Name _____

Midterm Exam

Intertemporal Choice

Fall, 2017

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Question

Stimulative Effects of Transitory Versus Permanent Tax Cuts. Consider a buffer stock model like the one presented in class but with a permanent 'payroll' wage tax τ so that the employed consumer's income is wage income times the untaxed proportion $\not\tau$. This model relates the ratio of consumption to permanent income, $c = C/(\not\tau \mathsf{W})$, to the ratio of market resources to permanent income $m = M/(\not\tau \mathsf{W})$. Use this model to rank the relative magnitudes (from largest to smallest) of the effects on consumption you would expect from each of the following policies: (1) A transfer of size X targeted to people with large values of m (so the person receives income of $\not\tau \mathsf{W}+X$); (2) a permanent tax cut targeted to people with large values of m; (3) a transitory transfer (as described above) targeted to people with small values of m; (4) a permanent tax cut targeted to people with small values of m. (Your answer will be graded not just on whether you rank the four options correctly, but also on how well you *explain why* you obtain the answer you do. A diagram or two may be helpful in explaining your reasoning).

Answer:

The model implies that the marginal propensity to consume out of permanent changes in income will be fairly high (close to one) for both rich and poor households. However, it implies a much higher MPC out of transitory shocks for poor than for rich households; this is an implication of the concavity of the consumption function.

The correct ordering is (4),(2),(3),(1). The only bit that could possibly be in doubt is whether (4)<(2) but it is certainly true in the limit as the 'poor' person approaches $m = 0$ because for that person the MPC approaches 1 and it will be less than 1 for someone with larger m. (You could get full credit if you said that (4) versus (2) was ambiguous).

Part II: Long Question

Rational Inattention and Consumption Dynamics.

Reis (2006) argues that the 'excess smoothness' of aggregate consumption can be understood using a model in which consumers are 'rationally inattentive': Because they face costs of obtaining the information needed to make a perfect decision, they (optimally) only 'do the research' needed to set the level of c occasionally.

For a continuous-time consumer with CARA utility $\mathrm{u}(c) = -(1/\alpha)e^{-\alpha c}$ who faces an information acquisition cost K, Reis shows that it will be optimal to adjust c only at fixed intervals of length d. If the consumer's time preference rate is equal to the interest rate, c will remain constant between these adjustment dates. Designating the level of c as a function of wealth o, the information cost as K, and the variance of shocks to permanent income as σ^2, Reis shows that at dates of adjustment the consumption function is

$$\mathrm{c}(o;K) \;=\; \mathrm{c}(o;0) - \left(\frac{\mathsf{r}K}{e^{\mathsf{r}d}-1}\right) - \left(\frac{\alpha \mathsf{r}\sigma^2}{4}\right)\left(e^{\mathsf{r}d}-1\right) \tag{1}$$

and that the length of the intervals of inattentiveness (that is, the intervals during which c does not adjust) is

$$d \;=\; \left(\frac{1}{\mathsf{r}}\right)\log\left(1+\sqrt{\frac{4K}{\alpha\sigma^2}}\right). \tag{2}$$

1. Provide an interpretation for each of the two terms that equation (1) says must be subracted from the frictionless consumer's consumption function for a consumer who faces information-gathering costs K; be sure to discuss why each term takes the form it does. Hint: K is a real monetary expenditure that the consumer must pay in each period of adjustment.

   Answer:

   a) The $\left(\frac{\mathsf{r}K}{e^{\mathsf{r}d}-1}\right)$ term reflects the fact that the consumer is paying a cost K on a regular basis. The consumer therefore cannot afford to do as much spending on items other than the information gathering process. (Think of this as being like the fees one might pay to a personal financial advisor).

   b) The $\left(\frac{\alpha\mathsf{r}\sigma^2}{4}\right)\left(e^{\mathsf{r}d}-1\right)$ term reflects the precautionary motive induced by the fact that the consumer's c will deviate from the optimal level during intervals when news has arrived but not been processed. This increases the degree of uncertainty about future c, which is why the term depends on the risk aversion parameter α: A consumer who is not risk averse does not care about the increased uncertainty of his c and does not adjust spending as a consequence of the extra risk. The magnitude of the increase in uncertainty is measured by the $e^{\mathsf{r}d}-1$ term.

2. Give intuitive explanations for the sign of the effects of r, K, α, and σ in (2).

   Answer:

   a) r: When interest rates are low, the wealth penalty for errors in the level of c (which is the accumulated interest on the erroneous expenditures) is small. Therefore the consumer is willing to wait a longer time between adjusting.

   b) K: The higher is the information cost, the longer the consumer waits between periods of adjustment, in order to minimize the total amount spent on acquiring information

   c) α and σ^2: These jointly measure the size of the precautionary motive; with a higher degree of risk aversion or a larger amount of risk, the consumer will adjust more frequently.

3. Now consider an entire economy populated by inattentive consumers of this kind. Reis shows that if consumers' decision dates are randomly distributed in the population, and the maximum length of an inattentiveness interval is D, then

$$\mathbb{E}_{t-D}[C_{t+1} - C_t] \quad = \quad \text{constant.} \tag{3}$$

   Provide an intuition for this equation, and explain how it relates to the Hall (1978) model. Contrast this result with the predictions of a model of sticky expectations or habit formation.

   Answer:

   Hall's model is a special case of Reis's model with costs of adjustment of 0 so that c adjusts at every instant. In Reis's model, consumers cannot predict whether they will be adjusting their c up or down at the next adjustment date; so if we take expectations from a period longer in the past than the maximum interval of inattentiveness of any consumer, it must be the case that the changes in c of every consumer are unpredictable with respect to information available at that past date. If each consumer's changes are unpredictable, aggregate c changes must be unpredictable.

   In the model of sticky expectations presented in class, a fraction of consumers updates their information exogenously in each period. But even after an arbitrarily long time interval, there will still be a few consumers who have not adjusted their expectations. Hence the random walk proposition does not hold even over long stretches of time. (Of course, in practice if the interval is long enough, the fraction of consumers who have not adjusted will get so small as to be undetectable).

   In the model of habit formation, dynamics of c were similar to those in the sticky expectations model, and again there is no time interval long enough to eliminate all predictability in c growth.

4. Reis shows that even a small cost of obtaining information can produce long intervals of optimal inattention. What is the key intuition for why the cost of remaining inattentive is likely to be small if c is set optimally during the brief instants of attention?

Answer:

Since c at the instants of attention is set to the optimal level, the deviations of optimal c from actual c are likely to be quite small for a long time; this is an implication of the general point that the costs of small deviations of c from its optimum are second-order small: $\mathrm{u}(c) \approx \mathrm{u}(c^*)+\mathrm{u}'(c^*)(c-c^*)+\mathrm{u}''(c^*)((c-c^*)^2/2)$, but the Envelope theorem says that the marginal utility cost of changing c a little bit at the optimum is zero, so the cost of deviations will be on the order of $\mathrm{u}''(c^*)((c-c^*)^2/2)$.

The interval of adjustment that would be required for this model to explain the 'excess smoothness' facts is roughly $d = 1$. Now we want to calibrate the model, to see if $d = 1$ is a plausible implication of the model.

a) Solve (2) for K under the assumption that $d = 1$

Answer:

$$\begin{aligned} e^{\mathsf{r}d} - 1 &= \sqrt{\frac{4K}{\alpha\sigma^2}} \\ \sigma\mathsf{r}\sqrt{\alpha/4} &\approx \sqrt{K} \\ K &\approx (\alpha/4)\sigma^2\mathsf{r}^2 \end{aligned}$$

b) Explain why, in a model in which the level of c has been normalized to 1, for fluctuations around $c = 1$ the CARA model can be interpreted as an approximation to a CRRA model with $\alpha = \rho$. Since $\rho = 4$ is plausible in the general model, we can conveniently calibrate $\alpha = 4$.

Answer:

Relative risk aversion is absolute risk aversion times the level of c. If $c = 1$ the two are (locally) the same.

c) Now we want to calibrate σ. Suppose that permanent income $\mathbf{p} = pP$ where p is the person-specific component of permanent income (did I just get fired? promoted? did I change jobs) and P is the aggregate component (what is the aggregate level of GDP per capita? the aggregate wage?). Shocks to both p

and P are unpredictable: In a discrete-time version of the model,

$$p_{t+1} = p_t \psi_{t+1} \quad (4)$$
$$P_{t+1} = P_t \Psi_{t+1} \quad (5)$$

so that

$$\boldsymbol{p}_{t+1} = \boldsymbol{p}_t \psi_{t+1} \Psi_{t+1} \quad (6)$$

and in this case it can be shown that the variance of total shocks to the log of permanent income is the sum of the variances of the idiosyncratic and the aggregate shocks:

$$\sigma^2_{\log \Delta \mathbf{p}} = \sigma^2_{\log \psi} + \sigma^2_{\log \Psi} \quad (7)$$

Considering the two possibilities below,

i. **total ignorance** The consumer has no ability to distinguish idiosyncratic from aggregate shocks, and observes only the total $\mathbf{p}$; in this case the appropriate choice would be to set $\sigma = \sigma_{\log \Delta \mathbf{p}} = \sqrt{\sigma^2_\psi + \sigma^2_\Psi}$

ii. **aggregate ignorance** The consumer can observe perfectly the idiosyncratic component of $\boldsymbol{p}$ ('oops, I just got fired') but not the aggregate component, which can only be observed by paying the cost K; in this case the appropriate choice would be to set
$\sigma = \sqrt{\sigma^2_\Psi} = \sigma_\Psi$

discuss the plausibility of the values of K that arise for a reasonable calibration of interest rates (say, $\mathsf{r} = 0.02$), and using the empirical fact that plausible calibrations for the variances of the permanent shocks are $\sigma^2_\psi = 0.01$ and $\sigma^2_\Psi = 0.0001$. That is, the variance of the idiosyncratic shocks is about 100 times larger than the aggregate ones. (Hint 1: In our normalized model, the absolute size of K is the number of years' worth of consumption required to pay for the information about the true state of $\boldsymbol{p}$; Hint 2: you may find it easier to do your calculations if you assume that annual income is, say, \$100,000 which will help some of the tiny numbers to come into perspective)

Answer:

Calibrating with $\mathsf{r} = 0.02$ and $\alpha = 4$ simplifies the solution above to

$$K \approx \sigma^2 \mathsf{r}^2$$

Supposing a typical income of \$100000, for $\mathsf{r}^2 = 0.0004$, if σ were 1 we would have $K = 0.0004 \times 100000 = \$40$

Ignorance level	Cost	proportion of income	dollars
aggregate:	$K =$	$0.0004\ 0.0001 = 0.0000004$	$= 0.004$
total:	$K =$	$0.0004\ 0.01 = 0.00004$	$= 0.40$

That is, to obtain the desired aggregate smoothness under the 'aggregate' assumption, the cost of obtaining the information has to be

about 4 tenths of a cent; to obtain the desired aggregate smoothness under the 'total' assumption, the cost has to be about 40 cents.

5. Now suppose that the K that you extracted from the prior question which yielded $d = 1$ is a plausible guess about the true cost of obtaining the necessary information. Use equation (2) to discuss how sensitive the duration of adjustment should be to interest rates. Critique this implication of the model using the fact that the degree of 'excess smoothness' seems to be fairly stable across times and countries.

   Answer:

   The model says that the duration between adjustment should be extremely sensitive to interest rates; indeed, as interest rates go to zero, the duration approaches infinity. Interest rates have varied widely across countries and time periods; if this model were right, we should have expected different levels of interest rates to have resulted in very different degrees of excess smoothness of consumption. Since we do not see that, this implication of the model is wrong.

References

HALL, ROBERT E. (1978): "Stochastic Implications of the Life-Cycle/Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 96, 971–87, Available at http://www.stanford.edu/~rehall/Stochastic-JPE-Dec-1978.pdf.

REIS, RICARDO (2006): "Inattentive Consumers," *Journal of Monetary Economics*, 53(8), 1761–1800.