A Behavioral New Keynesian Model

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Abstract

This paper presents a framework for how to analyze how bounded rationality affects monetary and fiscal policy. The model is a tractable and parsimonious enrichment of the widely-used New Keynesian model – with one main new parameter, which quantifies how poorly agents understand future policy and of its impact. That myopia parameter in turn affects the power of monetary and fiscal policy in a microfounded general equilibrium.

A number of consequences emerge. First, fiscal stimulus is powerful, and, indeed, can pull the economy out of the zero lower bound. More generally, the model allows analyzing jointly optimal monetary and fiscal policy. Second, the model helps solve the "forward guidance puzzle," the fact that in the rational model, shocks to very distant rates have a very powerful impact on today's consumption and inflation: because the agent is de facto myopic, this effect is muted. Third, the zero lower bound is much less costly than in the traditional model. Fourth, even with passive monetary policy, equilibrium is determinate, whereas the traditional rational model generates multiple equilibria, which reduce its predictive power. Fifth, the model is also "neo-Fisherian" in the long run, but Keynesian in the short run – something that has proven difficult for other models to achieve: a permanent rise in the interest rate decreases inflation in the short run and increases it in the long run. The non-standard behavioral features of the model seem warranted by the empirical evidence.

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1 Introduction

The New Keynesian (NK) agent is a rational agent in a world where firm prices are sticky. Here, I explore the consequences of making this agent, or firms, more behavioral. The agent is partially myopic to "unusual events" and doesn't anticipate the future perfectly. The formulation takes the form of a parsimonious amendment of the traditional model that allows to analyze monetary and fiscal policy. This has a number of strong consequences for aggregate outcomes:

1. Forward guidance is much less powerful than the traditional model, resolving the "forward guidance puzzle".

2. Fiscal policy is much more powerful: in the traditional model, rational agents are Ricardian and don't react to tax cuts. In the behavioral, agents are partly myopic, and consume more when receiving tax cuts.

3. The zero lower bound is much less costly.

4. Equilibrium selection issues vanish in many cases: for instance, even with a constant nominal rate there is just one (bounded) equilibrium.

5. A number of neo-Fisherian paradoxes vanish. A permanent rise in the nominal interest rates makes inflation falls in the short run (a Keynesian effect), and rise in the long run (so that long-run Fisher neutrality holds with respect to inflation).

In addition, there is, I will argue below that there is reasonable empirical evidence for the "non-standard" main features of the model.

Fiscal policy. In the traditional NK model, agents are fully rational. So Ricardian equivalence holds, and tax cuts have no impact. So, fiscal policy (lump-sum taxes, as opposed to government expenditure) has no impact. Here, in contrast, the agent isn't Ricardian because he doesn't anticipate future taxes well. As a result, tax cuts and transfers are unusually stimulative, particularly if they happen in the present. As the agent is partially myopic, taxes are best enacted in the present.

Equilibrium determinacy. When agents are boundedly rational enough, there is just one (bounded) equilibrium. This contrasts with the traditional model, which has a continuum of (bounded) equilibria. Hence, the response to a simple question like "what happens when the interest rates are kept constant" is well-defined – it is not mired in morass of equilibrium selection. [1]

[1] However, I do need to rule out "bubble" and explosive equilibria.

Forward guidance. With rational agents, "forward guidance" by the central bank is predicted to work very powerfully, probably too much so, as emphasized by Del Negro, Giannoni, Patterson (2015) and McKay, Nakamura and Steinsson (forth.). The reason is that the traditional consumer unflinchingly respects his Euler equation and expects other agents to do the same, so that a movement of the interest rate far in the future has a strong impact today. However, in the behavioral model I put forth, this impact is muted by the agent's myopia, which makes forward guidance less powerful. The model, in reduced form, takes the form of a "discounted Euler equation," where the agent reacts in a discounted manner to future consumption growth.

Zero lower bound (ZLB). Depressions due to the ZLB are moderate and bounded, even though they are unboundedly large in a rational model (Werning 2012).

A number of neo-Fisherian paradoxes vanish. A number of authors, especially John Cochrane (2015), highlight that in the strict (rational) NK model, a rise in interest rates (even temporary) leads to a rise in inflation (though this depends on which equilibrium is selected, leading to some cacophony in the dialogue). They call that a "neo-Fisherian" property. This is true, in this model, in the long run: the long-run real rate is independent of monetary policy (Fisher neutrality holds in that sense). However, in the short run, raising rates does lower inflation and output, as in the Keynesian model. Cochrane (2015, p.1) summarizes the situation:

> "If the Fed raises nominal interest rates, the [New Keynesian] model predicts that inflation will smoothly rise, both in the short run and long run. This paper presents a series of failed attempts to escape this prediction. Sticky prices, money, backward-looking Phillips curves, alternative equilibrium selection rules, and active Taylor rules do not convincingly overturn the result."

This paper proposes a way to overturn this result, coming naturally from agents' bounded rationality.

Literature review. I build on the large New Keynesian literature, as summarized in Gali (2015). I am also indebted to the identification of the "forward guidance puzzle" in Del Negro, Giannoni, Patterson (2015) and McKay, Nakamura and Steinsson (forth.). I discuss their proposed resolution (based on rational agents) below. I was also motivated by the paradoxes in the New Keynesian model outlined in Cochrane (2015).

For the behavioral model, I rely on the general dynamic setup derived in Gabaix (2016), itself building on a general static setup laid out in Gabaix (2014).

There are other ways to model bounded rationality, including rules of thumbs (Campbell and Mankiw 1989), limited information updating (Caballero 1995, Gabaix and Laibson 2002, Mankiw and Reis 2002, Reis 2006) and noisy signals (Sims 2003, Maćkowiak and Wiederholt forth., Woodford 2012). This sparsity approach aims at being tractable and unified, as it applies to both microeconomic problems like basic consumer theory and Arrow-Debreu-style general equilibrium (Gabaix 2014), dynamic macroeconomics (Gabaix 2016) and public economics (Farhi and Gabaix 2015).

At any rate, this is the first paper to study how behavioral considerations affect forward guidance in the New Keynesian model, along Garcia-Schmidt and Woodford (2015). That paper was circulated simultaneously and offers a very different modelling. [2] Woodford (2013) explores non-rational expectations in the NK model, particularly of the learning type. He doesn't distill his analysis into something compact like the 2-equation NK model of Proposition 3.6.

Section 2 elementary 2-period model with a behavioral agents. This lays out in an elementary manner some conceptual issues. Then, Section 3 considers an infinite-horizon economy. It lays out the set, and derives the basic model, summarizes in Proposition 3.6. Section 4 then derives the consequences for the policy issues mentioned above. Section 5 then considers policy experiments with long run changes. Section 6 concludes.

2 Behavioral Keynesian Macro: A Two-period economy

Here I present a 2-period model that captures some of the basic features of this behavioral model. It is similar to undergraduate models, but with rigorous microfoundations. It highlight the complementarity between New Keynesian and behavioral features. It is a useful model in its own right: to consider extensions and variants, I found it easiest to start with this 2-period model.

The reader knowledgeable with the NK model may skip directly to Section 3. The present section is more for the reader who wants an elementary exposition of the economics in a compact and self-contained way.

Basic setup Utility is:

$$\sum_{t=0}^{1} \beta^t u(C_t, N_t) \text{ with } u(C,N) = \frac{C^{1-\gamma}-1}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}.$$

[2] The core of the forward guidnance part of this paper was originally contained in what became Gabaix (2016), but was split off later.

The economy is a made of a Dixit-Stiglitz continuum of firms. Firm i produces $q_{it} = N_{it}$ with unit productivity (there is no capital), and sets a price P_{it}. A corrective wage subsidy $\tau = \frac{1}{\varepsilon}$ (financed by lump-sum transfers) ensures that there are no distortions on average. When prices are flexible, the privately optimal price is $P_{it} = w_t$.[3]

The final good is in quantity $q_t = \left(\int_0^1 q_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, and its price is set competitively at:

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{1}$$

Calling GDP Y_t, the aggregate resource constraint is:

$$\text{Resource constraint: } Y_t = C_t + G_t = N_t \tag{2}$$

The real wage is ω_t. Labor supply is frictionless, so the agent respects his first order condition: $\omega_t u_c + u_N = 0$, i.e.

$$\text{Labor supply: } N_t^{\phi} = \omega_t C_t^{-\gamma} \tag{3}$$

The economy at time 1. Let us assume that the time-1 economy has flexible prices and no government consumption. Then, the real wage must be productivity, $\omega_t = Z_t = 1$. The labor supply equation (3) and $C_t = N_t$ give: $N_t^{\phi} = N_t^{-\gamma}$, so

$$N_1 = C_1 = 1$$

The economy at time 0. Now, consider the consumption demand at time 0, for the rational consumer. Taking for now personal income y_t as given, he maximizes $\max_{(C_t)_{t=0,1}} \sum_{t=0}^{1} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$ s.t. $\sum \frac{C_t}{R^t} = y_0 + \frac{y_1}{R}$. That gives

$$C_0 = b\left(y_0 + \frac{y_1}{R}\right) \tag{4}$$
$$b := \frac{1}{1+\beta}$$

with log utility. In the general case, $b := \frac{1}{1+\beta^{\psi} R_0^{\psi-1}}$, calling $\psi = \frac{1}{\gamma}$ the intertemporal elasticity of substitution (IES). But in this section I just use $\psi = 1$. Here b is the marginal propensity to consume (given the labor supply).[4]

[3] This is well-known: $P_{it} = (1-\tau)\,\mu w_t = w_t$ with $\mu = \frac{\varepsilon}{\varepsilon-1}$.

[4] This is different from the more subtle MPC inclusive of labor supply movements, which is $\frac{\phi}{\gamma+\phi}\frac{1}{1+\beta}$ when evaluated

Without taxes, we also have the (real) income: $y_t = C_t$. Hence,

$$C_0 = b\left(C_0 + \frac{C_1}{R_0}\right) \tag{5}$$

which yields the Euler equation $\beta R_0 \frac{C_0}{C_1} = 1$. I use the consumption function formulation (4) rather than this Euler equation. Indeed, the consumption function is the formulation that generalizes well to behavioral agents (Gabaix 2016).

Monetary policy is effective with sticky prices At time $t = 0$, a fraction θ of firms have sticky prices – their prices is pre-determined at a value we'll call P_0^d (if prices are sticky, then $P_0^d = P_{-1}$, but we could have $P_0^d = P_{-1}e^{\pi_0^d}$, where π_0^d is an "automatic" price increase pre-programmed at time 1, not reactive to time-0 economic conditions, e.g. as in Mankiw and Reis (2002)). The other firms optimize freely their price, so optimally choose a price

$$P_0^* = \frac{\omega_0}{Z_0} P_0 \tag{6}$$

where ω_0 is the real wage. Indeed, prices will be flexible at $t = 1$, so only current conditions matter for the optimal price. By (1), the aggregate price level is:

$$P_0 = \left(\theta\left(P_0^d\right)^{1-\varepsilon} + (1-\theta)\left(P_0^*\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{7}$$

as a fraction θ of firms set the price P_0^d and a fraction $1-\theta$ set a price P_0^*.

To solve the problem, there 6 unknowns: $C_0, N_0, \omega_0, P_0, P_0^*, R_0$, and 5 equations (2)-(3) for $t = 0$ and (4)-(7). What to do?

In the model with flexible prices ($\theta = 0$), this just means that P_0 is indeterminate (as in the basic Arrow-Debreu model). However, the real variables are determinate: with no government consumption, $C_0 = N_0 = 1$.

In the model with sticky prices ($\theta > 0$), there is a one-dimensional continuum of equilibria. Having monetary policy is mind, we can say that by picking the real interest rate R_0, the central bank chooses the equilibrium.

The behavioral consumer and fiscal policy We can now consider the behavioral consumer. If his true income is $y_1 = y_1^d + \widehat{y}_1$, he sees only $y_1^s = y_1^d + \bar{m}\widehat{y}_1$ for some $\bar{m} \in [0, 1]$, which is the

at $C = N = 1$.

attention to future income shocks. Here the default is the frictionless default, $y_1^d = C_1 = 1$.

Here I assume a representative agent. This analysis complements other analyses assume heterogeneous agents to model non-Ricardian agents, in particular rule-of-thumb agents à la Campbell-Mankiw (1989), as Gali, Lopez-Salido and Valles (2007), Mankiw (2000), Mankiw and Weinzierl (2011) and Woodford (2013).[5] When dealing with complex situations, a representative agents is often simpler.

But now suppose that (4) becomes:[6]

$$C_0 = b\left(y_0 + \frac{y_1^d + \bar{m}\hat{y}_1}{R_0}\right) \tag{8}$$

Suppose that the government consumes G_0 at 0, nothing at time 1, and makes a transfer $\mathcal{T}_t$ to the agents at times $t = 0, 1$. Call $d_0 = G_0 + \mathcal{T}_0$ the deficit at time 0. The government must pay its debts at the end of time 1, which yields the fiscal balance equation:

$$R_0 d_0 + \mathcal{T}_1 = 0 \tag{9}$$

The real income of a consumer at time 0 is

$$y_0 = C_0 + G_0 + \mathcal{T}_0 = C_0 + d_0$$

Indeed, labor and profit income equal to the sales of the firms, $C_0 + G_0$, plus the transfer from the government, $\mathcal{T}_0$. Income at time 1 is $y_1 = y_1^d + \mathcal{T}_1$. Hence, (8) gives:

$$C_0 = b\left(C_0 + d_0 + \frac{y_1 + \bar{m}\mathcal{T}_1}{R_0}\right)$$

[5] Gali, Lopez-Salido and Valles (2007)'s rich analysis cannot distilled into a simple 2-equation model, as the present model – instead it is largely solved numerically. Mankiw and Weinzierl (2011) have a form of representative agent with a partial rule of thumb behavior. They derive an instructive optimal policy in a 3-period model with capital (which is different from the standard New Keynesian model), but do not analyze an infinite horizon economy. Another way to have non-Ricardian agents is via rational credit constraints, as in Kaplan, Moll and Violante (2016). The analysis is then rich and complex.

[6] Formally in terms of behavioral dynamic programming as in Gabaix (2016), this comes from the consumers maximizing:

$$(C_0, N_0) = \arg \underset{C_0, N_0 | m}{\text{smax}}\, u(C_0, N_0) + V\left(y_1^d + m_{\mathcal{T}}\widehat{y}_1 + R_0(\mathcal{T}_0 + \omega_0 N_0 - C_0)\right)$$

where V is the continuation value function. To make things very straightforward, consider that N_1 is fixed at 1. Then, $V(x) = u(x, 1)$.

Using the fiscal balance equation (9) we have:

$$C_0 = b\left(C_0 + (1-\bar{m})\,d_0 + \frac{Y_1}{R}\right)$$

and solving for C_0:

$$C_0 = \frac{b}{1-b}\left((1-\bar{m})\,d_0 + \frac{Y_1}{R_0}\right) \tag{10}$$

We see how the "Keynesian multiplier" $\frac{b}{1-b}$ arises.

When consumers are fully attentive, $\bar{m} = 1$, and deficits don't matter in (10). However, take the case of really behavioral consumers, $\bar{m} \in [0, 1)$. Consider a transfer by the government $\mathcal{T}_0$, with no government consumption, $G_0 = 0$. Equation (10) means that a positive transfer $d_0 = \mathcal{T}_0$ stimulates activity. If the government gives him $\mathcal{T}_0 > 0$ dollars at time 0, he doesn't fully see that they will be taken back (with interest rates) at time 1, so that this is a wash. Hence, given $\frac{Y_1}{R_0}$, the consumer is tempting to consume more.

To see the full effect, when prices are not frictionless, we need to take stand on monetary policy to determine R_0. Here assume that the central bank does not change the interest rate R_0.[7] Then, given (10) implies that GDP ($Y_0 = C_0 + G_0$) changes as:

$$\frac{dY_0}{d\mathcal{T}_0} = \frac{b}{1-b}\,(1-\bar{m}) \tag{11}$$

With rational models, $\bar{m} = 1$, and fiscal policy has no impact. With a behavioral agent, $\bar{m} < 1$ and fiscal policy has an impact: the Keynesian multiplier $\frac{b}{1-b}$, times $(1-\bar{m})$, a measure of deviation from full rationality.

I record those results in the next proposition.

Proposition 2.1 *Suppose that we have (partially) sticky prices, and the central bank keeps the real interest rate constant. Then, a lump-sum transfer $\mathcal{T}_0$ from the government at time 0 creates an increase in GDP:*

$$\frac{dY_0}{d\mathcal{T}_0} = \frac{b}{1-b}\,(1-\bar{m})$$

where $b = \frac{1}{1+\beta}$ is the marginal propensity to consume. Likewise government spending G_0 has the

[7] With flexible prices ($\theta = 0$), we still have $\omega_0 = 1$ hence we still have $C_0 = N_0 = 1$. Hence, the interest rate R_0 has to increase. Hence, to obtain an effect of a government transfer, we need both monetary fricitions (partially sticky prices) and cognitive frictions (partial failure of Ricardian equivalence).

GDP multiplier:

$$\frac{dY_0}{dG_0} = 1 + \frac{b}{1-b}(1-\bar{m})$$

We see that $\frac{dY_0}{dT_0} > 0$ *and* $\frac{dY_0}{dG_0} > 1$ *if and only if consumers are non-Ricardian,* $\bar{m} < 1$*.*

This proposition also announces a result on government spending, that I now derive. Consider an increase in G_0, assuming a constant monetary policy (i.e., a constant real interest rate R_0). Equation (10) gives $\frac{dC_0}{dG_0} = \frac{b}{1-b}(1-\bar{m})$, so that GDP $Y_0 = C_0 + G_0$, has a multiplier

$$\frac{dY_0}{dG_0} = 1 + \frac{b}{1-b}(1-\bar{m})$$

When $\bar{m} = 1$ (Ricardian equivalence), a change in G_0 creates no change in C_0. Only labor demand N_0 increase, hence, via (3), the real wage increases, and inflation increases. GDP is $Y_0 = C_0 + G_0$, so that the multiplies $\frac{dY_0}{dG_0}$ is equal to 1.

However, when $\bar{m} < 1$ (so Ricardian equivalence fails), the multiplier $\frac{dY_0}{dG_0}$ is greater than 1. This is for the reason evoked in undergraduate textbooks: people feel richer, so spend more, which creates more demand. Here, we can say that with full good conscience – provided we allow behavioral consumers.

Without Ricardian equivalence, the government consumption multiplier is greater than 1. [8] Again, this relies on monetary policy here being passive, in the sense of keeping a constant real rate R_0. If the real interest rate rises (as it would do with frictionless pricing), then the multiplier would fall to a value less than 1.

This idea is known in the old Keynesian literature. Mankiw and Weinzierl (2011) also consider non-Ricardian agents, and find indeed a multiplier greater than 1. Their modelling method is different in a number of respects (in particular, they do not generalize their model to the infinite-horizon canonical NK model, and they have capital). The methodology here generalizes well to static and dynamic contexts.

Old vs New Keynesian model: a mixture via bounded rationality The above derivations show that the model is a mix of old and new Keynesian models. To see the difference in Cochrane (2013).

[8] This idea is known in the old Keynesian literature. Mankiw and Weinzierl (2011) consider late in their paper non-Ricardian agents, and find indeed a multiplier greater than 1. But to do that they use two types of agents, which makes the analytics quite complicated when generalizing to a large number of periods. The methodology here generalizes well to static and dynamic contexts.

> The old-Keynesian model is driven completely by an income effect with no substitution effect. Consumers don't think about today vs. the future at all. The new-Keynesian model based on the intertemporal substitution effect with no income effect at all. [...] This [old-Keynesian] model captures a satisfying story. More government spending, even if on completely useless projects, 'puts money in people's pockets.' Those people in turn go out and spend, providing more income for others, who go out and spend, and so on. [...] But, alas, the old-Keynesian model of that story is wrong. It's just not economics. A 40 year quest for "microfoundations" came up with nothing. (Cochrane (2013))

Here, we do obtain a microfoundation for the old Keynesian story (somewhat modified). We do see what is needed: some form of non-Ricardian behavior (here via bounded rationality), and of sticky prices. This behavioral model allows for a simple (and I think realistic) mixture of the two ideas.

For completeness, I describe the behavior of realized inflation – the Phillips curve. I describe other features in Section 7.

The Phillips curve Taking a log-linear approximation around $P_t = 1$, with $p_t = \ln P_t$, (7) becomes: $p_0 = \theta p_0^d + (1-\theta)\, p_0^*$, i.e.

$$p_0 - p_0^d = \frac{1-\theta}{\theta}\left(p_0^* - p_0\right)$$

Recall that $P_0^d = P_{-1} e^{\pi_0^d}$, so inflation is

$$\pi_0 = p_0 - p_{-1} = \left(p_0 - p_0^d\right) + \left(p_0^d - p_{-1}\right) = \frac{1-\theta}{\theta}\left(p_0^* - p_0\right) + \pi_0^d \tag{12}$$

Via (6),

$$p_0^* - p_0 = \hat{\omega}_0 \tag{13}$$

where $\hat{\omega}_0 = \frac{\omega_0 - \omega_0^*}{\omega_0^*}$ is the percentage deviation of the real wage from the frictionless real wage, ω_0^*. Because of the labor supply condition (3), and $C_0 = N_0$, we have $\omega_0 = C_0^{\phi+\gamma}$. Hence, $\hat{\omega}_0 = (\phi+\gamma)\,\hat{C}_0$. Hence (13) becomes $p_0^* - p_0 = (\phi+\gamma)\,\hat{C}_0$, and (12) yields:

$$\text{Phillips curve: } \pi_0 = \kappa \hat{C}_0 + \pi_0^d \tag{14}$$

with $\kappa := \frac{1-\theta}{\theta}(\phi+\gamma)$. Hence, we obtain the elementary "New Keynesian Phillips curve": increases in economic activity $\hat{C}_0$ leads to inflation. Inflation comes also from the automatic adjustment π_0^d.

To synthesize, we gather the results. Here $x_0 = \left(C_0 + G_0 - Y_0^d\right)/Y_0^d$ is the deviation of GDP from its frictionless value, $Y_0^d = 1$,while π_0 is the inflation between time -1 (the pre-time 0 price level) and time 0.[9]

Proposition 2.2 (Two-period behavioral Keynesian model) *In this 2-period model, we have, for time-0 consumption and inflation between periods 0 and 1:*

$$\hat{x}_0 = \hat{G}_0 + b_d\hat{d}_0 - \sigma\hat{r}_0 \text{ (IS curve)} \tag{15}$$

$$\pi_0 = \kappa\hat{C}_0 + \pi_0^d \text{ (Phillips curve)} \tag{16}$$

where $\hat{G}_0$ is government consumption, $\hat{d}_0$ the budget deficit, $b_d = \frac{b}{1-b}(1-\bar{m})$ is the sensitivity to future deficits, $b = \frac{1}{1+\beta}$ is the marginal propensity to consumer (given labor income) and $\hat{r}_0 = i_0 - \mathbb{E}\pi_1$ is the real interest rate between periods 0 and 1, and $\sigma = \frac{1}{R} = \beta$ with log utility.

3 A Behavioral New Keynesian Model

Let us recall the traditional NK model, there is no capital or government spending, so output is consumption. Calling $x_t = (C_t - C_t^*)/C^d = \hat{c}_t$ the output gap, the traditional NK model is:

$$x_t = \mathbb{E}_t[x_{t+1}] - \sigma(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) \tag{17}$$

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa x_t \tag{18}$$

where r_t^n is the real risk-free rate in the frictionless economy.

3.1 A Behavioral Agent: Microeconomic Behavior

In this section I present a way to model boundedly rational agents, drawing results from Gabaix (2016), which lays out its microfoundations.

[9]If the agent perceived only part of the change in the real rates, replacing R_0 by $(1-m_r)R_0^d + m_r R_0$ in (10), then the expression in (15) is the same, replacing $\sigma = \frac{1}{R}$ by $\sigma = \frac{m_r}{R}$.

Setup I consider an agent with standard utility

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t\right) \text{ with } u\left(C, N\right) = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}$$

where C_t is consumption, and N_t labor supply. The price level is p_t, the nominal wage is w_t, so that the real wage is $\omega_t = \frac{w_t}{p_t}$. The real interest rate is r_t and his real income is y_t (it is labor income $\omega_t N_t$ plus potential government transfers $\mathcal{T}_t$, normally set to 0). His real financial wealth k_t, evolves as:[10]

$$k_{t+1} = \left(1 + r_t\right)\left(k_t - c_t + y_t\right) \tag{19}$$

$$y_t = \omega_t N_t + \mathcal{T}_t \tag{20}$$

The agent's problem is $\max_{(C_t, N_t)_{t \geq 0}} U$ s.t. (19)-(20), and the usual transversality condition.

Consider first the case where the economy is deterministic, so that the interest rate, income, and real wage are at $\bar{r}$ (which soon I will call r), $\bar{y}$, and ϖ. Define $R := 1 + \bar{r}$ and assume that R satisfies the macro equilibrium condition $\beta R = 1$. We have a simple deterministic problem, whose solution is: $c_t^d = \frac{r k_t}{R} + \bar{y}$, and labor supply is $g'\left(\bar{N}\right) = \varpi$. If there are no taxes, as in the baseline model, $\bar{y} = \bar{\omega} \bar{N}$. In the New Keynesian model, there is no capital, so that in most expressions the reader can see $k_t = 0$. Still, to track the consumption policy, it is useful to consider general wealth holdings.

In general, there will be deviations from the steady state. I decompose the values as:

$$r_t = \bar{r} + \hat{r}_t,\ y_t = \bar{y} + \hat{y}_t,\ \omega_t = \bar{\omega} + \hat{\omega}_t,\ N_t = \bar{N} + \hat{N}_t$$

Rational agent I first present a simple lemma describing the rational policy using Taylor expansions.[11] To signify "up to second order term," I use the notation $O\left(\|x\|^2\right)$, where $\|x\|^2 := \mathbb{E}\left[\hat{y}_\tau^2\right]/\bar{y}^2 + \mathbb{E}\left[\hat{r}_\tau^2\right]/\bar{r}^2$ (the constants $\bar{y}, \bar{r}$ are just here to keep valid units).

Lemma 3.1 (Traditional rational consumption function) *In the rational policy, optimal consump-*

[10] I change a bit the timing convention compared to Gabaix (2016): income innovation $\hat{y}_t$ is received at t, not $t+1$. The wealth k_t is the wealth at the beginning of the period, before receiving any income.

[11] Auclert (2015) and Woodford (2013) derive related Taylor expansions.

tion is: $c_t = c_t^d + \hat{c}_t$, *with* $c_t^d = \frac{\bar{r}k_t}{R} + \bar{y}$ *and*

$$\hat{c}_t = \mathbb{E}_t \left[\sum_{\tau \geq t} \frac{1}{R^{\tau - t}} \left(b_r \left(k_t \right) \hat{r}_\tau + b_y \hat{y}_\tau \right) \right] + O\left(\|x\|^2 \right), \tag{21}$$

$$b_r \left(k_t \right) := \frac{\frac{r}{R} k_t - \frac{1}{\gamma} c^d}{R^2}, \qquad b_y := \frac{r}{R}. \tag{22}$$

and labor supply is $\frac{\hat{N}_t}{\bar{N}} = \frac{1}{\phi} \frac{\hat{\omega}_t}{\bar{\omega}} - \frac{\gamma}{\phi} \frac{\hat{c}_t}{c_t^d}$.

Consumption reacts to future interest rates and income changes, according to the usual income and substitutions effect (multiplied by $\frac{1}{\gamma}$). Note that here $\hat{y}_\tau$ is the change in total income, which includes the changes from endogenous (current and future) labor supply. To keep things tractable, it is best not to expand the endogenous $\hat{y}_\tau$ at this stage.

Behavioral agent In the behavioral model, the agent is partially inattentive to the variable part of the interest rate $\hat{r}_\tau$ and of income $\hat{y}_\tau$. The behavioral policy is then as follows – see Gabaix (2016) for the derivation.

Proposition 3.2 (Sparse consumption function) *In the behavioral model, consumption is:* $c_t = c_t^d + \hat{c}_t$, *with* $c_t^d = \frac{\bar{r}k_t}{R} + \bar{y}$ *and*

$$\hat{c}_t = \mathbb{E}_t^s \left[\sum_{\tau \geq t} \frac{\bar{m}^{\tau - t}}{R^{\tau - t + 1}} \left(b_r \left(k_t \right) m_r \hat{r}_t + b_y m_y \hat{y}_\tau \right) \right] + O\left(\|x\|^2 \right) \tag{23}$$

where $\mathbb{E}_t^s$ *is the expectation taken with respect to the agent's beliefs under the subjective model. Labor supply is:*

$$\frac{\hat{N}_t}{\bar{N}} = \frac{1}{\phi} \frac{\hat{\omega}_t}{\bar{\omega}} - \frac{\gamma}{\phi} \frac{\hat{c}_t}{c_t^d}. \tag{24}$$

Parameters $m_r, m_y, \bar{m}$ are attention parameters in $[0, 1]$. When they are all equal to 1, the agent is the traditional, rational agent. Here, m_r and m_y capture the attention to the interest rate and income, respectively. Parameter $\bar{m}$ is a form of "cognition discounting" – discounting future innovations more as they're more distant in the future.[12,13]

[12] See Gabaix (2016), Section 12.10 in the online appendix.

[13] Here the labor supply comes from the first order condition. One could develop a more general version $\frac{\hat{N}_t}{\bar{N}} = \frac{\frac{\hat{\omega}_t}{\bar{\omega}} - \gamma m_c^N \frac{\hat{c}_t}{c_t^d}}{\phi}$, where $m_c^N \in [0, 1]$ is attention to consumption when choosing labor supply. When $m_c^N = 0$, wealth effects are eliminated. Then, we have a behavioral microfoundation for the labor supply coming traditionally from the Greenwood, Hercowitz and Huffman (1988) preferences, $u\left(C - \frac{N^{1+\phi}}{1+\phi} \right)$.

If the reader seeks maximum parsimony, I recommend setting $m_r = m_y = 1$, and keeping $\bar{m} \in (0,1]$ at the main parameter governing inattention.

There is mounting microeconomic evidence for the existence of inattention, for macroeconomic variables (Coibon and Gorodnichenko 2015), taxes (Chetty, Looney and Kroft 2009, Taubinsky and Rees-Jones 2015), and, more generally, small dimensions of reality (Brown, Hossain, and Morgan, 2010, Caplin, Dean and 2011). Those are represented in a compact way by the inattention parameters m_y, m_r and $\bar{m}$.

3.2 Behavioral IS curve: First, Without Fiscal Policy

I start with a behavioral New Keynesian IS curve, in the case without fiscal policy. The derivation is instructive, and very simple. Proposition 3.2 gives:

$$x_t = \frac{\hat{c}_t}{c^d} = \frac{1}{c^d}\mathbb{E}_t\left[\sum_{\tau\geq t}\frac{\bar{m}^{\tau-t}}{R^{\tau-t}}\left(b_y m_y \hat{y}_\tau + b_r\left(k_t\right) m_r \hat{r}_\tau\right)\right]$$

The agent sees only partially income innovations, with a dampening m_y for income and m_r for the interest rate. The cognitive dampening features $\bar{m}$ will mostly be useful for calibration purposes.

Now, because in the NK model there is no capital, we have $\hat{y}_\tau = \hat{c}_\tau$: income is equal to aggregate demand. Conceptually, this $\hat{c}_\tau$ is the consumption of the other agents in the economy, a distinction that the representative agent framework obliterates. Hence, using $x_\tau = \frac{\hat{y}_\tau}{c^t}$, we have with $b_y = \frac{r}{R}$ and $\tilde{b}_r := \frac{b_r(k_t)_{|k_t=0} m_r}{c^d} = -\frac{\frac{1}{\gamma}m_r}{R^2}$,

$$x_t = \mathbb{E}_t\left[\sum_{\tau\geq t}\frac{\bar{m}^{\tau-t}}{R^{\tau-t}}\left(b_y m_y x_\tau + \tilde{b}_r \hat{r}_\tau\right)\right] = \frac{r}{R}m_y x_t + \tilde{b}_r\hat{r}_\tau + \frac{\bar{m}}{R}\mathbb{E}_t\left[x_{t+1}\right]$$

Multiplying by R and gathering the x_t terms, we have:

$$x_t = \frac{\bar{m}\mathbb{E}_t\left[x_{t+1}\right] + R\tilde{b}_r\hat{r}_\tau}{R - r m_y}$$

Using $M := \frac{\bar{m}}{R - r m_y}$ and $\sigma := \frac{-R\tilde{b}_r}{R - r m_y} = \frac{m_r/\gamma}{R(R - r m_y)}$, we obtain the "discounted IS curve," with discount M:

$$x_t = M\mathbb{E}_t\left[x_{t+1}\right] - \sigma\hat{r}_t \tag{25}$$

The next proposition records the result. The innovation in the interest rate is written in terms of the nominal rate,

$$\hat{r}_t := i_t - \mathbb{E}_t \pi_{t+1} - r_t^n. \tag{26}$$

Proposition 3.3 (Discounted Euler equation) *In equilibrium, the output gap x_t follows:*

$$x_t = M \mathbb{E}_t [x_{t+1}] - \sigma (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{27}$$

where $M := \frac{\bar{m}}{R - r m_y} \in [0, 1]$ is a modified attention parameter, and $\sigma := \frac{m_r}{\gamma R (R - r m_y)}$. In the rational model, $M = 1$.

Any kind of inattention (to aggregate variables via m_y, cognitive discounting via $\bar{m}$) creates $M < 1$. When the inattention to macro variables is the only force ($\bar{m} = 1$), then $M \in \left[\frac{1}{R}, 1\right]$. Hence, the cognitive discounting gives a potential powerfully quantitative boost.

This behavioral NK IS curve (36) implies:

$$x_t = -\sigma \sum_{\tau \geq t} M^{\tau - t} \mathbb{E}_t [\hat{r}_\tau] \tag{28}$$

i.e. it's the discounted value of future interest rates that matters, rather than the undiscounted sum. This will be important soon when we study forward guidance.

McKay, Nakamura and Steinnsson (forth.) find that this equation fits better. They provide a microfoundation based on heterogeneous rational agents with limited risk sharing. In their model, wealthy, unconstrained agents with no unemployment risk would still satisfy the usual Euler equation. Werning (2015)'s analysis yields a modified Euler equation with rational heterogeneous agents, which often yields $M > 1$. Piergallini (2006), Nistico (2012), and Del Negro, Giannoni, Patterson (2015) offer micro-foundations with heterogeneous mortality shocks, as in perpetual-youth models (this severely limits how myopic agents can be, given that life expectancies are quite high). Caballero and Farhi (2015) offer a different explanation of the forward guidance puzzle in a model with endogenous risk premia and a shortage of safe assets (see also Caballero, Farhi and Gourinchas 2015).

My take, in contrast, is behavioral: the reason that forward guidance doesn't work well is that it's in some sense "too subtle" for the agents. In independent work, Garcia-Schmidt and Woodford (2015) offer another, distinct, behavioral take on the NK IS curve.

Understanding discounting in rational and behavioral models. It is worth pondering what where the discounting comes from in (28). What is the impact at time 0 of a one-time fall of the real interest rate $\hat{r}_\tau$, in partial and general equilibrium, in both a rational and a behavioral model?

Let us start with the *rational model*. In partial equilibrium (i.e., taking future income as given), a change in the future real interest rate $\hat{r}_\tau$ changes time-0 consumption by[14]

$$\text{Rational agent: } \hat{c}_0^{\text{direct}} = -\frac{\psi \hat{r}_\tau}{R^\tau}$$

Hence, there is discounting by $\frac{1}{R^\tau}$. However, in general equilibrium, the impact is (see (28) with $M = 1$), so that

$$\text{Rational agent: } \hat{c}_0^{\text{total}} = -\psi \hat{r}_\tau$$

so that there is no discounting by $\frac{1}{R^\tau}$. The reason is the following: the agent sees the "first round of impact", $-\frac{\psi \hat{r}_\tau}{R^\tau}$: a future interest rate cut will raise consumption. But he also sees how this increase in consumption will increase other agents' future consumptions, hence increase his future income, hence his own consumption: this is a second-round effect. Iterating other all rounds (as in the Keynesian cross of equation 10), the initial impulse is greatly magnified: though the first round (direct) impact is $-\frac{\psi \hat{r}_\tau}{R^\tau}$, the full impact (including indirect channels) is $-\psi \hat{r}_\tau$.This means that the total impact is larger than the direct effects by a factor $\frac{\hat{c}_0^{\text{total}}}{\hat{c}_0^{\text{direct}}} = R^\tau$. At large horizons τ, this is a large multiplier. Note that this large general equilibrium effect relies common knowledge of rationality: the agent needs to assume that other agents fully rational. This is a very strong assumption, typically rejected in most experimental setups (see the literature on the $p-$beauty contest, e.g. Nagel 1995).

In contrast, in this *behavioral model*, the agent isn't fully attentive to future innovations. So, first, the direct impact of a change in interest rates is smaller:

$$\text{Behavioral agent: } \hat{c}_0^{\text{direct}} = -m_r \bar{m}^\tau \frac{\psi \hat{r}_\tau}{R^\tau}$$

which comes from (23). Next, the agent is not fully attentive to indirect effects (including general equilibrium) of future polices. This results in the total effect in (28):

$$\text{Behavioral agent: } \hat{c}_0^{\text{total}} = -m_r M^\tau \psi \hat{r}_\tau$$

with $M = \frac{\bar{m}}{R - r m_y}$. So the multiplier for general equilibrium effect is: $\frac{\hat{c}_0^{\text{total}}}{\hat{c}_0^{\text{direct}}} = \left(\frac{R}{R - r m_y}\right)^\tau \in [1, R^\tau]$, and it is smaller than the multiplire R^τ in economies with common knowledge of rationality.

[14] See equation (21). I use continuous time notations, so replace $\frac{1}{\gamma R^2}$ by $\psi = \frac{1}{\gamma}$ to unclutter the analysis; I take the case with not capital.

3.3 Behavioral IS Curve with Fiscal Policy

I call B_t the real value of government debt at period t, before period-t taxes. It evolves as $B_{t+1} = \frac{1+i_t}{1+\pi_{t+1}} \left(B_t + \mathcal{T}_t\right)$ where $\mathcal{T}_t$ is the lump-sum transfer given by the government to the agent (so that $-\mathcal{T}_t$ is a tax), and $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$ is the realized real interest rate.[15] Here, I take the Taylor expansion, neglecting the variations of the real rate (i.e. second-order terms $O\left(|\hat{r}_t|\left(|B_t| + |d_t|\right)\right)$) around $r = R - 1$. Hence, debt evolves as:

$$B_{t+1} = R\left(B_t + \mathcal{T}_t\right)$$

I also define d_t, the budget deficit (after payment of interest rates) in period t:

$$d_t := \mathcal{T}_t + rB_t$$

which implies for the evolution of public debt[16]

$$B_{t+1} = B_t + Rd_t$$

Iterating gives $B_\tau = B_t + R\sum_{u=t}^{\tau-1} d_u$, so that the transfer at time τ is: $\mathcal{T}_\tau = -\frac{r}{R}B_\tau + d_\tau$, i.e.

$$\mathcal{T}_\tau = -\frac{r}{R}B_t + \left(d_\tau - r\sum_{u=t}^{\tau-1} d_u\right) \tag{29}$$

Consumption satisfies, again from Proposition 3.2:

$$x_t = \mathbb{E}_t^s\left[\sum_{\tau\geq t} \frac{1}{R^{\tau-t}}\left(b_y\left(x_\tau + \mathcal{T}_\tau\right) + b_r\left(k_t\right)\hat{r}_\tau\right)\right] + \frac{r}{R}B_t$$

where $\mathbb{E}_t^s$ is the expectation under the subjective model.

The general formalism gives the following behavioral version of (29):[17]

$$\mathbb{E}_t^s\left[\mathcal{T}_\tau\right] = -\frac{r}{R}B_t + m_y\bar{m}^{\tau-t}\left(d_\tau - r\sum_{u=t}^{\tau-1} d_u\right) \tag{30}$$

This reflects a partially rational consumer. Given initial debt B_t, the consumer will see that it will

[15] The debt is short-term. Debt maturity choice is interesting, but well beyond the scope of this paper.
[16] Indeed, $B_{t+1} = R\left(B_t - \frac{r}{R}B_t + d_t\right) = B_t + Rd_t$:
[17] See in the derivation of Proposition 3.4 for details.

have to be repaid: he accurately foresees the part $\mathbb{E}_t^s [\mathcal{T}_\tau] = -\frac{r}{R} B_t$. If there were not future deficits, he would be rational. However, he does sees dimly future deficits, those that come beyond the service of the public debt. This is capture by the term $m_y \bar{m}^{\tau-t}$.

Calculations in the appendix give the following modifications of the IS curve. Note that here we have only deficits, not government consumption.

Proposition 3.4 (Discounted Euler equation with sensitivity to budget deficits) *We have the following IS curve reflecting the impact of both fiscal and monetary policy:*

$$x_t = b_d d_t + M\mathbb{E}_t [x_{t+1}] - \sigma \left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \tag{31}$$

where d_t is the budget deficit and

$$b_d = \frac{r m_y}{R - m_y r} \frac{R(1-\bar{m})}{R - \bar{m}} \tag{32}$$

is the sensitivity to deficits. When agents are rational, $b_d = 0$, but with behavioral agents $b_d > 0$.

Hence, bounded rational gives both a discounted IS curve and an impact of fiscal policy.

3.4 Phillips Curve with Behavioral Firms

I next explore what happens if firms don't fully process the future either; this is not essential, so the reader may wish to skip this during the first reading.

I assume that firms are partially myopic to the value of future markup. To do so, I adapt the classic derivation (e.g. chapter 3 of Galí (2015)) to behavioral agents. Firms can reset their prices with probability $1-\theta$ each period. The general price level is p_t, and a firm that resets its price at t sets a price p_t^* according to:

$$p_t^* - p_t = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k m^f \mathbb{E}_t [\psi_{t+k} - p_t] \tag{33}$$

i.e. the price is equal to the present value of future marginal costs $\psi_{t+k} - p_t$. The log marginal cost is simply the log nominal wage (if productivity is constant), $\psi_{t+k} = \ln(\omega_{t+k} P_{t+k})$. Here $m^f \in [0,1]$ indicates the imperfect attention to future markup innovations. Otherwise, the setup is as in Galí. When $\bar{m} = m^f = 1$, we have the traditional NK framework.

The details of the derivation follow Galí's, with behavioral firms (see Section 10): it traces the implication of the microeconomic equation (33) for the macro outcomes. The result is as follows. [18]

Proposition 3.5 (Phillips curve with behavioral firms) *When firms are partially inattentive to future macro conditions, the Phillips curve becomes:*

$$\pi_t = \beta M^f \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa x_t \tag{34}$$

with the attention coefficient M^f:

$$M^f := \bar{m} \left[\theta + (1-\theta) \frac{1-\beta\theta}{1-\beta\theta\bar{m}} m^f \right] \tag{35}$$

and

$$\kappa = \bar{\kappa} m^f$$

where $\bar{\kappa}$ (given in 73) is independent of attention. Firms are more forward-looking in their pricing (M^f is higher) when prices are sticky for a longer time (θ higher) and firms are more attentive to future macroeconomic outcomes (m^f is higher). When $m^f = \bar{m} = 1$ (traditional firms), we recover the usual model, and $M^f = 1$.

In the traditional model, the coefficient of inflation on future inflation in (34) is exactly β and, miraculously, does not depend on the adjustment rate of prices θ. In the behavioral model, in contrast, the coefficient (βM^f) is higher when prices are stickier for a longer time (higher θ).

Firms can be fully attentive to all idiosyncratic terms (something that will be easy to include in a future version of the paper), e.g. idiosyncratic part of productivity of demand. They simply have to pay attention m^f to macro outcomes. If we include idiosyncratic terms, and firms are fully attentive to them, the aggregate NK curve doesn't change.

The behavioral elements simply changes β into βM^f, where $M^f \leq 1$ is an attention parameter. Empirically, this "extra discounting" (replacing β by βM^f in (34)) seems warranted, as we shall see in the next section.

Let me reiterate that firms are still forward-looking (with discount parameter β rather than βM^f) in the deterministic steady state. It's only their sensitivity to deviations around the deterministic steady state that is partially myopic.

[18] Here I state the Proposition in that case where firms are constant return to scale ($\alpha = 0$). The expressions are similar (and available upon request) in the case where $\alpha \in [0, 1)$.

3.5 Synthesis: Behavioral New Keynesian Model

I now gather the above results.

Proposition 3.6 (Behavioral New Keynesian model – two equation version) *The behavioral version of the New Keynesian model gives:*

$$x_t = M\mathbb{E}_t\left[x_{t+1}\right] + b_d d_t - \sigma\left(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n\right) \text{ (IS curve)} \tag{36}$$

$$\pi_t = \beta M^f \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa x_t \text{ (Phillips curve)} \tag{37}$$

where $M, M^f \in [0,1]$ are the attention of consumers and firms, respectively, to macroeconomic outcomes, and $b_d \geq 0$ is an impact of deficits:

$$M := \frac{\bar{m}}{R - rm_y}$$

$$M^f := \bar{m}\left[\theta + (1-\theta)\frac{1-\beta\theta}{1-\beta\theta\bar{m}}m^f\right]$$

$$b_d = \frac{rm_y}{R - m_y r}\frac{R(1-\bar{m})}{R-\bar{m}}$$

In the traditional model, $M = M^f = 1$ and $b_d = 0$. In addition, $\sigma := \frac{m_r\psi}{R(R-rm_y)}$, and $\kappa = \bar{\kappa}m^f$, where $\bar{\kappa}$ (given in 73) is independent of attention.

Empirical Evidence on the Model's Deviations from Pure Rationality The empirical evidence, we will now see, appears to support the main deviations of the model from pure rationality.

In the Phillips curve, firms don't appear to be fully forward looking: $M^f < 1$. Empirically, the Phillips curve is not very forward looking. For instance, Galí and Gertler (1999) find that we need $\beta M^f \simeq 0.75$ at the annual frequency; given that $\beta \simeq 0.95$, that leads to an attention parameter of $M^f \simeq 0.8$. If we have $\theta = 0.2$ (so that 80% of prices are reset after a year), then this corresponds to $m^f = 0.75$.

In the Euler equations consumers don't appear to be fully forward looking: $M < 1$. The literature on the forward guidance puzzle concludes, plausibly I think, that $M < 1$.

Ricardian equivalence doesn't fully hold. There is much debate about Ricardian equivalence. The provisional median opinion is that it only partly holds. For instance, the literature on tax rebates (Johnson, Parker and Souleles 2006) appears to support $b^d > 0$.

All three facts come out naturally from a model with cognitive discounting $\bar{m} < 1$, even without the auxiliary parameters m_y, m_r, m^f. Those could be set to 1 (the rational value) in most cases.

Continuous time version In continuous time, we write $M = 1 - \xi \Delta t$ and $\beta M^f = 1 - \rho \Delta t$. In the small time limit ($\Delta t \to 0$), $\xi \geq 0$ is the cognitive discounting parameter due to myopia in the continuous time model, while ρ is the discount rate inclusive of firm's myopia. The model (36) becomes, in the continuous time version:

$$\dot{x}_t = \xi x_t - b_d d_t + \sigma \left(i_t - r_t - \pi_t \right) \tag{38}$$

$$\dot{\pi}_t = \rho \pi_t - \kappa x_t \tag{39}$$

When $\xi = b_d = 0$, we recover Werning (2012)'s formulation, in which agents are all rational.[19]

We now study several consequences of these modifications for the forward guidance puzzle.

4 Consequences of this Behavioral Model

4.1 Equilibria are Determinate Even with a Fixed Interest Rate

The traditional model suffers from the existence of a continuum of multiple equilibria, when monetary policy is passive. We will now see that if consumers are boundedly rational enough, there is just one unique (bounded) equilibrium.[20]

Let us start by considering a Taylor rule of the type:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + j_t \tag{41}$$

We express Proposition 3.6 with the notations: $z_t = (x_t, \pi_t)'$, forcing variable $a_t := j_t - r_t^n$,

[19] Note that the way to read (38) is as a forward equation. Calling $\delta_t := i_t - r_t - \pi_t$ the excess interest rate. We take into account the fact that the model is stationary ($\lim_{t \to \infty} x_t = 0$). We have:

$$\dot{x}_t = -\sigma \int_t^\infty e^{-\xi(s-t)} \delta_s ds \tag{40}$$

Hence, the effect of future rate changes is dampened by myopia.

[20] This theme that bounded rationality reduces the scope for multiple equilibria is general, and also holds in simple static models. I plan to develop it separately.

$m = \left(M, M^f\right)$, $\beta^f := \beta M^f$. For simplicity, I set fiscal policy to 0.[21] Calculations yield:[22]

$$z_t = A(m)\,\mathbb{E}_t\left[z_{t+1}\right] + b(m)\,a_t \tag{42}$$

$$\begin{aligned} A(m) &= \frac{1}{1+\sigma\left(\phi_x+\kappa\phi_\pi\right)}\begin{pmatrix} M & \sigma\left(1-\beta^f\phi_\pi\right) \\ \kappa M & \beta^f\left(1+\sigma\phi_x\right)+\kappa\sigma \end{pmatrix}, \\ b(m) &= \frac{-\sigma}{1+\sigma\left(\phi_x+\kappa\phi_\pi\right)}(1,\kappa)',\ a_t = j_t - r_t^n \end{aligned} \tag{43}$$

The next proposition generalizes the well-known Taylor stability criterion to behavioral agents:

Proposition 4.1 (Equilibrium determinacy with behavioral agents) *There is a unique equilibrium (all of A's eigenvalues are less than 1 in modulus) if and only if:*

$$\phi_\pi + \frac{\left(1-\beta^f\right)}{\kappa}\phi_x + \frac{\left(1-\beta^f\right)(1-M)}{\kappa\sigma} > 1 \tag{44}$$

In continuous time, the criterion (44) becomes: $\phi_\pi + \frac{\rho}{\kappa}\phi_x + \frac{\rho\xi}{\kappa\sigma} > 1$.

In particular, with passive monetary policy, we have a stable economy[23] if and only if bounded rationality is strong enough, in the sense that

$$\frac{\left(1-\beta M^f\right)(1-M)}{\kappa\sigma} > 1 \tag{45}$$

in the discrete time version; in the continuous time version this condition is:

$$\frac{\rho\xi}{\kappa\sigma} > 1 \tag{46}$$

[21] Given a rule for fiscal policy, the sufficient statistics is the behavior of the "monetary and fiscal policy mix" $i_t - \frac{b_d d_t}{\sigma}$. For instance, suppose that: $i_t - \frac{b_d d_t}{\sigma} := \phi_\pi\pi_t + \phi_x x_t + j_t$, with an some (unimportant) decomposition between i_t and d_t. The analysis is then the same. More general analyses might add the total debt D_t as a state variable in the rule for d_t.

[22] It is actually slightly easier (especially when considering higher-dimensional variants) to proceed with the matrix $(A(m))^{-1}$, and a system $\mathbb{E}_t\left[z_{t+1}\right] = (A(m))^{-1} z_t + \widetilde{b}(m)\,a_t$, and to reason on the roots of $(A(m))^{-1}$.

$$(A(m))^{-1} = \frac{1}{M\beta^f}\begin{pmatrix} \beta^f\left(1+\sigma\phi_x\right)+\kappa\sigma & -\sigma\left(1-\beta^f\phi_\pi\right) \\ -\kappa M & M \end{pmatrix}$$

[23] So, $\rho(A(m)) < 1$, where $\rho(V)$ is the the "spectral radius" of a matrix V, i.e. the maximum of the modulus of its eigenvalues. If $\rho(V) < 1$, then $\sum_{k=1}^{\infty} V^k$ converges.

Condition (45) does not hold in the traditional model, where $M = 1$. It basically means that agents are boundedly rational enough, that M is sufficiently less than 1.

Condition (45) implies that the two eigenvalues of A are less than 1.[24] This implies that the equilibrium is determinate.[25] This is different from the traditional NK model, in which there is a continuum of non-explosive monetary equilibria, given that one root is greater than 1 (as condition (45) is violated in the traditional model).

This absence of multiple equilibria is important. Indeed, take a central bank following a deterministic (e.g. constant) interest path – for instance in a period of prolonged ZLB. Then, in the traditional model, there is always a continuum of (bounded) equilibria – technically, because matrix $A(m)$ has a root greater than 1 (in modulus) when $M = 1$. As a result, there is no definite answer to the question "what happens if the central bank raises the interest rate" – as one needs to select a particular equilibrium (e.g. Cochrane 2015). In this papers' behavioral model, however, we do get a definite equilibrium.

Then, we can simply write, with $A(m)$

$$z_t = \mathbb{E}_t[\sum_{\tau \geq t} A(m)^{\tau - t} b(m) a_\tau] \tag{47}$$

The detailed solution is in Section 8 of the Appendix. [26]

4.2 Application to the Forward Guidance Puzzle

Here I follow McKay, Nakamura and Steinsson (forth.). Suppose that the central bank announces at time 0 that it will cut the rate at time T, following a policy $\delta_t = 0$ for $t \neq T$, $\delta_T < 0$, where δ is the interest rate gap. What is the impact?

Figure 1 illustrates the effect. We see that the biggest qualitative deviation comes from consumer attention M, while the firm's attention M^f makes a more quantitative difference.

Formally, we have $x_t = Mx_{t+1} - \sigma\delta_t$, so $x_t = -\sigma M^{T-t}\delta_T$ for $t \leq T$ and $x_t = 0$ for $t > T$. This

[24] Indeed, (45) is equivalent to $\phi(1) > 0$, so both eigenvalues are either below or above 1. Given the sum of the two eigenvalues is $\lambda_1 + \lambda_2 = \beta + M < 2$, this implies that both eigenvalues are below 1.

[25] The condition does not prevent unbounded or explosive equilibria, the kind that Cochrane (2011) wrestles with. My take is that this issue is interesting (as are rational bubbles in general), but that the largest practical problem is to eliminate bounded equilibria. The behavioral model does that well.

[26] Here, I rule out any extra "explosive bubble" term.

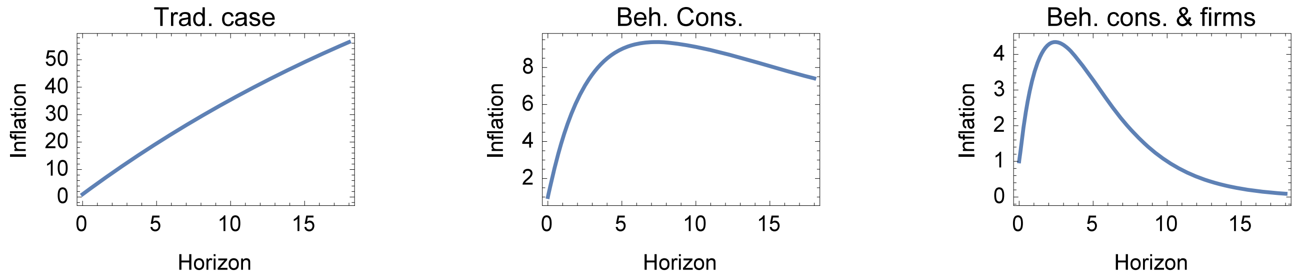

Figure 1: This Figure shows the response of current inflation to forward guidance about interest rate in T periods, compared to a immediate rate change of the same magnitude. Units are yearly. Left panel: traditional New Keynesian model. Middle panel: model with behavioral consumers and rational firms. Right panel: model with behavioral consumers and firms. Parameters are the same in both models, except that (annualized) attention is $M = M^f = e^{-\xi} = 0.7$ in the behavioral model, and $M = 1$ in the traditional model.

implies that inflation is:

$$\pi_0(T) = \kappa \sum_{t\geq 0} \mathcal{B}^t x_t = -\kappa\sigma \sum_{t=0}^{T} \mathcal{B}^t M^{T-t} \delta_T = -\kappa\sigma \frac{M^{T+1} - \mathcal{B}^{T+1}}{M - \mathcal{B}} \delta_T$$

where $\mathcal{B} := \beta M^f$ is the discount factor adjusted for firms' inattention. A rate cut in a very distant future has a powerful impact on today's inflation ($\lim_{T\to\infty} \pi_0(T) = \frac{-\kappa\sigma}{1-\mathcal{B}}$) in the rational model ($M = 1$), and no impact at all in the behavioral model ($\lim_{T\to\infty} \pi_0(T) = 0$ if $M < 1$).

When attention is endogenous, the analysis could become more subtle. Indeed, if other agents do think more about forward Fed announcement, their impact will be bigger, and a consumer will want to think more about them. This positive complementarity in attention could create multiple equilibria in effective attention M, m_r. I don't pursue that here.

4.3 The ZLB is Less Costly with Behavioral Agents

I follow a thought experiment in Werning (2012), but with behavioral agents. I take $r_t = \underline{r}$ for $t \leq T$, and $r_t = \bar{r}$ for $t > T$, with $\underline{r} < 0 < \bar{r}$. I assume that for $t > T$, the central bank implements $x_t = \pi_t = 0$ by setting $i_t = \bar{r}$. At time $t < T$, I suppose that the CB is at the ZLB, so that $i_t = 0$.

Proposition 4.2 *In the traditional rational case ($\xi = 0$), we obtain an unboundedly intense recession as the length of the ZLB increases:* $\lim_{t\to-\infty} x_t = -\infty$. *This also holds when myopia is mild,* $\frac{\rho\xi}{\sigma\kappa} \leq 1$.

However, suppose cognitive myopia is strong enough ($\frac{\rho\xi}{\sigma\kappa} > 1$), which is the continuous-time

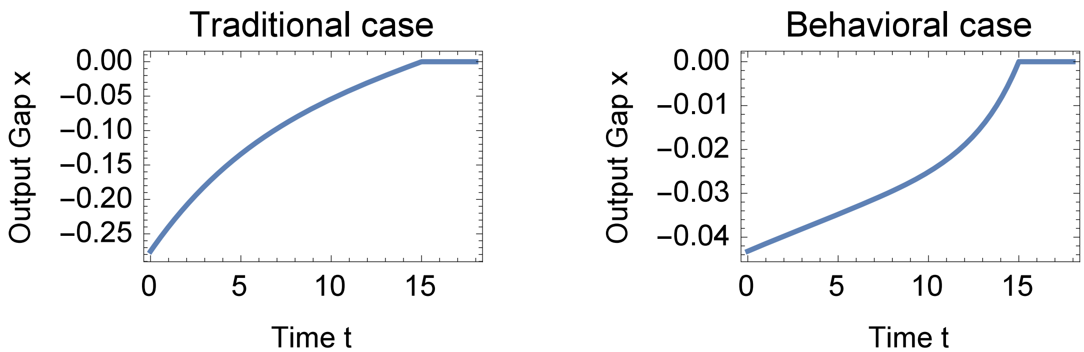

Figure 2: This Figure shows the output gap x_t. The economy is at the Zero Lower Bound during times 0 to $T = 15$ years. The left panel is the traditional New Keynesian model, the right panel the behavioral model. Parameters are the same in both models, except that (annualized) attention is $M = e^{-\xi} = 0.7$ in the behavioral model, and $M = 1$ in the traditional model.

version of condition (45). Then, we obtain a boundedly intense recession: $\lim_{t\to-\infty} x_t = \frac{\rho\sigma\underline{r}}{\rho\xi-\sigma\kappa} < 0$.

We see how impactful myopia can be. We see that myopia has to be stronger when the agents are highly sensitive to the interest rate (σ high) and price flexibility is high (high κ). High price flexibility makes the system very reactive, and a high myopia is useful to counterbalance that.

Figure 2 shows the dynamics. The left panel shows the traditional model, the right one the behavioral model. The parameters are the same in both models, except that attention is lower (set an annualized rate of $M = e^{-\xi} = 0.7$) in the behavioral model (against its value $M = 1$ in the traditional model).[27] On the left panel, we see how costly the ZLB is (mathematically it is unboundedly costly as it is more long-lasting) , while on the right panel, we see a finite, though prolonged cost. Reality looks more like the prediction of the behavioral model (right panel) – something like Japan since the 1990s – rather than the prediction of the rational model (left panel) – which is something like Japan 1945-46 or Rwanda.

I note that this quite radical change of behavior is likely to hold in other contexts. For instance, in those studied by Kocherlakota (2016) where the very long run matters a great deal, it is likely that a some modicum of bounded rationality would change a lot make the economy's behavior.

4.4 Optimal Mix of Fiscal and Monetary Policy

Suppose that we have a "crisis period" $I = (T_1, T_2)$, with $r_t^n < 0$ there, so that the ZLB binds. With only monetary policy, the situation is dire. However, with fiscal policy, the first best can be restored.

[27] The other parameters are: $\rho = 3\%, \kappa = 0.1, \sigma = 0.2, \underline{r} = -5\%$.

Proposition 4.3 (Optimal mix of fiscal and monetary in a ZLB environment). *Using a mix of monetary and fiscal policy yields the first best, $x_t = \pi_t = 0$ at all dates. The policy is as follows.* During the crisis *$(t \in (T_1, T_2))$, use fiscal policy*

$$d_t = -\frac{\sigma r_t^n}{b_d},\ i_t = 0$$

i.e. run a deficit with low interest rates. After the crisis *$(t \leq T_2)$, pay back accumulate debt, running government surplus, keeping the economy afloat with low rates, e.g. $d_t = R^{-1}(B_{T_2} - B_0)(1-\rho_d)\rho_d^{t-T_2} < 0$ for some $\rho_d \in (0,1)$, and adjusts $i_t = \frac{b_d d_t}{\sigma} < 0$ to ensure full macro stabilization, $x_t = \pi_t = 0$.* Before the crisis *$(t < T_2)$, there is no preventive action to do, so set $i_t = d_t = 0$.*

Proof The proof is simply by examination of the basic equations of the NK model, (36)-(37). We adjust the instruments so that $x_t = \pi_t = 0$ at all dates. Note that there are multiple ways to soak up the debt after the crisis, so that $d_t = R^{-1}(B_{T_2} - B_0)(1-\rho_d)\rho_d^{t-T_2}$ is simply indicative. □

In general, monetary and fiscal policy are substitutes (d_t and i_t enter symmetrically in (36), so a great number of policies achieve the first best. However, fiscal policy d_t helps when monetary policy is constraint (e.g. at the ZLB).

The ex ante preventive benefits of potential ex post fiscal policy Proposition 4.3 shows that "the possibility of fiscal policy as ex post cure produces ex ante benefits". Imagine that fiscal policy is not available. Then, the economy is depressed, at the ZLB, during (T_1, T_2). However, it is also depressed before: because the IS curve is forward looking, output threatens to be depressed before T_1, and that can put the economy at the ZLB at a time T_0 before T_1. Hence, the threat of a ZLB-depression at (T_1, T_2) creates a earlier recession at (T_0, T_2) with $T_0 < T_1$. Intuitively, agents feel "if something happens, monetary policy will be impotent, so large dangers loom". However, if the government has fiscal policy in its arsenal, the agents feel "worse case, the government will use fiscal policy, so there's no real threat", and there is no recession in (T_0, T_1). Hence, the possibility of Fiscal policy as ex post cure producing ex ante benefits.

5 Long Run Expectations and Fisher neutrality

In behavioral models, agents's actions and thoughts are anchored at a "default".[28] The "default" corresponds to: if the agent don't think, what kind of inflation do they expect? So far, I have assumed a constant default at 0 – this streamlines the analysis, at little cost in most situations. However, let us explore how to have a richer default, and what are the consequences.

This section is more speculative, and is likely to change with future iterations of this project.

5.1 Modelling the impact of long run policy expectation

For clarity, it is useful to be a bit general and abstract. Suppose that we have a system:

$$z_t = A(m)\,\mathbb{E}_t[z_{t+1}] + b(m)\,a_t \tag{48}$$

where a_t is some exogenous "action" by the external world (e.g. the central bank), and z_t by endogenous variables. We assume, for the subjective model m of the agents, $A(m)$ has only stable roots, less than 1 in modulus. Also, $A^r = A(\iota)$ and $b^r = b(\iota)$ are the response that would happen in agent where rational (with ι a vector of ones, representing full rationality); but A^r could have unstable roots (with eigenvalues greater than 1 in modulus). For instance, in our NK setup in (43) with passive policy ($\phi_x = \phi_\pi = 0$), the behavioral response is $A(m) = \begin{pmatrix} M & \sigma \\ \kappa M & \beta M^f + \kappa\sigma \end{pmatrix}$, and the rational response is $A^r = A(\iota) = \begin{pmatrix} 1 & \psi \\ \kappa & \beta + \kappa\psi \end{pmatrix}$, where (from 17.3, I define

$$\psi := \frac{1}{\gamma R} \tag{49}$$

which is basically the rational IES in the continuous time limit.

Suppose that we have a constant long run action: $a_t = a$ for all t. Then, (42) gives that the rational response z should satisfy:[29]

$$z = A^r z + b^r a$$

[28] In Bayesian models, the "default" is basically called the "prior" – which is a more complex probability distribution, whereas the default is more typically a point estimate.

[29] Here I make a (arguably mild) equilibrium selection: I assume that a constant impulse a generates a a constant response z.

hence, $z = H^r a$, with

$$H^r := (1 - A^r)^{-1} b^r \tag{50}$$

Now consider an agent who forms, at time t, some view of the long run action, e.g.

$$a_t^{LR} = \lim_{\tau \to \infty} \mathbb{E}_t [a_{t+\tau}] \tag{51}$$

but we will shortly consider smoother version of this concept. Given the long run action a_t^{LR}, the rational action is $z_t^{LR} = b^r a_t^{LR}$.

Next, I posit that agents reason about the economy in "deviation from the long run", e.g. they think about a world:[30]

$$\hat{z}_{\tau|t} = A(m) \hat{z}_{\tau+1|t} + b(m) \hat{a}_{\tau|t} \tag{52}$$

where $\hat{z}_{\tau|t}$ and $\hat{a}_{\tau|t}$ are the deviations from the time-t default:

$$\hat{Z}_{\tau|t} := Z_\tau - m_{LR} Z_t^{LR} \text{ for } Z = z, a$$

Here, again $m_{LR} \in [0, 1]$ is the weight on the LR as an anchor. In the formulation so far, we had $m_{LR} = 0$. If $m_{LR} = 1$, they think of economic outcomes as a deviation from the long run. I assume here that in their simulation at time t, then set an anchor for the whole future path $\hat{z}_{\tau|t}$ at times τ after t. I assume that agents do have access to this notion of "normatively correct long run response" $H(\iota) a_t^{LR}$. It's a bit of a strong assumption, though less strong than that of the traditional model. In future drafts, I plan to reexamine this assumption, and perhaps change it.

Calling $H(m) = \sum_{i \geq 0} A(m)^i b(m) = (1 - A(m))^{-1} b(m)$, so that $H(\iota) = H^r$.

Proposition 5.1 *In the model with a non-zero long-run a_t^{LR}, we have*

$$z_t = \sum_{\tau \geq t} \mathbb{E}_t \left[A(m)^{\tau - t} b(m) a_\tau \right] + m_{LR} (H(\iota) - H(m)) a_t^{LR}$$

[30] This is in the traditional of much cognitive modelling, where thinking is anchored at a "default" and the agent considers partial adjustments from it (cf. Tversky and Kahneman (1974), Gabaix (2014, 2016)). Here, the default is the long run, which itself is influenced by the past, as we shall soon see.

Proof We have

$$\begin{aligned}
z_t &= \hat{z}_{t|t} + m_{LR} z_t^{LR} = \sum_{\tau \geq t} \mathbb{E}_t \left[A(m)^{\tau - t} b(m) \left(a_\tau - m_{LR} a_t^{LR} \right) \right] + m_{LR} z_t^{LR} \\
&= \sum_{\tau \geq t} \mathbb{E}_t \left[A(m)^{\tau - t} b(m) a_\tau \right] - m_{LR} [\sum_{\tau \geq t} A(m)^{\tau - t} b(m)] a_t^{LR} + m_{LR} H(\iota) a_t^{LR} \\
&= \sum_{\tau \geq t} \mathbb{E}_t \left[A(m)^{\tau - t} b(m) a_\tau \right] + m_{LR} \left(H(\iota) - H(m) \right) a_t^{LR}
\end{aligned}$$

□

This is what we had before, $z_t = \sum_{\tau \geq t} \mathbb{E}_t \left[A(m)^{\tau - t} b(m) a_t \right]$ (equation (47)) with a new term, $\left(H(\iota) - H(m) \right) a_t^{LR}$, for the adjustment to long run impact. When agents are rational, $m = \iota$ and this term is 0: there is no need for an extra adjustment term. When the agents are less than rational, the anchor on the long term helps them be more rational.

To get clean expressions, I use the notations:

$$M = 1 - \xi,\ M_f = \frac{1 - \rho}{\beta}, \beta = 1 - \rho + \chi \tag{53}$$

so that the rational case corresponds to $\xi = 0$ for for consumers to $\chi = 0$ for firms, and the expressions are similar in discrete and continuous time. Indeed, $\left(1 - \beta M^f \right) (1 - M) = \rho \xi$ then. Simple calculations show that we have:[31]

$$H(m) = \frac{1}{\rho \xi - \kappa \sigma} (-\rho, \kappa) \sigma, \qquad H(\iota) = \left(\frac{\rho - \chi}{\kappa}, 1 \right)$$

and $b^{LR}(m) = H(\iota) - H(m)$ is equal to

$$b^{LR}(m) = \frac{\rho \xi}{\rho \xi - \kappa \sigma} \left(\frac{\rho}{\kappa}, 1 \right)' - \left(\frac{\chi}{\kappa}, 0 \right) \tag{54}$$

Here, I gather the model.

Proposition 5.2 (Behavioral New Keynesian model, with adjustment for the long run) *The generalization of the model with adjustment for the long run is as follows: Define* $\widehat{x}_\tau$*,* $\widehat{\pi}_\tau$ *to be the*

[31] Actually, the value of κ is a bit different in the rational model. This little bug will be fixed in a next iteration.

solutions of the model of Proposition 3.6:

$$\widehat{x}_{\tau} = M\mathbb{E}_{\tau}\left[\widehat{x}_{\tau+1}\right] + b_d d_t - \sigma\left(i_{\tau} - \mathbb{E}_{\tau+1}\widehat{\pi}_{\tau} - r_{\tau}^{n}\right) \tag{55}$$

$$\widehat{\pi}_{\tau} = \beta M^{f}\mathbb{E}_{\tau}\left[\widehat{\pi}_{\tau+1}\right] + \kappa\widehat{x}_{\tau} \tag{56}$$

The actual values of output and inflation are:

$$(x_t, \pi_t) = (\widehat{x}_t, \widehat{\pi}_t) + m_{LR} b^{LR} i_t^{LR}$$

where b^{LR} is given in (54), and i_t^{LR}, the perception of long run policy, is given by $i_t^{LR} = \lim_{\tau\to\infty}\mathbb{E}_t\left[i_{t+\tau}\right]$ in the "strict long run" case; or (58) applied to $a_t = i_t$ in the "smoothed long run" case.

If indeed the action is constant, then $a^{LR} = \bar{a}$, and the true long run is

$$\bar{z}^{LR} = \left[(1 - m_{LR}) H(m) + m_{LR} H(\iota)\right]\bar{a}.$$

Lemma 5.3 (In the long run, does inflation increase or decrease with interest rates?) *Suppose that the nominal interest rates (minus the RBC normative interest rate) constant at $\bar{\imath}$ in the long run. Then, the steady state inflation is is:*

$$\bar{\pi} = \left[-(1 - m_{LR})\frac{\kappa\sigma}{\rho\xi - \kappa\sigma} + m_{LR}\right]\bar{\imath} \tag{57}$$

Hence, if $m_{LR} = 1$, long-run Fisher neutrality holds. More generally, if m_{LR} is close enough to 1, inflation increases with the interest rate in the long run.

Let me now detail the "smoothed long run" case.

5.2 A "Smoothed Long Run"

The simplest way to model the long run is $a_t^{LR} = \lim_{\tau\to\infty}\mathbb{E}_t\left[a_{t+\tau}\right]$ (equation (51)). But, this notion captures only "mathematical infinity" and will not capture for policies that last for 80 years, rather than forever. In addition, expectations of BR agents may be slow to adjust. Hence, I use a smooth generalization of (51):

$$a_t^{LR} = \sum_{D,\tau\geq 0}\mathbb{E}_{t-D}\left[a_{t-D+\tau}\right] g(D) f(\tau) \tag{58}$$

Here, D represents a delay in the adjustment of the information set (as in Gabaix and Laibson 2002, Mankiw Reis 2002), distributed according to $g(D) = \phi e^{-\phi D}$ (i.e. $g(D) = \phi (1-\phi)^D$ in discrete time). Also, $f(\tau)$ is the weight put on the future τ periods ahead. In practice, I take $f(\tau) = \zeta^2 e^{-\zeta\tau}\tau$, which puts more weight on the future than the immediate present.

When $\zeta \to 0$ and $\phi \to \infty$, a_t^{LR} converges to $\lim_{\tau\to\infty} \mathbb{E}_t [a_{t+\tau}]$. Let us evaluate its value for a typical case.

Lemma 5.4 *Suppose that a policy change $a_t = e^{-\alpha t} a_0$ is announced at time time 0. Then agents' perception of its long-run value is (in continuous time):*

$$a_t^{LR} = \frac{\zeta^2 \phi}{(\zeta+\alpha)^2 (\phi - \alpha)} \left(e^{-\alpha t} - e^{-\phi t} \right) a_0$$

For instance, if this is a permanent change, $\alpha = 0$, then $a_t^{LR} = \left(1 - e^{-\phi t}\right) a_0$. There is a delayed adjustment captured by ϕ. When $\phi = \infty$ (no delay in expectations), then $a_t^{LR} = a_0$, expectations adjust immediately.

When the policy change will mean-revert ($\alpha > 0$), then the "strict long run" is just 0: $\lim_{\tau\to\infty} \mathbb{E}_t [a_{t+\tau}] = 0$. However, the "smoothed long run" a_t^{LR} is not 0. Hence, a shock lasting say 50 years but not an infinite amount of years is captured by the smoothed long run.

5.3 Impact of a Permanent Rise in the Nominal Interest Rate

We can now study the impact of a permanent rise in the nominal interest rate: i_t increase by $J = 1\%$.

I take the following parameters, in continuous time with yearly units: $\xi_0 = 0.45$, $\rho^f = -\ln\left(\beta M^f\right) = 0.2$, $m_{LR} = 1$, $\kappa_0 = 0.6$, $\sigma = 0.1$, $\phi = 0.25$, $\zeta = 0.05$. They have not been particularly optimized.

The impact is in Figure 3. On impact, the rise in rate lowers inflation and output – this is the conventional Keynesian effect. In the long run, however, the Fisherian prediction holds: the 1% rise in interest rate goes with a 1% rise in inflation, so that the long term interest rate is unchanged.[32]

This effect is very hard to obtain in a conventional New Keynesian model. Cochrane (2015) documents this, and explores many variants: they all give that a rise in interest rate creates a rise in inflation (though Cochrane needs to select one particular equilibrium, as the traditional model generates a continuum of bounded equilibria). However, here the bounded rationality of the agents overturns this result, with just one bounded equilibrium.

[32] However, this neutrality is partial – as in the basic NK model, output does increase permanently if inflation is permanently higher. This effect, however, if quite small.

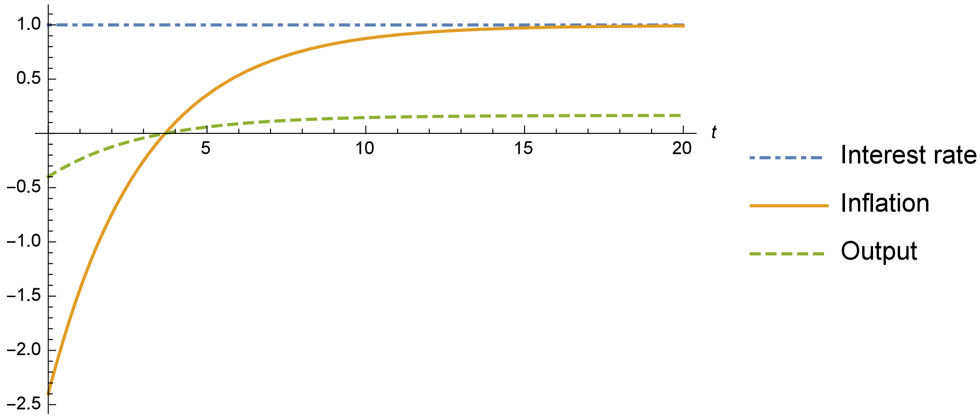

Figure 3: Impact of a permanent rise in the nominal interest rate. At time 0, the nominal interest rate is permanently increased by 1%. The Figure traces the impact on inflation and output. Units are percents.

To see analytically why, it is worth examining two polar cases. First, take the full rationality case in Proposition 3.6, we have

$$x = x - \sigma (J - \pi) \tag{59}$$

$$\pi = \beta \pi + \kappa x_t \tag{60}$$

Then, the solution with constant coefficient is : $\pi_t = J$, $x = \frac{(1-\beta)}{\kappa} J$. Hence, a higher interest rate leads to higher inflation, as in the Fisher neutrality.

However, take a completely myopic model, so that $M = M^f = 0$. Then, Proposition 3.6 reduces to:

$$x_t = -\sigma \left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n \right)$$

$$\pi_t = \kappa x_t$$

so

$$\pi_t = \kappa \sigma \mathbb{E}_t \pi_{t+1} - \sigma J$$

so that a higher interest rate leads to *lower* inflation.

The enriched model with long run expectation gives something in between those two polar models. Hence, it generates the first Keynesian dynamics, with high interest rate lowering inflation, and the long run Fisher effect of a higher inflation to restore constant real rates.

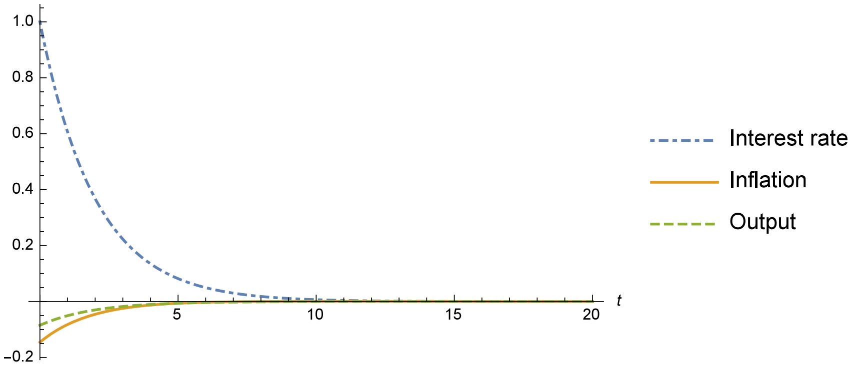

Figure 4: Impact of a permanent rise in the nominal interest rate. At time 0, the nominal interest rate is temporarily increased by 1%. The Figure traces the impact on inflation and output. Units are percents.

5.4 Impact of a Temporary Rise in Interest Rates

Let us now consider a short term rise in the interest rate, in Figure 4. We find indeed that a temporary rise in interest rates decreases inflation and output. This is a result that was hard to get in NK models (though again this depends on issues of equilibrium selection), see Cochrane (2015). Here, we get it easily, with a determinate equilibrium.

6 Conclusion

This paper gives simple way to think about the impact of bounded rationality on monetary and fiscal policy.

It is grounded on decent systematic microfoundations, that model a behavioral agent in basic microeconomic contexts à la Arrow-Debreu (Gabaix 2014) and to dynamic settings with general dynamic programming (Gabaix 2016). Those microfoundations not only should give the model user good conscience, but they ensure for instance that model parameters respond to incentives in a sensible way, and that the model generalizes.

Furthermore, we have seen that the model has good empirical support in its main non-standard elements. For instance, when Galí and Gertler (1999) estimate a Phillips curve, they estimate a coefficient inflation of $\beta M^f \simeq 0.75$ at the annual frequency, which leads to an attention parameter of $M^f \simeq 0.8$ (at the annual frequency). In the IS curve, the literature on the forward guidance puzzle, using a mix of market data and thought experiment, gives good evidence that we need $M < 1$, a main contention of the model. Finally, the notion that a higher interest lowers inflation in the short run (Keynesian effect), then raises it in the long run (classical Fisherian effect) is generally well accepted, using again a mix of historical episodes and empirical evidence. This is generated by the model, and is hard to generate by other models (Cochrane 2015).

In conclusion, we have a model with quite systematic microfoundations, empirical support in its non-standard features, that is also simple to use.

There are several obvious next steps for future research.

One is a more general normative analysis (see Farhi and Gabaix (2015) for optimal taxation with behavioral agents, in static settings). What is the optimal policy in that setup? Here, I derived it in a simple case with no constraints on fiscal policy, where policy is so powerful than the first best is achieved. In other contexts (e.g. with constraints on monetary and fiscal policy), things will be more complex, and require a full-fledged analysis. In particular, what happens with endogenous attention to monetary policy?

Another question is a more quantitative approach, possibly with a medium scale model rather than the most basic model studied here.

7 Appendix: Complements to the 2-period Model

This appendix gives complements to the 2-period model of Section 2

Derivation of (8) Call k_1 the wealth at the beginning of period 1 (before receiving labor income and profit), and $\mathcal{T}_1$ the transfer received from the government, and I_1 the profit income from the oligopolistic firms (so that $\omega_1 N_1 + I_1 = C_1$ when aggregating). The rational value function at time 1 is:

$$V^r(k_1, \mathcal{T}_1) = \max_{c_1, N_1} u(c_1, N_1) \text{ s.t. } c_1 \leq \omega_1 N_1 + I_1 + k_1 + \mathcal{T}_1$$

The decision at time 0 is

$$\operatorname*{smax}_{c_0, N_0; \bar{m}} u(c_0, N_0) + \beta V^r\left(R_0\left(\omega_0 N_0 + I_0 + \mathcal{T}_0\right), \bar{m}\mathcal{T}_1\right)$$

where $\bar{m}$ is optimized upon in the sparse max. Taking here provisionally the $\bar{m}$ as given, then the decision is simply:

$$\max_{c_0, N_0} u(c_0, N_0) + \beta V^r\left(R_0\left(\omega_0 N_0 + I_0 + \mathcal{T}_0 - c_0\right), \bar{m}\mathcal{T}_1\right)$$

The first order condition are:

$$u_{c_0} = \beta R_0 V_{k_1}$$
$$u_{N_0} = -\omega_0 \beta R_0 V_{k_1}$$

so that the intra-period labor supply condition $\omega_0 u_{c_0} + u_{N_0} = 0$ holds. Given we have $V_{k_1} = u_{c_1}$, we obtain

$$u_{c_0}\left(c_0, N_0\right) = \beta R_0 u_{c_1}\left(c_0, N_0\right)$$

Now, we have $V^r_{k_1} = u'\left(c_1\right) = u'\left(k_1 + y_1\right)$ with $y_1 = \omega_1 N_1 + I_1 + m\mathcal{T}_1$, so

$$\frac{1}{c_0} = \frac{\beta R_0}{c_1}$$

with $c_1 = y_1 + R\left(y_0 - c_0\right)$ i.e. $c_0 + \frac{c_1}{R} = y_0 + \frac{y_1}{R}$, and with the Euler equation $c_1 = \beta R_0 c_0$

$$C_0 = \frac{1}{1+\beta}\left(y_0 + \frac{y_1}{R}\right) = b\left(y_0 + \frac{y_1 + m\hat{y}_1}{R}\right)$$

Discounted Euler equation in the 2-period model Also, this consumer satisfies an Euler equation. Rewrite (5) as

$$C_0 = b\left(C_0 + \frac{C_1^d + m\hat{C}_1}{R_0^d + m_r\hat{R}_0}\right)$$

Where $C_0^d = b\left(C_0^d + \frac{C_1^d}{R_0^d}\right)$. Then, we have

$$\hat{C}_0 = b\hat{C}_0 + \frac{m\hat{C}_1 - m_r\hat{R}_0}{R_0^d}$$

i.e.

$$\hat{C}_0 = \frac{b}{1-b}\frac{1}{R_0^d}\left(m\hat{C}_1 - m_r\hat{R}_0\right)$$

In the rational model, we have $C_0 = \frac{b}{1-b}\frac{1}{R_0^d}C_1$ and $C_0 = C_1 = 1$. Hence, $\frac{b}{1-b}\frac{1}{R_0^d} = 1$. We obtain:

$$\hat{C}_0 = m\mathbb{E}_0\left[\hat{C}_1\right] - m_r\hat{R}_0 \tag{61}$$

This is a "discounted Euler equation" (with discount factor m m), i.e. instead of the rational Euler equation

$$\hat{C}_0 = \mathbb{E}\left[\hat{C}_1\right] - \hat{R}_0 \tag{62}$$

Hence, we see that the same factor m gives a power of fiscal policy, and a discounted Euler equation.

8 Appendix: Explicit Solutions

The correspondence between the discrete and continuous time version is that $A = I - \bar{A}\Delta t + o(\Delta t)$ and $\Lambda_k = 1 - \lambda_k \Delta t + o(\Delta t)$ in the limit of small time intervals Δt. I drew the graphs in continuous time.

Continuous time I use the notations $a_t = i_t - r_{nt}$ and $b = (-\sigma, 0)$. In continuous time, the system (42) is

$$0 = \dot{z}_t - \bar{A}z_t + ba_t$$

where $\bar{A}(m) = \begin{pmatrix} \xi & -\sigma \\ -\kappa & \rho \end{pmatrix}$, which has eigenvalues, λ_1 and λ_2, solutions of $\lambda^2 - (\xi + \rho)\lambda + \xi\rho - \kappa\sigma = 0$. Using the stability criterion (46) they are both positive. Hence, the solution is

$$z_t = \mathbb{E}_t[\int_t^\infty e^{-\bar{A}(s-t)} ba_s ds] \tag{63}$$

To make the expression $e^{-\bar{A}(s-t)}$ a bit more explicit, we proceed as follows. We call w_k an eigenvector corresponding to λ_k: $\bar{A}w_k = \lambda_k w_k$. We also find vectors c_k so that $\sum_k w_k c_k' = I$, where I is the identity matrix.

Lemma 8.1 *The solution can be written*

$$z_t = \mathbb{E}_t[\int_t^\infty \sum_{k=1}^{2} e^{-\lambda_k(s-t)} v_k a_s ds]$$

where $v_k := w_k c_k' b$. *Explicitly,* $v_1 = \frac{\lambda_1 - \rho}{\lambda_1 - \lambda_2}\left(1, \frac{\lambda_2 - \rho}{\sigma}\right)$ *and symmetrically for* v_2.

Proof. We diagonalize $\bar{A}$ and write $\bar{A} = V^{-1}D_\lambda V$, where $D_\lambda = diag(\lambda_k)$, and $w_k = V^{-1}\mathsf{e}_k$, with $\mathsf{e}_1 = (1,0)$ and $\mathsf{e}_2 = (0,1)$. We also find vectors c_k so that $\sum_k w_k c_k' = I$ (a "partition of unity"): explicitly, $c_k := V'\mathsf{e}_k$. This gives

$$Ve^{-\bar{A}\tau}V^{-1} = \left(diag\left(e^{-\lambda_k\tau}\right)\right)_{k=1...2} = \sum_k e^{-\lambda_k\tau}\mathsf{e}_k\mathsf{e}_k',$$

Hence, we have

$$\begin{aligned}
z_t &= \mathbb{E}_t[\int_t^\infty e^{-\bar{A}(s-t)}ba_s ds] = \mathbb{E}_t[\int_t^\infty V^{-1}\left(Ve^{-\bar{A}(s-t)}V^{-1}\right)Vba_s ds] \\
&= \mathbb{E}_t[\int_t^\infty V^{-1}\left(\sum_k e^{-\lambda_k\tau}\mathsf{e}_k\mathsf{e}_k'\right)Vba_s ds] \\
&= \mathbb{E}_t[\int_t^\infty \left(\sum_k e^{-\lambda_k\tau}V^{-1}\mathsf{e}_k\mathsf{e}_k'V\right)ba_s ds] = \mathbb{E}_t[\int_t^\infty \sum_{k=1}^2 e^{-\lambda_k(s-t)}w_k c_k' ba_s ds] \\
&= \mathbb{E}_t[\int_t^\infty \sum_{k=1}^2 e^{-\lambda_k(s-t)}v_k a_s ds]
\end{aligned}$$

as $v_k := w_k c_k' b$. $\square$

Discrete time In discrete time, the method is similar. The system is

$$z_t = \mathbb{E}_t\left[A(m)\,z_{t+1}\right] + ba_t$$

which yields, as $A(m)$ has eigenvalues less than 1 in modulus

$$z_t = \sum_{s\geq t}\mathbb{E}_t\left[A(m)^{s-t}\,b(m)\,a_s\right]$$

After diagonalization:

$$z_t = \mathbb{E}_t[\sum_{s=t}^\infty\sum_{k=1}^2 \Lambda_k^{s-t}v_k a_s ds]$$

where $\Lambda_k < 1$ are the eigenvalues of $A(m)$.

9 Appendix: Stability of Equilibrium, Rationality and Old Keynesian Models

Here I extend the analysis to consider at the same time the Old Keynesian, the New Keynesian, and this Behavioral Keynesian model. The question is the following: in those economies, suppose that we're at the ZLB forever; will the economy embark in a deflationary spiral? more generally, when is the equilibrium determinate?

Consider the basic model (Proposition 3.6), and extend it as follows:

$$x_t = M\mathbb{E}_t\left[x_{t+1}\right] + b_d d_t - \sigma\left(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n\right) \tag{64}$$

$$\pi_t = \chi\pi_{t-1} + \beta^f\mathbb{E}_t\left[\pi_{t+1}\right] + \kappa x_t \tag{65}$$

with $\beta^f := M^f\beta$. The new term is $\chi\pi_{t-1}$: inflation is partially backward-looking[33]. For instance, the Taylor (1999) model corresponds to $M = \beta^f = 0$, a backward looking model[34]. I assume that the central bank follows a basic Taylor rule (41) (with $j_t = 0$).

When does this system create determinacy? The answer is as follows.[35]

Proposition 9.1 *(Unified criterion for stability with Old, New and Behavioral Keynesian model). A necessary condition for stability is:*

$$\phi_\pi + \frac{1-\beta^f-\chi}{\kappa}\phi_x + \frac{(1-M)\left(1-\beta^f-\chi\right)}{\kappa\sigma} > 1 \tag{66}$$

which generalizes (44).

To simplify the discussion, let us assume $\phi_x = 0$.

In the old Keynesian Taylor model with $\chi = 1$ and $\beta^f = 0$, the condition (66) becomes $\phi_\pi > 1$, the basic Taylor criterion. In the New Keynesian model $M = 1$, so the condition (66) again is $\phi_\pi > 1$.

In this hybrid behavioral, old and new Keynesian ($M < 1$, χ free), we can obtain stability with

[33] This term could be microfounded with a partial indexation along the lines of Gali and Gertler (1999).

[34] It also replaces $i_t - \mathbb{E}_t\pi_{t+1}$ by $i_t - \pi_t$, and there is a x_{t-1} term in the IS equaiton, but that's a fairly immaterial difference.

[35] This condition is necessary. There are two other conditions that jointly ensure necessity and sufficiency. They are more minor (cf. Woodford (2003, p.670-676) or the general Ruth-Hurwitz criterion), but will be added in a next iteration of this paper.

a fixed interest rate iff

$$\frac{(1-M)\left(1-\beta^{f}-\chi\right)}{\kappa\sigma}>1$$

Hence, the economy is stable under a fixed rate because of bounded rationality.

If the economy is at the ZLB, *we avoid the deflationary spirals because of bounded rationality.* We need $M<1$, and also $\beta^{f}+\chi<1$, i.e. current inflation is not very responsive to its past and future values.[36] We also need a high enough degree of price stickiness (κ low) and people not to react too much to the interest rate (σ low). Those features, in turn, are guarantied by enough bounded rationality (e.g. in Proposition 3.6, $\kappa=\bar{\kappa}m^{f}$).

Intuitively, bounded rationality makes people's decision less responsive to the future (and in old Keynesian model, to the past). As a result, it reduces the degree of complementarities, and we can more easily have only one equilibrium (this is a quite general point).

As the same time, this model does features long run Fisher neutrality when it is enriched along the lines of Proposition 5.2.

10 Appendix: Proofs

Proof of Proposition 3.1 For consumption, this is simply a re-expression of Lemma 4.2 in Gabaix (2016), adapting the notations and the timing.

Labor supply is $g'(N_t)=\omega_t u'(c_t)$, i.e. $N_t^{\phi}=\omega_t c_t^{-\gamma}$. Taking logs and deviations from the constant values, $\phi\frac{\hat{N}_t}{\bar{N}}=\frac{\hat{\omega}_t}{\bar{\omega}}-\gamma\frac{\hat{c}_t}{c_t^{d}}$.

Proof of Proposition 3.4 *Derivation of (30).* This comes naturally for the general formalism. Call $z_s=(B_s,d_s,d_{s+1},d_{s+2},...)$ the state vector (more properly, the part of it that concerns deficits). Under the rational model, $z_{s+1}=Hz_s$ for a matrix $H:(Hz)(1)=z(1)+Rz(2)$ and $(Hz)(i)=z(i+1)$ for $i>0$, where $z(i)$ is the $i-$th component of vector z. Under the cognitive discounting model simulated by the agent at time t, set $z_t^{d}=(B_t,0,0,...)$ and the subjective model $z_s=z_t^{d}+\bar{m}H\left(z_s-z_t^{d}\right)$. This capture that the agent "sees" the debt B_t, but more dimly the deficits d_t.We also have

$$T_s=-\frac{r}{R}B_s+d_s=e^{T}z_s \text{ with } e^{T}:=\left(-\frac{r}{R},1,0,0,...\right)$$

[36] The Taylor model does feature a deflationary spiral, because it has $\chi=1$.

So,

$$\begin{aligned}
\mathbb{E}_t^{BR}[T_s] &= \mathbb{E}_t^{BR}\left[e^T \cdot z_s\right] = e^T \cdot \mathbb{E}_t^{BR}[z_s] = e^T \cdot \left(z_t^d + (\bar{m}A)^{s-t}\left(z_s - z_t^d\right)\right) \\
&= e^T \cdot \left(z_t^d + \bar{m}^{s-t}\mathbb{E}_t^{\text{rat}}\left[z_s - z_t^d\right]\right) = -\frac{r}{R}B_t + \bar{m}^{s-t}\left(T_s + \frac{r}{R}B_t\right). \\
&= -\frac{r}{R}B_t + \bar{m}^{s-t}\left(d_s - r\sum_{u=t}^{\tau-1} d_u\right).
\end{aligned}$$

Derivation of (31). We have:

$$\begin{aligned}
x_t &= \frac{r}{R}k_t + \mathbb{E}_t^{BR}\left[\sum_{s\geq t}\frac{1}{R^{s-t}}b_y\left(x_s + \mathcal{T}_s\right)\right] \\
&= \frac{r}{R}B_t + b_y\sum_{s\geq t}\frac{\mathbb{E}_t^{BR}[\mathcal{T}_s]}{R^{s-t}} + \mathbb{E}_t^{BR}\left[\sum_{s\geq t}\frac{\bar{m}^{s-t}}{R^{s-t}}m_y b_y x_s\right] \\
&= \frac{r}{R}B_t + \frac{r}{R}\sum_{s\geq t}\frac{-\frac{r}{R}B_t + m_y\bar{m}^{s-t}\left(d_s - r\sum_{\tau=t}^{s-1}d_\tau\right)}{R^{s-t}} + \mathbb{E}_t^{BR}\left[\sum_{s\geq t}\frac{\bar{m}^{s-t}}{R^{s-t}}m_y b_y x_s\right] \\
&= \mathbb{E}_t^{BR}\left[\sum_{s\geq t}\frac{\bar{m}^{s-t}}{R^{s-t}}m_y b_y\left(x_s + d_s - r\sum_{\tau=t}^{s-1}d_\tau\right)\right]
\end{aligned}$$

We see that the impact of B_t cancels out, a form of Ricardian equivalence. Old debt B_t does not make the agent feel richer. But a new deficit today (d_t) does.

This implies:

$$x_t = m_y\frac{r}{R}\left(x_t + Ad_t\right) + \frac{\bar{m}}{R}x_{t+1}$$

with

$$A = 1 - r\sum_{s\geq t+1}\frac{\bar{m}^{s-t}}{R^{s-t}} = 1 - r\frac{\frac{\bar{m}}{R}}{1-\frac{\bar{m}}{R}} = 1 - \frac{r\bar{m}}{R-\bar{m}} = \frac{R(1-\bar{m})}{R-\bar{m}}$$

So, rearranging as in the derivation leading up to Proposition 3.3,

$$x_t = \frac{1}{R - m_y r}\left(Arm_y d_t + \bar{m}x_{t+1}\right) = b_d d_t + \frac{\bar{m}}{R - m_y r}x_{t+1}$$

with $b_d = \frac{rm_y}{R-m_y r}\frac{R(1-\bar{m})}{R-\bar{m}}$.

Proof of Proposition 3.5 The proof follows the steps and notations of Gali (2015, Chapter 3). I simplify the matters by assuming constant return to scale ($\alpha = 0$ in Gali's notations). [37] So,

[37] The expressions are similar (and available upon request) in the case where $\alpha \in [0, 1)$.

the marginal cost at $t+k$ is simply ψ_{t+k}, not a more complicated $\psi_{t+k|k}$.

Notations. When referring to equation 10 of that chapter 3 in Gali (2015), I write "equation (G10)" and do the same for (G11) and other equations. Lower-case letters denotes logs. I replace his coefficient of relative risk aversion (σ in his notations) by γ (as in $u'(C) = C^{-\gamma}$).

The law of motion is correctly perceived, but in the profit function, firm see only a fraction m^f of the (variable component) of the markup: they replace the real markup $\psi_{t+} - p_t$ by $m^f(\psi_{t+k} - p_t) + (1 - m^f)\left(\psi_{t+k}^d - p_t^d\right)$.

Firms can reset there price with probability $1-\theta$. Mimicking Galí's calculations, with behavioral firms, shows that the firm i's optimal price p_t^* satisfies:

$$p_t^* - p_t = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k \mathbb{E}_t^{BR}[\psi_{t+k} - p_t]$$

Given our assumption that firms underperceive the departure of the markup from the baseline:

$$p_t^* - p_t = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k \bar{m}^k m_f \mathbb{E}_t[\psi_{t+k} - p_t]$$

It is useful to define the following constants:

$$\bar{\lambda} := (1 - \theta)(1 - \beta\theta) m^f \tag{67}$$

$$C := (1 - \theta) m^f \tag{68}$$

$$C' = (1 - \theta) \frac{1 - \beta\theta}{1 - \beta\theta\bar{m}} m^f \tag{69}$$

Equation (G15) still holds, with $\mu_t := p_t - \psi_t$, and becomes simply (because of the CRS assumption, $\alpha = 0$; we have $\psi_{t+k|t} = \psi_{t+k}$ simply)

$$\psi_{t+k} = p_{t+k} - \mu_{t+k}$$

so

$$p_t^* - p_t = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta\bar{m})^k m^f \mathbb{E}_t[p_{t+k} - \mu_{t+k} - p_t]$$

i.e., using $\pi_t = (1 - \theta)(p_t^* - p_{t-1}) = \frac{1-\theta}{\theta}(p_t^* - p_t)$,

$$\frac{\theta}{1 - \theta}\pi_t = p_t^* - p_t = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta\bar{m})^k m^f \mathbb{E}_t[p_{t+k} - p_t - \mu_{t+k}]$$

which is a close cousin of the equation right before (G16). Using $\bar{\lambda} = (1-\theta)(1-\beta\theta)\, m^f$ (when $m^f = 1$, $\bar{\lambda}$ is Gali's λ times θ)

$$\begin{aligned}
\theta\pi_t &= \sum_{k\geq 0} (\beta\theta\bar{m})^k\, m^f \mathbb{E}_t \left[(1-\beta\theta)(1-\theta)(p_{t+k}-p_t)\right] - \sum_{k\geq 0} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\bar{\lambda}\mu_{t+k}\right] \\
&= CA - \sum_{k\geq 0} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\bar{\lambda}\mu_{t+k}\right]
\end{aligned}$$

with

$$\begin{aligned}
A &:= (1-\beta\theta) \sum_{i\geq 0} (\beta\theta\bar{m})^i\, \mathbb{E}_t \left[p_{t+i}-p_t\right] = (1-\beta\theta) \sum_{i\geq 0} (\beta\theta\bar{m})^i\, \mathbb{E}_t \left[\pi_{t+i} + ... + \pi_{t+1}\right] \\
&= (1-\beta\theta) \sum_{k\geq 1} \mathbb{E}_t \left[\pi_{t+k} \sum_{i\geq k} (\beta\theta\bar{m})^i\right] \\
&= \frac{1-\beta\theta}{1-\beta\theta\bar{m}} \sum_{k\geq 1} \mathbb{E}_t \left[(\beta\theta\bar{m})^k\, \pi_{t+k}\right]
\end{aligned}$$

Hence, $\theta\pi_t = CA - \sum_{k\geq 0} (\beta\theta)^k\, \mathbb{E}_t \left[\bar{\lambda}\mu_{t+k}\right]$ gives: (using $C' := C\frac{1-\beta\theta}{1-\beta\theta\bar{m}}$)

$$\begin{aligned}
\theta\pi_t &= C' \sum_{k\geq 1} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\pi_{t+k}\right] - \sum_{k\geq 0} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\bar{\lambda}\mu_{t+k}\right] \\
&= C'\beta\theta\bar{m}\mathbb{E}_t \left[\pi_{t+1}\right] - \bar{\lambda}\mu_t + \left\{ C' \sum_{k\geq 2} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\pi_{t+k}\right] - \sum_{k\geq 1} (\beta\theta\bar{m})^k\, \mathbb{E}_t \left[\bar{\lambda}\mu_{t+k}\right] \right\} \\
&= C'\beta\theta\bar{m}\mathbb{E}_t \left[\pi_{t+1}\right] - \bar{\lambda}\mu_t + \beta\theta\bar{m} \left\{\theta\mathbb{E}_t \left[\pi_{t+1}\right]\right\} \\
&= \beta\theta\bar{m} (\theta + C')\, \mathbb{E}_t \left[\pi_{t+1}\right] - \bar{\lambda}\mu_t
\end{aligned} \tag{70}$$

i.e. we obtain the key equation (which is a behavioral version of (G17)):

$$\pi_t = \beta M^f \mathbb{E}_t \left[\pi_{t+1}\right] - \lambda\mu_t \tag{71}$$

with

$$\begin{aligned}
M^f &:= \bar{m} (\theta + C') = \bar{m} \left[\theta + (1-\theta)\frac{1-\beta\theta}{1-\beta\theta\bar{m}} m^f\right] \\
\lambda &:= \frac{\bar{\lambda}}{\theta} = \frac{(1-\theta)(1-\beta\theta)\, m^f}{\theta}
\end{aligned}$$

When $m^f = 1$, we have

$$\frac{M}{\bar{m}} = \theta + (1-\theta)\frac{1-\beta\theta}{1-\beta\theta\bar{m}} = \frac{\theta - \beta\theta^2\bar{m} + 1 - \theta - \beta\theta + \beta\theta^2}{1-\beta\theta\bar{m}} = \frac{1-\beta\theta\left(1-(1-\bar{m})\theta\right)}{1-\beta\theta\bar{m}}.$$

The rest of the proof is as in Gali. The labor supply is still (3), $N_t^{\phi} = \omega_t C_t^{-\gamma}$, and as the resource constraint it $C_t = N_t$, $\omega_t = C_t^{-(\gamma+\phi)}$, i.e. $\hat{\omega}_t = -(\gamma+\phi)x_t$. This gives $\mu_t = \hat{\omega}_t = -(\gamma+\phi)x_t$, and we obtain the behavioral version of (G22):

$$\pi_t = \beta M^f \mathbb{E}_t[\pi_{t+1}] + \kappa x_t$$

with $\kappa = \lambda(\gamma+\phi)$, i.e.

$$\kappa = \bar{\kappa} m^f \tag{72}$$

$$\bar{\kappa} = \left(\frac{1}{\theta} - 1\right)(1-\beta\theta)(\gamma+\phi). \tag{73}$$

Proof of Proposition 4.2 We have: $\dot{x}_t = \xi x_t - \sigma(\underline{r} + \pi_t)$. To solve for the system, note:

$$\begin{aligned}\ddot{x}_t &= \xi\dot{x}_t - \sigma\dot{\pi}_t = \xi\dot{x}_t - \sigma(\rho\pi_t - \kappa x_t) = \xi\dot{x}_t + \sigma\kappa x_t - \rho\sigma\pi_t \\ &= \xi\dot{x}_t + \sigma\kappa x_t + \rho(\dot{x}_t - \xi x_t + \sigma\underline{r}) = (\rho+\xi)\dot{x}_t + (\sigma\kappa - \rho\xi)x_t + \rho\sigma\underline{r}\end{aligned}$$

so that:

$$\ddot{x}_t - (\rho+\xi)\dot{x}_t + (\rho\xi - \sigma\kappa)x_t = \rho\sigma\underline{r} \tag{74}$$

and the boundary conditions are: $x_T = \pi_T = 0$, hence (taking the left derivative):

$$x_T = 0, \dot{x}_T = -\sigma\underline{r}. \tag{75}$$

To analyze (74), we look for solutions of the type $x_t = e^{\lambda t}$. Call $\lambda \leq \lambda'$ the two roots of:

$$\lambda^2 - (\rho+\xi)\lambda + \rho\xi - \sigma\kappa = 0 \tag{76}$$

Then, with $D = \frac{\rho\sigma\underline{r}}{\rho\xi - \sigma\kappa}$ the solution is:

$$x_t = D + \frac{(D\lambda - \sigma\underline{r})e^{\lambda'(t-T)} - (D\lambda' - \sigma\underline{r})e^{\lambda(t-T)}}{\lambda' - \lambda}. \tag{77}$$

In the traditional case, $\xi = 0$, so that $\lambda < 0 < \lambda'$. As $D > 0$, this implies that, as $t \to -\infty$, $x_t \to -\infty$. We obtain an unbounded large recession. This is the logic that Werning (2012) analyzes.

However, take the case where cognitive myopia is strong enough, $\xi > \frac{\sigma\kappa}{\rho}$. Then, both roots of (76) are positive. Hence, we have a bounded recession. Indeed, as $D < 0$ in that case, x_t is increasing in t. $\square$

Proof of Proposition 9.1 The state vector is $z_t = (x_t, \pi_t, \pi_{t-1})$. We can write $\mathbb{E}_t z_{t+1} = B z_t$, with

$$B = \begin{pmatrix} \frac{\kappa\sigma+\beta^f(1+\sigma\phi_x)}{M\beta^f} & \frac{\sigma(\beta\phi_\pi-1)}{M\beta^f} & \frac{\sigma\chi}{M\beta^f} \\ \frac{-\kappa}{\beta^f} & \frac{1}{\beta^f} & \frac{-\chi}{\beta^f} \\ 0 & 1 & 0 \end{pmatrix} \tag{78}$$

Consider also the characteristic polynomial of B, $\Phi(\Lambda) = \det(\Lambda I_3 - B)$, which factorizes factorizes $\Phi(\Lambda) = \prod_{i=1}^{3}(\Lambda - \Lambda_i)$, where Λ_i are the eigenvalues of B.

When $\chi \neq 0$, inflation π_{t-1} is now a predetermined variable, not a jump variable. Hence, for determinacy, B needs to have 1 eigenvalue less than 1 in modulus (this corresponding to the predetermined variable π_{t-1}), and 2 greater than 1 (corresponding to the free variables x_t, π_t). This implies that $\Phi(1) > 0$. Some calculations show that this is equivalent to (66), which is exactly the stability criterion (44), replacing β^f by $\beta^f + \chi$. $\square$

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