MATLAB Codes: Small Open Economy Monetary Policy and Equilibrium Determinacy: New Lessons from a Model with Endogenous Monetary Policy Trade-Off

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1 Introduction

The recursive competitive equilibrium is characterized by a three-equation forward-looking dynamical system:

$$\pi_{H,t} = \widehat{\beta}\mathbb{E}_t\left\{\pi_{H,t}\right\} + \lambda\left(\kappa_1\widetilde{x}_t + \kappa_2\widetilde{q}_t\right), \tag{1}$$

$$\widetilde{x}_t = \varpi\mathbb{E}_t\left\{\widetilde{x}_{t+1}\right\} - \mu\left[i_t - \mathbb{E}_t\left\{\pi_{H,t+1}\right\}\right] + \chi\mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} + \epsilon_t, \tag{2}$$

and

$$\widetilde{q}_t = \mathbb{E}_t\left\{\widetilde{q}_{t+1}\right\} - (1-\gamma)\left[i_t - \mathbb{E}_t\left\{\pi_{H,t+1}\right\}\right] + u_t, \tag{3}$$

where

$$\lambda = \left[\frac{(1-\theta)(1-\theta\beta)}{\theta}\right]\left[\frac{(1-v)(1-\delta)}{1-v+\delta\varphi}\right],$$

$$\kappa_1 = \frac{\sigma}{1-\gamma} + \varphi,$$

$$\kappa_2 = \underbrace{\frac{\delta(1-v+\varphi)}{(1-\gamma)(1-v)(1-\delta)}}_{\text{Production effect}} \underbrace{- \frac{\sigma\eta\gamma(2-\gamma)}{(1-\gamma)^2} + \frac{\gamma}{1-\gamma}}_{\text{demand effect}},$$

$$\varpi = \frac{\sigma}{\sigma-\phi},$$

$$\mu = \left(\frac{1-\gamma}{\sigma-\phi}\right)\left[1-\gamma + \frac{\eta\gamma(2-\gamma)(\sigma-\phi)}{1-\gamma}\right],$$

and

$$\chi = \frac{\eta\gamma\phi(2-\gamma)}{(1-\gamma)(\sigma-\phi)}.$$

2 Calibration

Our baseline economy is defined by taking the same parameters as Llosa and Tuesta (2008) and McCallum and Nelson (1999). Llosa and Tuesta (2008) uses the same calibration as Gali and Monacelli (2005) with the exception of the inverse of IES (σ), the inverse of Frisch labor supply elasticity (φ), and the elasticity of substitution between domestic and foreign goods (η). However, the calibration of the former seems much convenient for our analysis for two reasons: (i) because of comparison for the stability analysis; and (ii) because is a much general calibration. Furthermore, these parameters does not affect qualitatively to the results, although they may have important quantitative effects. This is mainly true in the case of σ. In fact, we will perform some sensitivity analysis in this parameter when it would be required.

The next table summarizes this parametrization:

Parameter	Value	Source
Preferences		
σ	5	Llosa and Tuesta (2008)
ψ	-1	Gali and Monacelli (2005)
φ	0.47	Llosa and Tuesta (2008)
ϕ	10^{-6}	
ϑ	0	
β	0.99	Gali and Monacelli (2005)
Composition demand		
η	1.5	Llosa and Tuesta (2008)
γ	0.4	Gali and Monacelli (2005)
ε	6	Gali and Monacelli (2005)
sc	0.75	Cooley and Prescott (1995)
	0.89	McCallum and Nelson (1999)
Production		
θ	0.75	Gali and Monacelli (2005)
v	-2	McCallum and Nelson (1999)
δ	0.144	McCallum and Nelson (1999)

The parameter v is chosen by McCallum and Nelson (2005) to avoid a excessive variability of the output under flexible prices with respect to real exchange rate. In our model this variability is given by Ω_2. As in the aforementioned paper, this volatility is small for values of v smaller than -2.

The previous would be our baseline economy. However, we will compare in our numerical discussion and analysis between four different scenarios:

1. Our baseline economy: the open economy with incomplete markets.

2. Our extreme example followed to derive the optimal policy: $\phi = 0$, $\alpha = 1$ (i.e., $\delta = 0$).

3. The open economy with complete markets: $\kappa_2 = 0$, $\kappa_1 = \kappa_1^c$ and $\mu = \mu^c$. Moreover, the other characterization of this economy is given by the complete risk sharing condition, in place of the UIP condition.

4. The close economy: $\gamma = 0$ and $\alpha = 1$ (i.e., $\delta = 0$).

Afterwards we may also consider the intervals of the parameters that determines the stability of the equilibrium under different monetary policies. These parameters could be: (i) those determining the openness via demand, i.e., η and γ; (ii) those determining the openness via production, i.e., α and v; (iii) those determining the frictions, i.e., ε and θ; and (iv) the inverse of the intertemporal elasticity of substitution σ.

3 Numerical Experiments

We will consider three policy rules:

1. The *Domestic Inflation Taylor Rule* (DITR).- Where the domestic monetary authority adjust the domestic interest rate to both domestic inflation and the domestic output gap:
$$i_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t \tag{4}$$
where ϕ_π and ϕ_x are exogenous non-negative reaction parameters.

2. The *CPI Inflation Taylor Rule* (CPITR).- Where the monetary authority adjust the domestic interest rate to both CPI inflation and the domestic gap:
$$i_t = \phi_\pi \pi_t + \phi_x \widetilde{x}_t, \tag{5}$$
where ϕ_π and ϕ_x are again exogenous non-negative reaction parameters.

3. The *Managed Exchange Rate Taylor Rule* (MERTR).- Where the domestic monetary authority adjust the domestic interest rate to CPI inflation, the domestic output gap and the change in the nominal exchange rate:
$$i_t = \phi_\pi \pi_t + \phi_x \widetilde{x}_t + \phi_s \bigtriangleup s_t \tag{6}$$
where $\bigtriangleup s_t$ is the change in the nominal exchange rate, and ϕ_π, ϕ_x and ϕ_s are again exogenous non-negative reaction parameters.

4. The *Optimal Policy Rule* (OPR).- We have obtained that the optimal condition of the minimization problem of the Loss Function is
$$a\widetilde{q}_t + b\widetilde{x}_t + c\pi_{H,t} + \widetilde{z}_t = 0, \tag{7}$$
where a, b and c are reduced parameters that depend on the fundamental parameters, and $\widetilde{z}_t$ is an exogenous stochastic variable. This will be a monetary rule of the form
$$i_t = \widehat{\phi}_\pi E\left\{\pi_{H,t+1}\right\} + \widehat{\phi}_x E\left\{\widetilde{x}_{t+1}\right\} + \widehat{\phi}_q E\left\{\widetilde{q}_{t+1}\right\},$$
where $\widehat{\phi}_\pi$, $\widehat{\phi}_x$ and $\widehat{\phi}_q$ are now **endogenous** reaction parameters (i.e., they depend on the fundamental parameters of our model). For that reason we first study the stability of the following ad-hoc rule :
$$i_t = \phi_\pi E\left\{\pi_{H,t+1}\right\} + \phi_x E\left\{\widetilde{x}_{t+1}\right\} + \phi_q E\left\{\widetilde{q}_{t+1}\right\}, \tag{8}$$
where ϕ_π, ϕ_x and ϕ_q are again exogenous non-negative reaction parameters. After that, we should check if the parameters of the optimal rule $\widehat{\phi}_\pi$, $\widehat{\phi}_x$ and $\widehat{\phi}_q$ belong to the set of parameters $\{\phi_\pi, \phi_x, \phi_q\}$ that report determinacy in the previous *Forecast-Based Managed Exchange Rate Taylor Rule (FB-MERTR).*

4 `MATLAB` Scripts

Step 1. The relevant `MATLAB` scripts are stored in the directory `simulations/` of the accompanying files to the paper. Below we list a short description of each experiment/script:

- Domestic Inflation Targeting Rule (DITR) and Forecast-based DITR (FB-DITR):
  - `ditr_close.m`: For the closed economy special case of our model.
  - `ditr_complete.m`: For the Gali-Monacelli complete markets open economy special case of our model.
  - `ditr.m`: For our model with incomplete markets.
- CPI Inflation Targeting Rule (CPITR) and Forecast-based CPITR (FB-CPITR):
  - `cpitr_close.m`: For the closed economy special case of our model.
  - `mertrCPI_complete.m`: Set `POLICY = 0` for this rule in the script. For the Gali-Monacelli complete markets open economy special case of our model.
  - `mertrCPI.m`: Set `POLICY = 0` for this rule in the script. For our model with incomplete markets.
- Managed Exchange Rate Taylor Rule (MERTR) and Forecast-based MERTR (FB-MERTR):
  - `mertrCPI_complete.m`: Set `POLICY = 1` for this rule in the script. For the Gali-Monacelli complete markets open economy special case of our model.
  - `mertrCPI.m`: Set `POLICY = 1` for this rule in the script. For our model with incomplete markets.
- Optimal Time-Consistent Managed Exchange Rate Taylor Rule (MERTR) and Forecast-based MERTR (FB-MERTR):
  - `mertrCPI_complete.m`: Set `POLICY = 1` for the FB-MERTR rule in the script. For the Gali-Monacelli complete markets open economy special case of our model.
  - `mertrCPI.m`: Set `POLICY = 1` for the FB-MERTR rule in the script. For our model with incomplete markets.
  - `mertr_optimal`: Compute stability regions under a set of FB-MERTR where the ϕ_q parameter is changed; for the incomplete markets model. Also computes the point corresponding to the baseline calibrated model's optimal time-consistent policy rule, expressed as a point in the FB-MERTR family of rules.

Step 2. A summary script is found in `patchdraw.m`. This collects all the results above and performs a convex hull approximation of relevant regions of (ϕ_π, ϕ_x) that induces stable rational expectations equilibrium for each fixed policy behavior. This script then graphs and saves the output as "patch" diagrams. The figures are named and saved automatically according to the name of the family of policy rules considered (e.g. `ditr.eps` and `ditr.fig` for the DITR family of rules) in a sub-directory called `simulations/_figures/`.

4.1 User instructions

- Perform all the possible combinations of the script executions in Step1 above.
- Then run Step 2.