LC-1: Diffraction and Interference

5/15/12

Name:____________________________Section:____________________________

1 Introduction

Much of this lab is a repeat of what was covered in lecture. While that means some of this will be redundant, my hope is to help you build up intuition to support the lecture's equations.[1]

Diffraction and interference are phenomena that happen when waves interact. There's not a clear distinction between the two, but – for our purposes – we'll think about interference as the effect arising from point sources (wave sources that are very small – like the single oscillating charges in lecture) and diffraction as the effect from large sources. I realize that's vague, but it should become clearer by the end of the lab.

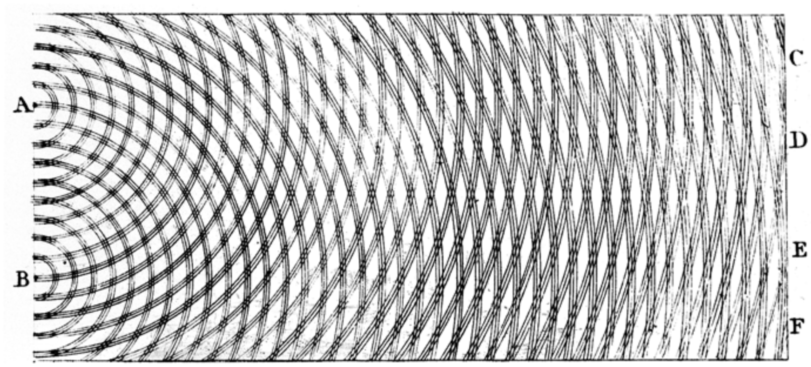


Figure 1: Thomas Young's illustration of interference – waves radiating out from point sources A and B.[5, Page 457-471 for chapter, 776 for figure.]

The ancestor to these experiments was first performed by Thomas Young in 1803, and it was instrumental in convincing scientists that light was a wave. Before that, most held Newton's view that light was made out of particles. See Figure 1 for his illustration of the double slit interference experiment; you'll make a similar picture soon on the computer. He did some interesting experiments before that too, including one which would make MacGyver proud: he measured the wavelength of light using only a hair and a candle.[3, 4] Thomas Young's exploits weren't limited to physics; he did everything from help decipher the Rosetta Stone to study how the heart works. The cited interestingly-titled book about him has been on my reading list for a while.[1]

We'll be using two tools in this lab, a computer simulation[2] and a laser based experiment. Both wave sources have two important properties:

1. All sources have the same frequency.
2. All sources have the same phase (they're all doing the same thing at every moment in time.)

The second isn't necessary (as shown in lecture, a constant phase difference will do), but it'll make the math simpler.

[1] Physics education research has shown that this material is tricky to teach. I've tried to model this lab on an approach shown to be effective.[2]

[2] The simulation software is available at https://github.com/lnmaurer/Interference-Inference-Interface.

2 Computer Simulations

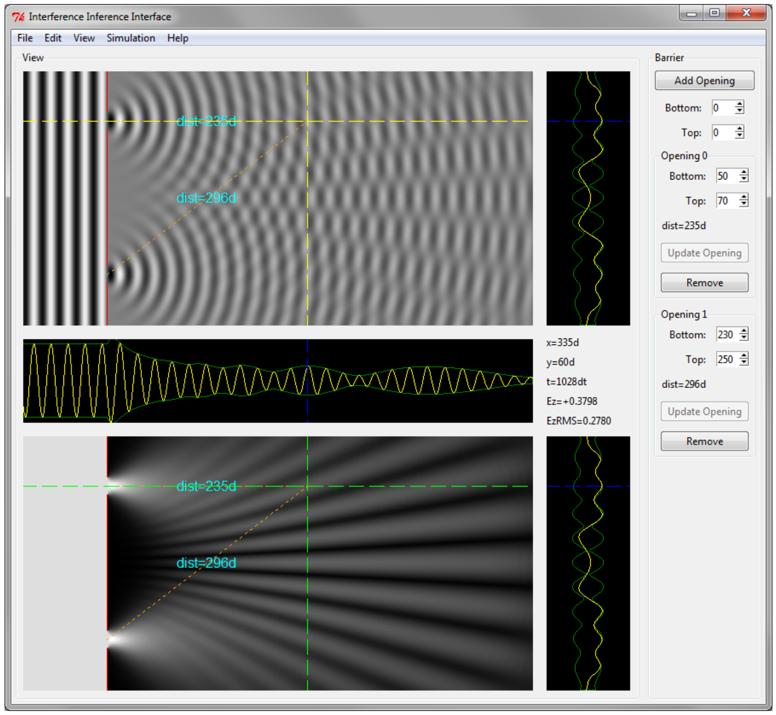


Figure 2: The simulation software's interface.

Let's review some key points about the simulation and its interface (Figure 2). You can think of the simulation displaying water waves, but the mathematics behind the scenes describes electromagnetic waves. Specifically, it simulates waves that are uniform in z; the value only depends on the x and y position. So, it only shows a 2D, xy slice in the top left and bottom left plots. The units are arbitrary; they're just called 'd'. Note that being uniform in z means that everything extends to infinity in the z direction – including the barrier and the slits in it!

Speaking of which, in the simulation, a plane wave propagating to the right hits a barrier (drawn in red) and is only let through the openings you make. These openings are the sources. They fulfill both of our requirements: the waves coming out of the openings are at the same frequency and phase. The waves from these sources then propagate to the right, mingle, and give us interference and diffraction effects.

The top left plot shows the instantaneous value of E_z (the z component of the electric field). Black is the minimum value, white is the maximum value, and shades of gray are in between. The bottom left plot showsE_{zRMS} – an average value instead of an instantaneous one. Here, black corresponds to a value of zero (since negative RMS values aren't possible). This plot can also be made to show E_{zRMS}^2, which is proportional to the light wave's intensity.

The other three plots show 1D slices taken along the vertical and horizontal dashed lines. The yellow curve shows E_z, and the green curve shows $\pm\sqrt{2}E_{zRMS}$, an envelope for E_z. The zero for both curves is right down the middle of the figures' long axes. However, the green curve can also be set to show E_{zRMS} or E_{zRMS}^2, in which case zero is along an edge of the plot.

The cyan text shows the distances from the center of each opening to the intersection of the slices. The text between the two vertical plots shows information about the point where the slices intersect: its position, E_z, and E_{zRMS} values.

The right part of the screen lets you set the openings in the barrier.

Note that for the 2D plots, y is zero at the top and increases as you go down.

2.1 First Simulation: A point source

First, let's try a simple example that doesn't really show interference or diffraction, but will get you familiar with our sources. Set up the barrier so that there's only one gap in the middle (top=160, bottom=140), and run the simulation until E_{zRMS} has stabilized.

Describe the waves in E_z. How does it differ from the water waves shown in Figure 3 caused by a drop of water? The point sources in lecture produce waves similar to that in some ways.

Figure 3: Ripples from a water drop, another type of point source.

What would E_{zRMS} look like for the lecture source and how is this different from the E_{zRMS} shown? Remember that the field from the sources in lecture decreased like $\frac{1}{r}$. Does our field do that? Look at a horizontal slice through the middle of the slit. What's the fundamental difference between that situation and our situation?

2.2 Second Simulation: Double Slit Interference

Now that we have a basis for comparison, set up the barriers with two slits: (top=70, bottom=50) and (top=250, bottom=230). Run the simulation until E_{zRMS} has stabilized. Now, find 4 points that are maxima of E_{zRMS} along the vertical 1D slice. This is known as constructive interference. In this lab, don't measure the global maxima that are right next to the openings in the barrier.

Measure the positions as accurately as possible; use the arrow keys for fine control and watch the value of E_{zRMS} at the intersection of the slices to home in on the maximum. For each maximum, record the distances from the two openings in the table, and then fill out the remaining columns (the wavelength, $\lambda = 20$).

	Distance from Opening 0 (d_0)	Distance from Opening 1 (d_1)	$\|d_0 - d_1\|$	$\frac{\|d_0-d_1\|}{\lambda}$
Maximum 1				
Maximum 2				
Maximum 3				
Maximum 4				

What type of numbers are $\frac{|d_0-d_1|}{\lambda}$ (approximately)?

Now, repeat the process except look for minima of E_{zRMS}. This is known as destructive interference.

	Distance from Opening 0 (d_0)	Distance from Opening 1 (d_1)	$\|d_0 - d_1\|$	$\frac{\|d_0-d_1\|}{\lambda}$
Minimum 1				
Minimum 2				
Minimum 3				
Minimum 4				

What type of numbers are $\frac{|d_0-d_1|}{\lambda}$ (approximately)?

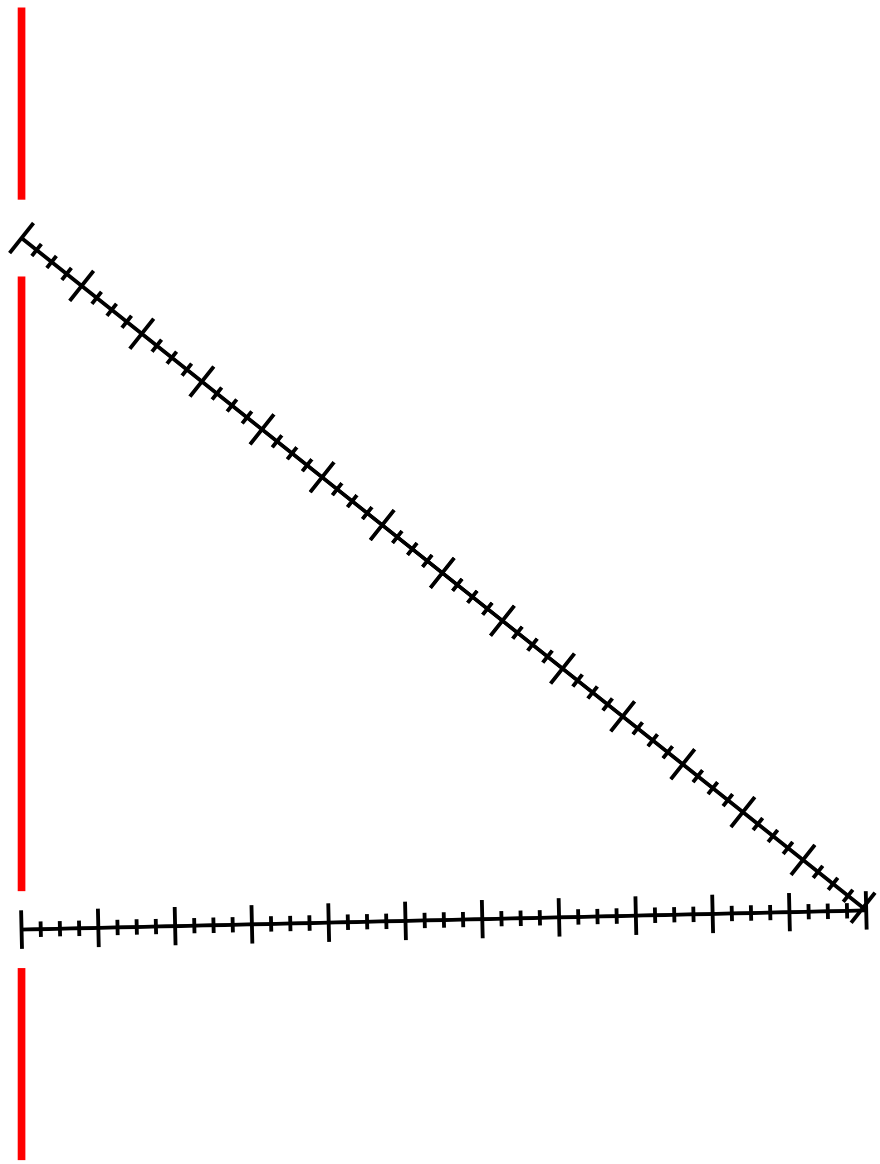


Figure 4: First case.

Now, let's figure out why that is. The intersection of the lines in Figure 4 is a point of constructive or destructive interference. There are marks **every quarter wavelength** and big marks are **every wavelength** along the lines connecting the openings' centers to the measurement point. Draw a sinusoidal wave along each of these lines, assuming that the wave is a **maximum at the opening** and that the waves' amplitude doesn't decrease with distance.

Note that you're drawing the wave at a snapshot in time. Even though the waves will be changing in time, the relation between the wave's value at the opening and the wave's value at the interference point will always remain the same.

In this case, would the measurement point be a maximum or a minimum in E_{zRMS}? Why?

Now, do the same for Figure 5.

In this case, would the measurement point be a maximum or a minimum in E_{zRMS}? Why?

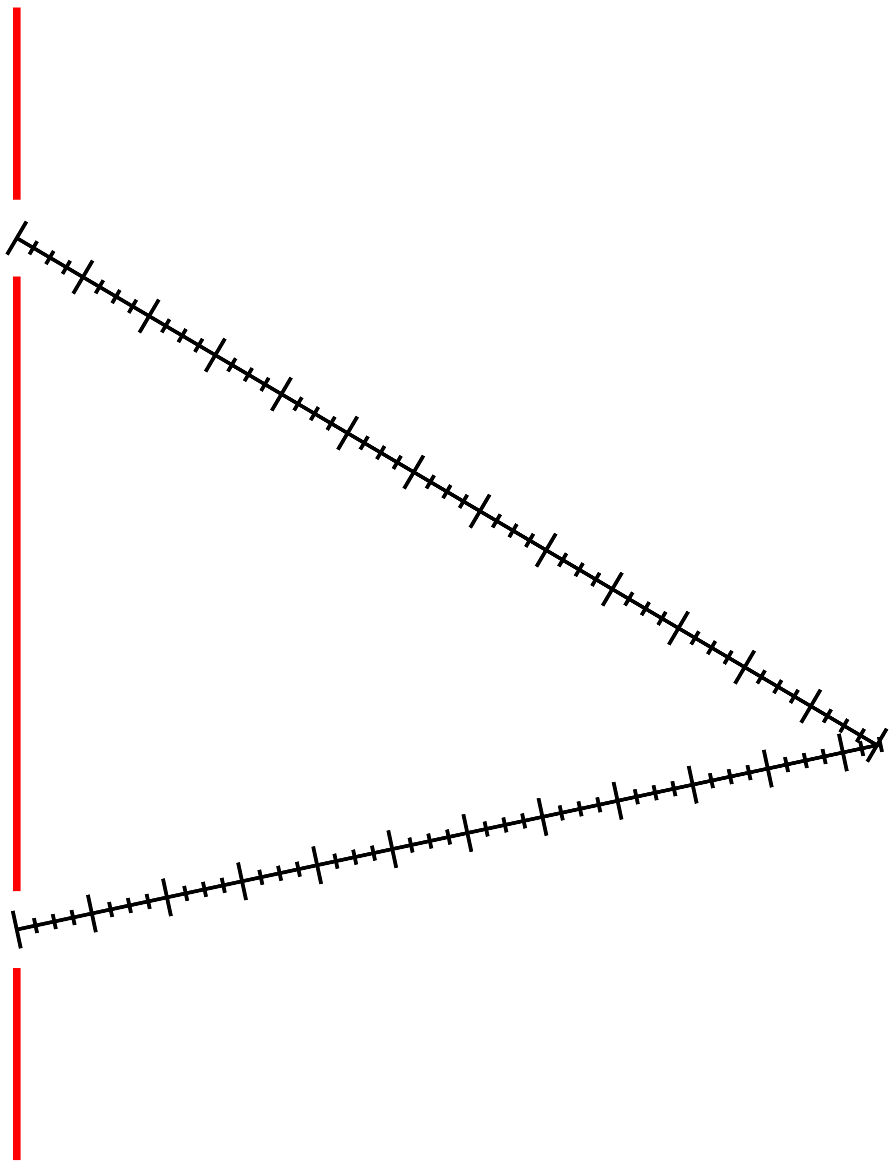


Figure 5: Second case.

Those situations were special cases. In one, both distances were an integer number of wavelengths, but that's not required for constructive interference; all that's necessary is that the difference between the lengths is an integral number of wavelengths. That insures that both waves are doing the same thing at the interference point; they're in phase (so that they add together instead of canceling one another).

Let's show that mathematically. Say x_0 is the distance along the first line and x_1 is the distance along the second line. Then the curves you drew were $\cos(kx_0)$ and $\cos(kx_1)$. Show that $\cos(kx_0) = \cos(kx_1)$ if $x_1 - x_0 = n\lambda$, where n is an integer. Hint: write $\cos(kx_1) = \cos(k(x_1 - x_0 + x_0))$ and then rewrite k.

Now, none of the minima in E_{zRMS} are zeros. We could show that they should be zeros using an argument similar to the one you just made. However, we made two assumptions earlier, in drawing the waves. Which one of them is wrong? How does that explain why the minima aren't zeros?

This effect is small, so the earlier conclusions are still excellent approximation.[3]

2.3 Third Simulation: Triple Slit Interference

Now, keep those two slits and add a slit right in the middle (top=160, bottom=140). Run the simulation until E_{zRMS} has stabilized. Find a point that's **a maximum along both the vertical and horizontal slices**. Again, find this point as accurately as possible.

Distance from Opening 0 (d_0)	d_1	d_2	$\frac{\lvert d_1 - d_0 \rvert}{\lambda}$	$\frac{\lvert d_2 - d_0 \rvert}{\lambda}$	$\frac{\lvert d_2 - d_1 \rvert}{\lambda}$

Do the differences in the distances follow the same rule as before? Explain why or why not?

Repeat that procedure with a minimum along both the vertical and horizontal slices.

Distance from Opening 0 (d_0)	d_1	d_2	$\frac{\lvert d_1 - d_0 \rvert}{\lambda}$	$\frac{\lvert d_2 - d_0 \rvert}{\lambda}$	$\frac{\lvert d_2 - d_1 \rvert}{\lambda}$

Do the differences in the distances follow the same rule as before? Explain why or why not? Hint: if two of the waves canceled each other out, what would happen to the third wave?

[3]For the sources in lecture, the interference term went like $\frac{1}{x_0 x_1}\cos(k(x_0 - x_1))$, so that's the term you'd have to maximize or minimize. However, its extrema are very close to the extrema of $\cos(k(x_0 - x_1))$ so long as $\frac{1}{x_0 x_1}$ is slowly varying (i.e. as long as x_0 and x_1 are large).

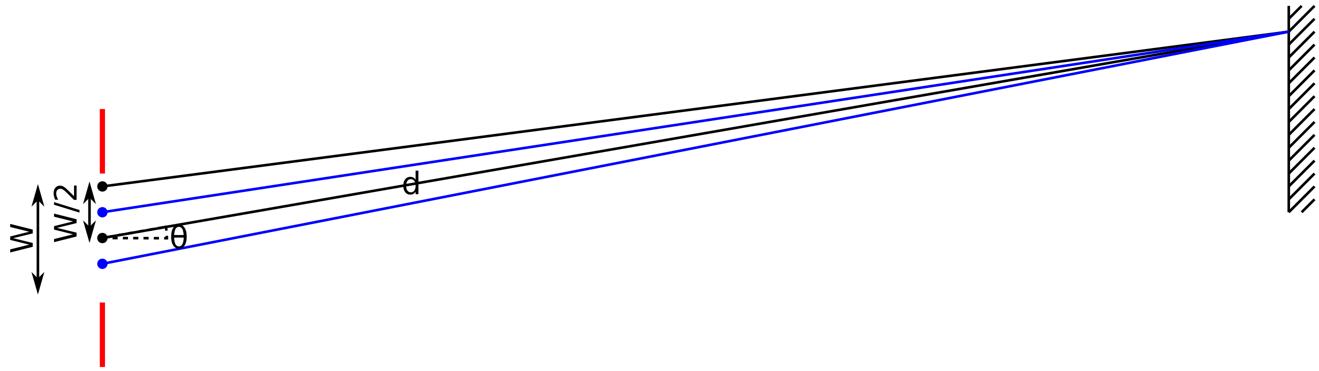


Figure 7: The top and middle source points are in black, while the ones below them are in blue.

2.4 Fourth Simulation: Single Slit Diffraction

2.4.1 Background

Before we run a simulation, we need to learn a bit about diffraction. Take the case of two point sources separated by $\frac{\lambda}{4}$ (Figure 6).

Is it possible to have constructive interference in this case? Is it possible to have destructive interference in this case? Explain. Hints: What's the smallest value $|d_0 - d_1|$ can be to get destructive interference? Does that value occur anywhere? Seemingly unrelated hint: $\left|\vec{a}+\vec{b}\right| \leq |\vec{a}| + \left|\vec{b}\right|$.

What's the minimum separation between the sources needed to get destructive interference? Where would the interference occur in that case? (Draw a picture.)

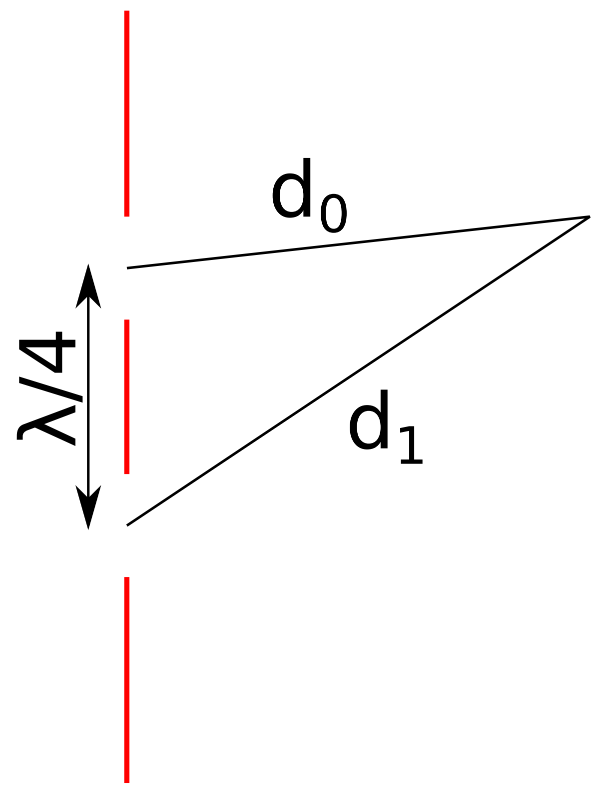


Figure 6: Two point sources separated by $\frac{\lambda}{4}$.

Now, if you had one wide slit, you'd expect different parts of the slit to act like point sources and interfere with one another. In fact, to rigorously analyze these slits, you'd say that there are an infinite number of infinitesimal point sources along the opening. I'm not going to go there.

However, we can make arguments in the the limit where $d \gg W$ and θ is small. See Figure 7. Suppose the top point in the slit and the middle point in the slit would interfere destructively. In the next section, we'll show that the smallest θ this happens for is when $\frac{\lambda}{s2} = \sin(\theta) \approx \theta$, where s is the distance between the sources. Here, $s = \frac{W}{2}$, where W is the width of the slit. Putting that together, the first minimum occurs at $\theta = \frac{\lambda}{W}$. Now, if the top point and middle point interfere destructively, then so do the second point from the top and the point below the middle because they form the same triangle as the previous two points.[4] So it goes, down the line; every point has a partner that cancels with it. So, the condition that holds for one pair of points holds overall: when $\theta = \frac{\lambda}{W}$, we have a minimum.

2.4.2 Simulation

Set up the simulation so that there's just one big slit in the middle (top=200, bottom=100), and run the simulation until E_{zRMS} has stabilized. Now, move the vertical slice all the way to the right edge of the plot. The vertical plot of E_{zRMS} should show a big bump in the middle, a minimum to each side, and then a smaller bump beyond that. Find the y position of the minimum between the two bumps, and calculate the angle the orange line makes with the horizontal. Compare this to $\frac{\lambda}{W}$. Are we in the $d \gg W$ limit?

3 Double Slit Interference and Diffraction Experiment

In this experiment, we're going to shine a laser through the two nearby slits. We'll see a mix of interference and diffraction because the slits are much wider than λ; there'll be the big center peak of diffraction modulated by the interference pattern and smaller peaks to the sides. See Figure 9. The plan is to measure the pattern and determine the laser's wavelength from that.

See Figure 8. We're working in the limit of $d_1, d_2 \gg W$. That makes $\theta_1 \approx \theta \approx \theta_2$, so the two lines are roughly parallel. That means the blue side of the triangle, $W\sin(\theta)$, must be $\approx d_2 - d_1$. For constructive interference we still need that $|d_2 - d_1| = n\lambda$ for some integer n. That means $\frac{n\lambda}{W} = \sin(\theta)$, which can further be approximated as $\frac{n\lambda}{W} \approx \theta$ since we'll be working with small θ. Similarly, for destructive interference, we'll need $\frac{(n+\frac{1}{2})\lambda}{W} \approx \theta$.

As a check, here's a quick derivation with the law of cosines and some Taylor expansion to show that it works out (similar to the derivation in lecture):

$$\begin{aligned}
d_1^2 &= d_2^2 + W^2 - 2d_2W\cos\left(\theta_2 + \frac{\pi}{2}\right) \\
&= d_2^2 + W^2 + 2d_2W\sin(\theta_2) \\
&\approx d_2^2 + 2d_2W\sin(\theta_2) \\
d_1 &= d_2\sqrt{1 + 2\frac{W}{d_2}\sin(\theta_2)} \\
&\approx d_2 - W\sin(\theta_2) \\
d_2 - d_1 &= W\sin(\theta_2)
\end{aligned}$$

[4] Ok, it's not exactly the same triangle, but it's very close to the same in the limit we're working in.

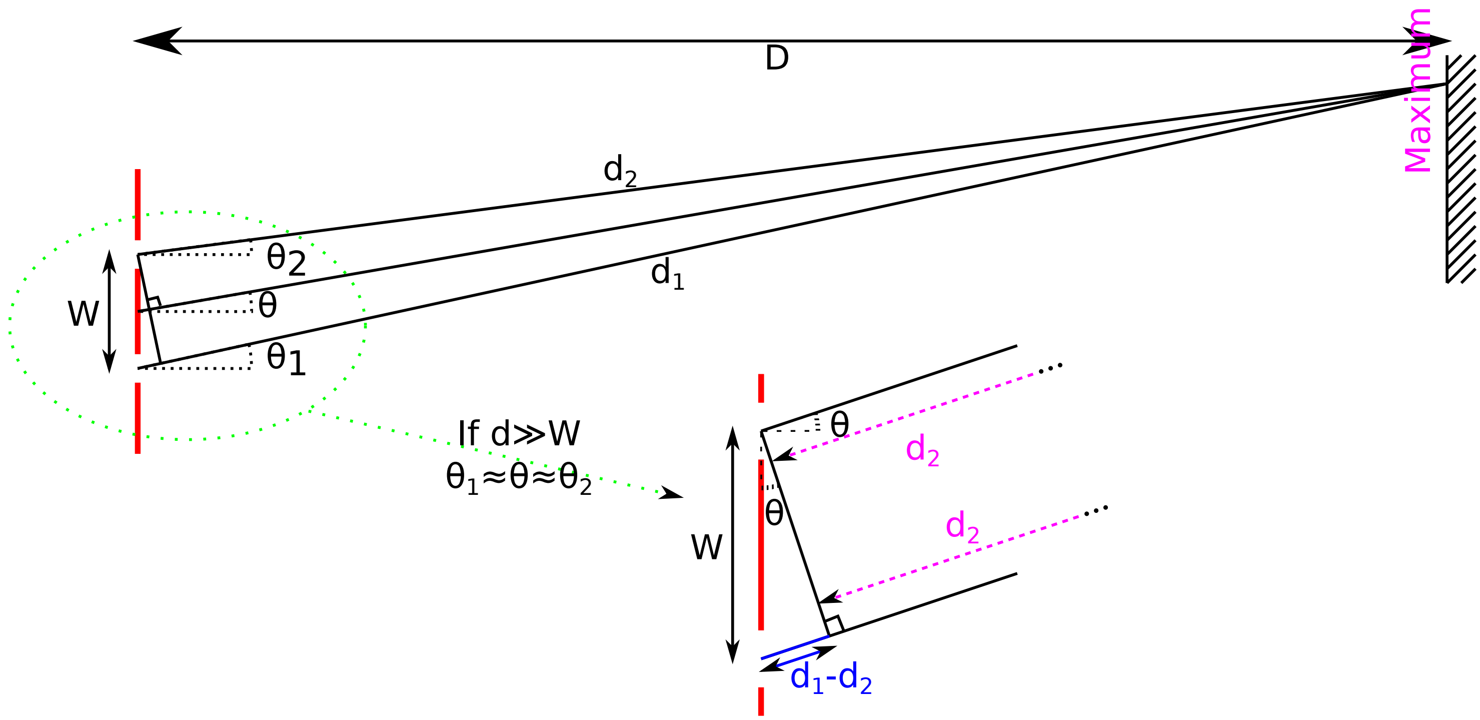


Figure 8: The double slit experiment. If $d_1, d_2 \gg W$, then $\theta_1 \approx \theta \approx \theta_2$.

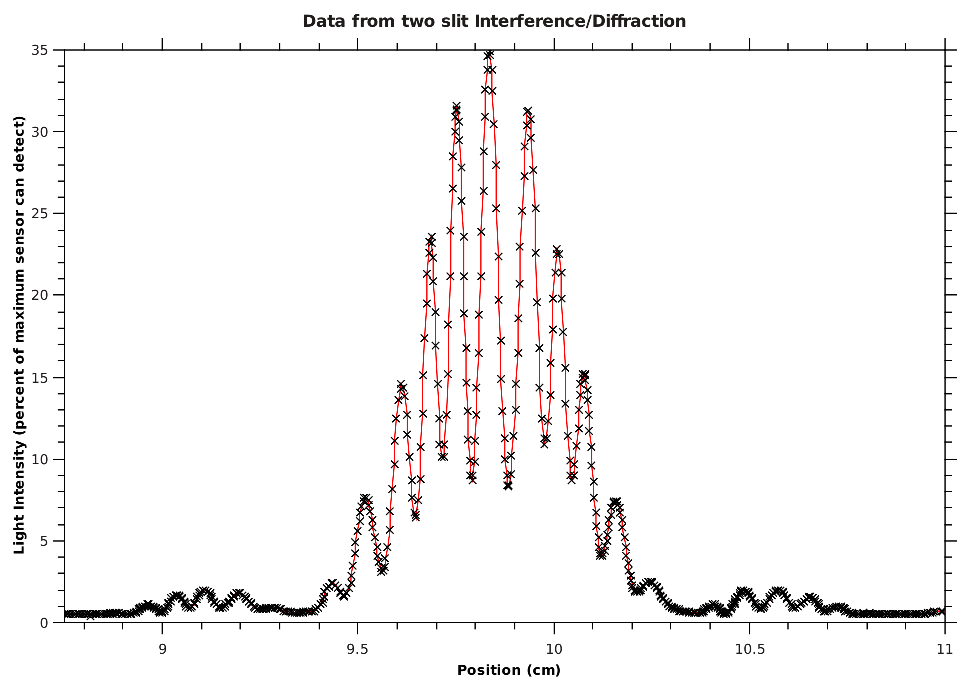


Figure 9: Example of good data from the two slit interference/diffraction experiment.

$$\approx \quad W \sin(\theta)$$

That derivation may seem a little sketchy because we're approximating the angles to be the same, but we're not doing the same for the distances. That's because a tiny change in the distances can have a large effect, so I used a more accurate approximation for them. For example, red light has $\lambda \approx 680nm$, so a change in $|d_1 - d_2|$ of only $340nm$ can get you from constructive to destructive interference – from light to darkness.

Here's the setup procedure:

1. Download the PASCO experiment file from http://badger.physics.wisc.edu/lab/datastudio/phys208/l01a_207.ds
2. Hook up everything
   (a) Power to the laser
   (b) Light Sensor to port A
   (c) Rotational Motion Sensor yellow plug to port 1 and black plug to port 2
3. Set the wheel on the light sensor so that slit 1 is at the bottom position.
4. Set light sensor gain switch to 10. If your signal ends up being too small or too large (i.e. it gets clipped at 50), then adjust this gain switch.
5. Remove the single slit holder, and set double slit holder about 50cm from the light sensor (the distance isn't critical).
6. Turn the double slit wheel so that the $a = 0.08mm$ $d = 0.50mm$ pattern is at the 3 o'clock position. a is the slit width. Their d is our W, the spacing between the slits.
7. Center the light sensor.
8. Turn on the laser. If you don't see the pattern on the light sensor or if the pattern isn't centered and level, talk to me; the setup may need some fine adjustment.

Now that everything is set up, move the light sensor to one side. Start taking data. Then, slowly and smoothly move the sensor to the other side by hand. When you're done, stop the measurement.

Now that you have the data, use it to find the laser's wavelength by filling out the following table. y is the distance from the first (central) maximum – the highest peak. $\theta \approx \frac{y}{D}$ where D is the distance from the slits to the sensor (see Figure 8). You know what $\frac{|d_1-d_2|}{\lambda}$ should be in each of the cases.

Extrema	y	θ	$\frac{\|d_1-d_2\|}{\lambda}$	λ in meters
Second Maximum				
First Minimum				
First (Central) Maximum	—0—	—0—	—0—	— —
First Minimum				
Second Maximum				

Check your numbers against the wavelength written on the back of the laser; you should be able to get pretty good agreement.

Finally, put in the single slit holder, and check out the patterns produced by the square and hexagon (you should see a square and hexagonal dot pattern). You can use the viewing screen for this. These are just a few of the cool patterns you can get using diffraction in 3D.

References

[1] Andrew Robinson. *The Last Man Who Knew Everything: Thomas Young, The Anonymous Polymath Who Proved Newton Wrong, Explained How We See, Cured the Sick, and Deciphered the Rosetta Stone, Among Other Feats of Genius.* Pi Press, 2005.

[2] Karen Wosilait, Paula R. L. Heron, Peter S. Shaffer, and Lillian C. McDermott. Addressing student difficulties in applying a wave model to the interference and diffraction of light. *American Journal of Physics*, 67(S1):S5–S15, 1999.

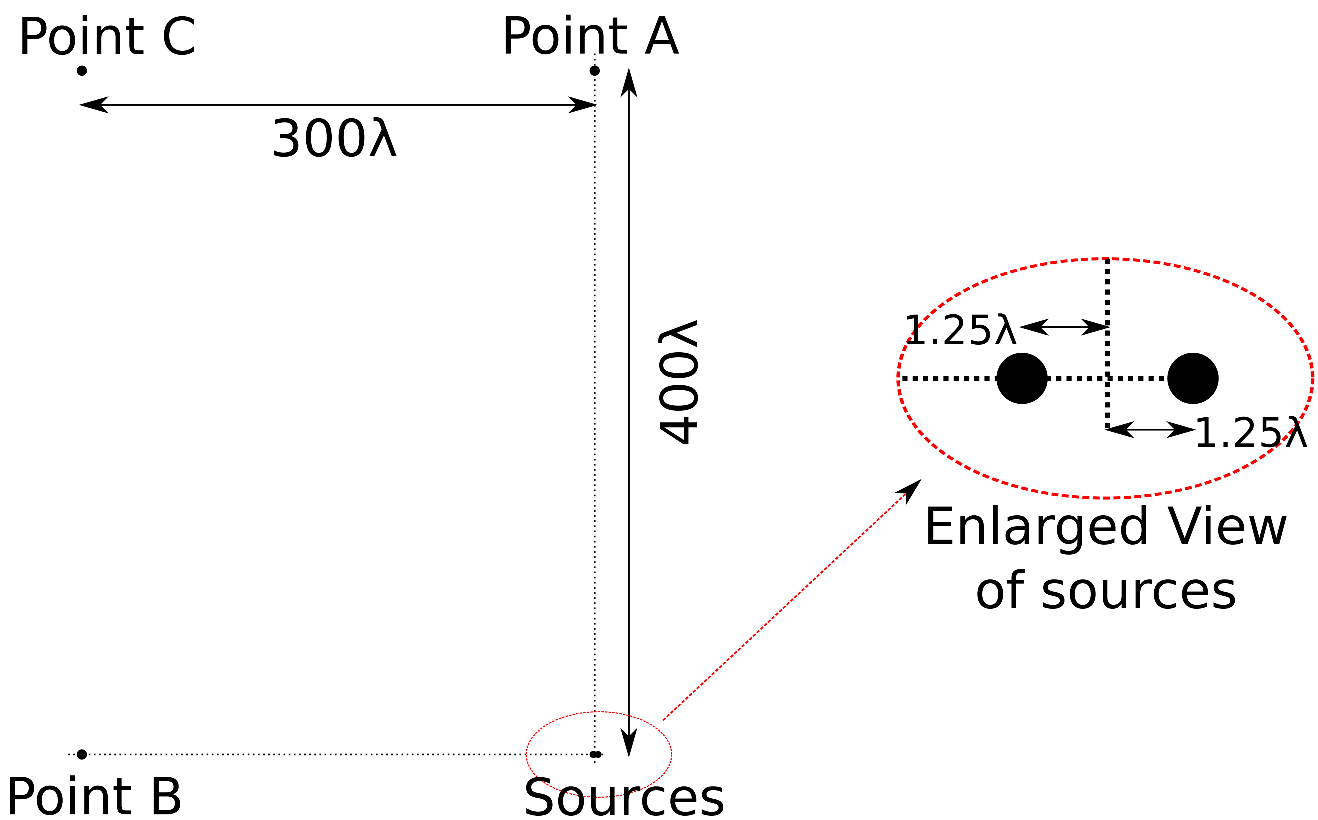
[3] Thomas Young. An account of some cases of the production of colours, not hitherto described. *Philosophical Transactions of the Royal Society of London*, 92:pp. 387–397, 1802.

[4] Thomas Young. The bakerian lecture: Experiments and calculations relative to physical optics. *Philosophical Transactions of the Royal Society of London*, 94:pp. 1–16, 1804.

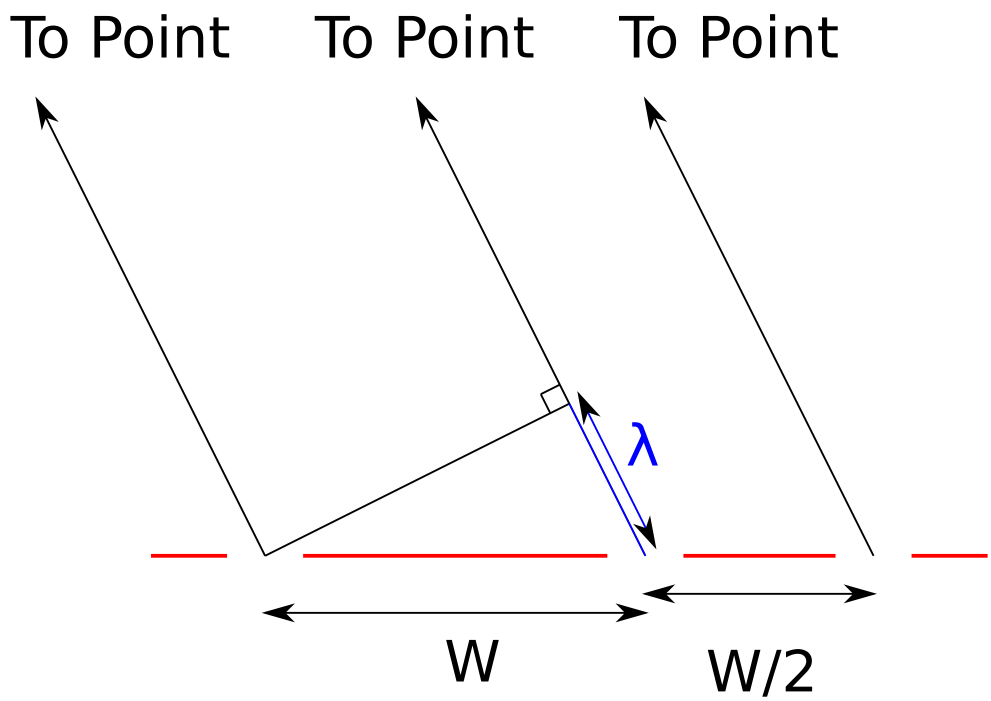
[5] Thomas Young. *A course of lectures on natural philosophy and the mechanical arts*. Number v. 1 in A Course of Lectures on Natural Philosophy and the Mechanical Arts. Johnson, 1807.

4 Anonymous Quiz

Now, it's time for a little quiz to see how effective this lab was. Answer the questions on it without looking back at previous parts of the lab, and turn in this sheet. Don't put your name on it; it won't effect your grade.



The two sources shown in the figure are in phase and 2.5λ apart. Determine if there is constructive interference, destructive interference, or neither at points A, B, and C. Show your work. You may want to use a calculator for at least one of them.



In the drawing above, the light from three slits is being observed at one far away point. Assume that the point is far enough away that the three lines to it are approximately parallel. First, consider just the left and middle sources. Do they constructively or destructively interfere at the observation point? If you now consider all three sources, will the intensity at the observation point increase or decrease relative the the case with just the left two sources?

5 Comments on the lab