Search and the Labor Market

Jesse Perla
University of British Columbia

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1 Quick Review

1.1 Markov Chain Model of Unemployment

Recall model of unemployment:

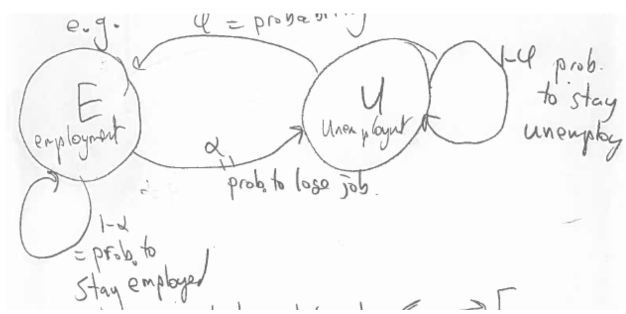

Figure 1: Lake Model with Markov Chains

where ϕ probability of U to E transition should come from consumer decisions.

- This model will endogenize based on dynamic decisions of consumers looking for, and potentially rejecting, jobs.
- This, in turn, endogenizes unemployment rate.

1.2 Random Variables Review

- p is a non-negative random variable with pdf (probability density function) $f(p)$ and cdf (cumulative distribution function):

$$F(p) \equiv \int_0^p f(s)ds \tag{1}$$

Assume $F(0) = 0$, $F(B) = 1$, where B is the upper bound.

- If p has continuous values on $[0, B]$, then:

$$\underbrace{\mathbb{E}[p]}_{\text{mean of } p} = \int_0^B p f(p) dp \tag{2}$$

An alternative formula using the CDF,[1]

$$\mathbb{E}[p] = \int_0^B (1 - F(p))\, dp \tag{3}$$

- Consider two <u>independent</u> random variables: $\{p_1, p_2\}$, both from F. Event $\underbrace{\{p_1 < p\}}_{\text{follows } F(p)} \underbrace{\cap}_{\substack{\text{intersection} \\ \text{"or"}}} \{p_2 < p\}$ denotes event $\max\{p_1, p_2\} < p$.

- Since independent $F(p)F(p) = F(p)^2$, as probability $\max\{p_1, p_2\} < p$. Then:

$$\mathbb{E}[\max\{p_1, p_2\}] = \int_0^B \left[1 - F(p)^2\right] dp \tag{8}$$

where $\mathbb{E}[\max\{p_1, p_2\}] > \mathbb{E}[p_1]$

[1]To derive, Recall integration by parts,

$$\int_a^b u dv = \underbrace{uv \mid_a^b}_{\substack{\text{evaluated at } b \text{ -} \\ \text{evaluated at } a}} - \int_a^b v du \tag{4}$$

Using our definition of expectation:

$$\int_0^B \underbrace{p}_{\substack{u=p \\ \Downarrow \\ du=dp}} \underbrace{f(p)dp}_{\substack{dv=f(p)dp \\ \Downarrow \\ v=F(p)=\int_0^p f(s)ds}} = \int_0^B u dv \tag{5}$$

By formula,

$$= pF(p) \mid_0^B - \int_0^B F(p)dp \tag{6}$$

$$= \underbrace{BF(B)}_{B} - 0 - \int_0^B F(p)dp = \underbrace{\boxed{\int_0^B (1 - F(p))\, dp = \mathbb{E}[p]}}_{\text{Alternative formula for expectation}} \tag{7}$$

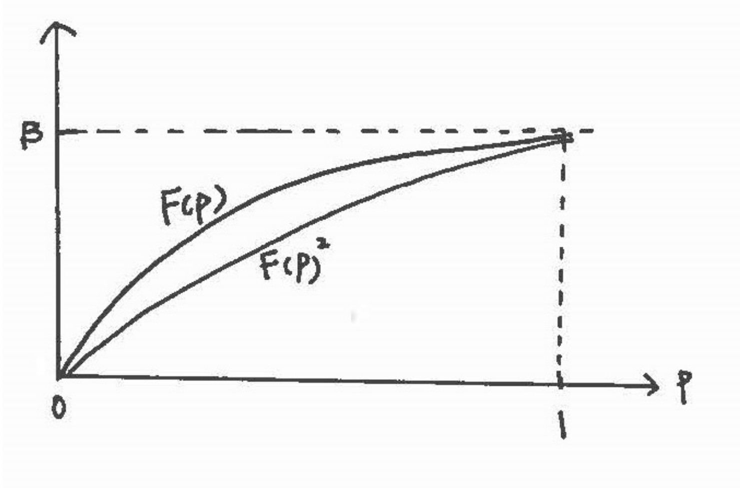

Figure 2: Expectation among $\max(p_1, p_2)$

2 McCall Search Model

2.1 Setup

- Background:
  - Infinitely lived risk-neutral worker wants to maximize the expected P.D.V:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t y_t\right], \text{ with } y_t = \begin{cases} w_t \text{ if employed} \\ c \text{ if unemployed} \end{cases} \tag{9}$$

    where $\beta \in (0,1)$
  - Each period, an unemployment worker draws one offer to work at wage w forever, the wage is drawn from the distribution of wages in the economy $F(w)$, where $F(0) = 0$, $F(B) = 1$
  - There is recall of previous wages each period
  - Each period, can accept current wage draw from $F(w)$, or reject it and collect $c > 0$ (unemployment benefits) and draw again next period.
  - If accept, the worker will work forever at w and can neither be fired nor quit.
- Problem:
  - Find the worker's optimal strategy for accepting or rejecting draws
- Solution:

– Let Q = optimal value of the problem for a worker about to draw a wage offer

$$Q \equiv \underbrace{\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t y_t\right]}_{\text{expectation \underline{before} wage is drawn}} \tag{10}$$

– Let $v(w)$ be the optimal value of the problem for a previously unemployed worker who has just drawn w and is about to decide what to do,

$$\underbrace{Q}_{\substack{\text{value}\\\text{about to}\\\text{draw}}} = \underbrace{\int_0^B v(w) f(w) dw}_{\text{expected \underline{value} over draws}} \tag{11}$$

– Key observation: write recursively as value today in terms of tomorrow

$$\underbrace{v(w)}_{\substack{\text{value if}\\\text{draw } w}} = \max_{\{\text{accept,reject}\}} \left\{ \underbrace{w}_{\text{Wage today}} + \underbrace{\beta v(w)}_{\substack{\text{Discounted value}\\\text{keeping wage}}} , \underbrace{c}_{\substack{\text{Unemployment}\\\text{Benefits today}}} + \underbrace{\beta \cdot Q}_{\substack{\text{Discounted value}\\\text{drawing wage}}} \right\} \tag{12}$$

Recognizing if you accepting w once, you would keep accepting it, $v(w|\text{accept}) = w + \beta v(w|\text{accept})$ for accepted wage. So $v(w|\text{accept}) = \frac{w}{1-\beta}$, and

$$\underbrace{v(w)}_{\substack{\text{value if}\\\text{draw } w}} = \max_{\{\text{accept,reject}\}} \left\{ \underbrace{\frac{w}{1-\beta}}_{\substack{\text{PDV of}\\\text{wage forever}}} , \underbrace{c}_{\substack{\text{unemployment}\\\text{benefits today}}} + \underbrace{\beta \cdot Q}_{\substack{\text{discounted}\\\text{value tomorrow}}} \right\} \tag{13}$$

Note: $c + \beta \cdot Q$ is independent of w, since $Q = \int v(w')f(w')dw'$

– Plug in for Q to find a <u>functional equation</u> in $v(w)$,

$$v(w) = \max\left\{\frac{w}{1-\beta}, c + \beta \int_0^B v(w')f(w')dw'\right\} \tag{14}$$

2.2 Solve the Bellman Equation

Solution : A $v(w)$ and accept or reject policy which fulfills (14) for β, $f(.)$, c.

- <u>Method 1</u>: Numerically iterate.

$$v_{j+1}(w) = \max\left\{\frac{w}{1-\beta}, c + \beta \int_0^B v_j(w')f(w')dw'\right\} \tag{15}$$

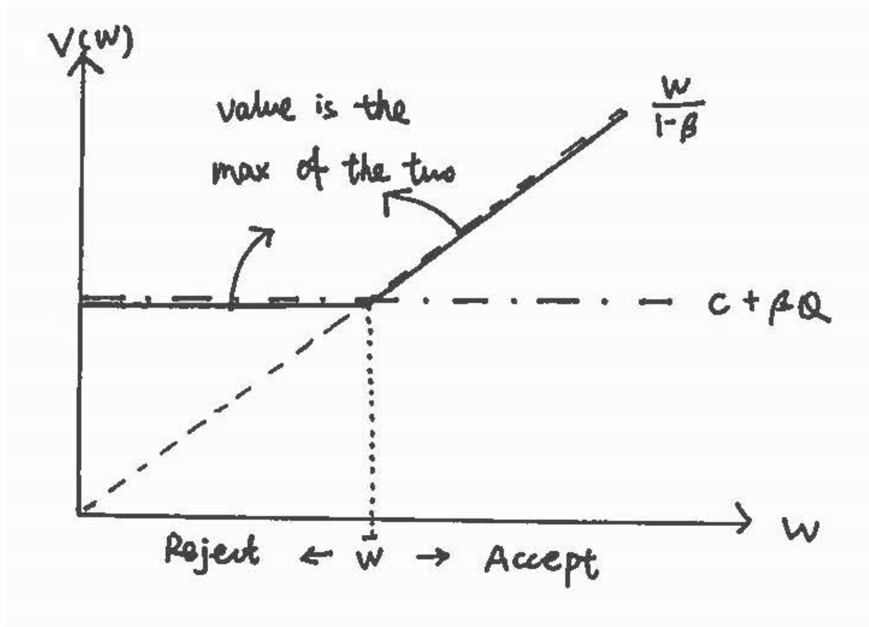

Figure 3: McCall model

Let $v_0(w) = 0$, evaluate $v_1(w)$, etc.
Guaranteed to converge to a stationary $v(w)$

- Method 2 Guess-and-verify

$$v(w) = \max\left\{\frac{w}{1-\beta}, c + \beta\int_0^B v(w')f(w')dw'\right\} \tag{16}$$

  - Note, $\bar{w}$ is point where indifferent between accept and reject.

$$v(\bar{w}) = \frac{\bar{w}}{1-\beta} = c + \beta\int_0^B v(w')f(w')dw', \text{ where } \begin{cases}\text{reject if } w' < \bar{w} \\ \text{accept if } w' > \bar{w}\end{cases} \tag{17}$$

$$= c + \beta\int_0^{\bar{w}} v(w')f(w')dw' + \beta\int_{\bar{w}}^B v(w')f(w')dw' \tag{18}$$

$$= c + \beta\int_0^{\bar{w}} \underbrace{\frac{\bar{w}}{1-\beta}}_{\substack{\text{reject value by} \\ \text{definition of } \bar{w}}} f(w')dw' + \beta\int_{\bar{w}}^B \underbrace{\frac{w'}{1-\beta}}_{\text{accept value}} f(w')dw' \tag{19}$$

$$= \frac{\bar{w}}{1-\beta} \tag{20}$$

- Expand LHS:

$$\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw' + \int_{\bar{w}}^{B} \frac{\bar{w}}{1-\beta} f(w')dw' \tag{21}$$

$$= c + \beta \underbrace{\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw'}_{\text{subtract to LHS}} + \beta \int_{\bar{w}}^{B} \frac{w'}{1-\beta} f(w')dw' \tag{22}$$

$\Rightarrow$ [After the subtract and add above]

$$(1-\beta) \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} f(w')dw' - c = \frac{1}{1-\beta} \int_{\bar{w}}^{B} (\beta w' - \bar{w}) f(w')dw' \tag{23}$$

- By adding $\bar{w} \int_{\bar{w}}^{B} f(w')dw'$ to both sides:

$$\bar{w} \left[\int_0^{\bar{w}} f(w')dw' + \int_{\bar{w}}^{B} f(w')dw' \right] - c = \boxed{\underbrace{\bar{w} - c}_{\substack{\text{cost of searching} \\ \text{one more period}}} = \underbrace{\frac{\beta}{1-\beta} \int_{\bar{w}}^{B} (w' - \bar{w}) f(w')dw'}_{\text{benefit of searching one more period}}} \tag{24}$$

- Given $\beta, c, f(.)$, we can solve for $\bar{w}$, then use policy: $\begin{cases} \text{reject if } w < \bar{w} \\ \text{accept if } w > \bar{w} \end{cases}$

- Then let $h(w)$ = Benefit of search at $w = \frac{\beta}{1-\beta} \int_w^B (w' - w) f(w')dw'$
  $h(0) = \frac{\beta}{1-\beta} \mathbb{E}[w] > 0$
  $h(B) = 0$
  $h'(w) < 0, h'' > 0$

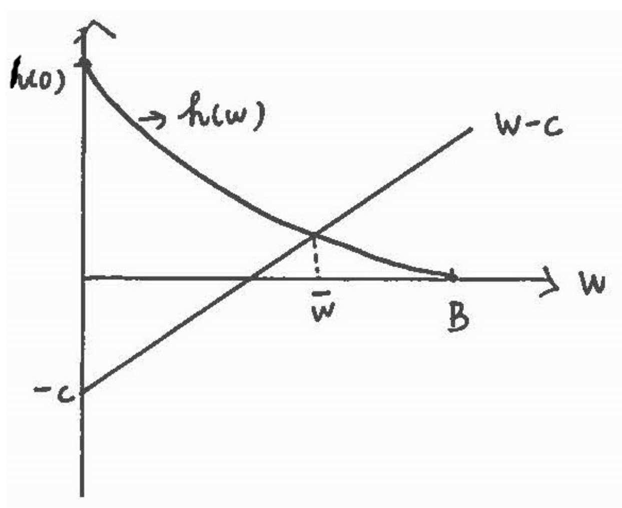

Figure 4: search model

2.3 The probability to accept

- Prob(accept | unemployed) = Prob($w \geq \bar{w}$) = $1 - F(\bar{w}) \equiv \pi$

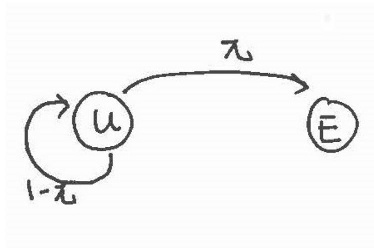

Figure 5: markov chain in search model

- In this basic setup, no firing or quitting, so E is absorbing. (Problem set includes firing)

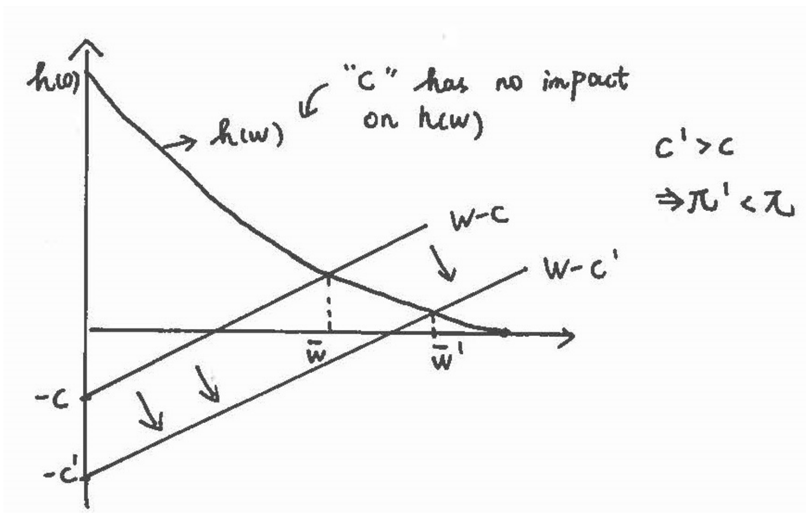

Figure 6: What is role of unemployment benefits?