Assignment 5: Random Preference Models

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The goal of this exercise is to introduce simulation methods. We use data from a field experiment. Each choice is binary, and the lotteries are denoted ($\mathcal{C}_1$ and $\mathcal{C}_2$). They are composed of a low and high payoffs. Subjects are presented with five gambles entailing binary choices. In each case, the first option has lower expected value and dispersion than the second. At the fifth choice, expected values are similar across lotteries, and the second option still has higher variance. In total, there is 25 decisions, which are stored in dat_choices.

Table 1: Risk Aversion Lotteries: Ordered Lotteries Selection

	$\mathcal{C}_1$				$\mathcal{C}_2$			
	$\mathbb{P}_1$	$\mathbb{P}_2$	EV	SD.	$\mathbb{P}_1$	$\mathbb{P}_2$	EV	SD.
List 1	48	48	48	0	40	64	52	16.97
	40	64	52	16.97	32	80	56	33.94
	32	80	56	33.94	24	96	60	50.91
	24	96	60	50.91	16	112	64	67.88
	16	112	64	67.88	8	120	64	79.2
List 2	48	48	48	0	42	66	54	16.97
	42	66	54	16.97	36	84	60	33.94
	36	84	60	33.94	30	102	66	50.91
	30	102	66	50.91	24	120	72	67.88
	24	120	72	67.88	16	128	72	79.2
List 3	48	48	48	0	38	62	50	16.97
	38	62	50	16.97	28	76	52	33.94
	28	76	52	33.94	18	90	54	50.91
	18	90	54	50.91	8	104	56	67.88
	8	104	56	67.88	0	112	56	79.2
List 4	42	42	42	0	36	60	48	16.97
	36	60	48	16.97	30	78	54	33.94
	30	78	54	33.94	24	96	60	50.91
	24	96	60	50.91	18	114	66	67.88
	18	114	66	67.88	10	122	66	79.2
List 5	54	54	54	0	44	68	56	16.97
	44	68	56	16.97	34	82	58	33.94
	34	82	58	33.94	24	96	60	50.91
	24	96	60	50.91	14	110	62	67.88
	14	110	62	67.88	6	118	62	79.2

Notes: Table lists five lotteries. $\mathcal{C}_1$ and $\mathcal{C}_2$ refer to the choices of the lottery, while $\mathbb{P}_1$ and $\mathbb{P}_2$ refers to the payoffs. $\mathbb{E}V$ is for expected value, while SD is the standard deviation across payoffs. Payoffs are in Canadian \$. Source: SRDC-CIRANO Field Experiment on Education Financing.

Exercise 1 Part 1

Consider an individual i endowed with an initial wealth w_i which is set to 20. Individuals have CRRA utility function. Consider the choice between $\mathcal{C}_{1l} = (c_{11l}, c_{12l})$ and $\mathcal{C}_{2l} = (c_{21l}, c_{22l})$. We assume that individuals are endowed with CRRA utility function. Further, we assume that at decision l, the risk aversion of individual i, denoted by θ_{il} is a random draw

$$\theta_{il} \sim \mathcal{N}(\theta_i, \sigma_i) \quad \forall \quad l \tag{1}$$

Specifically, we assume that attitudes toward risk is not summarized by a parameter, but a distribution with mean θ_i and standard deviation σ_i. The goal is to estimate $theta_i$ and σ_i. The practical consequence is that individual may use different draws to evaluate utilities.

Given this structure, the value of each alternative is given by $V_{il1}(\mathcal{C}_{1l})$ and $V_{il2}(\mathcal{C}_{l1})$:

$$V_{1i}(\mathcal{C}_{1l}, \theta_{il}) = 0.5\,(u(c_{11l}, \theta_{il}) + u(c_{12l}, \theta_{il})) \tag{2}$$

$$V_{2i}(\mathcal{C}_{2l}, \theta_{il}) = 0.5\,(u(c_{21l}, \theta_{il}) + u(c_{22l}, \theta_{il})) \tag{3}$$

θ_{il} represents the level of risk aversion used to evaluate lottery 1 and lottery 2 at decision l.

The likelihood of choosing lottery 1 is given by:

$$\Pr(y_{il} = 1) = \int V_{1i}(\mathcal{C}_{1l}, x) - V_{2i}(\mathcal{C}_{2l}, x) d\Phi(x)$$

Where Φ is the cdf of a normal distribution. The likelihood does not admit any closed form solution.

A frequency simulator is as follows:

Initialization Step Set an initial guess for θ_i and σ_i

Simulation Step At simulation step s, draw θ_{il} from $\mathcal{N}(\theta_i, \sigma_i)$ and evaluate $\delta = V_{1i}(\mathcal{C}_{1l}, x) - V_{2i}(\mathcal{C}_{2l}, x)$.

Let the simulated probability be defined as:

$$\overset{s}{\Pr}(y_{il} = 1) = \begin{cases} 1 & \text{if } \delta \geq 0 \\ 0, & \text{if } \delta < 0 \end{cases} \tag{4}$$

Repeat this step $S = 1000$ times and store $\Pr^s(y_{il} = 1)$ for $s \in 1, \ldots 1000$.

Likelihood Step Approximate the integral using simulations, and compute

$$\Pr(y_{il} = 1) = \frac{1}{S} \sum_{s=1}^{S} \overset{s}{\Pr}(y_{il} = 1)$$

Then evaluate the likelihood

$$\log \mathcal{L}_i = \sum_{l=1}^{L} y_{il} \cdot \log(\Pr(y_{il} = 1)) + (1 - y_{il}) \cdot \log(1 - \Pr(y_{il} = 1))$$

1. Assume the scale parameter is known ($\sigma = 0.5$), For a grid of risk aversion parameters (0.1 to 2), report the relationship between the choice probability and the risk aversion.

2. Implement the frequency simulator and estimate the risk aversion for individuals 120, 280 and 1200.