Low-Rank Tensor MMSE Equalization

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Motivation

- Massive MIMO
  - High spatial resolution
  - Large beamforming gain
  - Better interference rejection
  - **Spatial multiplexing boost!**
- Signal processing becomes more complex
- Opportunity for computationally efficient filtering solutions
- Proposal: **tensor** filters

What Are Tensor Filters?

- Linear and time-invariant filter: $\boldsymbol{w} = [w_1, \ldots, w_N]^{\mathsf{T}}$
- Multi-linear (tensor) and time-invariant filter:

$$\boldsymbol{w} = \boldsymbol{w}_1 \otimes \ldots \otimes \boldsymbol{w}_M \in \mathbb{C}^N$$

  where $\boldsymbol{w}_m \in \mathbb{C}^{N_m}$ with $\prod_{m=1}^{M} N_m = N$

- $\otimes$ denotes Kronecker (tensor) product
- Basic idea: design **each** factor instead of the **whole** vector
- Questions
  - Fewer computations?
  - How much performance loss, if any?
- Our first results:
  - "Low-complexity separable beamformers for massive antenna array systems," IET Signal Processing 13.4 (2019): 434-442;
  - "Separable linearly constrained minimum variance beamformers," Signal Processing, 158 (2019): 15-25.

Uniform Rectangular Array (URA) Beamforming

- Far-field propagation, narrowband signal, no coupling assumptions
- URA response vector:

$$\begin{aligned}\boldsymbol{a}(\phi_r, \theta_r) &= \begin{bmatrix} 1 \\ e^{j\pi \cos\theta_r} \\ \vdots \\ e^{j\pi(N_v-1)\cos\theta_r} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ e^{j\pi \sin\phi_r \sin\theta_r} \\ \vdots \\ e^{j\pi(N_h-1)\sin\phi_r \sin\theta_r} \end{bmatrix} \\ &= \boldsymbol{a}_v(q_r) \otimes \boldsymbol{a}_h(p_r)\end{aligned}$$

  with $N = N_h \cdot N_v$, $p_r = \sin\phi_r \sin\theta_r$ and $q_r = \cos\theta_r$.
- Response vector is **separable** in horizontal and vertical domains
- Separable filter $\boldsymbol{w} = \boldsymbol{w}_v \otimes \boldsymbol{w}_h$

Uniform Rectangular Array (URA) Beamforming

- Complexity reduction with slight source recovery performance degradation ✓
- However, practical antenna arrays are hardly exactly separable
  - Mutual coupling
  - Multipath propagation
- Consequently, separable beamformer performs poor: few degrees of freedom (DoF)
- How to increase DoF and keep complexity low? **Low-rank filter structure**

Low-Rank Filter

- Rank-1 (separable) order M filter

$$\boldsymbol{w} = \boldsymbol{w}_1 \otimes \ldots \otimes \boldsymbol{w}_M \tag{1}$$

- Rank-R (low-rank) order M filter

$$\boldsymbol{w} = \sum_{r=1}^{R} \boldsymbol{w}_{1,r} \otimes \ldots \otimes \boldsymbol{w}_{M,r} \tag{2}$$

- Degrees of freedom:
  - Linear filter: N
  - Separable tensor filter: $N_1 + \ldots + N_M$
  - Low-rank tensor filter: $R(N_1 + \ldots + N_M)$

Uplink MIMO System Model

- U users

$$\boldsymbol{x}[k] = \sum_{u=1}^{U} \boldsymbol{H}_u \boldsymbol{s}_u[k] + \boldsymbol{b}[k] \tag{3}$$

$$\boldsymbol{s}_u[k] = [s_u[k], \ldots, s_u[k-Q+1]]^{\mathsf{T}} \tag{4}$$

- Channel model

$$\boldsymbol{H}_u = \sum_{\ell=1}^{L} \alpha_{u,\ell} \boldsymbol{a}(\theta_{u,\ell}) \boldsymbol{g}(\tau_{u,\ell})^{\mathsf{T}} \in \mathbb{C}^{N \times Q} \tag{5}$$

$$\boldsymbol{a}(\theta_{u,\ell}) = \left[1, \ldots, e^{-\jmath\pi(N-1)\cos\theta_{u,\ell}}\right]^{\mathsf{T}} \in \mathbb{C}^{N} \tag{6}$$

$$\boldsymbol{g}(\tau_{u,\ell}) = [g(-\tau_{u,\ell}), \ldots, g((Q-1)T - \tau_{u,\ell})]^{\mathsf{T}} \in \mathbb{C}^{Q} \tag{7}$$

- $\boldsymbol{H}_u$ is not separable; but admits a **low-rank** structure
- **Low-rank equalizer** to filter the desired data stream $s_u[k]$

Some Algebra...

The filter coefficients can be written as

$$w_{n_1,\ldots,n_D} = \sum_{r=1}^{R} \prod_{d=1}^{D} [\boldsymbol{w}_{d,r}]_{n_d}, \tag{8}$$

which allows us to recast the equalizer output $y[k] = \boldsymbol{w}^{\mathsf{H}} \boldsymbol{x}[k]$ as follows

$$y[k] = \sum_{n_1,\ldots,n_D=1}^{N_1,\ldots,N_D} \left(\sum_{r=1}^{R} [\boldsymbol{w}_{1,r}]^*_{n_1} \ldots [\boldsymbol{w}_{D,r}]^*_{n_D} \right) x_{n_1,\ldots,n_D}[k]. \tag{9a}$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [\boldsymbol{w}_{d,r}]^*_{n_d} \left(\sum_{n_q=1}^{N_q} \prod_{q \neq d}^{D} [\boldsymbol{w}_{q,r}]^*_{n_q} x_{n_1,\ldots,n_D}[k] \right) \tag{9b}$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [\boldsymbol{w}_{d,r}]^*_{n_d} [\boldsymbol{u}_{d,r}[k]]_{n_d} = \boldsymbol{w}_d^{\mathsf{H}} \boldsymbol{u}_d[k] \tag{9c}$$

Output is linear w.r.t. each tensor filter factor w_d!

Low-Rank Tensor MMSE

- We formulate for each filter mode

$$\min_{\boldsymbol{w}_d} \mathbb{E}\left[\left|s_u[k-\delta] - \boldsymbol{w}_d^{\mathsf{H}}\boldsymbol{u}_d[k]\right|^2\right], \quad d \in \{1,\ldots,D\}.$$

where

$$\boldsymbol{u}_d[k] = \left[\boldsymbol{u}_{d,1}^{\mathsf{T}}[k],\ldots,\boldsymbol{u}_{d,R}^{\mathsf{T}}[k]\right]^{\mathsf{T}} \in \mathbb{C}^{RN_d} \tag{10}$$

$$\boldsymbol{u}_{d,r}[k] = \boldsymbol{X}_{(d)}[k] \bigotimes_{q\neq d}^{D} \boldsymbol{w}_{q,r}^{*} \in \mathbb{C}^{N_d} \tag{11}$$

$$\boldsymbol{w}_d = \left[\boldsymbol{w}_{d,1}^{\mathsf{T}},\ldots,\boldsymbol{w}_{d,R}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{RN_d} \tag{12}$$

- Solution:

$$\boldsymbol{w}_{d,\mathsf{MMSE}} = \boldsymbol{R}_{u_d,u_d}^{-1}\boldsymbol{p}_{u_d} \in \mathbb{C}^{RN_d}, \tag{13}$$

$$\boldsymbol{R}_{u_d,u_d} = \mathbb{E}\left[\boldsymbol{u}_d[k]\boldsymbol{u}_d^{\mathsf{H}}[k]\right] \in \mathbb{C}^{RN_d\times RN_d}, \tag{14}$$

$$\boldsymbol{p}_{u_d} = \mathbb{E}\left[\boldsymbol{u}_d[k]s_u^{*}[k-\delta]\right] \in \mathbb{C}^{RN_d} \tag{15}$$

- **Block coordinate descent sweeping between modes**

Computational Complexity

- N: number of antennas
- K: number of snapshots (covariance matrix estimation)
- MMSE filter

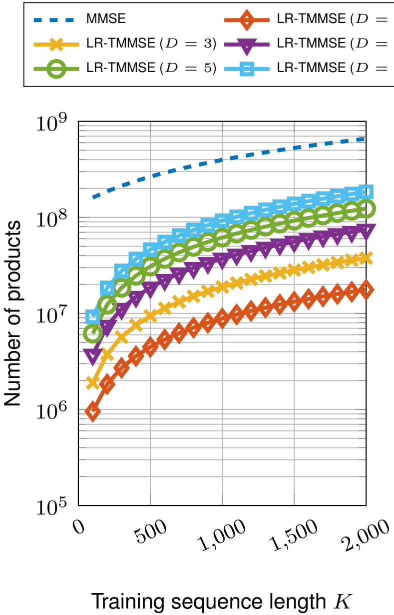
$$P_{\text{MMSE}}(N,K) = \underbrace{N^2K + NK}_{\text{statistics estimation}} + \overbrace{O(N^3)}^{\text{cov. matrix inversion}} + \underbrace{N^2}_{\text{filtering}}$$

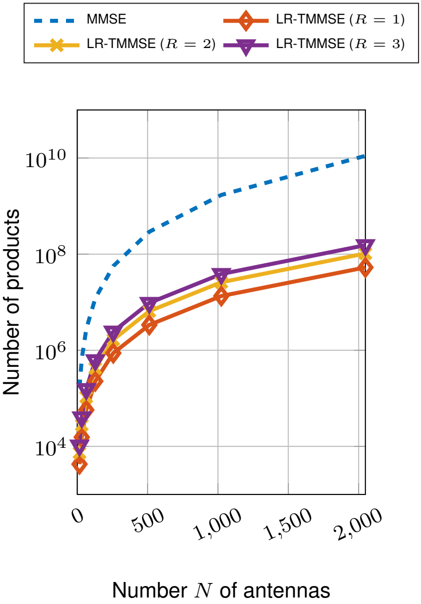
- LR-TMMSE filter

$$P_{\text{LR-TMMSE}}(\{N_d\}, D, I, K) =$$
$$I\left[\sum_{d=1}^{D} \underbrace{R(D-1)NK + N_d^2K + N_dK}_{\text{statistics estimation}} + \overbrace{O(N_d^3)}^{\text{cov. matrix inversion}} + \underbrace{N_d^2}_{\text{filtering}}\right]$$

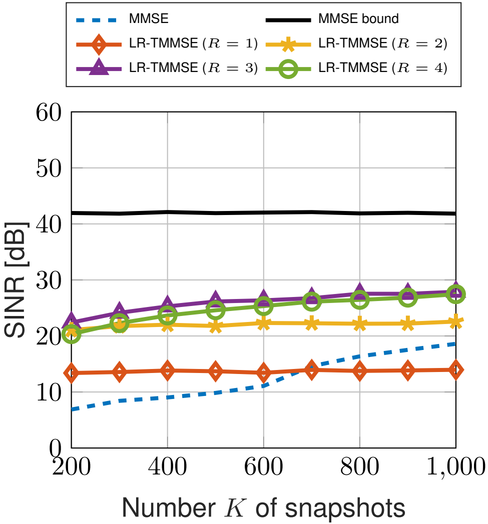
- I: iterations number

Computational Complexity





Symbol recovery performance



$$\mathrm{SINR}(\boldsymbol{w}) = \frac{\boldsymbol{w}^{\mathsf{H}} \boldsymbol{R}_{xx} \boldsymbol{w}}{\boldsymbol{w}^{\mathsf{H}} (\boldsymbol{R}_{ii} + \boldsymbol{R}_{bb}) \boldsymbol{w}}$$

$N = 512$ antennas

SNR = 20 dB

Filter order $D = 3$

$U = 4$ users

$L = 4$ paths

Conclusion

- $\uparrow$ rank $\uparrow$ equalization performance (up to a point)
- $\uparrow$ filter order $\uparrow$ calculations: **tensor overhead**
  - number of tensor products, unfoldings, etc, increase with tensor order!
- Fewer samples to estimate covariance matrices
- **Attractive complexity/performance trade-off**

Research perspectives

- Incorporate non-idealities with more realistic arrays (HFSS)
- Comprehensive multi-cell massive MIMO performance analysis

Thank you!
Questions?

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Slides available at `http://lnribeiro.github.io`