

Name _____

Midterm Exam
Intertemporal Choice
Fall, 2014
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Analytical Question

Saving Rates and Interest Rates. **PerfForesightCRRA** shows that in a perfect foresight infinite horizon CRRA-utility consumption model, the fact that

$$\mathbf{c}_t = (h_t \mathbf{p}_t + \mathbf{b}_t) \kappa \quad (1)$$

implies that

$$\mathbf{c}_t \approx \overbrace{\mathbf{p}_t (1 - \mathbf{p}_\gamma / (r - \gamma))}^{\text{from human wealth}} + \overbrace{(r - \rho^{-1}(r - \vartheta)) \mathbf{b}_t}^{\text{from nonhuman wealth}} \quad (2)$$

where

$$\begin{aligned} h_t &= \text{The ratio of human wealth to permanent noncapital income } \mathbf{p}_t \\ \gamma &= \text{Growth rate of permanent noncapital income} \\ \mathbf{p}_\gamma &= \rho^{-1}(r - \vartheta) - \gamma \\ &\equiv \text{'growth impatience rate'} \end{aligned} \quad (3)$$

1. Using the fact that the approximate MPC in this model is $\kappa \approx r - \rho^{-1}(r - \vartheta)$, derive (2) from (1)

Answer:

This is implicitly done in **PerfForesightCRRA**.

2. Explain, in words, why the ‘return impatience condition’ $\mathbf{p}_r < 0$ needs to be imposed in order for the model to have a sensible solution. Explain, in words, why the ‘finite human wealth’ condition, $\gamma < r$ needs to be imposed. Explain, in words, what imposing the ‘growth impatience condition’ $\mathbf{p}_\gamma < 0$ accomplishes and why it might be desirable to impose that condition in an infinite-horizon model. (Reminder: $\mathbf{p}_r \equiv \rho^{-1}(r - \vartheta) - r$).

Answer:

These points are all explained in **PerfForesightCRRA**.

3. Use (2) and the fact that the level of saving can be defined as total income minus total consumption:

$$\mathbf{s}_t \approx r \mathbf{a}_{t-1} + \mathbf{p}_t - \mathbf{c}_t \quad (4)$$

to show that the ratio of saving to permanent labor income \mathbf{p}_t is approximately

$$s_t \approx \mathbf{p}_\gamma / (r - \gamma) + \rho^{-1}(r - \vartheta) a_{t-1} \quad (5)$$

and that the ratio of saving to total income (the ‘saving rate’) is

$$\hat{s}_t = \left(\frac{\mathbf{p}_\gamma / (r - \gamma) + \rho^{-1}(r - \vartheta) a_{t-1}}{1 + r a_{t-1}} \right). \quad (6)$$

Answer:

These derivations are in **PerfForesightCRRA**.

4. Now use (6) and the following calibration:

$$r = 0.08 \quad (7)$$

$$\vartheta = 0.04 \quad (8)$$

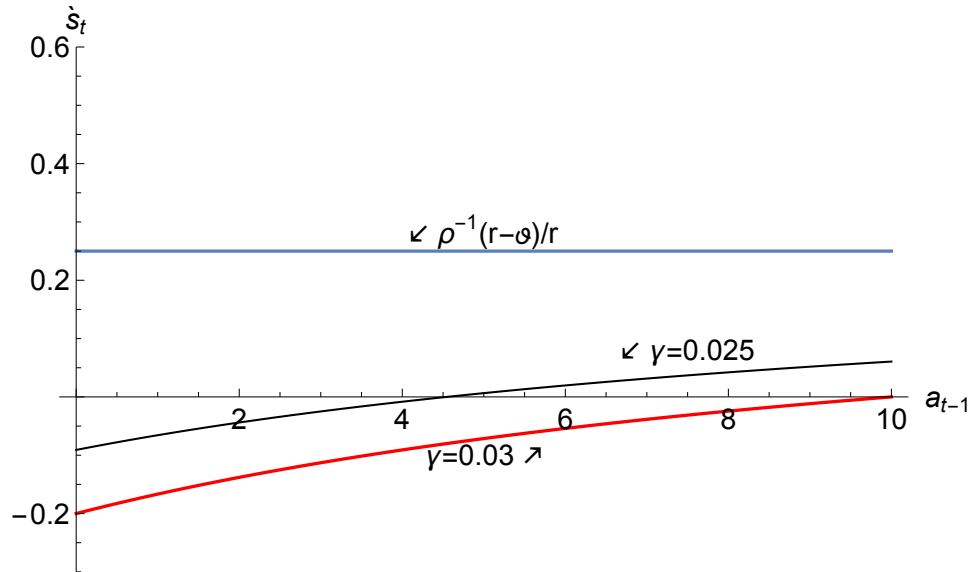
$$\rho = 2 \quad (9)$$

to calculate the saving rates at the following combinations of a_{t-1} and γ :

a_{t-1}	γ
0	0.025
0	0.030
10	0.025
10	0.030
∞	0.025
∞	0.030

and use the results to construct a graph showing the relationship between a_{t-1} and the saving rate for the range $a_{t-1} \in [0, 10]$, showing on the same graph the asymptote of the saving rate as $\lim_{a_{t-1} \rightarrow \infty}$.

Answer:



5. Opinions about the long-term growth rate of income, γ , are widely divergent today. Some scholars (e.g., Robert [Gordon \(2012\)](#)) believe that productivity growth in the U.S. is likely to be slow, perhaps 1 percent, over the next 50 years, while others (e.g., Erik [Brynjolfsson and McAfee \(2014\)](#)) are considerably more optimistic, projecting little if any slowdown from the growth rate of 2.5 percent a year that has characterized the postwar period. Furthermore, beliefs about long-run growth have become considerably more pessimistic in the period since the Great Recession.

- a) Explain why these facts cast doubt upon this model as a useful or reliable guide to understanding actual saving choices. Relate this point to the argument of [Summers \(1981\)](#) about the magnitude of the human wealth effect in perfect foresight models.

Answer:

Basically, the model says that the saving rate is insanely hypersensitive to the exact values of the various parameters and the relationship among them. If the model is taken (quantitatively) literally, it implies that even modest revisions in beliefs about long-run growth should result in massive shifts in saving rates. Or, minor differences across people in their beliefs about long-run growth should result in huge differences in their saving behavior. This is an illustration of the point made in class: Economists have gradually moved from merely asking their models to provide some *qualitative* guidance about what *kinds* of effects to expect to see, to expecting the models to have some *quantitative* plausibility. The perfect foresight model fails dismally on the quantitative side, not because it predicts the wrong quantity but because it is so sensitive to tiny calibration differences that it can predict almost anything.

- b) Explain why the saving rate is higher in a **TractableBufferStock** model than in the perfect foresight model

Answer:

The saving rate is higher in the model with risk because now the consumers have a *precautionary motive*. The impetus to save is not merely to smooth consumption across (an infinite number of) periods, but now also to provide a bulwark against catastrophe which takes the form of a transition into the unemployed state.

- c) Compare the degree of sensitivity of the saving rate to the growth rate of income in this model compared to the perfect foresight model. Give the intuition for any differences.

Answer:

The saving rate is relatively less responsive to the growth rate in the face of uncertainty than it is with perfect foresight. The intuition is that now changes in the growth rate have an additional effect on precaution. In the perfect foresight model, an increase in income growth results in less saving because the consumer no longer has to save as much to transfer the same amount of resources into the future – those resources are already there due to the up-tick in growth. However, in the buffer stock model this also means that the unemployed state is even scarier from the perspective of the employed – it is now a further fall, and this tends to increase the

precautionary motive. The presence of this countervailing force will tend to dampen the response of saving to changes in the growth rate.

Part II: Discussion Questions

1. **Secular Stagnation.** In 2013, Larry Summers delivered a widely discussed lecture at which he warned that the U.S. and European economies may face a risk that the next decade will be a period of “secular stagnation” like the one that has gripped Japan over the past 25 years. He proposed that a substantial increase in government spending could be an effective way to respond to this risk.

- a) If the extra spending advocated by Summers were directly in the form of transfers to households with a high marginal propensity to consume, can the argument described above be completely understood as an assertion that the U.S. is now in a state of “dynamic inefficiency?” (Hint: Mention Ricardian Equivalence in your answer).

Answer:

The economy is dynamically inefficient if the rate of return is less than the “natural” growth rate of the economy (usually, productivity growth plus population growth). If the rise in government spending resulted in a larger amount of government debt, which crowded out private capital (that is, Ricardian Equivalence fails – the transfers are spent by the high MPC consumers, rather than being saved), then the aggregate capital stock would lower and aggregate interest rates would be higher, moving the economy away from the dynamically inefficient state. Note that this is not just a static operation. Due to the presence of debt and GE effects this has intertemporal consequences; it is not just a shuffling around of resources *within* a period. Therefore it can address *dynamic* inefficiency.

- b) In fact, Summers did not advocate that the extra spending should all be directed to households who would spend it. He has advocated substantial increases in government investment in infrastructure, education, and other “investment” goods. Explain under what conditions this advice is what would be called for in a dynamically inefficient economy. Explain why this would or would not be an appropriate response.

Answer:

In an OLG model, an economy that is dynamically inefficient already has *too much* capital. If government investment in capital substitutes for private investment in capital (for example, if private firms would build toll roads if the government did not build free roads), the increase in capital from government investments might be counter-productive and drive the interest rate down further. If, however, government investment has the effect of boosting the productivity of private investment (for example, if investment in research spurs technological innovation), then more government investment might

help even a neoclassical economy move away from the dynamically inefficient region by boosting the interest rate.

- c) Elsewhere, [DeLong and Summers \(2012\)](#) have argued that when the economy is operating far below full employment, an increase in “aggregate demand” may be able to call forth a substantial increase in aggregate supply. This is essentially a Keynesian story. If [DeLong and Summers \(2012\)](#) are right, how would that relate to your answer to question (b) about dynamic inefficiency?

Answer:

If increased aggregate demand calls forth increased aggregate supply, the extra government spending could “pay for itself,” at least in part, even if the government capital substitutes (in part or in full) for private capital. The extra output would be a “bonus” that would further reinforce the case for increased government debt.

2. A recent literature stimulated by [Bloom \(2009\)](#) has argued that shocks to uncertainty are a major source of business cycle fluctuations; [Carroll, Slacalek, and Sommer \(2012\)](#) argue that a majority of the increase in the U.S. personal saving rate during the Great Recession reflected increases in labor income uncertainty. Explain, using both math and words, why this argument cannot be investigated using a model in which a representative consumer with quadratic utility makes optimal consumption choices.

Answer:

3. **Fisherian Separation ([Fisher \(1930\)](#))**. Explain what is meant by the “Fisherian separation” proposition, then explain how relaxing each of the following assumptions in the Fisher model might undermine the proposition:
- a) Liquidity constraints
 - b) Uncertainty
 - c) Failure to optimize intertemporally
 - d) Time inconsistent preferences

Answer:

Fisherian separation holds when the profile of consumption is not dependent on the profile of income. In other words, the timing of consumption is independent of the timing of income (up to PDV constraints).

- a) If there are liquidity constraints, then a dollar today and a dollar tomorrow are not equivalent. I could use the dollar today directly; but because of liquidity constraints I may be unable to borrow money based on future income and so my consumption is smaller. Thus not only the PDV of income matters for consumption, the timing does as well.

- b) Because consumers are risk averse, uncertain income in the future is not equivalent to income today for planning purposes. I will tend to save more and consume less when my income is in the risky future, and consume more and save less when my income is in the present, so the timing of income is important.
- c) If consumers fail to optimize intertemporally and are simple Keynesians, spending nine tenths of current income, then the timing of income will influence the timing of consumption directly.
- d) With time inconsistent preferences, if the consumer receives more income today she may not be able to help herself and consume more of it than she would have had she received the income later in life; thus not only the PDV of income matters.

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