Macro forecasting: ARDL-MIDAS regressions

August 29, 2019

ARDL-MIDAS regression

ARDL-MIDAS(p_y^Q, q_X^M) model

$$Y_{t+h}^{Q,h} = \mu_h + \sum_{j=0}^{p_y^Q - 1} \rho_{j+1}^h Y_{t-j}^Q + \beta^h \sum_{j=0}^{q_X^M - 1} \sum_{i=0}^{m-1} \omega_{i+j*m}(\theta_h) X_{m-i,t-j}^M + \epsilon_{t+h}^h \quad (1)$$

where

- $Y_{t+h}^{Q,h}$ low-frequency (quarterly) target variable
- $Y_{t+h}^{Q,h}$ low-frequency (quarterly) target variable $X_{m-i,t-j}^M$ high-frequency (monthly) predictor variable
- $\omega_{i+j*m}(\theta_h)$ weight function
- $\Theta := (\mu_h, \rho_1, \ldots, \rho_{p_y^Q}, \beta^h, \theta_h)$ model parameters estimated by NLS

Listing 1: Preliminaries

```
%clear workspace
clear all

%add MIDAS toolbox to filepath
addpath(genpath('MIDASv2.2/MIDASv2.2/privatex'));

% Load data for macro examples:
load('example1.mat')
```

Check if data is loaded in your Matlab workspace.

You should see: ads, cfnai, payems, rgdp.

Listing 2: Transform payems and rgdp

```
%log growth
logpayems = log(payems(2:end,2)./payems(1:end-1,2))*100;
logrgdp = log(rgdp(2:end,2)./rgdp(1:end-1,2))*100;
```

Macro forecasting

Listing 3: Transform dates to datetime format

```
%convert to datetime
datepayems= datetime(payems(2:end,1),'ConvertFrom','datenum','Format','yyyy-MM-dd');
datergdp= datetime(rgdp(2:end,1),'ConvertFrom','datenum','Format','yyyy-MM-dd');
```

MIDAS in Matlab

- There are several MIDAS toolboxes for Matlab to run different MIDAS regressions.
- For this example we will rely on the MIDASv2 toolbox. Specifically, the MIDASADL function will be used.

First example is with Nonfarm Payrolls data Set initial and last date for in-sample estimation:

Listing 4: Set intial and last date for in-sample estimation:

```
% Specify start and end dates
EstStart = '1985-01-01';
EstEnd = '2009-01-01';
```

Nonfarm Payrolls example

Set options for MIDAS estimation:

Listing 5: Options

```
% Specify lag structure , horizon and method
Xlag = 9;
Ylag = 1;
Horizon = 3;
Method = 'fixedWindow';

% set data- x and y variables
DataX=logpayems;
DataY=logrgdp;

% set data dates
DataYdate = datergdp;
DataXdate = datepayems;

% set polynomial
poly='expAlmon';
```

Nonfarm Payrolls example

Estimate ARDL-MIDAS regression using MIDAS ADL function from MIDAS package.

Listing 6: Estimating via MIDAS ADL function

```
[OutputForecast1,OutputEstimate1,MixedFreqData] = MIDAS_ADL(DataY,DataYdate,
    DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial',poly,'Method',Method,'PlotWeights',0);
```

Nonfarm Payrolls example

Estimate ARDL regression model using time-averaged Nonfarm Payrolls data.

Listing 7: Estimating via time-averaged data

```
%now compare to time averaged employment data
%form X vars first from MixedFreqData structure
Xvars=[MixedFreqData.EstLagY, mean(MixedFreqData.EstX(:,1:3),2), mean(
    MixedFreqData.EstX(:,4:6),2), mean(MixedFreqData.EstX(:,7:9),2)];
est_ta= fitlm(Xvars,MixedFreqData.EstY)
```

Nonfarm Payrolls example

Compare in-sample performance. Grab MIDAS residuals from OutputEstimate1 structure and time-averaged from est ta.Residuals.

Listing 8: In sample of MIDAS model

```
%compare in sample performance
sqrt(mean(OutputEstimate1.resid.^2))
```

ans =
0.5103

Listing 9: In sample of time-averaged model

```
sqrt(mean(table2array(est_ta.Residuals(:,1)).^2))
```

ans =
0.4945

Nonfarm Payrolls example

Compare out-of-sample performance. Grab MIDAS RMSE from OutputForecast1.RMSE. Forecast out time-averaged using MixedFreqData structure to grab data first.

Listing 10: Out-of-sample time-averaged model and MIDAS RMSE

```
% now compare out of sample
% first predict time-averaged model
Xnew=[MixedFreqData.OutLagY, mean(MixedFreqData.OutX(:,1:3),2), mean(
    MixedFreqData.OutX(:,4:6),2), mean(MixedFreqData.OutX(:,7:9),2)];
ta_oos = predict(est_ta,Xnew);

% compare RMSE
% grab MIDAS RMSE
midas_oos_rmse = OutputForecast1.RMSE
```

ans =
0.4461

Listing 11: Calculate time-averaged out-of-sample RMSE

```
ta_oos_rmse = sqrt(mean((ta_oos-MixedFreqData.OutY).^2))
```

ans =
0.4487

Nonfarm Payrolls example

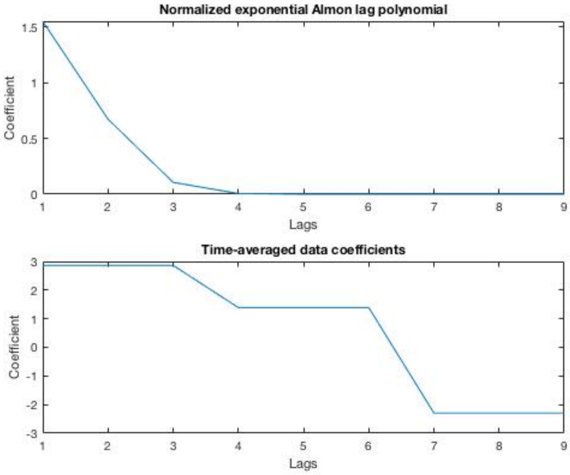
Plot lag polynomials.

Listing 12: Collect time-averaged weights

```
%set lags
lags=1:9;
%time averaged weights
ta_weights = [repelem(table2array(est_ta.Coefficients(3,1)),3) repelem(
    table2array(est_ta.Coefficients(4,1)),3) repelem(table2array(est_ta.
    Coefficients(5,1)),3)]';
%now plots...
    subplot(2,1,1)
    plot(lags,OutputEstimate1.estWeights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('Normalized exponential Almon lag polynomial');

    subplot(2,1,2)
    plot(lags,ta_weights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('Time-averaged data coefficients');
```

Nonfarm Payrolls example



Nonfarm Payrolls example

We will compare fixed, rolling, expanding windows predictions of GDP using these two models. For MIDAS our previous settings should be saved.

Listing 13: Fixed Window MIDAS

```
[OutputForecast1_Fixed,OutputEstimate1_Fixed,MixedFreqData_Fixed] = MIDAS_ADL(DataY,DataYdate,DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial','expAlmon','Method',Method,'PlotWeights',0);
```

Nonfarm Payrolls example

Use the structure MixedFreqData Fixed as inputs for time-averaged model.

Listing 14: Fixed Window time-averaged

```
%Now time averaged MIDAS- IS
Xvars_TA=[MixedFreqData_Fixed.EstLagY, mean(MixedFreqData_Fixed.EstX(:,1:3)
    ,2), mean(MixedFreqData_Fixed.EstX(:,4:6),2), mean(MixedFreqData_Fixed.
    EstX(:,7:9),2)];
est_ta_Fixed= fitlm(Xvars_TA,MixedFreqData_Fixed.EstY);
```

Recall, we have already compared predictions for fixed scheme above.

Nonfarm Payrolls example

Set Method to Rolling Window.

Listing 15: Rolling Window MIDAS

```
%rolling windows
Method='RollingWindow';
[OutputForecast1_Roll,OutputEstimate1_Roll,MixedFreqData_Roll,
    ExtendedForecast_Roll] = MIDAS_ADL(DataY,DataYdate,DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial','expAlmon','Method',Method);
```

Nonfarm Payrolls example

Listing 16: Rolling Window time-averaged

```
%rolling time averaged
nobs=length(MixedFreqData_Roll.EstY);
nforecast=length(MixedFreqData_Roll.OutY);
nroll=nforecast;

Xvars_TA=[MixedFreqData_Roll.EstLagY, mean(MixedFreqData_Roll.EstX(:,1:3),2),
    mean(MixedFreqData_Roll.EstX(:,4:6),2), mean(MixedFreqData_Roll.EstX
    (:,7:9),2)];
Xnew_TA_Roll=[MixedFreqData_Roll.OutLagY, mean(MixedFreqData_Roll.OutX(:,1:3)
    ,2), mean(MixedFreqData_Roll.OutX(:,4:6),2), mean(MixedFreqData_Roll.OutX
    (:,7:9),2)];
Xvars_temp = [Xvars_TA;Xnew_TA_Roll];
Yvars_temp = [MixedFreqData_Roll.EstY;MixedFreqData_Roll.OutY];

for t=1:nroll
    Xvars_TA_Roll = Xvars_temp(t:nobs-1+t,:);
    Yvars_Roll = Yvars_temp(t:nobs-1+t,:);
    est_ta_Roll = fitlm(Xvars_TA_Roll,Yvars_Roll);
    Xvars_OOS_Roll = Xnew_TA_Roll(t,:);
    ta_oos_Roll(t,:) = predict(est_ta_Roll,Xvars_OOS_Roll);
end
```

Nonfarm Payrolls example

Compare out-of-sample performance for rolling window.

Listing 17: MIDAS Rolling window RMSE

```
%compare forecasts of rolling
% compare RMSE
% grab MIDAS RMSE
OutputForecast1_Roll.RMSE
```

ans =
0.4284

Listing 18: Time-averaged Rolling window RMSE

```
% compute rolling rmse
sqrt(mean((ta_oos_Roll-MixedFreqData_Roll.OutY).^2))
```

ans =
0.4326

Nonfarm Payrolls example

Set Method to Recursive.

Listing 19: Expanding Window MIDAS

```
%Expanding Window
Method='Recursive';
[OutputForecast1_Exp,OutputEstimate1_Exp,MixedFreqData_Exp,
    ExtendedForecast_Exp] = MIDAS_ADL(DataY,DataYdate,DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial','expAlmon','Method',Method,'PlotWeights',0);
```

Nonfarm Payrolls example

Listing 20: Expanding Window time-averaged

```
% Expanding time averaged
nobs=length(MixedFreqData_Exp.EstY);
nforecast=length(MixedFreqData_Exp.OutY);
Xvars_TA_Exp=[MixedFreqData_Exp.EstLagY, mean(MixedFreqData_Exp.EstX(:,1:3)
     ,2), mean(MixedFreqData_Exp.EstX(:,4:6),2), mean(MixedFreqData_Exp.EstX
     (:,7:9),2)];
Xnew_TA_Exp=[MixedFreqData_Exp.OutLagY, mean(MixedFreqData_Exp.OutX(:,1:3),2)
     , mean(MixedFreqData_Exp.OutX(:,4:6),2), mean(MixedFreqData_Exp.OutX
     (:,7:9),2)];
Xvars_temp_Exp = [Xvars_TA_Exp;Xnew_TA_Exp];
Yvars_temp_Exp = [MixedFreqData_Exp.EstY;MixedFreqData_Exp.OutY];

for t=1:nroll

    Xvars_TA_Exp = Xvars_temp_Exp(1:nobs-1+t,:);
    Yvars_Exp = Yvars_temp_Exp(1:nobs-1+t,:);
    est_ta_Exp = fitlm(Xvars_TA_Exp,Yvars_Exp);
    Xvars_OOS_Exp = Xnew_TA_Exp(t,:);
    ta_oos_Exp(t,:) = predict(est_ta_Exp,Xvars_OOS_Exp);

end
```

Nonfarm Payrolls example

Compare out-of-sample performance for expanding window.

Listing 21: MIDAS expanding window RMSE

```
% compare forecasts of rolling
% compare RMSE
% grab MIDAS RMSE
OutputForecast1_Exp.RMSE
```

ans =
0.4296

Listing 22: Time-averaged expanding window RMSE

```
% compute expanding rmse
 sqrt(mean((ta_oos_Exp-MixedFreqData_Exp.OutY).^2))
```

ans =
0.4369

CFNAI example

First convert CFNAI dates to datetime format. Then set estimation dates, horizon, x and y variables, lags, etc.

Listing 23: Set options for MIDAS estimation

```
% convert cfnai dates to datetime
datecfnai= datetime(cfnai(1:end,1),'ConvertFrom','datenum','Format','yyyy-MM-
    dd');

% Specify lag structure and sample size
Xlag = 12;
Ylag = 1;
Horizon = 3;
EstStart = '1987-01-01';
EstEnd = '2011-12-01';
Method = 'fixedWindow';
% Data
DataYdate = datergdp;
DataXdate = datecfnai;
DataX=cfnai(:,2);
DataY=logrgdp;
```

CFNAI example

Estimate CFNAI example with exponential almon and u-midas polynomials. Compare the results.

Listing 24: Set polynomials and estimate MIDAS regressions

```
%Estimate ADL regression model using exp almon scheme
poly='expAlmon';
[OutputForecast1,OutputEstimate1,MixedFreqData] = MIDAS_ADL(DataY,DataYdate,
    DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial',poly,'Method',Method,'PlotWeights',0);


%Estimate ADL regression model using U-MIDAS scheme
poly='UMIDAS';
[OutputForecast_U,OutputEstimate_U,MixedFreqData_U] = MIDAS_ADL(DataY,
    DataYdate,DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial',poly,'Method',Method,'PlotWeights',0);
```

CFNAI example

Compare in-sample RMSE performance.

Listing 25: Exponential Almon polynomial

```
%compare in sample performance
sqrt(mean(OutputEstimate1.resid.^2))
```

ans =
0.4849

Listing 26: U-MIDAS polynomial

```
%compare in sample performance
sqrt(mean(OutputEstimate_U.resid.^2))
```

ans =
0.4552

CFNAI example

Compare out-of-sample RMSE performance.

Listing 27: Exponential Almon polynomial

```
%compare OOS performance now
%grab MIDAS RMSE
midas_oos_rmse = OutputForecast1.RMSE
```

ans =
0.3155

Listing 28: U-MIDAS polynomial

```
umidas_oos_rmse = OutputForecast_U.RMSE
```

ans =
0.3368

CFNAI example

Plot the different weighting schemes.

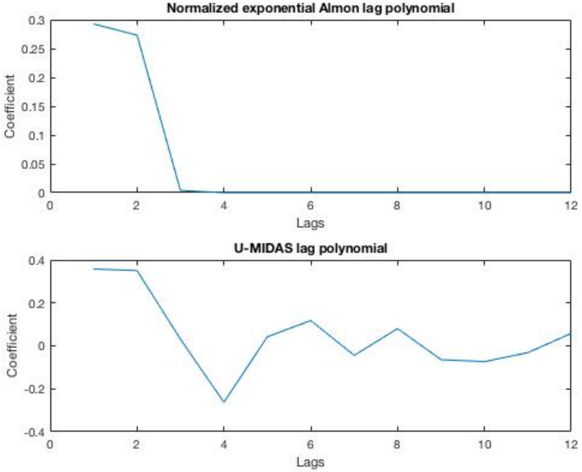
Listing 29: Plot weights for both polynomials.

```
%plot weights and compare
%set lags
lags=1:12;

    subplot(2,1,1)
    plot(lags,OutputEstimate1.estWeights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('Normalized exponential Almon lag polynomial');

    subplot(2,1,2)
    plot(lags,OutputEstimate_U.estWeights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('U-MIDAS lag polynomial');
```

CFNAI example



ADS example

Estimate ADS example with exponential almon and u-midas polynomials. Compare the results. We will focus on nowcasting vs. out-of-sample forecasting.
First convert ADS dates to datetime format. Then set estimation dates, horizon, x and y variables, lags, etc.

Listing 30: Set options for MIDAS estimation

```
%Third Example now with ADS data
dateads= datetime(ads(1:end,1),'ConvertFrom','datenum','Format','yyyy-MM-dd')
    ;

% Specify lag structure and sample size
Xlag = 66;
Ylag = 1;
Horizon = 66;
EstStart = '1987-01-01';
EstEnd = '2011-12-01';
Method = 'fixedWindow';

DataYdate = datergdp;
DataXdate = dateads;
DataX=ads(:,2);
DataY=logrgdp;
```

ADS example

Estimate ADS example with forecasting, horizon was set to 66.

Listing 31: Estimate MIDAS regressions

```
%let's estimate with exp almon polynomial first
poly='expAlmon';
[OutputForecast1,OutputEstimate1,MixedFreqData] = MIDAS_ADL(DataY,DataYdate,
    DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial',poly,'Method',Method,'PlotWeights',0);

%Estimate ADL regression model using U-MIDAS scheme
%these match
poly='UMIDAS';
[OutputForecast_U,OutputEstimate_U,MixedFreqData_U] = MIDAS_ADL(DataY,
    DataYdate,DataX,DataXdate,...
    'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
        EstEnd,'Polynomial',poly,'Method',Method,'PlotWeights',0);
```

Estimate ADS example with nowcasting. Set horizon = 22

Listing 32: Estimate MIDAS regressions

```
%nowcasts
Horizon = 22;
%first , expAlmon
poly='expAlmon';
[OutputForecast1_Now,OutputEstimate1_Now,MixedFreqData_Now] = MIDAS_ADL(DataY
     ,DataYdate,DataX,DataXdate,...
     'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
          EstEnd,'Polynomial',poly,'Method',Method);

%now Umidas
poly='UMIDAS';
[OutputForecast_UNow,OutputEstimate_UNow,MixedFreqData_UNow] = MIDAS_ADL(
     DataY,DataYdate,DataX,DataXdate,...
     'Xlag',Xlag,'Ylag',Ylag,'Horizon',Horizon,'EstStart',EstStart,'EstEnd',
          EstEnd,'Polynomial',poly,'Method',Method);
```

ADS example

Compare out-of-sample RMSE performance when forecasting with horizon set to 66.

Listing 33: Exponential Almon polynomial

```
%forecasts OOS
midas_oos_rmsefix = OutputForecast1.RMSE
```

ans =
0.3096

Listing 34: U-MIDAS polynomial

```
umidas_oos_rmsefix = OutputForecast_U.RMSE
```

ans =
0.6150

ADS example

Compare out-of-sample RMSE performance when nowcasting with horizon set to 22.

Listing 35: Exponential Almon polynomial

```
%nowcasts
midas_oos_rmsenow = OutputForecast1_Now.RMSE
```

ans =
0.3486

Listing 36: U-MIDAS polynomial

```
umidas_oos_rmsenow = OutputForecast_UNow.RMSE
```

ans =
0.7207

ADS example

Plot polynomial weights for forecasting when horizon set to 66.

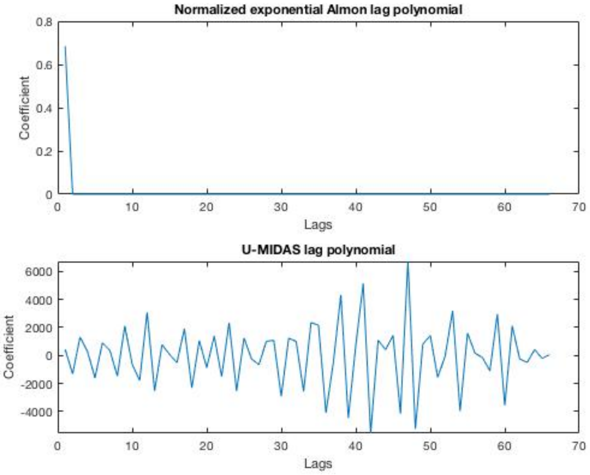
Listing 37: U-MIDAS polynomial

```
%plot weights and compare
%set lags
lags=1:66;

    subplot(2,1,1)
    plot(lags,OutputEstimate1.estWeights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('Normalized exponential Almon lag polynomial');

    subplot(2,1,2)
    plot(lags,OutputEstimate_U.estWeights)
    xlabel('Lags');
    ylabel('Coefficient');
    title('U-MIDAS lag polynomial');
```

ADS example



Normalized exponential Almon lag polynomial
Coefficient
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
Lags
U-MIDAS lag polynomial
Coefficient
6000
4000
2000
0
-2000
-4000
0
10
20
30
40
50
60
70
Lags

The End