Name ____________________________

Final Exam
Intertemporal Choice
Fall, 2017
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

Labor Supply and Consumption. This question asks you to explain some implications of models where labor supply and consumption c are both chosen freely subject to a budget constraint. Define z as the proportion of time spent in leisure activities (that is, not working) and W, r and ϑ as the wage, interest, and time preference rates.

1. Assuming that the 'hat' operator is equivalent to a difference in logs (e.g., $\hat{\bullet}_{t+1} \equiv \log \bullet_{t+1} - \log \bullet_t$), explain the intuition behind the result that

$$\hat{z}_{t+1} \quad \approx \quad -\hat{\mathsf{W}}_{t+1} + (\mathsf{r}_{t+1} - \vartheta) \tag{1}$$

   when preferences satisfy the "balanced growth" condition in a perfect foresight model and utility is logarithmic.

   Answer:

   > See `RBC-Prescott`. The model suggests z fluctuates for two reasons: The first reason is a response of labor supply to temporary deviations of wages from their permanent level (by assumption the response of leisure to permanent changes is zero, but that assumption does not rule out temporary increases in labor supply to take advantage of temporary high wages). The second reason is intertemporal trade off between consumption of different periods. The idea is that in a period with high interest rates you will want to reduce both your c and your z in order to earn more cash which can be invested to take advantage of the high interest rate (and vice versa).

2. Discuss the relationship between this result and empirical evidence under two hypotheses about the reasons for employment fluctuations over the business cycle:

   a) They are driven mostly by temporary fluctuations in wages

   b) They are driven mostly by optimizing responses to interest rate shocks

   Answer:

   > a) If this were true, we would see a large elasticity of labor supply with respect to wages; but microeconomic evidence has estimated that elasticity to be small;
   >
   > b) If this is true, there should at the same time be a strong consumption response. Business cycle fluctuations should show that consumption is high when work hours are low and vice versa, which is the opposite of the true pattern

3. Use the logic of the first order condition to discuss whether you would expect the addition of uncertainty about future consumption to change the implications of the frictionless model about the relationship between movements in consumption and equilibrium labor supply.

Answer:

The two first order conditions are:

$$\frac{\mathrm{u}'(z_t)}{\mathrm{u}'(c_t)} = \mathsf{W}_t \tag{2}$$

$$\mathrm{u}'(c_t) = \beta\,\mathbb{E}_t[\mathbf{R}_{t+1}\mathrm{u}'(c_{t+1})] \tag{3}$$

Introducing uncertainty about future consumption does not change any part of the equation other than adding an expectation sign. You might imagine two effects of uncertainty: A direct (precautionary) effect, and an effect that comes through any consequence uncertainty might have for interest rates. The direct effect implies that, for precautionary reasons, consumption would fall and leisure would fall – which makes sense for an individual, whose fears might make them both cut back on their spending and want to work extra hours, but which does not make sense in the aggregate, because periods of falling consumption tend to be periods of falling employment (and therefore rising leisure). The effects through interest rates are the same as in the model without uncertainty.

Capital Market Imperfections and the Fed. Over the period 2007-2008, the Federal Reserve took several unusual actions in response to developments in the capital markets, including orchestrating the takeover of Bear Sterns by JP Morgan, pledging to be a lender of last resort to investment banks, and joining with the Treasury in a plan for a government takeover of Fannie Mae and Freddie Mac if they should fail.

In the model presented in class on capital market imperfections, the following condition was presented:

$$\gamma > 1 + \mathsf{r} + A(c, \mathsf{r}, W, \gamma) \tag{4}$$

1. Explain this condition, and use that model to provide a variety of interpretations of either the reasons for the Fed's intervention or the reasons its actions might be expected to improve the functioning of capital markets.

   Answer:

   Subject to interpretation.

2. Suppose the "right" diagnosis of the credit market disruptions is that it has been discovered that the cost of verification of financial contracts is higher than had been anticipated. Discuss what this model would predict about the consequences of such an increase in verification costs.

   Answer:

   Discussed in handout.

3. Give an intuitive explanation for why a decrease in interest rates might not be an effective response to financial market problems caused by financial market imperfections.

Answer:

Effects of a reduction in r are discussed in handout. If the financial disruptions are caused by an increase in c, an increase in r is an imperfect fix.

'Steady State' In the Brock and Mirman (1972) Model

Models subject to stochastic shocks generally do not settle down to a fixed point; it can nevertheless be useful to define a 'pseudo-steady-state' for such models. One definition of such a pseudo-steady-state is the location (if one exists) to which the model will converge after an arbitrarily long period in which no shocks occurred.

Handout `BrockMirman` shows that the dynamics of the log of the capital stock are given by

$$k_{t+1} = \log \alpha\beta + a_t + \alpha k_t \tag{5}$$

where $a = \log A$ is the log level of productivity.

1. Show that if a remains constant $a_{t+n} = a_t \; \forall \; n > 0$ then the 'pseudo-steady-state' level of the capital stock will be $K = (A_t \alpha\beta)^{1/(1-\alpha)}$

   Answer:

$$
\begin{aligned}
k &= \log \alpha\beta + a_t + \alpha k & (6)\\
k(1-\alpha) &= \log \alpha\beta + a_t & (7)\\
k &= (\log \alpha\beta + a_t)\,/(1-\alpha) & (8)\\
K &= (A_t \alpha\beta)^{1/(1-\alpha)} & (9)
\end{aligned}
$$

2. Show that the consumption function can be written $\mathrm{C}_t(K) = (1-\alpha\beta)A_t K^\alpha$

   Answer:

   BrockMirman shows that in this model, the optimal consumption function can be written

   $$C_t = (1-\alpha\beta)Y_t \tag{10}$$

   from which (along with $Y_t = A_t K_t^\alpha$) the result follows immediately.

3. Show that if the actual productivity process is lognormally distributed, $a_{t+1} = a_t + \psi_{t+1}$ where $\psi_{t+1} \sim N(-\sigma_\psi^2/2, \sigma_\psi^2)$ then at the pseudo-steady-state level of the capital stock, $\mathbb{E}_t[C_{t+1}] = C_t$.

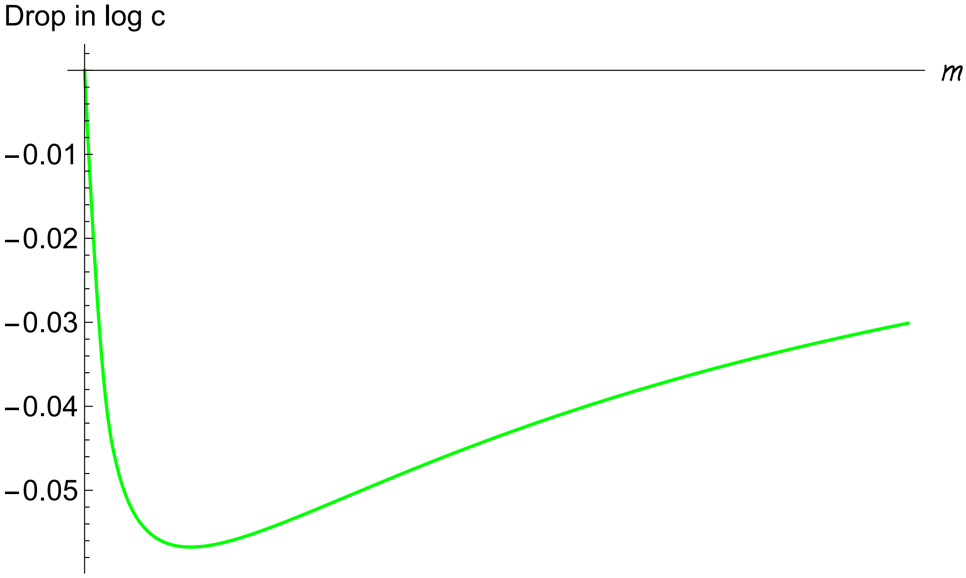
   Answer:

$$
\begin{aligned}
\mathbb{E}_t[C_{t+1}] &= (1-\alpha\beta)\underbrace{K_{t+1}^\alpha}_{=K_t^\alpha \text{ at SS}} A_t \overbrace{\mathbb{E}_t[\Psi_{t+1}]}^{=1 \text{ from MathFacts}} & (11)\\
&= (1-\alpha\beta)A_t K_t^\alpha & (12)\\
&= C_t & (13)
\end{aligned}
$$

Medium-Length Questions

Whose Consumption Falls Most When Uncertainty Increases?

In the model of TractableBufferStock, when there is an increase in the degree of uncertainty $\mho$, every employed consumer's consumption will fall. But the same increase in $\mho$ will affect people at different m's differently. The figure below shows the immediate change in log c that results from an unexpected doubling of the $\mho$ parameter from its baseline.



Characterizing the behavior of 'the poor' as being the limiting value as $m \downarrow 0$, 'the rich' as the limit as $m \uparrow \infty$, and 'the middle' as the people at points substantially away from either limit (for example, at the target level of wealth), answer the following.

1. To what limit does the drop in consumption go as $m \uparrow \infty$? Why?

   Answer:

   > The limiting behavior of the rich is the solution to the perfect foresight problem, both before and after the increase in $\mho$. So in the limit, the effect of increasing labor income uncertainty for 'the rich' is zero.

2. Explain why the drop in c gets very small as $m \downarrow 0$. Hint: an appendix to TractableBufferStock shows that

$$\lim_{m \downarrow 0} \kappa^e = \frac{\mho\varphi}{1+\mho\varphi} \tag{14}$$

   for some positive constant φ likely to be not too far from 1.

   Answer:

   > The limiting behavior of the poor is close to the spend-everything 45 degree line both before and after the increase in $\mho$ so at any given level of m the value of c is just a bit less than m. It drops a small amount in response to the increase in uncertainty, but not very much.

3. Explain intuitively (in words) why consumption of people in 'the middle' is the place to expect the largest precautionary effect.

   Answer:

   > The limiting behavior of the poor is close to the spend-everything 45 degree line both before and after the increase in $\mho$ so at any given level of m the value of c is just a bit less than m. It drops a small amount in response to the increase in uncertainty, but not very much.
   >
   > The 'middle' people whose buffer stock is at its target, are holding their buffer stock only for precautionary reasons. (They satisfy the growth impatience condition so in the absence of the precautionary motive their m would asymptote to zero).

4. In light of these results, explain the circumstances in which a Campbell-Mankiw model would or would not provide an adequate approximation to the behavior of the economy over the business cycle, if the true model is the tractable buffer stock model.

   Answer:

   > The Campbell-Mankiw model will certainly not be adequate if one of the things that happens over the business cycle is systematic movements in uncertainty, since that model cannot capture any effect of uncertainty. The only circumstances in which the model would be completely correct would be if the entire population were divided into 'the poor' with almost exactly zero wealth, and 'the rich' with so much wealth that any precautionary effects might be negligible.

A Variant of the Abel Abel (1981)-Hayashi Hayashi (1982) φ model. Consider a firm which faces convex costs of adjustment for investment. Capital depreciates at rate δ. There is no investment tax credit, $\eta = 0$. The purchase price of capital goods is constant at $\mathsf{P} = 1$. Firms are, however, allowed to deduct *all* investment expenditures, *including installation costs*, from profit for tax purposes (this is called 'expensing' of investment); assume the usual convex cost of adjustment function $\mathbf{j}(i_t, k_t)$. Thus, if the firm's expenditures on investment are $\xi_t = i_t + j_t$, the firm's after-tax profits in a given period will be $\pi_t = (\mathbf{f}(k_t) - \xi_t)(1 - \tau)$, in contrast with `qModel`'s assumption of $\pi_t = (\mathbf{f}(k_t)(1 - \tau) - \xi_t)$.

Assume that the firm is solving a continuous-time optimization problem of the form

$$v_0 = \max \int_0^\infty \pi_t e^{-\mathsf{r}t} dt \tag{15}$$

subject to the dynamic budget constraint

$$\dot{k}_t = i_t - \delta k_t \tag{16}$$

1. Explain how the Hamiltonian for this problem,

$$\mathcal{H}(k_t, i_t, \lambda_t) = (1 - \tau)(\mathrm{f}(k_t) - \xi_t) + (i_t - \delta k_t)\lambda_t \tag{17}$$

differs from the Hamiltonian for the problem solved in `qModel`.

Answer:

The Hamiltonian for `qModel` is

$$\mathcal{H}(k_t, i_t, \lambda_t) = \mathbf{f}(k_t)(1 - \tau) - \xi_t + ((1 - \delta)i_t - \delta k_t)\lambda_t \tag{18}$$

2. Using the optimality conditions

$$\mathcal{H}_i = 0 \tag{19}$$
$$\dot{\lambda}_t = \mathsf{r}\lambda_t - (1 - \tau)(f_t^k - \xi_t^k) + \delta\lambda_t \tag{20}$$

show that the equation of motion for the after-tax shadow price of capital, $\varphi = \lambda/(1 - \tau)$ is

$$\dot{\varphi}_t = (\mathsf{r} + \delta)\xi^i(i_t, k_t) - (\mathbf{f}'(k_t) - \xi_t^k) \tag{21}$$

Answer:

The optimality conditions are

$$i) \quad \frac{\partial \mathcal{H}(k_t^*, i_t^*)}{\partial i_t} = 0 \tag{22}$$
$$(1 - \tau)\xi^i(i_t, k_t) = \lambda_t \tag{23}$$
$$ii) \quad \dot{k}_t = i_t - \delta k_t \tag{24}$$
$$iii) \quad \dot{\lambda}_t = \mathsf{r}\lambda_t - (1 - \tau)(f_t^k - \xi_t^k) + \delta\lambda_t \tag{25}$$
$$iv) \quad \lim_{t \to \infty} e^{-\mathsf{r}}\lambda_t k_t = 0 \tag{26}$$

Define $\varphi = \lambda_t/\mathsf{P}(1-\tau) = \lambda_t/(1-\tau)$. Then the equation of motion for φ is

$$\begin{aligned}
\dot{\varphi}_t &= \dot{\lambda}_t/(1-\tau) && (27)\\
\dot{\varphi}_t(1-\tau) &= (\mathsf{r}+\delta)\lambda_t - (1-\tau)(f_t^k - \xi_t^k) && (28)\\
\dot{\varphi}_t(1-\tau) &= (\mathsf{r}+\delta)\overbrace{\xi^i(i_t,k_t)(1-\tau)}^{=\lambda_t \text{ from } (23)} - \overbrace{(\mathbf{f}'(k_t)-\xi_t^k)(1-\tau)}^{\equiv\pi_t^k} && (29)\\
\dot{\varphi}_t &= (\mathsf{r}+\delta)\xi^i(i_t,k_t) - (\mathbf{f}'(k_t)-\xi_t^k) && (30)
\end{aligned}$$

3. Assume that there are no costs of adjustment, and show that the target level of capital does not depend on the tax rate τ. Explain intuitively why this differs from the result in the standard model.

   Answer:

   If costs of adjustment are always zero, then $\xi^i = 1$ and $\dot{\varphi}_t = 0$; the expense of investment is simply equal to i_t and therefore firms always ensure that $\mathbf{f}'(k_t) = (\mathsf{r}+\delta)$ regardless of the tax rate. This is because the deductibility of investment removes tax considerations from the choice of optimal investment. The essential intuition is that the value of investment that maximizes $V(k_t)$ will also maximize $(1-\tau)V(k_t)$. This is an interesting point usually made in public finance courses: if investment is completely tax deductible, the rate of investment and the level of the equilibrium capital stock do not depend on the tax rate.

   With no costs of adjustment, it is not possible for φ to deviate from 1, and therefore $\dot{\varphi} = 0$ which implies from the $\dot{\varphi}$ equation above that

$$\begin{aligned}
(\mathsf{r}+\delta)\xi_t^i &= \mathbf{f}'(k_t) && (31)\\
(\mathsf{r}+\delta) &= \mathbf{f}'(k_t) && (32)
\end{aligned}$$

   which implicitly determines the optimal value of k_t. Since τ appears nowhere in this equation, the optimal capital stock is independent of τ.

4. Compare the qualitative results of 'expensing' of investment to the effects of the introduction of an equivalently-sized investment tax credit (of size τ). Assume that costs of adjustment are no longer zero; draw a phase diagram and show the adjustment toward the new steady state. Say something interesting about the quantitative results.

   Answer:

   The analysis of an investment tax credit is qualitatively the same in this model as in the standard model. With an ITC of $\eta > 0$, the $\dot{\varphi} = 0$ locus shifts up, inducing an increase in the saddle path and therefore an increase in the target level of capital. The interesting quantitative thing to say is that there will be *some* ITC which is mathematically equivalent to the expensing of investment.

5. Consider an economy which can be in one of these regimes:
    a) Bust (employment is below its natural rate)
    b) Boom (employment is above its natural rate)

   and suppose that costs-of-adjustment expenditures can boost employment and bid up 'aggregate demand.' Discuss the relative desirability of moving from a tax regime that does not allow 'expensing' of investment to one that does allow 'expensing' in these two circumstances. (You will have to use some basic knowledge of macroeconomics not taught in this class to answer this question.)

   Answer:

   > It should be obvious that the question was trying to hint that encouraging firms to spend more money on adjustment costs when the economy is slack is a (and the extra expenditures could boost aggregate demand) is a better idea than encouraging them to spend more when the conomy is already booming (and extra demand is likely only to bid up wages and increase inflation, rather than increasing output).

6. In practice, the tax code allows firms to deduct 'depreciation' of past capital expenditures from their taxes; if the tax allowances for depreciation worked perfectly, in steady state a system with depreciation would be equivalent to a system that allowed expensing. Even in a system with 'expensing,' however, it would be possible to adjust the attractiveness of investment by adding an investment tax credit when demand was slack. Given these points, discuss the desirability of two alternative tax reform proposals:
    a) Allowing the expensing of corporate investment
    b) Allowing the expensing of investment *and* cutting the corporate tax rate τ

   Answer:

   > The discussion above should have made you realize that once investment is expensed, the corporate tax rate should not matter in the long run, since the the k that maximizes π also maximizes $(1 - \tau)\pi$. So, cutting the corporate tax rate, in this model, has no effects on economic decisions. Of course, it is desirable from the standpoint of current owners of capital, for whom it is a windfall. But it does not move the economy toward efficiency (once the existence of expensing or an efficient system of depreciation are in place.).

References

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