Name ______________________________

Midterm Exam

Intertemporal Choice

Fall, 2018

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I

Consider a consumer with quadratic utility $\mathrm{u}(c) = -(1/2)(c - \cent)^2$ for whom the interest and discount factors satisfy $\mathsf{R}\beta = 1$. The consumer wishes to maximize the discounted sum of intertemporally separable utility from consumption

$$\sum_{s=t}^{\infty} \mathbb{E}_t \left[\beta^{s-t}\mathrm{u}(c_s)\right] \tag{1}$$

subject to a budget constraint

$$m_{t+1} = (m_t - c_t)\mathsf{R} + y_{t+1}$$

where m is market resources, y is labor income, and $\mathsf{R} = 1 + \mathsf{r}$ is the interest factor.

1. Write the problem in Bellman equation form, and show how to use the Envelope condition and the first order condition to obtain the Euler equation for consumption.

   Answer:

   Bellman:

   $$\mathrm{v}(m_t) = \max_{c_t} \mathrm{u}(c_t) + \beta\, \mathbb{E}_t[\mathrm{v}(m_{t+1})]$$

   Envelope:

   $$\mathrm{v}'_{t+1}(m_{t+1}) = \mathrm{u}'(c_{t+1})$$

   FOC:

   $$\mathrm{u}'(c_t) = \mathbb{E}_t[\mathrm{v}'_{t+1}(m_{t+1})]$$

   Combo: Substitute (2) into (2).

2. Show that the Euler equation $\mathrm{u}'(c_t) = \mathsf{R}\beta\, \mathbb{E}_t[\mathrm{u}'(c_{t+1})]$ implies that the level of consumption follows a random walk,

   $$\Delta c_{t+1} = \epsilon_{t+1}$$

   where $\mathbb{E}_t[\epsilon_{t+1}] = 0$.

   Answer:

   $$\begin{aligned} \mathrm{u}'(c) &= -(c - \cent) \\ \mathrm{u}'(c_t) &= \mathsf{R}\beta\, \mathbb{E}_t[\mathrm{u}'(c_{t+1})] \\ -(c_t - \cent) &= \mathbb{E}_t[-(c_{t+1} - \cent)] \\ c_t &= \mathbb{E}_t[c_{t+1}] \\ c_{t+1} &= c_t + \epsilon_{t+1} \end{aligned}$$

Now assume that income is subject to transitory and permanent shocks:

$$\begin{aligned} y_{t+1} &= p_{t+1} + \theta_{t+1} \\ p_{t+1} &= p_t \quad + \psi_{t+1} \end{aligned}$$

where the shocks are mutually and serially uncorrelated ($\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\theta_{t+n}] = \mathbb{E}_t[\psi_{t+m}\theta_{t+n}] = 0 \ \forall \ m, n > 0$).

3. Defining end-of-period human wealth as the PDV of expected future income,

$$\mathfrak{h}_t = \mathsf{R}^{-1}\,\mathbb{E}_t\left[\sum_{s=t+1}^{\infty} \mathsf{R}^{t+1-s} y_s\right]$$

show that

$$\mathfrak{h}_t = p_t/r$$

Answer:

$$\begin{aligned} \mathfrak{h}_t &= \mathsf{R}^{-1}\,\mathbb{E}_t[p_t + \mathsf{R}^{-1}(p_t + \psi_{t+1}) + \mathsf{R}^{-2}(p_t + \psi_{t+1} + \psi_{t+2}) + ...] \\ &= \mathsf{R}^{-1}\left(\frac{p_t}{1-\mathsf{R}^{-1}}\right) \\ &= \left(\frac{p_t}{\mathsf{R}-1}\right) = p_t/\mathsf{r} \end{aligned}$$

Define 'balances' b as the amount of resources the consumer has in hand before receiving income in the current period:

$$b_{t+1} = (b_t + y_t - c_t)\mathsf{R}$$

and define total wealth o as the sum of asset balances b, current income y, and human wealth $\mathfrak{h}$,

$$o_{t+1} = b_{t+1} + y_{t+1} + \mathfrak{h}_{t+1},$$

4. Use the intertemporal budget constraint to show that

$$c_t = p_t + (b_t + \theta_t)(\mathsf{r}/\mathsf{R}) \tag{2}$$

Explain why this equation implies a marginal propensity to consume out of shocks to permanent income ψ_t of 1, and out of transitory income θ of (r/R).

Answer:

$$\begin{aligned} \sum_{s=t}^{\infty} \mathbb{E}_t[\mathsf{R}^{t-s} c_s] &= b_t + y_t + \mathfrak{h}_t \\ c_t(\mathsf{R}/\mathsf{r}) &= b_t + \theta_t + p_t(1 + 1/r) \\ c_t &= p_t + (b_t + \theta_t)(\mathsf{r}/\mathsf{R}) \end{aligned}$$

MPC of one because $p_t = p_{t-1} + \psi_t$ and the MPC out of shocks to permanent income is $dc_t/d\psi_t$. The MPC of (r/R) out of transitory shocks is a direct implication of (2).

Part II

Predictable and Unpredictable Changes in Consumption in a Perfect Foresight Model.

In representative agent (RA) macroeconomic models, a key channel by which the central bank's control of interest rates affects aggregate demand is through its effect on consumption. For a small open economy, consumption behavior of the RA can be well approximated by the solution for a perfect foresight consumer, which `PerfForesightCRRA`[1] shows can be written approximately as:

$$c_t \approx \boldsymbol{p}_t\left(1 - Þ_\gamma/(\mathsf{r}-\gamma)\right) - Þ_{\mathsf{r}} b_t \tag{3}$$

where

$$Þ_\gamma \equiv \rho^{-1}(\mathsf{r}-\vartheta) - \gamma$$

is the 'growth patience' rate while the 'return patience' rate is

$$Þ_{\mathsf{r}} \equiv \rho^{-1}(\mathsf{r}-\vartheta) - \mathsf{r}.$$

1. a) Briefly:
   i. Explain why we must assume that $Þ_{\mathsf{r}} < 0$ and $Þ_\gamma < 0$

      Answer:

      These questions are answered in `PerfForesightCRRA`

   ii. Use these formulae to describe the three channels by which a change in r should affect the level of c

   Answer:

   `PerfForesightCRRA` articulates the three channels. The income effect is from the $\mathsf{r}b$ component, the substitution effect is from the $\rho^{-1}\mathsf{r}$ element of both $Þ_{\mathsf{r}}$ and $Þ_\gamma$, and the human wealth effect is reflected in the denominator of the $Þ_\gamma/(\mathsf{r}-\gamma)$ term.

   b) Discuss why, for quantitatively plausible calibrations in which r and ϑ are roughly the same size, this model implies that the effects of interest rates on consumption should be very large.

   Answer:

[1] Notation: Bank balances are b, noncapital income is $\boldsymbol{p}_t$, relative risk aversion is ρ, the permanent rate of income growth is γ, etc.

With $\gamma = 0$ we have that

$$\frac{dc}{d\mathsf{r}} \approx \underbrace{-\boldsymbol{p}\frac{\rho^{-1}\vartheta}{\mathsf{r}^2}}_{\text{human wealth effect}} - \underbrace{\rho^{-1}b}_{\text{substitution effect}} + \underbrace{\mathsf{r}b}_{\text{income effect}}$$

Because r is small, r^2 is very small, so the first term is large (if, say, the ϑ is roughly the same size as r). For $\rho = 2$ the intertemporal elasticity of substitution is 0.5 so even if this were the only channel it would be large (compared to empirical estimates of the IES that are around zero). The size of the income effect channel $\mathsf{r}b$ depends of course on the size of the consumer's bank balances; it could be large (for rich people) or small (for poor ones) or even negative (for debtors with $b < 0$).

Consider now an economy in which consumers always behave as though they *believe* that interest rates will remain constant at the current level. But, every now and then a completely unexpected shock occurs that changes the interest rate; subsequently, consumers believe interest rates will remain forever at their new level. (A shock of this kind is sometimes called an 'MIT shock'). Mathematically,

$$\mathbb{E}_t[\mathsf{r}_{t+n}] = \mathsf{r}_t \; \forall \; n > 0$$

2. Assume that leading up to time t, interest rates had been constant at some level $\underline{\mathsf{r}}$. At date t there is an MIT shock that causes interest rates to go up to $\bar{\mathsf{r}} > \underline{\mathsf{r}}$. Explain why this model implies the equation that Hall (1988) estimated:

$$\Delta \log c_{t+1} \approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + \epsilon_{t+1} \tag{4}$$

for $\mathbb{E}_t[\epsilon_{t+1} = 0$ if our expectations of interest rates come from surveys of consumers. (Hint: Consider separately the 'normal' periods in which there is no shock to interest rates, and the 'rare' periods in which there are such shocks).

Answer:

In the normal periods, when there is no shock to interest rates, the usual consumption Euler equation applies, and we have shown in class that under those circumstances $\Delta \log c_{t+1} \approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta)$. It is only in the rare cases where there is an MIT shock to interest rates that there is an $\epsilon_{t+1} \neq 0$. But because we have assumed that these shocks are completely unanticipated, it should be the case that $\mathbb{E}_t[\epsilon_{t+1}] = 0$.

3. Given your analysis above, explain what forces will be the chief contributors to the magnitude of the ϵ shocks?

   a) Answer this question analytically first, under the convenient assumption that the time preference rate is $\omega = \underline{\mathsf{r}}$ and that leading up to t bank balances were constant at $b = 1$ and permanent income was $\boldsymbol{p}_t = 1$.

   b) Now answer the question quantitatively, under the convenient assumption that $\underline{\mathsf{r}} = 0.02$ and $\bar{\mathsf{r}} = 0.04$ and $\rho = 2$ and $\gamma = 0$.

c) After you have done the algebraic and quantitative analysis, draw a diagram showing the level of consumption from the period leading up to t through an interval after t. On the diagram, label changes that correspond to the combined income, substitution, and human wealth effects in period $t+1$, and to the intertemporal substitution channel of predictable consumption growth in periods before t and after $t+1$.

Answer:

$$
\begin{aligned}
\mathbb{E}_t[c_{t+1}] &= 1 + \underline{\mathsf{r}} \\
c_{t+1} &= 1 - \rho^{-1}(\bar{\mathsf{r}} - \vartheta)/\bar{\mathsf{r}} - (\rho^{-1}(\bar{\mathsf{r}} - \vartheta) - \bar{\mathsf{r}})) \\
c_{t+1} &= 1 - \bar{\mathsf{r}} + \rho^{-1}(\bar{\mathsf{r}} - \vartheta)/\bar{\mathsf{r}} - \rho^{-1}(\bar{\mathsf{r}} - \vartheta) \\
c_{t+1} - \mathbb{E}_t[c_{t+1}] &= \underbrace{\bar{\mathsf{r}} - \underline{\mathsf{r}}}_{\text{income effect}} \quad - \underbrace{\rho^{-1}(\bar{\mathsf{r}} - \vartheta)/\bar{\mathsf{r}}}_{\text{human wealth}} - \underbrace{(\rho^{-1}(\bar{\mathsf{r}} - \vartheta))}_{\text{substitution}} \\
&= 0.02 - (1/2)(0.02/0.04) - (1/2)0.02 = 0.01 - 0.25
\end{aligned}
$$

so the effects on consumption from the shock to interest rates come almost entirely from the human wealth effect. But, after the *level* of consumption has adjusted downward in period $t+1$, it is still true that

$$
\mathbb{E}_{t+1}[\log c_{t+2}/c_{t+1}] = \rho^{-1}(\bar{\mathsf{r}} - \vartheta)
$$

The diagram should

- Show consumption being flat at $1 + \underline{r}$ leading up to period t
- In period $t+1$ there should be a big jump downward, of size 0.24, in the level of consumption, corresponding to the combined 'effects' of the interest rate shock on the level of consumption.
- In periods after $t+1$ there should be consumption growth at the rate $\rho^{-1}(\bar{\mathsf{r}} - \vartheta)$, corresponding to the intertemporal substitution channel.

References

Hall, Robert E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, XCVI, 339–357, Available at http://www.stanford.edu/~rehall/Intertemporal-JPE-April-1988.pdf.