Assessing The Limits of Synthetic Controls:

On the Estimation of Causal Effects in Time Series Data Structures

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Abstract

Potential framework: We argue that applications of Synthetic Controls (SC) are faced with a self-selection problem. That is, the method is primarily applied to non-complex data structures that are straightforward to forecast, given the availability of donors in the post-treatment period. Using Monte Carlo studies, we show that the high interpretability of SC comes at the costs of poor predictions and forecasts, which are especially pronounced if the data generating process contains a time series structure. To address this issue, we introduce the intricacy-statistics that informs the applied researcher whether or not the data at hand exceeds a level of time series structure that SC can handle. If the case, more flexible methodologies that combine the strengths of SC and conventional time series techniques promise more accurate predictions and forecasts. Hence we introduce the new VAR-SC estimator, that takes in account both the time series structure and the availability of donors. In order to implement these ideas, we introduce the R-package complex_synths that provides ready-to-use functions to compute the intricacy-statistics and, based on the magnitude of the statistics, the functionalities to estimate either the SC or the VAR-SC model. To probe the performance of our methodology outside the experimental setting, we apply it to existing application of SC and to a highly complex data structure: The inclusion of a stock in an index. Specifically, we find that the inclusion of the German multi-national eCommerce company Zalando in the German stock index (DAX) caused an excess capitalization of XXX milion euro.

Keywords: *Synthetic Control; Causality; VAR*

List of Acronyms

GDP Gross Domestic Product

SC Synthetic Control

USA United States of America

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1. Introduction

The method of Synthetic Controls (SC) is cool.

2. Literature Review 2-3 pages

2.1. Synthetic Control

The Synthetic Control (SC) method was developed by Alberto Abadie and colleagues in a series of influential papers ([Abadie and Gardeazabal, 2003], [Abadie et al., 2007], [Abadie et al., 2015]). The method is designed to estimate the causal effect of a treatment in a setting with a single treatment unit and a number of potential control units. Pre- and post-treatment data are observed for the treatment and control units for the outcome of interest as well as for a set of covariates. The SC-procedure combines aspects of the matching and difference-in-difference literature and can therefore be interpreted as a relative of the causal inference literature introduced by [Rubin, 1974]. Similar to many other microeconometric methods, the objective is to distinguish causation from correlation and to assess the magnitude and significance of treatments in observational case studies.

In their canonical 2003 article, Abadie and Gardeazabal evaluate the causal economic effects of conflict using terrorist conflicts in the Basque Country as a comparative case study. In their specific application example, they find that terrorist conflicts caused the per capita Gross Domestic Product (GDP) of the treatment unit (Basque Country) to decline by about 10% relative to the synthesized control unit. **some more words on other findis and things they did** The next appropriate setting for an application of the SC method was the introduction of a large-scale tobacco control program implemented in the state of California in the United States of America (USA) in 1988.

2.2. Overview

[Abadie, 2021] read.

[Athey and Imbens, 2016] read.

2.3. Application

[Born et al., 2019] read.

[Cho, 2020] read.

[Cunningham, 2021] read.

[Funke et al., 2020] read.

2.4. Methodological Background

[Hainmueller et al., 2011] read.

[Abadie and Imbens, 2006] not read.

[Abadie and Imbens, 2002] not read.

[Doudchenko and Imbens, 2016]

[Ferman, 2021] read.

[Frangakis and Rubin, 2002] not read.

[Rosenbaum and Rubin, 1983] not read

[Rubin, 1974] not read.

2.5. Extensions/ Developments

[Abadie and L'Hour, 2021] read.

[Amjad et al., 2018] read.

[Ben-Michael et al., 2021] read.

[Ben-Michael et al., 2021] not read.

[Kellogg et al., 2021] not read.

[Kuosmanen et al., 2021] not read.

[Muhlbach and Nielsen, 2019] read.

Developments

[Arkhangelsky et al., 2021] not read

[Athey et al., 2017] not read.

[Brodersen et al., 2015] read.

[von Brzeski et al., 2015] read.

[Hartford et al., 2017] read.

2.6. Testing

[Andrews, 2003] not read.

[Cattaneo et al., 2021] not read.

[Chernozhukov et al., 2019] not read.

[Chernozhukov et al., 2021] not read.

[Firpo and Possebom, 2018] not read.

[Hahn and Shi, 2017] read.

2.7. Time Series Econometrics

[Martin et al., 2012] read.

[Harvey and Thiele, 2020] read.

[Breitung and Knüppel, 2021] partially read.

3. Theory

3.1. Motivation

3.2. Static Framework

3.2.1. Simple Case

Consider a very simple framework for analyzing the causal effect of a single unit in the treatment unit $i = 0$ and two units in the control group $i = 1, 2$. It is assumed that before the intervention at time period $t = T_0$ the units have a joint distribution of the form

$$\boldsymbol{y} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ before } T_0$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)'$ and $\boldsymbol{\Sigma}$ is some positive definite covariance matrix with Choleski decomposition $\boldsymbol{\Sigma} = \boldsymbol{R}\boldsymbol{R}'$ and $\boldsymbol{R}$ is an *upper* triangular matrix. Assume that there is no contamination, meaning that the intervention effects the mean of the first variable such that $\mathbb{E}(Y_0) = \mu_0 + \delta$ after the intervention, whereas the means of the other two variables remain unaffected. Accordingly, δ represents the treatment effect on Y_0.

We are interested in deriving an optimal estimator for the counterfactual

3.2.2. General General Case

3.3. Dynamic Framework

4. Simulation Study (10pt, bold)

some text

5. Applications (if any)

6. Conclusion

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