Fermi Constants and "New Physics"

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Abstract

Various precision determinations of the Fermi constant are compared. Included are muon and (leptonic) tau decays as well as indirect prescriptions employing α, m_Z, m_W, $\sin^2\theta_W(m_Z)_{\overline{MS}}$, $\Gamma(Z \to \ell^+\ell^-)$, and $\Gamma(Z \to \nu\bar{\nu})$ as input. Their good agreement tests the standard model at the $\pm 0.1\%$ level and provides stringent constraints on new physics. That utility is illustrated for: heavy neutrino mixing, 2 Higgs doublet models, S, T, and U parameters and excited $W^{*\pm}$ bosons (Kaluza-Klein excitations). For the last of those examples, $m_{W^*} \gtrsim 2.9$ TeV is found.

The Fermi constant, G_F, is an important, venerable holdover from the old local theory of weak interactions [1]. Expressed in terms of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ standard model parameters, it is given by

$$G_F = g_2^2/4\sqrt{2}m_W^2 \tag{1}$$

where g_2 is an $\mathrm{SU}(2)_L$ gauge coupling and m_W is the $W^\pm$ gauge boson mass. To be more precise, G_F must be expressed in terms of physical observables or well prescribed renormalized parameters. Also, electroweak radiative corrections must be properly accounted for.

Traditionally, the muon lifetime, τ_μ, has been used to define the Fermi constant because of its very precise experimental value [2]

$$\tau_\mu = 2.197035(40) \times 10^{-6} s \tag{2}$$

and theoretical simplicity. Labeling that definition by G_μ, it is related to τ_μ via [3]

$$\tau_\mu^{-1} = \Gamma(\mu \to \text{all}) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)(1 + R.C.)\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)$$
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ell n x \tag{3}$$

In that expression, R.C. stands for radiative corrections and the $\frac{3}{5}m_\mu^2/m_W^2$ term is a small W boson propagator effect. The R.C. expression is somewhat arbitrary. Most quantum loop corrections to muon decay are absorbed into the renormalized parameter G_μ. For historical reasons and in the spirit of effective field theories, R.C. is defined to be the QED radiative corrections to muon decay in the local V-A four fermion description of muon decay. That separation is natural and practical, since those QED corrections are finite to all orders in perturbation theory [3]. In fact, they have been fully computed through $\mathcal{O}(\alpha^2)$ and are given by

$$R.C. = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{\alpha}{\pi}\left(\frac{2}{3}\ell n\frac{m_\mu}{m_e} - 3.7\right) + \left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{9}\ell n^2\frac{m_\mu}{m_e} - 2.0\ell n\frac{m_\mu}{m_e} + C\right) + \cdots\right) \tag{4}$$

where α is the fine structure constant

$$\alpha^{-1} = 137.03599959(40) \tag{5}$$

The leading $\mathcal{O}(\alpha)$ term in that expression has been known for 4 decades from the pioneering work of Kinoshita and Sirlin [4] and Berman [5]. Coefficients of higher order $\ell n\frac{m_\mu}{m_e}$ terms are determined by the renormalization group requirement [6]

$$\left(m_e\frac{\partial}{\partial m_e} + \beta(\alpha)\frac{\partial}{\partial\alpha}\right) R.C. = 0$$
$$\beta(\alpha) = \frac{2}{3}\frac{\alpha^2}{\pi} + \frac{1}{2}\frac{\alpha^3}{\pi^2} + \cdots \tag{6}$$

The -3.7 two loop term was very recently computed by van Ritbergen and Stuart [7]. Their result also implies the next-to-leading logs in (4) via (6), leaving C as the only unknown $\mathcal{O}(\alpha^3)$ contribution to R.C. Comparing (3) and (2), one finds

$$G_\mu = 1.16637(1) \times 10^{-5}\text{GeV}^{-2} \tag{7}$$

which is, by far, the best determination of the Fermi constant. In fact, it is more than 100 times better than the other prescriptions considered in this paper. Nevertheless, there have been several proposals to further reduce the uncertainty in τ_μ and G_μ by an additional factor of 10. Given the fundamental nature of G_μ, such measurements should certainly be encouraged. However, from the point of view of testing the standard model, some other independent determination of the Fermi constant would have to catch up to G_μ before a more precise τ_μ measurement could be fully utilized.

In the renormalization of G_μ, lots of interesting quantum loop effects have been absorbed. Included are top and Higgs loop corrections to the W boson propagator as well as potential new physics from supersymmetry, technicolor, etc. Even possible tree level contributions, for example from massive excited ${W^*}^\pm$ bosons or other effects, might be encoded in G_μ. To unveil such contributions requires comparison of G_μ with other independent determinations of the Fermi constant that could have different tree or loop level dependences.

Because of the renormalizability of the standard model, universality of bare gauge couplings among lepton generations [8]

$$g^e_{2_0} = g^\mu_{2_0} = g^\tau_{2_0} \tag{8}$$

and the bare natural relations [9]

$$\sin^2\theta^0_W = \frac{e_0^2}{g^2_{2_0}} = 1 - (m^0_W/m^0_Z)^2, \tag{9}$$

there are many ways to determine Fermi constants and compute very precisely their relationships with G_μ. Comparison of those quantities can then be used to test the standard model and probe for new physics.

The leptonic decay widths of the tau can provide, in close analogy with muon decay, Fermi constants $G_{\tau\ell}, \ell = e$ or μ. Including $\mathcal{O}(\alpha)$ QED corrections, one employs the radiative inclusive rate [10]

$$\Gamma(\tau \to \ell\nu\bar{\nu}(\gamma)) = \frac{G^2_{\tau\ell} m^5_\tau}{192\pi^3} f\left(\frac{m^2_\ell}{m^2_\tau}\right)\left(1 + \frac{3}{5}\frac{m^2_\tau}{m^2_W}\right)\left(1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right) \tag{10}$$

Those Fermi constants have been normalized, through $\mathcal{O}(\alpha)$, such that $G_{\tau e} = G_{\tau\mu} = G_\mu$ in the standard model. That is possible because $g^e_{2_0} = g^\mu_{2_0} = g^\tau_{2_0}$ and the $\mathcal{O}(\alpha)$ radiative corrections are the same (up to $\mathcal{O}(\alpha m_\tau^2/m_W^2)$).

Employing the experimental averages [11]

$$\tau_\tau = 290.5 \pm 1.0 \times 10^{-15} s \tag{11a}$$

$$B(\tau \to e\nu\bar{\nu}(\gamma)) = 0.1781(6) \tag{11b}$$

$$B(\tau \to \mu\nu\bar{\nu}(\gamma)) = 0.1736(6) \tag{11c}$$

implies

$$\Gamma(\tau \to e\nu\bar{\nu}(\gamma)) = 4.035(19) \times 10^{-13} \text{ GeV} \tag{12a}$$

$$\Gamma(\tau \to \mu\nu\bar{\nu}(\gamma)) = 3.933(19) \times 10^{-13} \text{ GeV} \tag{12b}$$

Used in conjunction with

$$m_\tau = 1777.0(3) \text{ MeV}, \tag{13}$$

those widths lead to

$$G_{\tau e} = 1.1666(28) \times 10^{-5} \text{ GeV}^{-2} \tag{14}$$

$$G_{\tau\mu} = 1.1679(28) \times 10^{-5} \text{ GeV}^{-2} \tag{15}$$

They are in very good accord with G_μ, but their errors are nearly 300 times larger. Nevertheless, collectively those Fermi constants test e-μ-τ universality at the $\pm 0.2\%$ level

$$g^e_{2_0} : g^\mu_{2_0} : g^\tau_{2_0} :: 1 : 1.0011(24) : 1.0006(24) \tag{16}$$

(employing (11b) and (11c) directly).

The good agreement between G_μ and the $G_{\tau\ell}$ can be used to constrain new physics. Consider, for example, the effect of a heavy fourth generation lepton doublet (ν_4, L) with masses $\gtrsim 95$ GeV; so, it would have escaped detection at existing colliders. Parametrizing the 3rd and 4th generation mixing by θ_{34}, one has (assuming no mixing with the first or second generations) [12–14]

$$\nu_\tau = \nu_3 \cos\theta_{34} + \nu_4 \sin\theta_{34} \tag{17}$$

That being the only mixing effect, one would expect $G_{\tau\ell} = G_\mu \cos\theta_{34}$. Combining (14) and (15) to get $G_{\tau\ell}^{\rm ave} = 1.1672(25) \times 10^{-5}$ GeV^{-2} and comparing with G_μ, one finds the rather stringent bound

$$\sin\theta_{34} < 0.075 \qquad (95\%\mathrm{CL}) \tag{18}$$

What value of $\sin\theta_{34}$ might be reasonable in such a scenario? If an analogy with quark mixing is appropriate, one might guess [12] $\sin\theta_{34} \approx \sqrt{m_\tau/m_L}$. If that is the case, (18) translates to $m_L \gtrsim 316$ GeV. An additional factor of 2 improvement in $G_{\tau\ell}$ would push that probe into the very interesting $m_L \gtrsim 850$ GeV region, under the above assumptions. A similar analysis could be applied to singlet neutrinos or more general mixing scenarios. Note, however, that heavy ν_4 mixing with the first two generations of neutrinos must be suppressed due to constraints from $\mu \to e\gamma$ and $\mu^- N \to e^- N$ searches.

As a second illustration of new physics, consider the general 2 Higgs doublet model with $\tan\beta = v_2/v_1$ and physical scalar masses m_h, m_H, m_A and $m_{H^\pm}$. Charged Higgs scalar exchange at the tree level would reduce the tau leptonic decay rates by a factor [15] $\left(1 - \frac{2m_\ell^2}{m_{H^\pm}^2}\tan^2\beta\right)$ and thus effectively imply $G_{\tau\mu} < G_{\tau e}$. However, the good agreement between (15) and (14) can be used to set the bound [12]

$$m_{H^\pm} \gtrsim 2\tan\beta \text{ GeV} \qquad (95\% \text{ CL}) \tag{19}$$

For large $\tan\beta \gtrsim 45$, that bound is competitive with direct e^+e^- collider searches as well as constraints from $B \to \tau\nu X$ [16]. However, $b \to s\gamma$ measurements generally give a more restrictive bound. Constraints on the spectrum of scalars can also be obtained by comparing G_μ and $G_{\tau\ell}$, but they will not be discussed here [17].

There are also a number of indirect prescriptions for obtaining Fermi constants. For example, using the natural relations in (9), one can define [18,19]

$$G_F^{(1)} = \frac{\pi\alpha}{\sqrt{2}m_W^2(1-m_W^2/m_Z^2)(1-\Delta r)} \tag{20}$$

$$G_F^{(2)} = \frac{\pi\alpha}{\sqrt{2}m_W^2\sin^2\theta_W(m_Z)_{\overline{MS}}(1-\Delta r(m_Z)_{\overline{MS}})} \tag{21}$$

$$G_F^{(3)} = \frac{4\pi\alpha}{\sqrt{2}m_Z^2\sin^2 2\theta_W(m_Z)_{\overline{MS}}(1-\Delta\hat{r})} \tag{22}$$

where Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$ represent the radiative corrections to those relationships. They have been normalized such that $G_\mu = G_F^{(1)} = G_F^{(2)} = G_F^{(3)}$ in the standard model [20]. To determine those quantities, requires calculations of the loop corrections to G_μ, α, m_Z, m_W, and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ as well as the reactions used to measure them. Fortunately, the complete one loop corrections in (20)–(22) are known and most leading higher order effects have also been computed [21].

Leptonic partial widths of the Z boson also provide useful Fermi constant determinations. Defining

$$G_F^{Z\ell^+\ell^-} = \frac{12\sqrt{2}\pi\Gamma(Z\to\ell^+\ell^-(\gamma))}{m_Z^3(1-4\sin^2\theta_W(m_Z)_{\overline{MS}}+8\sin^4\theta_W(m_Z)_{\overline{MS}})(1-\Delta r_Z(m_Z)_{\overline{MS}})} \tag{23}$$

$$G_F^{Z\nu\bar{\nu}} = \frac{4\sqrt{2}\pi\Gamma(Z\to\Sigma\nu\bar{\nu})}{m_Z^3(1-\Delta r_Z)} \tag{24}$$

with the radiative corrections $\Delta r_Z(m_Z)_{\overline{MS}}$ and Δr_Z again normalized such that $G_F^{Z\ell^+\ell^-} = G_F^{Z\nu\bar{\nu}} = G_\mu$ in the standard model. Note that $\Gamma(Z\to\ell^+\ell^-(\gamma))$ by definition corresponds to Z decay into massless charged leptons [22], $m_\ell = 0$, It is obtained from an average of $\ell = e$, μ, τ data, where only the $\tau^+\tau^-$ width requires a non-negligible phase space correction factor of 1.0023. For some new physics scenarios [23], a separate $G_F^{Z\tau^+\tau^-}$ could prove useful; however, those cases will not be considered here.

The electroweak radiative corrections in (20)–(24) are known. They depend with varying sensitivities on the top quark and Higgs masses. For example, $\Delta r(m_Z)_{\overline{MS}}$ exhibits very little dependence on those quantities while Δr is most sensitive. Also, the first three, Δr, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta\hat{r}$ have a common low energy hadronic vacuum polarization loop uncertainty [24] due to α. Here, a very small ± 0.0002 error from that source is assigned [25]. A more conservative approach might expand [26] that uncertainty by a factor of 2–4, but it would not affect our subsequent analysis significantly.

In the evaluation of electroweak radiative corrections, the following central values and uncertainty ranges are assumed

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$
$$m_H = 125^{+275}_{-35} \text{ GeV} \tag{25}$$

The Higgs mass range is bounded from below by LEP II results $m_H \gtrsim 89.8$ GeV. A conservative upper range of $m_H \sim 400$ GeV is assumed at the 1 sigma level. Using those input parameters, one finds [21]

$$\Delta r = 0.0358 \mp 0.0020^{+0.0049}_{-0.0012} \pm 0.0002 \tag{26a}$$
$$\Delta r(m_Z)_{\overline{MS}} = 0.0696 \pm 0.0001^{+0.0005}_{-0.0003} \pm 0.0002 \tag{26b}$$
$$\Delta \hat{r} = 0.0597 \mp 0.0005^{+0.0017}_{-0.0005} \pm 0.0002 \tag{26c}$$
$$\Delta r_Z(m_Z)_{\overline{MS}} = -0.0071 \mp 0.0005^{+0.0008}_{-0.0001} \tag{26d}$$
$$\Delta r_Z = -0.0048 \mp 0.0005^{+0.0008}_{-0.0001} \tag{26e}$$

where the first error corresponds to Δm_t, the second to Δm_H, and the third (when present) to hadronic vacuum polarization uncertainties. Increasing the last of those by a factor of 2–4 would make it comparable to other errors in $\Delta r(m_Z)_{\overline{MS}}$ and $\Delta \hat{r}$, but would not seriously impact our subsequent results.

Employing the values of α, m_t, and m_H given above, along with [27]

$$m_Z = 91.1867(21) \text{ GeV} \tag{27a}$$
$$m_W = 80.422(49) \text{ GeV} \tag{27b}$$
$$\sin^2\theta_W(m_Z)_{\overline{MS}} = \sin^2\theta_W^{\text{eff}} - 0.00028 = 0.23100(22) \tag{27c}$$
$$\Gamma(Z \to \ell^+\ell^-(\gamma)) = 83.91(10) \text{ MeV} \tag{27d}$$
$$\Gamma(Z \to \Sigma\nu\bar{\nu}) = 500.1(18) \text{ MeV} \tag{27e}$$

leads to

$$G_F^{(1)} = 1.1700(\mp 0.0036)(^{+0.0062}_{-0.0027}) \times 10^{-5} \text{ GeV}^{-2} \tag{28a}$$

$$G_F^{(2)} = 1.1661(\mp 0.0018)(^{+0.0005}_{-0.0004}) \times 10^{-5} \text{ GeV}^{-2} \tag{28b}$$

$$G_F^{(3)} = 1.1672(\mp 0.0008)(^{+0.0018}_{-0.0007}) \times 10^{-5} \text{ GeV}^{-2} \tag{28c}$$

$$G_F^{Z\ell^+\ell^-} = 1.1650(\pm 0.0014)(^{+0.0011}_{-0.0006}) \times 10^{-5} \text{ GeV}^{-2} \tag{28d}$$

$$G_F^{Z\nu\bar{\nu}} = 1.1666(\pm 0.0042)(^{+0.0011}_{-0.0006}) \times 10^{-5} \text{ GeV}^{-2} \tag{28e}$$

where the first error comes from the experimental input in (27) while the second is due to uncertainties in (26) from radiative corrections.

All derived Fermi constants in (28) are in excellent accord with $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2}, but their errors are more than 100 times larger. Nevertheless, they can be used to place tight constraints on new physics.

Consider the case of heavy new chiral SU(2)$_L$ doublets from a fourth generation of fermions or motivated by technicolor models of dynamical electroweak symmetry breaking. Such fermions contribute to the above radiative corrections via gauge boson self-energies. Their effects are conveniently described by the Peskin-Takeuchi S, T, and U parameters [28], where S represents isospin-conserving and T and U isospin-violating gauge boson loop contributions. Their presence would modify the relationships between G_μ and the other Fermi constants such that

$$G_\mu = G_F^{(1)}(1 + 0.017S - 0.026T - 0.020U) \tag{29a}$$

$$G_\mu = G_F^{(2)}(1 + 0.0085(S + U)) \tag{29b}$$

$$G_\mu = G_F^{(3)}(1 + 0.011S - 0.0078T) \tag{29c}$$

$$G_\mu = G_F^{Z\ell^+\ell^-}(1 - 0.0078T) \tag{29d}$$

$$G_\mu = G_F^{Z\nu\bar{\nu}}(1 - 0.0078T) \tag{29e}$$

No evidence for S, T, or $U \neq 0$ is apparent from (28). In fact, comparing (29b) with G_μ and $G_F^{(2)}$ in (28b) leads to

$$-0.28 < S + U < 0.33 \qquad (90\% \text{ CL}) \tag{30}$$

Comparing (29c) with (29d) and (29e) eliminates the dependence on T and gives the somewhat tighter constraint

$$-0.38 < S < 0.04 \qquad (90\% \text{ CL}) \tag{31}$$

In the case of a heavy fourth generation of fermions (4 chiral doublets), one expects $S = 2/3\pi \simeq 0.21$ which conflicts with (31). Generic technicolor models suggest [28] $S \sim \mathcal{O}(+1)$ which conflicts significantly with (31) and (30) for $U \simeq 0$ (as expected in those models). The bound on S provides an obstacle for electroweak dynamical symmetry breaking advocates or fourth generation scenarios. If high mass chiral fermion doublets exist, their dynamics must exhibit properties that preserve $S \sim 0$ or other loop effects must cause a cancellation.

¿From the comparison of (29d) and (29e) with G_μ, one also obtains the bound

$$-0.40 < T < 0.17 \qquad (90\% \text{ CL}) \tag{32}$$

on the isospin violating loop correction. The constraints in (30)–(32) are nearly as good as those obtained from global fits to all electroweak data [29].

The final example considered here is the possibility of excited $W^{*^\pm}$ bosons that arise in theories with extra compact dimensions (Kaluza-Klein excitations) [30] or models with composite gauge bosons. Assuming fermionic couplings to $W^{*^\pm}$ identical to those of the $W^\pm$, $g_2^* = g_2$, direct searches at the Tevatron lead to the bound [2]

$$m_{W^*} > 720 \text{ GeV} \qquad (95\% \text{ CL}) \tag{33}$$

For $g_2^* \neq g_2$, that bound is (roughly) multiplied by $1 + 0.3\ell n(g_2^*/g_2)$ and thus not so sensitive to shifts in g_2^*. If such bosons exist, they would also contribute to low energy charged current amplitudes such as muon or tau decays. Their effect would be encoded in G_μ and $G_{\tau\ell}$ but not the indirect Fermi constants in (28).

The effect of excited bosons would be to replace g_2^2/m_W^2 in low energy amplitudes by $g_2^2/\langle m_W^2\rangle$ where [27]

$$\frac{1}{\langle m_W^2 \rangle} = \frac{1}{m_W^2} + \frac{(g_2^*/g_2)^2}{m_{W^*}^2} + \frac{(g_2^{**}/g_2)^2}{m_{W^{**}}^2} + \cdots \tag{34}$$

As long as the relative signs are positive, $\langle m_W^2 \rangle$ is always smaller than m_W^2. The situation is analogous to adding resistors in parallel. In such a scenario, G_μ should be larger than the G_F in (28). There is no indication of such an effect. Quantitatively, one expects

$$\begin{aligned} G_\mu &= G_F^{(i)} \left(1 + C \left(\frac{g_2^*}{g_2} \right)^2 \frac{m_W^2}{m_{W^*}^2} \right) \\ C &= 1 + \left(\frac{g_2^{**}}{g_2^*} \right)^2 \frac{m_{W^*}^2}{m_{W^{**}}^2} + \cdots > 1 \end{aligned} \tag{35}$$

In the simplest single extra dimension theory [30], $C \simeq \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$. Additional compact space dimensions can further increase C.

Comparing (35) with (28), one finds (at 95% CL)

$$m_{W^*} > 2.9\sqrt{C}(g_2^*/g_2) \text{ TeV} \qquad (\text{from } G_F^{(3)}) \tag{36a}$$

$$m_{W^*} > 1.5\sqrt{C}(g_2^*/g_2) \text{ TeV} \qquad (\text{from } G_F^{(2)}) \tag{36b}$$

$$m_{W^*} > 1.4\sqrt{C}(g_2^*/g_2) \text{ TeV} \qquad (\text{from } G_F^{(1)}) \tag{36c}$$

$$m_{W^*} > 1.4\sqrt{C}(g_2^*/g_2) \text{ TeV} \qquad (\text{from } G_F^{Z\ell^+\ell^-}) \tag{36d}$$

$$m_{W^*} > 1.0\sqrt{C}(g_2^*/g_2) \text{ TeV} \qquad (\text{from } G_F^{Z\nu\bar{\nu}}) \tag{36e}$$

Note that $G_F^{Z\ell^+\ell^-}$ would lead to a better bound if its central value were not about 1 sigma below G_μ. Also, the bound from $G_F^{(2)}$ has less m_t and m_H sensitivity and probably provides the least model dependent constraint.

The above bounds can be relaxed if $g_2^* \ll g_2$ or increased for $C > 1$. Taking $m_{W^*} > 2.9$ TeV as representative, that corresponds to a bound on $W^\pm$ substructure at ~ 2.9 TeV and $R < 1/m_{W^*} \sim 7 \times 10^{-18}$ cm for the radii of extra dimensions [31].

How might the above constraints improve? Measurement of m_H and refinements in m_t will reduce the uncertainty in radiative corrections. At LEP II and the Tevatron, a reduction in Δm_W to ± 15 MeV is anticipated while at SLC, $\Delta \sin^2 \theta_W(m_Z)_{\overline{MS}}$ could be reduced to ± 0.00018. In the longer term, high statistics Z pole studies at a future $\ell^+\ell^-$ collider could reduce $\Delta \sin^2 \theta_W(m_Z)_{\overline{MS}}$ to about ± 0.00004 and significantly improve the leptonic Z partial widths. Such improvements will, for example, allow one to probe m_{W^*} beyond $5\sqrt{C}(g_2^*/g_2)$ TeV. For comparison, direct searches at the Tevatron with $2fb^{-1}$ will explore $m_{W^*} \lesssim 1.2$ TeV while LHC is sensitive to ~ 6 TeV. An advantage of direct collider searches for excited bosons is their reduced sensitivity to changes in g_2^*, as long as their leptonic branching ratio remains relatively fixed and is significant. On the other hand, indirect constraints obtained by comparing G_μ and $G_F^{(i)}$ are more sensitive to g_2^*, but independent of branching ratio assumptions. Hence, the two approaches are very complementary.

In addition to the above, one can define Fermi constants using quark beta decays and CKM unitarity or from low energy neutral current processes such as atomic parity violation. The latter case provides a powerful constraint on many examples of new physics. It will be examined in a subsequent paper which updates the radiative corrections to atomic parity violation.

The Fermi constant has played an important role in the history of weak interactions and development of the standard model. As demonstrated here, it continues to provide useful guidance for testing the standard model and probing new physics.

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hep-ph/9902323 which placed bounds on W^* boson effectively using $G_F^{(1)}$ and G_μ.