# Quantile Regression

August 23, 2019

# Quantile Regression Example

- The goal of this example is to show how to estimate conditional quantile functions at several horizons using MIDAS approach.
- We show quantile estimates at several quantile levels
- Compare with rolling-window unconditional quantile estimates
- Compare with single-frequency CAViaR model implied quantiles.

# MIDAS Quantile Regression

MIDAS quantile regression is:

$$q_{\tau,t}(r_{t,h}; (\beta_{\tau,h}, \theta_{\tau,h})) = \beta^0_{\tau,h} + \beta_{\tau,h} \times Z_t(\theta_{\tau,h})$$
$$Z_t(\theta_{\tau,h}) = \sum_{d=0}^{D} \omega_d(\theta_{\tau,h}) |r_{t-d}|$$

- Monthly S&P500 5-day returns (weekly, non-overlapping) regressed on absolute daily returns.
- Model specification: 22 lags of absolute daily returns ($D = 22$). Restricted Beta density is used for weighting function.

# MIDAS Quantile Regression

Listing 1: Preliminaries

```
%clear workspace
clear all
close all

%load S&P500 Data
load('example4.mat');
```

Check if data is loaded in your Matlab workspace.

You should see: snp500

# MIDAS Quantile Regression

Transform snp500 into log returns, grab dates and convert them.

Listing 2: Transform snp500

```
%computer log returns
returns= log(snp500(2:end,2)./snp500(1:end-1,2));
%set dates
dates=snp500(2:end,1);
dates = datenum(dates);
```

# Quantile Regression in Matlab

- For this example we will rely on the MIDASv2 toolbox. Specifically, the MidasQuantile_edited function will be used.
- MidasQuantile_edited constructs non-overlapping returns, while MidasQuantile does overlapping.
- We will also be using the Conditional Quantile Codes, specifically the EstimateConditionalQuantile function for CAViaR estimation.
- The condskewness function is also called.

# S&P500 Example

- Set y = returns and then fit 25% MIDAS conditional quantile

Listing 3: Estimate 25% conditional MIDAS quantile

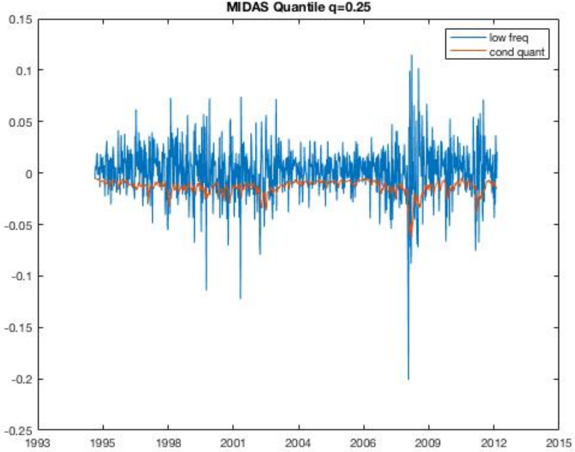
```
y=returns;
% Fit 25% conditional quantile
[estParams,condQuantile3,yLowFreq,xHighFreq,yDates] =
    MidasQuantile_edited(y,'Dates',dates,'Period',5,'NumLags',22,'
    Quantile',0.25);
```

# S&P500 Example

Listing 4: Plot the 25% conditional MIDAS quantile

```
plot(yDates,yLowFreq)
hold on;
plot(yDates,condQuantile3)
legend('low freq','cond quant')
title('MIDAS Quantile q=0.25')
dateaxis;
hold off;
```

# S&P500 Example



# S&P500 Example

- Fit 5% MIDAS conditional quantile

Listing 5: Estimate 5% conditional MIDAS quantile

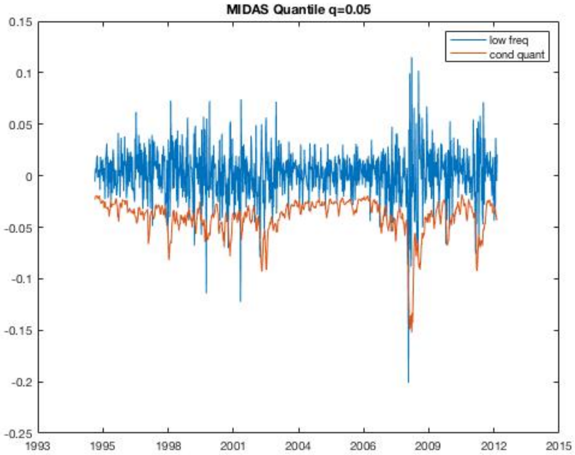
```
% Fit 5% conditional quantile
[estParams2,condQuantile32,yLowFreq2,xHighFreq2,yDates2] =
    MidasQuantile_edited(y,'Dates',dates,'Period',5,'NumLags',22,'
    Quantile',0.05);
```

# S&P500 Example

Listing 6: Plot the 5% conditional MIDAS quantile

```
plot(yDates2,yLowFreq2)
hold on;
plot(yDates2,condQuantile32)
dateaxis;
legend('low freq','cond quant')
title('MIDAS Quantile q=0.05')
hold off;
```

# S&P500 Example



# S&P500 Example

Estimate a rolling window 5% unconditional quantile for comparison.
Use the built in function "quantile"

Listing 7: 5% unconditional quantile estimation

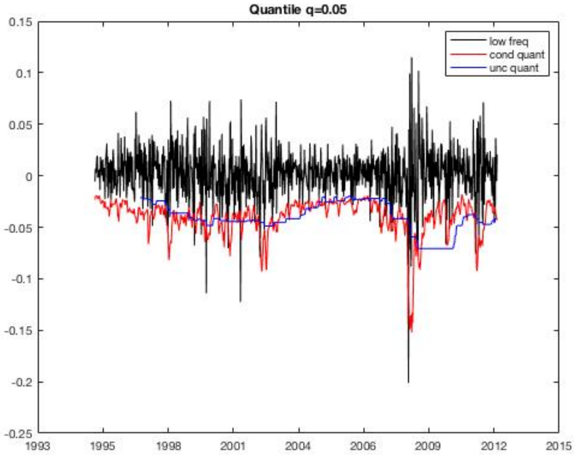
```
% now rolling window 5% unconditional quantile estimate for comparison
y = yLowFreq2;
windowsize = 100;

unquant = NaN(length(y),1);

for j = windowsize:length(y)
  unquant(j,:) = quantile(y(((j-windowsize+1):j),:), 0.05);
end
```

# S&P500 Example

Plot the unconditional quantile and MIDAS estimated quantile for comparison

```
% Plot the 5% conditional quantile and 5% unconditional quantile
plot(yDates2,yLowFreq2,'k-')
hold on;
plot(yDates2,condQuantile32,'r-')
plot(yDates2,unquant,'b-')
dateaxis;
legend('low freq','cond quant','unc quant')
title('Quantile q=0.05')
hold off;
```

# S&P500 Example



First set our empirical quantile from the unconditional rolling quantile
Estimate the 5% CAViaR quantile

Listing 8: CAViaR estimation

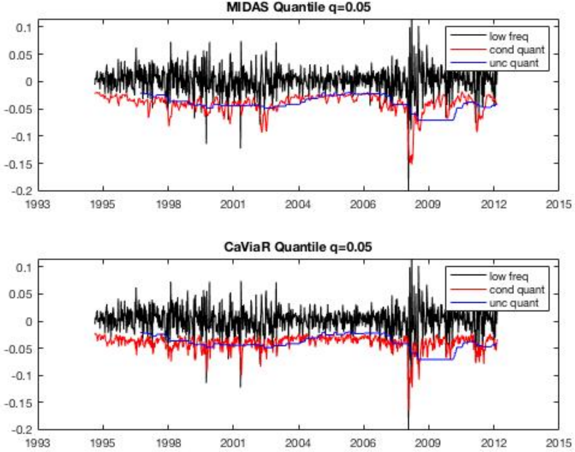
```
%now caviar estiamtion
empiricalQuantile =unquant(101); %to initialize loop
date=yDates;

[beta,condQ] = EstimateConditionalQuantile('C', 1, 0.05, y, empiricalQuantile
    , [], []);
```

# S&P500 Example

Plot the CAViar results and the MIDAS results to compare

```
% Plot the unconditional quantile and MIDAS
% and plot caviar results with unconditional
subplot(2,1,1)
plot(yDates2,yLowFreq2,'k-')
hold on;
plot(yDates2,condQuantile32,'r-')
plot(yDates2,unquant,'b-')
dateaxis;
legend('low freq','cond quant','unc quant')
title('MIDAS Quantile q=0.05')
hold off;

subplot(2,1,2)
plot(yDates2,yLowFreq2,'k-')
hold on;
plot(yDates2,condQ,'r-')
plot(yDates2,unquant,'b-')
dateaxis;
legend('low freq','cond quant','unc quant')
title('CaViaR Quantile q=0.05')
hold off;
```

# S&P500 Example

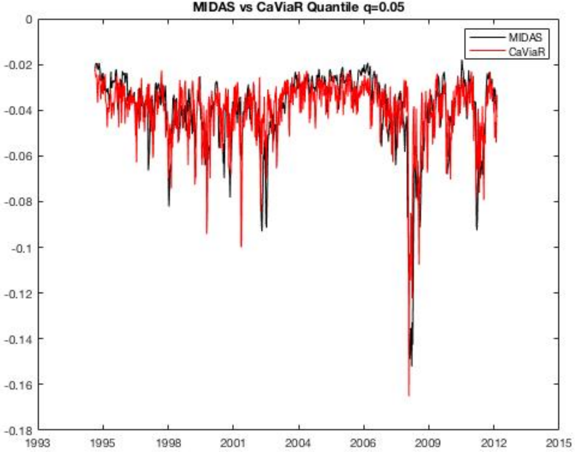


# S&P500 Example

Plot quantiles for MIDAS and CAViaR to compare

```
% Plot quantiles for MIDAS and CAViaR to compare

plot(yDates2,condQuantile32,'k-')
hold on;
plot(yDates2,condQ,'r-')
dateaxis;
legend('MIDAS','CaViaR')
title('MIDAS vs CaViaR Quantile q=0.05')
hold off;
```

# S&P500 Example



# S&P500 Example

- We will compute conditional skewness at 0.95 level for MIDAS and CAViaR and plot them.

Listing 9: Conditional Skewness function

```
function [cskew] = condskewness(condup,conddown,condmed,qlevel)
%Conditional Skewness function
  num = condup + conddown - 2 * condmed;
  den = condup - conddown;
  %Kornish-Fisher constant:
  const = 6/norminv(qlevel);
  cskew = num./den*const;
end
```

# S&P500 Example

- first compute CAViaR for 0.5 and 0.95 levels

Listing 10: CAViaR estimation for 0.5 and 0.95 levels

```
% compute conditional skewness at 0.95 level for MIDAS and CAViaR, plot
% first compute CAViaR for 0.5 and 0.95 levels
[beta_5 ,condQ_5] = EstimateConditionalQuantile('C', 1, 0.5, y, quantile(
    y(1:100) ,0.5), [], []);

[beta_95 ,condQ_95] = EstimateConditionalQuantile('C', 1, 0.95, y,
    quantile(y(1:100) ,0.95), [], []);
```

- Using the estimated CAViaR results, compute conditional skewness

```
% compute conditional skewness
[cskew_CaViaR] = condskewness(condQ_95 ,condQ ,condQ_5 ,0.95);
```

# S&P500 Example

- Now compute MIDAS for 0.5 and 0.95 levels
- for 0.50 level we use search algorithm instead of fmincon and set intial values to avoid local mins

Listing 11: MIDAS estimation for 0.5 and 0.95 levels

```
% MIDAS quantiles
% this one is tricky so we use search and set initial parameters to
    avoid
% falling into local mins
[estParams_50,condQuantile_50,yLowFreq_50,xHighFreq_50,yDates_50] =
    MidasQuantile_edited(returns,'Dates',dates,'Period',5,'NumLags',22,'
    Quantile',0.50,'Search',1,'Params0',[0.0009;.2;4]);

[estParams_95,condQuantile_95,yLowFreq_95,xHighFreq_95,yDates_95] =
    MidasQuantile_edited(returns,'Dates',dates,'Period',5,'NumLags',22,'
    Quantile',0.95);
```

# S&P500 Example

Using the estimated MIDAS results, compute conditional skewness

Use the condskewness function

```

%skew MIDAS
[cskew_MIDAS] = condskewness(condQuantile_95,condQuantile32,condQuantile_50,0.95);
```

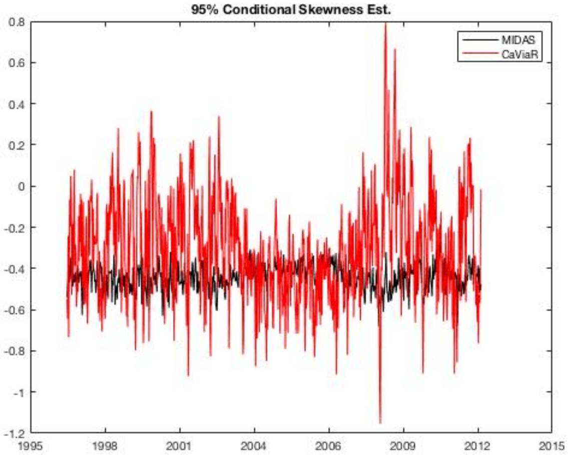
# S&P500 Example

Plot the MIDAS and CAViaR conditional skewness

Throw out the first 10% of the sample due to initialization

```
% plot conditional skewness
% throw out first 10% of sample due to initialization
idx=(round(length(cskew_CaViaR)*0.1)+1):length(cskew_CaViaR);

plot(yDates2(idx),cskew_MIDAS(idx),'k-')
hold on;
plot(yDates2(idx),cskew_CaViaR(idx),'r-')
dateaxis;
legend('MIDAS','CaViaR')
title('95% Conditional Skewness Est.')
hold off;
```

# S&P500 Example



The End