Name ____________________________

Final Exam
Intertemporal Choice
Fall, 2016
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I. Short Discussion Questions.

'Real' and 'Financial' Explanations of Asset Price Movements

A large literature at the intersection of finance and macroeconomics aims to explain movements of stock prices. Roughly speaking, much of that literature attempts to identify movements that can be explained by (a) 'real' and observable factors like news about a firm's profits ('cash flow') and dividends, versus (b) other unobservable factors that are often called 'discount rate' shocks. (The literature typically assumes that the 'discount rate' or, synonymously, the 'stochastic discount factor,' is impossible to observe *except* through its consequences for asset prices.)

1. Interpret the foregoing statement using the Lucas (1978) model. In that framework, what would you identify as 'real' and what would you identify as 'financial' factors?

   Answer:

   LucasAPM shows that in the version of Lucas's model with log utility, asset prices are given by:

   $$\mathsf{P}_t = \frac{d_t}{\vartheta} \tag{1}$$

   which provides a very direct interpretation: Movements in prices that are caused by changes in dividends (in the model, the productivity of the trees) are 'real' movements, while those due to unobserved factors must be attributed to changes in ϑ.

2. Explain why it is problematic, for the status of finance as a science, to attribute all movements in asset prices that are not explainable as reflecting observable 'real' factors to movements in a variable which is in principle unobservable ('the stochastic discount rate').

   Answer:

   The hypothesis that imputed movements of an unobservable variable (the 'stochastic discount factor') 'explain' those movements of an observable variable (asset prices) which cannot be explained by other observable phenomena ('real' measurable variables) begs the question of how the hypothesis can be disproven. If the stochastic discount factor can never be directly observed, and its movements can explain all movements in observable variables that are not otherwise explained, how can the theory be proven to be wrong?

3. Using the insights from the prior questions, explain why Robert Shiller's work in attempting to find credible ways of surveying market participants to measure their beliefs and preferences and attitudes would, if successful, be useful.

   Answer:

> If beliefs and preferences and attitudes could be directly measured by surveys or other methods, that would either bolster or undermine the presumption that movements in asset prices always reflect changes in beliefs or preferences or attitudes. Either outcome would be progress relative to the situation described above.

Dynamic Inefficiency and Japan's Woes.

Define dynamic inefficiency and explain why a plausible way of thinking about Japan's economic problems after 1990 is to argue that Japan's economy has been dynamically inefficient. Explain why the fact that Japan's capital/output ratio is not much higher than the U.S. capital/output ratio presents a challenge to the dynamic inefficiency interpretation of Japan's problems, and discuss how the question of dynamic inefficiency is connected not just to the level of aggregate capital but also to the efficiency with which financial markets allocate capital to productive uses.

Answer:

> An economy is dynamically inefficient in steady-state if the net rate of return on a marginal unit of capital accumulation (after depreciation) is less than the population growth rate plus the rate of productivity growth. While both population growth and productivity growth have been low in Japan and are projected to remain low, the rate of return may have been even lower or even negative. Thus the theory of dynamic inefficiency would say that the situation of Japanese consumers could be improved if there were less saving.
>
> In the standard model, the interest rate equals the marginal product of capital, i.e. $\mathsf{r} = \mathrm{f}'(k)$. If we have $k_U = k_J$ (where U and J stand for U.S. and Japan), then the fact that $\mathsf{r}_U >> \mathsf{r}_J$ suggests that the technology for converting marginal capital into marginal output is *different* in the U.S. and in Japan. That is, it is incorrect to assume that $\mathrm{f}_U = \mathrm{f}_J$ because if they have the same $\bar{k} = k_U = k_J$ yet different values of $\mathrm{f}'(k)$, only be captured by assuming $\mathrm{f}'_u \neq \mathrm{f}'_j$ so $\mathrm{f}_u \neq \mathrm{f}_j$.
>
> Thus, the possible explanation of Japan's problems is that Japan's high saving has been inefficient not because it increased the capital stock to an excessive level, but rather because it was being used inefficiently. If Japanese capital had been used better, perhaps the economy would not have been dynamically inefficient.
>
> This perspective suggests that if financial market reforms could boost the rate of return to saving in Japan, that could raise the net rate of return and eliminate the dynamic inefficiency.

Part II. Longer Analytical Questions

Brock and Mirman (1972) Multiplied

A large body of recent research has provided support for the old Keynesian idea that, at least under certain conditions,[1] the amount of aggregate output depends partly on the amount of 'aggregate demand.' Under such conditions, any shock that results in a change in a component of 'aggregate demand' will have a 'multiplier' effect (that is, the resulting change in output will be greater than the change in the component of aggregate demand).

Krueger, Mitman, and Perri (2016) propose that the role of a 'consumption multiplier' can be approximately captured by augmentation of the usual productivity term in the production function with a component that is increasing in the level of consumption:

$$\mathrm{F}_t(K_t, L_t) \quad = \quad C_t^{\omega} Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{2}$$

where $0 \leq \omega < 1$ determines the size of the multiplier, and $A_t = C_t^{\omega} Z_t$ would be measured as the level of 'productivity' using conventional approaches.

This question explores the implications of this production function in a growth model in which consumers are not aware of the existence of the multiplier; instead they interpret any changes in output that are attributable to the multiplier as reflecting the usual mysterious shocks to aggregate productivity that drive many other business cycle models. We assume that the 'multiplier effect' takes one time period to manifest itself (most attempts to provide deeper foundations for multipliers would suggest even longer lags).

We examine these ideas in a `BrockMirman` model. Thus, our consumers perceive their problem to be

$$\max \sum_{n=0}^{\infty} \beta^n \log C_{t+n} \tag{3}$$

s.t.

$$Y_t \quad = \quad A_t K_t^{\alpha} \tag{4}$$

$$K_{t+1} \quad = \quad Y_t - C_t \tag{5}$$

with Bellman equation

$$\mathrm{V}_t(K_t) = \max_{C_t} \; \log C_t + \beta \mathrm{V}_{t+1}(A_t K_t^{\alpha} - C_t). \tag{6}$$

However, in truth the level of income is given by

$$Y_t \quad = \quad C_{t-1}^{\omega} Z_t K_t^{\alpha} \tag{7}$$

(questions begin on the next page)

[1]Most notably, when interest rates are stuck at the zero lower bound

1. Show that the consumption function will be

$$C_t = \overbrace{(1-\alpha\beta)}^{\equiv\kappa} Y_t \tag{8}$$

Answer:

> Consumers perceive themselves to be solving a perfectly standard BrockMirman model, and so they will behave according to the solution of that model. See the handout for the derivation of the consumption function for that model.

For the rest of the question, suppose that consumers believe that the log of productivity $a \equiv \log A$ follows a random walk:

$$a_{t+1} = a_t + \varepsilon_{t+1} \tag{9}$$

and suppose that consumers observe the current level of output and derive their believed log level of 'productivity' as $a_t = y_t - \alpha k_t = z_t + \omega c_{t-1}$. Finally, assume that what we might call 'primitive' or 'structural' productivity z does indeed follow a random walk:

$$z_{t+1} = z_t + \zeta_{t+1}. \tag{10}$$

2. Show that if throughout an indefinitely long prior history the log-level of z has been $z = 0$, there will be 'pseudo-steady-state' values of the logs of the model's main variables, $\{\hat{c}, \hat{y}, \hat{k}, \hat{y}\}$ to which the economy will converge. (Note: Do *not* try actually to do the tedious algebra required to find the solutions; instead, you should give a mathematical argument for why it is likely that such a solution will exist, at least for some parameter values).

   Answer:

   > If we assume that there is a steady state, we have a linear system of four linear equations with four unknowns, which thus can be solved, unless the parameter values are too crazy:

$$\begin{aligned}
\hat{k} &= \log \alpha\beta + \hat{y} \\
\hat{y} &= \hat{a} + \alpha\hat{k} \\
\hat{c} &= \log(1-\alpha\beta) + \hat{y} \\
\hat{a} &= \underbrace{\hat{z}}_{=0 \text{ by assumption}} + \omega\hat{c}
\end{aligned}$$

3. Assume that the economy was in its pseudo-steady-state in period -1, $(\{c_{-1}, y_{-1}, k_{-1}, a_{-1}\} = \{\hat{c}, \hat{y}, \hat{k}, \hat{a}\})$. In period 1, a shock of size $\zeta_1 > 0$ occurs (and no other shocks occur thereafter: $\zeta_t = 0 \ \forall\ t \neq 1$). Show that the pattern of income growth is

$$\Delta y_1 = \zeta_1$$

$$\begin{aligned}
\Delta y_2 &= \zeta_1(\alpha+\omega) \\
\Delta y_3 &= \zeta_1(\alpha+\omega)^2 \\
\vdots &= \vdots \\
\Delta y_n &= \zeta_1(\alpha+\omega)^n
\end{aligned}$$

Hint: you will want to use these steady-state facts:

$$\hat{z} = 0 \tag{11}$$
$$\hat{a} = \hat{z} + \omega\hat{c} = \omega\hat{c} \tag{12}$$

and the dynamic equations:

$$z_t = z_{t-1} + \zeta_t \tag{13}$$
$$a_t = z_t + \omega c_{t-1} \tag{14}$$
$$k_t = \log\alpha\beta + y_{t-1} \tag{15}$$
$$y_t = a_t + \alpha k_t \tag{16}$$
$$c_t = \log(1-\alpha\beta) + y_t \tag{17}$$

Further hint: Begin by calculating how the shock ζ_1 changes the period 1 values of a, k, y, and c from their steady state values (for example, $a_t = \zeta_1 + \omega\hat{c}$), and then use the dynamic equations to move forward.

Answer:

Taking the 'Further hint':

$$a_1 = \omega\hat{c} + \zeta_1 \tag{18}$$
$$k_1 = \log\alpha\beta + \hat{y} \tag{19}$$
$$y_1 = \hat{y} + \zeta_1 \tag{20}$$
$$c_1 = \log(1-\alpha\beta) + y_1 = \hat{c} + \zeta_1 \tag{21}$$

and then using the 'dynamic equations'

$$z_2 = z_1 + \overbrace{\zeta_2}^{=0} \tag{22}$$
$$a_2 = z_2 + \omega c_1 \tag{23}$$
$$= \zeta_1 + \omega(\hat{c} + \zeta_1) \tag{24}$$
$$= \zeta_1(1+\omega) + \hat{a} \tag{25}$$
$$k_2 = \log\alpha\beta + y_1 \tag{26}$$
$$= \hat{k} + \zeta_1 \tag{27}$$
$$y_2 = a_2 + \alpha k_2 \tag{28}$$
$$= \zeta_1(1+\omega) + \alpha(\hat{k} + \zeta_1) \tag{29}$$
$$= \zeta_1(1+\omega+\alpha) + \hat{y} \tag{30}$$
$$\Delta y_2 = (\alpha+\omega)\zeta_1 \tag{31}$$
$$c_2 = \log(1-\alpha\beta) + y_2 \tag{32}$$

$$= \hat{c} + \zeta_1(1 + \omega + \alpha) \tag{33}$$

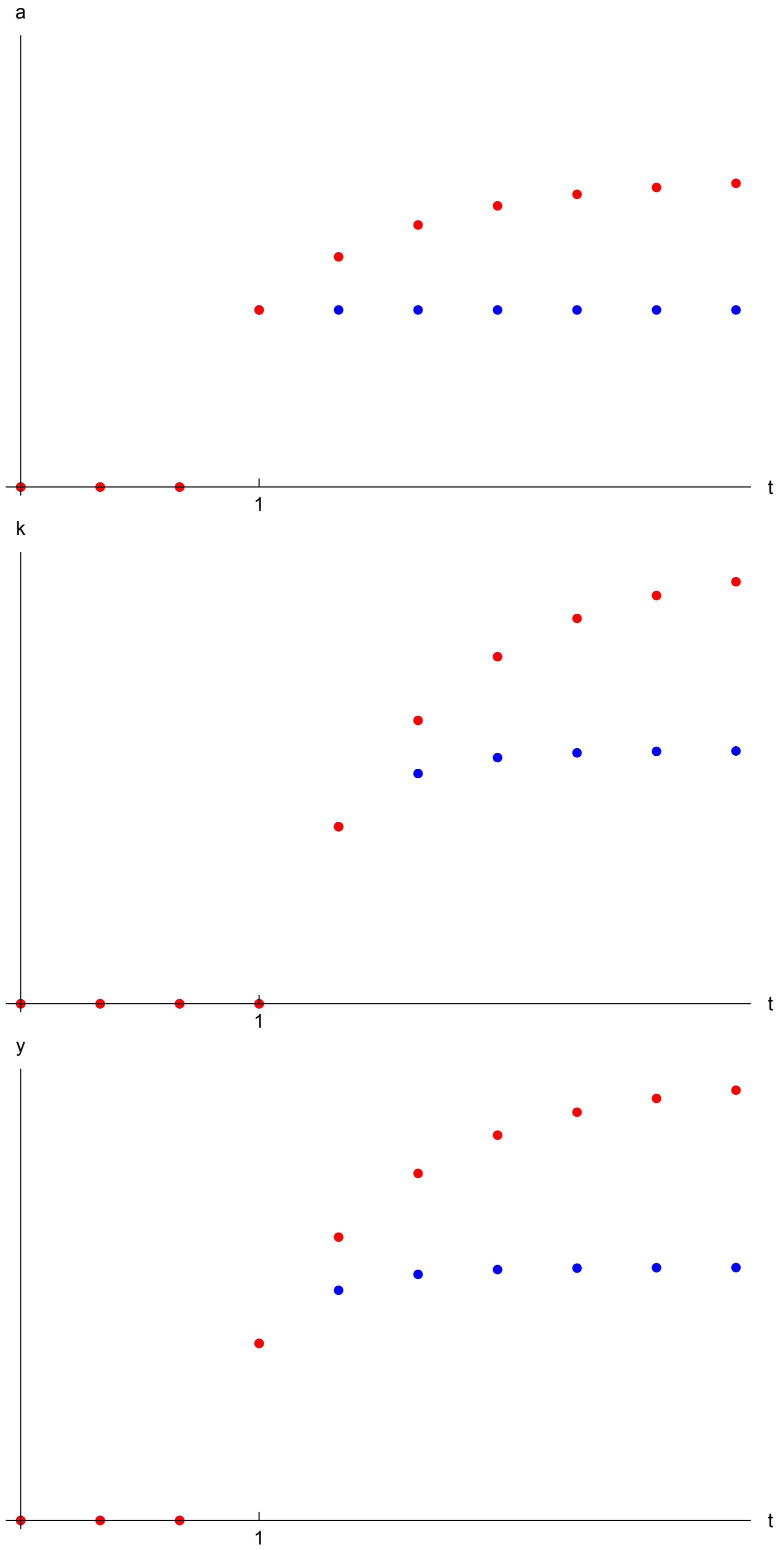
and thereafter generically

$$\begin{aligned}
a_3 &= z_3 + \omega c_2 && (34)\\
&= \zeta_1 + \omega(\hat{c} + \zeta_1(1 + (\alpha + \omega))) && (35)\\
&= \zeta_1(1 + \omega(1 + (\alpha + \omega))) + \hat{a} && (36)\\
&= a_2 + \zeta_1\omega(\alpha + \omega) && (37)\\
&= a_1 + \zeta_1\omega(1 + (\alpha + \omega)) && (38)\\
\Delta a_3 &= \zeta_1\omega(\alpha + \omega) && (39)\\
k_3 &= \log \alpha\beta + y_2 && (40)\\
&= \log \alpha\beta + \zeta_1(1 + \omega + \alpha) + \hat{y} && (41)\\
&= \hat{k} + \zeta_1(1 + (\omega + \alpha)) && (42)\\
\Delta k_3 &= \zeta_1(\omega + \alpha) && (43)\\
y_3 &= a_3 + \alpha k_3 && (44)\\
&= a_1 + \zeta_1\left(\omega(1 + (\alpha + \omega)) + \alpha(1 + (\alpha + \omega))\right) && (45)\\
&= a_1 + \zeta_1(\alpha + \omega)(1 + (\alpha + \omega)) && (46)\\
\Delta y_3 &= \Delta a_3 + \alpha\Delta k_3 && (47)\\
&= \zeta_1(\omega(\alpha + \omega)) + \alpha\zeta_1(\alpha + \omega) && (48)\\
&= \zeta_1\left(\alpha + \omega\right)^2 && (49)
\end{aligned}$$

and similar analyses show that the pattern continues indefinitely.

4. Draw diagrams showing the dynamics of a, k, and y in the experiment described above. On the same diagrams, show the dynamics that consumers *expect* these variables will follow. Using the comparison of the expected and the actual dynamics, explain the sense in which this model can be said to exhibit a 'consumption multiplier.'

Answer:



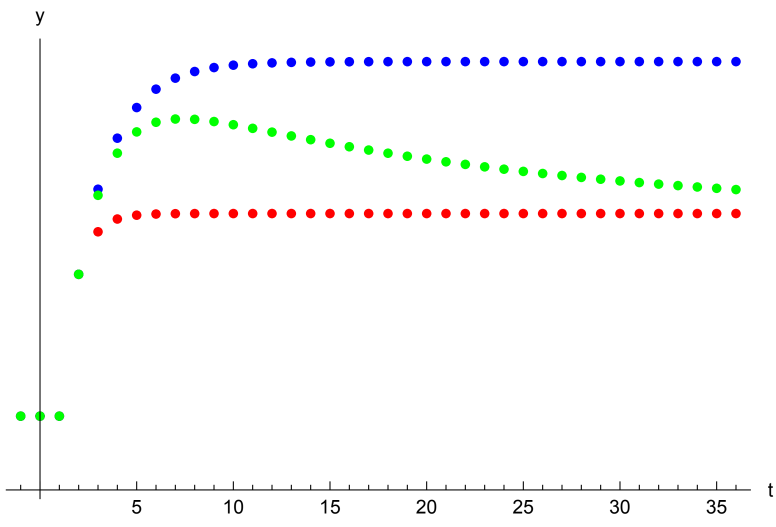
5. Most of the deeper mechanisms by which multipliers are hypothesized to work (e.g., having workers put in extra hours when demand surges, or stickiness of prices) imply that the stimulative effect on productivity eventually dies out. In the framework outlined above, this might be captured by supposing that the multiplier depends not upon the level of C but rather on its ratio to some new 'steady-state' value. That is, the 'productivity' term in the production function is $A_t(C_{t-1}/C^{\mathrm{SS}})^{\omega}$ where C^{SS} catches up slowly but inexorably to actual consumption (for example, if $C_t^{\mathrm{SS}} = \lambda C_{t-1}^{\mathrm{SS}} + (1-\lambda)C_{t-1}$ for some value of λ not much less than 1) so that eventually (in the absence of further shocks) the new 'pseudo-steady-state' would be reached at a point where $C_t/C^{SS} = 1$. Draw the diagrams you would expect this economy to exhibit in response to the same experiment performed earlier.

   Answer:

   If $\lambda = 1$, the model is essentially identical (except for a scale shift) to the one solved above since C^{SS} never moves. If $\lambda = 0$, the 'multiplier' term is $C_{t-1}/C_{t-1} = 1$ so we are back to the standard `BrockMirman` model.

   For a value of λ not much less than one, for the time shortly after the shock the diagrams for this model should look like the ones for the multiplier model; but gradually as C^{SS} catches up to C_{t-1} the levels of all the variables should erode back toward the steady-state values of the basic `BrockMirman` model.

   The figure below shows an approximation of what things might look like for y in this case, with the green dots representing the model in which the multiplier gradually wears off. The figures for k and a will be similar.



Infrastructure and Interest Rates.

Starting near the beginning of the Great Recession (henceforth, 'GR'), economists including Larry Summers, Paul Krugman, Brad deLong, and even Martin Feldstein argued that an appropriate response to a crisis that combined low aggregate demand and historically low interest rates would be a large increase in infrastructure investment.

From 2009 to 2015, Republican party dogma was that an increase in infrastructure investment was not desirable. But, in the 2016 presidential election campaign, Donald Trump criticized Hillary Clinton's proposals to boost infrastructure spending by saying that she was not calling for *enough of an increase* in infrastructure investment.

This question asks you to think about what a neoclassical model might say about infrastructure investment, and then to critique those insights.

Suppose that the private (that is, non-infrastructure) capital stock k is exogenously fixed at k_0 and does not depreciate but cannot be augmented by extra saving (there is an 'endowment' of capital).

Using n to designate the quantity of i$n$frastructure capital (which, like private capital, does not depreciate), suppose we can capture the long-run effect of n in the production function by:

$$\mathrm{f}(k, n) = k^{\alpha} n^{\mu} \tag{50}$$

so that (under the maintained assumption $0 < \mu < 1$), a country with more n gets higher productivity from its private capital k. (For the remainder of the question, assume the population is constant at 1).

1. Suppose that, leading up to the GR, the government had chosen a level of infrastructure capital $n = n_0$, paid for by borrowing on international capital markets, and had imposed lump sum taxes on households sufficient to pay the interest on the debt incurred thereby, $\tau_0 = \mathsf{r}_0 n_0$, so that after-tax national income was

$$y_0 = \mathrm{f}(k_0, n_0) - \tau_0. \tag{51}$$

   Calculate the level of the infrastructure capital stock $n_{0,*}$ that a government that wanted to maximize after-tax income would have chosen for this economy. *Explain intuitively* the reasons for the effects of the various parameters. Defining $\mathrm{f}_{0,*}$ as the production function that applies when n has been chosen optimally, and $\mathrm{f}^{n}_{0,*}$ as the marginal product of an additional unit of n, you should obtain the result that:

$$\mathrm{f}^{n}_{0,*} \equiv \mathrm{f}^{n}(k_0, n_{0,*}) = \mathsf{r}_0 \tag{52}$$

   Answer:

An optimizing government will choose a level of infrastructure capital to solve

$$n_{0,*} = \underset{\{n\}}{\arg\max} \quad \mathrm{f}(k_0, n) - \mathsf{r}n \tag{53}$$

whose first order condition is

$$\begin{aligned} \left(\frac{d\mathrm{f}(k,n)}{dn}\right) &= \mathsf{r} && (54) \\ \mu k^{\alpha} n^{\mu-1} &= \mathsf{r} && (55) \\ n &= \left(\frac{\mathsf{r}}{\mu k_0^{\alpha}}\right)^{1/(\mu-1)} && (56) \\ &= \left(\frac{\mu k^{\alpha}}{\mathsf{r}}\right)^{1/(1-\mu)} && (57) \end{aligned}$$

which says that optimal expenditures/taxes will be higher when

a) the capital stock is higher (because there is more productivity to "enhance" by infrastructure)

b) when the coefficient on capital is higher (α is larger), for similar reasons

c) μ is larger, because the larger is μ the smaller is the rate at which government efficiency improvements have diminishing marginal productivity effects

d) r is lower, because infrastructure is "cheaper" and so you want more of it

The question also asks you to explain why $\mathrm{f}^n_{0,*} = \mathsf{r}_0$. The explanation is that if the marginal product of additional n were greater than this, an optimizing government could borrow at r_0 and buy infrastructure with it and increase total output; while if $\mathrm{f}^n_{0,*} < \mathsf{r}_0$ the government could increase income by reducing n and borrowing less (thus cutting taxes).

2. Continue to assume that up to the GR, the government had chosen n optimally so that $y_{0,*} = \mathrm{f}_{0,*} - \mathsf{r}_0 n_{0,*} = k_0^{\alpha} n_{0,*}^{\mu} - \mathsf{r}_0 n_{0,*}$. Calculate explicit formulae for the level of after-tax income under the following alternative government policies, assuming that when the GR hits, interest rates drop to $\mathsf{r}_1 = \mathsf{r}_0(1-\epsilon)$ for some 'small' $\epsilon > 0$ (and are expected to stay at r_1 forever):

   a) Cut taxes to the level sufficient to pay the new, lower, interest rate costs associated with maintaining the original n_0

   b) Leave taxes at the old level of $\tau_{0,*} = \mathsf{r}_0 n_{0,*}$ and change n to the amount that can be paid for with those tax revenues. (Hints: (1) Begin by showing that your earlier FOC for $n_{0,*}$ implies that $\mu \mathrm{f}_{0,*} = \mathsf{r}_0 n_0$; (2) you will need to pay attention to the concavity of the production function in n either by using a

2nd order Taylor expansion or just by staring at the equation and seeing the relevant intuition)

c) Change the stock of infrastructure to the level optimal for the new level of interest rates, while changing the tax rate accordingly

Answer:

a. Under the tax cut alternative, production remains the same, while taxes decline by the amount

$$\begin{aligned}(\mathsf{r}_1 - \mathsf{r}_0)n_{0,*} &= (\mathsf{r}_0(1-\epsilon) - \mathsf{r}_0)\, n_{0,*} && (58)\\ &= \mathsf{r}_0 \epsilon n_{0,*} && (59)\end{aligned}$$

b. Under the tax maintenance alternative, the amount of extra n that can be paid for with the original tax revenues $\tau_{0,*}$ is directly proportional to the ratio of the interest rates:

$$\begin{aligned}\mathsf{r}_0/\mathsf{r}_1 &= \mathsf{r}_0/\mathsf{r}_0(1-\epsilon) && (60)\\ &\approx 1+\epsilon && (61)\end{aligned}$$

so the new level of production will be

$$\begin{aligned}\mathrm{f}_1 = k_0^{\alpha}(n_{0,*}(n_1/n_{0,*}))^{\mu} &= k_0^{\alpha}\,(n_{0,*}(1+\epsilon))^{\mu} && (62)\\ &= \mathrm{f}_{0,*}(1+\epsilon)^{\mu} && (63)\end{aligned}$$

and a 2nd-order Taylor expansion yields the approximation

$$\mathrm{f}_1 \approx \mathrm{f}_{0,*}(1+\mu\epsilon - \epsilon^2(1-\mu)/2 \qquad (64)$$

so the amount of change in production (and therefore income) is

$$\mathrm{f}_1 - \mathrm{f}_{0,*} \approx \mathrm{f}_{0,*}\left(\mu\epsilon - \epsilon^2(1-\mu)/2\right) \qquad (65)$$

but using the hint $\mathsf{r}_0 = \mathrm{f}_*^n = \mu \mathrm{f}_{0,*}/n_{0,*}$ so we can substitute here for $\mu \mathrm{f}_{0,*} = n_{0,*}\mathsf{r}_0$ to get

$$\mathrm{f}_1 - \mathrm{f}_{0,*} \approx n_{0,*}\mathsf{r}_0\epsilon - \mathrm{f}_{0,*}\left(\epsilon^2(1-\mu)/2\right) \qquad (66)$$

c. Under option (c), the choice is the optimal choice, per (57):

$$\begin{aligned}n_{1,c} &= \left(\frac{\mu k_0^{\alpha}}{\mathsf{r}_1}\right)^{1/(1-\mu)} && (67)\\ &= n_{0,*}\left(\frac{\mathsf{r}_0}{\mathsf{r}_1}\right)^{1/(1-\mu)} && (68)\end{aligned}$$

and after-tax income per capita is

$$y_{1,c} = k_0^{\alpha} n_{1,c}^{\mu} - \mathsf{r}_1 n_{1,c} \qquad (69)$$

generating total income of

$$y_{1,c} = \mathrm{f}_{0,*}\left(\frac{\mathsf{r}_0}{\mathsf{r}_1}\right)^{\mu/(1-\mu)} - \mathsf{r}_1 n_{0,*}(\mathsf{r}_0/\mathsf{r}_1)^{1/(1-\mu)}$$

$$
\begin{aligned}
&\approx \mathrm{f}_{0,*}(1+\epsilon\mu/(1-\mu)) - \mathsf{r}_0(1-\epsilon)n_{0,*}(1+\epsilon/(1-\mu)) \\
&\approx \mathrm{f}_{0,*}(1+\epsilon\mu/(1-\mu)) - \mathsf{r}_0 n_{0,*}(1-\epsilon+\epsilon/(1-\mu)-\epsilon^2/(1-\mu)) \\
&\approx \mathrm{f}_{0,*} + \epsilon\mathsf{r}_0 n_{0,*}/(1-\mu) - \epsilon\mathsf{r}_0 n_{0,*}/(1-\mu) - \mathsf{r}_0 n_{0,*}(1-\epsilon-\epsilon^2/(1-\mu)) \\
&\approx \mathrm{f}_{0,*} - \mathsf{r}_0 n_{0,*}(1-\epsilon-\epsilon^2/(1-\mu))
\end{aligned}
$$

so

$$
y_{1,c} - y_{0,*} = \mathsf{r}_0 n_{0,*}\epsilon(1+\epsilon/(1-\mu)) \tag{70}
$$

which is greater than the improvement in option (a) of merely $\mathsf{r}_0 n_{0,*}\epsilon$.

3. Can the three policies above be ranked from best to worst? If so, what is the ranking? If not, explain why and what additional information you would need to be able to rank them?

   Answer:

   Collecting the above results,

Option	$\mathrm{f}_1 - \mathrm{f}_{0,*}$
a.	$\mathsf{r}_0 n_{0,*}\epsilon$
b.	$\mathsf{r}_0 n_{0,*}\epsilon - \mathrm{f}_{0,*}(\epsilon^2(1-\mu)/2)$
c.	$\mathsf{r}_0 n_{0,*}\epsilon(1+\epsilon/(1-\mu))$

   A comparison of these to each other and to the original situation yields the following insights:

   a) Option (a) to cut taxes is ever-so-slightly better than option (b) to increase infrastructure. The difference between the two options comes from the second-order term in the case in which infrastructure is increased, which reflects the concavity of the production function; the increase in income due to a tax cut is linear in the size of the change while the increase in income due to infrastructure investment is sublinear and so slightly smaller

   b) Obviously, the optimizing choice c. yields the highest level of income. But full credit required you to identify why: In the tax cut case, the level of n remains the same so the marginal product of infrastructure remains at $\mathrm{f}_{0,*} = \mathsf{r}_0 > \mathsf{r}_1$ which is greater than the interest rate and so increasing infrastructure investment a bit will clearly increase output by more than it decreases after-tax income.

4. The preceding analysis assumed that the private capital stock k was fixed at k_0. We now want to consider how things might change if k is chosen optimally. (For the following analysis, suppose that k_0 was in fact the optimal level of the private capital stock at an interest rate of r_0 and with infrastructure of n_0).

   a) Suppose the government pursues option (a) above, keeping $n = n_0$. Using a φ model of investment, show the paths of investment and private capital k that you would expect to arise from optimizing firms after the drop in the world interest rate to r_1 (assuming that this is the only thing that has changed for the firms; that is, they are not expecting a fall in sales or other consequences of the GR)

Answer:

> This is a standard exercise with the ϱ model: qModel shows the results when there is a permanent cut in the interest rate.

b) Suppose now that when the GR hits, not only do interest rates drop to $\mathsf{r}_1 < \mathsf{r}_0$ but firms' expectations about their future productivity also deteriorate, by an amount that is just enough to keep their investment spending the same as it was before, i_0 (assuming that aggregate infrastructure remained at its pre-GR level n_0). That is, if firms expected the goverment to pursue the household tax-cut option in the prior question, firms would not change their behavior. Discuss the consequences for firm investment spending if, instead, the government pursues each of the other two options in the prior question. Can the size of any effect on firm investment spending be ranked between these two alternatives? Taking this into account, comment on any new insights about the relative desirability of the alternatives.

Answer:

> The setup of the question indicates a situation in which, if $n = n_{0,*}$, firms would leave their investment exactly unchanged. But $n_{1,b} > n_{1,c} > n_{1,a}$ among the three options above, and an increase in n unambiguously increases the marginal product of private capital because the exponent on n in f is $\mu > 0$. Hence, we can conclude that $i_{b,1} > i_{c,1} > i_{a,0}$. Thus, from the standpoint of the firms, the best option (in terms of their profitability) is (b), while from the standpoint of the economy as a whole the best option was (c). This conflict is because (by assumption) the firms are not paying for the infrastructure investment but they benefit from it; under these circumstances, the firms will always prefer the maximum amount of infrastructure because any addition to n increases the productivity of their private capital.

References

Brock, William, and Leonard Mirman (1972): "Optimal Economic Growth and Uncertainty: The Discounted Case," *Journal of Economic Theory*, 4(3), 479–513.

Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016): "Macroeconomics and Household Heterogeneity," Discussion paper.

Lucas, Robert E. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–1445, Available at http://www.jstor.org/stable/1913837.