Combining Synthetic Controls and VARs:

On the Estimation of Causal Effects in Time Series Data

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Abstract

We argue that applications of Synthetic Control (SC) are faced with a self-selection problem. That is, the method is primarily applied to non-complex data structures that are straightforward to forecast, given the availability of donors in the post-treatment period. Using simulation studies, we show that the high interpretability of SC comes at the cost of poor predictions and forecasts, which are especially pronounced if the data generating process contains a time series structure. To address this issue, we introduce the intricacy-statistics that informs the applied researcher whether or not the data at hand exceeds a level of time series structure that SC can handle. If the case, more flexible methodologies that combine the strengths of SC and conventional time series techniques promise more accurate predictions and forecasts. Hence we introduce the new Vector Autoregressive Synthetic Control (VARSC) estimator, that takes in account both the time series structure and the availability of donors. In order to implement these ideas, we introduce the R-package varsc that provides ready-to-use functions to compute the intricacy-statistics and, based on the magnitude of the statistics, the functionalities to estimate either the SC or the VARSC model. To probe the performance of our methodology outside the experimental setting, we apply it to three existing applications of SC: Specifically, we show that our proposed model performs equally well like the SC-method. The result is striking because in contrast to the SC-model, our models gets along without the informational contend of potential covariates.

Keywords: *Synthetic Control; Causality; VAR*

List of Acronyms

GDP Gross Domestic Product

SC Synthetic Control

USA United States of America

VAR Vector Autoregression

VARSC Vector Autoregressive Synthetic Control

Contents

1. Introduction

SC is cool. But Vector Autoregression (VAR) also.

2. Literature Review 2-3 pages

What must be clear by now

- **...**

2.1. Synthetic Control

The SC method was developed by Alberto Abadie and colleagues in a series of influential papers ([Abadie and Gardeazabal, 2003], [Abadie et al., 2007], [Abadie et al., 2015]). The method is designed to estimate the causal effect of a treatment in a setting with a single treatment unit and a number of potential control units. Pre- and post-treatment data are observed for the treatment and control units for the outcome of interest as well as for a set of covariates. The SC-procedure combines aspects of the matching and difference-in-difference literature and can therefore be interpreted as a relative of the causal inference literature introduced by [Rubin, 1974]. Similar to many other microeconometric methods, the objective is to distinguish causation from correlation and to assess the magnitude and significance of treatments in observational case studies.
In their canonical 2003 article, Abadie and Gardeazabal evaluate the causal economic effects of conflict using terrorist conflicts in the Basque Country as a comparative case study. In their specific application example, they find that terrorist conflicts caused the per capita Gross Domestic Product (GDP) of the treatment unit (Basque Country) to decline by about 10% relative to the synthesized control unit. **some more words on other findis and things they did** The next appropriate setting for an application of the SC method was the introduction of a large-scale tobacco control program implemented in the state of California in the United States of America (USA) in 1988.

2.2. Overview

[Abadie, 2021] read.
[Athey and Imbens, 2016] read.

2.3. Application

[Born et al., 2019] read.
[Cho, 2020] read.
[Cunningham, 2021] read.
[Funke et al., 2020] read.

2.4. Methodological Background

[Hainmueller et al., 2011] read.
[Abadie and Imbens, 2006] not read.

[Abadie and Imbens, 2002] not read.
[Doudchenko and Imbens, 2016] read.
[Ferman, 2021] read.
[Frangakis and Rubin, 2002] not read.
[Rosenbaum and Rubin, 1983] not read
[Rubin, 1974] not read.

2.5. Extensions/ Developments

[Abadie and L'Hour, 2021] read.
[Amjad et al., 2018] read.
[Ben-Michael et al., 2021] read.
[Ben-Michael et al., 2021] not read.
[Kellogg et al., 2021] not read.
[Kuosmanen et al., 2021] not read.
[Muhlbach and Nielsen, 2019] read.

Developments
[Arkhangelsky et al., 2021] not read
[Athey et al., 2017] not read.
[Brodersen et al., 2015] read.
[von Brzeski et al., 2015] read.
[Hartford et al., 2017] read.

2.6. Testing

[Andrews, 2003] not read.
[Cattaneo et al., 2021] not read.
[Chernozhukov et al., 2019] not read.
[Chernozhukov et al., 2021] not read.
[Firpo and Possebom, 2018] not read.
[Hahn and Shi, 2017] read.

2.7. Time Series Econometrics

[Martin et al., 2012] read.
[Harvey and Thiele, 2020] read.
[Breitung and Knüppel, 2021] partially read.

3. Theory

What must be clear by now

- **Consider case without covariates**
- **...**

In this chapter, we propose an alternative SC-estimator to assess the magnitude of treatment effects in observational settings. To establish a general basis, let us first describe the contextual environment of the estimation. We consider a framework with $k+1$ panel units indexed by $i = 0, 1, ..., k$ that are observed over a time horizon of T periods indexed by $t = 1, 2, ..., T_0, T_0 + 1, ..., T$. Without loss of generality, assume that unit $i = 0$, is exposed to the treatment at period $t = T_0$ and that there are no treatment anticipation and contamination. Our goal is to evaluate the causal effect of the treatment, the specific functional form of which has yet to be specified.[1]

The following theoretical argumentation is built up as follows: To gain some intuition, we first consider a very simple static scenario with only two donor units and one treatment unit. We then generalize this idea to the case with many potential donors. Since our main methodological improvement is rooted in the union of SC approaches with multivariate time series approaches, we finally present a general dynamic setup.

3.1. Simple Static Case

Consider a very simple framework for analyzing the causal effect of a treatment for unit $i = 0$ and two units in the control group $i = 1, 2$. It is assumed that before the intervention at time period $t = T_0$ the units have a joint distribution of the form

$$\boldsymbol{y} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ before } T_0.$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)'$ and $\boldsymbol{\Sigma}$ is some positive definite covariance matrix with Choleski decomposition $\boldsymbol{\Sigma} = \boldsymbol{R}\boldsymbol{R}'$ and $\boldsymbol{R}$ is an *upper* triangular matrix. Assume that the intervention affects the mean of the first variable such that $\mathbb{E}(Y_0) = \mu_0 + \delta$ after the intervention, whereas the means of the other two variables remain unaffected. Accordingly, δ represents the treatment effect on Y_0.

We are interested in deriving an optimal estimator for the counterfactual

$$\widehat{Y}_0^N = \mathbb{E}(Y_0 | \delta = 0, Y_1, Y_2) \text{ after } T_0.$$

[1] To ensure direct comparability with the SC literature, we adopt most of the commonly used labels. For example, control group units are labeled as 'donors'.

Let $Q = R^{-}1$ and q denotes the first row of Q, then

$$q'\boldsymbol{y} = q'\boldsymbol{\mu} + \epsilon,$$

where $\epsilon \sim \mathcal{N}(0,1)$ with $\mathbb{E}(\epsilon|Y_!, Y_2) = 0$. It follows that

$$\begin{aligned}\widehat{Y}_0^N &= w_1 Y_1 + w_2 + Y_2 + \mu^* \\ &= \mu_0 + w_1(Y_1 - \mu_1) + w_2(Y_2 - \mu_2),\end{aligned}$$

where $w_1 = -q_1/q_0$ and $w_2 = -q_2/q_0$ and $\mu^* = \mu_0 - w_1\mu_1 - w_2\mu_2$. These results imply that there is no reason to impose the restrictions $w_1 \leq 0, w_2 \leq 0$ (positivity) and $w_1 + w_2 = 1$ (adding-up). Furthermore, the construction of SC should include a constant term, as otherwise the SC may have a different mean, See also [Doudchenko and Imbens, 2016] for a careful discussion of these restrictions.

For illustration assume that

$$\boldsymbol{y} \sim \mathcal{N}\left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1 & 0.1 & 0.4\\0.1 & 1 & 0.5\\0.4 & 0.5 & 1\end{pmatrix}\right)$$

Elaborate here once understood. For this example the optimal weights for the SC result as $w_1 = -0.133$, $w_2 = 0.4667$ and $\mu^* = 1 - w_1 - w_2 = 0.667$. Note that w_1 is negative even all bivariate correlations between the panel units are positive. One may argue that this solution does not make much sense as from a economic perspective it is not clear what it means that Y_1 enters the SC with a negative sign. This demonstrates the trade-off between optimality in a statistical sense and the interpretability of the solution.

What happens if we impose the restrictions that all weights are positive and sum up to unity? In this case the restricted optimum yields the linear combination $\widetilde{Y}_0^N = 0.2Y_1 + 0.8Y_2$. The important difference lies in the variance of these estimates. For our example we obtain

$$\begin{aligned}var(Y_0 - \widehat{Y}_0^N) &= 0.827\\ var(Y_0 - \widetilde{Y}_0^N) &= 1.160\end{aligned}$$

It is interesting to note that the variance of the restricted estimate is even larger than the unconditional variance of Y_0. This is possible as $(w_1, w_2) = (0, 0)$ is not included in the restricted parameter space.

It is not difficult to see that if Y_0 is not correlated with Y_1 and Y_2, then the optimal estimate boils down to $\widehat{Y}_0^N = \mu_0$ and therefore it does not make sense to involve a SC. In microeconometric settings it is usually assumed that the individuals in the treatment group and individuals in th econtrol group are uncorrelated. In such cases we do not care about constructing a SC. The crucial feature of SC methods is the correlation between

the units in the treatment and the control group. In macroeconomic applications however, the variables in the treatment and control group (e.g. GDP) are typically correlated and it is therefore important to model the relationship between the variables. As the simple scenario with only two panel units in the donor pool is highly unrealistic in practice, we now move to the general static case with $k-1$ panel units.

3.2. General Static Case

What must be clear by now

- **Derive first analytical expressions for the case with k donors before talking about regularization**

In empirical practice it is often the case that the number of pre-intervention time periods $T_0 - 1$ is small and my even be smaller than the number of units in the donor pool, k.

3.3. General Dynamic Case

What must be clear by now

- **TBD**

When modeling macroeconomic time series it is often assumed that the $(k+1)x1$ vector of time series $y_t = (Y_{0t}, ..., Y_{kt})'$ can be represented by a VAR model given by

4. Simulation Study (10pt, bold)

some text

5. Applications (if any)

6. Conclusion

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