LỜI GIẢI MỘT SỐ BÀI TẬP
TOÁN CAO CẤP 2

Lời giải một số bài tập trong tài liệu này dùng để tham khảo. Có một số bài tập do một số sinh viên giải. Khi học, sinh viên cần lựa chọn những phương pháp phù hợp và đơn giản hơn. Chúc anh chị em sinh viên học tập tốt

BÀI TẬP VỀ HẠNG CỦA MA TRẬN

Bài 1:

Tính hạng của ma trận:

1) $$A=\begin{pmatrix} 2 & -4 & 3 & 1 & 0\\ 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 1 & -7 & 4 & -4 & 5\end{pmatrix} \xrightarrow{h1\leftrightarrow h2} \begin{pmatrix} 1 & -2 & 1 & -4 & 2\\ 2 & -4 & 3 & 1 & 0\\ 0 & 1 & -1 & 3 & 1\\ 1 & -7 & 4 & -4 & 5\end{pmatrix}$$

$$\xrightarrow[h1(-1)+h4]{h1(-2)+h2} \begin{pmatrix} 1 & -2 & 1 & -4 & 2\\ 0 & 0 & 1 & 9 & -4\\ 0 & 1 & -1 & 3 & 1\\ 0 & -5 & 3 & 0 & 3\end{pmatrix} \xrightarrow{h2\leftrightarrow h3} \begin{pmatrix} 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 0 & 0 & 1 & 9 & -4\\ 0 & -5 & 3 & 0 & 3\end{pmatrix}$$

$$\xrightarrow{h2(5)+h4} \begin{pmatrix} 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 0 & 0 & 1 & 9 & -4\\ 0 & 0 & -2 & 15 & 8\end{pmatrix} \xrightarrow{h3(2)+h4} \begin{pmatrix} 1 & -2 & 1 & -4 & 2\\ 0 & 1 & -1 & 3 & 1\\ 0 & 0 & 1 & 9 & -4\\ 0 & 0 & 0 & 33 & 0\end{pmatrix}$$

$$\Rightarrow r(A)=4$$

---

2)

$$A=\begin{pmatrix} 0 & 2 & -4\\ -1 & -4 & 5\\ 3 & 1 & 7\\ 0 & 5 & -10\\ 2 & 3 & 0\end{pmatrix} \xrightarrow{h1\leftrightarrow h2} \begin{pmatrix} -1 & -4 & 5\\ 0 & 2 & -4\\ 3 & 1 & 7\\ 0 & 5 & -10\\ 2 & 3 & 0\end{pmatrix} \xrightarrow[h1(2)+h4]{h1(3)+h3} \begin{pmatrix} -1 & -4 & 5\\ 0 & 2 & -4\\ 0 & -11 & 22\\ 0 & 5 & -10\\ 0 & -5 & 10\end{pmatrix}$$

$$\xrightarrow{h2\left(\frac{1}{2}\right)} \begin{pmatrix} -1 & -4 & 5\\ 0 & 1 & -2\\ 0 & -11 & 22\\ 0 & 5 & -10\\ 0 & -5 & 10\end{pmatrix} \xrightarrow{\substack{h2(11)+h3\\ h2(-5)+h4\\ h2(5)+h5}} \begin{pmatrix} -1 & -4 & 5\\ 0 & 1 & -2\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \Rightarrow r(A)=2$$

---

2) $A=\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix} \xrightarrow[\text{h1(-1)+h3}]{\text{h1(-2)+h2}} \begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 0 & 0 & -1 & 5 & -1 \\ 0 & 0 & -2 & 10 & -2 \end{pmatrix}$

$\xrightarrow{\text{h2(-2)+h3}} \begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 0 & 0 & -1 & 5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A)=2$

---

3)

$A=\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -5 & 4 \\ 5 & 1 & 1 & 7 \\ 7 & 7 & 9 & -1 \end{pmatrix} \xrightarrow{\substack{\text{h1}(-2)+\text{h2} \\ \text{h1}(-5)+\text{h3} \\ \text{h1}(-7)+\text{h4}}} \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -15 & 6 \\ 0 & -14 & -24 & 12 \\ 0 & -14 & -26 & 6 \end{pmatrix} \xrightarrow{\substack{\text{h2}(-2)+\text{h3} \\ \text{h2}(-2)+\text{h4}}} \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -15 & 6 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 4 & -6 \end{pmatrix}$

$\xrightarrow{\text{h3}\left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -15 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & -6 \end{pmatrix} \xrightarrow{\text{h4}(-4)+\text{h4}} \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -15 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -6 \end{pmatrix} \Rightarrow r(A)=4$

---

4)

$A=\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & 7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix} \xrightarrow{\text{h1}\leftrightarrow\text{h3}} \begin{pmatrix} 1 & -3 & -5 & 0 & 7 \\ 5 & -3 & 2 & 3 & 4 \\ 3 & -1 & 3 & 2 & 5 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix} \xrightarrow{\substack{\text{h1}(-5)+\text{h2} \\ \text{h1}(-3)+\text{h3} \\ \text{h1}(-7)+\text{h4}}} \begin{pmatrix} 1 & -3 & -5 & 0 & 7 \\ 0 & 12 & 27 & 3 & -31 \\ 0 & 8 & 18 & 2 & -16 \\ 0 & 16 & 36 & 4 & -48 \end{pmatrix}$

$\xrightarrow{\text{h3}\left(\frac{1}{2}\right)\leftrightarrow\text{h2}} \begin{pmatrix} 1 & -3 & -5 & 0 & 7 \\ 0 & 4 & 9 & 1 & -8 \\ 0 & 12 & 27 & 3 & -31 \\ 0 & 16 & 36 & 4 & -48 \end{pmatrix} \xrightarrow{\substack{\text{h2}(-3)+\text{h3} \\ \text{h2}(-4)+\text{h4}}} \begin{pmatrix} 1 & -3 & -5 & 0 & 7 \\ 0 & 4 & 9 & 1 & -8 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & -16 \end{pmatrix}$

$\xrightarrow{\text{h3}\left(-\frac{16}{7}\right)+\text{h4}} \begin{pmatrix} 1 & -3 & -5 & 0 & 7 \\ 0 & 4 & 9 & 1 & -8 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A)=3$

---

5)

$$A=\begin{pmatrix} 2 & 2 & 1 & 5 & -1 \\ 1 & 0 & 4 & -2 & 1 \\ 2 & 1 & 5 & -2 & 1 \\ -1 & -2 & 2 & -6 & 1 \\ -3 & -1 & -8 & 1 & -1 \\ 1 & 2 & -3 & 7 & -2 \end{pmatrix} \xrightarrow{h1\leftrightarrow h2} \begin{pmatrix} 1 & 0 & 4 & -2 & 1 \\ 2 & 2 & 1 & 5 & -1 \\ 2 & 1 & 5 & -2 & 1 \\ -1 & -2 & 2 & -6 & 1 \\ -3 & -1 & -8 & 1 & -1 \\ 1 & 2 & -3 & 7 & -2 \end{pmatrix}$$

$$\xrightarrow[\substack{h1(3)+h5 \\ h1(-1)+h6}]{\substack{h1(-2)+h2 \\ h1(-2)+h3 \\ h1+h4}} \begin{pmatrix} 1 & 0 & 4 & -2 & 1 \\ 0 & 2 & -7 & 9 & -3 \\ 0 & 1 & -3 & 2 & -1 \\ 0 & -2 & 6 & -8 & 2 \\ 0 & -1 & 4 & -5 & 2 \\ 0 & 2 & -7 & 9 & -3 \end{pmatrix} \xrightarrow{h2\leftrightarrow h3} \begin{pmatrix} 1 & 0 & 4 & -2 & 1 \\ 0 & 1 & -3 & 2 & -1 \\ 0 & 2 & -7 & 9 & -3 \\ 0 & -2 & 6 & -8 & 2 \\ 0 & -1 & 4 & -5 & 2 \\ 0 & 2 & -7 & 9 & -3 \end{pmatrix}$$

$$\xrightarrow[\substack{h2+h5 \\ h2(-2)+h6}]{\substack{h2(-2)+h3 \\ h2(2)+h4}} \begin{pmatrix} 1 & 0 & 4 & -2 & 1 \\ 0 & 1 & -3 & 2 & -1 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & -1 & 3 & -1 \end{pmatrix} \xrightarrow[h3(-1)+h6]{h3+h5} \begin{pmatrix} 1 & 0 & 4 & -2 & 1 \\ 0 & 1 & -3 & 2 & -1 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A)=4$$

---

6)

$$A=\begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 2 & 1 & -1 & 2 & 0 \\ -1 & 2 & 1 & 1 & 3 \\ 1 & 5 & -8 & -5 & -12 \\ 3 & -7 & 8 & 9 & 13 \end{pmatrix} \xrightarrow[\substack{h1(-1)+h4 \\ h1(-3)+h5}]{\substack{h1(-2)+h2 \\ h1+h3}} \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 3 & -5 & -4 & -8 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 6 & -10 & -8 & -16 \\ 0 & -4 & 2 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{h2\leftrightarrow h3} \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 3 & -5 & -4 & -8 \\ 0 & 6 & -10 & -8 & -16 \\ 0 & -4 & 2 & 0 & 1 \end{pmatrix} \xrightarrow[h2(4)+h5]{\substack{h2(-3)+h3 \\ h2(-6)+h4}} \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 0 & -8 & -13 & -29 \\ 0 & 0 & -16 & -26 & -58 \\ 0 & 0 & 6 & 12 & 29 \end{pmatrix}$$

$$\xrightarrow{\substack{h3(-1)+h4 \\ h3+h5}} \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 0 & -8 & -13 & -29 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{pmatrix} \xrightarrow{h5(-4)+h3} \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 0 & 0 & -9 & -29 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{pmatrix}$$

$$\xrightarrow{h5\leftrightarrow h4\leftrightarrow h3}\begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 & 7 \\ 0 & 0 & -2 & -1 & 0 \\ 0 & 0 & 0 & -9 & -29 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A) = 4$$

---

7)

$$A=\begin{pmatrix} -3 & 2 & -7 & 8 \\ -1 & 0 & 5 & -8 \\ 4 & -2 & 2 & 0 \\ 1 & 0 & 3 & 7 \end{pmatrix} \xrightarrow{h1\leftrightarrow h2} \begin{pmatrix} -1 & 0 & 5 & -8 \\ -3 & 2 & -7 & 8 \\ 4 & -2 & 2 & 0 \\ 1 & 0 & 3 & 7 \end{pmatrix} \xrightarrow[h1+h4]{\substack{h1(-3)+h2 \\ h1(4)+h3}} \begin{pmatrix} -1 & 0 & 5 & -8 \\ 0 & 2 & -22 & 32 \\ 0 & -2 & 22 & -32 \\ 0 & 0 & 8 & -1 \end{pmatrix}$$

$$\xrightarrow{h2(-1)+h3} \begin{pmatrix} -1 & 0 & 5 & -8 \\ 0 & 2 & -22 & 32 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & -1 \end{pmatrix} \xrightarrow{h3\leftrightarrow h4} \begin{pmatrix} -1 & 0 & 5 & -8 \\ 0 & 2 & -22 & 32 \\ 0 & 0 & 8 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A) = 3$$

---

8)

$$A=\begin{pmatrix} -1 & 3 & 3 & -4 \\ 4 & -7 & -2 & 1 \\ -3 & 5 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{pmatrix} \xrightarrow[h1(-2)+h4]{\substack{h1(4)+h2 \\ h1(-3)+h3}} \begin{pmatrix} -1 & 3 & 3 & -4 \\ 0 & 5 & 10 & -15 \\ 0 & -4 & -8 & 12 \\ 0 & -3 & -6 & 9 \end{pmatrix} \xrightarrow[h4\left(\frac{1}{3}\right)]{\substack{h2\left(\frac{1}{5}\right) \\ h3\left(\frac{1}{4}\right)}} \begin{pmatrix} -1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -3 \\ 0 & -1 & -2 & 3 \\ 0 & -1 & -2 & 3 \end{pmatrix}$$

$$\xrightarrow[h2+h4]{h2+h3} \begin{pmatrix} -1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A) = 2$$

---

9)

$$A=\begin{pmatrix}1&3&-1&6\\7&1&-3&10\\17&1&-7&22\\3&4&-2&10\end{pmatrix}\xrightarrow[h1(-3)+h4]{\substack{h1(-7)+h2\\h1(-17)+h3}}\begin{pmatrix}1&3&-1&6\\0&-20&4&-32\\0&-50&10&-80\\0&-5&1&-8\end{pmatrix}\xrightarrow[h3\left(\frac{1}{10}\right)]{h2\left(\frac{1}{4}\right)}\begin{pmatrix}1&3&-1&6\\0&-5&1&-8\\0&-5&1&-8\\0&-5&1&-8\end{pmatrix}$$

$$\xrightarrow[h2(-1)h4]{h2(-1)+h3}\begin{pmatrix}1&3&-1&6\\0&-5&1&-8\\0&0&0&0\\0&0&0&0\end{pmatrix}\Rightarrow r(A)=2$$

10)

$$A=\begin{pmatrix}0&1&10&3\\2&0&4&-1\\16&4&52&9\\8&-1&6&-7\end{pmatrix}\xrightarrow{h1\leftrightarrow h2}\begin{pmatrix}2&0&4&-1\\0&1&10&3\\16&4&52&9\\8&-1&6&-7\end{pmatrix}\xrightarrow[h1(-4)+h4]{h1(-8)+h3}\begin{pmatrix}2&0&4&-1\\0&1&10&3\\0&4&20&17\\0&-1&-10&-3\end{pmatrix}$$

$$\xrightarrow[h2+h4]{h2(-4)+h3}\begin{pmatrix}2&0&4&-1\\0&1&10&3\\0&0&-20&5\\0&0&0&0\end{pmatrix}\Rightarrow r(A)=3$$

Bài 2:

Biện luận theo tham số λ hạng của các ma trận:

1) $$A=\begin{pmatrix}3&1&1&4\\\lambda&4&10&1\\1&7&17&3\\2&2&4&1\end{pmatrix}\xrightarrow{\text{h2}\leftrightarrow\text{h4}}\begin{pmatrix}3&1&1&4\\2&2&4&1\\1&7&17&3\\\lambda&4&10&1\end{pmatrix}\xrightarrow{\text{c1}\leftrightarrow c4}\begin{pmatrix}4&1&1&3\\1&2&4&2\\3&7&17&1\\1&4&10&\lambda\end{pmatrix}$$

$$\xrightarrow{h1\leftrightarrow h2}\begin{pmatrix}1&2&4&2\\4&1&1&3\\3&7&17&1\\1&4&10&\lambda\end{pmatrix}\xrightarrow[h1(-1)+h4]{\substack{h1(-4)+h2\\h1(-3)+h3}}\begin{pmatrix}1&2&4&2\\0&-7&-15&-5\\0&1&5&-5\\0&2&6&\lambda-2\end{pmatrix}$$

$$\xrightarrow{h2\leftrightarrow h3}\begin{pmatrix}1&2&4&2\\0&1&5&-5\\0&-7&-15&-5\\0&2&6&\lambda-2\end{pmatrix}\xrightarrow{\substack{h2(7)+h3\\h2(-2)+h4}}\begin{pmatrix}1&2&4&2\\0&1&5&-5\\0&0&20&-40\\0&0&-4&\lambda+8\end{pmatrix}$$

$$\xrightarrow{h3\left(\frac{1}{5}\right)+h4}\begin{pmatrix}1&2&4&2\\0&1&5&-5\\0&0&20&-40\\0&0&0&\lambda\end{pmatrix}$$

Vậy :

- Nếu $\lambda = 0$ thì r(A) = 3
- Nếu $\lambda \neq 0$ thì r(A) = 4

$$2)\ A=\begin{pmatrix}3&1&1&4\\\lambda&4&10&1\\1&7&17&3\\2&2&4&3\end{pmatrix}\xrightarrow{h2\leftrightarrow h4}\begin{pmatrix}3&1&1&4\\2&2&4&3\\1&7&17&3\\\lambda&4&10&1\end{pmatrix}\xrightarrow{c1\leftrightarrow c4}\begin{pmatrix}4&1&1&3\\3&2&4&2\\3&7&17&1\\1&4&10&\lambda\end{pmatrix}$$

$$\xrightarrow{c1\leftrightarrow c2}\begin{pmatrix}1&4&1&3\\2&3&4&2\\7&3&17&1\\4&1&10&\lambda\end{pmatrix}\xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2\\h1(-7)+h3}}\begin{pmatrix}1&4&1&3\\0&-5&2&-4\\0&-25&10&-20\\0&-15&6&\lambda-12\end{pmatrix}$$

$$\xrightarrow{\substack{h2(-5)+h3\\h2(-3)+h4}}\begin{pmatrix}1&4&1&3\\0&-5&2&-4\\0&0&0&0\\0&0&0&\lambda\end{pmatrix}\xrightarrow{h3\leftrightarrow h4}\begin{pmatrix}1&4&1&3\\0&-5&2&-4\\0&0&0&\lambda\\0&0&0&0\end{pmatrix}$$

Vậy:

- Nếu $\lambda = 0$ thì r(A) = 2
- Nếu $\lambda \neq 0$ thì r(A) = 3

$$3)\ A=\begin{pmatrix}4&1&3&3\\0&6&10&2\\1&4&7&2\\6&\lambda&-8&2\end{pmatrix}\xrightarrow{C2\leftrightarrow C4}\begin{pmatrix}4&3&3&1\\0&2&10&6\\1&2&7&4\\6&2&-8&\lambda\end{pmatrix}\xrightarrow{h1\leftrightarrow h3}\begin{pmatrix}1&2&7&4\\0&2&10&6\\4&3&3&1\\6&2&-8&\lambda\end{pmatrix}$$

$$\xrightarrow[h1(-6)+h4]{h1(-4)+h3}\begin{pmatrix}1&2&7&4\\0&2&10&6\\0&-5&-25&-15\\0&-10&-50&\lambda-24\end{pmatrix}\xrightarrow{h2\left(\frac{1}{2}\right)}\begin{pmatrix}1&2&7&4\\0&1&5&3\\0&-5&-25&-15\\0&-10&-50&\lambda-24\end{pmatrix}$$

$$\xrightarrow[h2(10)+h4]{h2(5)+h3}\begin{pmatrix}1&2&7&4\\0&1&5&3\\0&0&0&0\\0&0&0&\lambda+6\end{pmatrix}\xrightarrow{h3\leftrightarrow h4}\begin{pmatrix}1&2&7&4\\0&-1&-5&-3\\0&0&0&\lambda+6\\0&0&0&0\end{pmatrix}$$

Vậy:

- Khi $\lambda+6=0\Leftrightarrow\lambda=-6$ thì r(A) = 2
- Khi $\lambda+6\neq0\Leftrightarrow\lambda\neq-6$ thì r(A) = 3

---

4) $$A=\begin{pmatrix}-3&9&14&1\\0&6&10&2\\1&4&7&2\\3&\lambda&1&2\end{pmatrix}\xrightarrow{C2\leftrightarrow C4}\begin{pmatrix}-3&1&14&9\\0&2&10&6\\1&2&7&4\\3&2&1&\lambda\end{pmatrix}\xrightarrow{h1\leftrightarrow h3}\begin{pmatrix}1&2&7&4\\0&2&10&6\\-3&1&14&9\\3&2&1&\lambda\end{pmatrix}$$

$$\xrightarrow[h1(-3)+h4]{h1(3)+h3}\begin{pmatrix}1&2&7&4\\0&2&10&6\\0&7&35&21\\0&-4&-20&\lambda-12\end{pmatrix}\xrightarrow{h2\left(\frac{1}{2}\right)}\begin{pmatrix}1&2&7&4\\0&1&5&3\\0&7&35&21\\0&-4&-20&\lambda-12\end{pmatrix}$$

$$\xrightarrow[h2(4)+h4]{h2(-7)+h3}\begin{pmatrix}1&2&7&4\\0&1&5&3\\0&0&0&0\\0&0&0&\lambda\end{pmatrix}\xrightarrow{h3\leftrightarrow h4}\begin{pmatrix}1&2&7&4\\0&1&5&3\\0&0&0&\lambda\\0&0&0&0\end{pmatrix}$$

Vậy :

- Nếu λ = 0 thì r(A) = 2
- Nếu $\lambda \neq$ 0 thì r(A) = 3

BÀI TẬP VỀ MA TRẬN NGHỊCH ĐẢO
VÀ PHƯƠNG TRÌNH MA TRẬN

Bài 1:

Tìm ma trận nghịch đảo của các ma trân sau:

1) $A=\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$

Ta có:

$$\left(A\middle|I\right)=\begin{pmatrix} 3 & 4 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{pmatrix}\xrightarrow{\text{h1}\left(-\frac{5}{3}\right)+\text{h2}}\begin{pmatrix} 3 & 4 & 1 & 0 \\ 0 & \frac{1}{3} & -\frac{5}{3} & 1 \end{pmatrix}\xrightarrow[h2(3)]{h1\left(\frac{1}{3}\right)}\begin{pmatrix} 1 & \frac{4}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -5 & 3 \end{pmatrix}$$

$$\xrightarrow{h2\left(-\frac{4}{3}\right)+h1}\begin{pmatrix} 1 & 0 & 7 & -4 \\ 0 & 1 & -5 & 3 \end{pmatrix}\Rightarrow A^{-1}=\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

---

2) $A=\begin{pmatrix} 1 & -2 \\ 4 & -9 \end{pmatrix}$

Ta có:

$$A^{-1}=\begin{pmatrix} 1 & -2 \\ 4 & -9 \end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}=\frac{1}{1.(-9)-(-2).4}\begin{pmatrix} -9 & 2 \\ -4 & 1 \end{pmatrix}=\begin{pmatrix} 9 & -2 \\ 4 & -1 \end{pmatrix}$$

---

3) $A=\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$

Ta có:

$$\left(A\middle|I\right)=\begin{pmatrix} 3 & -4 & 5 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 3 & -5 & -1 & 0 & 0 & 1 \end{pmatrix}\xrightarrow{\text{h2(-1)}+h1}\begin{pmatrix} 1 & -1 & 4 & 1 & -1 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 3 & -5 & -1 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow[\text{h1}(-3)+\text{h3}]{\text{h1}(-2)+\text{h2}}\begin{pmatrix} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 0 & -2 & -13 & -3 & 3 & 1 \end{pmatrix}\xrightarrow{\text{h2(-2)}+h3}\begin{pmatrix} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & -1 & -7 & -2 & 3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{h2(-1)}}\begin{pmatrix} 1 & -1 & 4 & 1 & -1 & 0 \\ 0 & 1 & 7 & 2 & -3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{pmatrix}\xrightarrow[\text{h3}(-4)+\text{h1}]{\text{h3}(-7)+\text{h2}}\begin{pmatrix} 1 & -1 & 0 & -3 & 11 & -4 \\ 0 & 1 & 0 & -5 & 18 & -7 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{h2+h1}}\begin{pmatrix} 1 & 0 & 0 & -8 & 29 & -11 \\ 0 & 1 & 0 & -5 & 18 & -7 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{pmatrix}$$

Vậy ma trận A là ma trận khả nghịch và $A^{-1} = \begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$

---

4) $A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}$

Ta có:

$$\left(A\middle|I\right) = \begin{pmatrix} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{h3 \leftrightarrow h1} \begin{pmatrix} 1 & 5 & 3 & 0 & 0 & 1 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 2 & 7 & 3 & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[h1(-2)+h3]{h1(-3)+h2} \begin{pmatrix} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -6 & -5 & 0 & 1 & -3 \\ 0 & -3 & -3 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{h3 \leftrightarrow h2} \begin{pmatrix} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -3 & 1 & 0 & -2 \\ 0 & -6 & -5 & 0 & 1 & -3 \end{pmatrix}$$

$$\xrightarrow{h2(-2)+h3} \begin{pmatrix} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -3 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{pmatrix} \xrightarrow{h2\left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow[h3(-3)+h1]{h3(-1)+h2} \begin{pmatrix} 1 & 5 & 0 & 6 & -3 & -2 \\ 0 & 1 & 0 & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{pmatrix} \xrightarrow{h2(-5)+h1} \begin{pmatrix} 1 & 0 & 0 & -\frac{7}{3} & 2 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

---

5) $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$

Ta có:

$$A=\left(\begin{array}{ccc|ccc}1 & 2 & 2 & 1 & 0 & 0\\ 2 & 1 & -2 & 0 & 1 & 0\\ 2 & -2 & 1 & 0 & 0 & 1\end{array}\right)\xrightarrow[h1(-2)+h3]{h1(-2)+h2}\begin{pmatrix}1 & 2 & 2 & 1 & 0 & 0\\ 0 & -3 & -6 & -2 & 1 & 0\\ 0 & -6 & -3 & -2 & 0 & 1\end{pmatrix}$$

$$\xrightarrow{h2(-2)+h3}\begin{pmatrix}1 & 2 & 2 & 1 & 0 & 0\\ 0 & -3 & -6 & -2 & 1 & 0\\ 0 & 0 & 9 & 2 & -2 & 1\end{pmatrix}\xrightarrow[h3\left(\frac{1}{9}\right)]{h2\left(-\frac{1}{3}\right)}\begin{pmatrix}1 & 2 & 2 & 1 & 0 & 0\\ 0 & 1 & 2 & \frac{2}{3} & -\frac{1}{3} & 0\\ 0 & 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{9}\end{pmatrix}$$

$$\xrightarrow[h3(-2)+h1]{h3(-2)+h2}\begin{pmatrix}1 & 2 & 0 & \frac{5}{9} & \frac{4}{9} & -\frac{2}{9}\\ 0 & 1 & 0 & \frac{2}{9} & \frac{1}{9} & -\frac{2}{9}\\ 0 & 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{9}\end{pmatrix}\xrightarrow{h2(-2)+h1}\begin{pmatrix}1 & 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{2}{9}\\ 0 & 1 & 0 & \frac{2}{9} & \frac{1}{9} & -\frac{2}{9}\\ 0 & 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{9}\end{pmatrix}$$

$$\Rightarrow A^{-1}=\begin{pmatrix}\frac{1}{9} & \frac{2}{9} & \frac{2}{9}\\ \frac{2}{9} & \frac{1}{9} & -\frac{2}{9}\\ \frac{2}{9} & -\frac{2}{9} & \frac{1}{9}\end{pmatrix}$$

Bài 2

Giải các phương trình ma trận sau

1) $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}X=\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}$

Đặt $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix};B=\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}$

Ta có: $AX=B\Leftrightarrow X=A^{-1}B$

$$A^{-1}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}=\frac{1}{1.4-2.3}\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}-2 & 1\\ \frac{3}{2} & \frac{-1}{2}\end{pmatrix}$$

$$\Rightarrow X=\begin{pmatrix}-2 & 1\\ \frac{3}{2} & \frac{-1}{2}\end{pmatrix}\begin{pmatrix}3 & 5\\ 5 & 9\end{pmatrix}=\begin{pmatrix}-1 & -1\\ 2 & 3\end{pmatrix}$$

2) $X\begin{pmatrix}3 & -2\\ 5 & -4\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -5 & 6\end{pmatrix}$

Đặt $A = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}; B = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$

Ta có: $XA = B \Leftrightarrow X = BA^{-1}$

$$A^{-1} = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{3.(-4) - 5.(-2)}\begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ \frac{5}{2} & -\frac{3}{2} \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} 2 & -1 \\ \frac{5}{2} & -\frac{3}{2} \end{pmatrix}\begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$$

---

3) $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

Giải:

Đặt $A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix}; B = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

Ta có: $AX = B \Leftrightarrow X = A^{-1}B$

Bằng phương pháp tìm ma trận nghịch đảo ta có: $A^{-1} = \begin{pmatrix} -4 & 3 & -2 \\ -8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}$

Suy ra: $X = \begin{pmatrix} -4 & 3 & -2 \\ -8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}\begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{pmatrix}$

---

4) $X\begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$

Đặt $A = \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix}; B = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$

Ta có: $XA = B \Leftrightarrow X = BA^{-1}$

Bằng phương pháp tìm ma trận nghịch đảo ta có:

$$A^{-1}=\begin{pmatrix} \frac{1}{19} & -\frac{1}{19} & -\frac{3}{19} \\ \frac{9}{19} & \frac{10}{19} & \frac{11}{19} \\ -\frac{13}{19} & -\frac{25}{19} & -\frac{18}{19} \end{pmatrix}$$

Suy ra:

$$X = BA^{-1} = A = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{19} & -\frac{1}{19} & -\frac{3}{19} \\ \frac{9}{19} & \frac{10}{19} & \frac{11}{19} \\ -\frac{13}{19} & -\frac{25}{19} & -\frac{18}{19} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

---

5) $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}$

Đặt $A=\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}; B=\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}; C=\begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}$

Ta có: $AXB = C \Leftrightarrow X = A^{-1}CB^{-1}$

$$A^{-1}=\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}^{-1}=\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$$

$$B^{-1}=\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}^{-1}=\begin{pmatrix} -4 & 3 \\ \frac{7}{2} & -\frac{5}{2} \end{pmatrix}$$

Suy ra:

$$X = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}\begin{pmatrix} -4 & 3 \\ \frac{7}{2} & -\frac{5}{2} \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}\begin{pmatrix} -4 & 3 \\ \frac{7}{2} & -\frac{5}{2} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

BÀI TẬP VỀ
HỆ PHƯƠNG TRÌNH TUYẾN TÍNH

Bài 1:

Giải các hệ phương trình sau:

1) $\begin{cases} 7x_1 + 2x_2 + 3x_3 = 15 \\ 5x_1 - 3x_2 + 2x_3 = 15 \\ 10x_1 - 11x_2 + 5x_3 = 36 \end{cases}$

Giải:

Ta có:

$$\left(A|B\right) = \left(\begin{array}{ccc|c} 7 & 2 & 3 & 15 \\ 5 & -3 & 2 & 15 \\ 10 & -11 & 5 & 36 \end{array}\right) \xrightarrow[h2(-2)+h3]{h2(-1)+h1} \left(\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 5 & -3 & 2 & 15 \\ 0 & -5 & 1 & 6 \end{array}\right) \xrightarrow{h1(-2)+h2} \left(\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 1 & -13 & 0 & 15 \\ 0 & -5 & 1 & 6 \end{array}\right)$$

$$\xrightarrow{h1\leftrightarrow h2} \left(\begin{array}{ccc|c} 1 & -13 & 0 & 15 \\ 2 & 5 & 1 & 0 \\ 0 & -5 & 1 & 6 \end{array}\right) \xrightarrow{h1(-2)+h2} \left(\begin{array}{ccc|c} 1 & -13 & 0 & 15 \\ 0 & 31 & 1 & -30 \\ 0 & -5 & 1 & 6 \end{array}\right) \xrightarrow{h3(6)+h2} \left(\begin{array}{ccc|c} 1 & -13 & 0 & 15 \\ 0 & 1 & 7 & 6 \\ 0 & -5 & 1 & 6 \end{array}\right)$$

$$\xrightarrow{h2(5)+h3} \left(\begin{array}{ccc|c} 1 & -13 & 0 & 15 \\ 0 & 1 & 7 & 6 \\ 0 & 0 & 36 & 36 \end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases} x_1 - 13x_2 = 15 \\ x_2 + 7x_3 = 6 \\ 36x_3 = 36 \end{cases} \Leftrightarrow \begin{cases} x_1 = 2 \\ x_2 = -1 \\ x_3 = 1 \end{cases}$$

---

2) $\begin{cases} 2x_1 + x_2 - 2x_3 = 10 \\ 3x_1 + 2x_2 + 2x_3 = 1 \\ 5x_1 + 4x_2 + 3x_3 = 4 \end{cases}$

Giải:

Ta có:

$$\left(A|B\right) = \left(\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 3 & 2 & 2 & 1 \\ 5 & 4 & 3 & 4 \end{array}\right) \xrightarrow{\substack{h1(-1)+h2 \\ h1(-2)+h3}} \left(\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 1 & 1 & 4 & -9 \\ 1 & 2 & 7 & -16 \end{array}\right) \xrightarrow{h1\leftrightarrow h2} \left(\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 2 & 1 & -2 & 10 \\ 1 & 2 & 7 & -16 \end{array}\right)$$

$$\xrightarrow{\substack{h1(-2)+h2 \\ h1(-1)+h2}} \left(\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 0 & -1 & -10 & 28 \\ 0 & 1 & 3 & -7 \end{array}\right) \xrightarrow{h2+h3} \left(\begin{array}{ccc|c} 1 & 1 & 4 & -9 \\ 0 & -1 & -10 & 28 \\ 0 & 0 & -7 & 21 \end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases} x_1 + x_2 + 4x_3 = -9 \\ -x_2 - 10x_3 = 28 \\ -7x_3 = 21 \end{cases} \Leftrightarrow \begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = -3 \end{cases}$$

---

3) $\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ 2x_1 + 5x_2 - 4x_3 = 5 \\ 3x_1 + 4x_2 + 2x_3 = 12 \end{cases}$

Giải:

Ta có:

$$\left(A|B\right) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & -4 & 5 \\ 3 & 4 & 2 & 12 \end{array}\right) \xrightarrow[h1(-3)+h3]{h1(-2)+h2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & -2 & 5 & 3 \end{array}\right) \xrightarrow{h2(2)+h3} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ x_2 - 2x_3 = -1 \\ x_3 = 1 \end{cases} \Leftrightarrow \begin{cases} x_1 = 2 \\ x_2 = 1 \\ x_3 = 1 \end{cases}$$

---

4) $\begin{cases} 2x_1 + x_2 - 3x_3 = 1 \\ 5x_1 + 2x_2 - 6x_3 = 5 \\ 3x_1 - x_2 - 4x_3 = 7 \end{cases}$

Giải:

Ta có:

$$\left(A|B\right) = \left(\begin{array}{ccc|c} 2 & 1 & -3 & 1 \\ 5 & 2 & -6 & 5 \\ 3 & -1 & -4 & 7 \end{array}\right) \xrightarrow[h3(-2)+h2]{h3(-1)+h1} \left(\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ -1 & 4 & 2 & -9 \\ 3 & -1 & -4 & 7 \end{array}\right) \xrightarrow[h1(3)+h3]{h1(-1)+h2} \left(\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ 0 & 2 & 1 & -3 \\ 0 & 5 & -1 & -11 \end{array}\right)$$

$$\xrightarrow{h2(-2)+h3} \left(\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ 0 & 2 & 1 & -3 \\ 0 & 1 & -3 & -5 \end{array}\right) \xrightarrow{h2\leftrightarrow h3} \left(\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ 0 & 1 & -3 & -5 \\ 0 & 2 & 1 & -3 \end{array}\right) \xrightarrow{h2(-2)+h3} \left(\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 7 & 7 \end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases} -x_1 + 2x_2 + x_3 = -6 \\ x_2 - 3x_3 = -5 \\ 7x_3 = 7 \end{cases} \Leftrightarrow \begin{cases} x_1 = 3 \\ x_2 = -2 \\ x_3 = 1 \end{cases}$$

---

5) $\begin{cases} 2x_1 + x_2 - 2x_3 = 8 \\ 3x_1 + 2x_2 - 4x_3 = 15 \\ 5x_1 + 4x_2 - x_3 = 1 \end{cases}$

Giải:
Ta có:

$$\left(A|B\right)=\left(\begin{array}{ccc|c}2 & 1 & -2 & 8\\ 3 & 2 & -4 & 15\\ 5 & 4 & -1 & 1\end{array}\right)\xrightarrow[h2(-2)+h3]{h2(-1)+h1}\left(\begin{array}{ccc|c}-1 & -1 & 2 & -7\\ 3 & 2 & -4 & 15\\ -1 & 0 & 7 & -29\end{array}\right)\xrightarrow[h1(-1)+h3]{h1(3)+h2}\left(\begin{array}{ccc|c}-1 & -1 & 2 & -7\\ 0 & -1 & 2 & -6\\ 0 & 1 & 5 & -22\end{array}\right)$$

$$\xrightarrow{h2+h3}\left(\begin{array}{ccc|c}-1 & -1 & 2 & -7\\ 0 & -1 & 2 & -6\\ 0 & 0 & 7 & -28\end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases}-x_1-x_2+2x_3=-7\\ -x_2+2x_3=-6\\ 7x_3=-28\end{cases}\Leftrightarrow\begin{cases}x_1=1\\ x_2=-2\\ x_3=-4\end{cases}$$

---

6) $\begin{cases}x_1+2x_2-3x_3=1\\ 2x_1+5x_2-8x_3=4\\ 3x_1+8x_2-13x_3=7\end{cases}$

Giải:
Ta có:

$$\left(A|B\right)=\left(\begin{array}{ccc|c}1 & 2 & -3 & 1\\ 2 & 5 & -8 & 4\\ 3 & 8 & -13 & 7\end{array}\right)\xrightarrow[h1(-3)+h3]{h1(-2)+h2}\left(\begin{array}{ccc|c}1 & 2 & -3 & 1\\ 0 & 1 & -2 & 2\\ 0 & 2 & -4 & 4\end{array}\right)\xrightarrow{h2(-2)+h3}\left(\begin{array}{ccc|c}1 & 2 & -3 & 1\\ 0 & 1 & -2 & 2\\ 0 & 0 & 0 & 0\end{array}\right)$$

Hệ phương trình đã cho tương đương với hệ phương trình:

$$\begin{cases}x_1+2x_2-3x_3=1\\ x_2-2x_3=2\end{cases}\Leftrightarrow\begin{cases}x_1=-3-x_3\\ x_2=2+2x_3\\ x_3\text{ tuỳ ý}\end{cases}\Leftrightarrow\begin{cases}x_1=-3-t\\ x_2=2+2t\left(t\in R\right)\\ x_3=t\end{cases}$$

Bài 2:
Giải các hệ phương trình sau:

1) $\begin{cases}2x_1+2x_2-x_3+x_4=4\\ 4x_1+3x_2-x_3+2x_4=6\\ 8x_1+5x_2-3x_3+4x_4=12\\ 3x_1+3x_2-2x_3+2x_4=6\end{cases}$

Giải:
Ta có:

$$\left(A|B\right)=\begin{pmatrix}2&2&-1&1&4\\4&3&-1&2&6\\8&5&-3&4&12\\3&3&-2&2&6\end{pmatrix}\xrightarrow[h1\left(-\frac{3}{2}\right)+h4]{\substack{h1(-2)+h2\\h1(-4)+h3}}\begin{pmatrix}2&2&-1&1&4\\0&-1&1&0&-2\\0&-3&1&0&-4\\0&0&-1/2&1/2&0\end{pmatrix}$$

$$\xrightarrow{h2(-3)+h3}\begin{pmatrix}2&2&-1&1&4\\0&-1&1&0&-2\\0&0&-2&0&2\\0&0&-1/2&1/2&0\end{pmatrix}\xrightarrow{h3(-1/4)+h4}\begin{pmatrix}2&2&-1&1&4\\0&-1&1&0&-2\\0&0&-2&0&2\\0&0&0&1/2&-1/2\end{pmatrix}$$

Khi đó (1) $\Leftrightarrow \begin{cases}2x_1+2x_2-x_3+x_4=4 & (1)\\ -x_2+x_3=-2 & (2)\\ -2x_3=-2 & (3)\\ \frac{1}{2}x_4=-\frac{1}{2} & (4)\end{cases}$

Từ (4) $\Rightarrow x_4=-1$

Thế $x_4=-1$ vào (3) $\Rightarrow x_3=-1$

Thế x_3 vào (2) ta được: $x_2=1$

Thế x_3, x_2, x_4 vào (1) ta được: $x_1=1$

Vậy nghiệm của phương trình đã cho là: $\begin{cases}x_1=1\\x_2=1\\x_3=-1\\x_4=-1\end{cases}$ hay (1, 1, -1, -1)

---

2) $\begin{cases}2x_1+3x_2+11x_3+5x_4=2\\x_1+x_2+5x_3+2x_4=1\\2x_1+x_2+3x_3+2x_4=-3\\x_1+x_2+3x_3+4x_4=-3\end{cases}$

Giải:

Ta có:

$$\left(A/B\right)=\left(\begin{array}{cccc|c}2&3&11&5&2\\1&1&5&2&1\\2&1&3&2&-3\\1&1&3&4&-3\end{array}\right)\xrightarrow{h1\leftrightarrow h2}\left(\begin{array}{cccc|c}1&1&5&2&1\\2&3&11&5&2\\2&1&3&2&-3\\1&1&3&4&-3\end{array}\right)$$

$$\xrightarrow[h1(-1)+h4]{\substack{h1(-2)+h2\\ h1(-2)+h3}} \left(\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -7 & -2 & -5 \\ 0 & 0 & -2 & 2 & -4 \end{array}\right) \xrightarrow{h2+h3} \left(\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -6 & -1 & -5 \\ 0 & 0 & -2 & 2 & -4 \end{array}\right) \xrightarrow{h3\leftrightarrow h4} \left(\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 2 & -4 \\ 0 & 0 & -6 & -1 & -5 \end{array}\right)$$

$$\xrightarrow{h3(-3)+h4} \left(\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 2 & -4 \\ 0 & 0 & 0 & -7 & 7 \end{array}\right)$$

Suy ra: (2) $\Leftrightarrow \begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1 & (1) \\ \quad x_2 + x_3 + x_4 = 0 & (2) \\ \quad\quad -2x_3 + 2x_4 = -4 & (3) \\ \quad\quad\quad -7x_4 = 7 & (4) \end{cases}$

Từ (4) $\Rightarrow x_4 = -1$

Thế $x_4 = -1$ vào (3) $\Rightarrow x_3 = 1$

Thế x_3, x_4 vào (2) ta được: $x_2 = 0$

Thế x_3, x_2, x_4 vào (1) ta được: $x_1 = -2$

Vậy nghiệm của phương trình đã cho là: $\begin{cases} x_1 = -2 \\ x_2 = 0 \\ x_3 = 1 \\ x_4 = -1 \end{cases}$ hay (-2, 0, 1, -1)

---

3) $\begin{cases} 2x_1 + 7x_2 + 3x_3 + x_4 = 6 \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4 \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2 \end{cases}$

$$\left(A/B\right) = \begin{pmatrix} 2 & 7 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{pmatrix} \xrightarrow{h2(-1)+h1} \begin{pmatrix} -1 & 2 & 1 & -1 & 2 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{pmatrix}$$

$$\xrightarrow[h1(3)+h3]{h1(3)+h2} \begin{pmatrix} -1 & 2 & 1 & -1 & 2 \\ 0 & 11 & 5 & -1 & 10 \\ 0 & 22 & 10 & -2 & 20 \end{pmatrix} \xrightarrow{h2(-2)+h3} \begin{pmatrix} -1 & 2 & 1 & -1 & 2 \\ 0 & 11 & 5 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Phöông trình ñaõ cho töông ñöông vôùi phöông trình:

$$\begin{cases} -x_1 + 2x_2 + x_3 - x_4 \ = 2 & (1) \\ \quad\ \ 11x_2 + 5x_3 - \ x_4 \ \ = 10 & (2) \end{cases}$$

$(2): x_4 = 11x_2 + 5x_3 - 10$

$(1) \Leftrightarrow -x_1 + 2x_2 + x_3 - \left(11x_2 + 5x_3 - 10\right) = 2 \Leftrightarrow x_1 = -9x_2 - 4x_3 + 8$

Vậy nghiệm của hệ phương trình đã cho là:

$$\begin{cases} x_1 = -9x_2 - 4x_3 + 8 \\ x_2 \text{ tuøy yù} \\ x_2 \text{ tuøy yù} \\ x_4 = 11x_2 + 5x_3 - 10 \end{cases} \quad \text{hay} \quad \begin{cases} x_1 = \ -9t - 4s + 8 \\ x_2 = t \\ x_3 = s \\ x_4 = 11t + 5s - 10 \end{cases} \qquad \left(\forall t, s \in R\right)$$

---

4) $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 7x_1 - 4x_2 + x_3 + 3x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$

Ta có:

$$\left(A/B\right) = \begin{pmatrix} 3 & -5 & 2 & 4 & 2 \\ 7 & -4 & 1 & 3 & 5 \\ 5 & 7 & -4 & -6 & 3 \end{pmatrix} \xrightarrow{\text{h1(-2)+h2}} \begin{pmatrix} 3 & -5 & 2 & 4 & 2 \\ 1 & 6 & -3 & -5 & 1 \\ 5 & 7 & -4 & -6 & 3 \end{pmatrix}$$

$$\xrightarrow{h1 \leftrightarrow h2} \begin{pmatrix} 1 & 6 & -3 & -5 & 1 \\ 3 & -5 & 2 & 4 & 2 \\ 5 & 7 & -4 & -6 & 3 \end{pmatrix} \xrightarrow[h1(-5)+h3]{h1(-3)+h2} \begin{pmatrix} 1 & 6 & -3 & -5 & 1 \\ 0 & -23 & 11 & 19 & -1 \\ 0 & -23 & 11 & 19 & -2 \end{pmatrix}$$

$$\xrightarrow{h2(-1)+h3} \begin{pmatrix} 1 & 6 & -3 & -5 & 1 \\ 0 & -23 & 11 & 19 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Suy ra: (4) $\Leftrightarrow \begin{cases} x_1 + 6x_2 - 3x_3 - 5x_4 \ = 0 \\ -23x_2 + 11x_3 + 19\ x_4 \ = -1 \\ \qquad\qquad\qquad\qquad 0 = -1 \end{cases} \quad \Rightarrow$ hệ vô nghiệm

---

5) $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1 \\ 2x_1 - x_2 \qquad - 3x_4 = 2 \\ 3x_1 \qquad - x_3 + x_4 = -3 \\ 3x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$

$$\left(A|B\right)\left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 1 \\ 2 & -1 & 0 & -3 & 2 \\ 3 & 0 & -1 & 1 & -3 \\ 3 & 2 & -2 & 5 & -6 \end{array}\right) \xrightarrow[h2(-1)+h1]{\substack{h2(-1)+h3 \\ h2(-1)+h4}} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 2 & -1 \\ 2 & -1 & 0 & -3 & 2 \\ 1 & 1 & -1 & 4 & -5 \\ 0 & 3 & -2 & 8 & -8 \end{array}\right)$$

$$\xrightarrow{h1\leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 4 & -5 \\ 2 & -1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 3 & -2 & 8 & -8 \end{array}\right) \xrightarrow{h1(-2)+h2} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 4 & -5 \\ 0 & -3 & 2 & -11 & 12 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 3 & -2 & 8 & -8 \end{array}\right)$$

$$\xrightarrow{h2+h4} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 4 & -5 \\ 0 & -3 & 2 & -11 & 12 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & -3 & 4 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1 + x_2 - x_3 + 4x_4 = -5 \\ \quad -3x_2 + 2x_3 - 11x_4 = 12 \\ \qquad x_3 + 2x_4 = -1 \\ \qquad\quad -3x_4 = 4 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 2 \\ x_3 = \dfrac{5}{3} \\ x_4 = -\dfrac{4}{3} \end{cases} \quad hay \quad \left(0, 2, \frac{5}{3}, -\frac{4}{3}\right)$$

---

6) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 11 \\ 2x_1 + 3x_2 + 4x_3 + x_4 = 12 \\ 3x_1 + 4x_2 + x_3 + 2x_4 = 13 \\ 4x_1 + x_2 + 2x_3 + 3x_4 = 14 \end{cases}$

Giaûi

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 11 \\ 2 & 3 & 4 & 1 & 12 \\ 3 & 4 & 1 & 2 & 13 \\ 4 & 1 & 2 & 3 & 14 \end{array}\right) \xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2 \\ h1(-3)+h3}} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 11 \\ 0 & -1 & -2 & -7 & -10 \\ 0 & -2 & -8 & -10 & -20 \\ 0 & -7 & -10 & -13 & -30 \end{array}\right)$$

$$\xrightarrow[h2(-7)+h4]{h2(-2)+h3} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 11 \\ 0 & -1 & -2 & -7 & -10 \\ 0 & 0 & -4 & 4 & 0 \\ 0 & 0 & 4 & 36 & 40 \end{array}\right) \xrightarrow{h3+h4} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 11 \\ 0 & -1 & -2 & -7 & -10 \\ 0 & 0 & -4 & 4 & 0 \\ 0 & 0 & 0 & 40 & 40 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1+2x_2+3x_3+4x_4=11 \\ \quad -x_2-2x_3-7x_4=-10 \\ \qquad -4x_3+4x_4=0 \\ \qquad\qquad 40x_4=40 \end{cases} \Leftrightarrow \begin{cases} x_1=2 \\ x_2=1 \\ x_3=1 \\ x_4=1 \end{cases} hay \quad (2,1,1,1)$$

---

7) $$\begin{cases} x_1-2x_2+3x_3-4x_4=4 \\ \quad x_2-x_3+x_4=-3 \\ x_1+3x_2 \quad -3x_4=1 \\ \quad -7x_2+3x_3+x_4=-3 \end{cases}$$

Giaûi

$$(A|B)=\left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 1 & 3 & 0 & -3 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{array}\right) \xrightarrow{h1(-1)+h3} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 0 & 5 & -3 & 1 & -3 \\ 0 & -7 & 3 & 1 & -3 \end{array}\right)$$

$$\xrightarrow[h2(7)+h4]{h2(-5)+h3} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 0 & 0 & 2 & -4 & 12 \\ 0 & 0 & -4 & 8 & -24 \end{array}\right) \xrightarrow{h3(2)+h4} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 0 & 0 & 2 & -4 & 12 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1-2x_2+3x_3-4x_4=4 \\ \qquad x_2-x_3+x_4=-3 \\ \qquad 2x_3-4x_4=12 \end{cases} \Leftrightarrow \begin{cases} x_1=-8 \\ x_2=x_4+3 \\ x_3=2x_4+6 \\ x_4 \text{ tuøy yù} \end{cases} \Leftrightarrow \begin{cases} x_1=-8 \\ x_2=t+3 \\ x_3=2t+6 \\ x_4=t \end{cases} \quad (t\in R)$$

---

8) $$\begin{cases} 3x_1+4x_2+x_3+2x_4=3 \\ 6x_1+8x_2+2x_3+5x_4=7 \\ 9x_1+12x_2+3x_3+10x_4=13 \end{cases}$$

Giaûi

$$(A|B)=\left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 6 & 8 & 2 & 5 & 7 \\ 9 & 12 & 3 & 10 & 13 \end{array}\right) \xrightarrow[h1(-3)+h3]{h1(-2)+h2} \left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 4 \end{array}\right) \xrightarrow{h2(-4)+h3} \left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} 3x_1+4x_2+x_3+2x_4=3 \\ \qquad\qquad\qquad\quad x_4=1 \end{cases} \Leftrightarrow \begin{cases} x_3=1-3x_1-4x_2 \\ x_4=1 \\ x_1, \text{x}_2 \ \text{tuøy yù} \end{cases} \Leftrightarrow \begin{cases} x_1=1-3t-4s \\ x_2=t \\ x_3=s \\ x_4=1 \end{cases} \qquad (t,s\in R)$$

---

9) $\begin{cases} 9x_1-3x_2+5x_3+6x_4=4 \\ 6x_1-2x_2+3x_3+4x_4=5 \\ 3x_1-x_2+3x_3+14x_4=-8 \end{cases}$

Giaûi

$$\left(A|B\right)=\left(\begin{array}{cccc|c} 9 & -3 & 5 & 6 & 4 \\ 6 & -2 & 3 & 4 & 5 \\ 3 & -1 & 3 & 14 & -8 \end{array}\right) \xrightarrow{h3\leftrightarrow h1} \left(\begin{array}{cccc|c} 3 & -1 & 3 & 14 & -8 \\ 6 & -2 & 3 & 4 & 5 \\ 9 & -3 & 5 & 6 & 4 \end{array}\right) \xrightarrow[h1(-3)+h3]{h1(-2)+h2} \left(\begin{array}{cccc|c} 3 & -1 & 3 & 14 & -8 \\ 0 & 0 & -3 & -24 & 21 \\ 0 & 0 & -4 & -36 & 28 \end{array}\right)$$

$$\xrightarrow[h3\left(\frac{1}{4}\right)]{h2\left(-\frac{1}{3}\right)} \left(\begin{array}{cccc|c} 3 & -1 & 3 & 14 & -8 \\ 0 & 0 & 1 & 8 & -7 \\ 0 & 0 & -1 & -9 & 7 \end{array}\right) \xrightarrow{h3+h4} \left(\begin{array}{cccc|c} 3 & -1 & 3 & 14 & -8 \\ 0 & 0 & 1 & 8 & -7 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} 3x_1-x_2+3x_3+14x_4=-8 \\ \qquad\qquad\quad x_3+8x_4=-7 \\ \qquad\qquad\qquad\quad x_4=0 \end{cases} \Leftrightarrow \begin{cases} x_1=\frac{1}{3}x_2+\frac{13}{3} \\ \text{x}_2 \ \text{tuøy yù} \\ x_3=-7 \\ x_4=0 \end{cases} \Leftrightarrow \begin{cases} x_1=\frac{1}{3}t+\frac{13}{3} \\ x_2=t \\ x_3=-7 \\ x_4=0 \end{cases} \qquad (t\in R)$$

---

10) $\begin{cases} 3x_1-2x_2-5x_3+x_4=3 \\ 2x_1-3x_2+x_3+5x_4=-3 \\ x_1+2x_2 \qquad -4x_4=-3 \\ x_1-x_2-4x_3+9x_4=22 \end{cases}$

Giaûi

$$\left(A|B\right)=\left(\begin{array}{cccc|c} 3 & -2 & -5 & 1 & 3 \\ 2 & -3 & 1 & 5 & -3 \\ 1 & 2 & 0 & -4 & -3 \\ 1 & -1 & -4 & 9 & 22 \end{array}\right) \xrightarrow{h1\leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 2 & 0 & -4 & -3 \\ 2 & -3 & 1 & 5 & -3 \\ 3 & -2 & -5 & 1 & 3 \\ 1 & -1 & -4 & 9 & 22 \end{array}\right)$$

$$\xrightarrow[h1(-1)+h4]{\substack{h1(-2)+h2 \\ h1(-3)+h3}} \left(\begin{array}{cccc|c} 1 & 2 & 0 & -4 & -3 \\ 0 & -7 & 1 & 13 & 3 \\ 0 & -8 & -5 & 13 & 12 \\ 0 & -3 & -4 & 13 & 25 \end{array}\right) \xrightarrow[h3(-1)+h4]{h3(-1)+h2} \left(\begin{array}{cccc|c} 1 & 2 & 0 & -4 & -3 \\ 0 & 1 & 6 & 0 & -9 \\ 0 & -8 & -5 & 13 & 12 \\ 0 & 5 & 1 & 0 & 13 \end{array}\right)$$

$$\xrightarrow[h2(-5)+h4]{h2(8)+h3}\left(\begin{array}{cccc|c}1 & 2 & 0 & -4 & -3\\ 0 & 1 & 6 & 0 & -9\\ 0 & 0 & 43 & 13 & -60\\ 0 & 0 & -29 & 0 & 58\end{array}\right)\xrightarrow{h4\left(\frac{1}{29}\right)\leftrightarrow h3}\left(\begin{array}{cccc|c}1 & 2 & 0 & -4 & -3\\ 0 & 1 & 6 & 0 & -9\\ 0 & 0 & -1 & 0 & 2\\ 0 & 0 & 43 & 13 & -60\end{array}\right)$$

$$\xrightarrow{h3(43)+h4}\left(\begin{array}{cccc|c}1 & 2 & 0 & -4 & -3\\ 0 & 1 & 6 & 0 & -9\\ 0 & 0 & -1 & 0 & 2\\ 0 & 0 & 0 & 13 & 26\end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases}x_1-2x_2-4x_4=-3\\ x_2+6x_3=-9\\ -x_3=2\\ 13x_4=26\end{cases}\Leftrightarrow\begin{cases}x_1=1\\ x_2=3\\ x_3=-2\\ x_4=2\end{cases}$$

---

11) $$\begin{cases}x_1+x_2-6x_3-4x_4=6\\ 3x_1-x_2-6x_3-4x_4=2\\ 2x_1+3x_2+9x_3+2x_4=6\\ 3x_1+2x_2+3x_3+mx_4=-7\end{cases}$$

Giaûi

$$\left(A|B\right)=\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 3 & -1 & -6 & -4 & 2\\ 2 & 3 & 9 & 2 & 6\\ 3 & 2 & 3 & 8 & -7\end{array}\right)\xrightarrow[h1(-3)+h4]{\substack{h1(-3)+h2\\ h1(-2)+h3}}\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 0 & -4 & 12 & 8 & -16\\ 0 & 1 & 21 & 10 & -6\\ 0 & -1 & 21 & 20 & -25\end{array}\right)$$

$$\xrightarrow{h2\left(\frac{1}{4}\right)}\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 0 & -1 & 3 & 2 & -4\\ 0 & 1 & 21 & 10 & -6\\ 0 & -1 & 21 & 20 & -25\end{array}\right)\xrightarrow[h2(-1)+h4]{h2+h3}\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 0 & -1 & 3 & 2 & -4\\ 0 & 0 & 24 & 12 & -10\\ 0 & 0 & 18 & 18 & -21\end{array}\right)$$

$$\xrightarrow{h4\left(\frac{1}{3}\right)\leftrightarrow h3\left(\frac{1}{2}\right)}\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 0 & -1 & 3 & 2 & -4\\ 0 & 0 & 6 & 6 & -7\\ 0 & 0 & 12 & 6 & -5\end{array}\right)\xrightarrow{h3(-2)+h4}\left(\begin{array}{cccc|c}1 & 1 & -6 & -4 & 6\\ 0 & -1 & 3 & 2 & -4\\ 0 & 0 & 6 & 6 & -7\\ 0 & 0 & 0 & -6 & 9\end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = -3 \\ x_2 + 3x_3 + 2x_4 = -4 \\ 6x_3 + 6x_4 = -7 \\ -6x_4 = 9 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 2 \\ x_3 = \dfrac{1}{3} \\ x_4 = -\dfrac{3}{2} \end{cases}$$

---

12) $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1 \\ 2x_1 - x_2 - 3x_4 = 2 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$

Giaûi

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 1 \\ 2 & -1 & 0 & -3 & 2 \\ 3 & 0 & -1 & 1 & -3 \\ 2 & 2 & -2 & 5 & -6 \end{array}\right) \xrightarrow[h1(-1)+h4]{\substack{h1(-1)+h2 \\ h1(-1)+h3}} \left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 1 \\ 1 & 1 & -2 & 2 & -4 \\ 0 & 3 & -3 & 6 & -7 \end{array}\right)$$

$$\xrightarrow{h1 \leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 2 & -4 \\ 0 & 0 & -1 & -2 & 1 \\ 2 & -1 & 1 & -1 & 1 \\ 0 & 3 & -3 & 6 & -7 \end{array}\right) \xrightarrow{h_1(-2)+h2} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 2 & -4 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & -3 & 5 & -5 & 9 \\ 0 & 3 & -3 & 6 & -7 \end{array}\right)$$

$$\xrightarrow{h3+h4} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 2 & -4 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & -3 & 5 & -5 & 9 \\ 0 & 0 & 2 & 1 & 2 \end{array}\right) \xrightarrow{h2 \leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 2 & -4 \\ 0 & -3 & 5 & -5 & 9 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 2 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{h3(2)+h4} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 2 & -4 \\ 0 & -3 & 5 & -5 & 9 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & -3 & 4 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1 + x_2 - 2x_3 + 2x_4 = -4 \\ -3x_2 + 5x_3 - 5x_4 = 9 \\ -x_3 - 2x_4 = 1 \\ -3x_4 = 4 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 2 \\ x_3 = \dfrac{5}{3} \\ x_4 = -\dfrac{4}{3} \end{cases}$$

---

13) $\begin{cases} 3x_1+5x_2-3x_3+2x_4=12 \\ 4x_1-2x_2+5x_3+3x_4=27 \\ 7x_1+8x_2-x_3+5x_4=40 \\ 6x_1+4x_2+5x_3+3x_4=41 \end{cases}$

Giaûi

$$(A|B)=\left(\begin{array}{cccc|c} 3 & 5 & -3 & 2 & 12 \\ 4 & -2 & 5 & 3 & 27 \\ 7 & 8 & -1 & 5 & 40 \\ 6 & 4 & 5 & 3 & 41 \end{array}\right) \xrightarrow[h1(-2)+h4]{\substack{h1(-1)+h2 \\ h1(-2)+h3}} \left(\begin{array}{cccc|c} 3 & 5 & -3 & 2 & 12 \\ 1 & -7 & 8 & 1 & 15 \\ 1 & -2 & 5 & 1 & 16 \\ 0 & -6 & 11 & -1 & 17 \end{array}\right)$$

$$\xrightarrow{h1\leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 1 & -7 & 8 & 1 & 15 \\ 3 & 5 & -3 & 2 & 12 \\ 0 & -6 & 11 & -1 & 17 \end{array}\right) \xrightarrow[h1(-3)+h3]{h1(-1)+h2} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -5 & 3 & 0 & -1 \\ 0 & 11 & -18 & -1 & -36 \\ 0 & -6 & 11 & -1 & 17 \end{array}\right)$$

$$\xrightarrow[h2(-1)+h4]{h2(2)+h3} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -5 & 3 & 0 & -1 \\ 0 & 1 & -12 & -1 & -38 \\ 0 & -1 & 8 & -1 & 18 \end{array}\right) \xrightarrow{h2\leftrightarrow h4} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 1 & -12 & -1 & -38 \\ 0 & -5 & 3 & 0 & -1 \end{array}\right)$$

$$\xrightarrow[h2(-5)+h4]{h2+h3} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 0 & -4 & -2 & -20 \\ 0 & 0 & -37 & 5 & -91 \end{array}\right) \xrightarrow{h3\left(-\frac{1}{2}\right)} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 0 & 2 & 1 & 10 \\ 0 & 0 & -37 & 5 & -91 \end{array}\right)$$

$$\xrightarrow{h3(18)+h4} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 0 & 2 & 1 & 10 \\ 0 & 0 & -1 & 23 & 89 \end{array}\right) \xrightarrow{h3\leftrightarrow h4} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 0 & -1 & 23 & 89 \\ 0 & 0 & 2 & 1 & 10 \end{array}\right)$$

$$\xrightarrow{h3(2)+h4} \left(\begin{array}{cccc|c} 1 & -2 & 5 & 1 & 16 \\ 0 & -1 & 8 & -1 & 18 \\ 0 & 0 & -1 & 23 & 89 \\ 0 & 0 & 0 & 47 & 188 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1-2x_2+5x_3+x_4=16 \\ -x_2+8x_3-x_4=18 \\ -x_3+23x_4=89 \\ 47x_4=188 \end{cases} \Leftrightarrow \begin{cases} x_1=1 \\ x_2=2 \\ x_3=3 \\ x_4=4 \end{cases}$$

14) $\begin{cases} 4x_1 + 4x_2 + 5x_3 + 5x_4 = 0 \\ 2x_1 \qquad\quad + 3x_3 - x_4 = 10 \\ x_1 + x_2 - 5x_3 \qquad\quad = -10 \\ \qquad 3x_2 + 2x_3 \qquad\quad = 1 \end{cases}$

Giaûi

Ta coù:

$$(A|B) = \left(\begin{array}{cccc|c} 4 & 4 & 5 & 5 & 0 \\ 2 & 0 & 3 & -1 & 10 \\ 1 & 1 & -5 & 0 & -10 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right) \xrightarrow{h1 \leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 2 & 0 & 3 & -1 & 10 \\ 4 & 4 & 5 & 5 & 0 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[h1(-4)+h3]{h1(-2)+h2} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & -2 & 13 & -1 & 30 \\ 0 & 0 & 25 & 5 & 40 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right) \xrightarrow{h4+h2} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 25 & 5 & 40 \\ 0 & 3 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{h2(-3)+h4} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 25 & 5 & 40 \\ 0 & 0 & -43 & 3 & -92 \end{array}\right) \xrightarrow{h3\left(\frac{1}{5}\right)} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 5 & 1 & 8 \\ 0 & 0 & -43 & 3 & -92 \end{array}\right)$$

$$\xrightarrow{h3(9)+h4} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 5 & 1 & 8 \\ 0 & 0 & 2 & 12 & -20 \end{array}\right) \xrightarrow{h4\left(\frac{1}{2}\right) \leftrightarrow h3} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 1 & 6 & -10 \\ 0 & 0 & 5 & 1 & 8 \end{array}\right)$$

$$\xrightarrow{h3(-5)+h4} \left(\begin{array}{cccc|c} 1 & 1 & -5 & 0 & -10 \\ 0 & 1 & 15 & -1 & 31 \\ 0 & 0 & 1 & 6 & -10 \\ 0 & 0 & 0 & -29 & 58 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1 + x_2 - 5x_3 = -10 \\ x_2 + 15x_3 - x_4 = 31 \\ x_3 + 6x_4 = -10 \\ -29x_4 = 58 \end{cases} \Leftrightarrow \begin{cases} x_1 = 1 \\ x_2 = -1 \\ x_3 = 2 \\ x_4 = -2 \end{cases}$$

---

15) $\begin{cases} 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 + 3x_2 + 3x_3 + 2x_4 = 6 \\ 3x_1 - x_2 - x_3 - 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$

Giải:

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 2 & -1 & 3 & 2 & 4 \\ 3 & 3 & 3 & 2 & 6 \\ 3 & -1 & -1 & -2 & 6 \\ 3 & -1 & 3 & -1 & 6 \end{array}\right) \xrightarrow{\substack{h2(-1)+h1 \\ h2(-1)+h3 \\ h2(-1)+h4}} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 3 & 3 & 3 & 2 & 6 \\ 0 & -4 & -4 & -4 & 0 \\ 0 & -4 & 0 & -3 & 0 \end{array}\right)$$

$$\xrightarrow[h3(-1)+h4]{h1(3)+h2} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 0 & -9 & 3 & 2 & 0 \\ 0 & -4 & -4 & -4 & 0 \\ 0 & 0 & 4 & 1 & 0 \end{array}\right) \xrightarrow{h3\left(-\frac{1}{4}\right)\leftrightarrow h2} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -9 & 3 & 2 & 0 \\ 0 & 0 & 4 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{h2(9)+h3} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 12 & 11 & 0 \\ 0 & 0 & 4 & 1 & 0 \end{array}\right) \xrightarrow{h4\leftrightarrow h3} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 12 & 11 & 0 \end{array}\right)$$

$$\xrightarrow{h3(-3)+h4} \left(\begin{array}{cccc|c} -1 & -4 & 0 & 0 & -2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 8 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} -x_1 - 4x_2 = -2 \\ x_2 + x_3 + x_4 = 0 \\ 4x_3 + x_4 = 0 \\ 8x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 2 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{cases}$$

---

16) $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$

Giải:

$$\left(A|B\right)=\left(\begin{array}{cccc|c}1&1&2&3&1\\3&-1&-1&-2&-4\\2&3&-1&-1&-6\\1&2&3&-1&-4\end{array}\right)\xrightarrow[h1(-1)+h4]{\substack{h1(-3)+h2\\h1(-2)+h3}}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&-4&-7&-11&-7\\0&1&-5&-7&-8\\0&1&1&-4&-5\end{array}\right)$$

$$\xrightarrow{h2\leftrightarrow h3}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&1&-5&-7&-8\\0&-4&-7&-11&-7\\0&1&1&-4&-5\end{array}\right)\xrightarrow[h2(-1)+h3]{h2(4)+h3}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&1&-5&-7&-8\\0&0&-27&-39&-39\\0&0&6&3&3\end{array}\right)$$

$$\xrightarrow[h4\left(\frac{1}{3}\right)]{h3\left(-\frac{1}{3}\right)}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&1&-5&-7&-8\\0&0&9&13&13\\0&0&2&1&1\end{array}\right)\xrightarrow{h4(-5)+h3}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&1&-5&-7&-8\\0&0&-1&8&8\\0&0&2&1&1\end{array}\right)$$

$$\xrightarrow{h3(2)+h4}\left(\begin{array}{cccc|c}1&1&2&3&1\\0&1&-5&-7&-8\\0&0&-1&8&8\\0&0&0&17&17\end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases}x_1+x_2+2x_3+3x_4=-2\\x_2-5x_3-7x_4=-8\\-x_3+8x_4=8\\17x_4=17\end{cases}\Leftrightarrow\begin{cases}x_1=-1\\x_2=-1\\x_3=0\\x_4=1\end{cases}$$

---

17) $$\begin{cases}x_1+2x_2+3x_3+4x_4=5\\2x_1+x_2+2x_3+3x_4=1\\3x_1+2x_2+x_3+2x_4=1\\4x_1+3x_2+2x_3+x_4=-5\end{cases}$$

Giải:

$$\left(A|B\right)=\left(\begin{array}{cccc|c}1&2&3&4&5\\2&1&2&3&1\\3&2&1&2&1\\4&3&2&1&-5\end{array}\right)\xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2\\h1(-3)+h3}}\left(\begin{array}{cccc|c}1&2&3&4&5\\0&-3&-4&-5&-9\\0&-4&-8&-10&-14\\0&-5&-10&-15&-25\end{array}\right)$$

$$\xrightarrow[h3(-1)+h3]{h3(-1)+h2}\left(\begin{array}{cccc|c}1&2&3&4&5\\0&1&4&5&5\\0&-4&-8&-10&-14\\0&-1&-2&-5&-11\end{array}\right)\xrightarrow[h2+h4]{h2(4)+h3}\left(\begin{array}{cccc|c}1&2&3&4&5\\0&1&4&5&5\\0&0&8&10&6\\0&0&2&0&-6\end{array}\right)$$

$$\xrightarrow{h3\leftrightarrow h4}\left(\begin{array}{cccc|c}1&2&3&4&5\\0&1&4&5&5\\0&0&2&0&-6\\0&0&8&10&6\end{array}\right)\xrightarrow{h3(-4)+h4}\left(\begin{array}{cccc|c}1&2&3&4&5\\0&1&4&5&5\\0&0&2&0&-6\\0&0&0&10&30\end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases}x_1+2x_2+3x_3+4x_4=5\\x_2+4x_3+5x_4=5\\2x_3=-6\\10x_4=30\end{cases}\Leftrightarrow\begin{cases}x_1=-2\\x_2=2\\x_3=-3\\x_4=3\end{cases}$$

---

18) $\begin{cases}x_1+x_2+x_3+x_4=2\\x_1+2x_2+3x_3+4x_4=2\\2x_1+3x_2+5x_3+9x_4=2\\x_1+x_2+2x_3+7x_4=2\end{cases}$

Giải:

$$\left(A|B\right)=\left(\begin{array}{cccc|c}1&1&1&1&2\\1&2&3&4&2\\2&3&5&9&2\\1&1&2&7&2\end{array}\right)\xrightarrow[h1(-1)+h4]{\substack{h1(-1)+h2\\h1(-2)+h3}}\left(\begin{array}{cccc|c}1&1&1&1&2\\0&1&2&3&0\\0&1&3&7&-2\\0&0&1&6&0\end{array}\right)\xrightarrow{h2(-1)+h3}\left(\begin{array}{cccc|c}1&1&1&1&2\\0&1&2&3&0\\0&0&1&4&-2\\0&0&1&6&0\end{array}\right)$$

$$\xrightarrow{h3(-1)+h4}\left(\begin{array}{cccc|c}1&1&1&1&2\\0&1&2&3&0\\0&0&1&4&-2\\0&0&0&2&2\end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases}x_1+x_2+x_3+x_4=2\\x_2+2x_3+3x_4=0\\x_3+4x_4=-2\\2x_4=2\end{cases}\Leftrightarrow\begin{cases}x_1=-2\\x_2=9\\x_3=-6\\x_4=1\end{cases}$$

Bài 3:

Giải các hệ phương trình tuyến tính thuần nhất sau:

1) $\begin{cases} 2x_1 + x_2 - 4x_3 = 0 \\ 3x_1 + 5x_2 - 7x_3 = 0 \\ 4x_1 - 5x_2 - 6x_3 = 0 \end{cases}$

$$(A/B) = \begin{pmatrix} 2 & 1 & -4 & 0 \\ 3 & 5 & -7 & 0 \\ 4 & -5 & -6 & 0 \end{pmatrix} \xrightarrow{\text{h3(-1) +h2+h1}} \begin{pmatrix} 1 & 11 & -5 & 0 \\ 3 & 5 & -7 & 0 \\ 4 & -5 & -6 & 0 \end{pmatrix} \xrightarrow[\text{h1(-4)+h3}]{\text{h1(-3)+h2}} \begin{pmatrix} 1 & 11 & -11 & 0 \\ 0 & -28 & 8 & 0 \\ 0 & -49 & 14 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{h2}\left(-\frac{49}{28}\right)\text{+h3}} \begin{pmatrix} 1 & 11 & -11 & 0 \\ 0 & -28 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ta có: (1) $\Leftrightarrow \begin{cases} x_1 + 11x_2 - 11x_3 = 0 & (1) \\ -28x_2 + 8x_3 = 0 & (2) \end{cases}$

Từ (2) $\Rightarrow x_3 = \dfrac{28}{8} x_2$

Thế x_3 vào (1), ta được: $x_1 = -11x_2 + 11\left(\dfrac{28}{8}\right)x_2 = \dfrac{55}{2}x_2$

Vậy nghiệm của hệ phương trình đã cho là: $\begin{cases} x_1 = \dfrac{55}{2} x_2 \\ x_2 \text{ tuỳ ý} \\ x_3 = \dfrac{28}{8} x_2 \end{cases}$

---

2) $\begin{cases} 3x_1 + 5x_2 + 2x_3 = 0 \\ 4x_1 + 7x_2 + 5x_3 = 0 \\ x_1 + x_2 - 4x_3 = 0 \\ 2x_1 + 9x_2 + 6x_3 = 0 \end{cases}$

$$(A/B) = \begin{pmatrix} 3 & 5 & 2 & 0 \\ 4 & 7 & 5 & 0 \\ 1 & 1 & -4 & 0 \\ 2 & 9 & 6 & 0 \end{pmatrix} \xrightarrow{h1 \leftrightarrow h3} \begin{pmatrix} 1 & 1 & -4 & 0 \\ 4 & 7 & 5 & 0 \\ 3 & 5 & 2 & 0 \\ 2 & 9 & 6 & 0 \end{pmatrix} \xrightarrow{\substack{h1(-4)+h2 \\ h1(-3)+h3 \\ h1(-2)+h4}} \begin{pmatrix} 1 & 1 & -4 & 0 \\ 0 & 3 & 21 & 0 \\ 0 & 2 & 14 & 0 \\ 0 & 7 & 14 & 0 \end{pmatrix}$$

$$\xrightarrow{h2\left(\frac{1}{3}\right), h3\left(\frac{1}{2}\right), h4\left(\frac{1}{7}\right)} \begin{pmatrix} 1 & 1 & -4 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix} \xrightarrow{\substack{h2(-1)+h3 \\ h2(-1)+h4}} \begin{pmatrix} 1 & 1 & -4 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & 0 \end{pmatrix} \xrightarrow{h3 \leftrightarrow h4} \begin{pmatrix} 1 & 1 & -4 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ta có: (2) $\Leftrightarrow \begin{cases} x_1 + x_2 - 4x_3 = 0 \\ x_2 + 7x_3 = 0 \Rightarrow x_1 = x_2 = x_3 = 0 \\ -5x_3 = 0 \end{cases}$

3) $\begin{cases} 2x_1 - x_2 + 3x_3 + 7x_4 = 0 \\ 4x_1 - 2x_2 + 7x_3 + 5x_4 = 0 \\ 2x_1 - x_2 + x_3 - 5x_4 = 0 \end{cases}$

Giaûi

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 2 & -1 & 3 & 7 & 0 \\ 4 & -2 & 7 & 5 & 0 \\ 2 & -1 & 1 & -5 & 0 \end{array}\right) \xrightarrow[h1(-1)+h3]{h1(-2)+h2} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 7 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & -2 & -12 & 0 \end{array}\right) \xrightarrow{h2(2)+h3} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 7 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & 0 & 6 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} 2x_1 - x_2 + 3x_3 + 7x_4 = 0 \\ x_3 - 9x_4 = 0 \\ x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_2 = 2x_1 \\ x_3 = 0 \\ x_4 = 0 \\ x_1 \text{ tuøy yù} \end{cases} \Leftrightarrow \begin{cases} x_1 = t \\ x_2 = 2t \\ x_3 = 0 \\ x_4 = 0 \end{cases} \quad (t \in R)$$

---

4) $\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0 \end{cases}$

Giaûi

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 1 & 2 & 4 & -3 & 0 \\ 3 & 5 & 6 & -4 & 0 \\ 4 & 5 & -2 & 3 & 0 \\ 3 & 8 & 24 & -19 & 0 \end{array}\right) \xrightarrow[h(-3)+h4]{\substack{h(-3)+h2 \\ h(-4)+h3}} \left(\begin{array}{cccc|c} 1 & 2 & 4 & -3 & 0 \\ 0 & -1 & -6 & 5 & 0 \\ 0 & -3 & -18 & 15 & 0 \\ 0 & 2 & 12 & -10 & 0 \end{array}\right)$$

$$\xrightarrow[h2(2)+h3]{h2(-3)+h3} \left(\begin{array}{cccc|c} 1 & 2 & 4 & -3 & 0 \\ 0 & -1 & -6 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Heä phöông trình ñaõ cho töông ñöông vôùi heä phöông trình:

$$\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0 \\ -x_2 - 6x_3 + 5x_4 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 8x_3 - 7x_4 \\ x_2 = -6x_3 + 5x_4 \\ x_3, x_4 \text{ tuøy yù} \end{cases} \Leftrightarrow \begin{cases} x_1 = 8t - 7s \\ x_2 = -6t + 5s \\ x_3 = t \\ x_4 = s \end{cases} \quad (t, s \in R)$$

BÀI TẬP VỀ ĐỊNH THỨC

Bài 1

Tính các định thức cấp 2:

1) $D = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 5.3 - 7.2 = 15 - 14 = 1$

2) $D = \begin{vmatrix} 3 & 2 \\ 8 & 5 \end{vmatrix} = 3.5 - 8.2 = 15 - 16 = -1$

3) $D = \begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix} = (n+1)(n-1) - n^2 = n^2 - 1 - n^2 = -1$

4) $D = \begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1$

Bài 2:

Tính các định thức cấp 3:

1) $D = \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix} = 18+2+60-9-16-15 = 40$

---

2) $D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = 30+18+8-15-36-8 = -3$

---

3) $D = \begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix} = 40-24-105+10+224-45=100$

---

4) $D = \begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix} =-9-20-32+20+12+24= -5$

---

5) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix} = 12 + 3 + 3 - 2 - 9 - 6 = 1$

---

6) $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{matrix} a & b \\ b & c \\ c & a \end{matrix}$

$= acb + bac + cba - c^3 - a^3 - b^3 = 3abc - c^3 - a^3 - b^3$

---

7) $D = \begin{vmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{vmatrix} = 0$

---

8) $D = \begin{vmatrix} a & x & x \\ x & b & x \\ x & x & c \end{vmatrix} \begin{matrix} a & x \\ x & b \\ x & x \end{matrix}$

$= abc + x^3 + x^3 - bx^2 - ax^2 - cx^2 = abc - 2x^3 - x^2\left(a+b+c\right)$

---

9) $D = \begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix} \begin{matrix} a+x & x \\ x & b+x \\ x & x \end{matrix}$

$= \left(a+x\right)\left(b+x\right)\left(c+x\right) + x^3 + x^3 - x^2\left(b+x\right) - x^2\left(a+x\right) - x^2\left(c+x\right)$

$= \left(ab + ax + bx + x^2\right)\left(c+x\right) + x^3 + x^3 - bx^2 - x^3 - x^2a - x^3 - x^2c - x^3$

$= abc + abx + acx + ax^2 + bcx + bx^2 + cx^2 + x^3 + x^3 + x^3 - bx^2 - x^3 - x^2a - x^3 - x^2c - x^3$

$= abc + abx + acx + bcx$

---

10) $D = \begin{vmatrix} a & b & c & 1 \\ b & c & a & 1 \\ c & a & b & 1 \\ \frac{b+c}{2} & \frac{c+a}{2} & \frac{a+b}{2} & 1 \end{vmatrix} \xrightarrow{c3+c2+c1} \begin{vmatrix} a+b+c & b & c & 1 \\ b+c+a & c & a & 1 \\ c+a+b & a & b & 1 \\ a+b+c & \frac{c+a}{2} & \frac{a+b}{2} & 1 \end{vmatrix}$

$= \left(a+b+c\right)\begin{vmatrix} 1 & b & c & 1 \\ 1 & c & a & 1 \\ 1 & a & b & 1 \\ 1 & \frac{c+a}{2} & \frac{a+b}{2} & 1 \end{vmatrix} = 0$

Bài 3

Tính các định thức:

1) $D = \begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix} \xrightarrow{h3} (-1)^{3+1}\left[a\left|M_{31}\right| - b\left|M_{32}\right| + c\left|M_{33}\right| - d\left|M_{34}\right|\right]$

$$* \ |M_{31}| = \begin{vmatrix} -3 & 4 & 1 \\ -2 & 3 & 2 \\ -1 & 4 & 3 \end{vmatrix} = -27 -8 -8 + 3 +24 + 24 = 8$$

$$* \ |M_{32}| = \begin{vmatrix} 2 & 4 & 1 \\ 4 & 3 & 2 \\ 3 & 4 & 3 \end{vmatrix} = 18 + 24 + 16 - 9 - 16 - 48 = -15$$

$$* \ |M_{33}| = \begin{vmatrix} 2 & -3 & 1 \\ 4 & -2 & 2 \\ 3 & -1 & 3 \end{vmatrix} = -12 - 18 - 4 + 6 +4 +36 = 12$$

$$* \ |M_{34}| = \begin{vmatrix} 2 & -3 & 4 \\ 4 & -2 & 3 \\ 3 & -1 & 4 \end{vmatrix} = -16 -27 - 16 + 24 + 6 +48 = 19$$

Vậy: D = 8a+15b+12c-19d

---

2) $$D = \begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix} \xrightleftharpoons{c2} (-1)^{2+1}\left[a|M_{21}| - b|M_{22}| + c|M_{23}| - d|M_{24}|\right]$$

$$* \ |M_{12}| = \begin{vmatrix} 4 & 4 & -3 \\ 2 & 3 & -2 \\ 4 & 5 & -4 \end{vmatrix} = -48 - 32 - 30 + 36 + 40 + 32 = -2$$

$$* \ |M_{22}| = \begin{vmatrix} 5 & 2 & -1 \\ 2 & 3 & -2 \\ 4 & 5 & -4 \end{vmatrix} = -60 -16 - 10 + 12 + 50 +16 = -8$$

$$* \ |M_{32}| = \begin{vmatrix} 5 & 2 & -1 \\ 4 & 4 & -3 \\ 4 & 5 & -4 \end{vmatrix} = -80 - 24 - 20 + 16 + 75 + 32 = -1$$

$$* |M_{42}| = \begin{vmatrix} 5 & 2 & -1 \\ 4 & 4 & -3 \\ 2 & 3 & -2 \end{vmatrix} = -40 -12 - 12 + 8 + 45 + 16 = 5$$

Vậy: D = - (-2a + 8b – c - 5d) = 2a - 8b + c + 5d

---

3) $D=\begin{vmatrix} a & 3 & 0 & 5\\ 0 & b & 0 & 2\\ 1 & 2 & c & 3\\ 0 & 0 & 0 & d\end{vmatrix} \xrightleftharpoons{h4} (-1)^{4+1}\left(-d\left|M_{44}\right|\right)=d\times\begin{vmatrix} a & 3 & 0\\ 0 & b & 0\\ 1 & 2 & c\end{vmatrix}=abcd$

---

4) $D=\begin{vmatrix} 1 & 0 & 2 & a\\ 2 & 0 & b & 0\\ 3 & c & 4 & 5\\ d & 0 & 0 & 0\end{vmatrix} \xrightleftharpoons{h4} (-1)^{4+1}d\left|M_{41}\right|=-d\times\begin{vmatrix} 0 & 2 & a\\ 0 & b & 0\\ c & 4 & 5\end{vmatrix}=abcd$

Bài 4

Tính các định thức sau:

1) $D=\begin{vmatrix} 1 & 1 & 1 & 1\\ 1 & -1 & 1 & 1\\ 1 & 1 & -1 & 1\\ 1 & 1 & 1 & -1\end{vmatrix} \xrightleftharpoons[h1(-1)+h4]{\substack{h1(-1)+h2\\ h1(-1)+h3}} \begin{vmatrix} 1 & 1 & 1 & 1\\ 0 & -2 & 0 & 0\\ 0 & 0 & -2 & 0\\ 0 & 0 & 0 & -2\end{vmatrix}=1\times(-2)\times(-2)\times(-2)=-8$

---

2)

$$D=\begin{vmatrix} 0 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\\ 1 & 1 & 0 & 1\\ 1 & 1 & 1 & 0\end{vmatrix} \xrightleftharpoons{c1\leftrightarrow c2} -\begin{vmatrix} 1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\\ 1 & 1 & 0 & 1\\ 1 & 1 & 1 & 0\end{vmatrix} \xrightleftharpoons[h1(-1)+h4]{h1(-1)+h3} -\begin{vmatrix} 1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 1 & -1 & 0\\ 0 & 1 & 0 & -1\end{vmatrix}$$

$$=-1\times\begin{vmatrix} 1 & 1 & 1\\ 1 & -1 & 0\\ 1 & 0 & -1\end{vmatrix}\begin{matrix} 1 & 1\\ 1 & -1\\ 1 & 0\end{matrix}$$

$$=-\left(1+1+1\right)=-3$$

---

3)

$$D=\begin{vmatrix}2&-5&1&2\\-3&7&-1&4\\3&-9&2&7\\4&-6&1&2\end{vmatrix}\xrightleftharpoons{c1\leftrightarrow c3}-\begin{vmatrix}1&-5&2&2\\-1&7&-3&4\\2&-9&3&7\\1&-6&4&2\end{vmatrix}\xrightleftharpoons[h1(-1)+h4]{\substack{h1+h2\\h1(-2)+h3}}-\begin{vmatrix}1&-5&2&2\\0&2&-1&6\\0&1&-1&3\\0&-1&2&0\end{vmatrix}$$

$$=-1\times\begin{vmatrix}2&-1&6\\1&-1&3\\-1&2&0\end{vmatrix}\begin{matrix}2&-1\\1&-1\\-1&2\end{matrix}$$

$$=-(3+12-6-12)=3$$

---

4)

$$D=\begin{vmatrix}3&-3&-5&8\\-3&2&4&-6\\2&-5&-7&5\\-4&3&5&-6\end{vmatrix}\xrightleftharpoons{h4+h1}\begin{vmatrix}-1&0&0&2\\-3&2&4&-6\\2&-5&-7&5\\-4&3&5&-6\end{vmatrix}\xrightleftharpoons[h1(-4)+h4]{\substack{h1(-3)+h2\\h1(2)+h3}}\begin{vmatrix}-1&0&0&2\\0&2&4&-12\\0&-5&-7&9\\0&3&5&-14\end{vmatrix}$$

$$=-1\times\begin{vmatrix}2&4&-12\\-5&-7&9\\3&5&-14\end{vmatrix}=-1\times2\times\begin{vmatrix}1&2&-6\\-5&-7&9\\3&5&-14\end{vmatrix}\begin{matrix}1&2\\-5&-7\\3&5\end{matrix}$$

$$=-2(98+54+150-126-45-140)=-2\times(-9)=18$$

---

5)

$$D=\begin{vmatrix}-3&9&3&6\\-5&8&2&7\\4&-5&-3&-2\\7&-8&-4&-5\end{vmatrix}\xrightleftharpoons[h3(-1)+h4]{\substack{h3+h1\\h3+h2}}\begin{vmatrix}1&4&0&4\\-1&3&-1&5\\4&-5&-3&-2\\3&-3&-1&-3\end{vmatrix}$$

$$\xrightleftharpoons[h1(-3)+h4]{\substack{h1+h2\\h1(-4)+h3}}\begin{vmatrix}1&4&0&4\\0&7&-1&9\\0&-21&-3&-18\\0&-15&-1&-15\end{vmatrix}=1\begin{vmatrix}7&-1&9\\-21&-3&-18\\-15&-1&-15\end{vmatrix}\begin{matrix}7&-1\\-21&-3\\-15&-1\end{matrix}$$

$$=315-270+189-405-126+315=18$$

---

6)

$$D=\begin{vmatrix}1&0&1&-1&2\\0&1&1&2&-1\\1&2&1&0&1\\-1&0&1&0&2\\-1&1&1&1&1\end{vmatrix}\xrightleftharpoons[h1+h5]{\substack{h1(-1)+h3\\h1+h4}}\begin{vmatrix}1&0&1&-1&2\\0&1&1&2&-1\\0&2&0&1&-1\\0&0&2&-1&4\\0&1&2&0&3\end{vmatrix}=1\times\begin{vmatrix}1&1&2&-1\\2&0&1&-1\\0&2&-1&4\\1&2&0&3\end{vmatrix}$$

$$\xrightleftharpoons[h1(-1)+h3]{h1(-2)+h2}\begin{vmatrix}1&1&2&-1\\0&-2&-3&1\\0&2&-1&4\\0&1&-2&4\end{vmatrix}=1\times\begin{vmatrix}-2&-3&1\\2&-1&4\\1&-2&4\end{vmatrix}\begin{matrix}-2&-3\\2&-1\\1&-2\end{matrix}$$

$$=8-12-4+1-16+24=1$$

---

7)

$$D=\begin{vmatrix}0&0&5&0&0\\0&2&0&-2&0\\1&3&18&-6&2\\4&17&9&-15&2\\19&20&24&3&5\end{vmatrix}\xrightleftharpoons{h1\leftrightarrow h3}-\begin{vmatrix}1&3&18&-6&2\\0&2&0&-2&0\\0&0&5&0&0\\4&17&9&-15&2\\19&20&24&3&5\end{vmatrix}$$

$$\xrightleftharpoons[h1(-19)+h5]{h1(-4)+h4}-\begin{vmatrix}1&3&18&-6&2\\0&2&0&-2&0\\0&0&5&0&0\\0&5&-63&9&-6\\0&-37&-318&117&-33\end{vmatrix}=-1\times\begin{vmatrix}2&0&-2&0\\0&5&0&0\\5&-63&9&-6\\-37&-318&117&-33\end{vmatrix}$$

$$\xrightleftharpoons{h2}-(-1)^{2+1}\left[-5\left|M_{22}\right|\right]=-5\times\begin{vmatrix}2&-2&0\\5&9&-6\\-37&117&-33\end{vmatrix}\begin{matrix}2&-2\\5&9\\-37&117\end{matrix}$$

$$=-5\left(-594-444+1404-330\right)=-5\times36=-180$$

---

8)

$$D=\begin{vmatrix}1&-2&1&4&10\\1&3&2&5&3\\0&5&3&7&9\\0&0&2&3&7\\0&0&0&3&15\end{vmatrix}\xrightleftharpoons{h1(-1)+h2}\begin{vmatrix}1&-2&1&4&10\\0&5&1&1&-7\\0&5&3&7&9\\0&0&2&3&7\\0&0&0&3&15\end{vmatrix}=1\times\begin{vmatrix}5&1&1&-7\\5&3&7&9\\0&2&3&7\\0&0&3&15\end{vmatrix}$$

$$\xrightleftharpoons{h1(-1)+h2}\begin{vmatrix}5&1&1&-7\\0&2&6&16\\0&2&3&7\\0&0&3&15\end{vmatrix}\xrightleftharpoons{h2(-1)+h3}\begin{vmatrix}5&1&1&-7\\0&2&6&16\\0&0&-3&-9\\0&0&3&15\end{vmatrix}\xrightleftharpoons{h3+h4}\begin{vmatrix}5&1&1&-7\\0&2&6&16\\0&0&-3&-9\\0&0&0&6\end{vmatrix}$$

$$=5\times2\times(-3)\times6=-180$$

---

9)

$$D=\begin{vmatrix}7&3&2&6\\8&-9&4&9\\7&-2&7&3\\5&-3&3&4\end{vmatrix}\xrightleftharpoons[h1(-1)+h4]{\substack{h1(-1)+h2\\h1(-1)+h3}}\begin{vmatrix}7&3&2&6\\1&-12&2&3\\0&-5&5&-3\\-2&-6&1&-2\end{vmatrix}\xrightleftharpoons{h1\leftrightarrow h2}-\begin{vmatrix}1&-12&2&3\\7&3&2&6\\0&-5&5&-3\\-2&-6&1&-2\end{vmatrix}$$

$$\xrightleftharpoons[h1(2)+h4]{h1(-7)+h2}-\begin{vmatrix}1&-12&2&3\\0&87&-12&-15\\0&-5&5&-3\\0&-30&5&4\end{vmatrix}=-1\times\begin{vmatrix}87&-12&-15\\-5&5&-3\\-30&5&4\end{vmatrix}=-3\times\begin{vmatrix}29&-4&-5\\-5&5&-3\\-30&5&4\end{vmatrix}\begin{matrix}29&-4\\-5&5\\-30&5\end{matrix}$$

$$=-3\left(580-360+125-750+435-80\right)=-3\times(-50)=150$$

BÀI TẬP VỀ HỆ PHƯƠNG TRÌNH KRAMER

Giải hệ phương trình bằng phương pháp Kramer:

1) $\begin{cases} 2x_1 \qquad + x_3 = -1 \\ x_1 + 4x_2 + 2x_3 = 7 \\ \qquad 5x_2 + \ x_3 = 5 \end{cases}$

Ta có:

* $D = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 4 & 2 \\ 0 & 5 & 1 \end{vmatrix}$ = 8 + 5 – 20 = -7

* $Dx_1 = \begin{vmatrix} -1 & 0 & 1 \\ 7 & 4 & 2 \\ 5 & 5 & 1 \end{vmatrix}$ = - 4 + 35 – 20 + 10 = 21

* $Dx_2 = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 7 & 2 \\ 0 & 5 & 1 \end{vmatrix}$ = 14 + 5 – 20 +1 = 0

* $Dx_3 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 7 \\ 0 & 5 & 5 \end{vmatrix}$ = 40 – 5 -70 = -35

Vì D $\neq 0$ nên hệ có nghiệm duy nhất:

$$\begin{cases} x_1 = \dfrac{Dx_1}{D} = -\dfrac{21}{7} = -3 \\ x_2 = \dfrac{Dx_2}{D} = -\dfrac{0}{7} = 0 \\ x_3 = \dfrac{Dx_3}{D} = -\dfrac{-35}{7} = 5 \end{cases}$$

---

2) $\begin{cases} x_1 - x_2 \quad + 3x_3 = 6 \\ \qquad 4x_2 - 5x_3 = -13 \\ 3x_1 \qquad - 2x_3 = 1 \end{cases}$

Ta có:

* $D = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 4 & -5 \\ 3 & 0 & -2 \end{vmatrix}$ = - 8 +15 – 36 = -29

$$* \quad Dx_1 = \begin{vmatrix} 6 & -1 & 3 \\ -13 & 4 & -5 \\ 1 & 0 & -2 \end{vmatrix} = -48 + 5 - 12 + 26 = -29$$

$$* \quad Dx_2 = \begin{vmatrix} 1 & 6 & 3 \\ 0 & -13 & -5 \\ 3 & 1 & -2 \end{vmatrix} = 26 - 90 + 117 + 5 = 58$$

$$* \quad Dx_3 = \begin{vmatrix} 1 & -1 & 6 \\ 0 & 4 & -13 \\ 3 & 0 & 1 \end{vmatrix} = 4 + 39 - 72 = -29$$

Vì D $\neq 0$ nên hệ có nghiệm duy nhất:

$$\begin{cases} x_1 = \dfrac{Dx_1}{D} = \dfrac{-29}{-29} = 1 \\ x_2 = \dfrac{Dx_2}{D} = -\dfrac{58}{29} = -2 \\ x_3 = \dfrac{Dx_3}{D} = \dfrac{-29}{-29} = 1 \end{cases}$$

---

3) $$\begin{cases} x_1 + 4x_2 - x_3 = 2 \\ 2x_2 - 3x_3 - 5x_4 = -8 \\ 2x_1 - x_3 = 5 \\ x_1 - 2x_2 - 3x_4 = 0 \end{cases}$$

Ta có:

$$D = \begin{vmatrix} 1 & 4 & -1 & 0 \\ 0 & 2 & -3 & -5 \\ 2 & 0 & -1 & 0 \\ 1 & -2 & 0 & -3 \end{vmatrix} \underset{h1(-1)+h4}{\overset{h1(-2)+h3}{=\!=\!=\!=\!=\!=}} \begin{vmatrix} 1 & 4 & -1 & 0 \\ 0 & 2 & -3 & -5 \\ 0 & -8 & 1 & 0 \\ 0 & -6 & 1 & -3 \end{vmatrix} = 1 \times \begin{vmatrix} 2 & -3 & -5 \\ -8 & 1 & 0 \\ -6 & 1 & -3 \end{vmatrix} \begin{matrix} 2 & -3 \\ -8 & 1 \\ -6 & 1 \end{matrix}$$

$$= -6 + 40 - 30 + 72 = 76$$

$$D_{x_1} = \begin{vmatrix} 2 & 4 & -1 & 0 \\ -8 & 2 & -3 & -5 \\ 5 & 0 & -1 & 0 \\ 0 & -2 & 0 & -3 \end{vmatrix} \overset{c1 \leftrightarrow c3}{=\!=\!=\!=} - \begin{vmatrix} -1 & 4 & 2 & 0 \\ -3 & 2 & -8 & -5 \\ -1 & 0 & 5 & 0 \\ 0 & -2 & 0 & -3 \end{vmatrix} \underset{h1(-1)+h3}{\overset{h1(-3)+h2}{=\!=\!=\!=\!=}} - \begin{vmatrix} -1 & 4 & 2 & 0 \\ 0 & -10 & -14 & -5 \\ 0 & -4 & 3 & 0 \\ 0 & -2 & 0 & -3 \end{vmatrix}$$

$$= -1 \times \begin{vmatrix} -10 & -14 & -5 \\ -4 & 3 & 0 \\ -2 & 0 & -3 \end{vmatrix} \begin{matrix} -10 & -14 \\ -4 & 3 \\ -2 & 0 \end{matrix}$$

$$= -(90 - 30 + 168) = -228$$

$$D_{x_2} = \begin{vmatrix} 1 & 2 & -1 & 0 \\ 0 & -8 & -3 & -5 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 0 & -3 \end{vmatrix} \overset{h1(-2)+h3}{\underset{h1(-1)+h4}{======}} \begin{vmatrix} 1 & 2 & -1 & 0 \\ 0 & -8 & -3 & -5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 1 & -3 \end{vmatrix} = 1 \times \left| \begin{matrix} -8 & -3 & -5 \\ 1 & 1 & 0 \\ -2 & 1 & -3 \end{matrix} \right| \begin{matrix} -8 & -3 \\ 1 & 1 \\ -2 & 1 \end{matrix}$$

$= 24 - 5 - 10 - 9 = 0$

$$D_{x_3} = \begin{vmatrix} 1 & 4 & 2 & 0 \\ 0 & 2 & -8 & -5 \\ 2 & 0 & 5 & 0 \\ 1 & -2 & 0 & -3 \end{vmatrix} \overset{h1(-2)+h3}{\underset{h1(-1)+h4}{======}} \begin{vmatrix} 1 & 4 & 2 & 0 \\ 0 & 2 & -8 & -5 \\ 0 & -8 & 1 & 0 \\ 0 & -6 & -2 & -3 \end{vmatrix} = 1 \times \left| \begin{matrix} 2 & -8 & -5 \\ -8 & 1 & 0 \\ -6 & -2 & -3 \end{matrix} \right| \begin{matrix} 2 & -8 \\ -8 & 1 \\ -6 & -2 \end{matrix}$$

$= -6 - 80 - 30 + 192 = 76$

$$D_{x_4} = \begin{vmatrix} 1 & 4 & -1 & 2 \\ 0 & 2 & -3 & -8 \\ 2 & 0 & -1 & 5 \\ 1 & -2 & 0 & 0 \end{vmatrix} \overset{h1(-2)+h3}{\underset{h1(-1)+h4}{======}} \begin{vmatrix} 1 & 4 & -1 & 2 \\ 0 & 2 & -3 & -8 \\ 0 & -8 & 1 & 1 \\ 0 & -6 & 1 & -2 \end{vmatrix} = 1 \times \left| \begin{matrix} 2 & -3 & -8 \\ -8 & 1 & 1 \\ -6 & 1 & -2 \end{matrix} \right| \begin{matrix} 2 & -3 \\ -8 & 1 \\ -6 & 1 \end{matrix}$$

$= -4 + 18 + 64 - 48 - 2 + 48 = 76$

Vì D $\neq 0$ nên hệ có nghiệm duy nhất:

$$\begin{cases} x_1 = \dfrac{Dx_1}{D} = \dfrac{228}{76} = 3 \\ x_2 = \dfrac{Dx_2}{D} = \dfrac{0}{76} = 0 \\ x_3 = \dfrac{Dx_3}{D} = \dfrac{76}{76} = 1 \\ x_4 = \dfrac{Dx_4}{D} = \dfrac{76}{76} = 1 \end{cases} \quad hay \quad (3,0,1,1)$$

---

4) $\begin{cases} x_1 \qquad - 3x_3 + x_4 = 2 \\ 2x_1 - x_2 \qquad - x_4 = 0 \\ \qquad 2x_2 - 5x_3 + 2x_4 = 5 \\ \qquad 3x_2 \qquad - x_4 = 4 \end{cases}$

Ta có:

$$D = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 2 & -1 & 0 & -1 \\ 0 & 2 & -5 & 2 \\ 0 & 3 & 0 & -1 \end{vmatrix} \overset{h1(-2)+h2}{======} \begin{vmatrix} 1 & 0 & -3 & 1 \\ 0 & -1 & 6 & -3 \\ 0 & 2 & -5 & 2 \\ 0 & 3 & 0 & -1 \end{vmatrix} = 1 \times \begin{vmatrix} -1 & 6 & -3 \\ 2 & -5 & 2 \\ 3 & 0 & -1 \end{vmatrix}$$

$= -5 + 36 - 45 + 12 = -2$

$$D_{x_1}=\begin{vmatrix}2&0&-3&1\\0&-1&0&-1\\5&2&-5&2\\4&3&0&-1\end{vmatrix}\xlongequal{c1\leftrightarrow c4}-\begin{vmatrix}1&0&-3&2\\-1&-1&0&0\\2&2&-5&5\\-1&3&0&4\end{vmatrix}\xlongequal[h1+h4]{\substack{h1+h2\\h1(-2)+h3}}-\begin{vmatrix}1&0&-3&2\\0&-1&-3&2\\0&2&1&1\\0&3&-3&6\end{vmatrix}$$

$$=-1\times\begin{vmatrix}-1&-3&2\\2&1&1\\3&-3&6\end{vmatrix}=-\left(-6-9-12-6-3+36\right)=0$$

$$D_{x_2}=\begin{vmatrix}1&2&-3&1\\2&0&0&-1\\0&5&-5&2\\0&4&0&-1\end{vmatrix}\xlongequal{h1(-2)+h2}\begin{vmatrix}1&2&-3&1\\0&-4&6&-3\\0&5&-5&2\\0&4&0&-1\end{vmatrix}=1\times\begin{vmatrix}-4&6&-3\\5&-5&2\\4&0&-1\end{vmatrix}$$

$$=-20+48-60+30=-2$$

$$D_{x_3}=\begin{vmatrix}1&0&2&1\\2&-1&0&-1\\0&2&5&2\\0&3&4&-1\end{vmatrix}\xlongequal{h1(-2)+h2}\begin{vmatrix}1&0&2&1\\0&-1&-4&-3\\0&2&5&2\\0&3&4&-1\end{vmatrix}=1\times\begin{vmatrix}-1&-4&-3\\2&5&2\\3&4&-1\end{vmatrix}$$

$$=5-24-24+45+8-8=2$$

$$D_{x_4}=\begin{vmatrix}1&0&-3&2\\2&-1&0&0\\0&2&-5&5\\0&3&0&4\end{vmatrix}\xlongequal{h1(-2)+h2}\begin{vmatrix}1&0&-3&2\\0&-1&6&-4\\0&2&-5&5\\0&3&0&4\end{vmatrix}=1\times\begin{vmatrix}-1&6&-4\\2&-5&5\\3&0&4\end{vmatrix}$$

$$=20+90-60-48=2$$

Vì D $\neq 0$ nên hệ có nghiệm duy nhất:

$$\begin{cases}x_1=\dfrac{Dx_1}{D}=-\dfrac{0}{2}=0\\[2mm] x_2=\dfrac{Dx_2}{D}=\dfrac{-2}{-2}=1\\[2mm] x_3=\dfrac{Dx_3}{D}=-\dfrac{2}{2}=-1\\[2mm] x_4=\dfrac{Dx_4}{D}=-\dfrac{2}{2}=-1\end{cases}$$

BAØI TAÄP BIEÄN LUAÄN THEO THAM SOÁ

Baøi 1:

Giaûi vaø bieän luaän:

$$\begin{cases} 3x_1+2x_2+5x_3+4x_4=3 \\ 2x_1+3x_2+6x_3+8x_4=5 \\ x_1-6x_2-9x_3-20x_4=-11 \\ 4x_1+x_2+4x_3+\lambda x_4=2 \end{cases}$$

Giaûi:

$$\left(A\middle|B\right)=\begin{pmatrix} 3 & 2 & 5 & 4 & 3 \\ 2 & 3 & 6 & 8 & 5 \\ 1 & -6 & -9 & -20 & -11 \\ 4 & 1 & 4 & \lambda & 2 \end{pmatrix} \xrightarrow{h1\leftrightarrow h3} \begin{pmatrix} 1 & -6 & -9 & -20 & -11 \\ 2 & 3 & 6 & 8 & 5 \\ 3 & 2 & 5 & 4 & 3 \\ 4 & 1 & 4 & \lambda & 2 \end{pmatrix}$$

$$\xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2 \\ h1(-3)+h3}} \begin{pmatrix} 1 & -6 & -9 & -20 & -11 \\ 0 & 15 & 24 & 48 & 27 \\ 0 & 20 & 32 & 64 & 36 \\ 0 & 25 & 40 & \lambda+80 & 46 \end{pmatrix} \xrightarrow[h3\left(\frac{1}{4}\right)]{h2\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & -6 & -9 & -20 & -11 \\ 0 & 5 & 8 & 16 & 9 \\ 0 & 5 & 8 & 16 & 9 \\ 0 & 25 & 40 & \lambda+80 & 46 \end{pmatrix}$$

$$\xrightarrow[h2(-5)+h4]{h2(-1)+h3} \begin{pmatrix} 1 & -6 & -9 & -20 & -11 \\ 0 & 5 & 8 & 16 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 1 \end{pmatrix} \xrightarrow{h3\leftrightarrow h4} \begin{pmatrix} 1 & -6 & -9 & -20 & -11 \\ 0 & 5 & 8 & 16 & 9 \\ 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(1)\Leftrightarrow\begin{cases} x_1-6x_2-9x_3-20x_4=-11 \\ \qquad 5x_2+8x_3+16x_4=9\,(2) \\ \qquad\qquad \lambda x_4=1 \end{cases}$$

$$1)\ Khi\ \lambda\neq 0:(2)\Leftrightarrow\begin{cases} x_1=-\frac{1}{5}\times\frac{\lambda+3\lambda t-4}{\lambda} \\ x_2=-\frac{1}{5}\times\frac{-9\lambda+8\lambda t+16}{\lambda} \\ x_3=t \\ x_4=\frac{1}{\lambda} \end{cases}\quad (t\in R)$$

$$2)\ Khi\lambda=0:(3)\Leftrightarrow\begin{cases} 1x_1-6x_2-9x_3-20x_4=-11 \\ 15x_2+24x_3+48x_4=27 \\ 0=1 \end{cases}$$: heä voâ nghieäm

Baøi 2:

Cho heä phöông trình:

$$\begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9 \\ mx_1 - 4x_2 + 9x_3 + 10x_4 = 11 \end{cases}$$

a) Tìm m ñeå heä phöông trình coù nghieäm

b) Giaûi heä phöông trình khi m = 10

Giaûi:

a) Ta coù:

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ m & -4 & 9 & 10 & 11 \end{array}\right) \xrightarrow{c1 \leftrightarrow c4 \leftrightarrow c1} \left(\begin{array}{cccc|c} -1 & 4 & 3 & 2 & 5 \\ -2 & 6 & 5 & 4 & 7 \\ -3 & 8 & 7 & 6 & 9 \\ -4 & 10 & 9 & m & 11 \end{array}\right)$$

$$\xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2 \\ h1(-3)+h3}} \left(\begin{array}{cccc|c} -1 & 4 & 3 & 4 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & -4 & -2 & 0 & -6 \\ 0 & -6 & -3 & m-8 & -9 \end{array}\right) \xrightarrow[h2(-3)+h4]{h2(-2)+h3} \left(\begin{array}{cccc|c} -1 & 4 & 3 & 4 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m-8 & 0 \end{array}\right)$$

$$\xrightarrow{h3 \leftrightarrow h4} \left(\begin{array}{cccc|c} -1 & 4 & 3 & 4 & 5 \\ 0 & -2 & -1 & 0 & -3 \\ 0 & 0 & 0 & m-8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Ta thaáy: $\forall m \in R : r\left(A|B\right) = r\left(A\right) < 4$. Suy ra heä coù nghieäm vôùi moïi giaù trò cuûa m

b) Giaûi heä khi m = 10:

Bieán ñoåi sô caáp treân haøng ta coù:

$$\left(A/B\right) = \begin{pmatrix} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ 10 & -4 & 9 & 10 & 11 \end{pmatrix} \to \ldots \to \begin{pmatrix} 2 & -1 & 3 & 4 & 5 \\ 0 & 1 & -6 & -10 & -14 \\ 0 & 0 & -2 & -4 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(1) \Leftrightarrow \begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5 \\ x_2 - 6x_3 - 10x_4 = -14 \\ -2x_3 - 4x_4 = -6 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 4 - 2t \\ x_3 = 3 - 2t \\ x_4 = t \end{cases} \quad \left(t \in R\right)$$

Baøi 3

Giaûi vaø bieän luaän heä phöông trình sau theo tham soá λ :

$$\begin{cases}(\lambda+1)x_1+x_2+x_3=1\\ x_1+(\lambda+1)x_2+x_3=\lambda\\ x_1+x_2+(\lambda+1)x_3=\lambda^2\end{cases}$$

Giaûi:

Ta coù

$$D=\begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix}\xrightleftharpoons{h3+h2+h1}\begin{vmatrix}\lambda+3 & \lambda+3 & \lambda+3\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix}=(\lambda+3)\begin{vmatrix}1 & 1 & 1\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix}$$

$$\xrightleftharpoons[h1(-1)+h3]{h1(-1)+h2}(\lambda+3)\begin{vmatrix}1 & 1 & 1\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{vmatrix}=(\lambda+3)\lambda^2$$

$$D_{x_1}=\begin{vmatrix}1 & 1 & 1\\ \lambda & \lambda+1 & 1\\ \lambda^2 & 1 & \lambda+1\end{vmatrix}\xrightleftharpoons[h1(-\lambda^2)+h3]{h1(-\lambda)+h2}\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 1-\lambda\\ 0 & 1-\lambda^2 & -\lambda^2+\lambda+1\end{vmatrix}=1\times\begin{vmatrix}1 & 1-\lambda\\ 1-\lambda^2 & -\lambda^2+\lambda+1\end{vmatrix}$$

$$=-\lambda^2+\lambda+1-\left(1-\lambda^2\right)(1-\lambda)=-\lambda^2+\lambda+1-1+\lambda+\lambda^2-\lambda^3=-\lambda^3+2\lambda=\lambda\left(2-\lambda^2\right)$$

$$D_{x_2}=\begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda & 1\\ 1 & \lambda^2 & \lambda+1\end{vmatrix}\xrightleftharpoons{c1\leftrightarrow c3}-\begin{vmatrix}1 & 1 & \lambda+1\\ 1 & \lambda & 1\\ \lambda+1 & \lambda^2 & 1\end{vmatrix}$$

$$\xrightleftharpoons[h1(-(\lambda+1))+h3]{h1(-1)+h2}-\begin{vmatrix}1 & 1 & \lambda+1\\ 0 & \lambda-1 & -\lambda\\ 0 & \lambda^2-\lambda-1 & -\lambda^2-2\lambda\end{vmatrix}=-1\times\begin{vmatrix}\lambda-1 & -\lambda\\ \lambda^2-\lambda-1 & -\lambda^2-2\lambda\end{vmatrix}$$

$$=-\left[(\lambda-1)\left(-\lambda^2-2\lambda\right)-\left(\lambda^2-\lambda-1\right)(-\lambda)\right]=-\left[-\lambda^3-2\lambda^2+\lambda^2+2\lambda+\lambda^3-\lambda^2-\lambda\right]$$

$$=2\lambda^2-\lambda=\lambda(2\lambda-1)$$

$$D_{x_3}=\begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda+1 & \lambda\\ 1 & 1 & \lambda^2\end{vmatrix}\xrightleftharpoons{c1\leftrightarrow c2}-\begin{vmatrix}1 & \lambda+1 & 1\\ \lambda+1 & 1 & \lambda\\ 1 & 1 & \lambda^2\end{vmatrix}$$

$$\xrightleftharpoons[h1(-1)+h3]{h1(-(\lambda+1))+h2}-\begin{vmatrix}1 & 1 & 1\\ 0 & -\lambda^2-2\lambda & -1\\ 0 & -\lambda & \lambda^2-1\end{vmatrix}=-1\times\begin{vmatrix}-\lambda^2-2\lambda & -1\\ -\lambda & \lambda^2-1\end{vmatrix}$$

$$=\lambda\times\begin{vmatrix}\lambda+2 & -1\\ 1 & \lambda^2-1\end{vmatrix}$$

$$=\lambda\left[(\lambda+2)\left(\lambda^2-1\right)+1\right]=\lambda\left(\lambda^3+2\lambda^2-\lambda-1\right)$$

Ta thaáy:

(1) $D=(\lambda+3)\lambda^2\neq 0 \Leftrightarrow \begin{cases}\lambda\neq -3\\ \lambda\neq 0\end{cases}$ Khi ñoù heä coù nghieäm duy nhaát:

$$\begin{cases} x_1=\dfrac{Dx_1}{D}=\dfrac{\lambda(2-\lambda^2)}{(\lambda+3)\lambda^2}=\dfrac{2-\lambda^2}{(\lambda+3)\lambda}\\ x_2=\dfrac{Dx_2}{D}=\dfrac{\lambda(2\lambda-1)}{(\lambda+3)\lambda^2}=\dfrac{2\lambda-1}{(\lambda+3)\lambda}\\ x_3=\dfrac{Dx_3}{D}=\dfrac{\lambda^3+2\lambda^2-\lambda-1}{(\lambda+3)\lambda} \end{cases}$$

(2) Neáu $\lambda=-3$ thì $D_{x_1}=3(2-9)=-21\neq 0$: Heä voâ nghieäm

(3) Neáu $\lambda=0$ thì heä trôû thaønh:

$$\begin{cases} x_1+x_2+x_3=1\\ x_1+x_2+x_3=0\\ x_1+x_2+x_3=0 \end{cases}$$

Heä voâ nghieäm

Baøi 4

Giaûi vaø bieän luaän heä phöông trình sau theo tham soá λ:

$$\begin{cases} 5x_1-3x_2+2x_3+4x_4=3\\ 4x_1-2x_2+3x_3+7x_4=1\\ 8x_1-6x_2-x_3-5x_4=9\\ 7x_1-3x_2+7x_3+17x_4=\lambda \end{cases}$$

Giaûi

$$(A|B)=\begin{pmatrix} 5 & -3 & 2 & 4 & 3\\ 4 & -2 & 3 & 7 & 1\\ 8 & -6 & -1 & -5 & 9\\ 7 & -3 & 7 & 17 & \lambda \end{pmatrix} \xrightarrow[\substack{h2(-2)+h3\\ h2(-1)+h4}]{h2(-1)+h1} \begin{pmatrix} 1 & -1 & -1 & -3 & 2\\ 4 & -2 & 3 & 7 & 1\\ 0 & -2 & -7 & -19 & 7\\ 3 & -1 & 4 & 10 & \lambda-1 \end{pmatrix}$$

$$\xrightarrow[h1(-3)+h4]{h1(-4)+h2} \begin{pmatrix} 1 & -1 & -1 & -3 & 2\\ 0 & 2 & 7 & 19 & -7\\ 0 & -2 & -7 & -19 & 7\\ 0 & 2 & 7 & 19 & \lambda-7 \end{pmatrix} \xrightarrow[h2(-1)+h4]{h2+h3} \begin{pmatrix} 1 & -1 & -1 & -3 & 2\\ 0 & 2 & 7 & 19 & -7\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$

$$\xrightarrow{h4\leftrightarrow h3} \begin{pmatrix} 1 & -1 & -1 & -3 & 2\\ 0 & 2 & 7 & 19 & -7\\ 0 & 0 & 0 & 0 & \lambda\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Heä phöông trình töông ñöông vôùi heä:

$$\begin{cases} x_1 - \quad x_2 - \quad x_3 - \quad 3x_4 = \quad 2 \\ \qquad 2x_2 + \quad 7x_3 + \quad 19x_4 = \quad -7 \\ \qquad\qquad\qquad\qquad 0 = \quad \lambda \end{cases}$$

Ta thaáy:

(1) Khi $\lambda \neq 0$ thì heä voâ nghieäm

(2) Khi $\lambda = 0$ thì heä trôû thaønh:

$$\begin{cases} x_1 - x_2 - x_3 - 3x_4 = 2 & (1) \\ \quad 2x_2 + 7x_3 + 19x_4 = -7 & (2) \end{cases}$$

$$(2): x_2 = -\frac{7}{2}x_3 - \frac{19}{2}x_4 - 7$$

$$(1) \Leftrightarrow x_1 + \frac{7}{2}x_3 + \frac{19}{2}x_4 + 7 - x_3 - 3x_4 = 2 \Leftrightarrow x_1 = -\frac{5}{2}x_3 - \frac{13}{2}x_4 - 5$$

Vaäy nghieäm cuûa heä khi ñoù laø:

$$\begin{cases} x_1 = -\frac{5}{2}x_3 - \frac{13}{2}x_4 - 5 \\ x_2 = -\frac{7}{2}x_3 - \frac{19}{2}x_4 - 7 \\ x_3, x_4 \ \text{tuøy yù} \end{cases}$$

Baøi 5

Giaûi vaø bieän luaän heä phöông trình sau theo tham soá λ

$$\begin{cases} 3x_1 + 2x_2 + 5x_3 + 4x_4 = 3 \\ 2x_1 + 3x_2 + 6x_3 + 8x_4 = 5 \\ x_1 - 6x_2 - 9x_3 - 20x_4 = -11 \\ 4x_1 + x_2 + 4x_3 + \lambda x_4 = 2 \end{cases}$$

Giải

Ta có:

$$\left(A|B\right) = \left(\begin{array}{cccc|c} 3 & 2 & 5 & 4 & 3 \\ 2 & 3 & 6 & 8 & 5 \\ 1 & -6 & -9 & -20 & -11 \\ 4 & 1 & 4 & \lambda & 2 \end{array}\right) \xrightarrow{h3 \leftrightarrow h1} \left(\begin{array}{cccc|c} 1 & -6 & -9 & -20 & -11 \\ 2 & 3 & 6 & 8 & 5 \\ 3 & 2 & 5 & 4 & 3 \\ 4 & 1 & 4 & \lambda & 2 \end{array}\right)$$

$$\xrightarrow[h1(-4)+h4]{\substack{h1(-2)+h2 \\ h1(-3)+h3}} \left(\begin{array}{cccc|c} 1 & -6 & -9 & -20 & -11 \\ 0 & 15 & 24 & 48 & 27 \\ 0 & 20 & 32 & 64 & 36 \\ 0 & 25 & 40 & \lambda+80 & 46 \end{array}\right) \xrightarrow{h3\left(\frac{1}{4}\right) \leftrightarrow h2} \left(\begin{array}{cccc|c} 1 & -6 & -9 & -20 & -11 \\ 0 & 5 & 8 & 16 & 9 \\ 0 & 15 & 24 & 48 & 27 \\ 0 & 25 & 40 & \lambda+80 & 46 \end{array}\right)$$

$$\xrightarrow[h2(-5)+h4]{h2(-3)+h3}\left(\begin{array}{cccc|c}1 & -6 & -9 & -20 & -11\\ 0 & 5 & 8 & 16 & 9\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \lambda & 1\end{array}\right)\xrightarrow{h3\leftrightarrow h4}\left(\begin{array}{cccc|c}1 & -6 & -9 & -20 & -11\\ 0 & 5 & 8 & 16 & 9\\ 0 & 0 & 0 & \lambda & 1\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

Khi đó:

(1) Nếu $\lambda \neq 0$ thì $r\left(A|B\right) = r\left(A\right) = 3 < 4$: hệ có vô số nghiệm (tìm nghiệm như bài trên)

(2) Nếu $\lambda = 0$ thì :

$$\left.\begin{array}{l} r\left(A|B\right)=3 \\ r\left(A\right)=2 \end{array}\right\} \Rightarrow r\left(A|B\right) \neq r\left(A\right): \text{ hệ vô nghiệm}$$

Baøi 6

Giaûi vaø bieän luaän heä phöông trình sau theo tham soá λ

$$\begin{cases}\left(\lambda+1\right)x_1 + x_2 + x_3 = \lambda^2 + 3\lambda \\ x_1 + \left(\lambda+1\right)x_2 + x_3 = \lambda^3 + 3\lambda^2 \\ x_1 + x_2 + \left(\lambda+1\right)x_3 = \lambda^4 + 3\lambda^3\end{cases}$$

Giaûi

Ta coù:

$$D = \begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix} \xleftrightarrow{h3+h2+h1} \begin{vmatrix}\lambda+3 & \lambda+3 & \lambda+3\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix} = \left(\lambda+3\right)\begin{vmatrix}1 & 1 & 1\\ 1 & \lambda+1 & 1\\ 1 & 1 & \lambda+1\end{vmatrix}$$

$$\xleftrightarrow[h1(-1)+h3]{h1(-1)+h2}\left(\lambda+3\right)\begin{vmatrix}1 & 1 & 1\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{vmatrix} = \left(\lambda+3\right)\lambda^2$$

$$D_{x_1} = \begin{vmatrix}\lambda^2+3\lambda & 1 & 1\\ \lambda^3+3\lambda^2 & \lambda+1 & 1\\ \lambda^4+3\lambda^3 & 1 & \lambda+1\end{vmatrix} = \begin{vmatrix}\lambda\left(\lambda+3\right) & 1 & 1\\ \lambda^2\left(\lambda+3\right) & \lambda+1 & 1\\ \lambda^3\left(\lambda+3\right) & 1 & \lambda+1\end{vmatrix} = \lambda\left(\lambda+3\right)\begin{vmatrix}1 & 1 & 1\\ \lambda & \lambda+1 & 1\\ \lambda^2 & 1 & \lambda+1\end{vmatrix}$$

$$\xleftrightarrow[h1(-\lambda^2)+h3]{h1(-\lambda)+h2}\lambda\left(\lambda+3\right)\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 1-\lambda\\ 0 & 1-\lambda^2 & -\lambda^2+\lambda+1\end{vmatrix} = \lambda\left(\lambda+3\right)\times 1\begin{vmatrix}1 & 1-\lambda\\ 1-\lambda^2 & -\lambda^2+\lambda+1\end{vmatrix}$$

$$= \lambda\left(\lambda+3\right)\left[-\lambda^2+\lambda+1-\left(1-\lambda^2\right)\left(1-\lambda\right)\right] = \lambda\left(\lambda+3\right)\left[-\lambda^2+\lambda+1-1+\lambda+\lambda^2-\lambda^3\right]$$

$$= \lambda\left(\lambda+3\right)\left[-\lambda^3+2\lambda\right] = \lambda^2\left(\lambda+3\right)\left(2-\lambda^2\right)$$

$$D_{x_2}=\begin{vmatrix}\lambda+1 & \lambda^2+3\lambda & 1\\ 1 & \lambda^3+3\lambda^2 & 1\\ 1 & \lambda^4+3\lambda^3 & \lambda+1\end{vmatrix}=\begin{vmatrix}\lambda+1 & \lambda(\lambda+3) & 1\\ 1 & \lambda^2(\lambda+3) & 1\\ 1 & \lambda^3(\lambda+3) & \lambda+1\end{vmatrix}=\lambda(\lambda+3)\begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda & 1\\ 1 & \lambda^2 & \lambda+1\end{vmatrix}$$

$$\xrightleftharpoons{c1\leftrightarrow c3}-\lambda(\lambda+3)\begin{vmatrix}1 & 1 & \lambda+1\\ 1 & \lambda & 1\\ \lambda+1 & \lambda^2 & 1\end{vmatrix}\xrightleftharpoons[h1(-(\lambda+1))+h3]{h1(-1)+h2}-\lambda(\lambda+3)\begin{vmatrix}1 & 1 & \lambda+1\\ 0 & \lambda-1 & -\lambda\\ 0 & \lambda^2-\lambda-1 & -\lambda^2-2\lambda\end{vmatrix}$$

$$=-\lambda(\lambda+3)\times 1\begin{vmatrix}\lambda-1 & -\lambda\\ \lambda^2-\lambda-1 & -\lambda^2-2\lambda\end{vmatrix}$$

$$=-\lambda(\lambda+3)\left[(\lambda-1)\left(-\lambda^2-2\lambda\right)-\left(\lambda^2-\lambda-1\right)(-\lambda)\right]$$

$$=-\lambda(\lambda+3)\left[-\lambda^3-2\lambda^2+\lambda^2+2\lambda+\lambda^3-\lambda^2-\lambda\right]$$

$$=-\lambda(\lambda+3)\left(-2\lambda^2+\lambda\right)=\lambda^2(\lambda+3)(2\lambda-1)$$

$$D_{x_3}=\begin{vmatrix}\lambda+1 & 1 & \lambda^2+3\lambda\\ 1 & \lambda+1 & \lambda^3+3\lambda^2\\ 1 & 1 & \lambda^4+3\lambda^3\end{vmatrix}=\begin{vmatrix}\lambda+1 & 1 & \lambda(\lambda+3)\\ 1 & \lambda+1 & \lambda^2(\lambda+3)\\ 1 & 1 & \lambda^3(\lambda+3)\end{vmatrix}=\lambda(\lambda+3)\begin{vmatrix}\lambda+1 & 1 & 1\\ 1 & \lambda+1 & \lambda\\ 1 & 1 & \lambda^2\end{vmatrix}$$

$$\xrightleftharpoons{c1\leftrightarrow c2}-\lambda(\lambda+3)\begin{vmatrix}1 & \lambda+1 & 1\\ \lambda+1 & 1 & \lambda\\ 1 & 1 & \lambda^2\end{vmatrix}\xrightleftharpoons[h1(-1)+h3]{h1(-(\lambda+1))+h2}-\lambda(\lambda+3)\begin{vmatrix}1 & 1 & 1\\ 0 & -\lambda^2-2\lambda & -1\\ 0 & -\lambda & \lambda^2-1\end{vmatrix}$$

$$=-\lambda(\lambda+3)\times 1\begin{vmatrix}-\lambda^2-2\lambda & -1\\ -\lambda & \lambda^2-1\end{vmatrix}=\lambda^2(\lambda+3)\begin{vmatrix}\lambda+2 & -1\\ 1 & \lambda^2-1\end{vmatrix}$$

$$=\lambda^2(\lambda+3)\left[(\lambda+2)\left(\lambda^2-1\right)+1\right]=\lambda^2(\lambda+3)\left(\lambda^3+2\lambda^2-\lambda-1\right)$$

Ta thaáy:

(1) Khi: $\begin{cases}\lambda\neq 0\\ \lambda\neq -3\end{cases}\Rightarrow D\neq 0$. Suy ra heä coù nghieäm duy nhaát:

$$\begin{cases}x_1=\dfrac{Dx_1}{D}=\dfrac{\lambda^2(\lambda+3)\left(2-\lambda^2\right)}{(\lambda+3)\lambda^2}=2-\lambda^2\\[2ex] x_2=\dfrac{Dx_2}{D}=\dfrac{\lambda^2(\lambda+3)(2\lambda-1)}{(\lambda+3)\lambda^2}=2\lambda-1\\[2ex] x_3=\dfrac{Dx_3}{D}=\dfrac{(\lambda+3)\lambda^2\left(\lambda^3+2\lambda^2-\lambda-1\right)}{(\lambda+3)\lambda^2}=\lambda^3+2\lambda^2-\lambda-1\end{cases}$$

(2) Khi $\left[\begin{array}{l}\lambda=0\\ \lambda=-3\end{array}\right.\Rightarrow D=0$ vaø $D_{x_1}=D_{x_2}=D_{x_3}=0$ suy ra heä coù voâ soá nghieäm