(Dis)Inflation Targeting

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The preferences of the government are given by

$$z_t = \frac{a}{2}(\pi_t)^2 - b_t(\pi_t - \pi_t^e)$$

and expectations formation are as follows:

- If $\pi_t \leq \pi_t^e$ then $\pi_{t+1}^e = \max\left\{\gamma\pi_t, \pi_{t+1}^*\right\}$, where π_{t+1}^* is the announced inflation target at $t+1$. For $\gamma = 0$ we have the Barro-Gordon case.
- If $\pi_t > \pi_t^e$ then $\pi_{t+1}^e = \frac{\bar{b}}{a}$ the discretionary outcome ($\bar{b}$ is the expected b_{t+1})

Case 1: Disinflation follow the threshold $\pi_{t+1}^* = \gamma\pi_t$

IN this case the discounted value for the government is

$$V_0 = \sum_{t=0}^{\infty}\beta^t\left[\frac{a}{2}\left(\gamma^t\frac{b}{a}\right)^2\right] = \sum_{t=0}^{\infty}\beta^t\left[\frac{b^2}{2a}\gamma^{2t}\right] = \frac{b^2}{2a}\frac{1}{1-\beta\gamma^2}$$

and starting form any arbitrary $t = s$ we have

$$V_s = \sum_{t=0}^{\infty}\beta^s\left[\frac{a}{2}\left(\gamma^{s+t}\frac{b}{a}\right)^2\right] = \sum_{t=0}^{\infty}\beta^s\left[\frac{b^2}{2a}\gamma^{2(s+t)}\right] = \frac{b^2}{2a}\gamma^{2(s)}\frac{1}{1-\beta\gamma^2} = \gamma^{2(s)}V_0$$

Is it whorty to deviate? the value in case the government deviates is

$$\begin{aligned} V_{d,s} &= \frac{a}{2}\left(\frac{b}{a}\right)^2 - b_t(\frac{b}{a} - \gamma^s\frac{b}{a}) + \beta V_0 \\ &= \frac{b^2}{2a} - \frac{b^2}{a}(1-\gamma^s) + \beta V_0 \end{aligned}$$

then it is optimal to deviate if

$$\begin{aligned}
\frac{b^2}{2a} - \frac{b^2}{a}(1-\gamma^s) + \beta V_0 &\le \gamma^{2(s)} V_0 \\
\frac{b^2}{2a} - \frac{b^2}{a}(1-\gamma^s) &\le \left(\gamma^{2(s)} - \beta\right) V_0 \\
\frac{\frac{b^2}{a}\left(\gamma^s - \frac{1}{2}\right)}{\left(\beta - \gamma^{2(s)}\right)} &\le V_0 \\
\frac{(2\gamma^s - 1)}{\left(\beta - \gamma^{2(s)}\right)} &\le \frac{1}{1-\beta\gamma^2}
\end{aligned}$$

with $\gamma = 0$ we recover the Barro-Gordon case where zero inflation is not sustainable as

$$\frac{(-1)}{(\beta)} \le 1$$

holds.

When $s = \infty$ we have

$$\begin{aligned}
\frac{b^2}{2a} - \frac{b^2}{a} + \beta V_0 &\le 0 \\
V_0 &\le \frac{1}{\beta}\frac{b^2}{2a} \\
\frac{b^2}{2a}\frac{1}{1-\beta\gamma^2} &\le \frac{1}{\beta}\frac{b^2}{2a} \\
\gamma^2 &\le \frac{1}{\beta} - 1
\end{aligned}$$

this sets a lower bound on γ. So to have no incentives to deviate at the limit it has to be that

$$\begin{aligned}
\gamma^2 &\ge \frac{1}{\beta} - 1 \\
\gamma &\ge \left(\frac{1}{\beta} - 1\right)^{1/2}
\end{aligned}$$

Going back to the previous condition

$$\begin{aligned}
\frac{b^2}{2a} - \frac{b^2}{a}(1-\gamma^s) &\le \left(\gamma^{2(s)} - \beta\right) V_0 \\
-\frac{1}{2}\frac{b^2}{a} &\le \left(\gamma^{2(s)} - \beta\right) V_0 - \frac{b^2}{a}\gamma^s \\
-\frac{1}{2} &\le \left(\gamma^{2(s)} - \beta\right) \frac{1}{2}\frac{1}{1-\beta\gamma^2} - \gamma^s
\end{aligned}$$

is there a γ for which this condition is not satisfied for any s? In this case we would be able to implement the efficient allocation.

Yes! There are values of γ for which the condition above is never satisifed for any s and consequently $\pi = 0$ can be implemented in the long run. See the numerical solutions in the matlab file test.m

The minimum level of inflation achievable with $\gamma = 0$ is given by

$$\pi^{BG} = \frac{1-\beta}{1+\beta}\frac{b}{a}$$

and the value is then given by

$$V^{BG} = \frac{1}{2}\frac{b^2}{a}\frac{1-\beta}{(1+\beta)^2}$$

so the Barro Gordon porvides higher welfare ex-ante than our case with $\gamma > 0$. Down the road for $s > 1$ then the welfare in our case is greater and at the limit $s \to \infty$ the loss in our case is zero.

So the plan is not dinamically consistent without the constraint of γ.

New setup

Expectations

$$\pi^e_{t+1} = \gamma_t \pi_t + (1-\gamma_t)\pi^a_{t+1}$$

IS

$$y_t = b(\pi_t - \pi^e_t)$$

updating exp

$$\gamma_{t+1} = (1-\theta)\gamma_t + \theta(\pi_{t+1} - \pi^a_{t+1})$$