Data: Mon, 24 May 2010 07:49:20 +0200 [07:49:20 CEST]
De: jalonso@uvigo.es
Para: "T. Kam" <mortheus@gmail.com>
Asunto: Re: Hello from Tokyo

Hi Tim,
How is going the visit?
I am working in our paper. The stability analysis is so hard... But we will converge.
Please confirm that I am right with the following.
The solution of the model after imposing a monetary policy can be written in two equivalent forms:

1.- E(y_t+1)=M*y_t+C*w_t (Form of Blanchard and Kahn, 1980)
In this case the equilibrium is determinate if the number of eigenvalues of M OUTSIDE the unit circle is equal to the number of pre-determinated variables (dimension of y).

2.- y_t=A*E(y_t+1)+D*w_t (Bullard and Mirta (2002)
In this case the equilibrium is determinate if the number of eigenvalues of A INSIDE the unit circle is equal to the number of pre-determinated variables (dimension of y).

My thought comes from the fact A=M^(-1) and D=M^(-1)*C.

That's all.
Give regards to Tina.
best,
Jaime

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----- Mensaxe de mortheus@gmail.com ---------
Data: Tue, 11 May 2010 10:10:05 +0900
De: "T. Kam" <mortheus@gmail.com>
Responder-A: "T. Kam" <mortheus@gmail.com>
Asunto: Hello from Tokyo
Para: @MISSING_DOMAIN

> Greetings from Kunitachi, Tokyo. I got here yesterday after a sleepless flight accompanied by Yann Martel's new novel, and then Le Concert (about post-communism ex-Bolshoi Jewish musicians who conned the Parisian Le Chatelet into thinking they're the current Bolshoi Symphony Orchestra and to let them perform there) and The Fantastic Mr Fox (a refreshing make of a Roald Dahl children's story by Wes Anderson of The Royal Tenenbaums, the Life Aquatic of Steve Zissou and The Darjeeling Limited.)
>
> It's been very wet here since this morning. Everything here moves too fast for my slothful pace of life now! Tomorrow we go to Kyoto for a seminar and a bit of look-see. We come back via Nara, sister city of Canberra, with also a stopover in Takayama (some hot springs place in the mountains bewteen Kyoto and Tokyo).
>
> Lucien
>
> (Inset: View outside my cubicle at Hitotsubashi's Institute of Economic Research.)

----- Terminar mensaxe de mortheus@gmail.com -----

A Scalar Expectational Difference Equation Example

para J. Alonso-Carrera

May 25, 2010

Consider the following scalar example of a linear or linearly approximated expectational difference equation as represented in Blanchard-Kahn:

$$\mathbb{E}_t x_{t+1} = m x_t + w_t \tag{1}$$

where w.l.o.g., assume that

$$w_t = \rho w_{t-1} + v_t,$$

$v \sim$ i.i.d.$(0, \sigma^2)$, $\sigma < \infty$, $m \in \mathbb{R}$ and $0 < \rho < 1$. By construction, any random sequence $(x_t)_{t=0}^{\infty}$ that satisfies (1) does not possess an initial condition. That is x_0 is part of the solution to (1), unlike a backward looking stochastic difference equation.

To characterize the solution to (1), we re-write it as:

$$x_t = m^{-1}\mathbb{E}_t x_{t+1} - m^{-1} w_t,$$

which is the alternative representation in Bullard-Mitra. Now solving this forward, since we do not have any initial condition, but we can impose some sort of transversality condition, we have

$$x_t = m^{-1}\mathbb{E}_t[\mathbb{E}_{t+1}(m^{-1}x_{t+2} - m^{-1}w_{t+1})] - m^{-1}w_t.$$

Applying the law of iterated expectations (under some conditions on information sets – i.e. sequence of information sets must be a filtration), we have

$$x_t = m^{-2}\mathbb{E}_t x_{t+2} - [m^{-2}\mathbb{E}_t w_{t+1} + m^{-1} w_t]$$

Recursive forward substituion yields to solution up to time $T > 0$ as:

$$x_t = m^{-T}\mathbb{E}_t x_{t+T} - \mathbb{E}_t \sum_{\tau=1}^{T} [m^{-\tau} w_{t+\tau-1}].$$

Imposing a choice of a transversality condition that $\lim_{T \nearrow +\infty} m^{-T}\mathbb{E}_t x_{t+T} = 0$, then a sufficient condition is to have $|m| > 1$. When $|m| > 1$, the solution is a random sequence $(x_t)_{t=0}^{\infty}$ generated by the underlying stochastic process $(v_t)_{t=0}^{\infty}$ such that:

$$x_t = -\mathbb{E}_t \sum_{\tau=1}^{T} [m^{-\tau} w_{t+\tau-1}].$$

Since we have a structure on the law for (w_t), specifically, we know $\mathbb{E}_t w_{t+\tau-1} = \rho^{\tau-1} v_t$, this is further simplified as

$$\begin{aligned} x_t &= -\rho^{-1} \sum_{\tau=1}^{T} \left(\frac{\rho}{m}\right)^{\tau} v_t \\ &= \frac{1}{\rho(1-\rho m^{-1})} v_t. \end{aligned}$$

So in this simple example, I have shown that the decision rule (i.e. also a minimal state variable representation solution) is a feedback rule of x_t in response to the current shock v_t, where the elasticity of response (assuming the variables are in logs) is given by the coefficient $[\rho(1-\rho)(1-m^{-1})]^{-1}$.

I hope my algebra above was correct, but this is basically the intuition that carries over to a system with more than one variable.