(Dis)Inflation Targeting

Mridula Duggal[*] and *Luis Eduardo Rojas*[†]

Abstract

TO BE COMPLETED

[*]Universitat Autònoma de Barcelona (UAB) and Barcelona Graduate School of Economics (BGSE). Email: mriduduggal@gmail.com.

[†]Markets, Organisation and Votes in Economics(MOVE), Universitat Autònoma de Barcelona (UAB), Barcelona Graduate School of Economics (BGSE). Email:luis.rojas@MOVEbarcelona.eu

1 Introduction

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2 Motivation

As Figure 1 and Figure 2 portray, Colombia and Brazil are examples of the process of disinflation that economies experience when adopting Inflation Targeting. Both countries adopted inflation targeting when they were experiencing episodes of high inflation post the oil crisis in the late 1980s and the East Asian Financial crisis in 1997.

Figure 1: Inflation Target, Actual Inflation and Inflation Expectations: Colombia

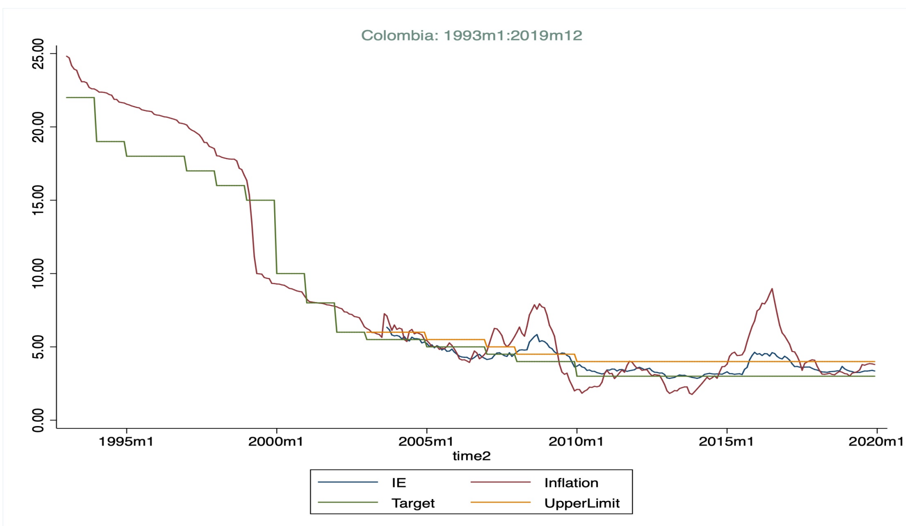
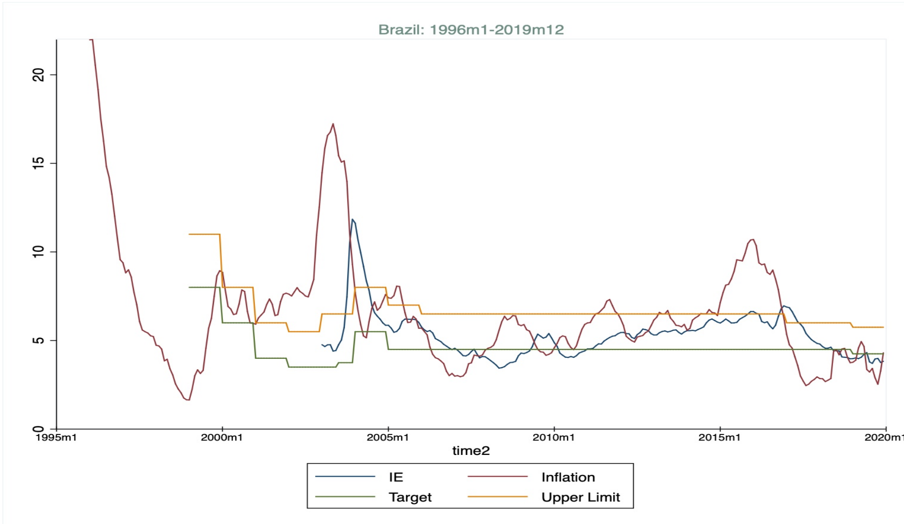


Figure 2: Inflation Target, Actual Inflation and Inflation Expectations: Brazil



3 Model

The preferences of the government are given by

$$z_t = \frac{a}{2}\left(\pi_t\right)^2 - b_t(\pi_t - \pi_t^e)$$

and expectations formation are as follows:

- If $\pi_t \leq \pi_t^e$ then $\pi_{t+1}^e = \max\left\{\gamma\pi_t, \pi_{t+1}^*\right\}$, where π_{t+1}^* is the announced inflation target at $t+1$.
- For $\gamma = 0$ we have the Barro-Gordon case.
- If $\pi_t > \pi_t^e$ then $\pi_{t+1}^e = \frac{\bar{b}}{a}$ the discretionary outcome ($\bar{b}$ is the expected b_{t+1})

Case 1: Disinflation follow the threshold $\pi_{t+1}^* = \gamma\pi_t$

In this case the discounted value for the government is

$$V_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{a}{2}\left(\gamma^t \frac{b}{a}\right)^2\right] = \sum_{t=0}^{\infty} \beta^t \left[\frac{b^2}{2a}\gamma^{2t}\right] = \frac{b^2}{2a}\frac{1}{1-\beta\gamma^2}$$

and starting form any arbitrary $t = s$ we have

$$V_s = \sum_{t=0}^{\infty} \beta^s \left[\frac{a}{2} \left(\gamma^{s+t} \frac{b}{a} \right)^2 \right] = \sum_{t=0}^{\infty} \beta^s \left[\frac{b^2}{2a} \gamma^{2(s+t)} \right] = \frac{b^2}{2a} \gamma^{2(s)} \frac{1}{1 - \beta\gamma^2} = \gamma^{2(s)} V_0$$

Is it worthy to deviate? the value in case the government deviates is

$$\begin{aligned} V_{d,s} &= \frac{a}{2} \left(\frac{b}{a} \right)^2 - b_t \left(\frac{b}{a} - \gamma^s \frac{b}{a} \right) + \beta V_0 \\ &= \frac{b^2}{2a} - \frac{b^2}{a} (1 - \gamma^s) + \beta V_0 \end{aligned}$$

then it is optimal to deviate if

$$\begin{aligned} \frac{b^2}{2a} - \frac{b^2}{a}(1 - \gamma^s) + \beta V_0 &\leq \gamma^{2(s)} V_0 \\ \frac{b^2}{2a} - \frac{b^2}{a}(1 - \gamma^s) &\leq \left(\gamma^{2(s)} - \beta \right) V_0 \\ \frac{\frac{b^2}{a} \left(\gamma^s - \frac{1}{2} \right)}{(\beta - \gamma^{2(s)})} &\leq V_0 \\ \frac{(2\gamma^s - 1)}{(\beta - \gamma^{2(s)})} &\leq \frac{1}{1 - \beta\gamma^2} \end{aligned}$$

with $\gamma = 0$ we recover the Barro-Gordon case where zero inflation is not sustainable as

$$\frac{(-1)}{(\beta)} \leq 1$$

holds.

When $s = \infty$ we have

$$\begin{aligned} \frac{b^2}{2a} - \frac{b^2}{a} + \beta V_0 &\leq 0 \\ V_0 &\leq \frac{1}{\beta} \frac{b^2}{2a} \\ \frac{b^2}{2a} \frac{1}{1 - \beta\gamma^2} &\leq \frac{1}{\beta} \frac{b^2}{2a} \\ \gamma^2 &\leq \frac{1}{\beta} - 1 \end{aligned}$$

this sets a lower bound on γ. So to have no incentives to deviate at the limit it has to be that

$$\gamma^2 \geq \frac{1}{\beta} - 1$$
$$\gamma \geq \left(\frac{1}{\beta} - 1\right)^{1/2}$$

Going back to the previous condition

$$\begin{aligned}
\frac{b^2}{2a} - \frac{b^2}{a}(1-\gamma^s) &\leq \left(\gamma^{2(s)} - \beta\right) V_0 \\
-\frac{1}{2}\frac{b^2}{a} &\leq \left(\gamma^{2(s)} - \beta\right) V_0 - \frac{b^2}{a}\gamma^s \\
-\frac{1}{2} &\leq \left(\gamma^{2(s)} - \beta\right) \frac{1}{2}\frac{1}{1-\beta\gamma^2} - \gamma^s
\end{aligned}$$

is there a γ for which this condition is not satisfied for any s? In this case we would be able to implement the efficient allocation.

Yes! There are values of γ for which the condition above is never satisifed for any s and consequently $\pi = 0$ can be implemented in the long run. See the numerical solutions in the matlab file test.m

The minimum level of inflation achievable with $\gamma = 0$ is given by

$$\pi^{BG} = \frac{1-\beta}{1+\beta}\frac{b}{a}$$

and the value is then given by

$$V^{BG} = \frac{1}{2}\frac{b^2}{a}\frac{1-\beta}{(1+\beta)^2}$$

so the Barro Gordon porvides higher welfare ex-ante than our case with $\gamma > 0$. Down the road for $s > 1$ then the welfare in our case is greater and at the limit $s \to \infty$ the loss in our case is zero.

So the plan is not dinamically consistent without the constraint of γ.

3.1 Optimal Policy

We are asusming that the solution to the problem is time consistent by assuming that γ_t is expgenous and deterministic and the policy function that solves the optimality conditions does not depend on the period which the central bank optimises. NEED TO CHECK!

Expectation Formation

$$\pi^e_{t+1} = \gamma_t \pi_t + (1 - \gamma_t)\pi^a_{t+1} \tag{1}$$

IS Curve

$$y_t = b_t(\pi_t - \pi^e_t) \tag{2}$$

Updating Equation

$$\gamma_{t+1} = (1 - \theta)\gamma_t + \theta(\pi_{t+1} - \pi^a_{t+1}) \tag{3}$$

Preferences of the Central Bank are given by,

$$z_t = \frac{a}{2}(\pi_t)^2 - b_t(\pi_t - \pi^e_t) = \frac{a}{2}(\pi_t)^2 - y_t \tag{4}$$

Central bank is optimising the inflation by announcing the target, given inflation expectations. It is an infinite hoirzon problem and they will optimise every period.

The central bank problem is stated as follows,

$$\min_{\{\pi_t, \pi^a_{t+1}, \pi^e_{t+1}, \gamma_{t+1}\}^\infty_{t=0}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{a}{2}(\pi_t)^2 - b_t(\pi_t - \pi^e_t)\right] \tag{5}$$

subject to (1), (2) and (3) and a given γ_0.

The Lagrangian is given by,

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{a}{2}(\pi_t)^2 - b_t(\pi_t - \pi^e_t)] \\ + \lambda_{1t}[\pi^e_{t+1} - \gamma_t \pi_t - (1 - \gamma_t)\pi^a_{t+1}] \\ + \lambda_{2t}[\gamma_{t+1} - (1 - \theta)\gamma_t - \theta(\pi_{t+1} - \pi^a_{t+1})\Big] \end{aligned} \tag{6}$$

The first order conditions at every $t \geq 0$ are,

$$
\begin{aligned}
\pi_t :&\ \ a\pi_t - b_t - \lambda_{1t}\gamma_t = 0 && (7)\\
\gamma_{t+1} :&\ \ \lambda_{2t} - E[\lambda_{1t+1}\pi_{t+1} + \lambda_{1t+1}\pi^a_{t+2} + (1-\theta)\lambda_{2t+2}] = 0 && (8)\\
\pi^e_{t+1} :&\ \ b_{t+1} + \lambda_{1t} = 0 && (9)\\
\pi^a_{t+1} :&\ \ -\lambda_{1t}(1-\gamma_t) + \theta\lambda_{2t} = 0 && (10)
\end{aligned}
$$

Combining (7) and (9), we get

$$\pi_t = \frac{1}{a}\big[b_t + b_{t+1}\gamma_t\big] \tag{11}$$

If $\gamma_t = 0\ \forall t$ then $\pi_t = \frac{b_t}{a}$, which is the Barro-Gordon equilibrium.

If we assume that b is constant, then for any $\gamma > 0$, inflation will grow faster than one-to-one. For $\gamma < 0$, the economy will enter a disinflationary path. At $\gamma = -1$, we will have $\pi_t = 0$

References

Echavarría, J. J., Rodríguez, N., and Rojas, L. E. (2011). The target of the central bank and inflation persistence in colombia. *Ensayos sobre POLÍTICA ECONÓMICA*, 29(65):198–222.