Network Recovery Scheduling Problem

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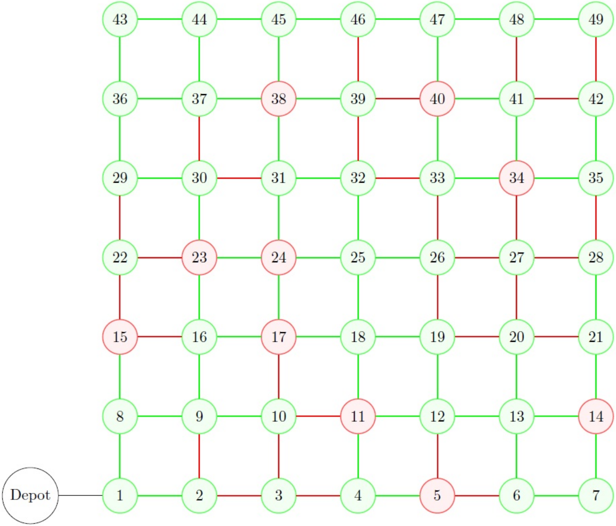
Network recovery scheduling problem

- Highway network:
  - Nodes = infrastructure elements (power generators, water substations, gas stations, etc.).
  - Edges = road segments.
- When disaster strikes, some edges and nodes are disrupted.
- We want to schedule the recovery (repair) of edges and nodes over a time horizon.
  - $\Rightarrow$ Post-event model.

Network recovery scheduling problem

- We have a fixed number of repair crews in each period.
- In each time period, we decide crews should be sent to which edges and nodes.
- We want to maximize the ”up time” of edges and nodes

Example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Parameters

- $F_{ij} \subseteq E$ = neighbor set of edge ij.
- $G_i \subseteq N$ = neighbor set of node i.
- D_{ij} = total workload required on edge $ij \in E$.
- n_1 = available number of repair crews for arcs
- T = length of time horizon

Variables

- State variables:
  - $u_{ij}^t = 1$ if edge ij is *accessible* at the start of period t, 0 otherwise
  - $v_{ij}^t = 1$ if edge ij is *functional* at the end of period t, 0 otherwise.
- Decision variables:
  - $x_{ij}^t = 1$ if a crew is sent to repair edge ij in period t, 0 otherwise
- Random variables:
  - p_{ij}^t = probability of completely repairing edge ij at the end time t, given that $x_{ij}^t = 1$.

Objective function

$$\text{maximize} \quad \sum_{t} \sum_{ij \in E} c_{ij} v_{ij}^{t}$$

- c_{ij} and d_i are weights for edges and nodes, respectively.
- The objective function maximizes a weighted sum of total "up time" across all edges and nodes.

Constraints

$$x_{ij}^t \leq u_{ij}^t \qquad \forall ij, t \qquad (1)$$

Constraint (1) requires that repair crews can only be sent to accessible arcs.

$$\sum_{ij} x_{ij}^t \leq n_1 \qquad \forall t \qquad (2)$$

Constraint (2) requires that the total number of repair crews across all edges cannot exceed n_1.

Constraints

$$u_{ij}^t \leq \sum_{i'j' \in F_{ij}} v_{i'j'}^t \qquad \forall ij, t \tag{3}$$

Constraint (3) requires that an edge is accessible only if at least an edge in its neighbor set is functional.

$$D_{ij} v_{ij}^t \leq \sum_{r=1}^{t-1} x_{ij}^r \qquad \forall ij, t \tag{4}$$

Constraint (4) requires that an edge is functional only if the workload on that edge is completed.

Constraints

$$v_{ij}^t \leq u_{ij}^t \qquad \forall ij, t \tag{5}$$

Constraint (5) requires that an edge is functional only if it is accessible.

$$y_i^t \leq n_2 \sum_{j \in G_i} v_{ij}^t \qquad \forall i, t \tag{6}$$

Constraint (6) requires that repair crews can be sent to a node only if at least an edge in its neighbor set is functional.

Constraints

$$\sum_{i} y_i^t \leq n_2 \qquad \forall t \tag{7}$$

Constraint (7) requires that the total number of repair edges across all nodes cannot exceed n_2.

$$W_i w_i^t \leq \sum_{r=1}^{t-1} y_i^r \qquad \forall i, t \tag{8}$$

Constraint (8) requires that a node is functional if its workload is completed.

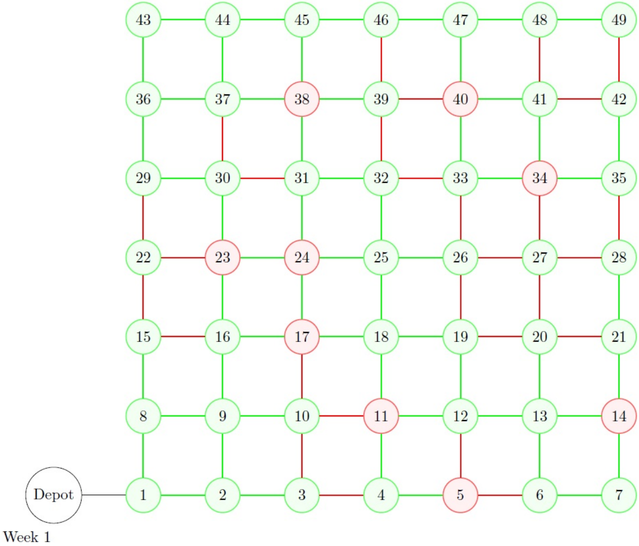
Constraints

$$x_{ij}^t \in \{0, 1, 2, \dots\} \qquad \forall ij, t \tag{9}$$

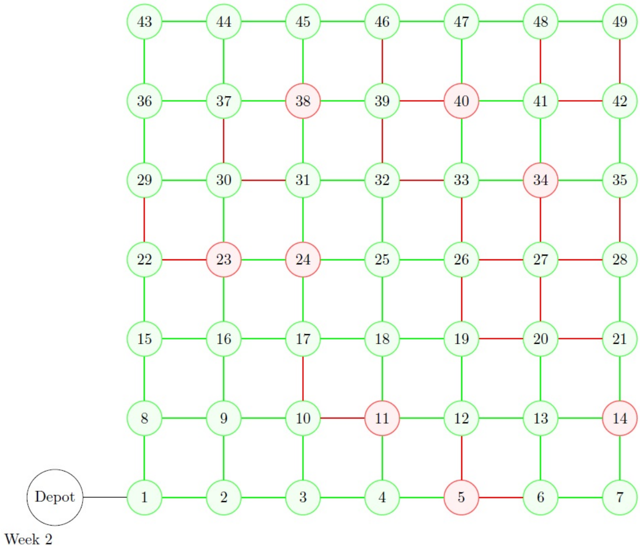
$$y_i^t \in \{0, 1, 2, \dots\} \qquad \forall i, t \tag{10}$$

Constraint (9) and (10) are the integer constraints.

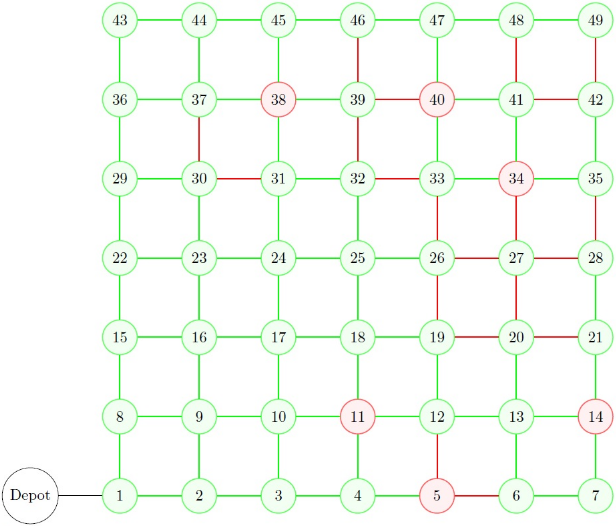
Solution - Week 1



Solution - Week 2

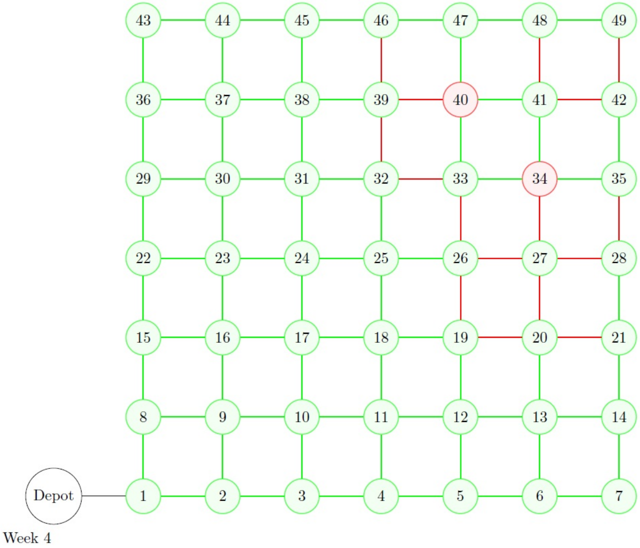


Solution - Week 3

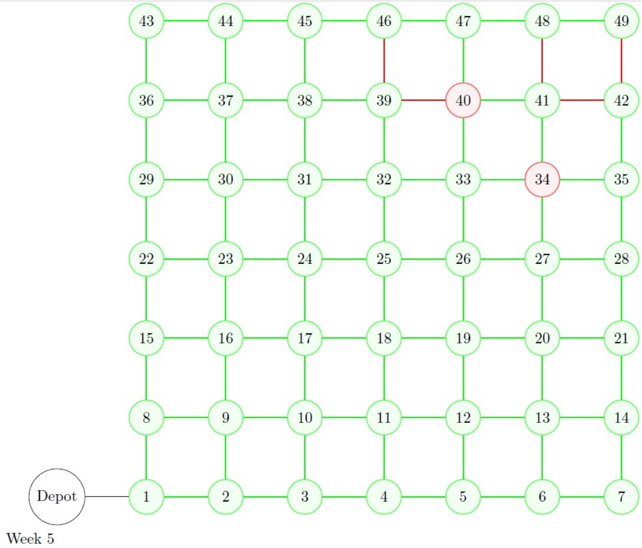


Week 3

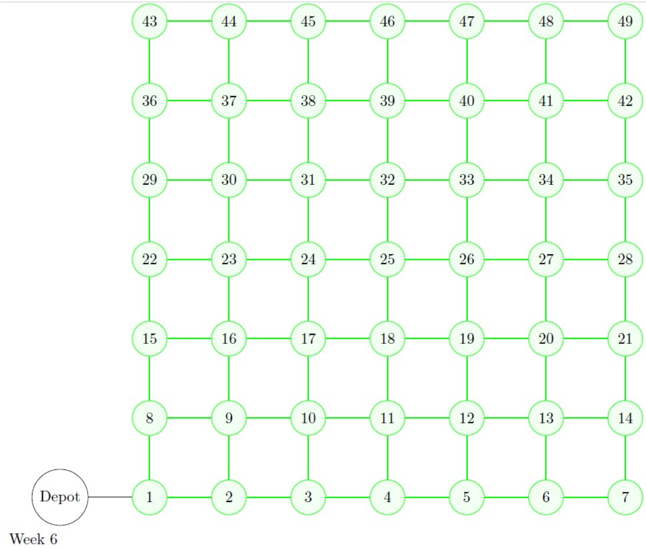
Solution - Week 4



Solution - Week 5



Solution - Week 6



Some results

No. of nodes	Objective value	CPU time
25	5557	2.80
64	68045	28.46
100	234021	434.34
144	521023	829.12
196	1313606	1003.54

Next step

- Model interdependency between transportation system (arcs) and power system (nodes) by:
  - Using objective function weights
  - Adding resource constraints
- Faster heuristic algorithms