# FODO AI PROBABILITY & STATS

HYPOTHESIS TESTING

# HYPOTHESIS TESTING

Depends Upon

- Design of the Experiment
- Nos. of Sample
- $H_0$ i.e Null Hypothesis
- Test-statistics

# TWO SAMPLE Z-TEST

If you want to compare 2 Test Statistics of 2 Independent SAMPLES when $\sigma_{population}$ is known

Example

Researchers wants to compare avg Marks from 2 different Teaching Methods.

Method 1 : 50 Students, $\mu^1_{sample} = 78 = \bar{x}_1$ , $\sigma_1 = 25$ (Population)

Method 2 : 45 Students, $\mu^2_{sample} = 82 = \bar{x}_2$ , $\sigma_2 = 20$ (population)

$H_0 : \mu_1 = \mu_2$ (i.e Both Methods are good)

$H_1 : \mu_1 \neq \mu_2$

Test Statistics

$$\boxed{z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

# TWO SAMPLE T-TEST

If you want to compare 2 Test Statistics of 2 Independent SAMPLES when $\sigma_{population}$ is UNKNOWN

Example

Researchers wants to compare avg Height of 2 Species of PLANTS.

Species A: 12 plants, $\bar{x}_1 = 60cm$, $s = 5cm$ (std deviation of Sample)

Species B: 15 plants, $\bar{x}_2 = 64cm$, $s = 6cm$

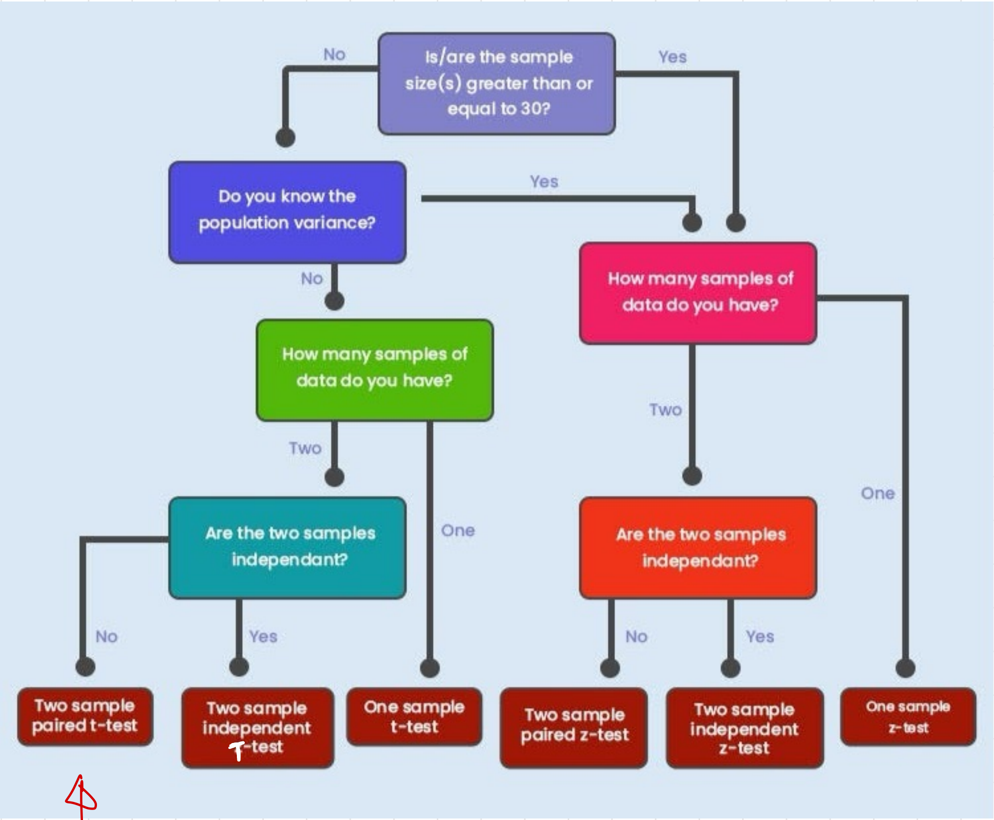
$H_0 : \mu_1 = \mu_2$ (i.e Heights of Both Species are same)

$H_1 : \mu_1 \neq \mu_2$

Test Statistics

$$\boxed{t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

$$\boxed{df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_1}\right)^2}{\frac{\frac{s_1^2}{n_1}}{n_1 - 1} + \frac{\frac{s_2^2}{n_1}}{n_2 - 1}}}$$

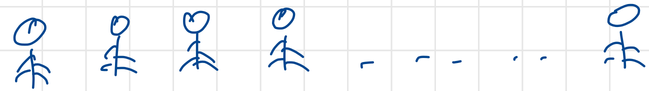



difference of Samples

# TWO SAMPE PAIRED Z-Test / T-Test

Case Study 4: GYM & WEIGHT LOSS

A Gym claims that in their current weight loss program, Participants have lost ~~greater that~~ 5 kgs at an avg.



| | | | | | | |
|---|---|---|---|---|---|---|
| Before → | 55 | 60 | 65 | 70 | - - . . | 100 |
| After → | 58 | 63 | 55 | 66 | - - . - | 97 |

$H_0: d_0 = 5$ $\sigma_0$ ✓ known

$H_1: d_1 \neq 5$

Difference → [ -3, 3, 10, .... 3]

$$\bar{d} = \frac{\Sigma d_i}{n} = 4.7$$

$$z = \frac{\bar{d}}{\sigma_d / \sqrt{n}}$$ if $n > 30$ & $\sigma_d$ is known

$$t = \frac{\bar{d}}{S_d / \sqrt{n}}$$ if $n < 30$ or $\sigma_d$ is Not known

$$S_d = \sqrt{\frac{\Sigma (d_i - \bar{d})^2}{(n-1)}}$$

$df = n - 1$

# RESAMPLING & PERMUTATION TESTING

Permutation Testing (or Randomization test) is used to determine if an observed effect is significant by comparing it to the distribution of effects generated by Random permutations of the Data.

example: Testing the Difference in Means Between Two Groups

Group A: 25, 30, 28, 21, 27

Group B: 35, 40, 38, 37, 36

Group A $\perp$ Group B

Step 1: find Group statistic.

$\bar{A} = 28.2$

$\bar{B} = 37.2$

$H_0$: No significant diff

$H_1$: There is diff.

$S_{original} = \bar{B} - \bar{A} = 9$

## Step 2: Combine, Permute (Random shuffle) & Split in 2 groups.

Original A + Original B $\rightarrow$ Shuffle $\rightarrow$ $A'$, $B'$

$A' \rightarrow$ mean $\overline{A'}$

$B' \rightarrow$ mean $\overline{B'}$

$\delta'$ (difference) $= \overline{B'} - \overline{A'}$

## Step 3: Repeat step 2 for large number of times.

$\delta_1, \delta_2, \delta_3 \dots \delta_{1000}$ (for 1000 times)

$\downarrow$ sort

$\delta'_1 \quad \delta'_2 \quad \delta'_3 \quad \dots \quad \uparrow \quad \dots \quad \delta'_{999} \quad \delta'_{1000}$

$\delta_{original}$ lies here in the range $[\delta'_1, \delta'_{1000}]$

## Step 4: Calculate p-value

$$\delta_1', \delta_2' \; . \; . \; . \; . \; . \quad \delta_{original} \; . \; . \; . \; . \quad \delta_{1000}'$$

$$\delta_{original} \; \underbrace{\ldots \; \delta_{1000}'}_{K \text{ points}}$$

$$Pr(\text{ difference} \geq \delta_{original} \mid H_0 \text{ is true}) = \frac{K}{1000} = x\%$$

$\alpha = 0.05$ ( i.e 5%)

Case 1: if $x = 10\%$

$p > \alpha \Rightarrow$ Accept $H_0$

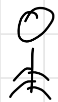
Case 2: if $x = 3\%$

$p < \alpha \Rightarrow$ Reject $H_0$

Accept $H_1$

** Permutation Testing is a Non-parametric approach to hypothesis testing without assuming a specific Distribution!

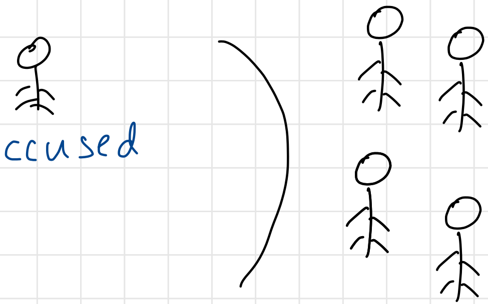
# ERRORS in HYPOTHESIS TESTING



Judge

↓

gives verdict & says



Accused

What if Accused is

Accused is Guilty — ? — Innocent
Guilty

Innocent — ? — Guilty
Innocent

Accused is (in Reality)

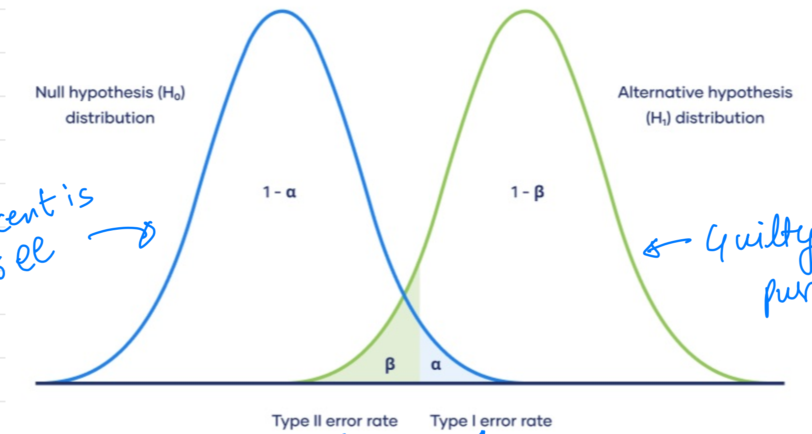
| Judge says | | innocent | Guilty |
|---|---|---|---|
| | innocent | NO ERROR <br> $1-\alpha$ | In favor of Accused <br> (TYPE II) <br> $\beta$ |
| | Guilty | Against Accused <br> (TYPE I) <br> $\alpha$ | NO ERROR <br> $1-\beta$ |

Acc. to Presumption of Innocence, the person is innocent until proven Guilty. That means Judge Must find the evidence which convince him "Beyond a Reasonable doubt" Level of Significance "$\alpha$"

i.e $Pr\left(\text{Judge says Guilty} \mid \text{Person is innocent}\right)$ should be Less!! Cond$^n$ Probability

Hypothesis Testing is Analogous to this Setup!

# Probability of Making Type I & Type II Error




Innocent is free

Guilty is punished

Guilty is free

innocent is punished

$H_0$: by default, Accused is innocent

$H_1$: Accused is Guilty

Accused is (in Reality)

| Judge says | innocent | Guilty |
|---|---|---|
| innocent | NO ERROR<br>$1-\alpha$ | In favor of Accused<br>(TYPE II)<br>$\beta$ |
| Guilty | Against Accused<br>(TYPE I)<br>$\alpha$ | NO ERROR<br>$1-\beta$ |

→ We Say $H_0$ is true (accept $H_0$) until we find strong evidence against it.

→ otherwise we accept $H_1$ & Reject $H_0$

$$\alpha = Pr\left(\frac{\text{Reject } H_0}{H_0 \text{ is True}}\right)$$

← Cond$^n$ Prob.

if $\alpha \downarrow$ smaller, then more evidence is require to reject $H_0$.

Truth about Population

**H0: Null Hypothesis**
*(Neutral / Status-quo)*

Reality

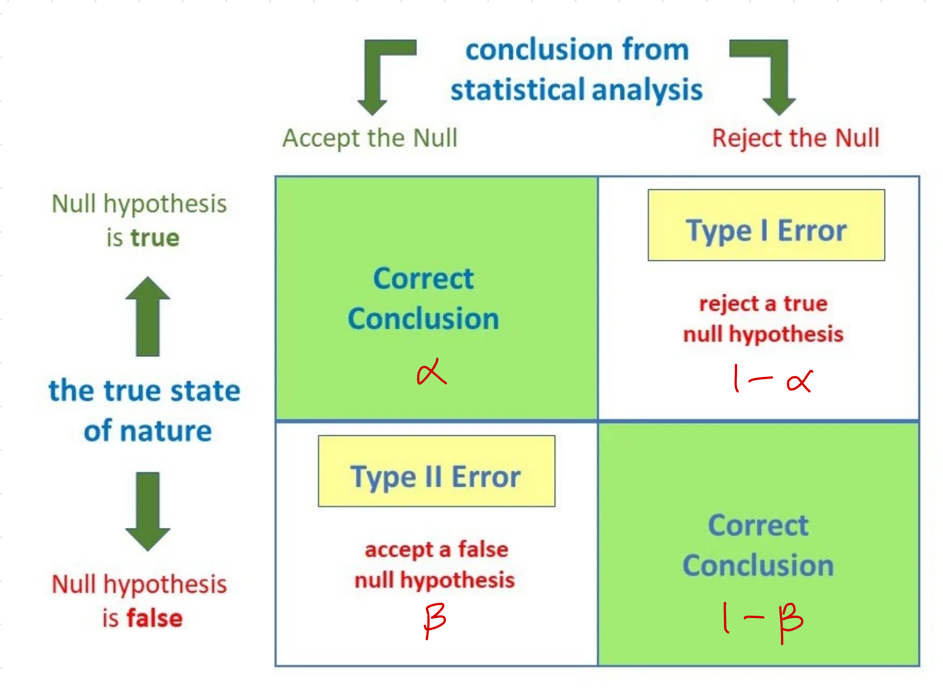
Decision based on Sample

Conclusions from observed data

| | (H0 is True) | (H0 is False) |
|---|---|---|
| **Reject H0** | **Type-I error (Alpha, α)**<br>- Reject a True Null Hypothesis.<br>→ False-positive conclusion | Correct Conclusion ☺<br>$(1-\beta)$ |
| **Do not Reject H0** | Correct Conclusion ☺<br>$(1-\alpha)$ | **Type-II error (Beta, β)**<br>- Do not reject a False Null Hypothesis<br>→ False-Negative conclusion |

$$\alpha + (1-\alpha) = 1$$

$$(1-\beta) + \beta = 1$$


conclusion from statistical analysis
Accept the Null
Reject the Null
Null hypothesis is true
the true state of nature
Null hypothesis is false
Correct Conclusion
$\alpha$
Type I Error
reject a true null hypothesis
$1-\alpha$
Type II Error
accept a false null hypothesis
$\beta$
Correct Conclusion
$1-\beta$