CP VIOLATION AND THREE GENERATIONS

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ABSTRACT

The development of the notion of flavor before the discovery of the third generation is briefly reviewed. Then the basic mechanism of CP violation in the six-quark model is explained. In particular, phase factors in the CP transformation and the phase redefinition of field operators and their role in CP violation are discussed. Representations of the flavor mixing matrix are also discussed in some detail and an exactly unitary Wolfenstein-type parameterization is given. Finally, a few remarks are made on the present status of the study of CP violation.

1 Introduction

CP violation is a very fundamental problem that may be represented by a simple question: "What is the essential difference between particles and antiparticles?" The beginning of the problem goes back to around 1930, when Dirac derived the Dirac equation, from which the existence of the antiparticles for spin-one-half particles was concluded. Obviously, on a macroscopic scale, apparent asymmetry exists between particles and antiparticles, or matter and anti-matter. This must have been a big mystery for the people of that time. In fact, Pauli provided a skeptical view about the interpretation of the Dirac equation in Handbuch der Physik,[1] which was probably written in 1932.

It seems, however, that this mystery was forgotten in the subsequent development of particle physics. Particles and antiparticles had been believed to be symmetrical in the microscopic world until 1964, when CP violation was first discovered in $\mathrm{K}_L \to \pi + \pi$ decay.[2] Parity violation was discovered almost a decade before the discovery of CP violation, and it was also found that charge conjugation invariance is violated. However, at that time it was considered that the nature is invariant under the combined transformation CP. Violation of C implies asymmetry between particles and antiparticles, but if CP is conserved, the difference between particles and antiparticles can be resolved by looking at them through a mirror. So C violation in this case is not an essential difference which creates the asymmetry between matter and antimatter in the universe.

After the discovery of CP violation, Sakharov discussed the necessary conditions for the generation of the matter-dominant universe.[3] Actually, C and CP violations are one of the three necessary conditions, together with the baryon number nonconservation and the nonequiribrium universe. This issue was revived by Yoshimura[4] and Ignatiev, Krasnikov, Kuzmin and Tavkhelidze[5] in connection with grand unified theories some years later.

As for the theoretical understanding of CP violation in the neutral kaon system, it remained at a phenomenological level until the beginning of the 1970's. Although the attractive idea of the super weak model[6] emerged from these phenomenological analyses, it was not easy to develop theoretical ideas given the limited amount of experimental information that was available. In early 1970's, however, the development of a renormalizable gauge theory of the weak interactions changed the situation. The requirements of gauge invariance and renormalizability reduced the degrees of arbitrariness in the theoretical understanding of CP violation. Furthermore, subsequent experimental discoveries of new flavors gave a special position to the six-quark model of CP violation.[7]

In the six-quark model, CP violation arises from flavor-mixing. Since flavor-mixing is an important subject of particle physics in its own right, we review the development of the notion of the flavor mixing in the next section. The basic mechanism of CP violation in the six-quark model is discussed in Section 3.

2 History of Flavor Mixing

Following the suggestion of Lee and Yang,[8] and the subsequent discovery by Wu[9] that parity was violated, our understanding of the weak interactions developed rapidly. After some initial confusion about the type of interaction at play in β decay, the structure of the weak interactions was established as a V-A type interaction.

In this course, the universality of the strength of the weak interactions had already been recognized, in particular in the similarity of the strength of the p-n transitions and muon decay. This fact lead Feynman and Gell-Mann[10] to propose the conserved vector current hypothesis: If the vector part of the weak current is the charged component of an isomultiplet electro-magnetic current, then its conservation guarantees the non-renormalization of the vector coupling of the p-n transitions.

It turned out, however, that there is slight discrepancy between the vector coupling of p-n transition and the muon-decay. Namely, the strength of the p-n transition is a few percent weaker than that for muon-decay. In addition, it was also observed that the coupling constants of the strangeness-changing weak interactions are considerably smaller than their strangeness non-changing counterparts.

In these circumstances, Gell-Mann and Levy[11] noted that the weak current could take the structure of the following form;

$$GV_\alpha + GV_\alpha^{(\Delta S=1)} = \frac{G_\mu}{(1+\epsilon^2)^{\frac{1}{2}}}\bar{p}\gamma_\alpha(n+\epsilon\Lambda)+\cdots.$$

This observation was made in a note-added-in-proof of their famous 1960 paper on the σ-model.

In 1963, Cabibbo gave a new notion of the universality of the weak interaction by generalizing the isospin current to the SU(3) current:[12]

$$J_\mu = \cos\theta(j_\mu^{(0)}+g_\mu^{(0)})+\sin\theta(j_\mu^{(1)}+g_\mu^{(1)}),$$

where the superscripts denote ΔS. The mixing angle θ is now known as the Cabibbo angle. Then the algebraic structure of the weak current began to attract attention, and it eventually lead to the gauge theory of weak interaction.

The current structure of the weak interaction and the success of current algebra are most easily understood in the quark model. However, there was difficulty in understanding the weak interaction field theoretically as the interaction between quarks and intermediate bosons. The difficulty was related to the strangeness changing neutral current, which had to be suppressed at the tree level as well as at an induced current.

In 1970, Glashow, Illiopoulos and Maiani[13] pointed out that this difficulty could be avoided by the introduction of a fourth quark and an extended weak current that is now known as the GIM current. This observation paved the way to the gauge theory of the weak interactions.

What we have seen so far is the main stream history as experienced in the western world. It is interesting to note that there was another stream in the story of the flavor mixing, one from which the present author personally received strong influence.

In 1956, Sakata[14] proposed a model, which we now call the Sakata model, to explain the existence of the many types of strange particles. His claim was that the triplet of the proton, the neutron and the Λ particle are the fundamental objects and that all the hadrons are composite states of them.

In addition to reproducing the strange particle spectrum, the Sakata model explained the weak interaction as well. In particular, the $\Delta S = \Delta Q$ rule is quite nicely explained by considering that strangeness-changing processes take place via a Λ-p transition. In 1959, Gamba, Marshak and Okubo[15] pointed out the following correspondence between p, n, Λ triplet and the leptons;

$$\begin{array}{ccc} \mathrm{p} & \mathrm{n} & \Lambda \\ | & | & | \\ \nu & \mathrm{e} & \mu \end{array} .$$

This observation prompted Sakata and his group to propose so-called Nagoya model, in which the triplet baryons are considered to be made of the leptons and an object called B-matter:[16]

$$\mathrm{p} = (\nu \mathrm{B}),\ \mathrm{n} = (e\mathrm{B}),\ \Lambda = (\mu \mathrm{B}).$$

In 1962, Lederman's group revealed that the neutrinos associated with the muon are different from the electron neutrino.[17] When this news reached Japan, two Japanese groups immediately proposed that the proton corresponds to a mixed state of ν_{e} and ν_μ

and therefore the couplings of p-n and p-Λ are suppressed by a mixing angle.[18] Furthermore, they pointed out the possible existence of the fourth baryon p' corresponding to the orthogonal two neutrino combination:

$$
\begin{aligned}
\mathrm{p} &= (\nu_1 \mathrm{B}), \qquad \nu_1 = \cos\theta\,\nu_{\mathrm{e}} + \sin\theta\,\nu_\mu\,, \\
\mathrm{n} &= (\mathrm{eB}), \\
\Lambda &= (\mu \mathrm{B}), \\
\mathrm{p}' &= (\nu_2 \mathrm{B}), \qquad \nu_2 = -\sin\theta\,\nu_{\mathrm{e}} + \cos\theta\,\nu_\mu\,.
\end{aligned}
$$

This scheme is called the extended Nagoya model.

It should be noted that these papers were published in 1962, even before Cabibbo's paper. This work was developed from a very different view points from that which prevailed in the western world. The Sakata model considered the actual p, n, Λ particles as the fundamental triplet and the SU(3) or U(3) symmetry was regarded as an approximate symmetry of the dynamics of those fundamental particles. In contrast, in the western world, the baryons are considered to belong to an octet representation, with p, n, Λ being parts of the octet, instead of a triplet, and, at least in the early stages, SU(3) symmetry was considered as something with a more abstract character.

Eventually the octet scheme of the baryons was confirmed experimentally, and two approaches were merged, in some sense. In 1964, the quark model was proposed, although it took several years before most people accepted the actual existence of quarks. On the other hand, in order to reconcile it with the octet nature of the baryons, the Nagoya model was reinterpreted in terms of a new layer of fundamental particles that were distinct from the actual p, n and Λ baryons.

The idea of Nagoya model was discussed by the people of Sakata's group from time to time. However, these prescient works of the Japanese groups were not developed in the direction of the gauge theory of the weak interactions.

Meanwhile, in 1971, there was unexpected development. In this year, Niu and his group found a few new kind of events in the emulsion chambers exposed to cosmic rays.[19]

One of those events had two distinct kinks, which suggested the pair production of short-lived unstable particles, and a pair of gamma ray shower which is consistent with π^0 production. The direction of π^0 is pointing one of the kinks and energy of π^0 can be estimated from the shower development. Assuming a two body decay process, they estimated the life time of the unstable particle as around 10^{-14} sec. If the charged decay product is the pion or the kaon, then the mass of the parent particle is around 2 GeV.

Hearing this news, Shuzo Ogawa of Hiroshima University (at that time) immediately pointed out that those events could be the productions of new particles which contain the fourth fundamental particle, i.e. the p'. When Ogawa proposed this, he clearly had the extended Nagoya model in his mind.[20]

Following Ogawa's suggestion, several groups in Japan began to study Niu's events and also various general aspects of the quartet models. At the time, I was a graduate student of Nagoya University. I was one of those who are working on the study of Niu's events and became familiar with the quartet model. The rest of this section are my own personal recollections.

In the same year, the Weinberg-Salam-Glashow model of the electroweak interactions began to attract wide attentions, especially because the nonabelian gauge theories were proved to be renormalizable by 'tHooft. In order to extend this type of theory to the quark sector, it was necessary to consider GIM-type quartet models. This might have been thought of as a drawback of the model, but, at least to me, everything looked to be neatly placed.

Actually, I thought that it might be possible to describe everything field theoretically along this line. Of course, this was before the discovery of the asymptotic freedom and, therefore, prior to the notion of quark confinement. Therefore, I could not pin down a specific interaction for the strong interactions, but I thought that the strong interactions were not incompatible with the field theoretical description. In that case, what was left was CP violating interactions.

After I received my Ph.D. from Nagoya University in 1972, I moved to Kyoto University where I started to investigate this problem with Maskawa, who was a research associate of Kyoto University at the time.

The problem was how to accommodate CP violation in the model in a gauge invariant and renormalizable manner. Once the problem was set, however, it was rather straightforward to solve. Rather than to propose a particular model, we tried to seek the logical consequences as much as possible. We quickly realized that the minimal quartet model scheme could not accommodate CP violation and we needed to add new particles. The six-quark model was pointed out as one of such possibilities.

3 Six-Quark Model

In this section, we discuss the mechanism of CP violation in the six-quark model. We will start with some very basic observations; one on the phase factor of the CP transformation and the other on the phase redefinition of field operators. Then, the way the six-quark model accommodates CP violations will be explained, and, finally, various representations of the quark mixing matrix will be discussed.

3.1 The Phase Factor of the CP Transformation

Let us consider the CP transformation of a certain complex field ϕ. The CP conjugate of ϕ is given by hermitian conjugate ϕ^*, which, in general, could be multiplied by an arbitrary phase factor:

$$U_{\rm cp}\ \phi\ U_{\rm cp}^{-1} = e^{i\alpha}\ \phi^*.$$

Taking the hermitian conjugate of this relation, we have

$$U_{\rm cp}\ \phi^*\ U_{\rm cp}^{-1} = e^{-i\alpha}\ \phi,$$

from which we can confirm that, when we apply $U_{\rm cp}$ twice, it yields the identity transformation, even if the phase factor has an arbitrary value:

$$U_{\rm cp}^2 = 1.$$

The existence of an arbitrary phase factor implies that there are infinitely many candidates of CP transformation. The question is how to fix the phase factor.

To illustrate how we can do it in the CP conserving case, we consider, as an example, the following interaction Hamiltonian,

$$H^{(1)} = \int d^3x\{g\ \phi\ O + g^*\phi^*O^*\}, \tag{1}$$

where g is a coupling constant and O is a certain operator. For example, in the case of Higgs type interaction, it looks like $O = \bar{\psi}\psi$. Here we assume that only $H^{(1)}$ changes the number of particles described by the field ϕ. This means that the rest of the Hamiltonian consists of $\phi^*\phi$ combinations.

Now we apply $U_{\rm cp}$ to this interaction Hamiltonian;

$$U_{\rm cp}\ H^{(1)}\ U_{\rm cp}^{-1} = \int d^3x\{g\ e^{i\alpha}\phi^*O^* + g^*e^{-i\alpha}\phi\ O\},$$

where we have assumed that the CP transformation of the operator O is fixed somewhere else, and, for the purpose of simplicity, is given by the following form, without an extra phase factor;

$$U_{\rm cp}\ O\ U_{\rm cp}^{-1} = O^*.$$

We note that, in general, O contains some other fields than ϕ and the CP transformation applied to them may have arbitrary phase factors like α. If they exist, in order to fix them we have to repeat similar arguments. How it ends up is an interesting question that is left as an exercise.

Now we find that if we choose the phase factor as

$$\alpha = -2\mathrm{arg.}(g),$$

then the Hamiltonian is invariant under such a U_{cp}. This means that the phase factor is fixed by the coupling constant. In other words, the Hamiltonian selects the CP transformation among infinitely many candidates. Only the selected CP transformation is the symmetry of the system, and the other candidates are not symmetries of the system. Note that for the system to be CP symmetric, we need only one CP transformation under which the system is invariant. Note also that a complex coupling constant does not necessarily imply CP violation. The phase of g_1 is made harmless by absorbing it into the phase factor α of the CP transformation.

So far we have been considering the CP conserving case. Now we turn to CP violating cases. A simple example of CP violating systems is the following:

$$H^{(2)} = \int d^3x\{g_1\phi\ O_1 + g_1^*\phi^* O_1^* + g_2\phi\ O_2 + g_2^*\phi^* O_2^*\}. \tag{2}$$

The difference from the previous example is that the interaction consists of two terms, or two coupling constants.

Now we consider the CP transformation of $H^{(2)}$;

$$\begin{aligned} U_{\mathrm{cp}}\ H^{(2)}\ U_{\mathrm{cp}}^{-1} &= \int d^3x\{g_1 e^{i\alpha}\phi^* O_1^* + g_1^* e^{-i\alpha}\phi\ O_1\} \\ &\quad + \int d^3x\{g_2 e^{i\alpha}\phi^* O_2^* + g_2^* e^{-i\alpha}\phi\ O_2\}, \end{aligned}$$

where we have assumed again that the CP transformations of O_1 and O_2 are given by their hermitian conjugates without any extra phase factors.

In this case, however, if the two coupling constants have different phases, we can not make the Hamiltonian invariant by choosing the phase factor α properly. If we choose α so that the first interaction is invariant, then the second interaction is not invariant, and vice versa. Whatever α we choose, U_{cp} is not a symmetry of the system. This is a typical mechanism for CP violation.

The following can be said in general. In CP violating cases, Hamiltonians do not single out α. None of the candidate CP transformations can be a symmetry of the system. Therefore, we have no unique CP transformation in CP-violating cases. This means that, when looking at a single interaction, we can not say whether that particular interaction violates CP symmetry or not, because the CP transformation itself is not defined uniquely.

Actually we need not fix α. The usual experimental measures of CP violation are defined without reference to the CP transformation. The origin of CP violation is a mismatch of phases of two or more coupling constants, which prevents the singling out of the value of α. Experimentally measurable quantities are related to the relative phases of the coupling constants.

3.2 Phase Redefinition

Next we discuss the phase convention of complex fields. Although this is closely related to the CP transformation phase factor, it is still an independent notion.

Let us consider the following phase redefinition of a complex field ϕ;

$$\phi = e^{i\theta}\tilde{\phi}.$$

We consider again $H^{(1)}$ as an example. If we rewrite the total Hamiltonian in terms of $\tilde{\phi}$, the Hamiltonian takes the same form as the original one except for a slight change of the coupling constants g in $H^{(1)}$;

$$\begin{aligned} H^{(1)} &= \int d^3x\{g\,\phi\,O + g^*\phi^*O^*\} \\ &= \int d^3x\{\tilde{g}\,\tilde{\phi}\,O + \tilde{g}^*\tilde{\phi}^*O^*\}, \end{aligned}$$

where

$$\tilde{g} = e^{i\theta}g.$$

Since we are assuming that the rest of the Hamiltonian consists of $\phi^*\phi$, there is no change in them. Only the coupling constants of ϕ number changing interactions obey the phase change. Under this phase redefinition, CP transformations with a phase factor changes as

$$U_{\rm cp}\,\tilde{\phi}\,U_{\rm cp}^{-1} = e^{i\alpha}e^{-2i\theta}\tilde{\phi}^*.$$

The new field $\tilde{\phi}$ has the same ability as ϕ to describe the particle, and, if we use $\tilde{g}$ instead of g, the physics is the same as the original Hamiltonian. Therefore, this phase redefinition is a matter of convention.

Here let us choose θ as

$$\theta = -\mathrm{arg.}(g),$$

so that $\tilde{g}$ is real;

$$\tilde{g} = e^{i\theta} g \ : \ \mathrm{real}\,.$$

Then under the CP-conserving transformation in which $\alpha = -2\mathrm{arg.}(g)$, the phase factor disappears for $\tilde{\phi}$;

$$U_{\rm cp}\, \tilde{\phi}\, U_{\rm cp}^{-1} = \tilde{\phi}^*.$$

What we learn from this simple example is that when CP is conserved, the coupling constants and phase factors of the CP transformation can be made real using the phase redefinition of the field operators.

In the CP-violating case, however, we can not do this. In the Hamiltonian (2), we can not make both coupling constants real at the same time by choosing phase convention of ϕ, as long as the two coupling constants have different phases. What we can do is to make one or the other of the coupling constants real.

When g_1 is made real, it is sometimes said that the imaginary part of g_2 violates CP invariance. But this is not a correct statement, because we did not fix the phase factor of the CP transformation in CP violating case, so that CP transformation itself is not defined uniquely. Since the CP violation arises from relation among various interactions, it is not adequate to say that a particular interaction is CP conserving or not.

For practical purposes, however, it may be convenient to make the coupling constant of the stronger interaction real. For example, if g_1 is much larger than g_2, then, by making g_1 real, we may treat the imaginary part of g_2 as the origin of CP violation for some purpose. We must be careful in applying this argument, however, because, in general, the strength of the interaction can not be compared simply by the coupling constants.

In conclusion, we note again that observable CP violating quantities are defined so that they are independent of a particular choice of the phase factor α and also independent of the phase convention of the field operators.

3.3 Quark Mixing

On the basis of the observations made in the previous subsections, we consider the mechanism of CP violation in the standard model. Since all the coupling constants of the gauge interactions are real numbers, there exists a chance of complex coupling constants only in the quark mixing which appears in the charged current weak interactions.

We note that there is another possibility for CP violation arising from a non-zero value of the θ parameter of the QCD vacuum. This is a very different mechanism for CP violation and it is usually thought that θ is very small, even if it is not zero. So we do not consider this type of CP violation in the following.

The problem we are going to consider is whether the complex parameters appearing in the quark mixing violate CP or not. Let us consider the case where the quark sector consists of n-generations of left-handed doublets and all the right-handed quarks are singlets:

$$\underbrace{\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L, \cdots}_{\longleftarrow \quad n \quad \longrightarrow}.$$

Here u, c and t denote the mass eigenstates of the u-type quarks, while d', s' and b', which are the doublet partners of the u-type quarks, are not necessarily mass eigenstates. Instead, they can be linear combinations of the d-type mass eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \\ \vdots \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix},$$

where V is an $n \times n$ mixing matrix. In this case the charged current weak interactions look like

$$H_{\rm cc} = \frac{g}{2\sqrt{2}} W_\mu^+ (\bar{u}, \bar{c}, \bar{t}, \cdots)\gamma^\mu (1-\gamma^5)\, V \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix} + h.c.\,.$$

Note that the coupling constants for $\bar{q}qW$ interactions are given by g times the matrix elements of V.

In order that the kinetic parts of the quarks are flavor diagonal and properly normalized, the mixing matrix V must be unitary. Since, in general, the elements of a unitary matrix are complex numbers, the charged current weak interactions are a possible source of CP violation.

The difference between the number of parameters of an $n \times n$ unitary matrix and those of an orthogonal matrix, $n^2 - n(n-1)/2 = n(n+1)/2$ can be regarded as the number of parameters arising from the complexness of the unitary matrix. According to the previous discussion, we have to investigate whether these complex numbers can be removed by the phase redefinition of the quark fields. The number of quark fields is $2n$, but the overall phase will have nothing to do with the matrix elements of V, because the interaction has the form of $\bar{q}Vq$. Therefore the number of those phases that can be used to absorb the phase factor of the matrix elements of V are $2n-1$. Subtracting $2n-1$ from $n(n+1)/2$, we have $(n-1)(n-2)/2$ as the number of unremovable phase parameters. Therefore, if $n=2$, complex coupling constants would be removed, whereas for $n=3$ they are not. Now we confirm this explicitly.

First we consider the case of two generations. Here we make the phase redefinition in the following way: First we adjust the relative phase of the u and d quark fields so that the corresponding u-d transition is described by a positive real matrix element, i.e. V_{ud} is real and positive. Then we adjust the phase of the s quark field so that V_{us} is real and positive. Finally the phase of the c quark field can be adjusted to make V_{cs} real and positive. Thus, with the phase redefinition for these quark fields, the three matrix elements of V are made real and positive as follows:

$$\begin{array}{ccc} \mathrm{u} & & \mathrm{c} \\ | & \diagdown & | \\ \mathrm{d} & & \mathrm{s} \end{array} \qquad \Rightarrow \qquad V = \begin{pmatrix} R & \underline{R} \\ * & R \end{pmatrix} .$$

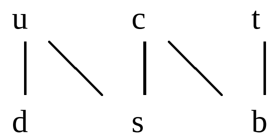
This shows that the matrix elements corresponding to the transitions connected by the lines are positive real numbers. The R symbols in the matrix imply elements that are positive real numbers and the asterisk indicates an element that could, so far, be a complex number. We see, however, that this element must be real as a result of the unitarity relation for the elements of V.

There is only one independent parameter and it can be chosen as V_{us}, indicated by $\underline{R}$ in the above expression. Parameterizing it by an angle variable, we have the familiar GIM form for V;

$$V = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

where θ lies in the first quadrant because $\underline{R}$ is positive by the phase convention.

Next we consider the case of three generations. Following a similar method to the previous case, we adjust the phase convention of the quark fields to remove the complex matrix elements as far as possible. Our choice here is such that five relative phases indicated by the following diagram are adjusted so that the corresponding matrix elements are real and positive:



Then the mixing matrix V looks like

$$V = \begin{pmatrix} R & \underline{R} & \underline{*} \\ * & R & \underline{R} \\ * & * & R \end{pmatrix}, \tag{3}$$

where R indicates elements that are real positive numbers and asterisks those that could be complex numbers. From the general structure of the unitary matrix, we can see that, when the diagonal elements are real, a unitary matrix can be parameterized by the elements of the upper-right small triangle indicated by the underlines. Namely the rest of elements can be expressed in terms of the underlined elements by using the unitarity relations and the reality of the diagonal elements.

Let us parameterize the triangle in the following way, as suggested by Wolfenstein:[21]

$$V = \begin{pmatrix} * & \lambda & A\lambda^3(\rho - i\eta) \\ * & * & A\lambda^2 \\ * & * & * \end{pmatrix}.$$

It should be noted that this is not an approximate expression for a small λ, but a definition of the four parameters, λ, A, ρ and η. We have three real parameters and one corresponding to the imaginary part, as we expect from the general argument given above.

The rest of matrix elements can be expressed in terms of these four parameters in an exactly unitary manner.[22] The following is the result of such expression:

$$V=\begin{pmatrix} D_1 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda\dfrac{D_2+A^2\lambda^4(\rho+i\eta)}{D_1} & D_2 & A\lambda^2 \\ A\lambda^3\dfrac{D_2-(\rho+i\eta)D_0^2}{D_1D_3} & -A\lambda^2\dfrac{D_2+\lambda^2(\rho+i\eta)}{D_3} & D_3 \end{pmatrix},$$

where

$$\begin{aligned} D_1 &= \sqrt{1-\lambda^2-A^2\lambda^6(\rho^2+\eta^2)}, \\ D_2 &= \frac{-A^2\lambda^6\rho+\sqrt{D_1^2D_3^2-A^4\lambda^{12}\eta^2}}{1-A^2\lambda^6(\rho^2+\eta^2)}, \end{aligned}$$

$$
\begin{aligned}
D_3 &= \sqrt{1 - A^2\lambda^4 - A^2\lambda^6(\rho^2+\eta^2)}, \\
D_0 &= \sqrt{1 - \lambda^2 - A^2\lambda^4 - A^2\lambda^6(\rho^2+\eta^2)}.
\end{aligned}
$$

The result is slightly complicated but the matrix elements can be expressed in terms of the elementary functions and the derivation is quite straight forward. If we expand these expressions with respect to λ we find the familiar expression given by Wolfenstein:

$$
V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.
$$

When λ is small, this expression is approximately unitary. We can easily obtain a better approximation for the unitarity from the above exact unitary expression.

Now a few technical remarks are in order. The first concerns the phase convention. So far we have been considering the particular phase convention in which real elements lie in a special pattern. Of course there are other possible phase conventions, some of which will be discussed below. It should be remembered that the physics consequences are the same for any phase convention.

In the original paper with Maskawa, we took another phase convention, in which the first line and the first row are made real, and the mixing matrix is parameterized in the following way;

$$
V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + s_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix},
$$

$$
s_i = \sin\theta_i, \;\; c_i = \cos\theta_i, \;\; i = 1,2,3 \,.
$$

Although this is one of the natural choices of the phase convention, it is not necessarily convenient in practical applications.

We note that the Particle Data Group is adopting the following representation as a standard one.

$$
V_{PDG} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix},
$$

$$
s_{ij} = \sin\theta_{ij}, \;\; c_{ij} = \cos\theta_{ij}, \;\; i,j = 1,2,3 \,.
$$

This was originally proposed by Chau and Wang[23] and given by successive rotations of two generations. In this representation, the real elements are located as follows:

$$V_{PDG} = \begin{pmatrix} R & R & * \\ * & * & R \\ * & * & R \end{pmatrix}$$

and the elements indicated by the asterisks are actually complex if δ_{13} is nontrivial. The number of real matrix elements are four and, in contrast to Eq.3, the central element V_{cs} is not real, Therefore this is a different phase convention from the previous one.

Since in V_{PDG}, the V_{us} and V_{cb} elements are real and V_{ub} is complex, we can use this representation to define a set of Wolfenstein parameters:

$$V_{PDG} = \begin{pmatrix} * & \hat{\lambda} & \hat{A}\hat{\lambda}^3(\hat{\rho} - i\hat{\eta}) \\ * & * & \hat{A}\hat{\lambda}^2 \\ * & * & * \end{pmatrix},$$

Now we have two sets of Wofenstein parameters defined in two different phase conventions. We should note that although they are defined with the same matrix elements, they are different parameter sets and their values are actually different. The relation between the two sets of parameters is

$$\begin{aligned} \hat{\lambda} &= \lambda, \\ \hat{A} &= A, \\ \hat{\rho} + i\hat{\eta} &= e^{i\xi}(\rho + i\eta), \end{aligned}$$

$$\xi = O(\lambda^6).$$

Namely the values of $\hat{\rho}$ and $\hat{\eta}$ are different from ρ and η by a small rotation in the $\rho - \eta$ plane. In this case. the difference is quite small and it is not so important for the purpose of the phenomenological analysis at the present level of experimental accuracy. Nevertheless, this is a good lesson for understanding the role of the phase convention.

Next we consider the generalization to four or more generations. Our previous choice of phase convention can be easily generalized to any number of generations:

$$\begin{array}{ccccccc} \mathrm{u} & & \mathrm{c} & & \mathrm{t} & & \\ | & \diagdown & | & \diagdown & | & & \cdots \\ \mathrm{d} & & \mathrm{s} & & \mathrm{b} & & \end{array}$$

In this case we have the following form for the mixing matrix;

$$V = \begin{pmatrix} R & \underline{R} & \underline{*} & \underline{*} & \cdot \\ * & R & \underline{R} & \underline{*} & \cdot \\ * & * & R & \underline{R} & \cdot \\ * & * & * & R & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

where the notations are the same as for the previous cases. The general pattern can be read off from this expression. Namely, the diagonal elements and their right-hand neighbors are real and positive. Independent parameters can be chosen as indicated by the underlines. From this pattern, we can easily count the number of the independent parameters as follows;

$$\begin{aligned} \text{real part} &: \frac{n(n-1)}{2}, \\ \text{imaginary part} &: \frac{(n-1)(n-2)}{2}. \end{aligned}$$

4 Remarks

This section contains a few remarks on what happened after the publication of our original paper in 1973. Of course the six-quark scheme is not an only possible mechanism of violating CP invariance and the point of our paper was that the minimal scheme based on the four-quark GIM-type model was too small to accommodate CP violation. Some new ingredients were needed. If we added some new fields, not necessarily new quarks, it was easy to violate CP. Roughly speaking, this is because the number of possible coupling constants increases with some power of the number of the fields—quadratically or maybe even faster—while the number of phases absorbed into the phase conventions of the field operators increases only linearly. This results in unremovable complex coupling constants with the increasing number of fields.

The special position of the six-quark model emerged from the subsequent experimental discoveries of the third generation particles. In 1975, the first evidence of the τ lepton was reported. It was after this report that the six-quark model began to attract attention, and a few papers that discussed the model appeared in 1976.[24,25,26] In particular, the first extensive study of the model was made in Ref.26, where the basic mechanism of CP violation in the neutral kaon system was given.

The discovery of the third generation continued. In 1977, the Υ particle was discovered and it soon became clear that it is the bound state of a fifth quark with charge

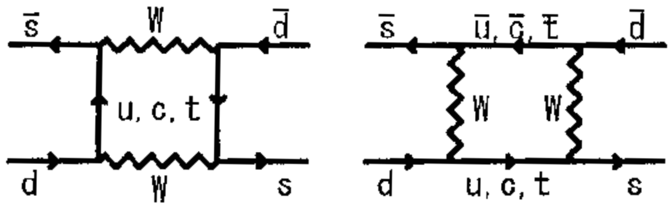


Fig. 1. Box diagrams

$-1/3$ and its antiparticle. After some detours to topless models, the existence of the t quark became a common belief. However we had to wait for long time to have an experimental evidence of the production of the t quark, until CDF group found the t quark in 1994.

Now we turn to what we know about CP violation in the six-quark model. Since this is a subject of other lectures, we will not enter into much detail here.

The present status of our understanding of the neutral K-meson system is summarized as follows. In the usual phase convention, CP violation in the $\mathrm{K}-\bar{\mathrm{K}}$ mixing is determined dominantly by the imaginary part of the dispersive part of the mass matrix of the neutral kaon system:

$$|\mathrm{K_L}\rangle = \frac{1}{\sqrt{2}}\{(1+\epsilon)\ |\mathrm{K}^0\rangle + (1-\epsilon)\ |\bar{\mathrm{K}}^0\rangle\},$$

$$|\mathrm{K_S}\rangle = \frac{1}{\sqrt{2}}\{(1+\epsilon)\ |\mathrm{K}^0\rangle - (1-\epsilon)\ |\bar{\mathrm{K}}^0\rangle\},$$

$$\epsilon \approx \frac{1}{2}\frac{i\mathrm{Im}M_{12}}{M_{12}-\frac{i}{2}\Gamma_{12}},$$

where M_{12} and Γ_{12} are the dispersive and absorptive parts of the off-diagonal elements of the mass matrix, respectively. The value of $\mathrm{Im}M_{12}$ is estimated by evaluating the contribution of the box diagram, and then we have

$$\begin{aligned}|\epsilon| \ \ \approx \ \ & \frac{G_F^2 m_W^2 B_K F_K^2 m_K}{6\sqrt{2}\pi^2\Delta m}\eta A^2\lambda^6[-E(x_c)\eta_c \\ & + E(x_c,x_t)\eta_{ct} + A^2\lambda^4(1-\rho)E(x_t)\eta_t],\end{aligned}$$

where

$$x_c = m_c^2/m_W^2,\ x_t = m_t^2/m_W^2,$$

and η_c, η_{ct} and η_t are QCD correction factors of $O(1)$.

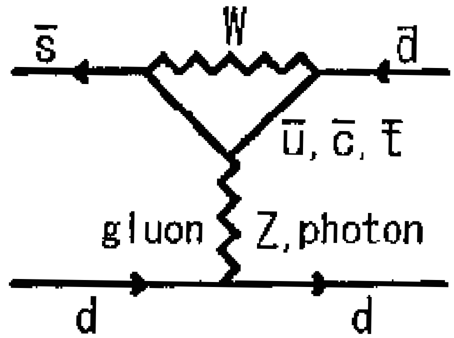


Fig. 2. Penguin diagram

The parameter B_K, which represents the bound state effect of the kaon, is still under the study, in particular by lattice QCD calculations. Nevertheless, a comparison of the above expression with the experimental value of ϵ yields relatively good bounds for the quark mixing parameters, particularly ρ and η.

Another observable CP violating quantity in the kaon system is the ϵ' parameter in $\mathrm{K_L} \to \pi + \pi$ decay, which is defined by

$$\begin{aligned} \eta_{+-} &= \frac{A(\mathrm{K_L} \to \pi^+\pi^-)}{A(\mathrm{K_S} \to \pi^+\pi^-)} &= \epsilon + \epsilon', \\ \eta_{00} &= \frac{A(\mathrm{K_L} \to \pi^0\pi^0)}{A(\mathrm{K_S} \to \pi^0\pi^0)} &= \epsilon - 2\epsilon'. \end{aligned}$$

Specifically, ϵ' is related to the weak phase difference of the decay amplitudes for the two final states with the isospin $I = 0$ and 2:

$$\sqrt{2}\epsilon' \approx ie^{i(\delta_2 - \delta_0)}\mathrm{Im}\frac{A_2}{A_0},$$

where A_0 and A_2 are defined by

$$\langle (\pi\pi)_I | T | \mathrm{K}^0 \rangle = e^{i\delta_I} A_I, \qquad I = 0, 2$$

and δ_I is the phase shift of the S-wave $\pi\pi$ scattering for $I = 0$ and 2. The phase difference arises from so-called Penguin diagrams, because they contribute to A_0 and A_2 differently. In particular the gluonic Penguin diagram contributes only to the $I = 0$ final state.

Considerable experimental effort has been expended measuring ϵ'. Recently new results are reported from KTeV group at FNAL and NA48 group at CERN:[27][28]

$$\begin{aligned} \mathrm{Re}(\epsilon'/\epsilon) &= (28.0 \pm 4.1) \times 10^{-4} \quad (\mathrm{KTeV}), \\ \mathrm{Re}(\epsilon'/\epsilon) &= (18.5 \pm 7.3) \times 10^{-4} \quad (\mathrm{NA48}). \end{aligned}$$

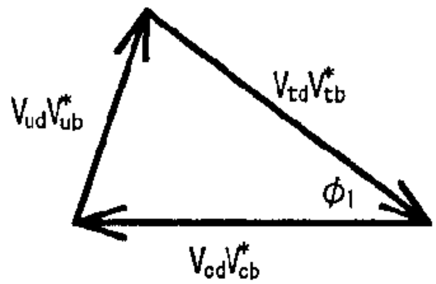


Fig. 3. Unitarity triangle

Even if we take into account these new experimental results, it seems that ϵ' does not give a stringent constraint on the mixing parameters due to ambiguities in the theoretical calculations.

The experimental value of ϵ, together with the available information about the B-meson system, such as the $\mathrm{B}-\bar{\mathrm{B}}$ mixing and the charm-less decays of the B-meson, constrain the mixing parameters ρ and η to a rather small range,

$$\begin{aligned} -0.14 &< \rho < 0.32, \\ 0.23 &< \eta < 0.39. \end{aligned}$$

Existence of a consistent solution for the mixing parameters itself has a non-trivial implication on the validity of the standard model, but more importantly, a charactereistic feature of the six-quark model CP violation resides in this result.

Implications of this constraint can be expressed nicely in the form of so-called unitarity triangle, which is a triangle consisting of vectors in the complex plane corresponding to each term of the following unitarity relation;

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$

Since all the terms are $O(\lambda^3)$, the shape of the triangle, barring the overall size, is determined by the values of ρ and η. Drawing the triangle with the above range of the parameters, we note that the triangle is fairly fat. Here it is particularly important that η is not small due to the constraint from ϵ.

A fat unitarity triangle implies relatively large phase difference among the matrix elements of V, so that we expect fairly large CP asymmetries for certain processes. We may say that CP violations are large in the six-quark model.

Potentially large CP asymmetries are suppressed in $\mathrm{K} \to \pi + \pi$ decays by factors of order $O(10^{-3})$, but should be observable in the B-meson system. The most promising process is $\mathrm{B}(\bar{\mathrm{B}}) \to \mathrm{J}/\psi + \mathrm{K_S}$ decay in which we expect a large asymmetry in the time profile of two decays starting from B and $\bar{\mathrm{B}}$ at $t = 0$. The decay comprises two processes, a direct decay and a decay via $\mathrm{B} - \bar{\mathrm{B}}$ mixing. The asymmetry arises because the interference of these two processes takes place differently for B and $\bar{\mathrm{B}}$. A remarkable fact is that this asymmetry can be related to the angle ϕ_1 of the above unitarity triangle without much theoretical ambiguity. Besides this “golden” mode, the B-meson system has many other channels in which we can expect large CP asymmetry.

As we have seen, the six-quark model can explain CP violation in the neutral kaon system. But, explaining one CP-violating phenomenon with an adjustable CP violating parameter is not enough to test the model. Still there is a possibility that the quark-mixing parameters are quite different and the CP violation comes from very different origins. Definitely we need a further test of the model. It is particularly important to test the above mentioned characteristic features in the B-meson system.

For this reason, CP violations in the B meson system are currently undergoing extensive experimental study. OPAL at LEP and CDF at Tevatron already reported preliminary results on CP asymmetry in $\mathrm{B}(\bar{\mathrm{B}}) \to \mathrm{J}/\psi + \mathrm{K_S}$.[29 30] Quite recently, Asymmetric B-factories started operating at SLAC and KEK.

The future direction of the study of CP violation depends on what comes out from these experiments. If the results include some discrepancy from the prediction of the standard model, then we need an immediate cure that must be a breakthrough to “beyond standard” physics. On the other hand, if the results are consistent with the standard model, we will be left with much harder questions: “What determines the Higgs couplings?” “What is the origin of the generations?” etc.. Another challenging problem is the baryogenesis of the early universe. It is likely that we need some other CP violating mechanism in order to explain the matter dominance of the universe. In that case, an interesting question is whether or not there is a common origin for such a new CP violation mechanism and that of the six-quark model. These are not the issue of CP violation alone, but CP violation could be a good clue to attack such fundamental questions.

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