Name ______________________

Midterm Exam

Intertemporal Choice

Fall, 2015

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I

Consumption, Saving, and the Great Recession.

In the U.S. during the Great Recession, household consumption expenditures fell even more than household income, leading to a sharp rise in the saving rate. This question explores potential explanations within the context of the partial equilibrium/small-open-economy model described in PerfForesightCRRA, which provides an approximate formula for consumption that can be rewritten as:

$$\boldsymbol{c}_t \quad \approx \quad \mathbf{p}_t\left(1 - \text{þ}_\gamma/(\mathsf{r}-\gamma)\right) - \text{þ}_{\mathsf{r}} b_t$$

where noncapital income is $\boldsymbol{p}_t$, and

$$\text{þ}_\gamma \quad \equiv \quad \rho^{-1}(\mathsf{r}-\vartheta) - \gamma$$

is the 'growth patience' rate while the 'return patience' rate is

$$\text{þ}_{\mathsf{r}} \quad \equiv \quad \rho^{-1}(\mathsf{r}-\vartheta) - \mathsf{r}.$$

1. Briefly:
   a) Explain in words why we need to impose $\text{þ}_{\mathsf{r}} < 0$
   b) Explain in words why we need to impose $\gamma < \mathsf{r}$
   c) Explain in words why we might want to impose $\text{þ}_\gamma < 0$

   Answer:

   These questions are answered in PerfForesightCRRA

2. Capital income is the interest rate multiplied by the amount of financial assets, so the saving rate (income minus consumption, divided by income) is approximately:

$$s_t \quad \approx \quad \left(\frac{\mathsf{r}\boldsymbol{b}_t + \boldsymbol{p}_t - \boldsymbol{c}_t}{\mathsf{r}\boldsymbol{b}_t + \boldsymbol{p}_t}\right). \tag{1}$$

   Assuming various quantities are 'small' in convenient ways,[1] for people whose financial wealth ratio $b_t \equiv \boldsymbol{b}_t/\boldsymbol{p}_t$ is small (that is, for most people), show that the saving rate will be approximately:

$$s_t \quad = \quad \left(\frac{\text{þ}_\gamma}{\mathsf{r}-\gamma}\right) + \text{þ}_{\mathsf{r}} b_t. \tag{2}$$

   Interpret (2) *in words*, explaining why each component has the effect that it does.

   Answer:

   Divide both sides of (1) by $\boldsymbol{p}_t$ then calculate the saving rate and approximate it (using MathFactList [OverPlus]: $\bullet/(1+\mathsf{r}b_t) \approx \bullet(1-\mathsf{r}b_t)$):

$$c_t \quad \approx \quad \left(1 - \text{þ}_\gamma/(\mathsf{r}-\gamma)\right) + (\mathsf{r} - \rho^{-1}(\mathsf{r}-\vartheta))b_t \tag{3}$$

[1] Warning: Some of the required assumptions will be bold ones that are not easy to justify.

$$
\begin{aligned}
s_t &\approx \left(\frac{1 + \mathsf{r} b_t - \left[\left(1 - \text{þ}_\gamma / (\mathsf{r} - \gamma)\right) + (\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)) b_t \right]}{1 + \mathsf{r} b_t} \right) && (4) \\
&\approx 1 - \left((1 - \text{þ}_\gamma / (\mathsf{r} - \gamma)) - \text{þ}_\mathsf{r} b_t \right) && (5) \\
&= \left(\frac{\text{þ}_\gamma}{\mathsf{r} - \gamma} \right) + \text{þ}_\mathsf{r} b_t && (6) \\
&= \left(\frac{\rho^{-1}(\mathsf{r} - \vartheta) - \gamma}{\mathsf{r} - \gamma} \right) + \text{þ}_\mathsf{r} b_t && (7)
\end{aligned}
$$

and the difficult-to-justify approximation is that the numerator in equation (4) is 'close enough' to 1 that the interaction term $-\left[\left(1 - \text{þ}_\gamma / (\mathsf{r} - \gamma)\right) + (\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)) b_t \right] \mathsf{r} b_t$ can be neglected (since both $[\bullet]$ and $\mathsf{r} b_t$ are 'small.' This approximation was made by a trained professional under carefully controlled laboratory conditions; do not try it at home (or in a research-strength manuscript).

In words: The imposition of the growth impatience condition $\text{þ}_\gamma < 0$ implies that the first term's numerator is negative; and the finite human wealth condition implies that the denominator is positive. Thus, the first term captures the effects on the saving rate of the degree of 'growth impatience' (the numerator) and of human wealth (the denominator).

An increase in growth impatience (say, from an increase in ϑ) makes the numerator a larger negative number and thus amplifies the term's (negative) contribution to the saving rate.

The effect of a change in growth on saving that comes through this term is given by the derivative of the term with respect to γ. Rewriting and differentiating the first term as

$$
\left(\frac{d (\text{þ} - \gamma)(\mathsf{r} - \gamma)^{-1}}{d\gamma} \right) = \left(\frac{-(\mathsf{r} - \gamma) - (\text{þ} - \gamma)}{(\mathsf{r} - \gamma)^2} \right) \qquad (8)
$$

the magnitude is easiest to assess in the special case where the consumer is 'growth poised' ($\text{þ}_\gamma = 0$). Take plausible values $\mathsf{r} = 0.05$ and $\gamma = 0.02$; this says that a 1 percentage point increase in expected growth will decrease the saving rate by $0.01/(\mathsf{r} - \gamma) \approx 0.33$ (33 percentage points). The effect is even larger for consumers who are growth impatient rather than poised.

The second term captures the effects of financial wealth analogously, but the relevant reference point is the *return* impatience rate. A return impatient consumer wants to consume more than the amount yielded by the return on the asset, and thus has a negative þ_r. The larger and more negative, the greater the dissaving rate out of assets. The effect is proportional to actual bank balances, because the wealthier the consumer is today the more she can dissave.

3. $\texttt{PerfForesightCRRA}$ shows that b_t gets very large, the saving rate approaches

$$s = \left(\frac{\rho^{-1}(\mathsf{r}-\vartheta)}{\mathsf{r}}\right). \tag{9}$$

a) Explain (in words as well as math) why the saving rate for very (financially) rich people does not depend on þ_γ or on b.

Answer:

They are so rich that labor income hardly has an effect on their behavior; hence their relative patience for future labor income þ_γ does plays a negligible role in determining their behavior. The level of saving is linearly related to bank balances, and the level of income is dominated by the size of bank balances (also linearly); in the ratio the two will cancel out as b gets large. As noted, the mathematical derivation is located in $\texttt{PerfForesightCRRA}$.

b) Show that, for these people, the response of the saving rate to the interest rate is

$$\left(\frac{ds}{d\mathsf{r}}\right) = \rho^{-1}\vartheta\mathsf{r}^{-2}, \tag{10}$$

and, for plausible parameter values, describe what this implies about the likely nature *and magnitude* of the response of the saving rate to changes in the interest rate.

Answer:

If we consider almost any plausible configuration of parameter values, say $\mathsf{r} = \vartheta = 0.05$ and $\rho = 2$, this translates to a very large elasticity of the saving rate with respect to interest rates (in the case of the parameter values mentioned above, $(1/2)(20) = 10$.)

4. Good economists do not want to explain the increase in the saving rate during the Great Recession by saying something silly like 'suddenly everyone just become more patient' or 'everyone's ρ parameters suddenly changed.' What do these formulas suggest are the only three other potential explanations? Explain *in words as well as math* how the changes in each of these *ceteris paribus* would affect saving *rates* of the very (financially) rich (with b approaching ∞) and of the non-rich (with reasonably 'small' values of $\mathsf{r}b_t$). Explain what kind of microeconomic data you would ideally like to have in order to test each of these ideas.

Answer:

The obvious answer to the kind of microeconomic data you would want is that you would want data on saving rates for people with very different levels of b_t. A less obvious answer is that you would like to have data on people's expectations about interest rates and income growth rates. And, of course, it would be very useful to see whose values of nonhuman wealth b_t changed and how their saving behavior changed. The three objects whose value the theory reasonably might allow to have changed are beliefs about γ, beliefs about r, and the level of financial assets b_t:

a) γ: A decline in the expected growth rate of labor income, γ holding interest rates fixed, would have essentially no effect on the saving rate of the very rich because they expect to finance essentially none of their consumption out of future labor income. However, for the nonrich, the 'human wealth effect' of a change in expected income growth can be very large. So, a decline in expected future income growth could help to explain a decline in saving by the nonrich.

b) b_t: For the financially rich, the saving *rate* is not much affected by the level of b (which is why it does not appear in (9)). And for the people for whom b_t is small ('the poor'), their saving rate is also not much affected by changes in b_t. However, if there are people whose value of b_t is moderate, they might cut their consumption substantially in response to a big decline in wealth, which could result in an increase in the saving rate for those people.

c) r: For both the rich and the nonrich, the saving rate is highly sensitive to the interest rate r.

  - For the nonrich (approximate by $b_t = 0$), the main reason is the human wealth effect. An increase in r reduces the size of their human wealth, and therefore their consumption, but (because they have little financial wealth) has little effect on their (interest) income and none on their labor income. With about the same income but less consumption, their saving rate rises.
  - For the rich, the main reason is that, while their spending goes up with higher r, their consumption goes up by even more, so the saving *rate* rises.

5. In practice, the explanations available from the formulas above are not very satisfying. In particular, interest rates seem to have moved in the wrong direction for the theories, and movements in the other explanators do not seem very persuasive. Of course, the perfect foresight, perfect-capital-markets model of `PerfForesightCRRA` leaves out two categories of explanation: Uncertainty, and changes in capital market conditions (e.g., credit availability).

   a) Explain intuitively, and using the consumption Euler equation, how an increase in the degree of uncertainty about future consumption could increase the saving rate.

   Answer:

   The consumption Euler equation is $u'(c_t) = \mathsf{R}\beta\,\mathbb{E}[u'(c_{t+1})]$. Since u' is a convex function, its expectation will rise as the variance of its argument rises. A larger RHS implies that the LHS, $u'(c_t)$ must also rise; this happens via a fall in c_t. For a fixed income today, this implies a rise in the saving rate.

b) Explain intuitively why restrictions on people's ability to borrow might change the implications of the theories for either or both categories of people discussed above.

Answer:

For the rich, they are not borrowing anyway and so their consumption and saving decision is not affected. The non-rich may be borrowing financially against their human wealth (most of which lies in the future); hence their consumption will likely be constrained by contemporaneous labor income, and will therefore fall.

c) Discuss what kinds of data you might want to obtain in order to test the proposition that either greater uncertainty or capital market imperfections could explain the change in saving behavior.

Answer:

Ideally we would have access to consumer financial data (bank balances, interest rates, debt, etc), as well as credit ratings (which help determine how tightly the credit constraints bind) and survey measures of how uncertain consumers think their future income stream is.

Part II

Dynamic Inefficiency and the Capital Share Coefficient in an OLG Model.
Consider a Diamond (1965) OLG economy like the one in the handout `OLGModel`, assuming logarithmic utility and a Cobb-Douglas aggregate production function,

$$Y = \mathrm{F}(K, PL) \tag{11}$$

where P is a measure of labor productivity that grows according to

$$P_{\tau+1} = \mathsf{G}P_{\tau}. \tag{12}$$

Population growth is zero ($\Xi = 1$; for convenience normalize the population at $L_\tau = 1 \; \forall \; \tau$), and until date t productivity growth has occurred at the rate $\mathsf{g} > 0$ (equivalently, $1 + \mathsf{g} = \mathsf{G} \geq 1$) forever. Under these assumptions, it can be shown that the dynamic process for aggregate $k \equiv K/PL$ is

$$k_{\tau+1} = \left(\frac{(1-\alpha)\beta}{\mathsf{G}_{\tau+1}(1+\beta)}\right) k_\tau^\alpha \tag{13}$$

1. Derive the steady-state level of k_τ that the economy will have achieved by date t if the rate of productivity growth has always been $\mathsf{G}_\tau = \mathsf{G} \; \forall \; \tau$.

   Answer:

   In the steady state $k_{\tau+1} = k_\tau = \bar{k}$. Substituting into equation (13):

$$\bar{k} = \bar{k}^\alpha \left[\frac{(1-\alpha)\beta}{\mathsf{G}(1+\beta)}\right] \tag{14}$$

$$\bar{k}^{1-\alpha} = \left[\frac{(1-\alpha)\beta}{\mathsf{G}(1+\beta)}\right] \tag{15}$$

$$\bar{k} = \left[\frac{(1-\alpha)\beta}{\mathsf{G}(1+\beta)}\right]^{1/(1-\alpha)} \tag{16}$$

   Now suppose that, after an eternity of remaining in the steady state, all of a sudden at the beginning of period t everybody learns that henceforth and forever more, the exponent on capital in the production function will change to $\hat{\alpha} > \alpha$.

2. Define the new steady-state as $\bar{\hat{k}}$. Will this be larger or smaller than the original steady state $\bar{k}$? *Explain your answer.*

   Answer:

   Note first that for $0 < \alpha < 1$, increasing α makes $1-\alpha$ a smaller number and therefore makes $1/(1-\alpha)$ a larger number. So, whether $\bar{\hat{k}}$ rises when this happens depends on whether the term in [] is greater or less than 1.

   But both $(1-\alpha)$ and $\frac{\beta}{1+\beta}$ are less than one and we assume $\mathsf{G} \geq 1$, so the term in brackets is certainly less than one. Exponentiating it to a larger value will make the result smaller.

The reason is that, with logarithmic utility, the increase in the marginal product of capital (and hence the incentive to save) has a net zero effect on consumption, as the income and substitution effects offset each other. However, as labor's share has dropped, the reduction in wages leads to lower income for the young. The old spend all their income, so all aggregate saving comes from the young. With lower saving by the young and unchanged saving by the old, the total amount of saving declines, resulting in a lower capital to output ratio.

3. Next, use a diagram to show how the $k_{\tau+1}(k_\tau)$ curve changes when the new α takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period t.

   Answer:

   Defining the original steady-state capital stock as $\bar{k}$ and the new steady-state capital stock as $\bar{\hat{k}}$, the convergence process looks as indicated in the figure in `OLGModel`.

4. Define an index of aggregate consumption per efficiency unit of labor in period τ as $\chi_\tau = c_{1,\tau} + c_{2,\tau}/\mathsf{G}$, and derive a formula for the sustainable level of χ associated with a given level of k.

   Answer:

$$
\begin{aligned}
K_{\tau+1} &= K_\tau + K_\tau^\alpha P_\tau^{1-\alpha} - C_{1,\tau} - C_{2,\tau} && (17)\\
\left(\frac{K_{\tau+1}}{P_\tau}\right) &= k_\tau + k_\tau^\alpha - c_{1,\tau} - \frac{c_{2,\tau} P_{\tau-1}}{P_\tau} && (18)\\
\left(\frac{K_{\tau+1}}{P_{\tau+1}} \frac{P_{\tau+1}}{P_\tau}\right) &= k_\tau + k_\tau^\alpha - c_{1,\tau} - c_{2,\tau}/\mathsf{G} && (19)\\
k_{\tau+1}\mathsf{G} &= k_\tau + k_\tau^\alpha + \chi_\tau && (20)
\end{aligned}
$$

   The sustainable level of χ is the level $\bar{\chi}$ such that $k_{\tau+1} = k_\tau = \bar{k}$:

$$
\begin{aligned}
(1+\mathsf{g})\bar{k} &= \bar{k} + \bar{k}^\alpha - \bar{\chi} && (21)\\
\bar{\chi} &= \bar{k}^\alpha - \mathsf{g}\bar{k}. && (22)
\end{aligned}
$$

5. Derive the conditions under which a marginal increase in α will result in an increase in the steady-state level of χ, and explain in words why this result holds.

   Answer:

   First, if $\bar{k}$ is the steady state level of capital, (22) implies

$$
\begin{aligned}
\bar{\chi} &= f(\bar{k}) - \mathsf{g}\bar{k} && (23)\\
\frac{d\bar{\chi}}{d\alpha} &= \left(\frac{\partial f(\bar{k})}{\partial k} - \mathsf{g}\right)\frac{d\bar{k}}{d\alpha} && (24)
\end{aligned}
$$

$$\frac{d\bar{\chi}}{d\alpha} = (\bar{\mathsf{r}} - \mathsf{g})\frac{d\bar{k}}{d\alpha}. \tag{25}$$

We know from the question above that if $\mathsf{G} \geq 1$, then $\frac{d\bar{k}}{d\alpha}$ is always strictly negative. Hence, the sign of the total derivative depends on whether or not $\bar{\mathsf{r}} < \mathsf{g}$.

If $\bar{\mathsf{r}} < \mathsf{g}$ holds, in which case the economy is dynamically inefficient, then the reduction in $\bar{k}$ that accompanies a rise in α (which we derived above) is a Pareto improvement and the steady state level of consumption rises. If the economy was *not* dynamically inefficient, then a reduction in the steady state level of capital will result in a reduction in the steady state level of consumption. For more intuition on dynamic inefficiency see the `OLGModel`.

References

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150, http://www.jstor.org/stable/1809231.