?'s Permanent Income Hypothesis. Answer the following questions under the assumption that Friedman's Permanent Income Hypothesis $c_i = p_i$ is true. Assume that income in each period for each household is $y_{t,i} = p_i + \xi_{t,i}$ where $\xi_{t,i}$ is a mean-0 random ('white noise') transitory shock to income.

1. Suppose that you have consumption and income data for a cross-section of households in a particular year. Suppose further that you know that the variance of transitory income is higher for farmers than it is for members of other occupational groups. Now imagine estimating a Keynesian consumption function $c_i = \alpha_0 + \alpha_1 y_i + \epsilon_i$. How would you expect the estimated coefficients α_1 to differ between the farmers in your sample and the rest of the population? Do you need to know any other information to answer the question definitively?

   Make a copy of the "KeynesFriedmanModigliani" Jupyter notebook, and rename your copy to "FriedmanPIH-[Your-Last-Name]." Using tools in that notebook simulate 50 observations of y_i for each group (farmers and non-farmers), with:

   a) $\sigma^2_{p,\text{farmers}} = \sigma^2_{p,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$

   b) $\sigma^2_{p,\text{farmers}} = 10\sigma^2_{p,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$

   c) $\sigma^2_{p,\text{farmers}} = 0.1\sigma^2_{p,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$

   For purposes of these simulations, assume that $\{\sigma^2_{p,\text{workers}}, \sigma^2_{\xi,\text{workers}}\} = 0.01, 0.02$. For each scenario, estimate coefficient α_1 of the Keynesian consumption function, draw the observations and estimated consumption function on a graph with y on the horizontal axis and p on the vertical axis (use different colors for different groups). Make sure you include a 45-degree line (corresponding to $c = y$) on each graph. Relate your results to the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?
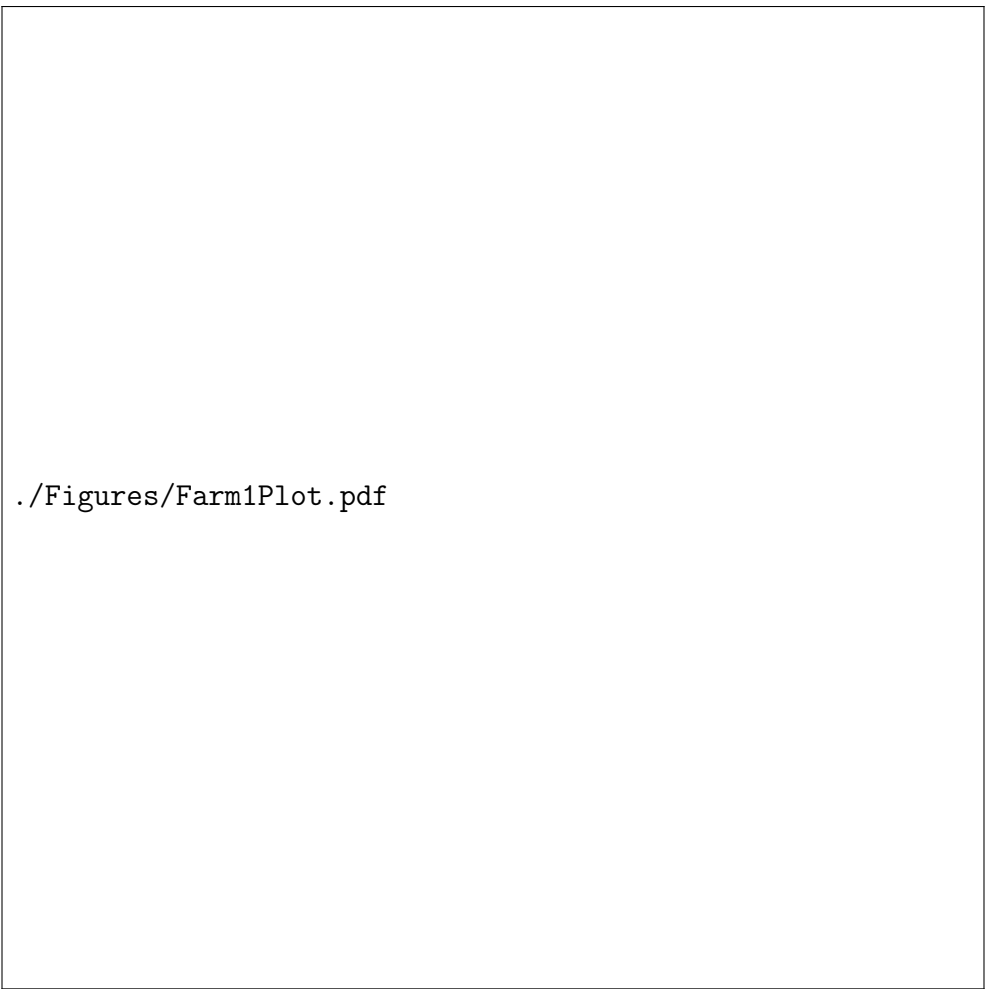
   Answer:

   In class a formula was given relating α_1 to the variance of transitory and permanent shocks:

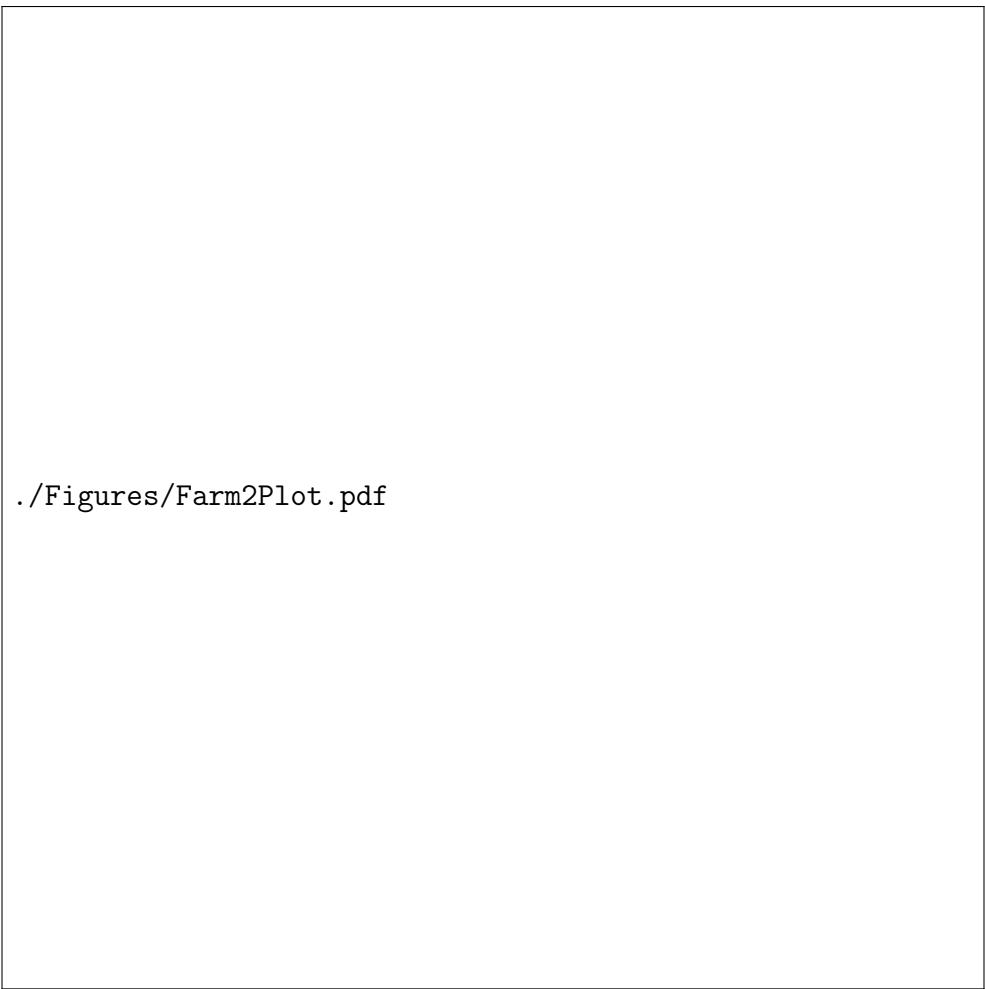 $$\alpha_1 = \sigma_p^2/(\sigma_p^2 + \sigma_\xi^2)$$

   where σ_p^2 is the variance of permanent income and σ_ξ^2 is the variance of transitory shocks to income. If σ_p^2 is the same for farmers and non-farmers, then because σ_ξ^2 is greater for farmers $\alpha_{1,\text{farmers}}$ should be less than $\alpha_{1,\text{workers}}$. However, the question provided no information about whether $\sigma^2_{p,\text{farmers}}$ is larger or smaller than $\sigma^2_{p,\text{workers}}$, so purely on the basis of the information provided in the question it is not possible to know whether $\alpha_{1,\text{farmers}}$ will be greater than or less than (or equal to) $\alpha_{1,\text{workers}}$.

   This point is vividly reflected in Figure 1, where the blue line represents the estimated consumption function for workers and red represents estimated consumption function for farmers. The σ_p^2 is assumed the same for farmers and non-farmers, but the variance of transitory shock differs:
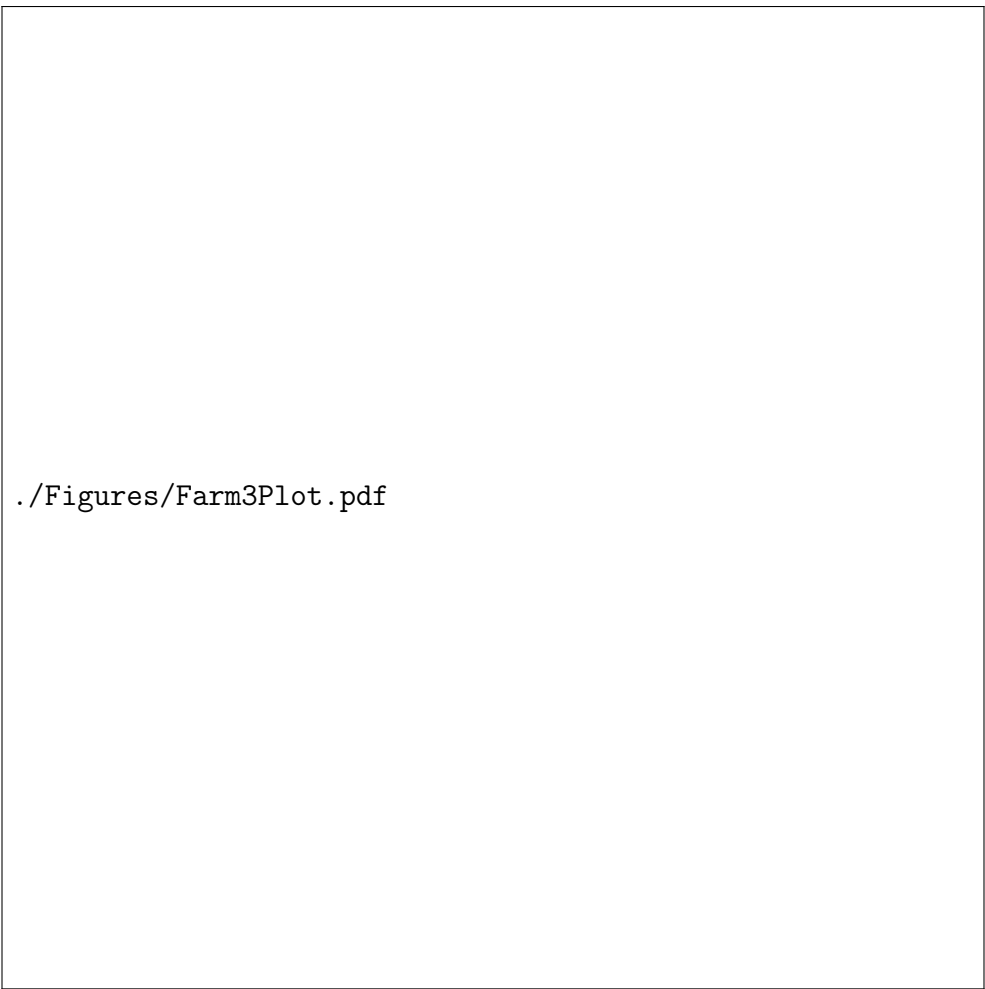
./Figures/Farm1Plot.pdf

Figure 1 Experiment 1

./Figures/Farm2Plot.pdf

Figure 2 Experiment 2

./Figures/Farm3Plot.pdf

Figure 3 Experiment 3

σ_ξ^2 for farmers is higher than the σ_ξ^2 for workers. The simulations confirm that the MPC for workers is larger than the MPC for farmers (i.e. the blueline is steeper).

The other figures similarly support the intuition behind Friedman's result.

2. Imagine that you observe a set of households for two consecutive periods, t and $t + 1$. What relationship should you find between the saving rate at time t and income growth between t and $t + 1$?

Using your Jupyter notebook again, simulate 50 observations of y_i for each of two consecutive periods t and $t+1$. Assume that any individual household's permanent income does not change from period t to $t + 1$.

For each household, calculate the simulated saving rate in period t, the growth rate of income between t and $t + 1$, and draw them on a graph (Hint: Put the saving rate in period t on the x-axis, and income growth between t and $t + 1$ on the y-axis. Your graph should consist of 50 points). Relate your results to the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?

Answer:

Think of a household which experienced a negative transitory shock in period t of amount ϵ, i.e. $y_{t,i} = \boldsymbol{p}_i - \epsilon$. That household would have a negative saving rate in period t, according to Friedman's PIH. Furthermore, in period $t + 1$ that household's income is expected to be $\mathbb{E}_t[y_{t+1,i}] = \boldsymbol{p}_i > y_{t,i} = \boldsymbol{p}_i - \epsilon_{t,i}$, so expected income growth is positive. A similar argument demonstrates that households with positive saving rates in period t should expect negative income growth between t and $t + 1$. The conclusion is that there should be a negative correlation between the saving rate in period t and the growth rate of income between t and $t + 1$.

The next figure plots the simulated saving rate and income growth rate in next period. Overall, we observe a negative correlation between the saving rate in period t and the growth rate of income between t and $t + 1$. A closer look at the figure reveals that this is not *always* the case, however. For instance, if the household after a negative transitory shock suffers from another more severe negative transitory shock, then the *realized* income growth may be negative, although the *expected* income growth is positive.

3. In his 1992 State of the Union address, President George H.W. Bush said that, in order to spur consumer spending, he was instructing the IRS to reduce the rate at which income taxes were withheld from taxpayers' wages. Income tax rates were not changed, only the timing of when consumers would pay those taxes. What effect might you expect this change in withholding to have had on consumption?

Figure 4 Simulated saving and income growth rate

Answer:

According to the PIH, changing withholding rates should have no effect on consumption if the total tax burden is unchanged. Any sensible definition of "transitory" versus "permanent" effects on income would have to classify a change in withholding schedules as having a "transitory" effect on income, since whatever the change in take-home pay is today, come April 15th that change will be exactly offset. One minor caveat: it is true that consumers now got to earn interest on the money that would have been in the government's hands if not for the change in the withholding rate, so perhaps a very small increase in consumption would be justifiable.

References

FRIEDMAN, MILTON A. (1957): *A Theory of the Consumption Function.* Princeton University Press.