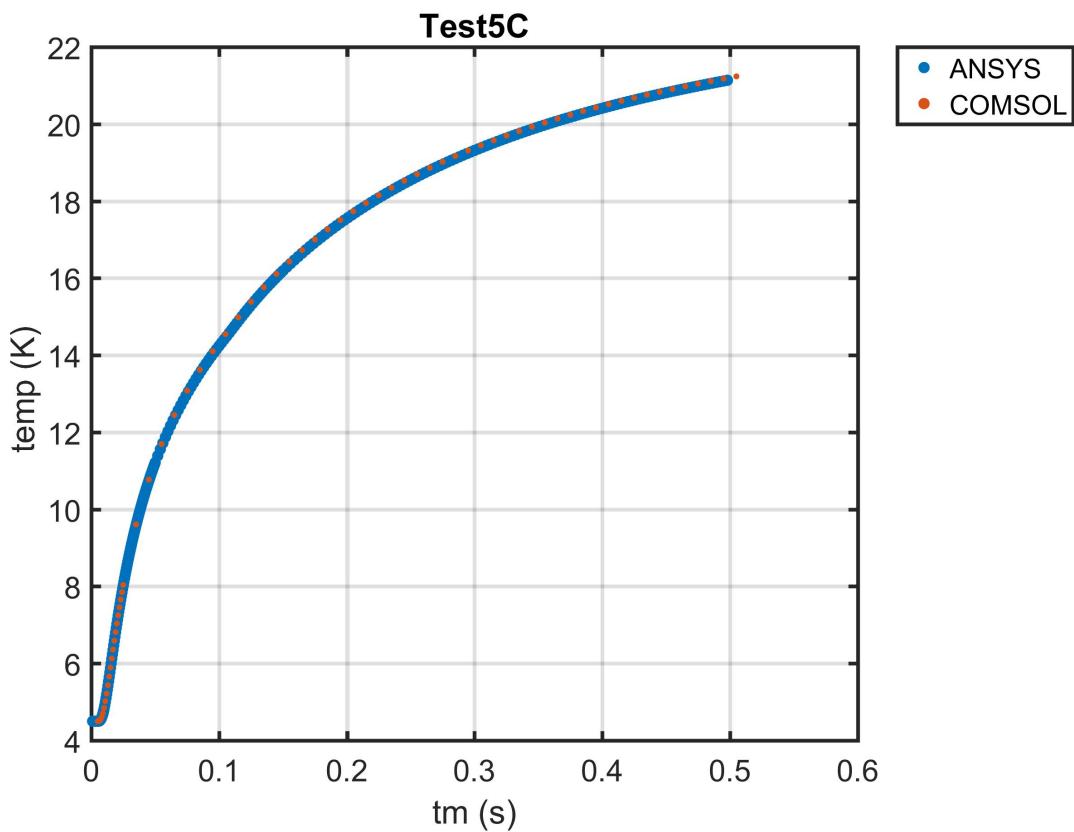
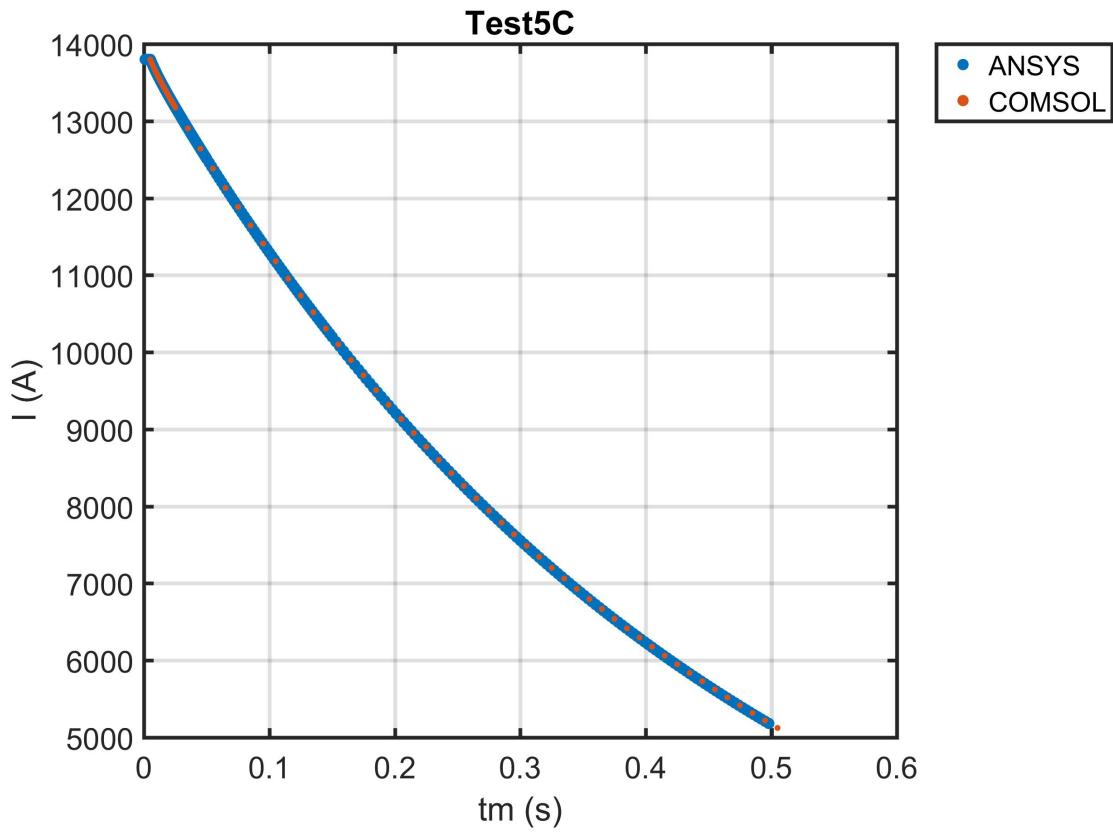


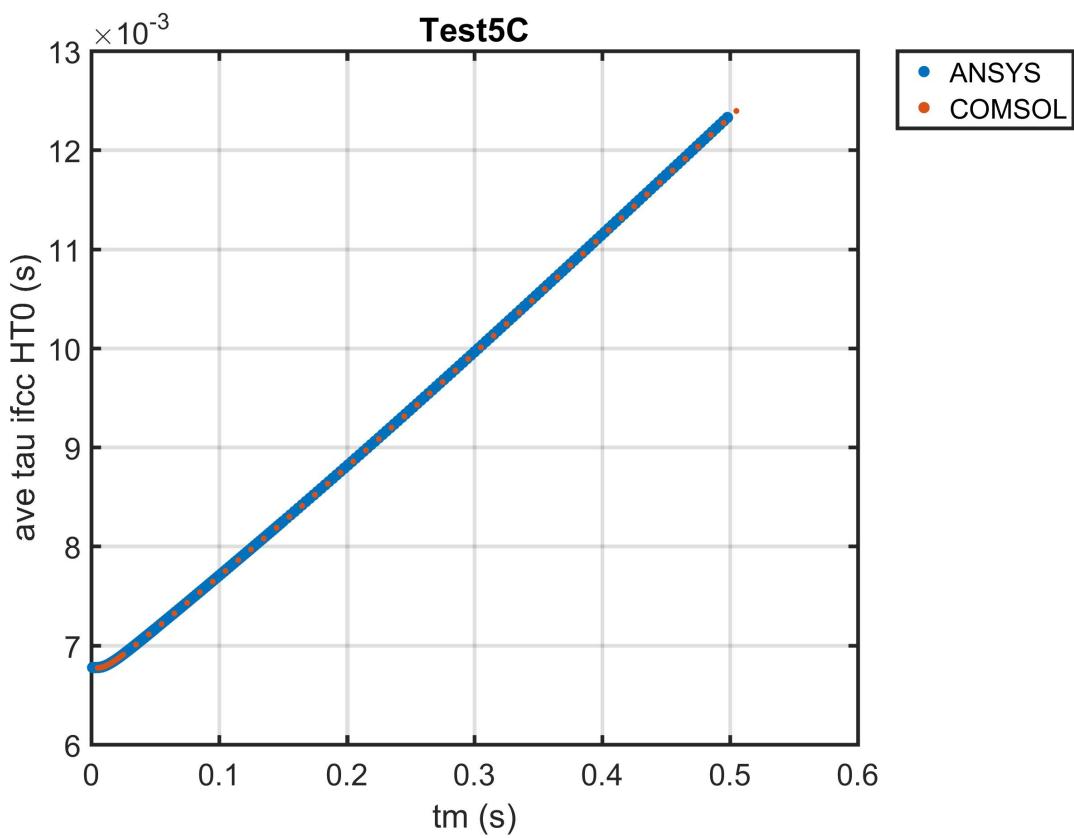
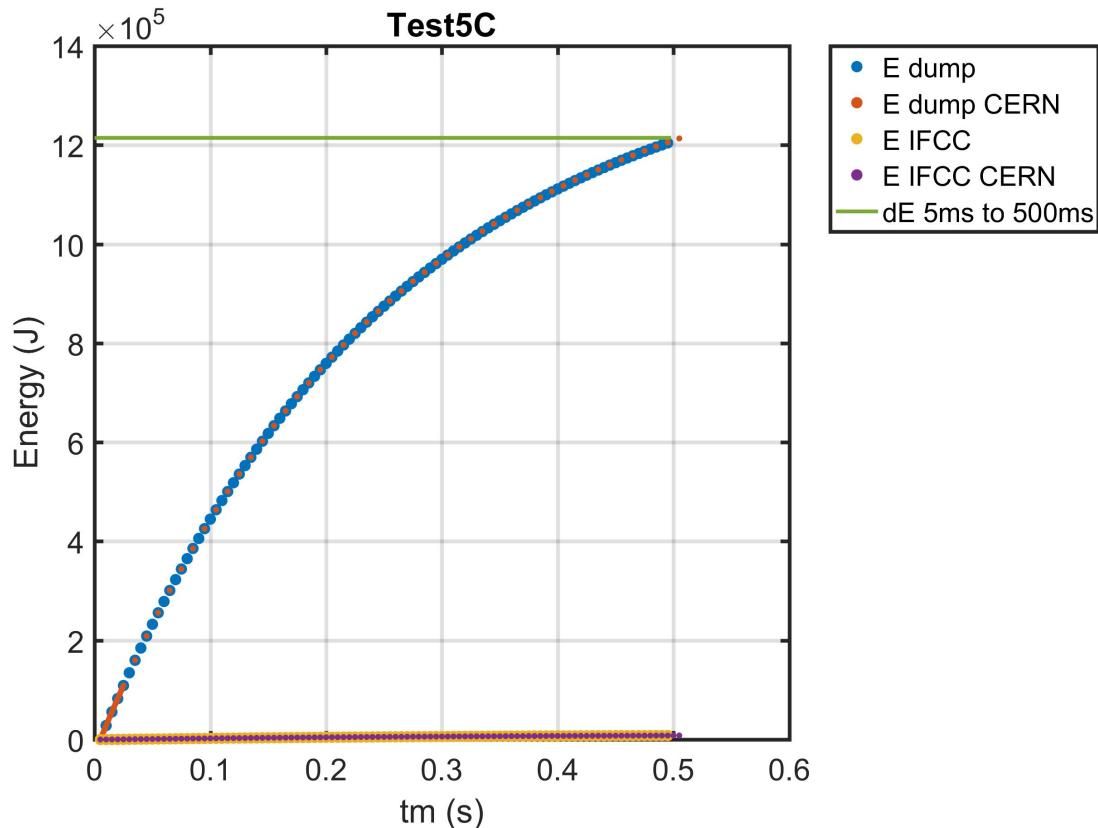
### 6.3 Test 5.C LR Decay With IFCC Loss and Thermal

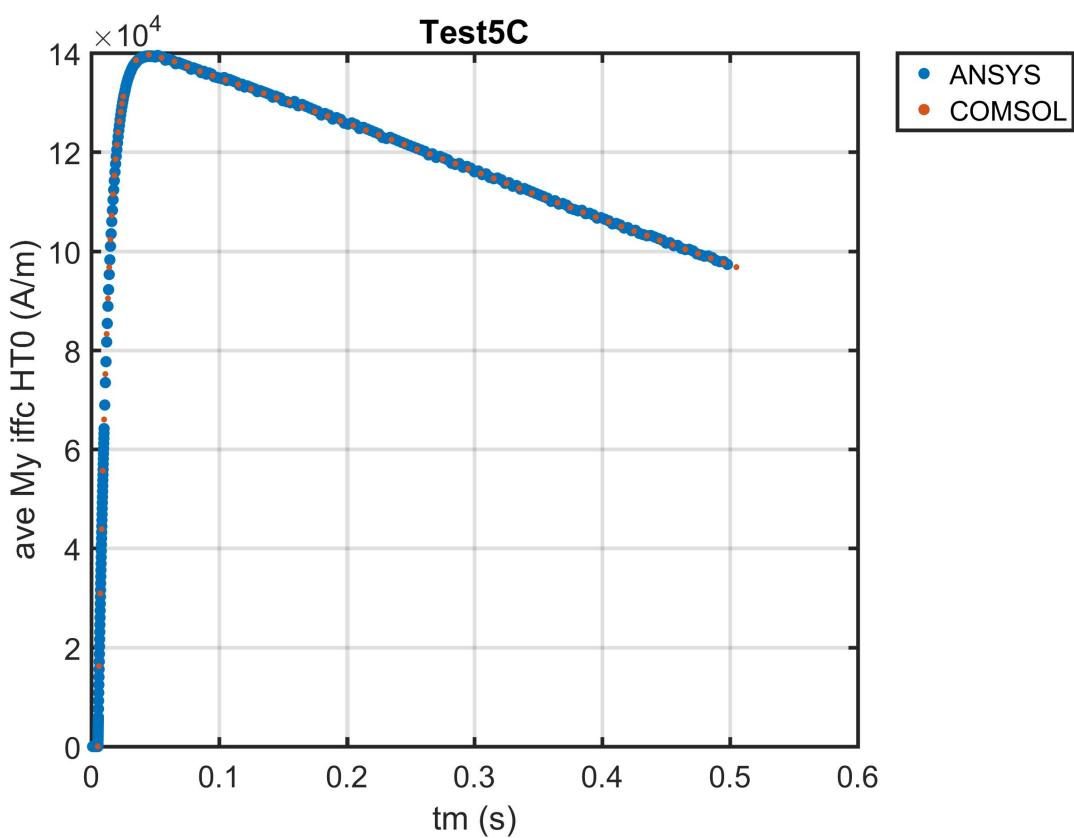
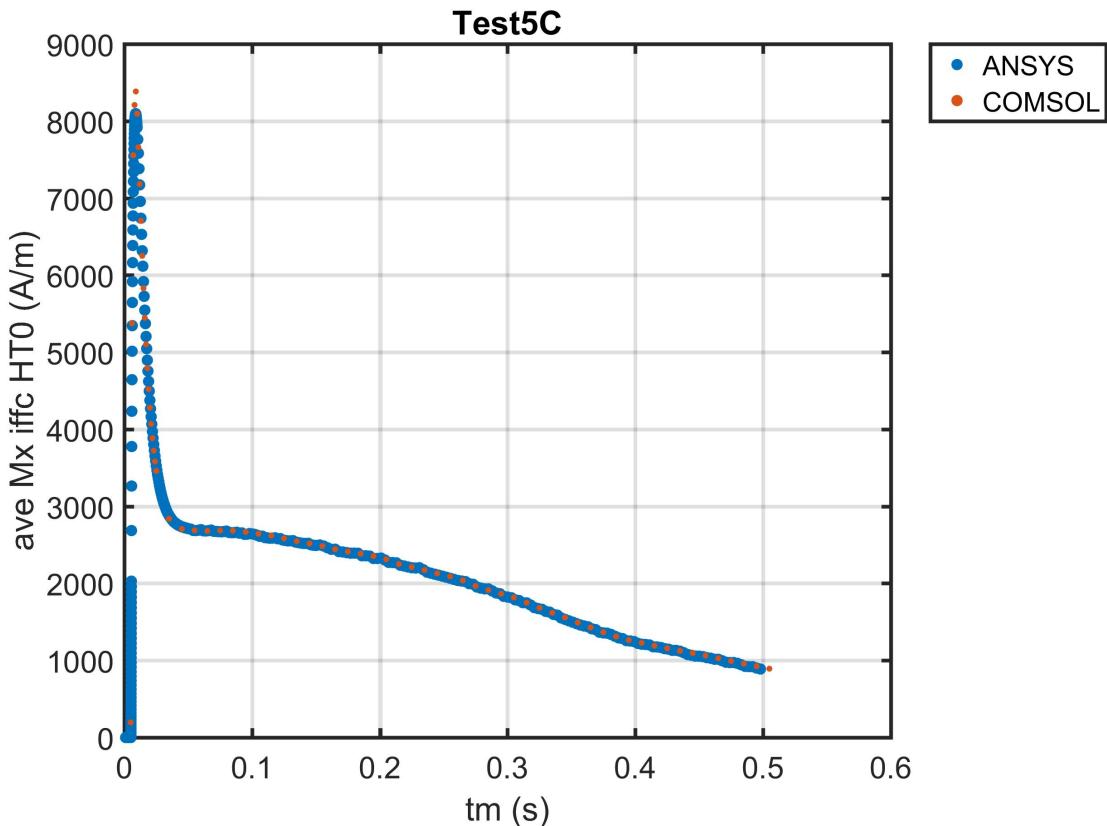
Test 5B is repeated with coupling to thermal. This allows the conductor to heat up due to the IFCC losses with the variation of material properties with temperature and field included. This is the first step towards a fully coupled case of both IFCC losses and quench.

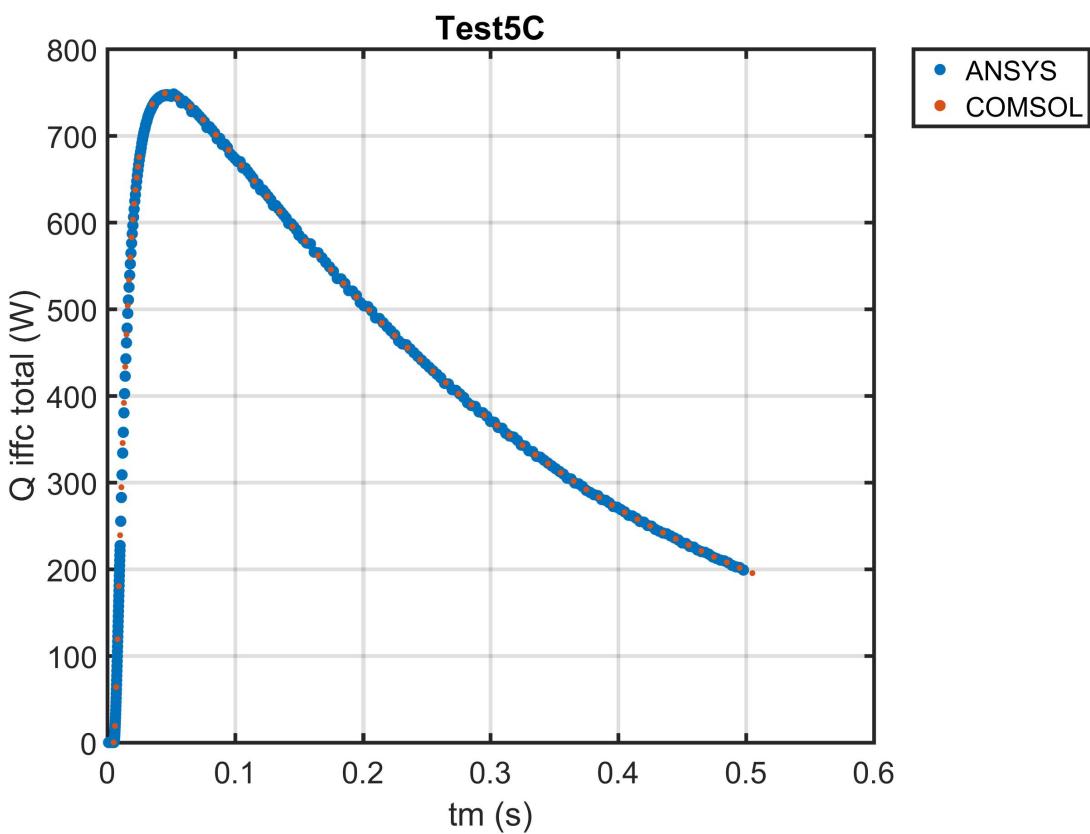
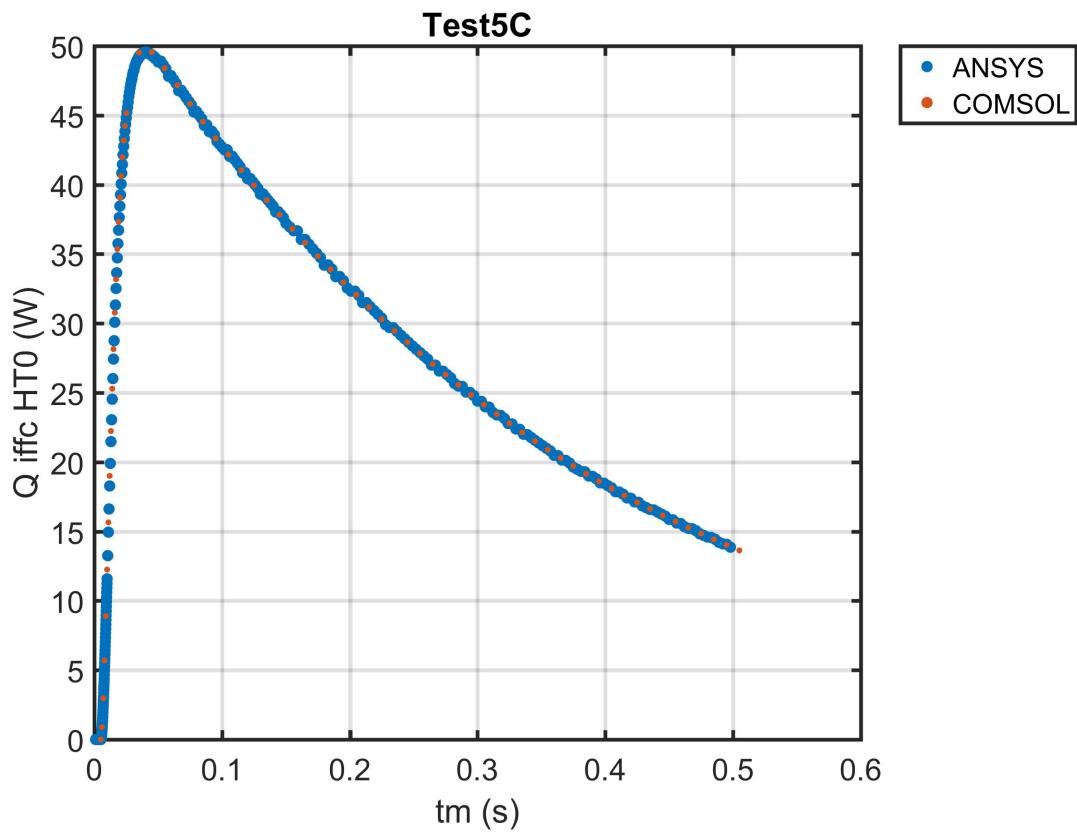
Active physics	EM, CIRCU, THERM
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Always SC
Cable insulation material	None
Cable voids filling material	GLASS FIBER
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	13.8 kA
Rdump	30 mΩ
$L_c$	N/A
$L_i$	9.20 m
$\rho_{cu}$ fit	NIST, see Appendix E
$C_v$ -Cu fit	CUDI, see Appendix E
$C_v$ -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
$C_v$ -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
$\tau$ -IFCC	Internally calculated
<b>Output</b>	I(t) [A]
	$T_{aveHT0}$ [K]
	E-dump(t) [J]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]

ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	1.469 MJ
Stored Energy at 500ms	0.254 MJ
$\Delta E$	1.215 MJ
Edump	1.207 MJ (99.36% of $\Delta E$ )
Eifcc	8.07 kJ (0.66% of $\Delta E$ )
Edump-CERN	1.205 MJ
Eifcc-CERN	8.05 kJ







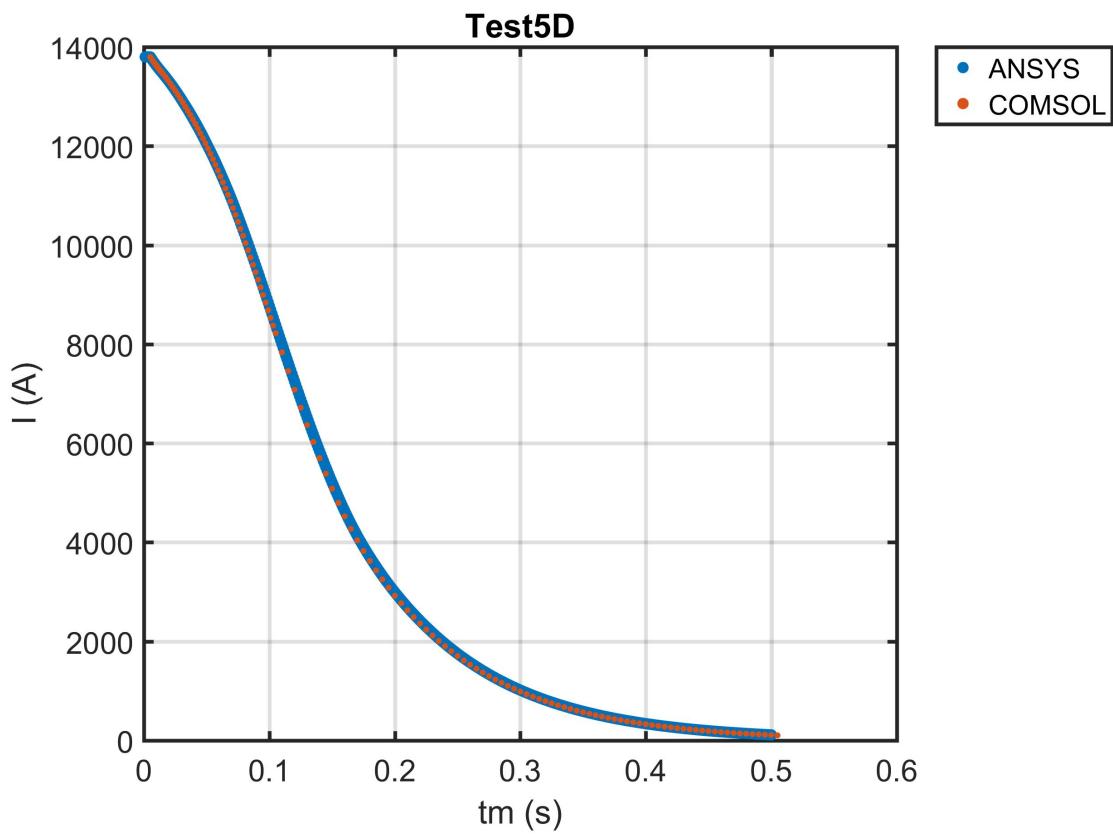


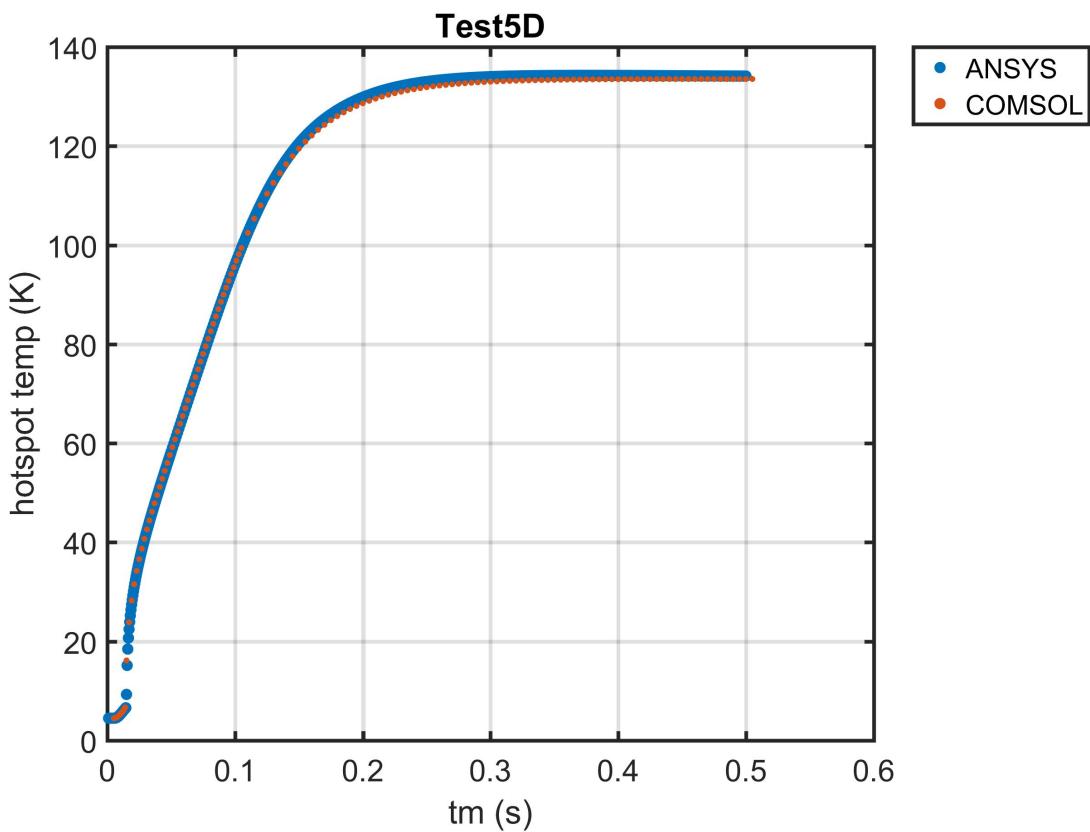
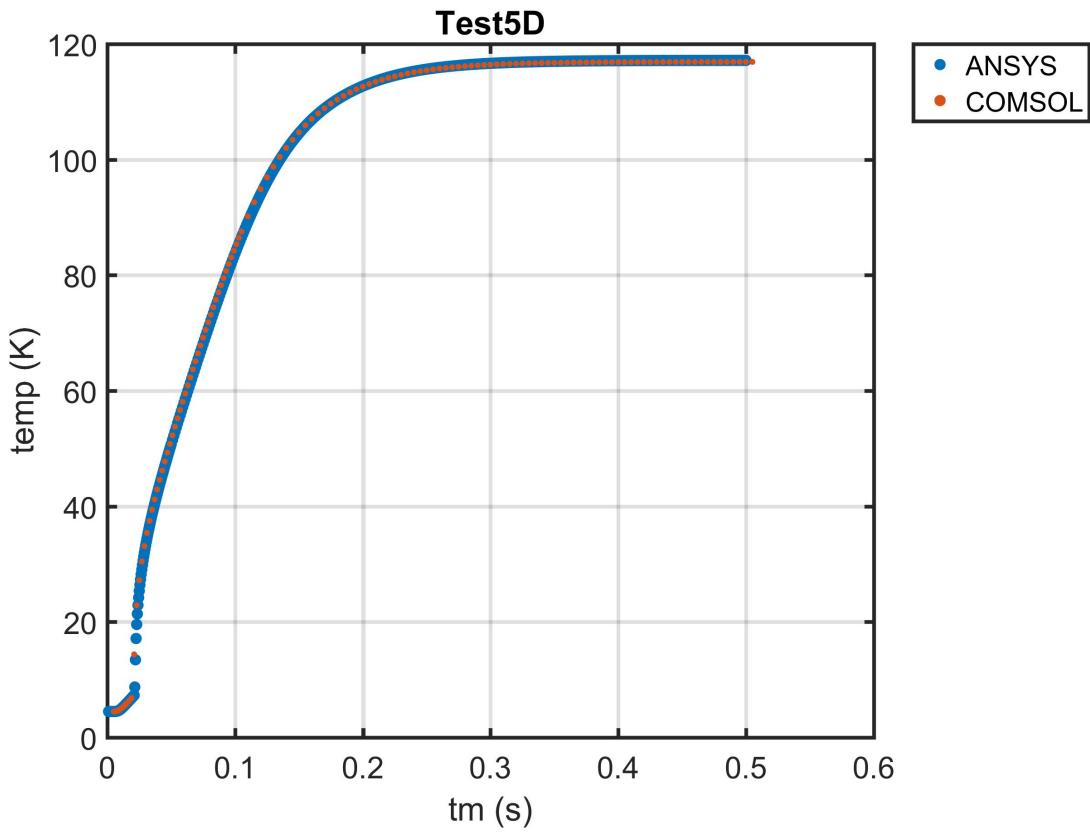
## 6.4 Test 5.D LR Decay With IFCC Induced Quench Back

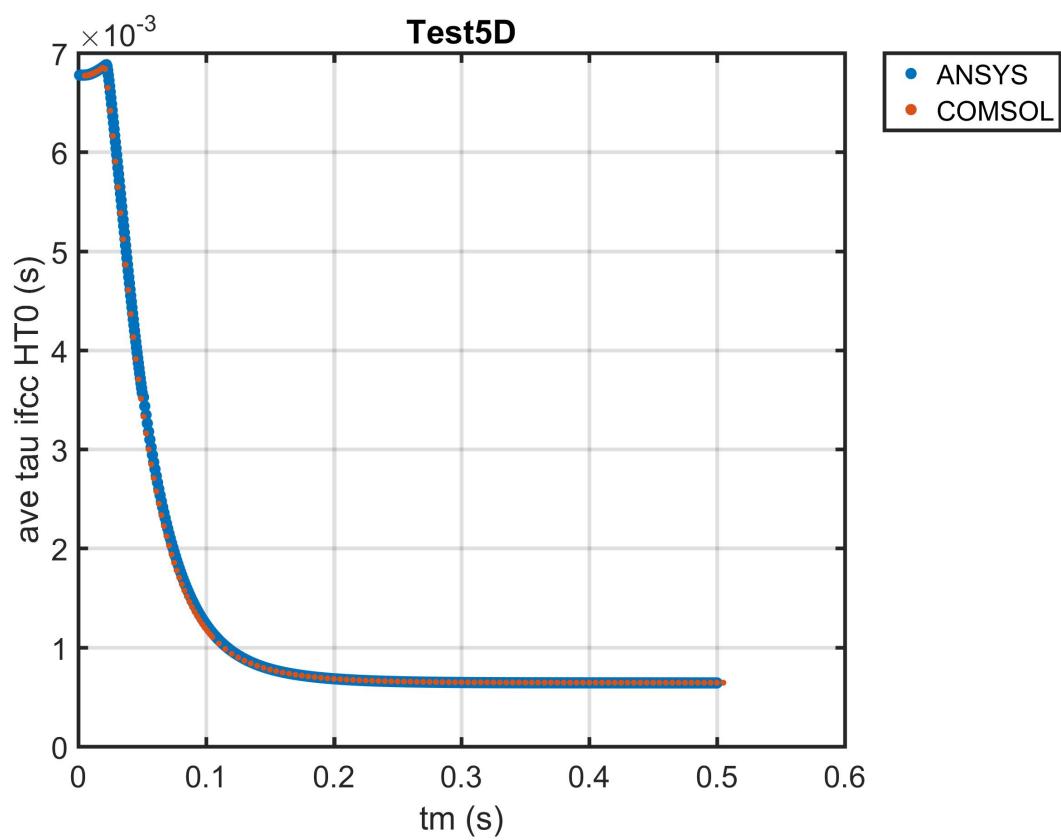
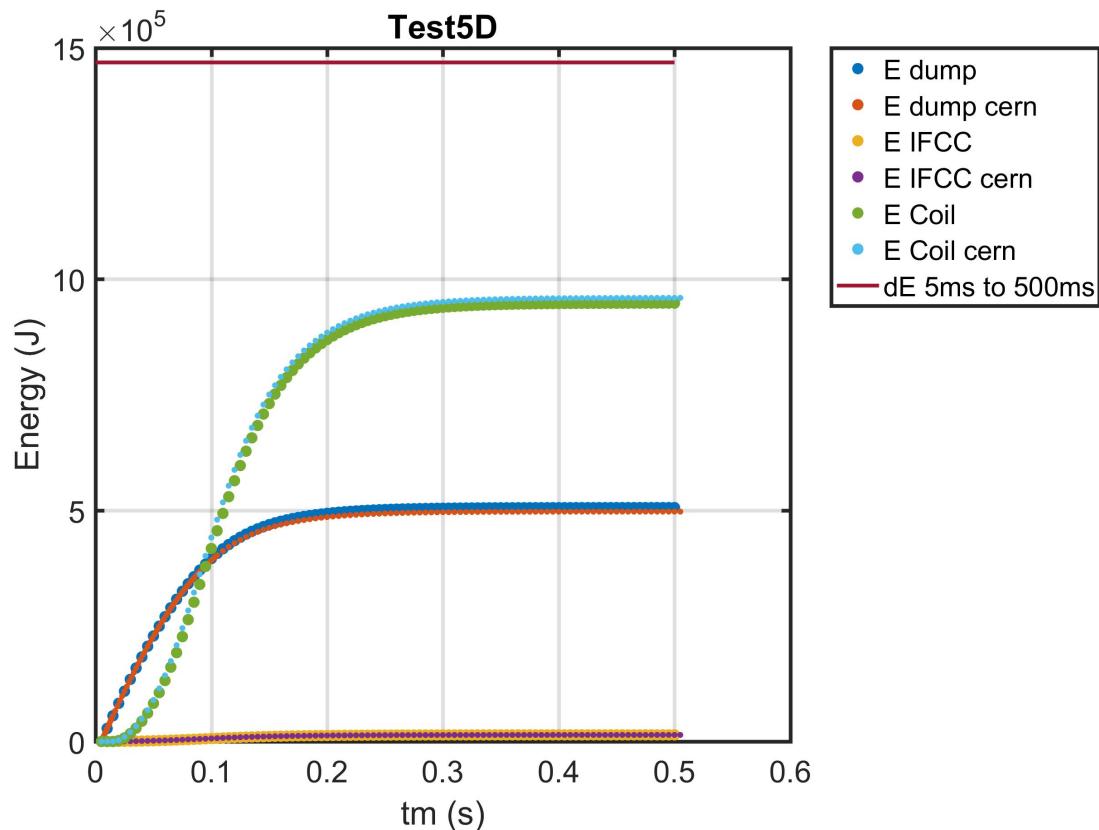
This is the first case where all the effects are combined for a quench back scenario. The high dB/dt due to the dump resistor extraction initiates IFCC losses which heats the coil to the point of quench. Then the coil resistance grows quickly and dominates the decay.

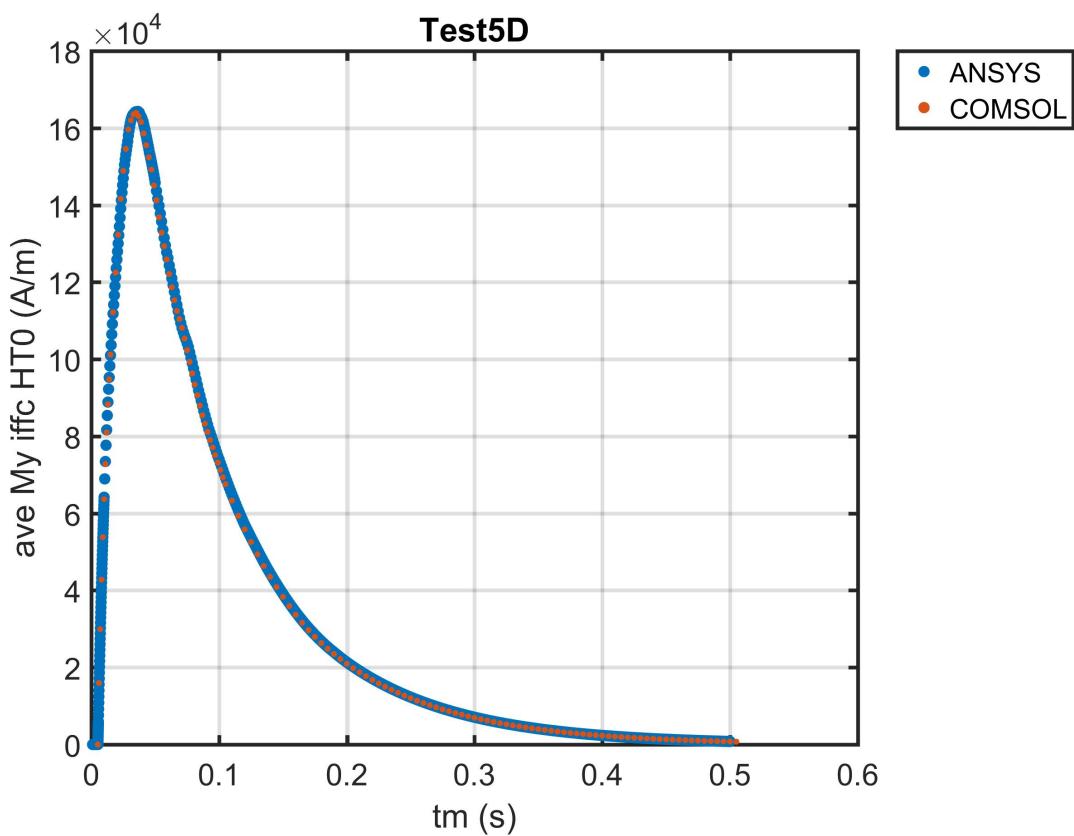
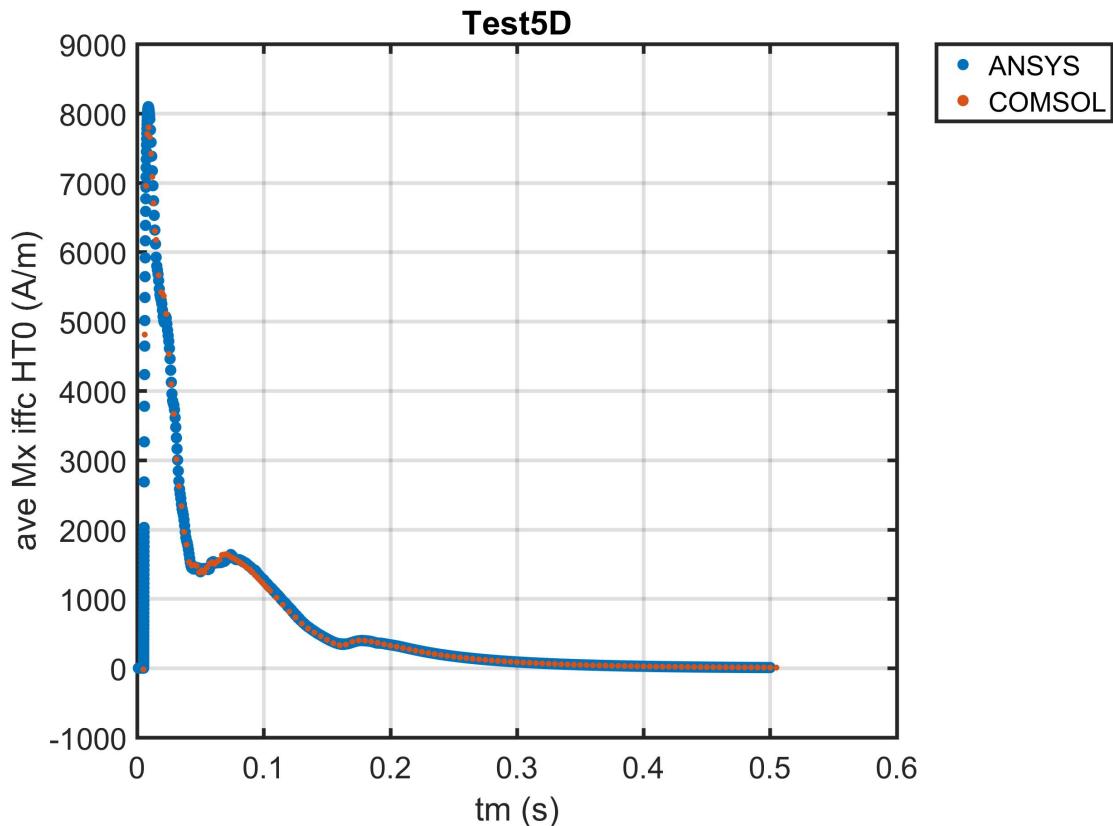
Active physics	EM, CIRCU, THERM
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Allow Quench with no Current Sharing
Cable insulation material	None
Cable voids filling material	GLASS FIBER
Time range for comparison	t=(0,500) [ms], Δt=suff. for conv.
I <sub>0</sub>	13.8 kA
R <sub>dump</sub>	30 mΩ
L <sub>c</sub>	10.11 m
L <sub>i</sub>	9.20 m
ρ <sub>cu</sub> fit	NIST, see Appendix E
C <sub>v</sub> -Cu fit	CUDI, see Appendix E
C <sub>v</sub> -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
C <sub>v</sub> -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
τ-IFCC	Internally calculated
<b>Output</b>	I(t) [A]
	T <sub>aveHT0</sub> [K]
	T <sub>hot</sub> [K]
	E-dump(t) [J]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]
	Q-Ohm <sub>totHT0</sub> [W/m] (per quad)
	Q-Ohm <sub>totcoil</sub> [W/m] (per quad)
	R-coil(t) [Ω]
	E-Ohm(t) [J]

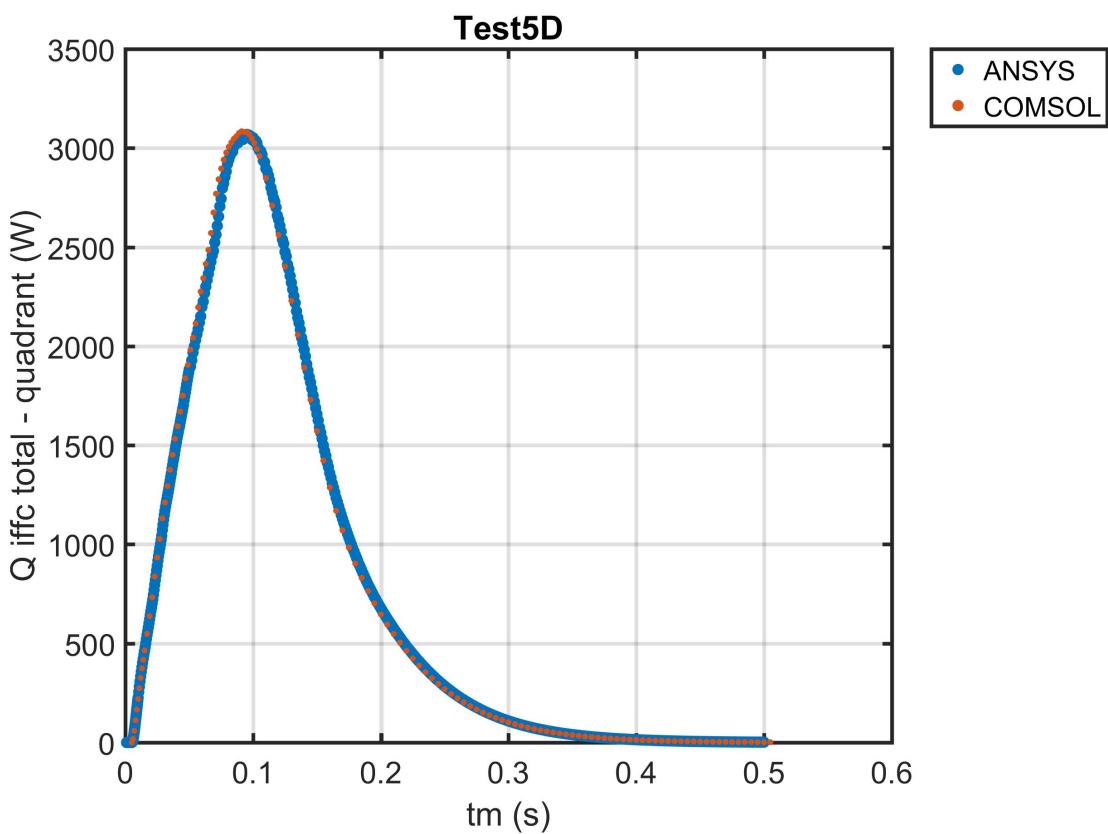
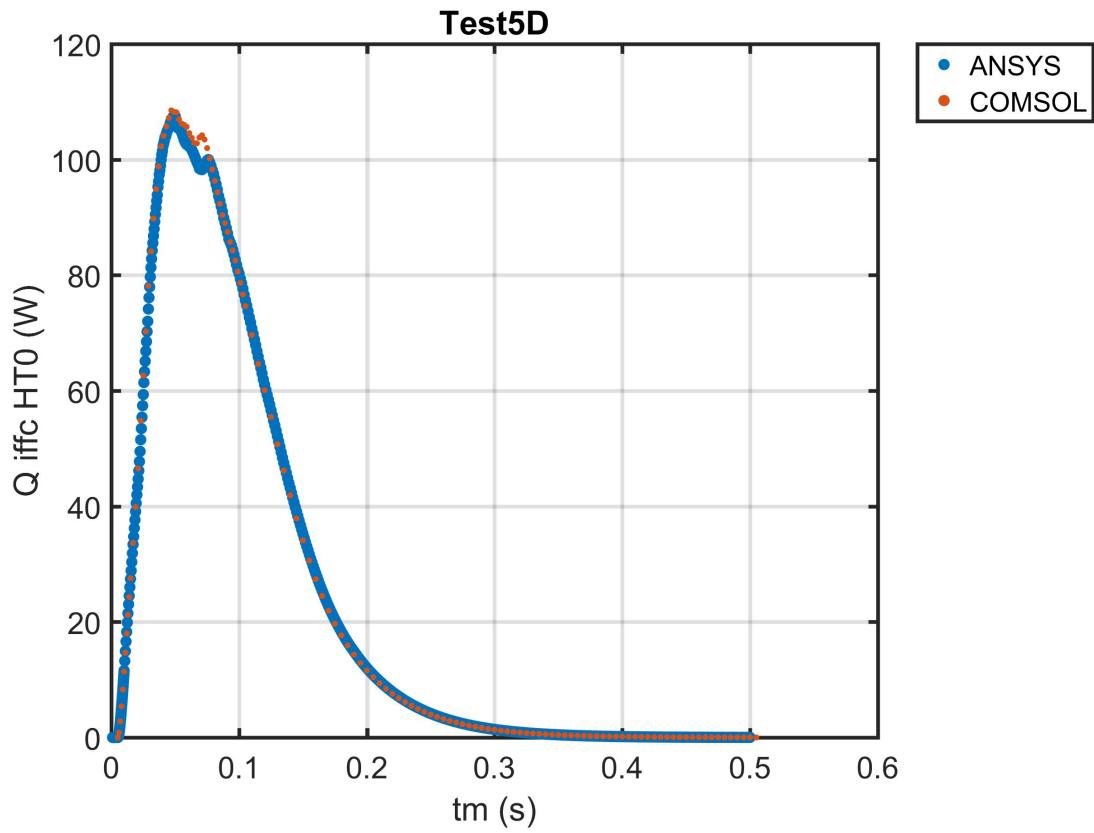
ANSYS: Summary of Energy Loss	
Stored Energy at 5ms	1.469 MJ
Stored Energy at 500ms	0.00 kJ
$\Delta E$	1.469 MJ
Edump	505.96 kJ (34.45% of $\Delta E$ )
Eifcc	14.32 kJ (0.98% of $\Delta E$ )
Eohm	948.24 kJ (64.56% of $\Delta E$ )
Edump-CERN	497.25 kJ
Eifcc-CERN	14.24 kJ
Eohm-CERN	959.55 kJ

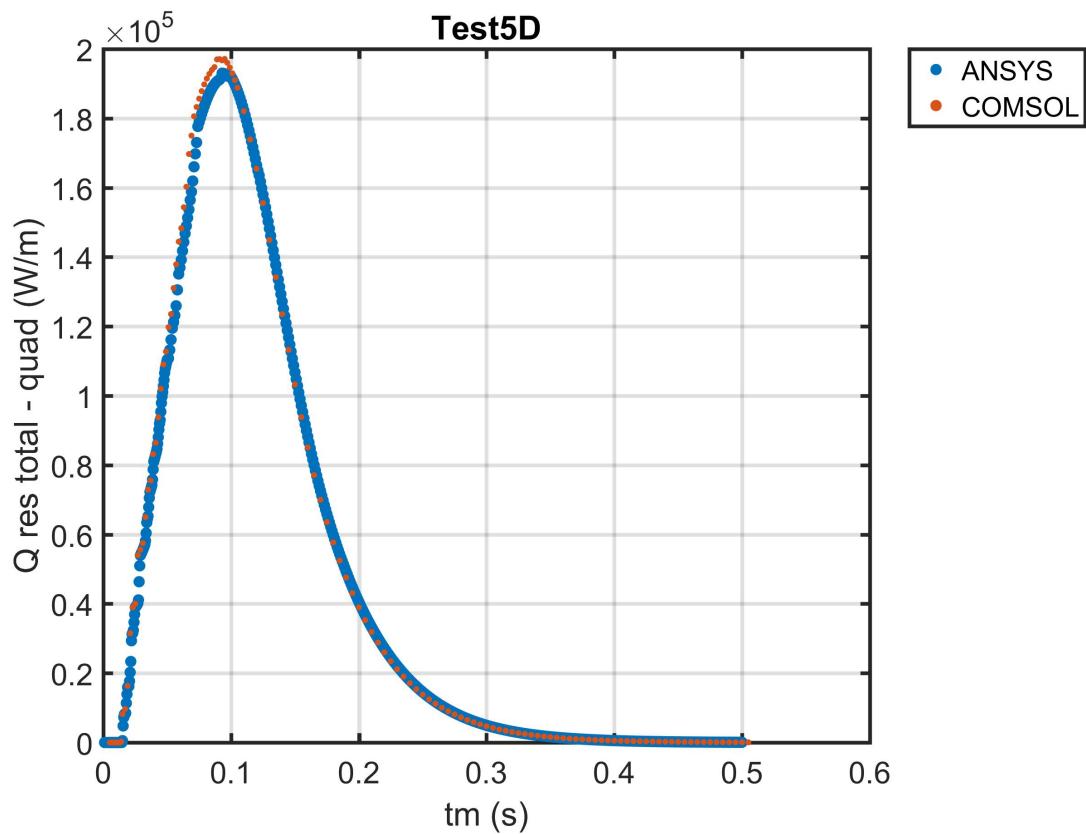
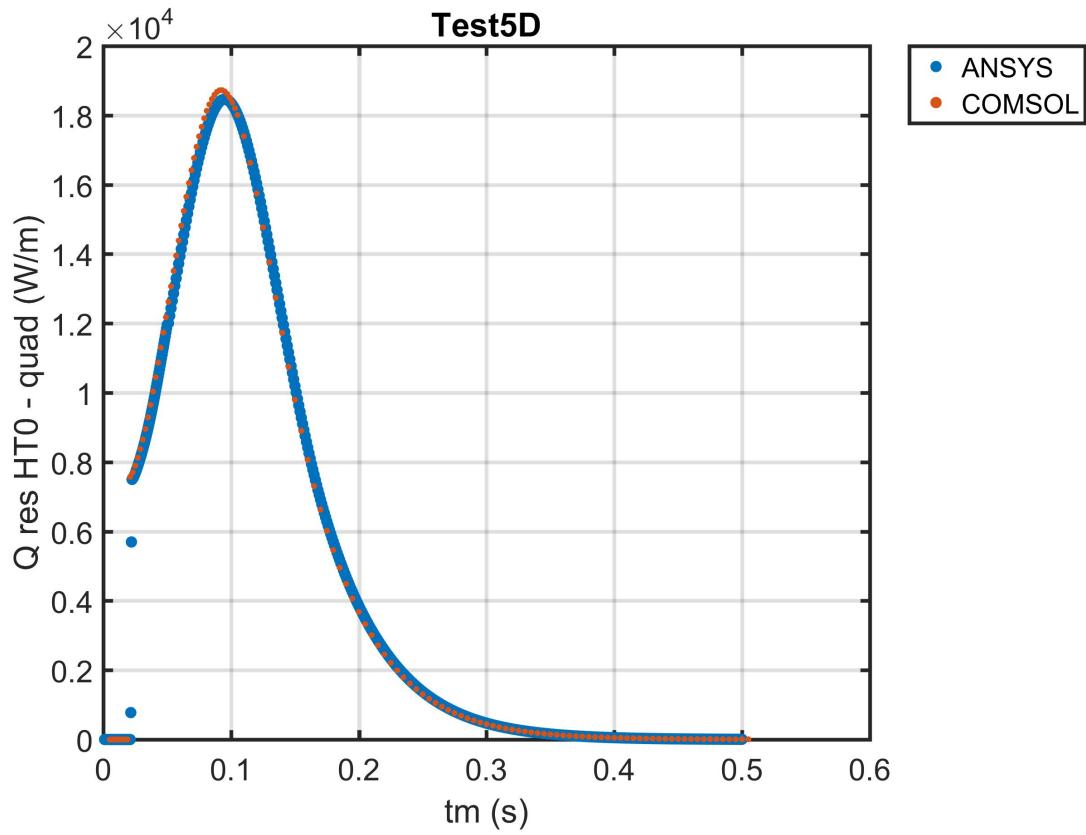


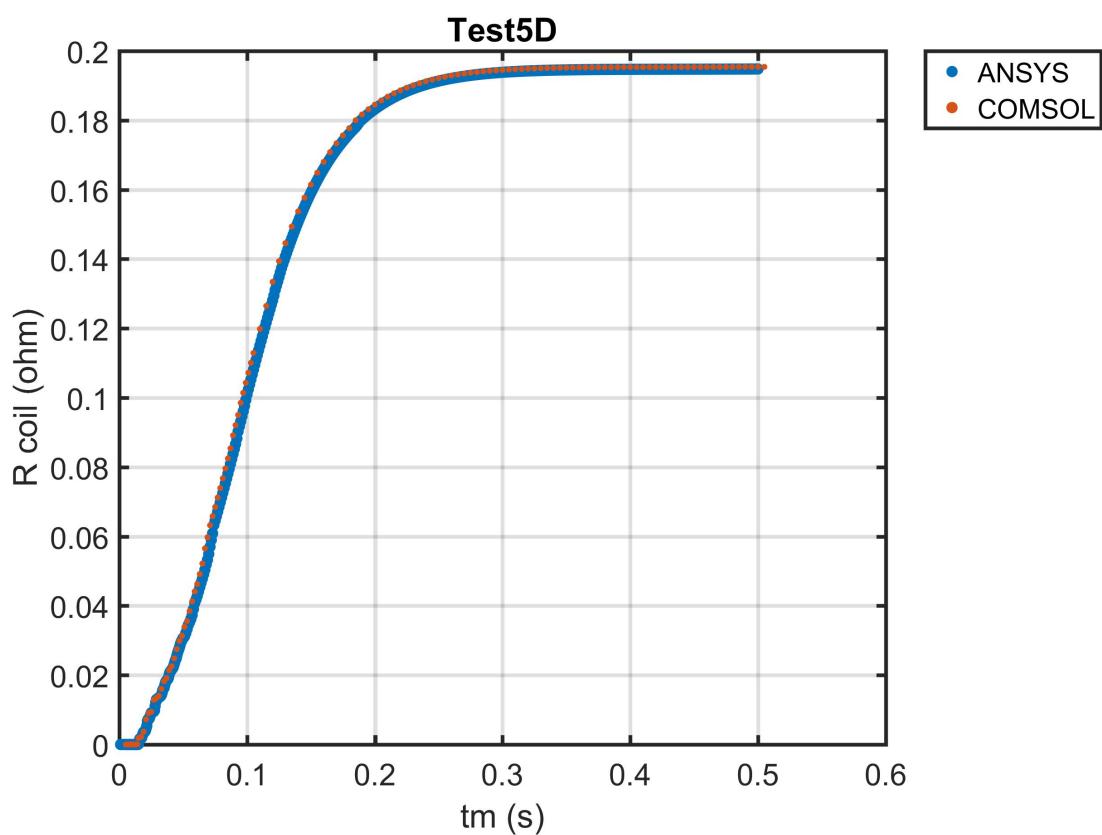






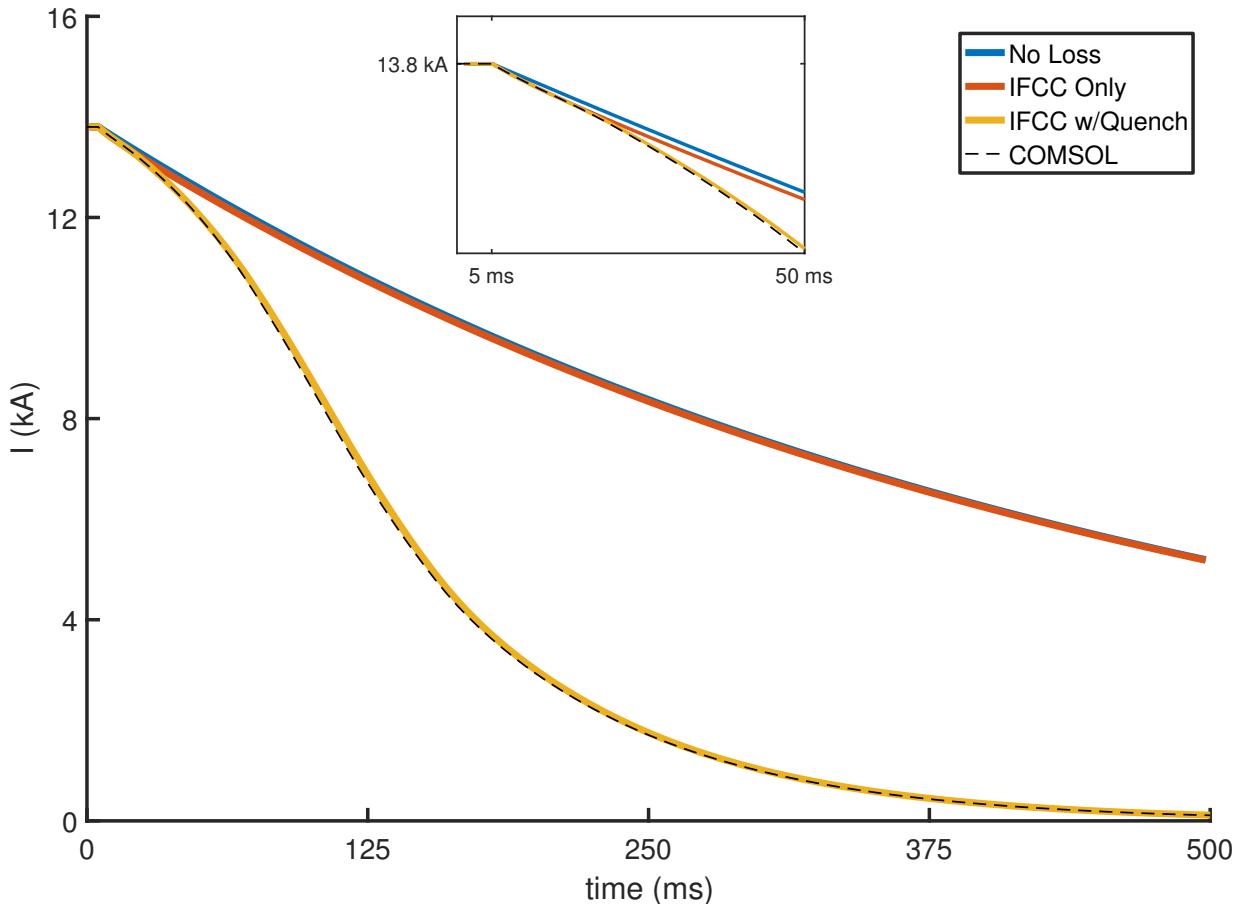




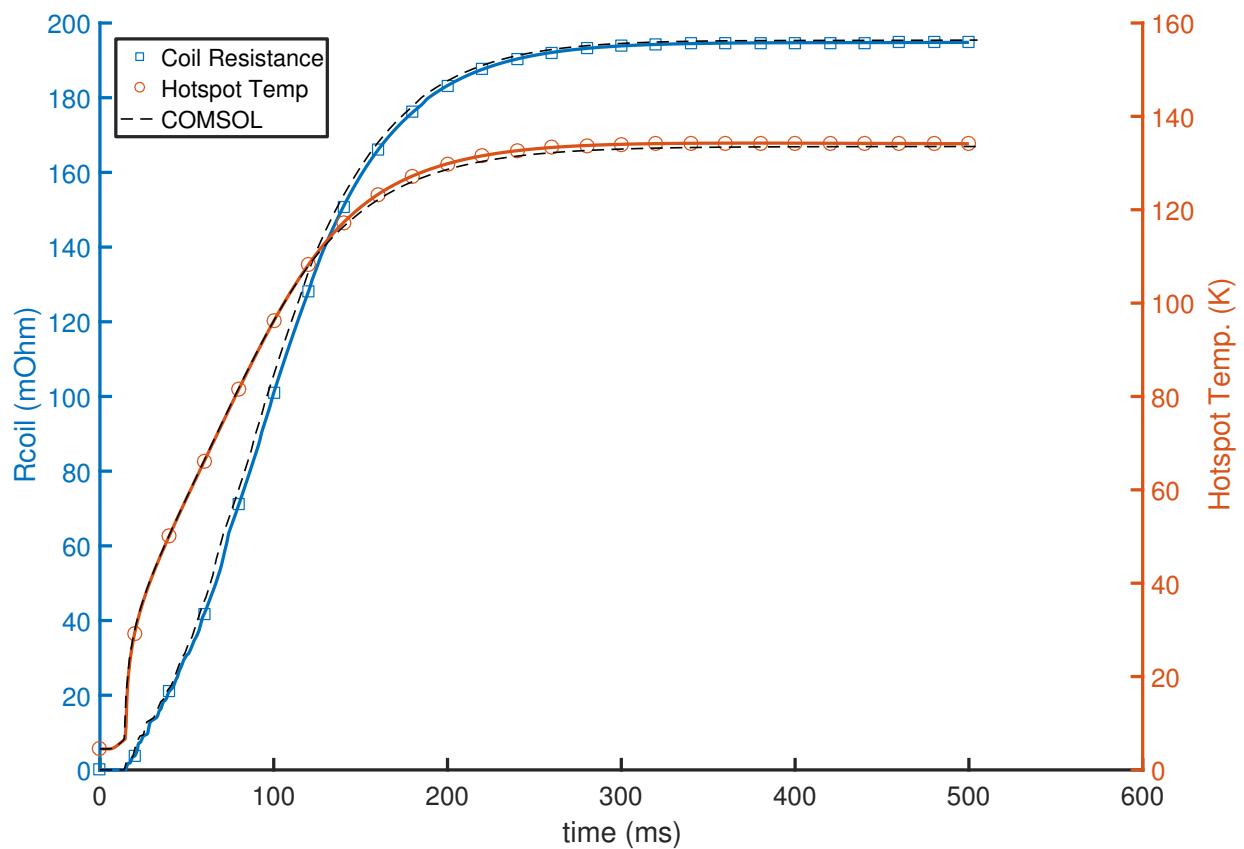


## 6.5 Comparison of LR Quench Back

The following figures and tables compare the results between tests 5B, 5C, and 5D. This slowly builds up from “No Loss”, to including “IFCC Only”, to including “IFCC w/Quench”. A slight difference in the initial current decay between the “No Loss” and the other two cases is seen up to about 20 ms. While the coil is not quenched, this difference is due to the impact of the coupling currents on the magnet’s differential inductance. At 20 ms, the impact of quench resistance growth is clearly seen as the “IFCC w/Quench” begins to quickly decay away from the “IFCC Only”.

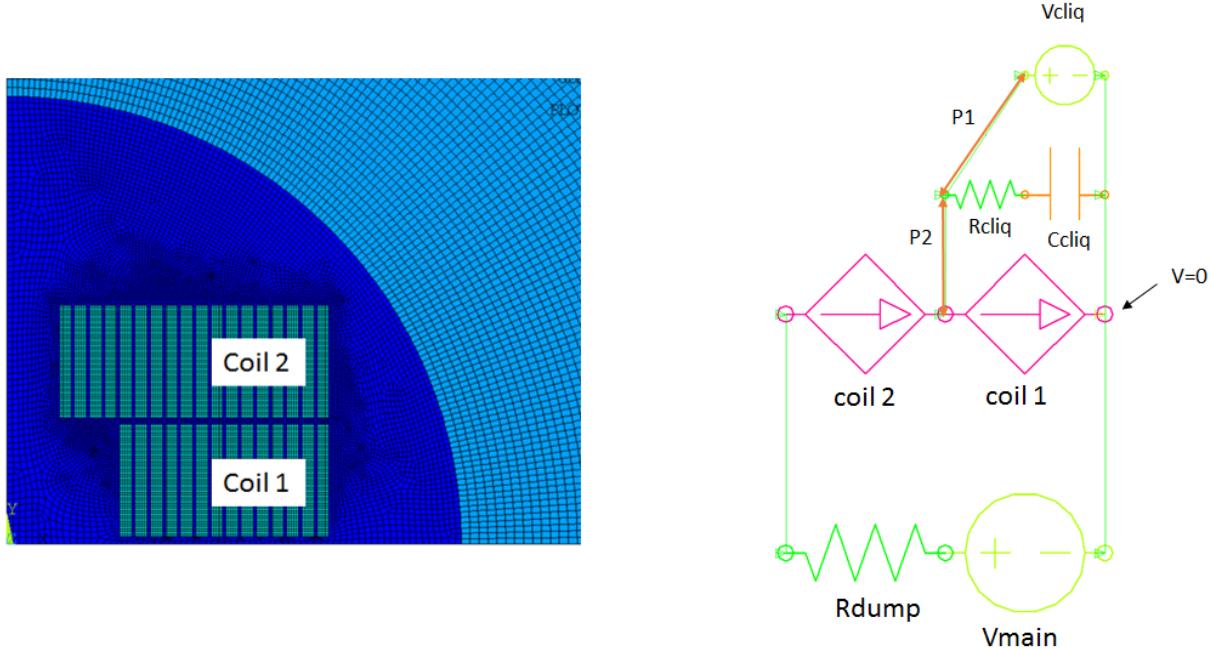


Comparison of Energy Loss for Test 5D				
Energy at 5ms	1.469 MJ			
Energy at 500ms	0.00 MJ			
$\Delta E$	1.469 MJ			
	ANSYS		COMSOL	
	kJ	%	kJ	%
Edump	505.96	34.45	497.25	33.80
Eifcc	14.32	0.98	14.24	0.97
Eohm	948.24	64.56	959.55	65.23



## 7 Double Layer CLIQ Tests

Tests starting with the label “6” introduce CLIQ [8]. A similar approach is taken here where first no losses are assumed in order to debug potential issues with the circuit set up. The CLIQ is connected across the inner coil as shown in the schematic below. The CLIQ circuit parameters and charge voltage are given in the table for each simulation.



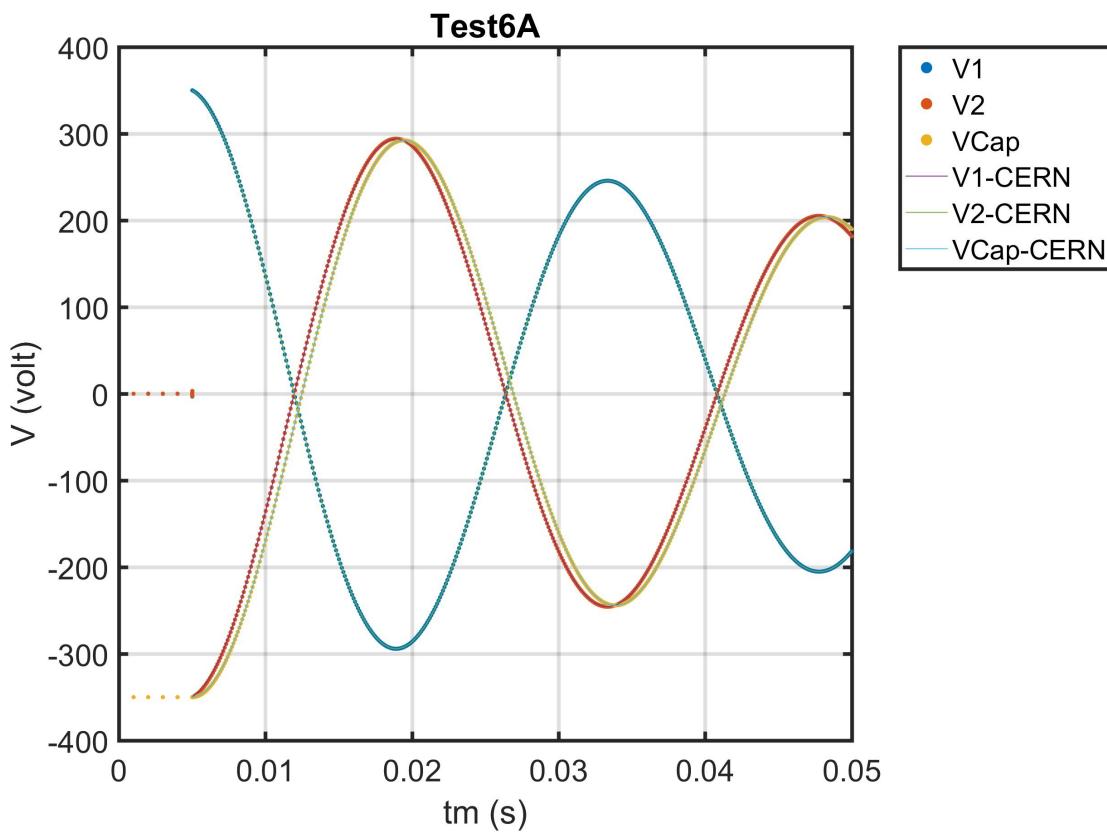
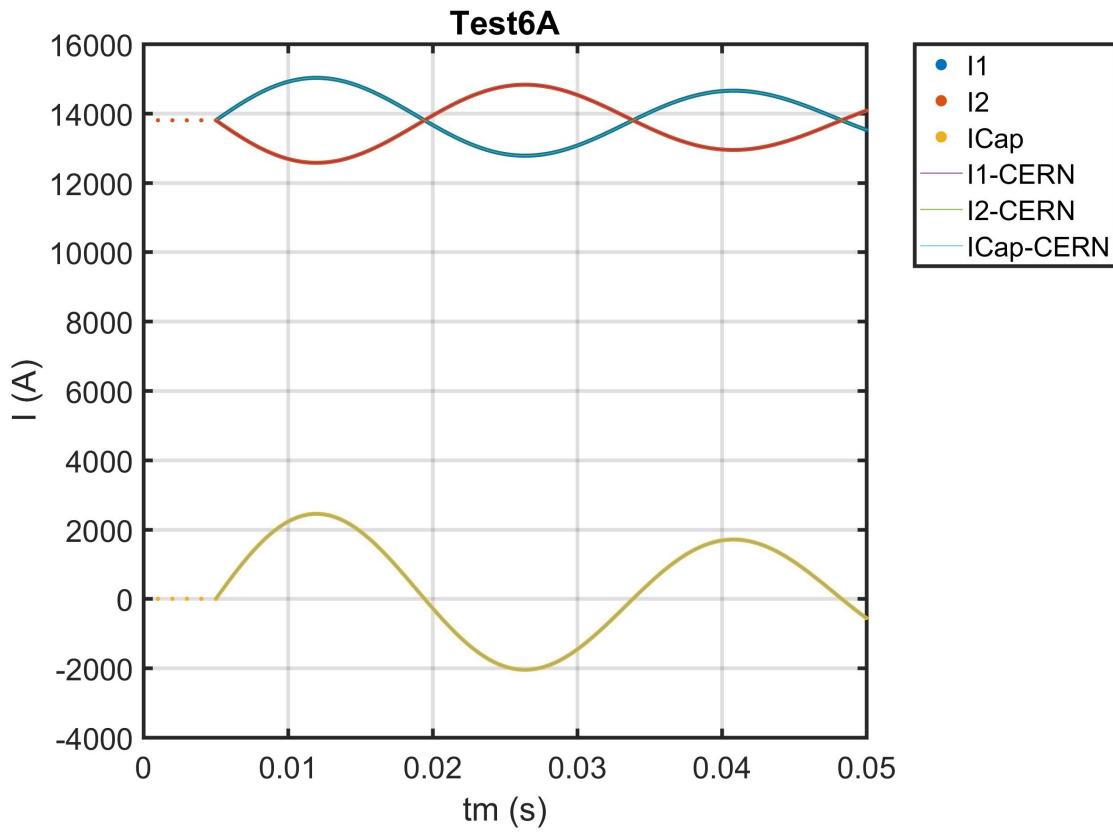
The simulations proceed with the following steps:

1. 0-5 ms
  - (a) P2 is broken
  - (b) P1 exists
  - (c) a “static” solution is solved to charge the cliq and set up magnet current
2. 5-5.01 ms
  - (a) turn on solution transients
  - (b) Vmain is ramped to zero
3. 5.01-5.02 ms
  - (a) P2 is added
  - (b) P1 is broken
  - (c) Vcliq is ramped to zero (with extra dump if numerical problem)
4. 5.02 ms +
  - (a) solve decay

## 7.1 Test 6.A CLIQ With No Losses

This first simulation is intended to test the circuit coupling and the CLIQ set up with the coil only contributing inductive voltage. Interfilament coupling currents and quench are turned off so there are no losses. Since there are no heating terms, thermal coupling is not needed and the coil will stay superconducting and at the initial temperature of 4.5 K. The length  $L_i$  is the effective magnet length used for inductive voltage.

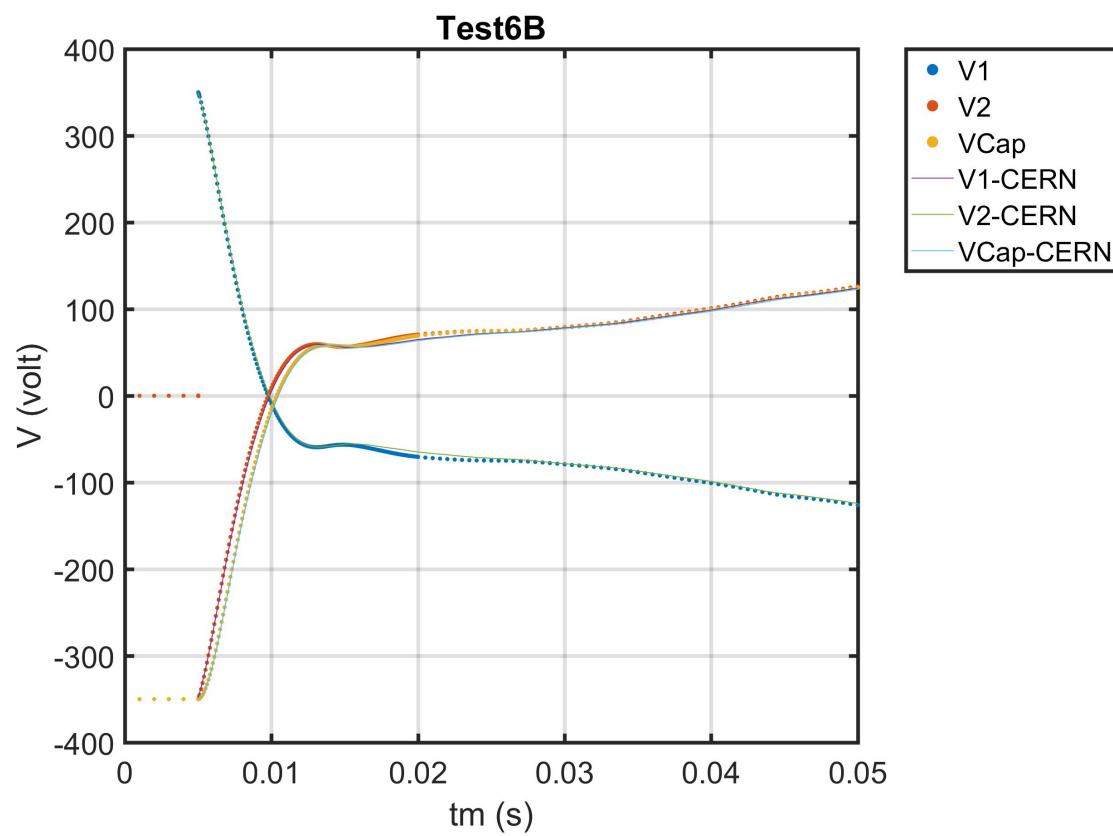
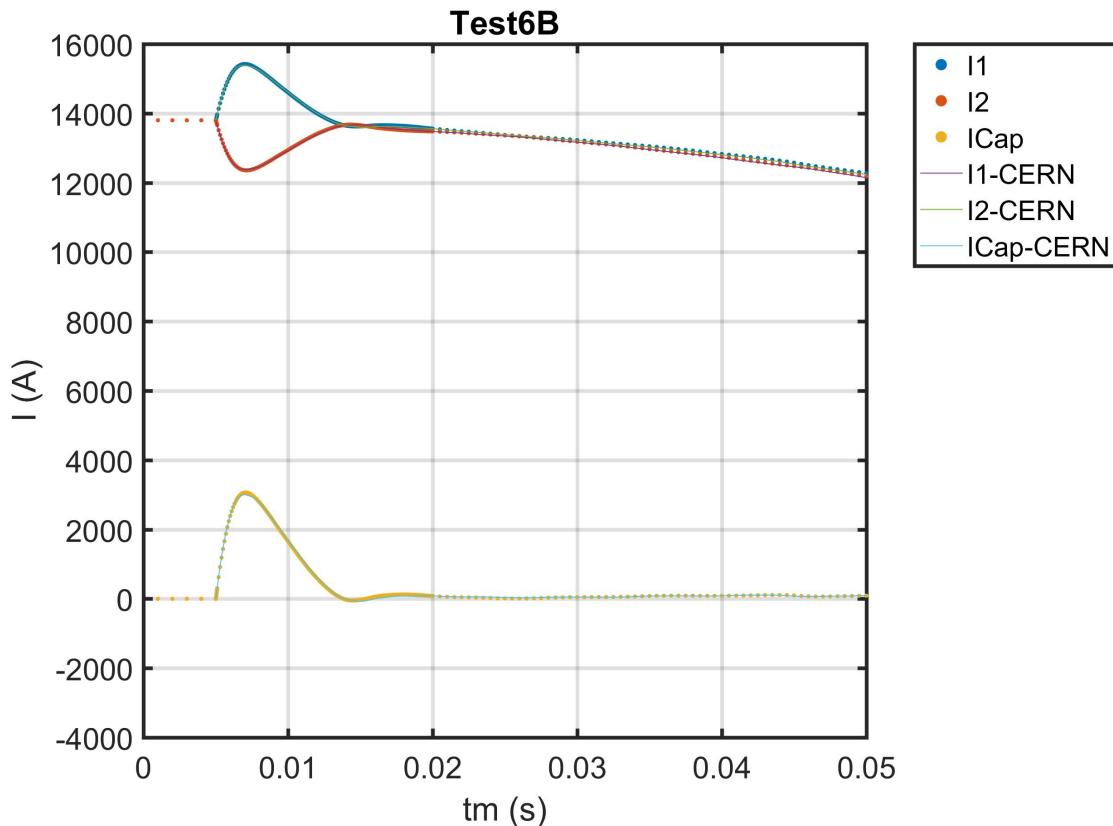
Active physics	EM, CIRCU
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	NO
Quench state	Always SC
Time range for comparison	$t=(0,50)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	13.8 kA
Rdump	0.0 m $\Omega$
$L_c$	N/A
$L_i$	9.20 m
$R_{CLIQ}$	15 m $\Omega$
$C_{CLIQ}$	35 mF
$V_{CLIQ}$	350 V
<b>Output</b>	$I_1$ [A]
	$I_2$ [A]
	$I_{CLIQ}$ [A]
	$V_1$ [V]
	$V_2$ [V]
	$V_{Cap}$ [V]

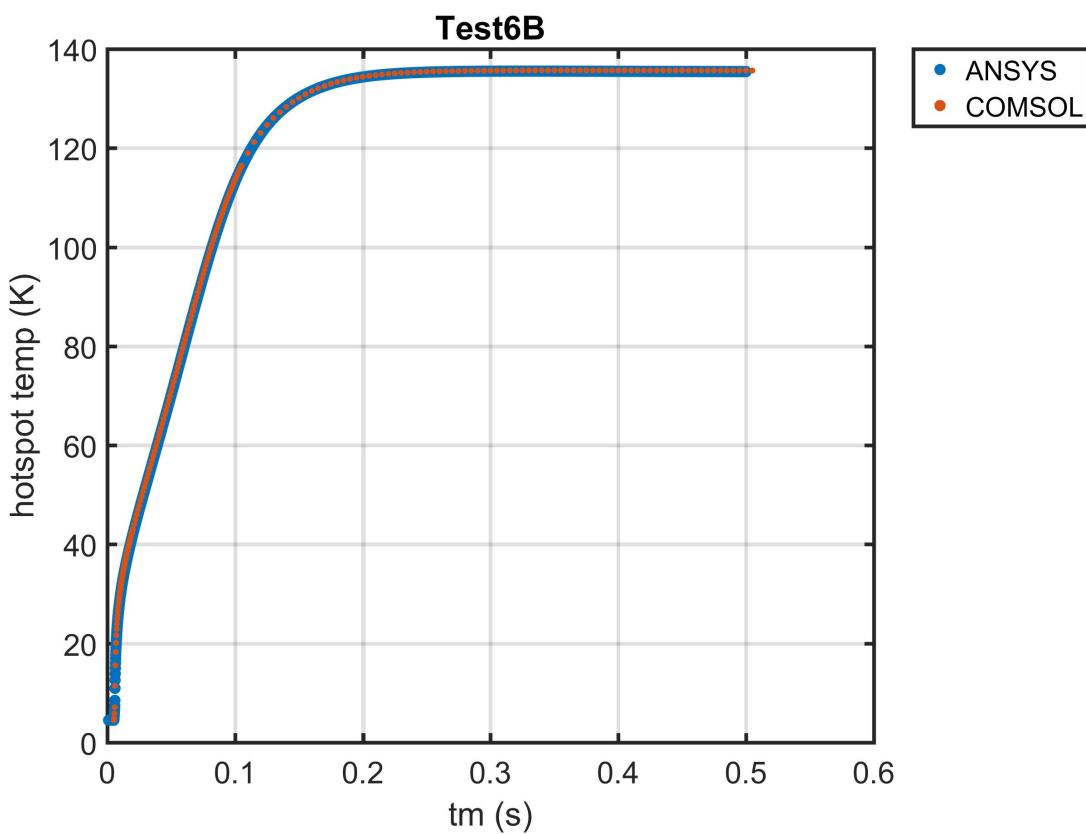
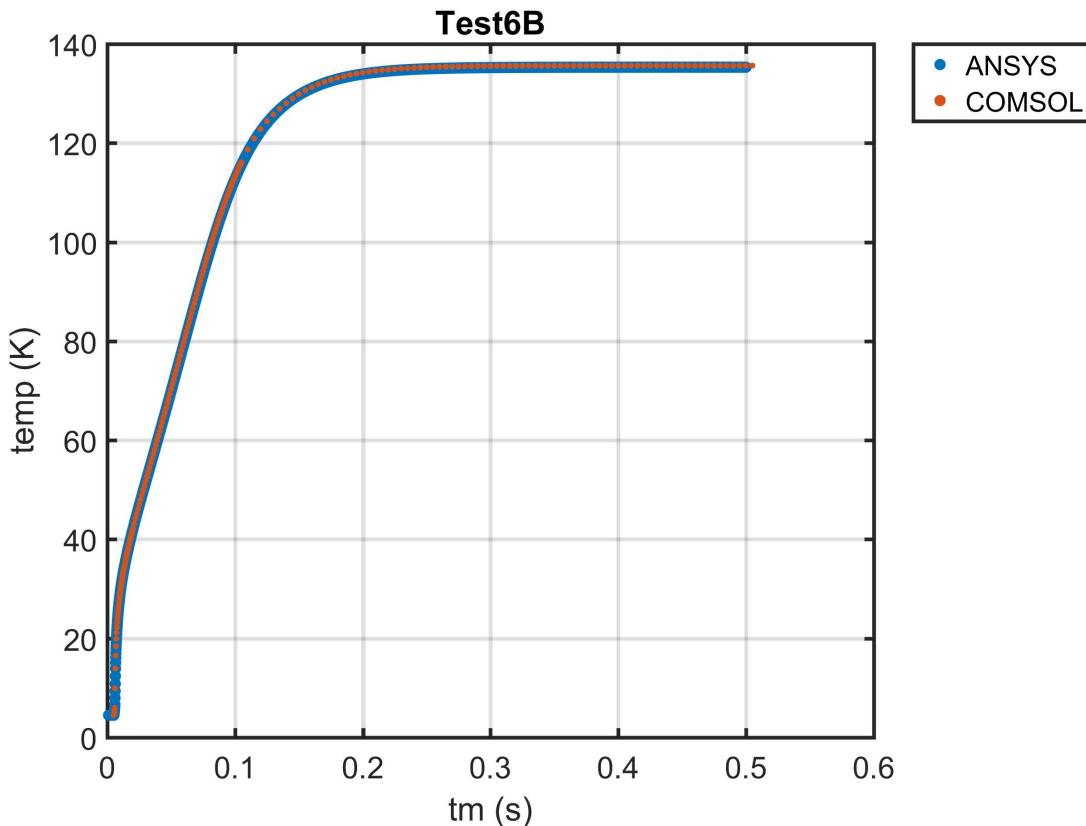


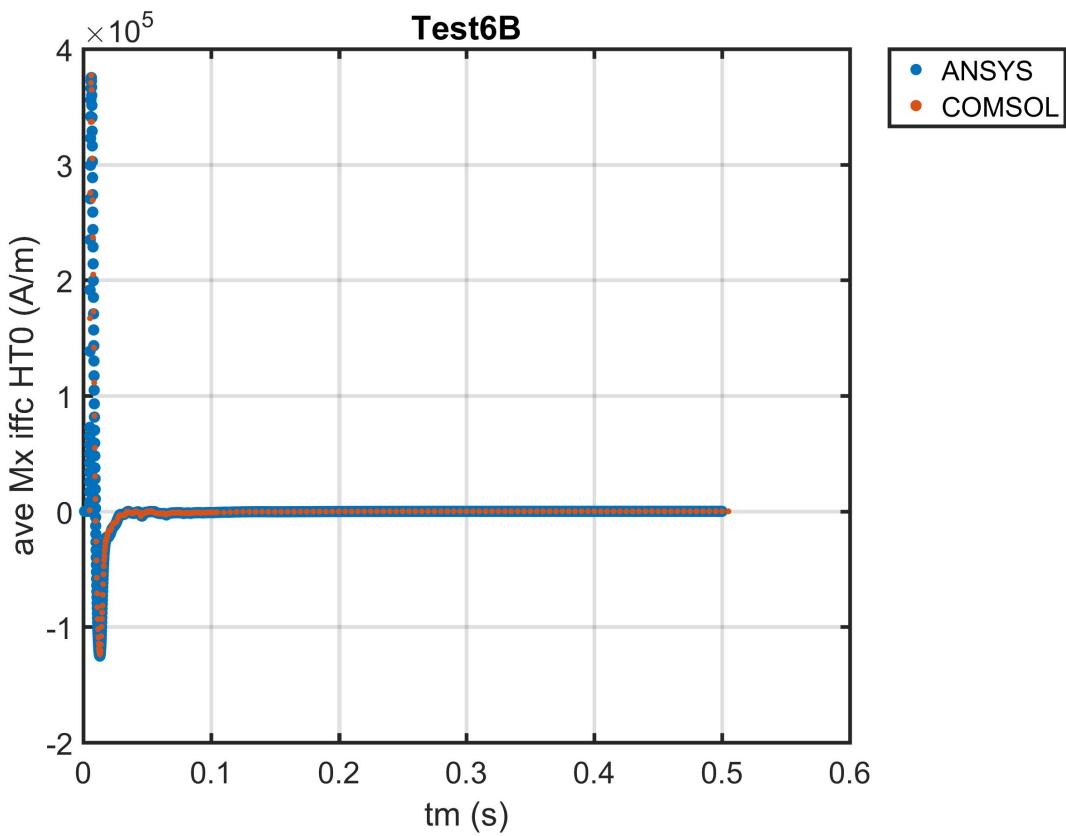
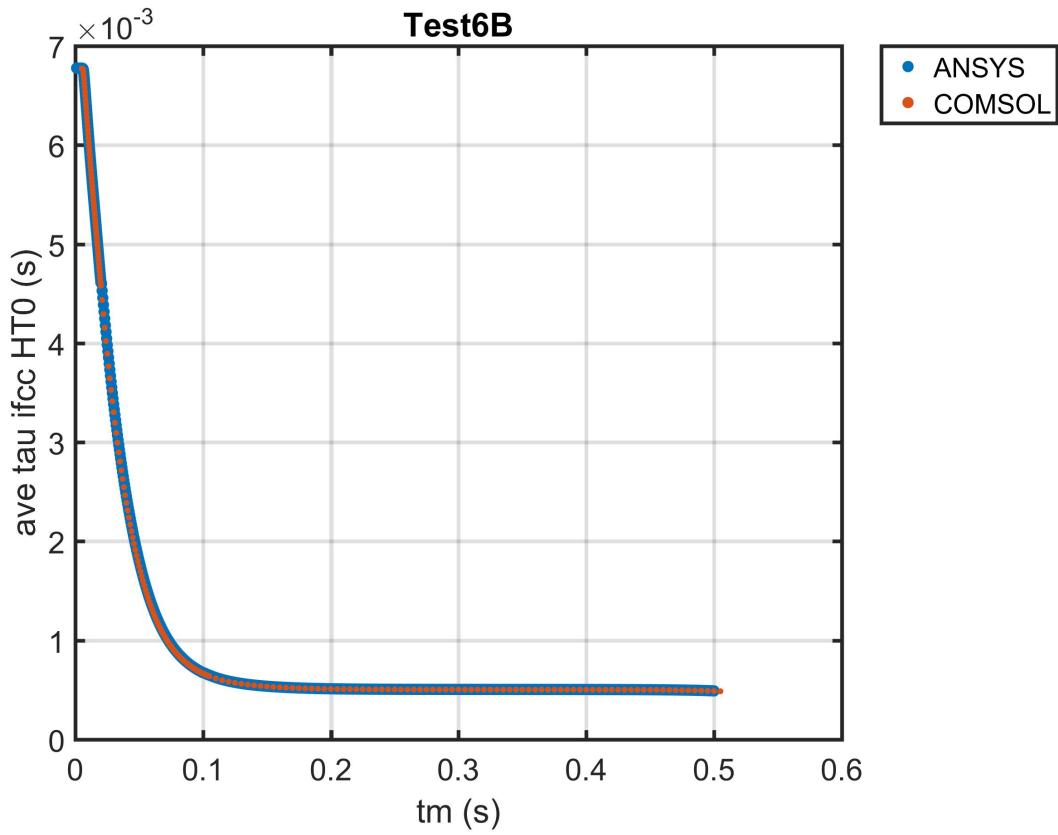
## 7.2 Test 6.B CLIQ Induced Quench Back

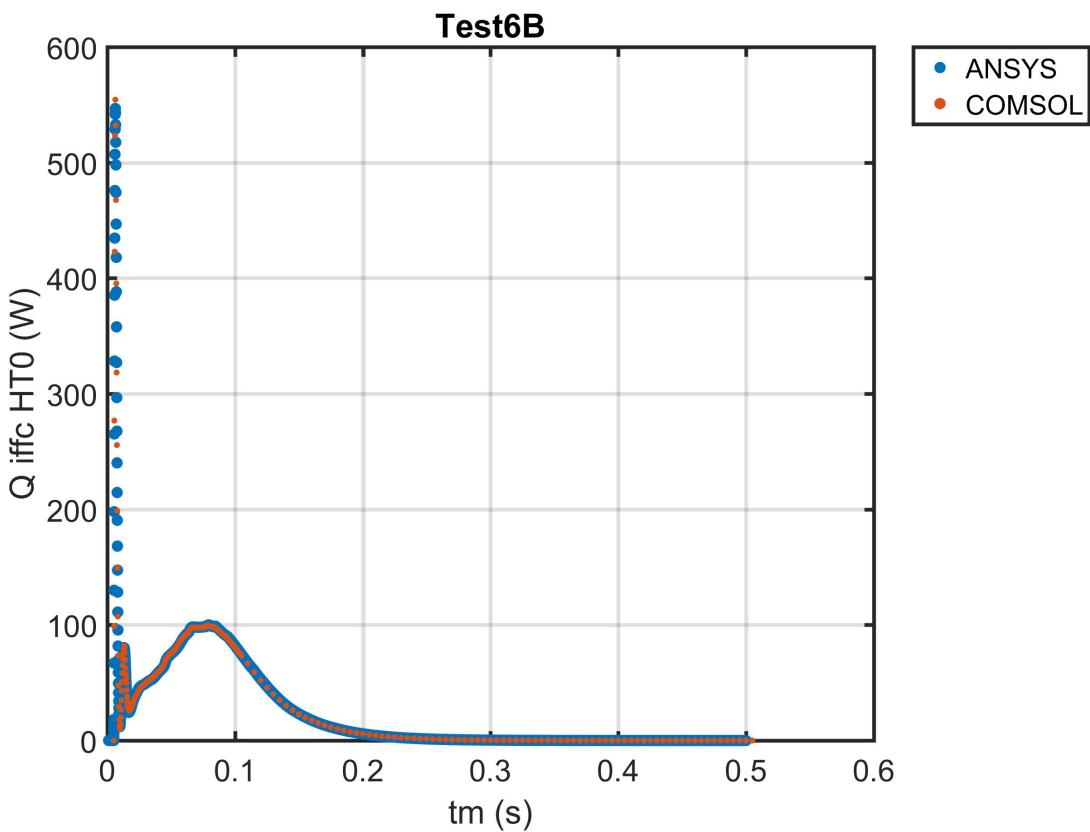
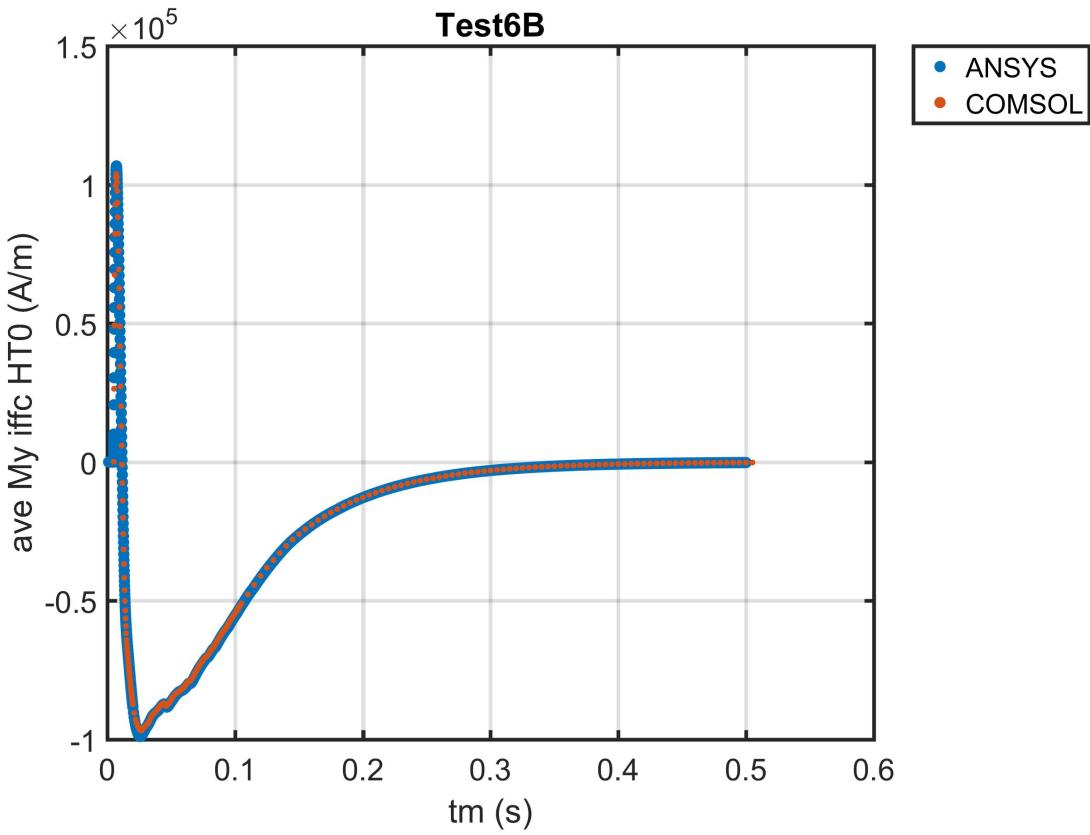
Test 6A is rerun with all effects included. The CLIQ induced current and field oscillation quenches the coil. The coil resistance growth then brings down the magnet current.

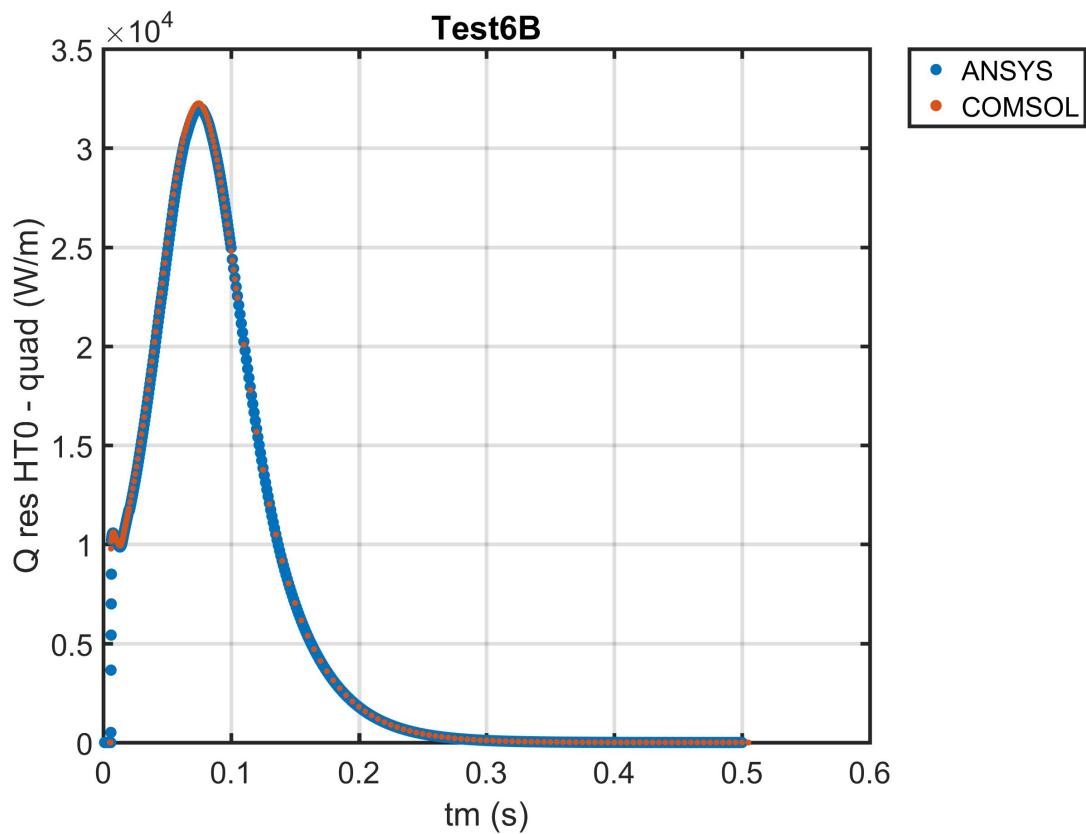
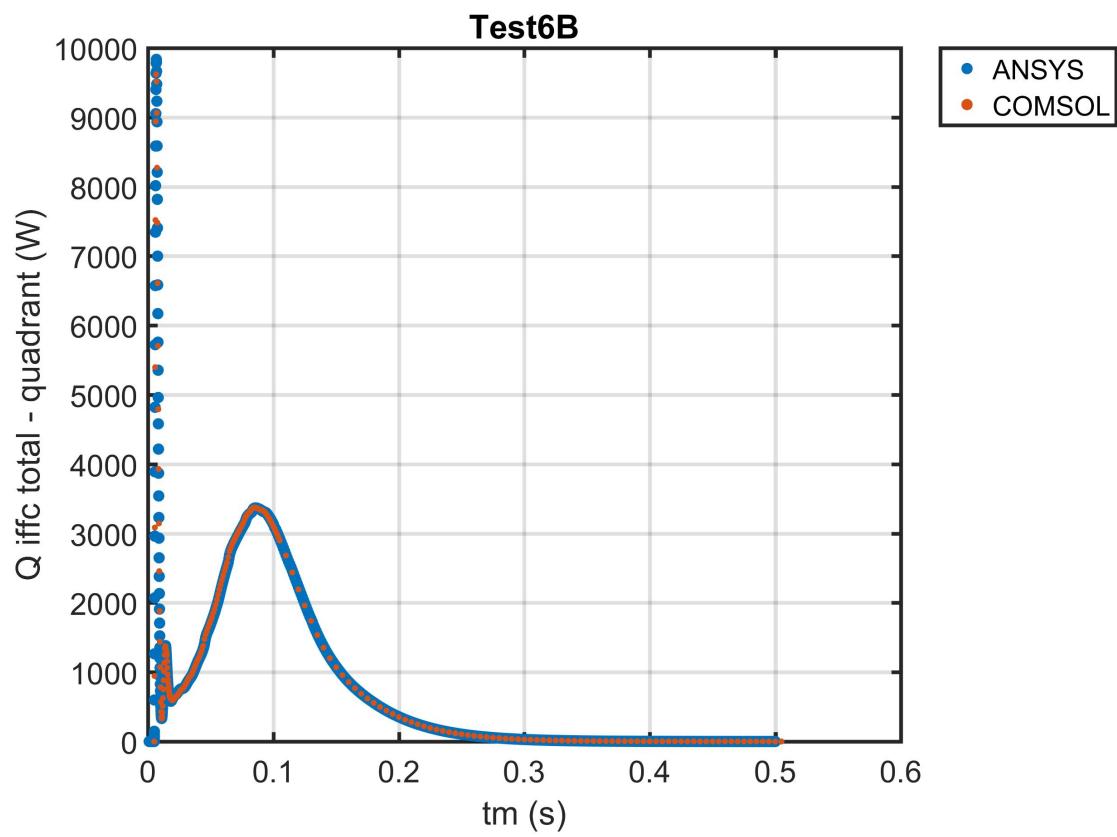
Active physics	EM, CIRCU, THERM
Coil initial temp	4.5 [K]
Iron yoke	YES
IFCC	YES
Quench state	Allow Quench with no Current Sharing
Cable insulation material	None
Cable voids filling material	GLASS FIBER
Time range for comparison	$t=(0,500)$ [ms], $\Delta t$ =suff. for conv.
$I_0$	13.8 kA
$R_{dump}$	0.0 mΩ
$L_c$	10.11 m
$L_i$	9.20 m
$R_{CLIQ}$	15 mΩ
$C_{CLIQ}$	35 mF
$V_{CLIQ}$	350 V
$\rho_{cu}$ fit	NIST, see Appendix E
$C_v$ -Cu fit	CUDI, see Appendix E
$C_v$ -Nb <sub>3</sub> Sn fit	NIST, see Appendix E
$C_v$ -G10 fit	NIST, see Appendix E
k-Cu fit	NIST, see Appendix E
$\tau$ -IFCC	Internally calculated
<b>Output</b>	$I_1, I_2, I_{CLIQ}$ [A]
	$V_1, V_2, V_{Cap}$ [V]
	$T_{aveHT0}$ [K]
	$T_{hot}$ [K]
	Tau-IFCC <sub>aveHT0</sub> [s]
	Mx-IFCC <sub>aveHT0</sub> [A/m]
	My-IFCC <sub>aveHT0</sub> [A/m]
	Q-IFCC <sub>totHT0</sub> [W/m] (per quad)
	Q-IFCC <sub>totcoil</sub> [W/m] (per quad)
	E-IFCC(t) [J]
	Q-Ohm <sub>totHT0</sub> [W/m] (per quad)
	Q-Ohm <sub>totcoil</sub> [W/m] (per quad)
	R-coil(t) [Ω]
	E-Ohm(t) [J]

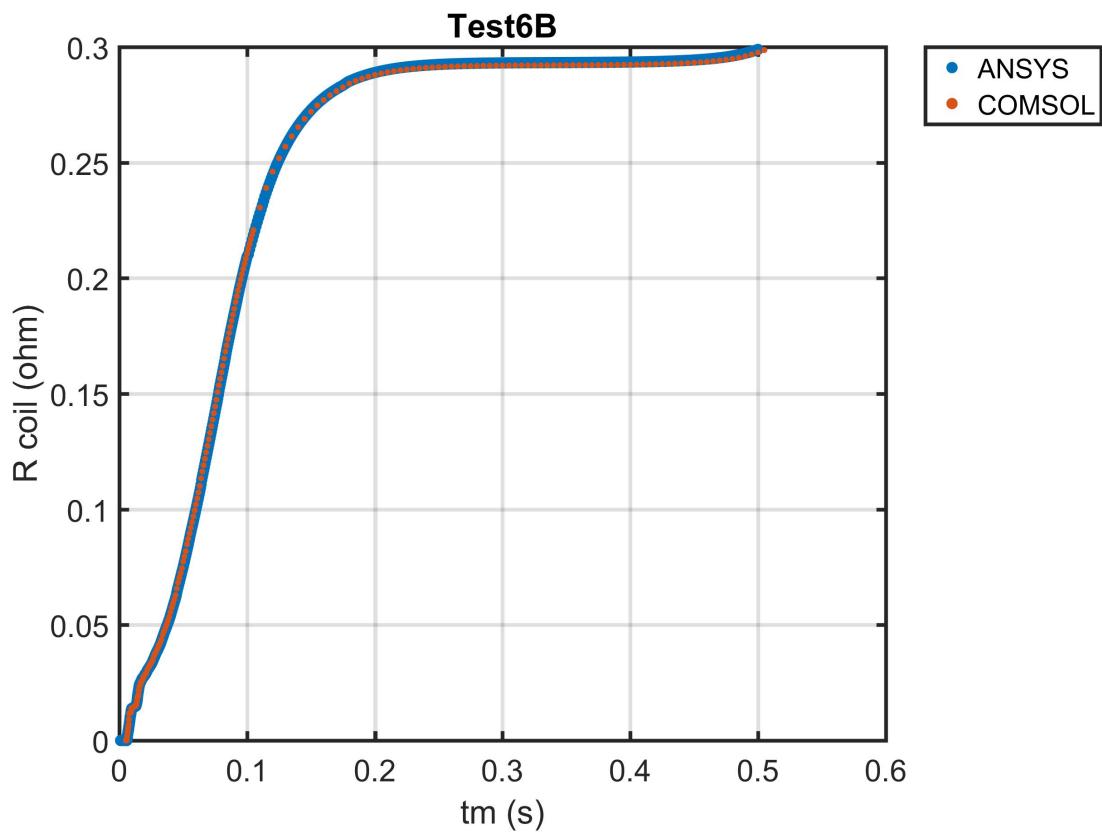
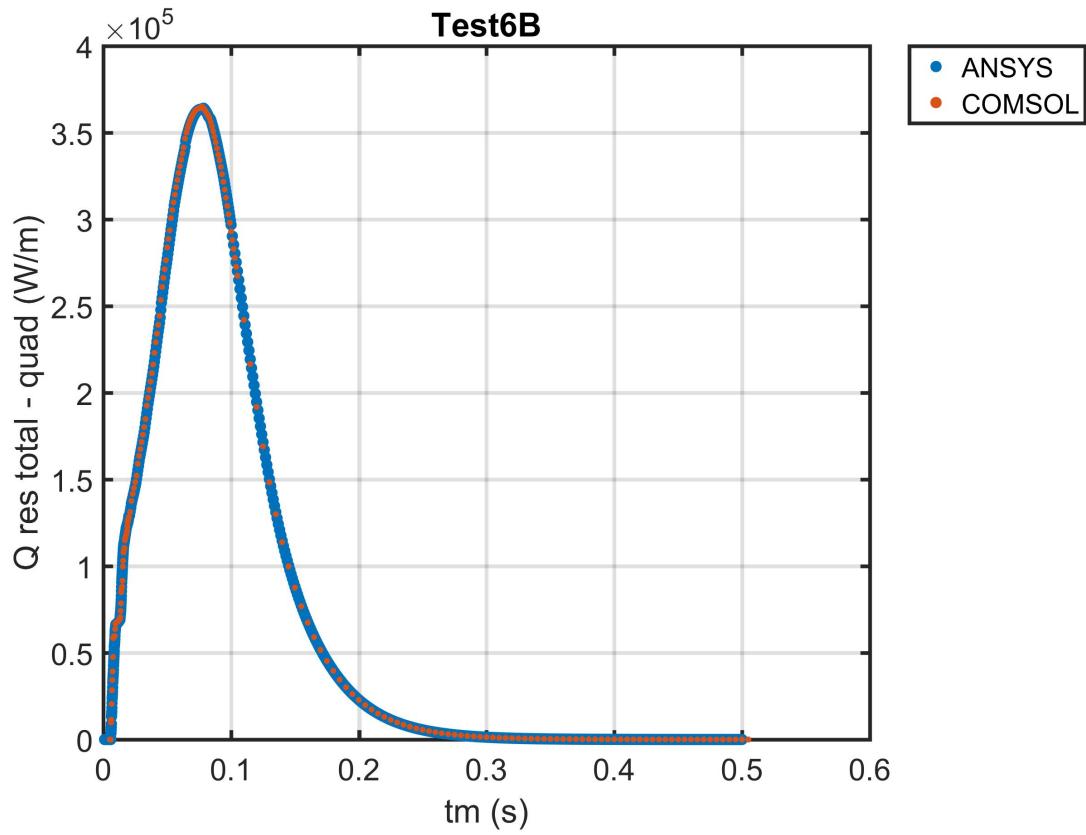


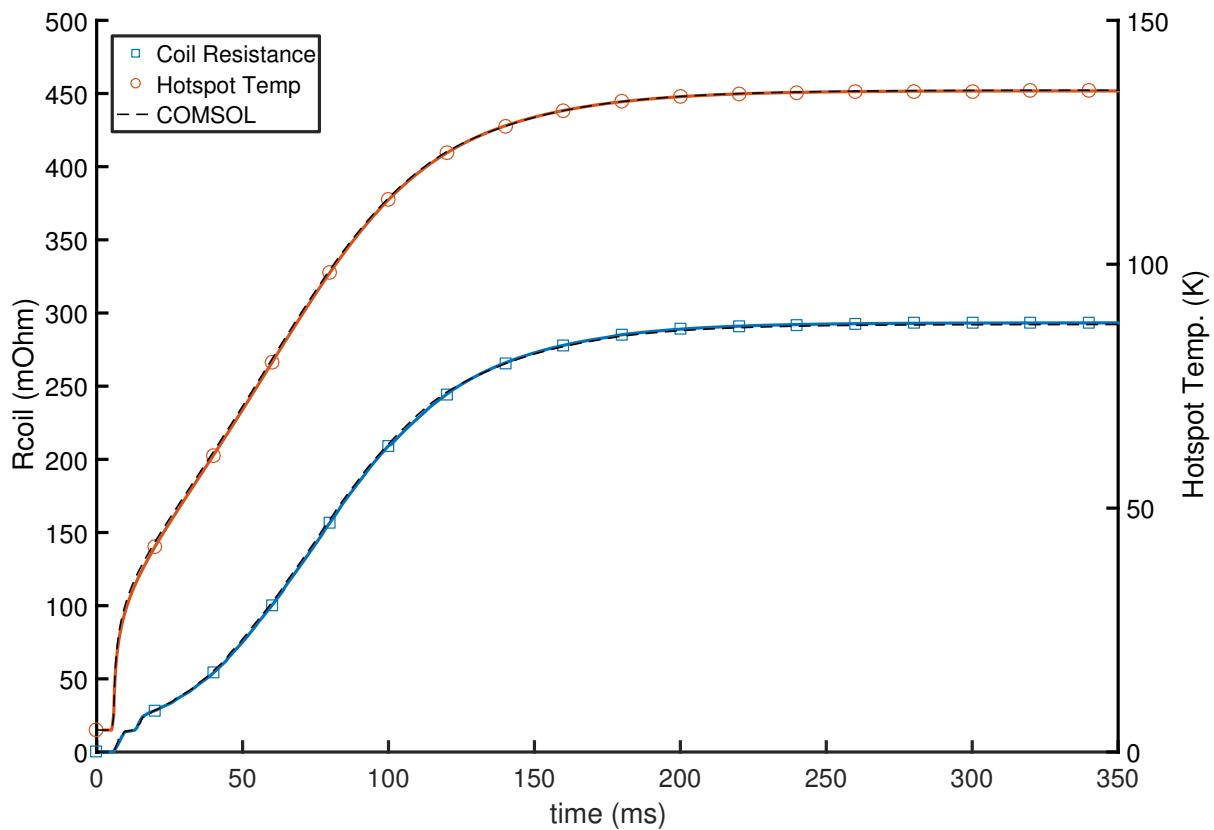












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- [11] G. Manfreda, “Review of ROXIE’s Material Properties Database for Quench Simulation,” *CERN Internal Note: EDMS NR*, vol. 1178007, 2018.

## A Strand and Cable Parameters

The cable parameters which stay constant across all tests are included in the table below. If additional assumptions are made they will be given in the description of the particular test.

<b>Filament</b>		
Diameter	50	$\mu\text{m}$
<b>Strand</b>		
Diameter	0.75	mm
Non-Cu Fraction	0.4	
Cu RRR	200	from 295 K
Effective resistivity	1.0	
Filament twist pitch	14	mm
<b>Cable</b>		
Width	15	mm
Height	1.5	mm
Number of strands	40	
<b>Cable Insulation</b>		
Radial thickness	100	$\mu\text{m}$
Azimuthal thickness	100	$\mu\text{m}$

## B Nb<sub>3</sub>Sn Jc Fit

When needed for determining current sharing and quench, the follow fit for the  $J_c$  of Nb<sub>3</sub>Sn is used.

T <sub>c0</sub>	16.0	K
B <sub>c0</sub>	28.11	T
J <sub>c0</sub>	6190	A/mm <sup>2</sup>
p	1.52	
α	0.96	

$$J_c = J_{c0}(1 - t^p)^{\alpha-1}(1 - t^2)^\alpha h^{-0.5}(1 - h)^2$$

where

$$t = \frac{T}{T_{c0}}$$

$$B_c = B_{c0}(1 - t^p)$$

$$h = \frac{B}{B_c}$$

## C Interfilament Coupling Currents

Interfilament coupling currents (IFCC) are an important loss mechanism for superconducting magnets. Their relatively short time constant (tens of milliseconds or less) and localized heat deposition within the strand make them an effective mechanism for magnet quench back. As shown in Fig. 1, these induced currents flow along the superconducting filaments which are twisted along the length of the strand and then across the strand matrix.

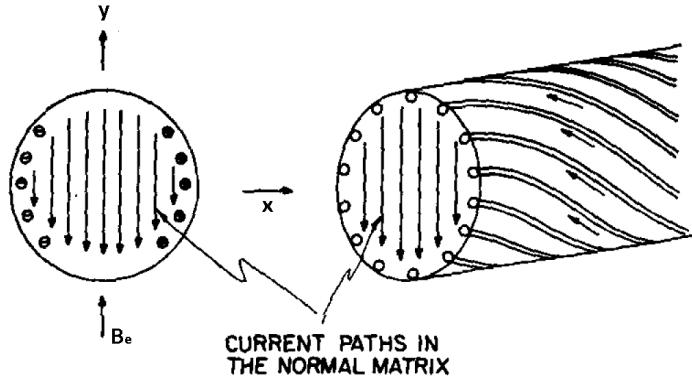


Figure 1: The path of interfilament coupling currents induced by a changing vertical field is shown (Fig. from [9]). The current flows along the twisted filaments and then across the strand matrix. This results in strand magnetization as well as resistive losses in the matrix material.

For a uniform field change within the strand, the induced axial current is typically approximated using a cosine theta (dipole producing) current density [9, 10]

$$g_z(\theta) = \dot{B}_i \left( \frac{L}{2\pi} \right)^2 \frac{\cos(\theta)}{\rho_{et}}, \quad (1)$$

where  $\dot{B}_i$  is the vertical field change within the strand,  $L$  is the filament twist pitch, and  $\rho_{et}$  is the effective resistivity of the strand matrix. To include contact resistance between matrix and filaments, the effective resistivity is defined as  $\rho_{et} = f_{eff}\rho_{cu}$  where  $\rho_{cu}$  is the stabilizer resistivity and  $f_{eff}$  is a single scaling parameter. If  $B_e$  is an applied, uniform external vertical field, the field within the strand is depressed by the induced IFCC currents as

$$B_i = B_e - \frac{1}{2}\mu_0 g_z(\theta = 0) \quad (2)$$

$$B_i = B_e - \tau \dot{B}_i, \quad (3)$$

where the natural time constant for the induced current is clearly

$$\tau = \frac{\mu_0}{2\rho_{et}} \left( \frac{L}{2\pi} \right)^2. \quad (4)$$

The strand magnetization is then given by

$$M_e = -\frac{4}{\mu_0 \pi} \int_0^{\pi/2} g_z(\theta) \cos(\theta) d\theta \quad (5)$$

$$M_e = -\frac{2\tau}{\mu_0} \dot{B}_i. \quad (6)$$

For those reviewing this approach in Wilson's textbook [10], it is helpful to note there is a missing  $\mu_0$  in Equation 8.51 (to match Equation 6). See the derivation of the finite element matrices in [6] to see this equivalent magnetization approach added to the vector potential formulation, leading to the addition of a curl-curl term in the damping matrix of the FEM. These induced currents deposit energy as heat within the strand matrix with a power per unit volume of

$$P_e = \vec{M}_e \cdot \frac{\partial \vec{B}}{\partial t}, \quad (7)$$

which in many cases leads to IFCC losses being an effective quench back mechanism.

## D Iron BH Curve

H(A/m)	B(T)
7.941831506	0.00712
15.88366301	0.0167
23.82549452	0.02925
31.76732602	0.04522
47.53162283	0.08891
63.36753936	0.15434
79.21937139	0.24066
158.8127569	0.76507
317.649387	1.29683
475.1809466	1.45217
633.6276471	1.52043
792.1061786	1.55841
1192.094374	1.60954
1589.758907	1.63892
1987.081257	1.66055
2386.281635	1.67826
2782.290957	1.69427
3179.493941	1.70828
3577.039107	1.72175
3971.377302	1.73414
4765.210312	1.75706
5559.099026	1.77805
6354.069994	1.79764
7149.486595	1.81567
7944.083548	1.8333
10590	1.85
13160	1.9
21170	2
33760	2.1
66000	2.2
120960	2.28
169600	2.3443
212170	2.3996
283130	2.4905
339890	2.5627
425040	2.6706
566950	2.8498
850760	3.2074
1702300	4.2782
2128000	4.8134
2837700	5.7052
3405100	6.4186
4256700	7.4887

## E Material Property Fits

Many of these fits originate from the ROXIE Material Database [11].

```

function rhocunist_Tref(TT,RRR,BB,TREF)
  DOUBLE PRECISION TT,RRR,BB,b,rhocunist_Tref,TREF,tref_RRR,t
  DOUBLE PRECISION c0,c0_scale,P1,P2,P3,P4,P5,P6,P7
  DOUBLE PRECISION a0,a1,a2,a3,a4
  DOUBLE PRECISION rho_0,rho_i,rho_i0,rho_n
  DOUBLE PRECISION x,log_x,f_exp,corr

  c0 = 1.553e-8
  tref_RRR = 273

  P1 = 1.171e-17
  P2 = 4.49
  P3 = 3.841e10
  P4 = 1.14
  P5 = 50
  P6 = 6.428
  P7 = 0.4531

  a0 = -2.662
  a1 = 0.3168
  a2 = 0.6229
  a3 = -0.1839
  a4 = 0.01827

  b=abs(BB)
  T = TT

  c0_scale = TREF/tref_RRR
  rho_0 = c0_scale*c0/RRR
  rho_i = P1*(T**P2)/(1+P1*P3*(T**P2-P4))*exp(-((P5/T)**P6))
  rho_i0 = P7*rho_i*rho_0/(rho_i+rho_0)
  rho_n = rho_0+rho_i+rho_i0

  if (b.gt.0.01d0) then
    x = c0 * B / rho_n
    log_x = log10(x)
    f_exp = a0+a1*log_x+a2*(log_x**2)+a3*(log_x**3) + a4*(log_x**4)
    corr = 10.0d0**f_exp
  else
    corr=0.0d0
  endif

  rhocunist_tref=rho_n*(1+corr);

end function rhocunist_Tref

function cvcucudi(TT)
  DOUBLE PRECISION, INTENT(IN) :: TT
  DOUBLE PRECISION :: cvcucudi

  IF (TT<9.441D0)THEN
    cvcucudi=-0.0308*TT**4.0D0+7.229*TT**3.0D0-2.1286*TT**2.0D0+101.89*TT+2.5631
  ELSE
    IF (TT<31.134D0)THEN
      cvcucudi=-0.3045*TT**4.0D0+29.871*TT**3.0D0-455.61*TT**2.0D0+3469.5*TT-8250.3
    ELSE
      IF (TT<123.34D0)THEN
        cvcucudi=0.0419*TT**4.0D0-14.024*TT**3.0D0+1508.9*TT**2.0D0-31595*TT+178432
      ELSE
        IF (TT<306.12D0)THEN
          cvcucudi=-8.48E-4*TT**4.0D0+0.8419*TT**3.0D0-325.52*TT**2.0D0+60590*TT-1.2851E6
        ELSE
          IF (TT<498.15D0)THEN
            cvcucudi=-4.80E-5*TT**4.0D0+0.09173*TT**3.0D0-64.12*TT**2.0D0+20363*TT+1.028E6
          ELSE
            cvcucudi=12E-5*TT**3.0D0-0.21486*TT**2.0D0+1003.84*TT+3.1823E6
          ENDIF
        END IF
      END IF
    END IF
  END IF

end function cvcucudi

function cvnb3snnist(TT,BB)
  DOUBLE PRECISION, INTENT(IN) :: TT

```

```

DOUBLE PRECISION, INTENT(IN) :: BB
DOUBLE PRECISION cvnb3snnist

DOUBLE PRECISION a0,a1,a2,a3,a4,a5,a6,a7
DOUBLE PRECISION beta,gamma,rho
DOUBLE PRECISION T1,T2,logT,fexp
DOUBLE PRECISION Tc,Tc0,Bc20

Tc0 = 17.8d0
Bc20 = 27.012d0
T1 = 20d0
T2 = 400d0

a0 = 79.78547d0
a1 = -247.44839d0
a2 = 305.01434d0
a3 = -186.90995d0
a4 = 57.48133d0
a5 = -6.3977d0
a6 = -0.6827738d0
a7 = 0.1662252d0
beta = 1.241d0/(1000.0d0)
gamma = 0.138d0
rho = 8950d0

IF (BB .lt. Bc20)THEN
    Tc = Tc0*((1.0d0-BB/Bc20)**0.59d0)
ELSE
    Tc = 0.0d0
END IF

IF (TT .lt. Tc)THEN      ! Superconducting
    cvnb3snnist = rho*((beta+3.0d0*gamma/(Tc0**2d0))*(TT**3d0) + gamma*BB/Bc20*TT)
ELSE IF ((TT .gt. Tc) .AND. (TT .le. T1))THEN
    cvnb3snnist = rho*(beta*(TT**3.0d0) + gamma*TT)
ELSE IF ((TT .gt. T1) .AND. (TT .le. T2))THEN
    logT = LOG10(TT)
    fexp = (a0+a1*logT+a2*(logT**2d0)+a3*(logT**3d0)+a4*(logT**4d0)+a5*(logT**5d0)+a6*(logT**6d0)+a7*(logT**7d0))
    cvnb3snnist = rho*(10d0**fexp)
ELSE
    cvnb3snnist = rho*(234.89d0+0.0425d0*TT)
END IF

end function cvnb3snnist

function cvG10nist(TT)

double precision :: dc_a, dc_b, dc_c, dc_d, dc_e, dc_f, dc_g, dc_h
double precision :: density
double precision, intent(in) :: TT
DOUBLE PRECISION :: cvG10nist, p

dc_a = -2.4083
dc_b = 7.6006
dc_c = -8.2982
dc_d = 7.3301
dc_e = -4.2386
dc_f = 1.4294
dc_g = -0.24396
dc_h = 0.015236

density = 1900.0 ! No value in NIST database, so equal to Cryocomp and Fermilab

p = dc_a+dc_b*(LOG10(TT))*1.0D0+dc_c*(LOG10(TT))*2.0D0+dc_d*(LOG10(TT))*3.0D0
p = p+dc_e*(LOG10(TT))*4.0D0+dc_f*(LOG10(TT))*5.0D0+dc_g*(LOG10(TT))*6.0D0+dc_h*(LOG10(TT))*7.0D0
cvG10nist = density * 10**p

end function cvG10nist

function kxxnist_tref(TT,RRR,BB,TREF)

DOUBLE PRECISION TT, RRR, BB,TREF, kxxnist_tref
kxxnist_tref = kxxnist_B0(TT,RRR)*rhocunist_tref(TT,RRR,0.0D0,TREF)/rhocunist_tref(TT,RRR,BB,tref)

end function kxxnist_tref

function kxxnist_B0(TT,RRR)

DOUBLE PRECISION TT, RRR, kxxnist_B0
DOUBLE PRECISION beta, k_0, k_i, k_i0
DOUBLE PRECISION P1, P2, P3, P4, P5, P6, P7

beta = 0.634/RRR
P1 = 1.754E-8

```

```
P2 = 2.763
P3 = 1102
P4 = -0.165
P5 = 70
P6 = 1.756
P7 = 0.838/( beta/0.0003)**0.1661

k_0 = beta/TT
k_i = P1*TT**P2/(1+P1*P3*TT** (P2+P4)* exp(-(P5/TT)**P6))
k_i0 = P7*k_i*k_0/( k_i+k_0)
kxxnist_B0 = 1/(k_0+k_i+k_i0)

end function kxxnist_B0
```

## F Fixed Material Properties

Tests 2A, 2B, and 2C require fixed material properties. These are given below.

Copper		
$\rho$	1e-8/pi	$\Omega\text{m}$
k	300	W/m/K
$c_v$	500	J/m <sup>3</sup> /K

Nb <sub>3</sub> Sn		
$\rho$	0	$\Omega\text{m}$
k	300	W/m/K
$c_v$	250	J/m <sup>3</sup> /K

Glass Fiber (G10)		
$\rho$	0	$\Omega\text{m}$
k	0.01	W/m/K
$c_v$	750	J/m <sup>3</sup> /K