

Question 1**(Basic Unemployment Calculations)**

There are two states: U for unemployment and E for employment.

- With probability $\lambda \in (0, 1)$, a person unemployed today becomes employed tomorrow.
 - With probability $\alpha \in (0, 1)$, a person employed today becomes unemployed tomorrow.
- (a) Let $N \geq 1$ be the number of periods until a currently unemployed person becomes employed. Calculate $\mathbb{E}[N]$.
 - (b) Let $M \geq 1$ be the number of periods until a currently employed person becomes unemployed. Calculate $\mathbb{E}[M]$.
 - (c) Please compute the fraction of time an infinitely lived person can expect to be unemployed and the fraction of time they can expect to be employed.

Question 2**(Multiple Employment States)**

An economy has 3 states for workers:

- U : unemployment.
- V : if they have found a potential employer and are being verified to see if they are a good fit.
- E : if a worker has been verified and is employed.

The probabilities that they jump between these states each period is:

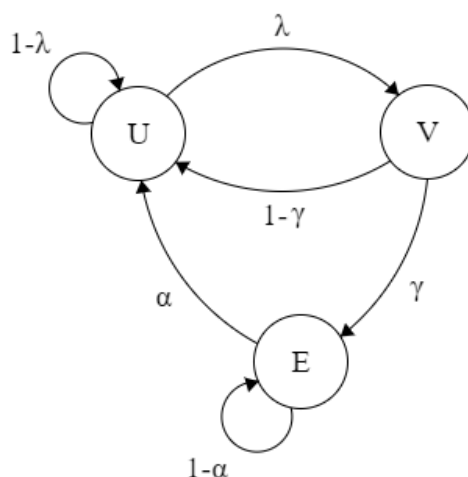


Figure 1: Markov Chain

i.e. probability γ they are a good fit, and the verification takes 1 period.

- (a) Write a Markov transition matrix for this process, P .
- (b) Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of P).
- (c) If a worker is U today, write an expression for the probability they will be employed exactly j periods in the future (considering any possible transitions which end in employment at j periods).¹.
- (d) Assume that $\alpha = 0, \lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist.

Question 3

(Asset Price Forecasts)

Let $y_t \in \mathbb{R}$ be a sequence of dividends such that

$$y_{t+1} = y_t + w_{t+1}$$

where $w_{t+1} \sim N(0, \sigma^2)$ for some $\sigma > 0$. i.e. $\mathbb{E}_t[w_{t+1}] = 0$ and $\mathbb{E}_t[w_{t+1}^2] = \sigma^2$. An agent prices an asset as the expected PDV of dividends,

$$p_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j y_{t+j} \right]$$

Define the forecast error as the deviation in the expected price tomorrow calculated at time t from the actual price tomorrow. i.e.,

$$FE_{t+1|t} \equiv p_{t+1} - \mathbb{E}_t[p_{t+1}]$$

- (a) Setup in our canonical Linear Gaussian State Space model.
- (b) Solve for p_t in terms of y_t and model intrinsics.
- (c) Find the expected forecast error: $\mathbb{E}_t[FE_{t+1|t}] = \mathbb{E}_t[p_{t+1} - \mathbb{E}_t[p_{t+1}]]$
- (d) Find the variance of forecast errors:

$$\begin{aligned} \mathbb{V}_t(FE_{t+1|t}) &\equiv \mathbb{E}_t[FE_{t+1|t}^2] - \left(\mathbb{E}_t[FE_{t+1|t}]\right)^2 \\ &= \mathbb{E}_t[(p_{t+1} - \mathbb{E}_t[p_{t+1}])^2] - (p_{t+1} - \mathbb{E}_t[p_{t+1}])^2 \end{aligned}$$

- (e) Setup the problem recursively as p_t define in terms of p_{t+1} . Test, using your solution from early in the question, if the stochastic process p_t a martingale (i.e., just a random walk in this case)? Recall that a martingale is a process such that $\mathbb{E}_t[p_{t+1}] = p_t$. Give any intuition you can on this result.

¹Note: This is only looking at j periods into the future. i.e. this is **not** the probability that they become at least employed once during the j periods.