Question 1

(**Welfare Cost of Financial Frictions**)

Let $\beta = .95$, $R = 1.04$.

Scenario 1 for Consumer The consumer maximizes the following welfare

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{1}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{2}$$

$$y_t = y_0 \delta^t, \quad \forall t \geq 0 \tag{3}$$

$$F_0 = 0 \tag{4}$$

$$\text{(transversality condition)} \tag{5}$$

Scenario 2 for Consumer The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of y_0 is potentially different (defined as y_0^{NB}. Define the PDV of utility (i.e., the welfare) of this as U^{NB}

(a) Assume $\delta = 1.02, y_0 = 1$, and $y_0^{NB} = 1$. Calculate U and U_{NB}.

(b) Let $y_0 = 1$. Now find a y_0^{NB} such that $U = U^{NB}$. The difference between y_0 and y_0^{NB} is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

(c) Maintain $y_0 = 1$. Now, let $\beta = .99, R = 1.04$, and $\delta = 1.01$. What is c_0 and F_1 here under Scenario 1? Repeat part (b) to find y_0^{NB} such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

Question 2

(**Sequential and Recursive**)

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0, B \geq 0, \beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{6}$$

$$\text{s.t. } F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \tag{7}$$

$$F_{t+1} \geq -B \tag{8}$$

$$F_0 = 0 \tag{9}$$

$$\text{(transversality condition)} \tag{10}$$

(a) Derive the euler equation as an inequality, and the condition for it holding with equality.

(b) Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

(c) Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(d) Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

(e) Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

(f) Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1.[1] Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$.

[1]Note that this equation deliberately is avoiding any t subscripts! This makes it a truly recursive expression.

Question 3

(Search with Firing)

Take the search model we did in class with an endogenous choice of accepting a job, but add in the following elements:

- If you are unemployed and reject a wage, there is only a $\theta \in (0, 1)$ probability to get a wage offer. Otherwise, you can recall a wage you previously rejected last period. (hint: they would never choose to recall a wage in equilibrium, but it may help you write down the Bellman equation cleanly.)
- There is an exogenous probability $\alpha \in (0, 1)$ of being fired at the end of any period you are working. You then draw a new wage as an unemployed agent entering the next period with certainty (i.e., bypass the θ probability going into the next period.

To summarize the timing here: As in our example in class, let $v(w)$ be the value of coming a period unemployed with wage offer w and when they are about to choose to accept or reject.

If they reject an offer, they gain unemployment insurance c, and have the probability θ to gain the draw with expected value

$$Q = \int_0^B v(\hat{w}) f(\hat{w}) \mathrm{d}\hat{w}$$

If they accept they gain the wage w that period, and then have the α chance of being fired as they come into the next period—at which point they get the wage offer draw with certainty, as discussed. (Hint: if they are not fired, the value next period is $v(w)$, the same as if they were first offered w.)

(a) Draw a Markov chain with two states E and U. Let the probability of staying unemployed be λ which will end up endogenous. You will also need the α transition probability

(b) Write the value of a worker with wage offer w who chooses to reject the offer (hint: if they reject they don't necessarily gain a new draw, but could have $v(w)$ as their value next period since they can recall the rejected w).

(c) Write the value of a worker accepting the offer of w. (hint: may need to be recursive now, unlike what we did in class)

(d) Combine the values in the previous two parts to form a Bellman equation with $v(w)$ and the max for the choice.

(e) Write the equation for an indifference point $\bar{w}$, where they are at the threshold of accepting (or rejecting) the wage.[2]

(f) Assuming that you could numerically solve the previous equation to find a $\bar{w}$, what is the expression for the stationary proportion of unemployed workers as a function of $\alpha, \theta, f(\cdot)$, and $\bar{w}$. (Hint: derive the λ, then use older notes on unemployment. However, recall that the $U \to E$ transition only occurs if both a wage offer and endogenous acceptance occur).

[2]As in the case of class, the Bellman equation could be reorganized to eliminate the $v(\cdot)$ function to give an implicit equation in $\bar{w}$ and parameters. This is significantly trickier than what we did, so only try to simplify the indifference equation if you wish.

Question 4

(GE with 2 goods)

A household supplies 1 unit of labor inelastically (i.e., doesn't value leisure) and consumes two types of goods in quantities: c_1 (e.g., apples) and c_2 (e.g., oranges).[3] The preferences are

$$u(c_1, c_2) = a c_1^{\alpha} c_2^{1-\alpha}$$

with $\alpha \in (0, 1)$ and $a > 0$.

The production technology uses labor (ℓ) as its only input and can produce good 1 or good 2 with separate constant returns to scale production functions: $y_1 = z_1 \ell_1$ and $y_2 = z_2 \ell_2$. with productivities $z_1 > 0$ and $z_2 > 0$.

The total labor allocated to producing each good cannot add to more than the total labor endowment: $\ell_1 + \ell_2 = 1$ (i.e., holds at equality due to inelastic supply).

(a) Carefully define a feasible allocation in this economy

(b) Formulate the planner's problem. (hint: how many constraints and choice variables are there compared to the examples in class?)

(c) Provide a system of equations which would solve the planning problem, and solve for the allocation.

Now, assume that firms operate the production technology, and that both firms and consumers are price takers for the market prices of labor and goods. Let q_1 and q_2 be the prices of the 2 goods. Let w be the wage of the consumer per unit of labor.

(d) What is a price system for this economy?

(e) What is the consumer's budget constraint? What are the choice variables of the consumer? (Hint: careful on what they are not allowed to choose). Write down the consumer's problem.

(f) What are the profits of firm 1 with output y_1? What are the choice variables of the firm? (Hint: again, careful on what they are not allowed to choose). Write down the full profit maximization problem of firm type 1. Repeat for firm type 2.

(g) Carefully define a competitive equilibrium for this economy.

(h) Solve for the competitive equilibrium. Does this decentralize the planner's problem?

[3] Hint: Equations combining "apples" and "oranges" directly without any prices may be correct, but they may not be useful as they are physically district objects.