FORMATIVE ASSESSMENT 5

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Problem 1

8.18. List all samples of size n ¼ 2 that are possible (with replacement) from the population in Problem 8.17. Using R, plot the sampling distribution of the mean to show that

$$\mu_{\bar{x}} = \mu$$

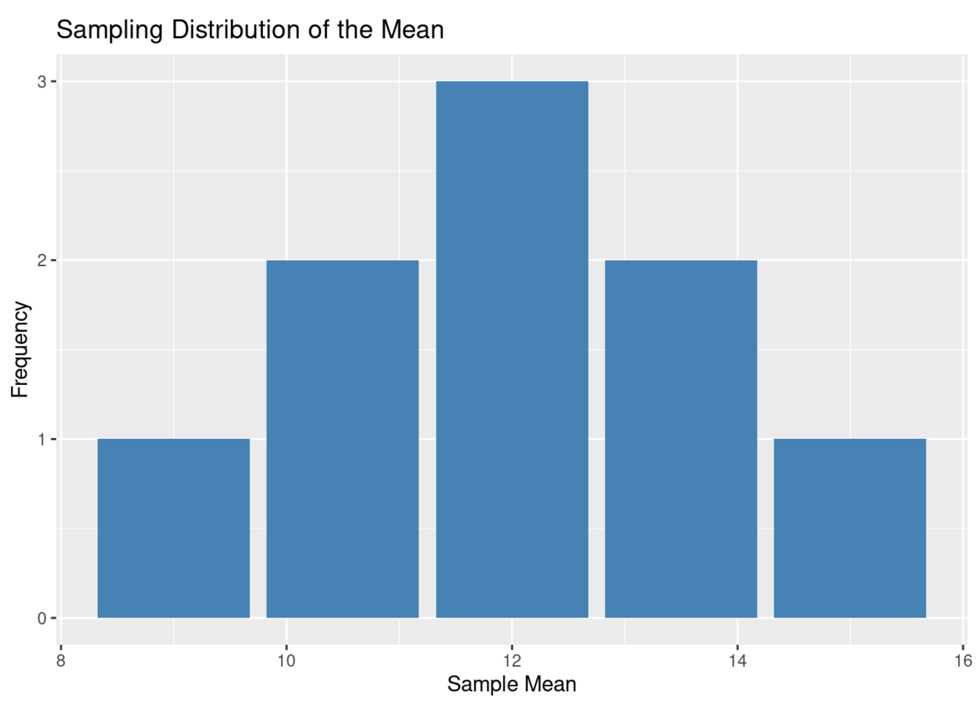
, and show that

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{2}$$

.

Sampling Distribution Table

X1	X2	mean	xbar	p(xbar)	xbar * p(xbar)	xbar^2 * p(xbar)
9	9	9.0	9.0	0.1111111	1.000000	9.00
12	9	10.5	10.5	0.1111111	1.166667	12.25
15	9	12.0	12.0	0.1111111	1.333333	16.00
9	12	10.5	10.5	0.1111111	1.166667	12.25
12	12	12.0	12.0	0.1111111	1.333333	16.00
15	12	13.5	13.5	0.1111111	1.500000	20.25
9	15	12.0	12.0	0.1111111	1.333333	16.00
12	15	13.5	13.5	0.1111111	1.500000	20.25
15	15	15.0	15.0	0.1111111	1.666667	25.00



Problems 2

8.21. A population consists of the four numbers 3, 7, 11, and 15. Consider all possible samples of size 2 that can be drawn with replacement from this population. Find (a) the population mean,(b) the population standard deviation, (c) the mean of the sampling distribution of means, and (d) the standard deviation of the sampling distribution of means. Verify parts (c) and (d) directly from (a) and (b) by using suitable formulas.

Population Mean and Standard Deviation

The population consists of the numbers 3, 7, 11, and 15.

(a) Population Mean μ

The population mean is calculated as:

$$\mu = \frac{3+7+11+15}{4} = \frac{36}{4} = 9$$

(b) Population Standard Deviation σ

The population standard deviation is calculated as:

$$\sigma = \sqrt{\frac{(3-9)^2+(7-9)^2+(11-9)^2+(15-9)^2}{4}}$$

First, compute the squared differences:

$$(3-9)^2 = 36, \quad (7-9)^2 = 4, \quad (11-9)^2 = 4, \quad (15-9)^2 = 36$$

Summing these values gives:

$$36+4+4+36 = 80$$

Now, divide by the number of elements:

$$\sigma = \sqrt{\frac{80}{4}} = \sqrt{20} = 4.47$$

(c) Mean of the Sampling Distribution of Means $\mu_{\bar{X}}$

Since we are sampling with replacement, the mean of the sampling distribution of means is equal to the population mean:

$$\mu_{\bar{X}} = \mu = 9$$

(d) Standard Deviation of the Sampling Distribution of Means $\sigma_{\bar{X}}$

The standard deviation of the sampling distribution of means is calculated as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4.47}{\sqrt{2}} = \frac{4.47}{1.414} = 3.16$$

Verification:

The calculations confirm that:

1. $$\mu_{\bar{X}} = \mu = 9$$
2. $$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 3.16$$

8.34. Find the probability that of the next 200 children born, (a) less than 40% will be boys, (b) between 43% and 57% will be girls, and (c) more than 54% will be boys. Assume equal probabilities for the births of boys and girls.

We are given that the probability of a child being a boy is $p = 0.5$, and the number of births is $n = 200$. The number of boys X follows a binomial distribution:

$$X \sim \text{Binomial}(n = 200, p = 0.5)$$

The binomial distribution can be approximated by a normal distribution:

$$X \sim N(\mu = np, \sigma^2 = np(1-p))$$

The mean and standard deviation are:

$$\mu = np = 200 \times 0.5 = 100$$

$$\sigma^2 = np(1-p) = 200 \times 0.5 \times 0.5 = 50$$

$$\sigma = \sqrt{50} \approx 7.071$$

(a) Probability that less than 40% will be boys

We want to compute $P(X < 0.40 \times 200)$, which translates to:

$$P(X < 80)$$

To approximate this using the normal distribution, we convert to the z-score:

$$Z = \frac{X-\mu}{\sigma} = \frac{80-100}{7.071} = \frac{-20}{7.071} \approx -2.828$$

Now, using the cumulative distribution function of the standard normal distribution:

$$P(Z < -2.828) \approx 0.0023$$

Thus, the probability that less than 40% will be boys is approximately:

$$P(X < 80) \approx 0.0023$$

(b) Probability that between 43% and 57% will be girls

We want to compute $P(0.43 \times 200 \leq X_{\text{girls}} \leq 0.57 \times 200)$, which translates to:

$$P(86 \leq X_{\text{girls}} \leq 114)$$

Since $X_{\text{girls}} = 200 - X_{\text{boys}}$, we are actually looking for:

$$P(86 \leq X_{\text{boys}} \leq 114)$$

To compute this, we standardize both limits:

For $X = 86$:

$$Z = \frac{86-100}{7.071} = \frac{-14}{7.071} \approx -1.98$$

For $X = 114$:

$$Z = \frac{114-100}{7.071} = \frac{14}{7.071} \approx 1.98$$

Thus, we compute the probability:

$$P(-1.98 < Z < 1.98) = P(Z < 1.98) - P(Z < -1.98)$$

From standard normal tables:

$$P(Z < 1.98) \approx 0.9761, \quad P(Z < -1.98) \approx 0.0239$$

So:

$$P(-1.98 < Z < 1.98) = 0.9761 - 0.0239 = 0.9522$$

Thus, the probability that between 43% and 57% will be girls is approximately:

$$P(86 \leq X_{\text{girls}} \leq 114) \approx 0.9522$$

(c) Probability that more than 54% will be boys

We want to compute $P(X > 0.54 \times 200)$, which translates to:

$$P(X > 108)$$

Standardize this value:

$$Z = \frac{108-100}{7.071} = \frac{8}{7.071} \approx 1.13$$

Now, using the cumulative distribution function:

$$P(Z > 1.13) = 1 - P(Z < 1.13)$$

From the standard normal table:

$$P(Z < 1.13) \approx 0.8708$$

So:

$$P(Z > 1.13) = 1 - 0.8708 = 0.1292$$

Thus, the probability that more than 54% will be boys is approximately:

$$P(X > 108) \approx 0.1292$$

Summary of Results

- **(a)**: The probability that less than 40% of the next 200 children born will be boys is approximately 0.0023.
- **(b)**: The probability that between 43% and 57% will be girls is approximately 0.9522.
- **(c)**: The probability that more than 54% will be boys is approximately 0.1292.

Problem 3

8.49 The credit hour distribution at Metropolitan Technological College is as follows:

x	p(x)
6	0.1
9	0.2
12	0.4
15	0.2
18	0.1

Find μ and σ^2. Give the 25 (with replacement) possible samples of size 2, their means, and their probabilities.

The mean is 12

The mean μ is calculated as:

$$\mu = \sum x \cdot p(x) = (6 \cdot 0.1) + (9 \cdot 0.2) + (12 \cdot 0.4) + (15 \cdot 0.2) + (18 \cdot 0.1)$$

$$\mu = 12$$

The variance is 10.8

The variance σ^2 is calculated as:

$$\sigma^2 = \sum (x-\mu)^2 \cdot p(x) = (6-12)^2 \cdot 0.1 + (9-12)^2 \cdot 0.2 + (12-12)^2 \cdot 0.4 + (15-12)^2 \cdot 0.2 + (18-12)^2 \cdot 0.1$$

Substitute the differences:

$$\sigma^2 = (-6)^2 \cdot 0.1 + (-3)^2 \cdot 0.2 + (0)^2 \cdot 0.4 + (3)^2 \cdot 0.2 + (6)^2 \cdot 0.1$$

Now calculate the squares and multiply:

$$\sigma^2 = 36 \cdot 0.1 + 9 \cdot 0.2 + 0 \cdot 0.4 + 9 \cdot 0.2 + 36 \cdot 0.1$$

$$\sigma^2 = 3.6 + 1.8 + 0 + 1.8 + 3.6$$

Finally, sum the values:

$$\sigma^2 = 10.8$$

The 25 (with replacement) possible samples of size 2, their means, and their probabilities.

Possible Samples of Size 2, Their Means, and Probabilities

Sample 1	Sample 2	Mean	Probability
6	6	6.0	0.01
9	6	7.5	0.02
12	6	9.0	0.04
15	6	10.5	0.02
18	6	12.0	0.01
6	9	7.5	0.02
9	9	9.0	0.04
12	9	10.5	0.08
15	9	12.0	0.04
18	9	13.5	0.02
6	12	9.0	0.04
9	12	10.5	0.08
12	12	12.0	0.16
15	12	13.5	0.08
18	12	15.0	0.04
6	15	10.5	0.02
9	15	12.0	0.04
12	15	13.5	0.08
15	15	15.0	0.04
18	15	16.5	0.02
6	18	12.0	0.01
9	18	13.5	0.02
12	18	15.0	0.04
15	18	16.5	0.02
18	18	18.0	0.01