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Analytical Expressions for Per-Unit-Length Parameters of Finite Length Transmission Lines With Discontinuities

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***Abstract*—Analytical expressions for per-unit-length parameters of finite-length transmission lines are derived and verified by the method of moment in this paper. The line can be a conductor with bends and varying sizes. The expressions are explicit in form and can be used to model and analyze the effects from ends and bends on the line effectively.**

***Index Terms*—Distributed parameters, method of moment, nonuniform transmission line, signal integrity.**

I. Introduction

When a transmission line is of finite length, the end effects of the line may cause disturbing electromagnetic fields. On the other hand, bends and other discontinuities also result in disturbing fields. These disturbing fields may limit the performance of interconnects in dense circuit and package for IC therefore become an important issue in signal integrity (SI) analysis and emission reduction.

For an infinite long, uniform, and nonlossy line, its transverse field has the same distribution along the line if we ignore the propagation factor $\exp[j(\omega t \pm \gamma s)]$, where γ is the propagation constant and s is the measure of distance along the line. When the line is of finite length or with discontinuities such as bends, due to the disturbing fields, the line will appear as a nonuniform transmission line. For a line that is relatively long and when a section apart from the ends or bends of the line needs to be considered, then this section can be regarded as uniform. As there are always two ends for any real transmission line and a line may often encounter bends and other discontinuities, therefore, the study for effects from ends, bends, and discontinuities is a practical issue. One way to account for these effects of a long line is to use excess capacitance and inductance in circuit models. But for short line, the disturbing electromagnetic fields due to these effects will play an important role on the whole line, which makes the line more like a nonuniform transmission line, and therefore, a methodology for modeling the line more precisely will be useful.

For finite length lines, King and Tomiyasa developed a theoretical approach based on electromagnetic potentials [1], [2]. But the research was specified for the analysis of various terminations. More recently,

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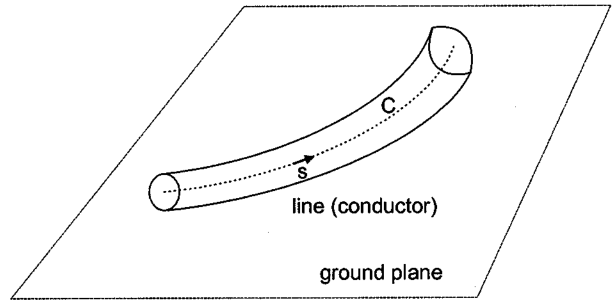


Fig. 1. A transmission line of finite length over ground plane.

W Getsinger developed a model for end-effects in quasi-TEM transmission lines [3]. Han and Smith investigated the terminal effect of cables underground [4]. For the finite-length line with discontinuity, Liu and Kami studied the reflection on the line and emission from the line [5]. For the finite-length nonuniform line, Grivet-Talocia and Canavero calculated current and voltage distribution numerically [6] while Miao and Canavero discussed the crosstalk of lossy line with arbitrary loads [7]. As the study for end effects, finite length, bends, and discontinuities has attracted interests for many years, here we will not intend to have an overall literature review for the issue of transmission lines such as microstrip lines, waveguides, and millimeterwave circuits, etc. Only a few, which are relevant to this paper, are highlighted above.

In our previous work [8], a model for analyzing bend and interconnection of conductors was proposed, but the issue for end effects was not addressed. In this paper, we will study the issue for a finite length conductor with bend and varying radii. The scopes and methods in this paper are different from those in the aforementioned references. By further developing the methodology in [8], analytical expressions for per-unit-length parameters of finite length transmission line with bend will be given. Results by these expressions will be compared with those by method of moments (MOM), and a good correlation has been achieved. The expressions for the per-unit-length parameters are explicit in form and easy to use. To the best of our knowledge, no such expressions have been introduced for the problem elsewhere. The results introduced in this paper are very useful to the modeling of interconnects on printed circuit boards (PCB) and to parameter extraction of IC package for EMC concerns as well as for other applications.

II. Formulation

The configuration for the problem is shown in Fig. 1, where a transmission line of finite length sits over infinite ground plane. The line is thin compared with its distance to the ground. Both the line and the ground are perfect electric conductors while the cross section of the conductor is circular. One basic assumption is that the system support quasi-TEM waves. For this problem, in our previous work [8], nonuniform transmission line equations have been derived as

$$\frac{dI(s)}{ds} = -j\omega c(s)V(s) \tag{1a}$$

$$\frac{dV(s)}{ds} = -j\omega l(s)I(s) \tag{1b}$$

where $I(s)$ represents the total current on the axis C of the conductor and V is the voltage. s is the local coordinate along the line. $l(s)$ and $c(s)$ are the per-unit-length inductance and capacitance of the line.

It should be mentioned that conventional ways of deriving transmission line equations are from the integral or differential forms of Maxwell's equations or from the per-unit-length equivalent circuit, where intermediate parameters such as the magnetic flux and total electric charge are required [9]. For (1), the derivation is based on the thin-wire approximation for the current, quasi-TEM wave assumption, for the fields and the matching boundary condition on the conductor. No intermediate parameters, such as magnetic flux, are used.

The above formulation is for an arbitrary line as shown in Fig. 1. Now, we will focus on a line that is formed by connecting two conductors of different radii at an angle Φ_0 over the ground plane, as shown in Fig. 2, in which the axis of the conductors is parallel to the ground. Based on the formulations in [8], the per-unit-length parameters can be derived as

$$l(s) = \frac{\mu}{4\pi}\begin{cases}\Lambda_1^- + \cos\Phi_0\Lambda_2^+, & s<0\\ \cos\Phi_0\Lambda_2^- + \Lambda_1^+, & s\geq 0\end{cases} \tag{2a}$$

$$c(s) = 4\pi\varepsilon\begin{cases}1/[\Lambda_1^- + \Lambda_2^+], & s<0\\ 1/[\Lambda_2^- + \Lambda_1^+], & s\geq 0\end{cases} \tag{2b}$$

where

$$\Lambda_1^- = E_i(-jkt)\Big|_{\sqrt{s^2+(2h)^2}-s}^{\sqrt{s^2+a_1^2}-s} - E_i(-jkt)\Big|_{\sqrt{(L_1+s)^2+(2h)^2}-(L_1+s)}^{\sqrt{(L_1+s)^2+a_1^2}-(L_1+s)} \tag{3a}$$

$$\Lambda_2^- = e^{jks(1-\cos\Phi_0)}\left[E_i(-jkt)\Big|_{\sqrt{s^2+(2h)^2+a_1^2}-s\cos\Phi_0}^{\sqrt{s^2+a_2^2}-s\cos\Phi_0} - E_i(-jkt)\Big|_{\sqrt{(L_1+s\cos\Phi_0)^2+(s\sin\Phi_0)^2+(2h)^2}-(L_1+s\cos\Phi_0)}^{\sqrt{(L_1+s\cos\Phi_0)^2+(s\sin\Phi_0)^2+a_1^2}-(L_1+s\cos\Phi_0)}\right] \tag{3b}$$

$$\Lambda_1^+ = E_i(-jkt)\Big|_{\sqrt{(L_2-s)^2+(2h)^2}+(L_2-s)}^{\sqrt{(L_2-s)^2+a_2^2}+(L_2-s)} - E_i(-jkt)\Big|_{\sqrt{s^2+(2h)^2}-s}^{\sqrt{s^2+a_2^2}-s} \tag{3c}$$

$$\Lambda_2^+ = e^{jks(1-\cos\Phi_0)}\left[-E_i(-jkt)\Big|_{\sqrt{s^2+(2h)^2}-s\cos\Phi_o}^{\sqrt{s^2+a_1^2}-s\cos\Phi_o} + E_i(-jkt)\Big|_{\sqrt{(L_2-s\cos\Phi_0)^2+(s\sin\Phi_0)^2+(2h)^2}+(L_2-s\cos\Phi_o)}^{\sqrt{(L_2-s\cos\Phi_0)^2+(s\sin\Phi_0)^2+a_1^2}+(L_2-s\cos\Phi_o)}\right] \tag{3d}$$

In these expressions, k is the free-space wave number and $E_i(-jkt)$ can be expressed by sine and cosine integration as

$$E_i(-jkt)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2}\frac{e^{-jkt}}{t}\,dt = [C_i(kt) - jS_i(kt)]\Big|_{t_1}^{t_2}. \tag{4}$$

The closed form expressions (2) and (3) for the per-unit-length parameters of the line can be further simplified for different cases, such as for the straight line of finite length or for the line of same radius, etc. Since $l(s)$ and $c(s)$ as given above vary with the distance s, so that (1) is a nonuniform transmission line (NTML) equation, we will denote our method of calculating $l(s)$ and $c(s)$ as NTML method.

III. Results and Discussions

In (2) and (3), end-effects due to finite length of conductors and influences from the bend are included. In the special case of infinite long straight line of same radius, by using the asymptotic expressions for sine and cosine integration, it can be obtained that $l(s) \approx (\mu/2\pi\ln(2h/a_1)$ while $c(s) = \varepsilon\mu/l(s)$, which are well-known results [9]. When there is a bend on the infinite long line, by using (2) and (3) to get the per-unit-length parameters, nonuniform transmission line equation (1) is determined and can be solved by using the matrix approach. For this case, the obtained voltage distribution along the line coincides well with the experimental results [8], [10].

For a more general case of the finite-length conductor with bend, the per-unit-length parameters calculated from (2) and (3) will be compared with the results of MOM to verify the applicability of the formulation.

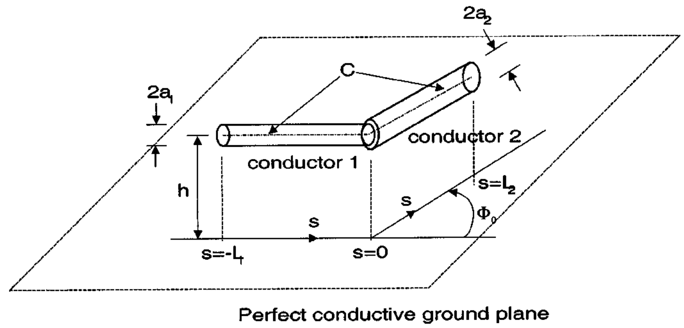


Fig. 2. Conductors with bend and different radii over ground plane.

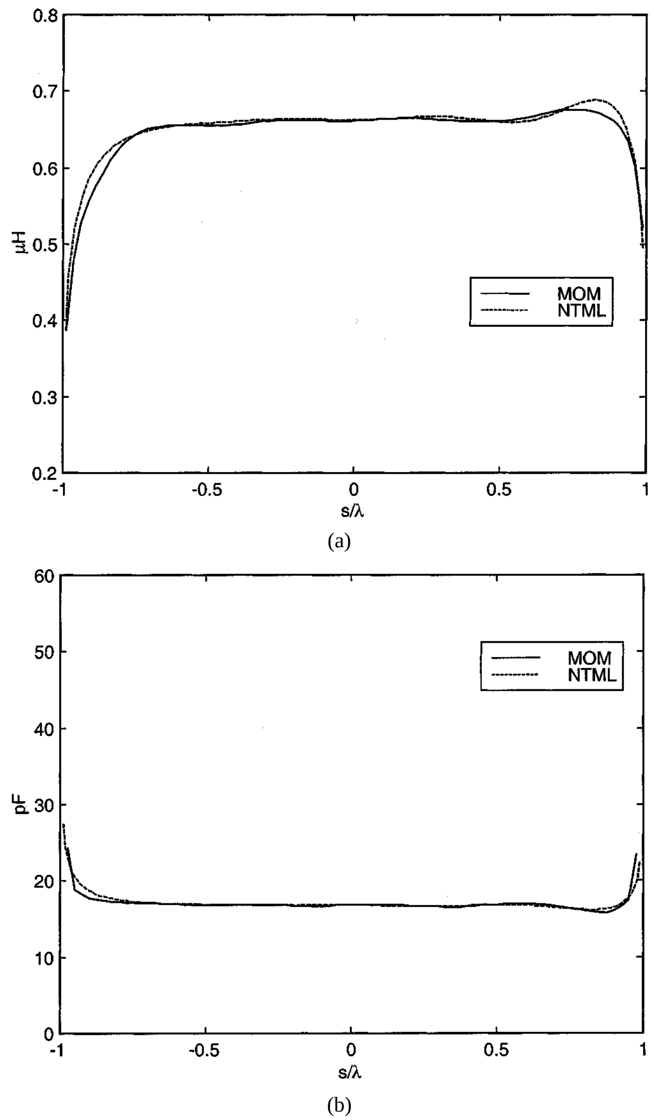


Fig. 3. Per-unit-length parameters for straight line ($\Phi_0 = 0$) of finite length $L_1 = L_2 = 1\lambda$ and same radius with $a_1 = a_2 = 0.0084\lambda$. (a) Inductance (b) Capacitance.

In order for MOM analysis, the domain for the solution is partitioned into sections. Voltage and current are discretized and point matching is used. Once the voltage sources at the two terminals are known, the voltage and current at any point along the line can be determined. It is noteworthy to point out that the voltage-excitation matrix in this MOM

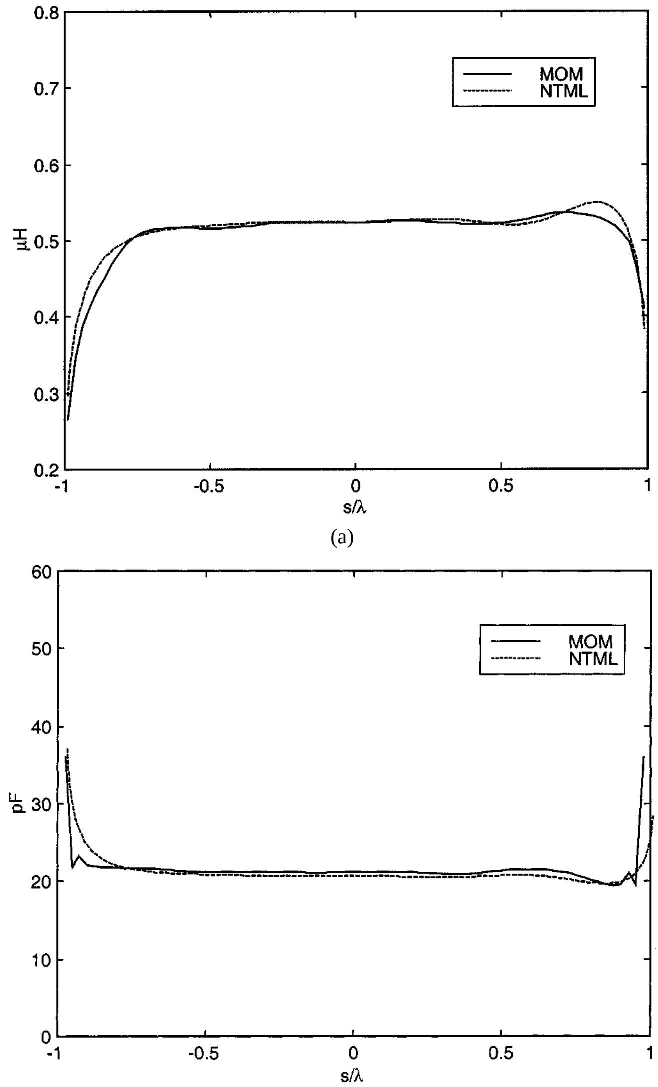


Fig. 4. Per-unit-length parameters for straight line ($\Phi_0 = 0$) of finite length $L_1 = L_2 = 1\lambda$ with larger radii $a_1 = a_2 = 0.0167\lambda$. (a) Inductance. (b) Capacitance.

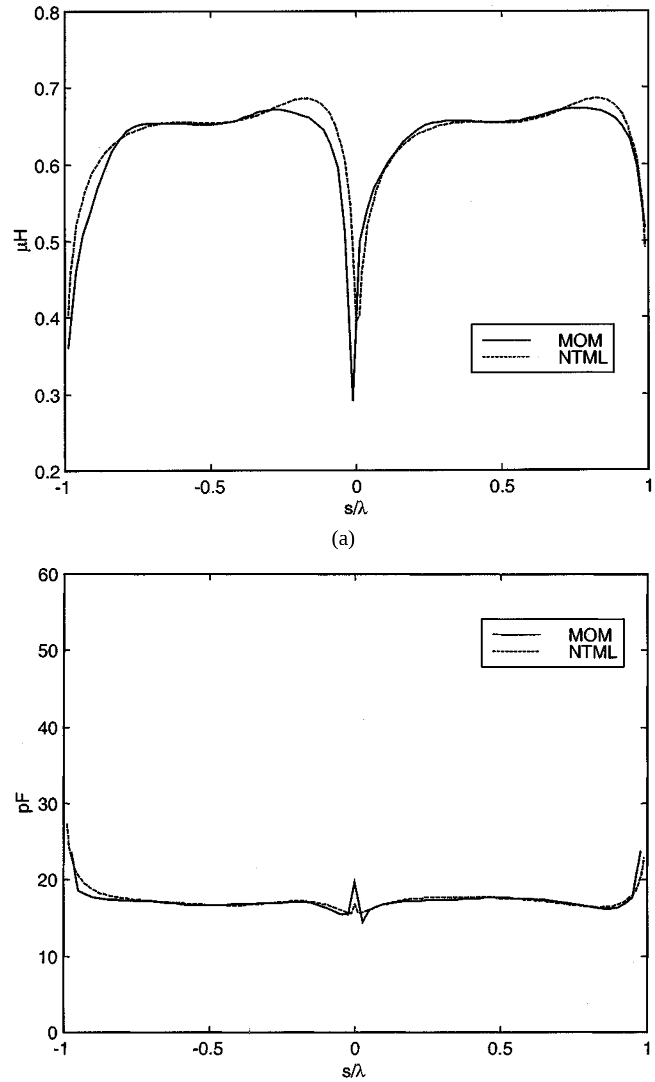


Fig. 5. Per-unit-length parameters for line with a right angle bend $\Phi_0 = \pi/2$ of same radii $a_1 = a_2 = 0.0084\lambda$ and finite length $L_1 = L_2 = 1\lambda$. (a) Inductance. (b) Capacitance.

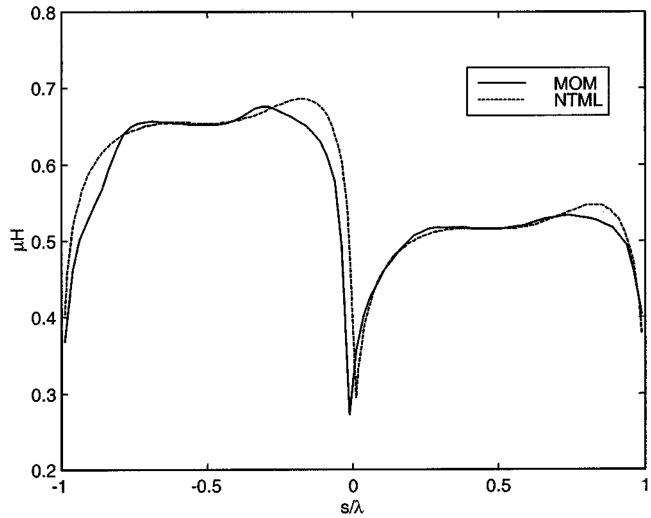
analysis is a little bit different from that in conventional MOM for antenna and scattering problems, where a term in the voltage excitation matrix usually has the form of $\bar{E} \cdot \Delta\bar{l}$. Unfortunately, if we do so in this conventional way, then, a problem will be encountered, i.e., additional conductors at two terminals will be needed to support the line, in order to impose the voltage sources. These additional conductors will incur unexpected influence to the analyzed line, while the discussion will also be more like an antenna or a scattering issue. In our MOM approach, we directly use voltage and current as unknowns to bypass the problem. Once the voltage and current have been determined, the per-unit-length capacitance and inductance of the line can be obtained approximately via lumped-circuit model [9], [11].

In the following discussion, the height h of the line is fixed to be 0.1165λ, where λ is the free-space wavelength. Fig. 3 gives the results by MOM and by NTML for $l(s)$ and $c(s)$ of a finite length straight line. In the figure, the steady-state values for $l(s)$ and $c(s)$ coincidence with the infinite long straight line parameters that are $c = 16.7$ pF and $l = 0.67$ μH.

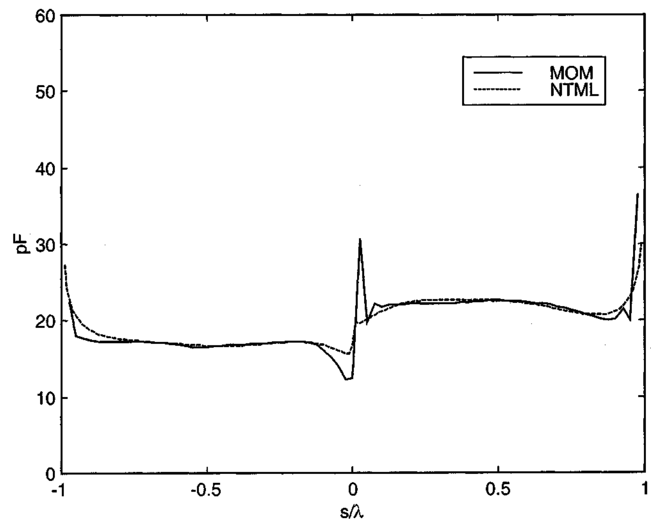
When the line is short or when the radii of conductors are relatively large, it is noted in the numerical simulation that the results by NTML and by MOM agree very well for the inductance. While for the capacitance, general agreement is achieved. Fig. 4 gives an example, in which parameters are the same as in Fig. 3, except that the radii of the conductors in Fig. 4 are twice as much as that in Fig. 3. There is small fluctuation in the results of MOM at the two ends as shown in Fig. 4(b). One reason for this phenomenon is possibly due to the short line and/or large radii, where the actual field distribution is more likely to deviate from the assumed TEM field. Though there is such discrepancy, overall, the results by MOM agree very well with those by NTML, for the inductance, as well as for the capacitance.

For a bend in the line, it is often regarded as a discontinuity. Fig. 5 clearly shows the variation of $c(s)$ and $l(s)$ along a line of same radius with a right angle bend. For a line that extends to infinity at two sides, the bend effects can be easily extracted by using (2) with asymptotic expressions to the integration in (3), while excess capacitance and inductance due to the bend can also be evaluated based on a given circuit model.

Finally, for a more general case of a finite-length line with a bend and various radii, Fig. 6 plots its per-unit-length parameters. In this case, there are effects from ends, bend and different radii. Again, the results by MOM agree well with those by NTML.



(a)



(b)

Fig. 6. Per-unit-length parameters for a line with a right-angle bend $\Phi_0 = \pi/2$ and different radii $a_1 = 0.0084\lambda$, $a_2 = 0.0167\lambda$ of finite length $L_1 = L_2 = 1\lambda$. (a) Inductance. (b) Capacitance.

In the previous calculation, the value for capacitance and inductance are complex with positive real part and small imaginary part. When the line is straight and infinitely long, there will be no such imaginary part. For simplicity, the data depicted in the figures are the amplitude.

For the NTML approach, terminal conditions (voltage sources and loads) will not be included in formulation. While in MOM analysis, data for the terminal conditions will be used. These data will affect current and voltage distributions along the line, but will not influence our results for the per-unit-length parameters.

It should be mentioned that both MOM and NTML analyses in this paper are based on the integral equation of [8, eqn. (4)]. The integral equation is strictly valid under the thin-wire approximation for current distribution and quasi-TEM approximation for fields. From the same integral equation, modeling for MOM and formulation for NTML are developed independently. As it is believed that MOM is feasible for thin-wire problems, therefore the general agreement between the results by MOM and by NTML for the problem reveals that expressions given by (2) and (3) are applicable and give a good prediction for the parameters.

For further usage, by using NTML approach developed in this paper, various equivalent lumped-circuit models can be established in order to account for the effects from ends or bends or abrupt changes of line radius, while excess inductance and capacitance are commonly used in these circuit models. By using the expressions for per-unit-length parameters given here, derivation or calculation of the excess inductance and capacitance are, in accordance with circuit models, straightforward and will not be studied in this paper.

Although the above discussions are mainly for cases of straight conductor connection, the method developed in this paper can also be used for transmission line of curvilinear conductor, conductor with multi-bends and conductors of continuously varying sizes.

IV. Conclusion

In conclusion, analytical expressions for the per-unit-length parameters of transmission line of finite length and various radii with bend have been derived. The expressions are verified by the method of moment. The method used in this paper can be further developed for lines of various configurations. Based on this research, lumped-circuit models can be established to account for the effects from line ends, bends or other discontinuities.

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