On a tractable Small Open Economy Model with Endogenous Monetary-Policy Trade-offs

Numerical analysis

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1 Approximation of the competitive equilibrium

The competitive equilibrium is now represented by three equations:

$$\pi_{H,t} = \widehat{\beta}\mathbb{E}_t\left\{\pi_{H,t}\right\} + \lambda\left(\kappa_1\widetilde{x}_t + \kappa_2\widetilde{q}_t\right), \tag{1}$$

$$\widetilde{x}_t = \varpi\mathbb{E}_t\left\{\widetilde{x}_{t+1}\right\} - \mu\left[i_t - \mathbb{E}_t\left\{\pi_{H,t+1}\right\}\right] + \chi\mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} + \epsilon_t, \tag{2}$$

and

$$\widetilde{q}_t = \mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} - (1-\gamma)\left[i_t - \mathbb{E}_t\left\{\pi_{H,t+1}\right\}\right] + u_t, \tag{3}$$

where

$$\lambda = \left[\frac{(1-\theta)(1-\theta\beta)}{\theta}\right]\left[\frac{(1-v)(1-\delta)}{1-v+\delta\varphi}\right],$$

$$\kappa_1 = \frac{\sigma}{1-\gamma} + \varphi,$$

$$\kappa_2 = \underbrace{\frac{\delta(1-v+\varphi)}{(1-\gamma)(1-v)(1-\delta)}}_{\text{Production effect}} - \underbrace{\frac{\sigma\eta\gamma(2-\gamma)}{(1-\gamma)^2} + \frac{\gamma}{1-\gamma}}_{\text{demand effect}},$$

$$\varpi = \frac{\sigma}{\sigma-\phi},$$

$$\mu = \left(\frac{1-\gamma}{\sigma-\phi}\right)\left[1-\gamma+\frac{\eta\gamma(2-\gamma)(\sigma-\phi)}{1-\gamma}\right],$$

and

$$\chi = \frac{\eta\gamma\phi(2-\gamma)}{(1-\gamma)(\sigma-\phi)}.$$

What is new in our model?:

1. **Exchange rate now matters**. First, the current value of this rate appears in the New Keynesian Phillips curve. In addition, the expected value of the exchange rate also appears in the IS equation. However, this last effect should be discarded because depends on the assumption of endogenous discounting. Since we assume that the value of ϕ is very close to zero, this effect tends to disappear. Focusing on the effect of exchange rate on the inflation, this is given by $\lambda\kappa_2$. Observe that κ_2 can be decomposed into two effects: *production effect* and *demand effect*. Let us explain them:

   (a) Production effect.- A depreciation of the exchange rate increases the prices of the imported inputs, which increases the marginal cost of producing and then the domestic inflation.

   (b) Demand effect.- This effect has two components. A deprecation of the exchange rate provokes a substitution of home consumption goods by foreign goods, which increases the marginal cost of producing and then the domestic inflation. In addition the depreciation of exchange rate has also an income effect, the depreciation forces the consumers to reduce the consumption of domestic goods if they want to maintain the level of consumption of foreign goods. This reduction of the demand of domestic goods, reduces the marginal cost and then the domestic inflation rate. A NEW TASK: We hace to decompose the second part of κ_2 into the two components of this demand effect.

2. In our economy **openness depends on two parameters**: γ, which determines the exterior dependence of demand; and δ, which determines the exterior dependence of production. This arises some important issues:

(a) The effects of γ are different to the original model of Gali and Monacelli (2005). As in this paper, in our economy γ also affects the response of the output gap to the real interest rate (μ) and the response of inflation to the current output gap (κ_1). However, this parameter γ also affect in our model to the response of inflation to the current exchange rate (κ_2).

(b) In contrast with Gali and Monacelli (2005), and as McCallum and Nelson (1999), openness also matters in the production side. The parameter δ first affects the response of inflation to the current output gap (λ), and to the current exchange rate (λ and κ_2). However, in contrast with the demand openness γ, this channel of openness does not affect the response of output gap to the real interest rate (μ).

(c) Regarding the effect of openness on the response of domestic inflation to output gap, note that the effect is ambiguous in our model because openness in demand (γ) positively affects κ_1, whereas openness in production (δ) negatively affects λ.

(d) These channels of openness effect should be considered to analyze the policy trade-off and the stability of the competitive equilibrium.

3. **What is the role of incomplete markets in the equilibrium relations?** To answer this question, we should derive the complete version of our model, which is a little be different than the one in Gali and Monacelli (2005) because our assumption on the openness in the production side (i.e., $\delta \neq 0$). In complete markets the international risk sharing imposes that $q_t = \sigma\left(c_t + c_t^*\right)$, Using this relationship we obtain that the expression (30) in the paper transforms into

$$y_t = \left[1 - \gamma + \frac{\gamma\sigma\eta\left(2-\gamma\right)}{1-\gamma}\right] c_t + \left[\gamma + \frac{\gamma\sigma\eta\left(2-\gamma\right)}{1-\gamma}\right] c_t^*.$$

This implies that output is a linear function of exchange rate, so that they are perfect correlated. By using this new expression, we obtain that the differences between the incomplete market model and the complete market model is in three points:

(a) In the complete version the exchange rate does not affect the inflation, i.e., $\kappa_2^c = 0$.[1] This implies that the complete version is an isomorphism of the close model. This is true because even when the expected exchange rate affects the output gap, this effect is negligible because ϕ is almost zero, and so also χ. Interestingly, remember that the fact that $\kappa_2 \neq 0$ implies that incompleteness introduces endogenously a cost push in the New Keynesian Phillips curve.

(b) Incompleteness also affects the response of inflation to the output gap. Effectively, the parameter κ_1 is different in the complete version. In particular, it would be

$$\kappa_1^c = \left\{\frac{1+\delta\left(\varphi - v\right)}{\left(1-\delta\right)\left[1-\gamma+\frac{\gamma\sigma\eta(2-\gamma)}{1-\gamma}\right]}\right\}\left(\frac{\sigma}{1-\gamma}\right) + \varphi.$$

The difference between κ_1 (incomplete markets) and κ_1^c (complete markets) is the first expression in the right-hand side of κ_1^c. This difference has an ambiguous sign because a priory is difficult to sign $\varphi - v$. However, we will consider negative values of v. Hence, this first expression in the right-hand side of κ_1^c will be positive. However, we do not yet know whether this expression is larger or smaller than one. In the next section we will quantify this difference.

[1]We will use the super-script c to denote the parameters that are different in the complete market version of the model.

(c) Finally, incompleteness also affects the response of output gap to the real interest rate given by μ. In the complete market version this parameter would be

$$\mu^c = \left[\frac{1-\gamma}{\sigma - \phi(1-\gamma)}\right]\left[1-\gamma+\frac{\eta\gamma\sigma(2-\gamma)}{1-\gamma}\right]$$

However, provided that ϕ takes values very close to zero, then $\mu = \mu^c$. Therefore, the effect of incompleteness in the response of output gap to real interest rate is negligible.

Therefore, the relevant difference introduced by assuming incomplete markets is in the New Keynesian Phillips curve. WE SHOULD FIND INTUITIONS FOR THESE DIFFERENCES.

4. **A caveat**.- Our model with complete markets is not equal to the model in Gali and Monacelli (2005). One obvious reason is that we incorporate the openness in production, which affects the response of inflation rate to output gap $\lambda\kappa_1^c$. However, this discrepancy with Gali and Monacelli (2005) also arise in the response of output gap to real interest rate μ^c even when this parameter does not depend on openness in production δ. In particular, this parameter μ^c in Gali and Monacelli (2005) framework with endogenous discounting would be

$$\mu^{gm} = \left[\frac{1-\gamma}{\sigma - \phi(1-\gamma)}\right]\left[1-\gamma+\frac{\sigma\gamma\left[\widehat{\gamma}+\eta(1-\gamma)\right]}{1-\gamma}\right].$$

The difference is the parameter $\widehat{\gamma}$ that in our model is equal to η. What is the economic meaning of this parameter $\widehat{\gamma}$? Gali and Monacelli (2005) consider that each imported good j is a CES aggregator of the quantity purchased in each foreign country i, where $\widehat{\gamma}$ is the elasticity of substitution across countries, i.e.,

$$C_F(j) = \left[\int_0^1 \left[C_i(j)\right]^{\frac{\widehat{\gamma}-1}{\widehat{\gamma}}}\right]^{\frac{\widehat{\gamma}}{\widehat{\gamma}-1}}.$$

We follow Clarida et al. (2001) in treating the rest of the world as a single, non-small country. This implies to assume that $\widehat{\gamma} = \eta$.

QUESTION: what does this mean in terms of substitutability between countries? Therefore, our model with complete markets coincides with Gali and Monacelli (2005) under they calibration: $\widehat{\gamma} = \eta = 1$. However, both models do not exactly coincide under the calibration in Llosa and Tuesta (2006): $\widehat{\gamma} = 1$ and $\eta = 1.5$. We will consider in our calibration that $\eta = 1.5$, so that we will be assuming that $\widehat{\gamma} = 1.5$. This point should be clear. In any case, we think that this point is negligible for the numerical results.

TO BE DISCUSSED.- Should we change our model to be compared with Gali and Monacelli (2005) and Llosa and Tuesta (2006). I think that this change is not necessary because the negligible quantitative consequences of this deviation. We can say that we consider the Clarida et al. (2001) version of the demand side to consider the rest of the world as a non-small country.

Note.- The second part of λ and the first pat of κ_2 is wrong in the paper. The true expressions are those given above.

2 Calibration

Our baseline economy is defined by taking the same parameters as Llosa and Tuesta (2008) and McCallum and Nelson (1999). Llosa and Tuesta (2008) uses the same calibration as Gali and Monacelli (2005) with the exception of the inverse of IES (σ), the inverse of Frisch labor supply elasticity (φ), and the elasticity of substitution between domestic and foreign goods (η). However, the calibration of the former seems much convenient for our analysis for two reasons: (i) because of comparison for the stability analysis; and (ii) because is a much general calibration. Furthermore, these parameters does not affect qualitatively to the results, although they may have important quantitative effects. This is mainly true in the case of σ. In fact, we will perform some sensitivity analysis in this parameter when it would be required.

The next table summarizes this parametrization:

Parameter	**Value**	**Source**
Preferences		
σ	5	Llosa and Tuesta (2008)
ψ	-1	Gali and Monacelli (2005)
φ	0.47	Llosa and Tuesta (2008)
ϕ	10^{-6}	
ϑ	0	
β	0.99	Gali and Monacelli (2005)
Composition demand		
η	1.5	Llosa and Tuesta (2008)
γ	0.4	Gali and Monacelli (2005)
ε	6	Gali and Monacelli (2005)
sc	0.75	Cooley and Prescott (1995)
	0.89	McCallum and Nelson (1999)
Production		
θ	0.75	Gali and Monacelli (2005)
v	-2	McCallum and Nelson (1999)
δ	0.144	McCallum and Nelson (1999)

The parameter v is chosen by McCallum and Nelson (2005) to avoid a excessive variability of the output under flexible prices with respect to real exchange rate. In our model this variability is given by Ω_2. As in the aforementioned paper, this volatility is small for values of v smaller than -2.

The previous would be our baseline economy. However, we will compare in our numerical discussion and analysis between four different scenarios:

1. Our baseline economy: the open economy with incomplete markets.

2. Our extreme example followed to derive the optimal policy: $\phi = 0$, $\alpha = 1$ (i.e., $\delta = 0$).

3. The open economy with complete markets: $\kappa_2 = 0$, $\kappa_1 = \kappa_1^c$ and $\mu = \mu^c$. WE SHOULD COMPUTE THE EQUILIBRIUM FROM THE BEGINING USING THE FACT THAT PERFEC SHARING IMPLIES $c_t = q_t/\sigma$, WHICH SHOULD BE SUBSTITUTED INTO (30).

4. The close economy: $\gamma = 0$ and $\alpha = 1$ (i.e., $\delta = 0$).

Afterwards we may also consider the intervals of the parameters that determines the stability of the equilibrium under different monetary policies. These parameters could be: (i) those determining the openness via demand, i.e., η and γ; (ii) those determining the openness via production, i.e., α and v; (iii) those determining the frictions, i.e., ε and θ; and (iv) the inverse of the intertemporal elasticity of substitution σ.

3 Numerical values of equilibrium relations

By using our baseline calibration we can already sign the reduced parameters determining the equilibrium relations between variables in the approximation of the competitive equilibrium (1), (2) and (3). In particular we obtain

Baseline Economy	
Paramater	Value
λ	0.0719
κ_1	8.8033
κ_2	-12.3424
ϖ	1.0000
μ	1.0320
χ	3.2×10^{-7}

The unique unclear equilibrium relation is the dependence of the output gap from the real exchange rate in the New Keynesian Phillips curve, given by $\lambda\kappa_2$. From Mundell-Flemming one should expect that without imported inputs this value should be negative. We show that in our baseline economy, this value is also negative with import inputs. In fact, we obtain that κ_2 is positive if and only if $v \in (-46.086, 1)$. (Remember that $v \leq 1$, and $v = 0$ is the Cobb-Douglas production function). Therefore, the aforementioned income effects of the exchange rate variations dominates.

At this point, it is convenient quantify the differences between the incomplete market version and the complete market version in terms of equilibrium relations. Remember that this differences are in the relations $\lambda\kappa_1$, $\lambda\kappa_2$ and μ. The next table reports this differences with and without openness in production:

	Incomplete		Complete		Close
	$\delta = 0.114$	$\delta = 0$	$\delta = 0.114$	$\delta = 0$	$\delta = \gamma = 0$
$\lambda\kappa_1$	0.6325	0.7556	0.1440	0.1235	0.4695
$\lambda\kappa_2$	-0.8868	-1.0872	0	0	0
μ	1.0320	1.0320	1.0320	1.0320	0.2

From this table we conclude the following:

1. The positive response of the inflation rate to the output gap, given by $\lambda\kappa_1$, is much larger with incomplete markets. FIND AN EXPLANATION.

2. This response of the inflation rate to the output gap, given by $\lambda\kappa_1$, is decreasing with δ (openness in production) in the incomplete market version, whereas $\lambda\kappa_1$, is increasing with δ in the complete market economy. FIND AN EXPLANATION.

3. The response of the inflation rate to the output gap, given by $\lambda\kappa_1$, in the close economy is between the value in the incomplete market version and the complete market version.

4. The openness in production, given by δ, reduces the negative effect of the depreciation of the exchange rate on the inflation rate. This is obvious because with $\delta = 0$, the production effect determined κ_2 disappears.

5. Given that ϕ is very close to zero, the response of the output gap to the interest rate, given by μ, is the same in the two versions of open economies.

6. The response of the output gap to the interest rate, given by μ, is much smaller in the close economy.

Using the value of $\sigma = 1$ in Gali and Monacelli we obtain:

$\sigma = 1$	
Paramater	Value
λ	0.0719
κ_1	2.1367
κ_2	-1.6757
ϖ	1.0000
μ	1.0320
χ	1.6×10^{-6}

We observe that the important change with respect to the baseline economy is κ_2, which determines the effect of real exchange rate in the inflation. There is also an important change in κ_1; which determines the response of inflation to output gap. However these changes only affect the length but not the sign. We should mention this point. In general, the change in the calibration with respect to the original in Gali and Monacelli implies important variations in the equilibrium relations. Although the main source of this change is the new value of σ (i.e., the inverse of the intertemporal elasticity of substitution). The η (i.e., the elasticity of substitution between domestic and foreign goods) has important impacts on κ_2 but much smaller than σ, and φ (i.e., the inverse of the Frisch labor supply elasticity) has a very small impact on κ_2.

4 Policy rules

We will consider three policy rules:

1. The *Domestic Inflation Taylor Rule* (DITR).- Where the domestic monetary authority adjust the domestic interest rate to both domestic inflation and the domestic output gap:

$$i_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t \tag{4}$$

where ϕ_π and ϕ_x are exogenous non-negative reaction parameters.

2. The *Managed Exchange Rate Taylor Rule* (MERTR).- Where the domestic monetary authority adjust the domestic interest rate to both domestic inflation and the domestic output gap:

$$i_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t + \phi_s \bigtriangleup s_t \tag{5}$$

where $\bigtriangleup s_t$ is the change in the nominal exchange rate, and ϕ_π, ϕ_x and ϕ_s are again exogenous non-negative reaction parameters.

3. The *Optimal Policy Rule* (OPR).- We have obtained that the optimal condition of the minimization problem of the Loss Function is

$$a\widetilde{q}_t + b\widetilde{x}_t + c\pi_{H,t} + \widetilde{z}_t = 0, \tag{6}$$

where a, b and c are reduced parameters that depend on the fundamental parameters, and $\widetilde{z}_t$ is an exogenous stochastic variable. This will be a monetary rule of the form

$$i_t = \widehat{\phi}_\pi E\left\{\pi_{H,t+1}\right\} + \widehat{\phi}_x E\left\{\widetilde{x}_{t+1}\right\} + \widehat{\phi}_q E\left\{\widetilde{q}_{t+1}\right\},$$

where $\widehat{\phi}_\pi$, $\widehat{\phi}_x$ and $\widehat{\phi}_q$ are now **endogenous** reaction parameters (i.e, they depend on the fundamental parameters of our model). For that resason we first study the stability of the following ad-hoc rule (contemporaneous and forecast-based):

$$i_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t + \phi_q \widetilde{q}_t, \tag{7}$$

where ϕ_π, ϕ_x and ϕ_q are again exogenous non-negative reaction parameters. After that, we should check if the parameters of the optimal rule $\widehat{\phi}_\pi$, $\widehat{\phi}_x$ and $\widehat{\phi}_q$ belong to the set of parameters $\{\phi_\pi, \phi_x, \phi_q\}$ that report determinacy in the previous *Exchange Rate Taylor Rule*.

We should consider two things respect these simple Taylor policies. First, under the ad-hoc simple Taylor Rules we will use the same calibration for the policy reaction parameters as in Llosa and Tuesta (2008) when numerical analysis is required: $\phi_\pi \in [0,4]$, $\phi_x \in [0,4]$ and $\phi_e \geq 0$. Second, we must to decide whether or not we consider also the forecast-based versions of these Taylor rules as in Bullard and Mitra (2002) and Llosa and Tuesta (2008). This seems a natural exercise because the optimal policy at the end implies an optimal interest rate that depends on the expected values of the variables (see below).

We compare all the results with the complete market case by setting $\kappa_2 = 0$ and analyzing the system formed by (1) and 2.

5 Domestic Inflation Taylor Rule

By combining the policy rule (4) with (1), (2) and (3), we obtain after some tedious algebra the following representation of the competitive equilibrium under the DITR monetary policy:[2]

$$\begin{pmatrix} \mathbb{E}_t\left\{\widetilde{x}_{t+1}\right\} \\ \mathbb{E}_t\left\{\pi_{H,t+1}\right\} \\ \mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A} \begin{pmatrix} \widetilde{x}_t \\ \pi_{H,t} \\ \widetilde{q}_t \end{pmatrix} + \underbrace{\begin{pmatrix} -\frac{1}{\varpi} & 0 & \frac{\chi}{\varpi} \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{C} \begin{pmatrix} \varepsilon_t \\ 0 \\ u_t \end{pmatrix}, \tag{8}$$

where

$$a_{11} = \frac{1 + \left(\phi_x + \frac{\lambda\kappa_1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{12} = \frac{\left(\phi_\pi - \frac{1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{13} = \frac{\left(\frac{\lambda\kappa_2}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right] - \chi}{\varpi},$$

$$a_{21} = -\frac{\lambda\kappa_1}{\beta},$$

$$a_{22} = \frac{1}{\beta},$$

$$a_{23} = -\frac{\lambda\kappa_2}{\beta},$$

$$a_{31} = \left(1-\gamma\right)\left[\phi_x - a_{21}\right],$$

$$a_{32} = \left(1-\gamma\right)\left[\phi_\pi - a_{22}\right],$$

and

$$a_{33} = 1 - \left(1-\gamma\right)a_{23}.$$

[2]**A caveat**.- The stochatic part of this representation should be revised: the matrix C. For instance, the definition of ε_t includes u_t. However, this is not relevant for the stability of the competitive equilibrium, only for the simulation of the dynamics and the welfare cost of deviating from the optimal policy rule. In any case, this caveat applies for all the policy rules.

For stability we must characterize the sign of the eigenvalues of matrix A. Following Blanchard and Khan (1980), in our forward-looking solution (8) there are three non-predeterminated variables. Therefore, the equilibrium under DITR will be determinate if the three eigenvalues of A are outside the unit circle, whereas it will be indeterminate when at least one of the three eigenvalues of A is inside the unit circle. Unfortunately, there is difficult to obtain an analytical characterization of this stability. We next simulate this stability.

5.1 Numerical results

Following Bullard and Mitra (2002) and Llosa and Tuesta (2008) we consider the following policy reaction parameters: $\phi_\pi \in [0, 4]$ and $\phi_x \in [0, 4]$. In this case we obtain the first preliminary results:

1. In our baseline open economy with incomplete markets, the set of indeterminacy is similar to the obtained by Llosa and Tuesta (2008). See the next figureWe observe that the larger value ϕ_π for which indeterminacy arises is 1, which corresponds with $\phi_x = 0$. By the contrary the larger value of ϕ_x for which we find indeterminacy is 4 (the maximum value), which corresponds with $\phi_\pi = 0.96$. In fact this point $(\phi_\pi, \phi_x) = (1, 0.96)$, which we mark with H in the figure, determines the length of indeterminacy and, therefore, the constraint faced by the policy makers in setting a DITR as a stabilizing device. In particular, the monetary authority has not constraint if the policy reaction to inflation ϕ_π is larger than unity. However, provided that $\phi_\pi < 1$, the smaller this policy parameter is, the greater the authority's responses to the output gap.

   TASK.- We should check the sensitivity of the indeterminacy length (point H) to the degree of openess (γ and δ) like in Llosa and Tuesta (2008).

2. **Main result.-** The set of indeterminacy in our version of open economy with complete markets and close economy are similar to the previous one. However, the indeterminacy is larger in the incomplete market economy than in the complete market one. In addition, the length of indeterminacy in the close economy is between the length in the two open economies: with and without complete markets. The largest value of ϕ_π for which indeterminacy arises is 1 in the three economies. However the point H in the previous figure is different across economies. Next table provides the value of this point for the three economies:

   Indeterminacy threshold (point H)

	ϕ_π	ϕ_x
Open and incomplete	0.96	4
Open and complete	0.72	4
Close	0.91	4

   Therefore, the constrains faced by the policy markers is slightly tighter in the baseline economy (i.e., open economy with incomplete markets) than in the open economy with complete markets. This means that, whenever $\phi_\pi < 1$, open economies with incomplete markets needs greater responses to the output gap than open economies with complete markets. The mechanism should be found in the parameters κ_2 and κ_1 determining the response of inflation rate to the current exchange rate and the current output gap, respectively. WE MUST WORK THIS POINT. Furthermore, as in Llosa and Tuesta (2008), the constrains faced by the policy markers is slightly tighter in the close economy than in the open economy with complete markets. However, and this is a remarkable result, the constraints faced by the policy makers is slightly tighter in the open economy with incomplete markets than in the close economy. In other words, while openness reduces the constraint for policy makers if markets are complete, this opens increases the constraints if markets are incomplete.

3. As was mentioned before, the results largely depend on the value of sigma. First, the length of indeterminacy is increasing in σ. In addition, the quantitative differences in the set of indeterminacy across economies is slightly larger for smaller values of σ. The next table reports the results on the length of indeterminacy for $\sigma = 1$ (considered by Gali and Monacelli, 2005):

Indeterminacy threshold (point H)

	ϕ_π	ϕ_x
Open and incomplete	0.82	4
Open and complete	0.66	4
Close	0.68	4

4. The value of ϕ (endogenous discounting) has either a marginal role in the differences across the economies. However, by setting $\phi = 0$, the competitive equilibrium is almost indeterminate for all the combinations of the policy reaction parameters ϕ_π and ϕ_x. In any case, it is difficult to find any pattern in the indeterminacy when $\phi = 0$.

In summarizing, indeterminacy under this Domestic Inflation Taylor Rule in our model is more often than in the original Gali and Monacelli (2005), which was analyzed by Bullard and Mitra (2002) and Llosa and Tuesta (2008). Furthermore, contrary to those authors, openness with incomplete markets increases the combinations of parameter reactions ϕ_π and ϕ_x for which equilibrium in indeterminate.

5.2 Forecast-based DITR

We now analyze the stability with the monetary authority sets the domestic interest rate by responding to the expected domestic inflation and the domestic output gap:

$$i_t = \phi_\pi \mathbb{E}_t \left\{ \pi_{H,t+1} \right\} + \phi_x \mathbb{E}_t \left\{ \widetilde{x}_{t+1} \right\}. \tag{9}$$

With this policy rule, the competitive equilibrium is given by

$$\begin{pmatrix} \mathbb{E}_t \left\{ \widetilde{x}_{t+1} \right\} \\ \mathbb{E}_t \left\{ \pi_{H,t+1} \right\} \\ \mathbb{E}_t \left\{ \widetilde{q}_{t+1} \right\} \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A_1} \begin{pmatrix} \widetilde{x}_t \\ \pi_{H,t} \\ \widetilde{q}_t \end{pmatrix} + \underbrace{\begin{pmatrix} c_{11} & 0 & c_{13} \\ 0 & 0 & 0 \\ c_{31} & 0 & c_{33} \end{pmatrix}}_{C_1} \begin{pmatrix} \varepsilon_t \\ 0 \\ u_t \end{pmatrix}, \tag{10}$$

where

$$a_{11} = \frac{1 + \left(\frac{\lambda \kappa_1}{\beta} \right) (1 - \phi_\pi) \left[\mu - \chi (1 - \gamma) \right]}{\varpi - \phi_x \left[\mu - \chi (1 - \gamma) \right]},$$

$$a_{12} = -\frac{\left(\frac{1}{\beta} \right) (1 - \phi_\pi) \left[\mu - \chi (1 - \gamma) \right]}{\varpi - \phi_x \left[\mu - \chi (1 - \gamma) \right]},$$

$$a_{13} = \frac{-\chi + \left(\frac{\lambda \kappa_2}{\beta} \right) (1 - \phi_\pi) \left[\mu - \chi (1 - \gamma) \right]}{\varpi - \phi_x \left[\mu - \chi (1 - \gamma) \right]},$$

$$a_{21} = -\frac{\lambda \kappa_1}{\beta},$$

$$a_{22} = \frac{1}{\beta},$$

$$a_{23} = -\frac{\lambda \kappa_2}{\beta},$$

$$a_{31} = (1-\gamma)\left[a_{11}\phi_x - a_{21}(1-\phi_\pi)\right],$$
$$a_{32} = (1-\gamma)\left[a_{12}\phi_x - a_{22}(1-\phi_\pi)\right],$$
$$a_{33} = 1 + (1-\gamma)\left[a_{13}\phi_x - a_{23}(1-\phi_\pi)\right],$$
$$c_{11} = -\frac{1}{\varpi - \phi_x\left[\mu - \chi(1-\gamma)\right]},$$
$$c_{13} = \frac{\chi}{\varpi - \phi_x\left[\mu - \chi(1-\gamma)\right]},$$
$$c_{31} = -\frac{(1-\gamma)(1-\phi_\pi)}{\varpi - \phi_x\left[\mu - \chi(1-\gamma)\right]},$$

and

$$c_{33} = \frac{\chi(1-\gamma)(1-\phi_\pi)}{\varpi - \phi_x\left[\mu - \chi(1-\gamma)\right]} - 1.$$

The results of stability in this case are (**CHECK ALGEBRA**):

1. There are a subset of combinations of ϕ_π and ϕ_x where the competitive equilibrium is indeterminate. Now this subset of indeterminacy is much larger than in the case of DITR.
2. Indeterminacy is larger in incomplete than in complete.
3. Indeterminacy is larger in the open economy with complete markets than in close economy. This result contradicts Llosa and Tuesta (2008). MORE WORK IS REQURED.
4. BUT, the picture are quite different that those in Llosa and Tuesta (2008). What is wrong? MORE WORK IS REQUIRED.

These results can be obtained from the following figure

6 Managed exchange rate Taylor Rule

The problem with this rule is that our reduced form of equilibrium is in terms of $\widetilde{q}_t$ instead of $\triangle s_t$. However, we can find in the solution of the model a relations between these two variables that we can use to analyze the stability of this rule. By definition we know that

$$s_t = q_t + p_t - p_t^*.$$

From this expression we directly obtain that

$$\triangle s_t = \triangle q_t + \triangle p_t - \triangle p_t^*. \tag{11}$$

We derived in the paper (see Expression 28) that

$$p_t - p_{H,t} = \left(\frac{\gamma}{1-\gamma}\right) q_t.$$

From this expression we also directly obtain that

$$\triangle p_t = \left(\frac{\gamma}{1-\gamma}\right) \triangle\, q_t + \triangle p_{H,t},$$

which can be written by using the defintion of $\pi_{H,t}$ as

$$\triangle p_t = \left(\frac{\gamma}{1-\gamma}\right) \triangle\, q_t + \pi_{H,t}. \tag{12}$$

By combining (11) and (12), we get

$$\triangle s_t = \left(\frac{1}{1-\gamma}\right) \triangle q_t + \pi_{H,t} - \triangle p_t^*. \tag{13}$$

At this point, we assume that the rest of the world follows an optimal monetary policy such that $\triangle p_t^* = 0$. This is without lost of generality since $\triangle p_t^*$ is an exogenous process in general, so it does not affect stability of equlibria. We then obtain the following equlibrium condition

$$\triangle s_t = \left(\frac{1}{1-\gamma}\right) \triangle q_t + \pi_{H,t}. \tag{14}$$

By using (14) in the policy rule (5), we obtain

$$i_t = (\phi_\pi + \phi_s)\, \pi_{H,t} + \phi_x \widetilde{x}_t + \frac{\phi_s}{1-\gamma} \left(\widetilde{q}_t - \widetilde{q}_{t-1}\right). \tag{15}$$

By combining the policy rule (15) with (1), (2) and (3), we obtain after some tedious algebra the representation of the competitive equilibrium under the MERTR monetary policy.[3] For that purpose, we first introduce the following dummy variable $z_t = \widetilde{q}_{t-1}$. In this way, we obtain that

$$\begin{pmatrix} \mathbb{E}_t\left\{\widetilde{x}_{t+1}\right\} \\ \mathbb{E}_t\left\{\pi_{H,t+1}\right\} \\ \mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} \\ z_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{A} \begin{pmatrix} \widetilde{x}_t \\ \pi_{H,t} \\ \widetilde{q}_t \\ z_t \end{pmatrix} + \underbrace{\begin{pmatrix} -\frac{1}{\varpi} & 0 & \frac{\chi}{\varpi} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{C} \begin{pmatrix} \varepsilon_t \\ 0 \\ u_t \\ 0 \end{pmatrix}, \tag{16}$$

where

$$a_{11} = \frac{1 + \left(\phi_x + \frac{\lambda\kappa_1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{12} = \frac{\left(\phi_\pi + \phi_s - \frac{1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{13} = \frac{\left(\phi_q + \frac{\lambda\kappa_2}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right] - \chi}{\varpi},$$

$$a_{14} = -\frac{\left(\frac{\phi_s}{1-\gamma}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi}$$

$$a_{21} = -\frac{\lambda\kappa_1}{\beta},$$

$$a_{22} = \frac{1}{\beta},$$

$$a_{23} = -\frac{\lambda\kappa_2}{\beta},$$

$$a_{31} = (1-\gamma)\left[\phi_x - a_{21}\right],$$

$$a_{32} = (1-\gamma)\left[\phi_\pi + \phi_s - a_{22}\right],$$

$$a_{33} = 1 + \phi_s - (1-\gamma)\, a_{23},$$

and

$$a_{34} = -\phi_s.$$

[3] **A caveat**.- The stochatic part of this representation should be revised: the matrix C. For instance, the definition of ε_t includes u_t. However, this is not relevant for the stability of the competitive equilibrium, only for the simulation of the dynamics and the welfare cost of deviating from the optimal policy rule. In any case, this caveat applies for all the policy rules.

6.1 Results

For stability we must characterize the sign of the eigenvalues of matrix A. We next simulate this stability. Following Bullard and Mitra (2002) and Llosa and Tuesta (2008) we consider the following policy reaction parameters: $\phi_\pi \in [0,4]$, $\phi_x \in [0,4]$ and $\phi_s = 0.6$, even when we consider the real exchange rate as a target instead of the changes in the nominal exchange rate. WE SHOULD MAKE SENSITIVITY ANALYSIS RESPECT TO ϕ_s. In this case we obtain the first preliminary results:

1. Under incomplete markets, the equilibrium is determinate for large values of ϕ_π and small values of ϕ_x. See the figure

   This result seems to be robust to changes in ϕ_s. CHECK THIS POINT

2. Under complete markets we use the fact that $c_t = q_t/\sigma$. This implies that $q_t = \tau y_t$, so that the policy rule can be transformed into

$$i_t = (\phi_\pi + \phi_s)\,\pi_{H,t} + \left(\phi_x + \frac{\tau\phi_s}{1-\gamma}\right)\widetilde{x}_t - \frac{\tau\phi_s}{1-\gamma} - \widetilde{x}_{t-1},$$

   and we now use the dummy variable $z_t = \widetilde{x}_{t-1}$. With complete markets, the results coincide with Llosa and Tuesta (2008): indeterminacy only appears for low values of ϕ_π. The next figure illustrate this point:

3. So incomplete markets lead this manage exchange rate Taylor rule to be indeterminate for a large number of combinations of reaction parameters $\phi's$. We observe that this rule (15) is equivalente to the Exchange rate Taylor rule (7), where the central bank cares about the level of the real exchange rate. The differences is that in (15)) the central government puts more weigth in inflation than in (7), and in (15)) the government cares about the changes in the real exchange rate instead about the level. In the next section, we prove that the indeterminacy set in Exchange rate Taylor rule (7) is similar to the case of Domestic Inflation Taylor Rule (4). Hence, by comparing this later result with the result in this section for MERTER (15), we can conclude that the optimal policy should care more about the level of real exchange rate than about its change. In other words, caring about the level of real exchange rate is more effective than caring about the change in this exchange rate. However, this is in contradiction with the conclusion from the complete market model, but it is in the line of what the optimal policy rule states.

IT SHOULD BE COMPLETED

6.2 Forecast-based managed exchange rate Taylor rule

TO BE COMPLETED

7 Optimal rule

The problem is that this optimal rule was derived with $\phi = 0$, and the indeterminacy follows a strange pattern in this case and arises for almost the entire set of combinations of ϕ_π and ϕ_x. In any case, we can still find the endogenous values of the policy reaction parameters ϕ_π,ϕ_x and ϕ_q, and then see whether this combination belongs to the determinacy or indeterminacy region found for the Forecast-based Exchange Rate Taylor Rule. This FB-ERTR policy is of the same form as the optimal policy, with the difference that in the former rule the policy reaction parameters are exogenous. An important **caveat** of this procedure is that the stability of the FB-ERTR was derived with $\phi > 0$, while the optimal policy was derived with $\phi = 0$.

7.1 Exchange Rate Taylor Rule

Hence, we first analyze the case of the Exchange Rate Taylor Rule. By combining the policy rule (7) with (1), (2) and (3), we obtain after some tedious algebra the following representation of the competitive equilibrium under the ERTR monetary policy:[4]

$$\begin{pmatrix} \mathbb{E}_t\left\{\widetilde{x}_{t+1}\right\} \\ \mathbb{E}_t\left\{\pi_{H,t+1}\right\} \\ \mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A} \begin{pmatrix} \widetilde{x}_t \\ \pi_{H,t} \\ \widetilde{q}_t \end{pmatrix} + \underbrace{\begin{pmatrix} -\frac{1}{\varpi} & 0 & \frac{\chi}{\varpi} \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{C} \begin{pmatrix} \varepsilon_t \\ 0 \\ u_t \end{pmatrix}, \tag{17}$$

where

$$a_{11} = \frac{1 + \left(\phi_x + \frac{\lambda\kappa_1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{12} = \frac{\left(\phi_\pi - \frac{1}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right]}{\varpi},$$

$$a_{13} = \frac{\left(\phi_q + \frac{\lambda\kappa_2}{\beta}\right)\left[\mu - \chi\left(1-\gamma\right)\right] - \chi}{\varpi},$$

$$a_{21} = -\frac{\lambda\kappa_1}{\beta},$$

$$a_{22} = \frac{1}{\beta},$$

$$a_{23} = -\frac{\lambda\kappa_2}{\beta},$$

$$a_{31} = \left(1-\gamma\right)\left[\phi_x - a_{21}\right],$$

$$a_{32} = \left(1-\gamma\right)\left[\phi_\pi - a_{22}\right],$$

and

$$a_{33} = 1 + \left(1-\gamma\right)\left[\phi_q - a_{23}\right].$$

For stability we must characterize the sign of the eigenvalues of matrix A. We next simulate this stability. Following Bullard and Mitra (2002) and Llosa and Tuesta (2008) we consider the following policy reaction parameters: $\phi_\pi \in [0,4]$, $\phi_x \in [0,4]$ and $\phi_q = 0.6$, even when we consider the real exchange rate as a target instead of the changes in the nominal exchange rate. WE SHOULD MAKE SENSITIVITY ANALYSIS RESPECT TO ϕ_q. In this case we obtain the first preliminary results:

1. The subset of indeterminacy is the same that in the case of the Domestic Inflation Taylor Rule. This would imply that policy reaction of interest rate to movements in the nominal exchange rate does not affect the lowers limits of both ϕ_x and ϕ_π. In fact, if we increase ϕ_q to 1.6 the indeterminacy set increase a little bite: the point H goes from $(1, 0.96)$ to $(1, 0.97)$. This means that by increasing the policy reaction to variations in the exchange rate we increase the set of determinacy, i.e., we shrink the lower limits of both ϕ_x and ϕ_π.

7.2 Forecast-based Exchange Rate Taylor Rule

TO BE COMPLETED

[4] **A caveat**.- The stochatic part of this representation should be revised: the matrix C. For instance, the definition of ε_t includes u_t. However, this is not relevant for the stability of the competitive equilibrium, only for the simulation of the dynamics and the welfare cost of deviating from the optimal policy rule. In any case, this caveat applies for all the policy rules.

7.3 Stability of the optimal rule

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