Name ____________________

Midterm Exam

Intertemporal Choice

Fall, 2016

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I

Consumption Dynamics in a Deaton (1992)-Friedman (1957) Model With Transitory and Permanent Shocks.

Consider a consumer solving the maximization problem

$$\max_{\{\mathrm{C}\}_t^\infty} \quad \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s \mathrm{u}(C_{t+s})\right]$$

subject to the transition equations

$$\begin{aligned} B_{t+1} &= (M_t - C_t)\mathsf{R} \\ P_{t+1} &= P_t + \Psi_{t+1} \\ Y_{t+1} &= P_{t+1} + \Xi_{t+1} \\ M_{t+1} &= B_{t+1} + Y_{t+1} \end{aligned}$$

where Ξ and Ψ are respectively the unpredictable transitory and permanent shocks to the level of income, satisfying $\mathbb{E}_t[\Psi_{t+n}] = \mathbb{E}_t[\Xi_{t+n}] = 0 \ \forall \ n > 0$.

1. Rewrite the problem in Bellman equation form, and derive the Euler equation for consumption.

   Answer:

   This is in `Envelope`.

   For the remainder of the problem, assume that $\mathsf{R}\beta = 1$; the utility function is quadratic; and the consumer's level of resources is too small for consumption ever to reach the 'bliss point.'

2. Show that under these assumptions, $\mathbb{E}_t[C_{t+n}] = C_t \ \forall \ n > 0$.

   Answer:

   This is done in `RandomWalk`.

3. Use the fact that the intertemporal budget constraint must hold in expectation to prove that optimal consumption in period t is given by

   $$C_t = (\mathsf{r}/\mathsf{R})(B_t + H_t + \Xi_t) \tag{1}$$

   where, designating the infinite horizon present discounted value by the operator $\mathbb{P}_t(\bullet)$,

   $$\begin{aligned} H_t &= \mathbb{E}_t[\mathbb{P}_t(Y)] && (2) \\ &= (\mathsf{R}/\mathsf{r})P_t && (3) \end{aligned}$$

   Answer:

   This is essentially the same as in `PerfForeCRRAModel` because when the expected shocks are all zero, the expectations operator annihilates the future values of those shocks leading to the same formula for expected future income as the formula for actual future income in the perfect foresight model. Note that (1) reduces to

$$C_t = P_t + (\mathsf{r}/\mathsf{R})(B_t + \Xi_t) \tag{4}$$

4. Show that

$$\mathbb{E}_t[\Delta Y_{t+1}] = -\Xi_t \tag{5}$$

and explain the intuition for this result.

Answer:

$$\begin{aligned} Y_{t+1} - Y_t &= P_{t+1} + \Xi_{t+1} - (P_t - \Xi_t) \\ \mathbb{E}_t[\Delta Y_{t+1}] &= \mathbb{E}_t[\Psi_{t+1}] + \mathbb{E}_t[\Xi_{t+1}] - \Xi_t \\ &= -\Xi_t \end{aligned}$$

If income is high today due to a transitory income shock, it is expected to fall in the following period, and vice versa. This amounts to transitory shocks tending to revert to their mean.

5. The 'Haig-Simons' definition of saving is the change in resources from one period to the next. Consistent with this, define

$$S_t = B_{t+1}/\mathsf{R} - B_t$$

and show that

$$\begin{aligned} \mathbb{E}_t[\Delta Y_{t+1}] &= -S_t\mathsf{R} - \mathsf{r}B_t & (6) \\ &\approx -S_t\mathsf{R} & (7) \end{aligned}$$

if $\mathsf{r}B_t$ is small. Explain this result intuitively, and discuss its relationship to Friedman's PIH.

Answer:

$$\begin{aligned} S_t &= (M_t - C_t) - B_t \\ &= (B_t + P_t + \Xi_t - C_t) - B_t \\ &= (1 - (\mathsf{r}/\mathsf{R}))\Xi_t - (\mathsf{r}/\mathsf{R})B_t \\ &= (\Xi_t - \mathsf{r}B_t)/\mathsf{R} \\ \mathsf{R}S_t &= \Xi_t - \mathsf{r}B_t \\ -\mathsf{R}S_t - \mathsf{r}B_t &= -\Xi_t \\ &= \mathbb{E}_t[\Delta Y_{t+1}] \end{aligned}$$

The intuition is that a positive transitory shock to income today lifts the level of income above its 'normal' value. If r is 'small' then the amount of extra spending induced by the transitory shock is small and can be neglected (leading to the approximation). That is, the consumer here is setting consumption approximately equal to permanent income P, and spending little out of their transitory income Ξ_t – which is basically what Friedman proposed in his Permanent Income Hypothesis. Thus, according both to Friedman and to this model, when you get a transitory shock, you save almost all of it. So, the size of the savings is roughly the amount by which income will change next period as it reverts back to its permanent level.

6. Suppose we now construct a forecast of income growth using (7) and perform a regression

$$\Delta C_{t+1} \quad = \quad \alpha_0 + \alpha_1 \, \mathbb{E}_t[\Delta Y_{t+1}]$$

What should our empirical estimate of α_1 be? How do you reconcile this with the fact that the level of consumption is positively related to the level of income?

Answer:

Nothing about this problem changes the usual conclusion that consumption follows a random walk. So $\alpha_1 = 0$. The fact that the *level* of consumption is related to the *level* of income does not mean that consumption *changes* are predictable even if income changes are predictable (at least one period in advance).

7. Now suppose that consumption data are measured with iid (=white noise = unpredictable) error,

$$\tilde{C}_{t+1} \quad = \quad C_{t+1} + \chi_{t+1}$$

where $\mathbb{E}_t[\chi_{t+n}] = 0 \; \forall \; n > 0$ and the variance of χ is $\mathbb{E}_t[\chi^2_{t+1}] = \sigma^2_\chi$.

Suppose that we wish to use $\Delta\tilde{C}_t$ to forecast $\Delta\tilde{C}_{t+1}$ from an equation of the form

$$\mathbb{E}_t[\Delta\tilde{C}_{t+1}] \quad = \quad \gamma\Delta\tilde{C}_t.$$

Define the expected squared errors from the forecasting equation as

$$SSE \quad = \quad \mathbb{E}_t[(\Delta\tilde{C}_{t+1} - \mathbb{E}_t[\Delta\tilde{C}_{t+1}])^2]$$

Defining the variance of ‘true’ changes in consumption as $\mathbb{E}_t[(\Delta C_{t+1})^2] = \sigma^2_{\Delta C}$, show that the choice of γ which minimizes the sum of squared errors from such a forecast is

$$\gamma \quad = \quad -\left(\frac{\sigma^2_\chi}{\sigma^2_\chi + \sigma^2_{\Delta C}}\right)$$

Explain why this makes sense intuitively and discuss how it relates to the previous result about forecasting income growth. Hint: Your problem is to find the value of γ which minimizes the expected sum of squared errors:

$$\min_\gamma SSE \tag{8}$$

and you can use the fact that 'true' consumption growth and measurement error are both iid: $\mathbb{E}_t[\Delta C_{t+1}\Delta C_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_{t-1}] = \mathbb{E}_t[\chi_{t+1}\Delta C_t] = \mathbb{E}_t[\chi_{t+1}\chi_{t-1}] = 0$.

Answer:

The FOC yields:

$$
\begin{aligned}
0 &= -\gamma 2\,\mathbb{E}_t[(\Delta\tilde{C}_{t+1} - \gamma\Delta\tilde{C}_t)(\Delta C_t + \chi_t - \chi_{t-1})] \\
&= \mathbb{E}_t[(\Delta C_{t+1} + \chi_{t+1} - \chi_t - \gamma\Delta\tilde{C}_t)(\Delta C_t + \chi_t - \chi_{t-1})] \\
&= -\sigma_\chi^2 - \gamma(\sigma_\chi^2 + \sigma_{\Delta C}^2) \\
\gamma &= -\left(\frac{\sigma_\chi^2}{\sigma_\chi^2 + \sigma_{\Delta C}^2}\right)
\end{aligned}
$$

where we have used the fact that the measurement errors are iid and uncorrelated with true consumption growth, $\mathbb{E}_t[\Delta C_{t+1}\Delta C_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_{t-1}] = \mathbb{E}_t[\chi_{t+1}\Delta C_t] = \mathbb{E}_t[\chi_{t+1}\chi_{t-1}] = 0$.

Just as changes in income should be unrelated to changes in consumption, ΔC_{t+1} should be unrelated to ΔC_t when consumption is a random walk. Thus, any non-zero estimate of γ is brought about purely by measurement error. Note that in the equation for γ above, when measurement error is negligible ($\sigma_\chi^2 = 0$), we find the desired result.

Part II

Habit Formation, Sticky Expectations, and Measurement Error.

Although the baseline RandomWalk model of consumption implies that consumption growth is unforecastable, the models in the handouts Habits and StickyExpectations are both consistent with serial correlation in 'true' consumption growth that takes forms like:

$$\Delta C_{t+1} = \alpha_0 + \alpha_1 \Delta C_t + \epsilon_{t+1} \tag{9}$$

1. Explain what the coefficient α_1 is interpreted as measuring in each of the two theories, and give some intuition for why the coefficient in this regression can be interpreted as measuring that object.

   Answer:

   > In StickyExpectations with an interest rate $R = 1$, α_1 is interpreted as the proportion of the population which does not update their expectations in period $t+1$. In Habits, α_1 is the penalty (due to habit formation) that agents incur in period $t+1$ for their consumption in the previous period.

In practice, a difficulty of estimating either of these models is that actual reported consumption data from government statistical agencies contains measurement error. Suppose that we have data on measures of the true beliefs, collected at date t, about consumption growth from these sources:

- $\mathbb{B}_t^{\text{hhs}}[\Delta C_t]$: Answers from a survey of households who are asked directly about what they believe their consumption growth was
- $\mathbb{B}_t^{\text{fed}}[\Delta C_t]$: Estimates of produced by the Federal Reserve based on aggregate statistics like surveys of retailers

Consider performing regressions of the form

$$\Delta C_{t+1} = \alpha_0 + \alpha_1^{\bullet} \mathbb{B}_t^{\bullet}[\Delta C_t] + \zeta_{t+1} \tag{10}$$

where the $\bullet$ is a stand-in for the different methods of measuring beliefs. This generates a potentially different estimate of α_1 for each of the different measures of consumption beliefs.

1. Suppose first that the estimates of α_1 are similar for all measures of consumption growth beliefs, say $\{\alpha_1^{\text{hhs}}, \alpha_1^{\text{fed}}\} = 0.75$.

   a) Under these conditions and using only these data, the two theories are obviously basically indistinguishable. Can you think of any other kinds of data which might be more useful in distinguishing these theories from each other? How would you go about using those data to distinguish the theories?

      Answer:

Questions 3 and 4 provide an example. Answers were judged on a case-by-case basis to determine whether the suggested data and methodology could be used to reject one of the theories.

Suppose now that if we somehow had access to data on 'true' consumption data, we would be able to show that households have perfect contemporaneous knowledge of their own consumption, $\mathbb{B}_t^{\text{hhs}}[\Delta C_t] = \Delta C_t$, while the Fed's contemporaneous beliefs are equal to the truth plus some mean-zero measurement error, $\mathbb{B}_t^{\text{fed}}[\Delta C_t] = \Delta C_t + \xi_t$ with some variance $\sigma_\xi^2 > 0$

2. Now suppose the estimates using these beliefs produce different results. In particular, suppose the coefficient estimates fit the pattern $\{\alpha_1^{\text{hhs}} > \alpha_1^{\text{fed}}\}$. Can you reach any new conclusions about the validity of the two theories?

   Answer:

   Both theories would imply this ranking of results, because mean-zero measurement error biases the estimate of α_1 toward zero regardless of the model which produces the observed consumption patterns. Therefore, they still can't be distinguished from each other.

Suppose that, although Fed's direct measure of consumption expenditures is imperfect, its supervision of the banking sector allows it to measure past income Y_{t-n}, bank balances B_{t-n}, and saving S_{t-n} perfectly (where saving is income minus consumption: $S_{t-n} = \mathsf{r}B_{t-n} + Y_{t-n} - C_{t-n}$ for $n \geq 0$.

3. How can the Fed use these data to improve its $\mathbb{B}_t^{\text{fed}}[\Delta C_t]$?

   Answer:

   The Fed can back out households' information about their own consumption using:

$$C_{t-n} = \mathsf{r}B_{t-n} + Y_{t-n} - S_{t-n} \tag{11}$$

   and thus obtain households' information about their past C_{t-1}.

Now suppose the Fed, again through its banking regulatory powers, has microeconomic data about individual households indexed by i on the same variables that are measured in the aggregate data, for example $c_{t,i}$ etc.

4. Can the Fed use these data to distinguish the two theories? How?

   Answer:

   Yes! The habit formation theory says that the same equation will hold at the micro as at the macro level. So the Fed can estimate equation 9, now with individual-level data. The rejection of the hypothesis that α_1 at the aggregate level is equal to that at the individual level will be equivalent to the rejection of the habit formation model.

References

Deaton, Angus S. (1992): *Understanding Consumption.* Oxford University Press, New York.

Friedman, Milton A. (1957): *A Theory of the Consumption Function.* Princeton University Press.