A Spacial Autoregressive Stochastic Frontier Model with Inefficiency Heterogeneity for Panel Data

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Outline

Introduction

The global financial crisis has given rise to reexamine the performance of the banking sector.

- Pre-crisis: bigger is better
- Post-crisis: too-big-to-fail

Previous literature ignored the spacial spillover effect.

Research Question

A bank holding company (BHC) is a company that controls one or more banks, but does not necessarily engage in banking itself.

- Does there exist spacial spillovers in the U.S. BHC during global financial crisis?
- Controlling for the spacial spillovers, what determines the cost/profit inefficiency of BHC?

Stochastic Frontier Model

First introduced independently by Aigner et al. (1977), and Meeusen and van Den Broeck (1977).

$$\begin{aligned} y_i &= \boldsymbol{x_i}\boldsymbol{\beta} + \nu_i - u_i, \\ \nu_i &\sim N(0, \sigma^2), \\ i &= 1, 2 \ldots, N \end{aligned} \tag{SFM}$$

- y_i: the (logged) cost/profit
- x_i: the (logged) output, input prices and or environmental variables
- ν_i: two-sided, idiosyncratic error
- u_i: one-sided error, measuring efficiency ($\exp(-u_i)$): exponential, truncated normal, half-normal, gamma, ...

SFM with Heterogeneous Inefficiency for Panel Data

$\boldsymbol{z_{it}}$ are observed covariates vector that affects inefficiency.

$$
\begin{aligned}
y_{i,t} &= \boldsymbol{x_{i,t}}\boldsymbol{\beta} + \nu_{i,t} - u_{i,t}, \\
\nu_{i,t} &\sim N(0, \sigma^2), \\
u_{i,t} &\sim \exp(\lambda), \\
\lambda &= \exp(\boldsymbol{z_{i,t}}\gamma) \\
i = 1, 2, \ldots, N, &\quad t = 1, 2, \ldots, T,
\end{aligned}
\tag{SFMH}
$$

SFM with Spacial Autoregressive Effect

Glass et al. (2016) proposed the spacial autoregressive stochastic frontier model, similar as following :

$$
\begin{aligned}
y_{i,t} = \rho \sum_{j=1}^{N} W_{i,j} y_{j,t} + \boldsymbol{x_{i,t}} \boldsymbol{\beta} + \nu_{i,t} - u_{i,t}, \\
\nu_{i,t} \sim N(0, \sigma^2), \\
u_{i,t} \sim \exp(\lambda) \\
i = 1, 2, \ldots, N, \quad t = 1, 2, \ldots, T,
\end{aligned} \tag{SAR}
$$

W: a neighborhood structure, measuring geographic or economic distances

I propose the following SAR model with heterogeneous inefficiency for panel data:

$$
\begin{aligned}
y_{i,t} = \boldsymbol{x_{i,t}}\boldsymbol{\beta} + \rho \sum_{j=1}^{N} W_{i,j} y_{j,t} + \nu_{i,t} - u_{i,t}, \\
\nu_{i,t} \sim N(0, \sigma^2), \\
u_{i,t} \sim \exp(\lambda), \\
\lambda = \exp(\boldsymbol{z_{i,t}}\gamma) \\
i = 1, 2, \ldots, N, \quad t = 1, 2, \ldots, T,
\end{aligned}
\tag{SARH}
$$

Estimation Method

Bayesian inference techniques organized around Markove Chain Monte Carlo (MCMC) are applied.
The model is completed by specifying the following priors:

$$\begin{aligned} \boldsymbol{\beta} &\sim N(\mathbf{0}, \sigma^2 \boldsymbol{R}), \\ \rho &\sim Unif(\frac{1}{r_{min}}, 1), \\ \sigma^{-2} &\sim \Gamma(a_s, b_s), \\ \gamma &\sim N(\mathbf{0}, \boldsymbol{R_1}) \end{aligned} \tag{1}$$

Where r_{min} is the most negative real characteristic root of $W = (W_{i,j})$ to ensure the above equation (SARH) stable.

Posterior Distribution

$$
\begin{aligned}
p(\boldsymbol{\beta}, \sigma^2, \rho, u, \gamma | X, y) &\propto p(X, y | \boldsymbol{\beta}, \sigma^2, \rho, u, \gamma) p(u|\gamma) p(\boldsymbol{\beta}|\sigma^2) p(\sigma^2) p(\rho) p(\gamma) \\
&\propto (\sigma^{-2})^{\frac{NT}{2}} \exp[-\frac{(y^* - X\beta + u)'(y^* - X\beta + u)}{2\sigma^2}] \\
&\times (\exp(z_{it}\gamma))^{NT} \exp(-\exp(z_{it}\gamma) u' 1_{NT}) I(u \geq 0) \\
&\times (\sigma^{-2})^{\frac{p}{2}} \exp(-\frac{\beta' \boldsymbol{R}^{-1} \beta}{2\sigma^2}) \\
&\times (\sigma^{-2})^{a_s - 1} \exp(-\sigma^{-2} b_s) \\
&\times I(\rho \in (\frac{1}{r_{min}, 1})) \\
&\times \exp(-\frac{\gamma' \boldsymbol{R_1}^{-1} \gamma}{2})
\end{aligned} \tag{2}
$$

where $y^* = (I_{NT} - \rho I_T \otimes W_N) y$, and $\otimes$ is kronecker product.

Conditional Posterior Distribution

$$\boldsymbol{\beta}|else \sim N((X'X + R^{-\mathbf{1}})^{-\mathbf{1}} X'(y+u), \sigma^{\mathbf{2}}(X'X + R^{-\mathbf{1}})^{-\mathbf{1}}) \tag{3}$$

$$\sigma^{-\mathbf{2}}|else \sim \Gamma(\frac{NT+p}{2} + a_s, \frac{(y^* - X\beta + u)'(y^* - X\beta + u)}{2} + b_s + \frac{\beta' R^{-\mathbf{1}}\beta}{2}) \tag{4}$$

$$u|else \sim N(-(y^* - X\beta + \sigma^{\mathbf{2}} \exp(z_{it}\gamma)\mathbf{1}_{NT}), \sigma^{\mathbf{2}}\mathbf{1}_{NT}) \tag{5}$$

$$\rho|else \sim N(-(y'(IW)'(IW)y)^{-\mathbf{1}} y'(IW)'(y - X\beta + u), (y'(IW)'(IW)y)^{-\mathbf{1}}\sigma^{\mathbf{2}}) I(\rho \in (\frac{1}{r_{min}}, 1)) \tag{6}$$

$$p(\gamma|else) \propto (\exp(z_{it}\gamma))^{NT} \exp(-\exp(z_{it}\gamma)u'\mathbf{1}_{NT}) I(u \geq 0) \exp(-\frac{\gamma' R_{\mathbf{1}}\gamma}{2}) \tag{7}$$

where $IW = I_T \otimes W_N$.

Monte Carlo Experiments

Simulation data are generated according to Equation (SARH).

		— **SARH** —	
Param	True	Estimate	95%HPD
β_1	1	1.1893	[0.7146, 1.5705]
β_2	4	4.0469	[3.8602, 4.2376]
ρ	0.5	0.5048	[0.4949, 0.5168]
σ^2	1	0.9128	[0.5039, 1.4246]
γ_1	-2	-2.0264	[-2.1191, -1.9155]
γ_2	1	0.9719	[0.8835, 1.0754]

Monte Carlo Experiments

		— SAR —		— SFMH —		— SFM —	
Param	True	Estimate	95%HPD	Estimate	95%HPD	Estimate	95%HPD
β_1	1	2.1784	[1.8159, 2.5630]	-7.6708	[-8.6454,6.7861]	-5.0684	[-5.8480,-3.9995]
β_2	4	4.0403	[3.7775, 4.2874]	4.5026	[3.8493,5.1452]	4.6834	[3.9951,5.3196]
ρ	0.5	0.5091	[0.5002, 0.5189]	NA	NA	NA	NA
σ^2	1	0.4372	[0.1750, 0.7045]	15.9938	[10.5209,20.8060]	8.7776	[4.9596,12.7367]
γ_1	-2	NA	NA	-2.4489	[-2.5855,-2.3336]	NA	NA
γ_2	1	NA	NA	0.7995	[0.6833,0.8993]	NA	NA
λ	NA	0.0708	[0.0645, 0.0769]	NA	NA	0.0511	[0.0466,0.0563]

Data and Variables

Source: Federal Reserve FR Y-9C Bank Holding Company Reports

- Quarterly data from 1986 to 2018
- Effective March 31, 2006, the FRY-9C filing threshold was increased from $150 million to $500 million or more
- I use yearly end data over 2006-2014
- Remove BHC with NA and negative equity/profit

Year	No. of BHC
2006	290
2007	277
2008	218
2009	203
2010	230
2011	249
2012	268
2013	285
2014	298

- No. of BHC in banlanced panel: 143

Variable Description

X_1	Sum of deposits, federal funds purchased, trading liabilities and other borrowed money
X_2	Number of full-time equivalent employees at end of current period
X_3	Premises and fixed assets (including capitalized leases)
Y_1	Loans and lease financing receivables
Y_2	Securities: held-to-maturity and available-for-sale
Y_3	Total non-interest income
Z_1	Total equity capital
Z_2	Total assets
Z_3	Allowance for loan and lease losses (NPL)
W_1X_1	Total interest expense
W_2X_2	Salaries and employee benefits
W_3X_3	Expenses of premises and fixed assets
TC	$W_1X_1 + W_2X_2 + W_3X_3$
TP	Net income (loss) attributable to holding company

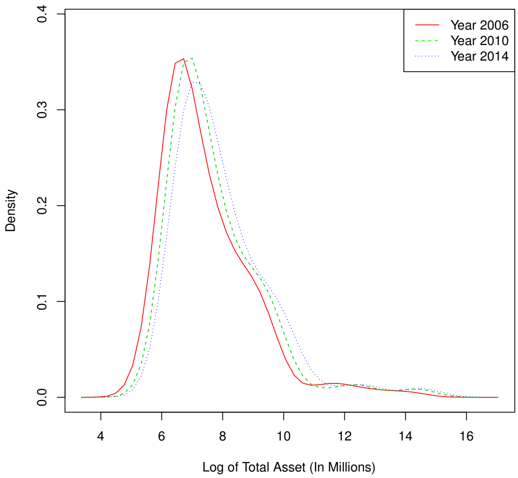
Inefficiency Determinants

The inefficiencies are assumed to depend upon covariates as in Tecles and Tabak (2010), they are:

- Market share of loans (MS)
- Non-performing loans/Asset (NPL)
- Equity/Asset (EQUITY)
- Asset size (SIZE)
- ...

$$\lambda = \exp(\gamma_1 MS + \gamma_2 NPL + \gamma_3 EQUITY + \gamma_4 SIZE + \dots) \tag{8}$$

Density of (log) Total Assets



Summary Statistics for Year 2014 (Dollar Amount in Millions)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Funds and Deposits (X_1)	143	35,279.47	217,535.10	379.56	824.74	5,072.30	2,134,102.00
Labor (X_2)	143	5,581.61	30,121.13	95	224	1,353.5	264,452
Fixed Asset (X_3)	143	295.91	1,355.71	2.63	17.39	87.91	12,936.00
Loans (Y_1)	143	18,602.25	100,025.80	158.28	560.06	3,790.52	865,677.00
Securities (Y_2)	143	7,444.89	39,989.74	2.53	259.54	1,715.29	346,901.00
Non-interest income (Y_3)	143	850.48	5,489.55	1.97	7.99	75.62	51,394.00
Equity (Z_1)	143	4,278.75	25,136.01	45.04	103.63	775.45	231,727.00
Asset (Z_2)	143	41,664.43	259,375.40	434.24	933.69	5,826.67	2,572,773.00
NPL (Z_3)	143	287.98	1,618.71	1.44	8.67	47.02	14,185.00
Total Cost (TC)	143	844.03	5,089.92	7.91	22.51	123.30	46,005.00
Total Profit (TP)	143	451.51	2,698.06	1.72	9.78	74.25	23,057.00

Neighborhood Matrix W

We have two possible definitions:

- distance between pairs of BHC (Glass and Kenjegalieva (2019)))
- k nearest neighbor (KNN)

Row sums of W are equal to 1.

SARH with Time-varying Spacial Spillovers

The changes of ρ_t can be interpreted as contagion.

$$\begin{aligned} y_{i,t} &= \boldsymbol{x_{i,t}}\beta + \rho_t \sum_{j=1}^{N} W_{i,j} y_{j,t} + \nu_{i,t} - u_{i,t}, \\ \nu_{i,t} &\sim N(0, \sigma^2), \\ u_{i,t} &\sim \exp(\lambda), \\ \lambda &= \exp(\boldsymbol{z_{i,t}}\gamma) \\ i = 1, 2, \ldots, N, \quad t &= 1, 2, \ldots, T, \end{aligned} \tag{9}$$

SAR with Inefficiency Spillovers

Inefficiency could also be spatially correlated. That is to say, inefficiency of one unit could also be affected by its neighbours.

$$
\begin{aligned}
y_{i,t} &= \boldsymbol{x_{i,t}}\boldsymbol{\beta} + \rho \sum_{j=1}^{N} W_{i,j} y_{j,t} + \nu_{i,t} - u_{i,t}, \\
\nu_{i,t} &\sim N(0, \sigma^2), \\
u_{i,t} &= \theta \sum_{j=1}^{N} W_{i,j} u_{j,t} + \epsilon_{i,t} \\
\epsilon_{i,t} &\sim \exp(\lambda), \\
i = 1, 2, \dots, N, &\quad t = 1, 2, \dots, T
\end{aligned}
\tag{10}
$$

Spatial-Temporal Stochastic Frontier Model

Inefficiency could also be time correlated. That is to say, there is adjustment cost for firms.

$$\begin{aligned} y_{i,t} &= \boldsymbol{x_{i,t}}\boldsymbol{\beta} + \nu_{i,t} - u_{i,t}, \\ \nu_{i,t} &\sim N(0, \sigma^2), \\ u_{i,t} &= \exp(\boldsymbol{z_{i,t}}\boldsymbol{\lambda} + \epsilon_{i,t}), \\ \boldsymbol{\epsilon_t} &= (\epsilon_{1,t}, \epsilon_{2,t}, \ldots, \epsilon_{N,t})^T, \\ \boldsymbol{\epsilon_t} = \theta\boldsymbol{\epsilon_{t-1}} + \boldsymbol{\phi_t}, \quad \text{for} \quad t &= 2, 3, \ldots, T, \\ \boldsymbol{\epsilon_1} &= \boldsymbol{\phi_1}, \\ \boldsymbol{\phi_t} \sim N(0, \tau^2(\boldsymbol{D} - \rho\boldsymbol{W})^{-1}), \quad \text{for} \quad t &= 1, 2, \ldots, T \end{aligned} \tag{11}$$

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