Long and Short Volatility: GARCH-MIDAS

August 19, 2019

GARCH-MIDAS

The GARCH-MIDAS model is:

$$r_{i,t} = \mu + \varepsilon_{i,t}, \tag{1}$$

with

$$\varepsilon_{i,t} = \sqrt{g_{i,t}\tau_t} Z_{i,t}. \tag{2}$$

The innovation $Z_{i,t}$ is assumed to be i.i.d. with mean zero and variance one. $g_{i,t}$ and τ_t denote the short- and long-term component of the conditional variance, respectively.

GARCH-MIDAS

The short-term component $g_{i,t}$ varies at the daily frequency and follows a unit-variance GARCH(1,1) process

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{\varepsilon_{i-1,t}^2}{\tau_t} + \beta g_{i-1,t}. \tag{3}$$

The long-term component varies at the monthly frequency and is given by

$$\tau_t = m + \varphi \sum_{k=1}^{K} \omega_k(\theta) X_{t-k}, \tag{4}$$

where X_t denotes the explanatory variable and the weighting scheme $\omega_k(\theta)$ parameterized by a low-dimensional θ parameter.

GARCH-MIDAS

Listing 1: Preliminaries

```
% housekeeping
clear all
close all

% First Example is with default RV calculated in the GARCH-MIDAS function
% load NASDAQ daily returns
load('NASDAQ_daily.mat');
```

Check if data is loaded in your Matlab workspace.

You should see: NASDAQ.

GARCH-MIDAS

Truncate the data, this will be useful when we add an exogonous variable.

Listing 2: Truncate the data

```
% trim data so start at same date..
start_indxy=find(NASDAQ.DATE=='2019-06-28');
NASDAQ_trunc=NASDAQ(1:start_indxy,:);
```

GARCH-MIDAS in Matlab

For this example we will rely on the MIDASv2 toolbox. Specifically, the GarchMidas function will be used.
First example is with NASDAQ daily return data. We will run with fixed window and rolling window.

Listing 3: Set data, aggregation and lag specifications:

```
y = NASDAQ_trunc.NASDAQCOM_PCH./100;

period = 22;
numLags = 24;
```

We set period to 22 so we have monthly RV and daily returns.

NASDAQ example

Run the GARCH-MIDAS model. Default specification sets our long-term component to RV.
We don't specify a polynomial because the default is the one-parameter Beta polynomial weighting scheme

Listing 4: Estimate GARCH-MIDAS

```
% Estimate the GARCH—MIDAS model, and extract the volatilities
% Notice we do not enter an X variable in the arguments of GarchMidas, so
% the default option is to calculate RV from the data

[estParams,EstParamCov,Variance,LongRunVar] = GarchMidas(y,'Period',period,'
    NumLags',numLags);
```

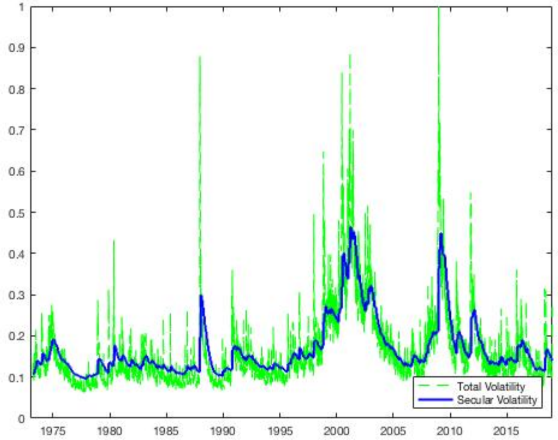
NASDAQ example

Plot the total volatility and secular volatility.

Listing 5: Plot volatility

```
% Plot the conditional volatility and its long-run component
figure(1)
nobs = size(y,1);
seq = (period*numLags+1:nobs)';
year = linspace(1971.2,2019.7,nobs);
plot(year(seq),sqrt(252*Variance(seq)),'g--','LineWidth',1);
hold on
plot(year(seq),sqrt(252*LongRunVar(seq)),'b-','LineWidth',2);
legend('Total Volatility','Secular Volatility','Location','SouthEast')
xlim([1973,2019])
ylim([0,1])
hold off
```

NASDAQ example



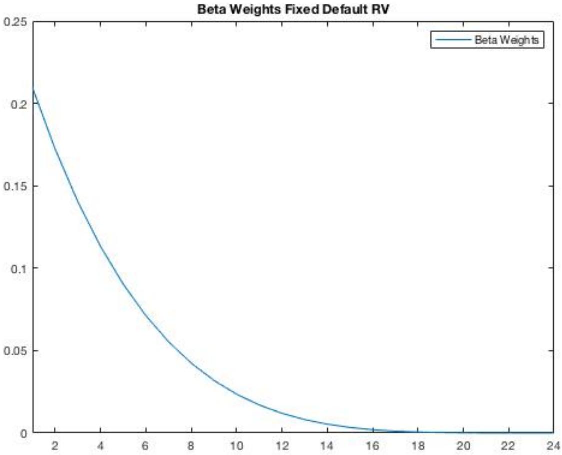
NASDAQ example

Recover the weighting scheme and plot it.

```
%%if we want to recover the weights, we can do the following:
param1 = estParams(5);
seq = numLags:-1:1;
weights = (1-seq./numLags+10*eps).^(param1-1);
%now re-scale weights...
weights = weights ./ nansum(weights);


% Plot the weights of its long-run component
figure(2)
plot(seq,weights','LineWidth',1);
legend('Beta Weights','Location','NorthEast')
xlim([1,24])
title('Beta Weights Fixed Default RV')
```

NASDAQ example



NASDAQ example

Run the GARCH-MIDAS model, but now with rolling windows.

Listing 6: Estimate GARCH-MIDAS

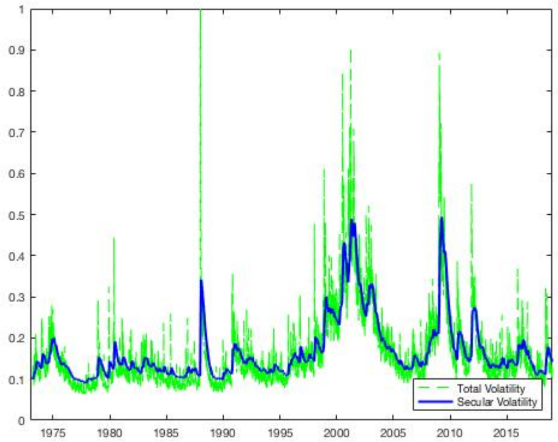
```
% Estimate the rolling window version of the GARCH-MIDAS model where RV is
% the long term component
[estParams,EstParamCov,Variance,LongRunVar]...
    = GarchMidas(y,'Period',period,'NumLags',numLags,'RollWindow',1);
```

Plot the total volatility and secular volatility from rolling window estimation.

Listing 7: Plot volatility

```
% Plot the conditional volatility and its long-run component
figure(1)
nobs = size(y,1);
seq = (period*numLags+1:nobs)';
year = linspace(1971.2,2019.7,nobs);
plot(year(seq),sqrt(252*Variance(seq)),'g--','LineWidth',1);
hold on
plot(year(seq),sqrt(252*LongRunVar(seq)),'b-','LineWidth',2);
legend('Total Volatility','Secular Volatility','Location','SouthEast')
xlim([1973,2019])
ylim([0,1])
hold off
```

NASDAQ example



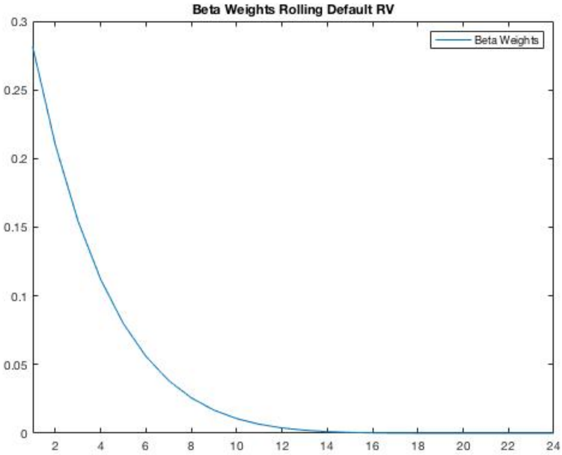
NASDAQ example

Recover the rolling window weighting scheme and plot it.

```
%%if we want to recover the weights, we can do the following:
param1 = estParams(5);
seq = numLags:-1:1;
weights = (1-seq./numLags+10*eps).^(param1-1);
%now re-scale weights...
weights = weights ./ nansum(weights);


% Plot the weights of its long-run component
figure(2)
plot(seq,weights','LineWidth',1);
legend('Beta Weights','Location','NorthEast')
xlim([1,24])
title('Beta Weights Fixed Default RV')
```

NASDAQ example



IP growth rate example

Now we estimate with IP growth rate driving our long term volatility. First, load in IP Index data and truncate it. Want to make sure it starts when NASDAQ daily data does.

Listing 8: Load and truncate data

```
%%Now we run GARCH-MIDAS where exogonous variable is IP growth.
% Industrial Production Index growth rate, 1971-2015
% Data Source: FRED database
% https://research.stlouisfed.org/fred2/series/INDPRO
load('INDPRO_monthly.mat');

%truncate the data
yDate=NASDAQ_trunc.DATE(1:end,:);
yDateDetails = datevec(yDate);
yDateMonth = yDateDetails(:,2); %grab all months..

%trim data so start at same date..
start_indx=find(IndPro.DATE=='1971-02-01');
IndPro_trunc=IndPro(start_indx:end,:);

xMonth = IndPro_trunc.INDPRO_PCH./ 100;
```

IP growth rate example

The Garchmidas function requires the exogenous variable formatted as a vector with the same length as the observation series y (NASDAQ returns). We just repeat the monthly values throughout the days. Then we can run the program with the name-value pair X.

Listing 9: Repeat monthly IP growth for daily returns

```
nobs = size(y,1);
xDay = NaN(nobs,1);
count = 1;

%now we repeat the monthly values to match the daily data
for t = 1:nobs
    if t > 1 && yDateMonth(t) ~= yDateMonth(t-1)
        count = count + 1;
        if count > length(xMonth)
            break
        end
    end
    xDay(t) = xMonth(count);
end
```

IP growth rate example

Estimate the GARH-MIDAS model with X = IP growth rate.
Add the options 'ThetaM',1 in the inputs. This let's the GarchMidas function know not to take squares for the parameter theta and m in the long-run volatility component. The default is false (they are squared).

Listing 10: Estimate GARCH-MIDAS

```
% Estimate the GARCH-MIDAS model with the exogenous regressor

[estParams,EstParamCov,Variance,LongRunVar] = GarchMidas(y,'Period',period,'
    NumLags',24,'X',xDay,'ThetaM',1);
```

IP growth rate example

Check parameter estimates. Theta is the coefficient for the long-term component.
Negative in front tells us we have a counter-cyclical relationship.

```
Method: Maximum likelihood
Sample size: 12625
Adjusted sample size: 12097
Logarithmic  likelihood:      39610.5
Akaike    info criterion:      -79209
Bayesian info criterion:     -79164.3
```

	Coeff	StdErr	tStat	Prob
mu	0.00077478	6.914e-05	11.206	0
alpha	0.10336	0.0030193	34.234	0
beta	0.88335	0.0033361	264.79	0
theta	-0.012587	0.0021613	-5.8238	0
w	1.6777	0.25641	6.5429	0
m	0.00015416	1.6544e-05	9.3184	0

```
RMSE of one-step variance forecast (period 1 to 12625): 4.487e-04.
```

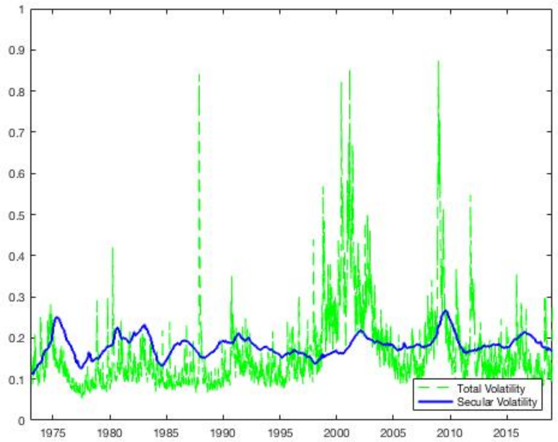
IP growth rate example

Plot the total volatility and secular volatility from rolling window estimation.

Listing 11: Plot volatility

```
% Plot the conditional volatility and its long-run component
figure(1)
nobs = size(y,1);
seq = (period*numLags+1:nobs)';
year = linspace(1971.2,2019.7,nobs);
plot(year(seq),sqrt(252*Variance(seq)),'g--','LineWidth',1);
hold on
plot(year(seq),sqrt(252*LongRunVar(seq)),'b-','LineWidth',2);
legend('Total Volatility','Secular Volatility','Location','SouthEast')
xlim([1973,2019])
ylim([0,1])
hold off
```

IP growth rate example



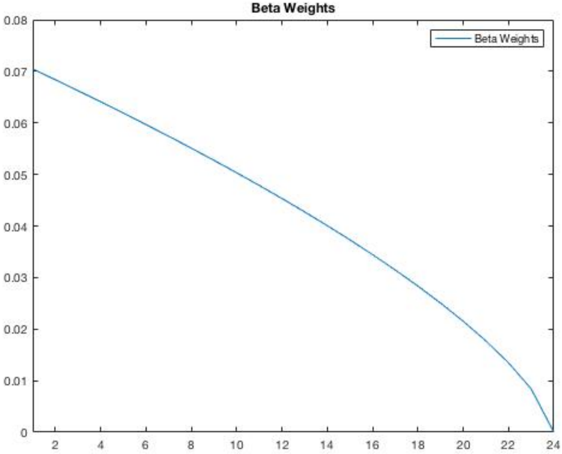
IP growth rate example

Recover the weighting scheme and plot it.

```
%%if we want to recover the weights, we can do the following:
param1 = estParams(5);
seq = numLags:-1:1;
weights = (1-seq./numLags+10*eps).^(param1-1);
%now re-scale weights...
weights = weights ./ nansum(weights);


% Plot the weights of its long-run component
figure(2)
plot(seq,weights','LineWidth',1);
legend('Beta Weights','Location','NorthEast')
xlim([1,24])
title('Beta Weights Fixed Default RV')
```

IP growth rate example



Consumer Sentiment example

Now we estimate with consumer sentiment driving our long term volatility. This is quarterly data.
First, load in sentiment data and truncate it. Want to make sure it starts when NASDAQ daily data does.

Listing 12: Load and truncate data

```
%%Now we run GARCH-MIDAS where exogonous variable is Consumer Sentiment.

load('UMCSENT_quarterly.mat')

yDate=NASDAQ_trunc.DATE(1:end,:);
yDateDetails = datevec(yDate);
yDateMonth = yDateDetails(:,2); %grab all months..
```

We also grab all of the months of the data.

Consumer Sentiment example

Now match months to quarters. We are setting the data up so we can repeat quarterly values for daily returns.

```
%Now need to translate months into corresponding quarters....
yQuartMonth((yDateMonth==1),:)= 1;
yQuartMonth((yDateMonth==2),:)= 1;
yQuartMonth((yDateMonth==3),:)= 1;

yQuartMonth((yDateMonth==4),:)= 4;
yQuartMonth((yDateMonth==5),:)= 4;
yQuartMonth((yDateMonth==6),:)= 4;

yQuartMonth((yDateMonth==7),:)= 7;
yQuartMonth((yDateMonth==8),:)= 7;
yQuartMonth((yDateMonth==9),:)= 7;

yQuartMonth((yDateMonth==10),:)= 10;
yQuartMonth((yDateMonth==11),:)= 10;
yQuartMonth((yDateMonth==12),:)= 10;
```

Consumer Sentiment example

Repeat the quarterly values throughout the days.

Listing 13: Repeat quarterly sentiment for daily returns

```
xMonth = UMCSENT.UMCSENT./100;
nobs = size(y,1);

xDay = NaN(nobs,1);
count = 1;
for t = 1:nobs
    if t > 1 && yQuartMonth(t) ~= yQuartMonth(t-1)
        count = count + 1;
        if count > length(xMonth)
            break
        end
    end
    xDay(t) = xMonth(count);
end
```

Consumer Sentiment example

Set period and lag specifications. Then estimate the GARCH-MIDAS model.

Listing 14: Estimate GARCH-MIDAS

```
% Estimate the GARCH-MIDAS model with the exogenous regressor
period = 66;
numLags = 12;

[estParams,EstParamCov,Variance,LongRunVar,ShortRunVar,logL] = GarchMidas(y,'
    Period',period,'NumLags',numLags,'X',xDay,'ThetaM',1);
```

Consumer Sentiment example

Check parameter estimates. Theta is the coefficient for the long-term component.
Negative in front tells us we have a counter-cyclical relationship.
This means that an increase in consumer sentiment is associated with a decline in long-term volatility.

```
Method: Maximum likelihood
Sample size: 12625
Adjusted sample size: 11833
Logarithmic  likelihood:      38717.6
Akaike   info criterion:     -77423.1
Bayesian info criterion:     -77378.5
```

	Coeff	StdErr	tStat	Prob
mu	0.00079391	7.0053e-05	11.333	0
alpha	0.10117	0.0029431	34.374	0
beta	0.88705	0.0032339	274.3	0
theta	-4.8039e-05	3.9704e-05	-1.2099	0.22631
w	5.9057	18.59	0.31768	0.75073
m	0.00017099	3.8211e-05	4.475	7.6417e-06

RMSE of one-step variance forecast (period 1 to 12625): 4.487e-04.

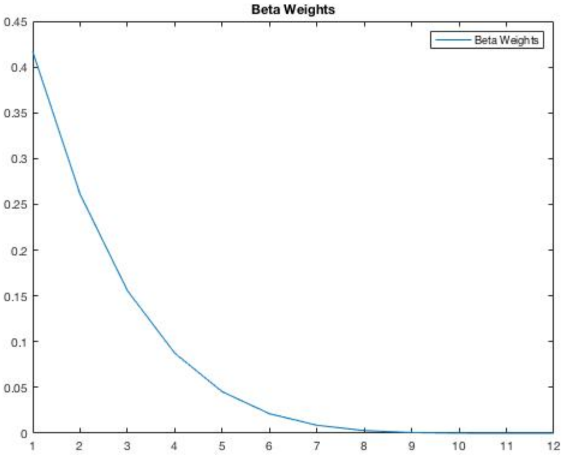
Consumer Sentiment example

Recover the weighting scheme and plot it.

```
%%if we want to recover the weights, we can do the following:
param1 = estParams(5);
seq = numLags:-1:1;
weights = (1-seq./numLags+10*eps).^(param1-1);
%now re-scale weights...
weights = weights ./ nansum(weights);


% Plot the weights of its long-run component
figure(2)
plot(seq,weights','LineWidth',1);
legend('Beta Weights','Location','NorthEast')
xlim([1,24])
title('Beta Weights Fixed Default RV')
```

Consumer Sentiment example



The End