

Rational and Adaptive Expectation

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Expectation: why do we care?

Think of basic model of money supply and demand (Cagan-Sargent-Wallace: **CSW**):

$$\frac{M_t}{P_t} = \bar{Y} \exp(-\alpha(\bar{r} + \pi_{t,t+1}^e))$$
$$\rightarrow p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} p_{t,t+1}^e$$

Where:

- ▶ p_t is price level, m_t is money supply and $p_{t,t+1}^e$ is your expectation for future price, all are deviation from s.s.
- ▶ $\pi_{t,t+1}^e = p_{t,t+1}^e - p_t$ is expected inflation.
- ▶ Suppose you are an investor, you want to buy forward contract of some good with price p_t ;
- ▶ Similar cases spans entire macroeconomics: Euler Equation, price setting behavior etc.

Type of Expectations

- ▶ **None**, agents don't forecast at all, or their forecast is purely noise(mean 0 disturbance).
- ▶ **Adaptive Expectation**: Ad-hoc decisions based on previous data (looking backwards).
- ▶ **Full Information Rational Expectation(FIRE)**: Dynamic decisions based on the structure of model, considering all the information at hand.
- ▶ **Bounded Rationality/Behavioral**: Discipline variations on rational expectations (e.g learning, information capacity etc).

Adaptive Expectation

$$p_{t,t+1}^e = F(p_t, p_{t-1} \dots) = \lambda p_t + (1 - \lambda) p_{t-1,t}^e$$

Iterating backwards we have:

$$p_{t,t+1}^e = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i p_{t-i}$$

This is one specific adaptive expectation, can be thought of as every period a fraction of agents λ update their expectation to p_t , whereas the rest don't update.

Adaptive Expectation

Plug into the CSW model we have:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} \lambda \sum_{i=0}^{\infty} (1-\lambda)^i p_{t-i} \quad (1)$$

$$= \frac{1}{1+\alpha(1-\lambda)} m_t + \frac{\alpha\lambda}{1-\alpha(1-\lambda)} \sum_{i=1}^{\infty} (1-\lambda)^i p_{t-i} \quad (2)$$

1. Current price depends on weighted average of past prices (backward looking);
2. temporary shock in m_t has the same effect as permanent one at time t .

Rational Expectation

Rational expectation assumes agents use the knowledge of the model and all the information to form expectation, so their expectation of price use the mathematical expectation:

$$p_{t,t+1}^e = E_t(p_{t+1})$$

As rational expectation is model specific, agents will use the model itself to form this expectation:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t p_{t+1}$$

$$\Rightarrow E_t p_{t+1} = \frac{1}{1+\alpha} E_t m_{t+1} + \frac{\alpha}{1+\alpha} E_t p_{t+2} \quad (3)$$

$$= \frac{1}{1+\alpha} \sum_{j=1}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^{j-1} E_t m_{t+j} \quad (4)$$

Rational Expectation

Plug (4) into CSW model we have:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} E_t p_{t+1} \quad (5)$$

$$= \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} \frac{1}{1+\alpha} \sum_{j=1}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^{j-1} E_t m_{t+j} \quad (6)$$

$$= \frac{1}{1+\alpha} \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j E_t m_{t+j} \quad (7)$$

1. Current price depends on expected future money supply (forward looking);
2. temporary shock in m_t has very different shocks as permanent ones (graph on board).

Now What?

Questions to think about:

1. Rational Expectation seems to be much more sophisticated than Adaptive Expectation, does it mean with FIRE we are always right?

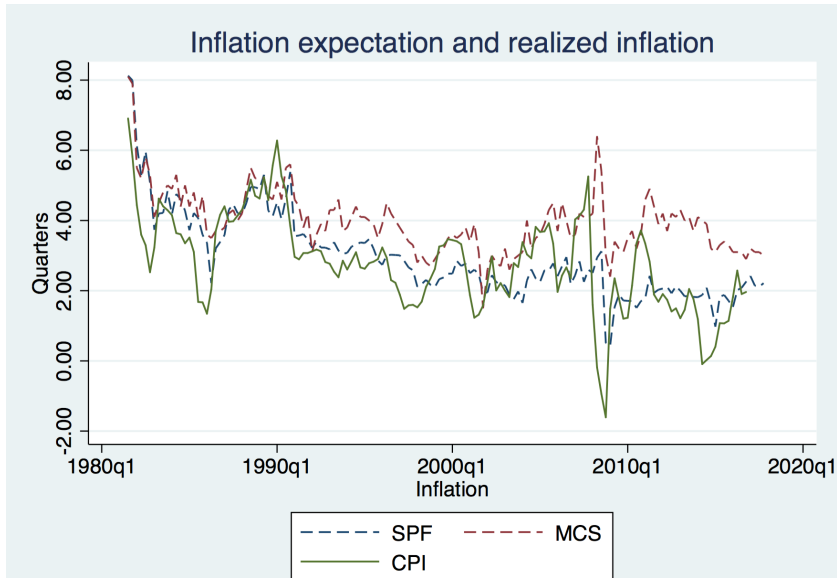
A: No, Forecasting Error: $FE_t = p_{t+1} - p_{t,t+1}^e$, both FIRE and Adaptive can have non-zero FE, but FIRE has zero FE **in expectation**. This means it has no systematic forecast error.

2. What if you are an investor in the market, does it mean you can do better with the FIRE rule we derived in (7)?

A: No! This is because how most people form expectation is **part of market structure**, a rational expectation in the market where most people use adaptive expectation will be different from (7)!

3. Now why do we care as economist?

Move Forward: Breaking FIRE



Move Forward: Breaking FIRE

	MSC	SPF	VAR2
FE_t	2.12	1.1	0.45
MSE_t	4.5	2.34	1.12

	(1) $Corr(E\pi_t, Eun_t)$	(2) $Corr(E\pi_t, Ebus_t)$	(3) $Corr(Eun_t, Ebus_t)$
MSC	0.14	-0.25	-0.63
SPF	0.02	-0.01	-0.79
	$Corr(\pi_t, un_t)$	$Corr(\pi_t, RGDP_t)$	$Corr(un_t, RGDP_t)$
Real time data	-0.00	0.08	-0.78

Table 1: Correlation Recovered Expectation v.s. Realized Data, 1981q3-2017q4

Questions to ask

1. Do agents really have rational expectation? (Huge literature on this)
2. What assumption have they failed?
 - × Do they know the structure of the model? (Learn about the structure)
 - × Then how can they know the model parameters? (Learn about the parameter)
3. If FIRE is too much to assume, then what affects their expectation? If it's adaptive (or some simple structure), do they even know what happened recently? (Inattention Literature)

News



Figure 2: Fraction that heard of news

News

