Lecture 9: Decomposition Method

Introduction ot Econometrics,Fall 2017

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11/27/2017

Review Previous Lectures

Topics covered

- Main Content

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- Main Content
  - Build a framework of Causal Inference

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    - Build a framework of Causal Inference
    - Review Basic Probability and Statistics

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  - Comprehensive Evaluations in Multiple OLS

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    - Comprehensive Evaluations in Multiple OLS
    - Nonlinear Regression model: Dummy dependent variable

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$$\beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$$

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     $$\beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$$

     for any group $G_m : \ m = 1, 2, ..., M$
  2. No Endogeneity(Internal Valid): there is no endogeneity in these estimating models. Essentially,**the 1st Assumption of indentification** in OLS model is satisfied.Thus

     $$E(u_i|X_1, X_2, ..., X_k) = 0$$

An simple Extension: Decomposition Method

1. Hetergeneity: Gap between two groups

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1. Hetergeneity: Gap between two groups
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    - Ignoriable or Conditional Independence Assumption(CIA)

Decomposition Methods

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    - Wage difference
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    - More unobservable characteristics
- The typical question is "what the pay(or other outcomes) would be *if women had* the same characteristics as men?"
- It will help us construct a counterfactual state by Counterfactual Exercises to recovery the causal effect((sort of causal) of a certain factor.

Decomposition Methods to Gaps: Two Categories

1. In Mean
   - Oaxaca-Blinder(1974):**OB**
   - Brown(1980):

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2. In Distribution(Skipped)
   - Juhn, Murphy and Pierce(1993): JMP
   - Machado and Mata(2005): MM
   - DiNardo, Fortin and Lemieux(1996): DFL
   - Firpo, Fortin and Lemieux(2007,2010): DFL
   - Donald et al(2000): DGF
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- Although some of methods listed above is quite sophisicated and frontier in the filed, the OB is so fundemental that all other methods can explained by it.Therefore,in our lecture, we will **only** cover **OB** and its extension version in nonlinear function.

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- Use a dummy variable in a regression function

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- So we want to know if there is the wage differential between male and female,then see if β_1 is large enough and significant statistically.
- but the result can only answer to the question: "is there a wage gap between men and women in the labor market"

Gender Wage Gap

```
##       ahe             yrseduc          female             age
##  Min.    :  2.00   Min.    : 6.0   Min.    :0.0000   Min.    :21.00
##  1st Qu.: 13.46   1st Qu.:12.0   1st Qu.:0.0000   1st Qu.:33.00
##  Median : 19.23   Median :13.0   Median :0.0000   Median :42.00
##  Mean   : 23.89   Mean   :14.1   Mean   :0.4385   Mean   :42.27
##  3rd Qu.: 29.81   3rd Qu.:16.0   3rd Qu.:1.0000   3rd Qu.:51.00
##  Max.   :400.64   Max.   :20.0   Max.   :1.0000   Max.   :64.00
##    northeast          midwest           south              west
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.00
##  Median :0.0000   Median :0.0000   Median :0.0000   Median :0.00
##  Mean   :0.1893   Mean   :0.2275   Mean   :0.3286   Mean   :0.2546
##  3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:1.00
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##      logahe            age2
##  Min.   :0.6931   Min.   : 441
```

Gender Wage Gap

```
##
## Call:
##   felm(formula = ahe ~ female + yrseduc + age + I(age^2) + west
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -42.28   -8.09   -2.14    5.34  354.06
##
## Coefficients:
##               Estimate Robust s.e t value Pr(>|t|)
## (Intercept) -4.280e+01  7.717e-01  -55.47   <2e-16 ***
## female      -5.979e+00  1.099e-01  -54.39   <2e-16 ***
## yrseduc      2.759e+00  2.537e-02  108.76   <2e-16 ***
## age          1.332e+00  3.540e-02   37.62   <2e-16 ***
## I(age^2)    -1.291e-02  4.307e-04  -29.97   <2e-16 ***
## west        -3.596e-01  1.771e-01   -2.03   0.0424 *
```

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- The aim of the OB decomposition is to explain *how much of the difference in mean outcomes* across two groups is due to *group differences in the levels of explanatory variables*, and how much is due to *differences in the magnitude of regression coefficients*(Oaxaca 1973; Blinder 1973).

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- The aim of the OB decomposition is to explain *how much of the difference in mean outcomes* across two groups is due to *group differences in the levels of explanatory variables*, and how much is due to *differences in the magnitude of regression coefficients*(Oaxaca 1973; Blinder 1973).
- Altough most applications of the technique can be found in the labor market and discrimination literature, it can also be useful in other fields. In general, the technique can be employed to study group differences in any (continuous or categorical) outcome variable.

Oaxaca-Blinder Decomposition

- Assume that a multiple OLS regression equation is

$$Y_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_k X_{ki} + u_i$$

where Y_i is dependent variable,$X_i s$ are a series independent(controlling) variables which affect Y_i.And u_i are error terms which satisfied by $E(u_i|X_1, ..., X_k) = 0$

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- use sample estimator to replace the population parameters and for the definition of error term, thus $\sum u_i = 0$, then

$$\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X}_1 + ... + \hat{\beta}_k \bar{X}_k$$

Oaxaca-Blinder Decomposition: Two groups

- If we assume that whole sample can be divided into 2 groups: A and B,then we could regress the similar regression using A and B subsamples, repectively. Thus,

$$
\begin{aligned}
Y_{Ai} &= \beta_{A0} + \beta_{A1} X_{1i} + ... + \beta_{Ak} X_{ki} + u_{Ai} \\
Y_{Bi} &= \beta_{B0} + \beta_{B1} X_{1i} + ... + \beta_{Bk} X_{ki} + u_{Bi}
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- Accordingly, we can obtain the means of outcome Y for group A and group B are

$$
\begin{aligned}
\bar{Y}_A &= \hat{\beta}_{A0} + \hat{\beta}_{A1} \bar{X}_1 + ... + \hat{\beta}_{Ak} \bar{X}_k \\
&= \bar{X}'_A \hat{\beta}_A \\
\bar{Y}_B &= \hat{\beta}_{B0} + \hat{\beta}_{B1} \bar{X}_1 + ... + \hat{\beta}_{Bk} \bar{X}_k \\
&= \bar{X}'_B \hat{\beta}_B
\end{aligned}
$$

Oaxaca-Blinder Decomposition: difference in mean

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- Then the second term is **characteristics effect** which describes how much the difference of outcome, Y, in mean is due to differences in the levels of explanatory variables(characteristics).

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- Then the second term is **characteristics effect** which describes how much the difference of outcome, Y, in mean is due to differences in the levels of explanatory variables(characteristics).
- the first term is **coefficients effect** which describes how much the difference of outcome, Y, in mean is due to differences in the magnitude of regression coefficients.

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2. Unexplained Part:due to differences in the coefficients to explanatory variables: such as **returns** to schooling years, experience and tenure and **premium** in industry and occupation,etc
 -**coefficients effect -returns effect -structure effect**

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Oaxaca-Blinder Decomposition: difference in mean

- use a *different reference group*: plus and minus a term $\bar{X}'_B\hat{\beta}_A$,then

$$\begin{aligned}
\bar{Y}_A - \bar{Y}_B &= \bar{X}'_A\hat{\beta}_A - \bar{X}'_B\hat{\beta}_B \\
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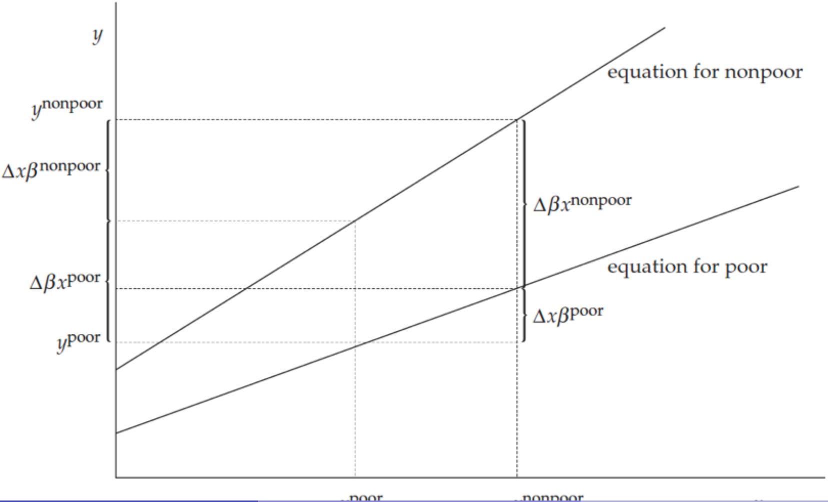
- Then the first term is **characteristics effect**.We also called it **endownment effect** as the amount of X_j can be seen as an endownment for group A or B.
- The second term is **coefficients effect**. We also called it **price(returns) effect** as the estimate coefficents $\hat{\beta}_j$ can be seen as the market price of or the returns to a certain X_j.

Oaxaca-Blinder Decomposition: Reference group problem

- What is the **ture** coefficient or characteristics effect ?



Oaxaca-Blinder Decomposition: a general framework

- Let β^* be such a nondiscriminatory coefficient vector. The outcome difference can then be written as

$$\begin{aligned}
\bar{Y}_A - \bar{Y}_B &= \bar{X}_A'\hat{\beta}_A - \bar{X}_B'\hat{\beta}_B \\
&= \bar{X}_A'\hat{\beta}_A - \bar{X}_A'\hat{\beta}^* + \bar{X}_A'\hat{\beta}^* - \bar{X}_B'\hat{\beta}^* + \bar{X}_B'\hat{\beta}^* - \bar{X}_B'\hat{\beta}_B \\
&= (\bar{X}_A' - \bar{X}_B')\hat{\beta}^* + [\bar{X}_A'(\hat{\beta}_A - \hat{\beta}^*) + \bar{X}_B'(\hat{\beta}^* - \hat{\beta}_B)]
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- However, the nondiscriminatory coefficients β^* is unknown. On the different circumstances, the value could be quite different.

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- Several suggestions have been made in the literature.

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- Several suggestions have been made in the literature.
  1. there may be reason to assume that discrimination is directed toward only **one** group.
     - For example: it is reasonable to assume that wage discrimination is directed only against women and there is no (positive) discrimination of men. And if we assume that members of group A are males and members of group B are females. Then we have $\beta^* = \beta_A$ and the wage gap can be decomposed into as

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     - Similarly, if there is only (positive) discrimination of men but no discrimination of women, the decomposition is

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Oaxaca-Blinder Decomposition: Weighted reference group

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Oaxaca-Blinder Decomposition: Weighted(Continued)

- As pointed out by Oaxaca and Ransom (1994), Using a special weighted matrix,the difference can also be expressed as

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- They show that

$$\hat{W} = \Omega = (X'_A X_A + X'_B X_B)^{-1}(X'_A X_A)$$

where X as the observed data matrix is equivalent to Neumark(1988),which use the coeffients from a *pooled model over both groups* as the reference coefficients.

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- where D as an indicator for group B, such as "female" in gender wage gap case

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- Then the **explained part** of the differential is

$$(\bar{X}_A - \bar{X}_B)\gamma^* = (\bar{X}_A - \bar{X}_B)[\gamma + \delta \frac{Cov(X, D)}{Var(X)}$$

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- However, deriving standard errors for the decomposition components seems to cause problems.
- Without reporting s.e. or C.I is problematic because it is hard to evaluate the significance of reported decomposition results without knowing anything about their sampling distribution.

Standard Errors for OB decomposition

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$$\hat{V}(\overline{X}'\hat{\beta}) = \overline{X}'\hat{V}(\hat{\beta})\overline{X} + \hat{\beta}'\hat{V}(\overline{X})\hat{\beta} + trace[\hat{V}(\overline{X})\hat{V}(\hat{\beta})]$$

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- where $\hat{V}(\hat{\beta})$ is simply the variance–covariance matrix obtained from the regression procedure.
- $V(\overline{X})$'s natural estimator is $\hat{V}(\overline{X})$ which is the sampling variance of $\overline{X}$.

Standard Errors for OB decomposition

- Then the variances for the components of the Blinder–Oaxaca decomposition can be derived analogously.

$$\begin{aligned}\hat{V}[(\overline{X}_A - \overline{X}_B)'\hat{\beta}_A] \approx (\overline{X}_A - \overline{X}_B)'\hat{V}(\hat{\beta}_A)(\overline{X}_A - \overline{X}_B) \\ +\hat{\beta}_A'[\hat{V}(\overline{X}_A) + \hat{V}(\overline{X}_B)]\hat{\beta}_A\end{aligned}$$

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- We could also obtain similar result for the alternative form of the components of the OB decomposition. $\hat{V}[\overline{X}'_B - (\hat{\beta}_A - \hat{\beta}_B)]$

Standard Errors for OB decomposition

2. Bootstrap Method

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- We will breifly introduce the topic later.

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- For example, one might want to evaluate how much of the gender wage gap is due to differences in education and how much is due to differences in work experience.
- Similarly, it might be informative to determine how much of the unexplained gap is related to differing returns to education and how much is related to differing returns to work experience.

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- The first summand reflects the contribution of the group differences in $X1$; the second, of differences in $X2$; and so on.
- Also the estimation of standard errors for the individual contributions is straightforward.

Detailed Decomposition:

- the individual contributions to the unexplained part are the summands in

$$\bar{X}_B'(\hat{\beta}_A - \hat{\beta}_B) = \bar{X}_{1B}'(\hat{\beta}_{1A} - \hat{\beta}_{1B}) + \bar{X}_{2B}'(\hat{\beta}_{2A} - \hat{\beta}_{2B})...$$

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- However, other than for the explained part of the decomposition, the contributions to the unexplained part is not evident.

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  - The first summand is the part of the unexplained gap that is due to "group membership"
  - the second summand reflects the contribution of differing returns to X.

Detailed Decomposition:

- Now assume that the zero point of X is shifted by adding a constant, a.The effect of such a shift on the decomposition results is as follows

$$\bar{X}_B(\hat{\beta}_A - \hat{\beta}_B) = [(\hat{\beta}_{0A} - a\hat{\beta}_{1A}) - (\hat{\beta}_{0B} - a\hat{\beta}_{1B})] - (\hat{\beta}_{1A} - \hat{\beta}_{1B})(\bar{X}_B + a)$$

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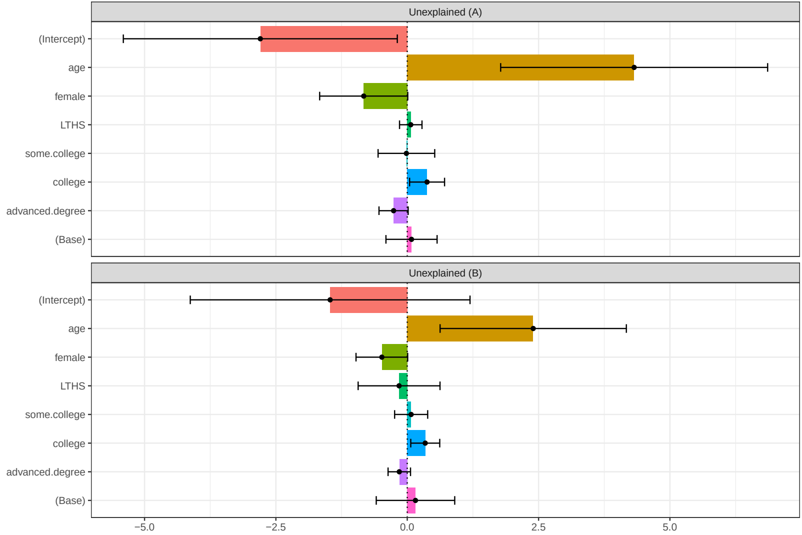
- The conclusion is that the detailed decomposition results for the unexplained part have a meaningful interpretation only for variables for which scale shifts are not allowed, that is, for variables that have a natural zero point.
- Luckily, in practice, it seems that people pay little attention on the issues.

Oaxaca-BLinder Decomposition: Native-Migrant Wage Gap

```
##        weight coef(explained) se(explained) coef(unexplained)
## [1,]  0.0000000       1.6165339     0.6763945          1.399040
## [2,]  1.0000000       0.1822482     0.7022436          2.833326
## [3,]  0.5000000       0.8993911     0.5700381          2.116183
## [4,]  0.5690691       0.8003263     0.5746951          2.215248
## [5,] -1.0000000       1.3557222     0.5258576          1.659852
## [6,] -2.0000000       0.9525717     0.5348045          2.063003
##      se(unexplained) coef(unexplained A) se(unexplained A)
## [1,]       0.9551153        1.399040e+00      9.551153e-01
## [2,]       0.8956480        0.000000e+00      0.000000e+00
## [3,]       0.8407323        6.995202e-01      4.775577e-01
## [4,]       0.8379134        6.028898e-01      4.115887e-01
## [5,]       0.6673190        9.445705e-01      3.806703e-01
## [6,]       0.8358226        4.840572e-14      3.844533e-14
##      coef(unexplained B) se(unexplained B)
```

Oaxaca-BLinder Decomposition: Native-Immigrant Wage Gap



Introduction to bootstrap

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- The general idea is to treat the observed data as a population that we can draw samples from. The most common resampling method is the **bootstrap**.

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- Instead of drawing from a specified distribution (such as the normal) by a random number generator, the bootstrap draws with replacement from the sample.
- It therefore takes the empirical distribution function (the step-function) as the true distribution function.
- The great advantage is that we neither make assumption about the distributions nor about the true values of the parameters.

The Method: Nonparametric Bootstrap

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- A very simple approach is to use the quantiles of the bootstrap sampling distribution of the estimator to establish the end points of a confidence interval nonparametrically.

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1. Draw B independent bootstrap samples (Y_i^*, X_i^*) of size N from original sample (Y_i, X_i).Usually $B = 100$ replications are sufficient.
2. Estimate the parameter β of interest for *each* bootstrap sample:

$$\hat{\beta}_b^* \; for \, b = 1, 2, ..., B$$

Bootstrap Standard Errors

3. Estimate $se(\hat{\beta})$ by

$$\hat{se}(\hat{\beta}) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(\hat{\beta}_b^* - \hat{\beta}^*)}$$

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- where $\hat{\beta}^* = \frac{1}{B-1}\hat{\beta}_b^*$
- In case, the estimator $\hat{\beta}$ is consistent and asymptotically normally distributed, bootstrap standard errors can be used to construct approximate confidence intervals and to perform asymptotic tests based on the normal distribution.

Bootstrap: Confidence Intervals

- We can construct a two-sided equal-tailed 1α confidence interval for an estimate $beta$ from the empirical distribution function of a series of bootstrap replications.

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- The estimated $1 - \alpha$ confidence interval of $\hat{\beta}$ is

$$\left[\hat{\beta}^*_{B\frac{\alpha}{2}}, \hat{\beta}^*_{B(1-\frac{\alpha}{2})}\right]$$

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- So the critical values are

$$t_{\frac{\alpha}{2}} = t^*_{B\frac{\alpha}{2}}, t_{1-\frac{\alpha}{2}} = t^*_{B(1-\frac{\alpha}{2})}$$

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- When the outcome of one of many small steps immediately affects the next, rapid results are important.

An Replicating Case Study

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- The sample is restricted to civilian wage and salary workers, thereby omitting self-employed workers.
- The wage rates are the hourly wage as reported directly by those paid by the hour. For those who are paid on another basis –day, week, month, usual weekly earnings are divided by usual weekly hours.

Know the distribution of interest

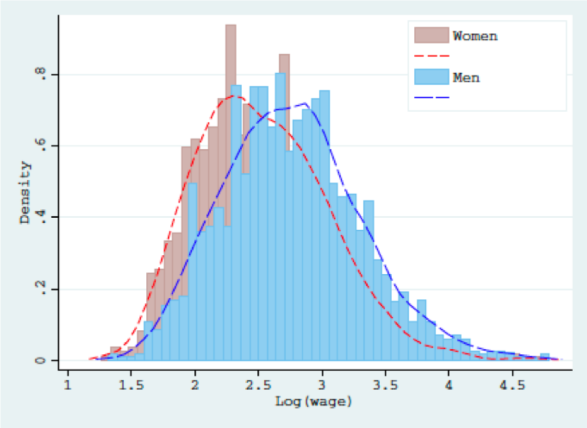

Figure 1: Densities of Male and Female Wages

Figure 2: Fig

Coefficients of Selected variables

Table 2. Means and OLS Regression Coefficients of Selected Variables from NLSY Log Wage Regressions for Workers Ages 35-43 in 2000

Explanatory Variables	(1) Means		(2) Male Coef.		(3) Female Coef.		(4) Male Coef		(5) Pooled Coef	
Female	0	1							-0.092	(0.014)
Education and skill level										
<10 yrs.	0.053	0.032	-0.027	(0.043)	-0.089	(0.05)	-0.027	(0.043)	-0.045	(0.033)
10-12 yrs (no diploma or GED)	0.124	0.104	---	---	---	---	---	---	---	---
HS grad (diploma)	0.326	0.298	-0.013	(0.028)	-0.002	(0.029)	-0.013	(0.028)	-0.003	(0.02)
HS grad (GED)	0.056	0.045	0.032	(0.042)	-0.012	(0.044)	0.032	(0.042)	0.006	(0.03)
Some college	0.231	0.307	0.164	(0.031)	0.101	(0.03)	0.164	(0.031)	0.131	(0.022)
BA or equiv. degree	0.155	0.153	0.380	(0.037)	0.282	(0.036)	0.380	(0.037)	0.330	(0.026)
MA or equiv. degree	0.041	0.054	0.575	(0.052)	0.399	(0.046)	0.575	(0.052)	0.468	(0.034)
Ph.D or prof. Degree	0.015	0.007	0.862	(0.077)	0.763	(0.1)	0.862	(0.077)	0.807	(0.06)
AFQT percentile score (x.10)	4.231	3.971	0.042	(0.004)	0.041	(0.004)	0.042	(0.004)	0.042	(0.003)
L.F. withdrawal due to family resp.	0.129	0.547	-0.078	(0.025)	-0.083	(0.019)	-0.078	(0.025)	-0.067	(0.015)
Lifetime Work Experience										
Years worked civilian	17.160	15.559	0.038	(0.003)	0.030	(0.002)	0.038	(0.003)	0.033	(0.002)
Years worked military	0.578	0.060	0.024	(0.005)	0.042	(0.013)	0.024	(0.005)	0.021	(0.004)
% worked part-time	0.049	0.135	-0.749	(0.099)	-0.197	(0.049)	-0.749	(0.099)	-0.346	(0.044)
Industrial Sectors										
Primary, Constr. & Utilities	0.186	0.087	---	---	---	---	0.059	(0.031)	---	---
Manufacturing	0.237	0.120	0.034	(0.026)	0.140	(0.035)	0.093	(0.029)	0.072	(0.021)
Education, Health, & Public Adm.	0.130	0.358	-0.059	(0.031)	0.065	(0.03)	---	---	-0.001	(0.02)
Other Services	0.447	0.436	0.007	(0.024)	0.088	(0.029)	0.066	(0.026)	0.036	(0.018)
Constant			2.993	(0.156)	2.865	(0.144)	2.934	(0.157)	2.949	(0.105)
Dependent Var. (Log Hourly Wage)	2.763	2.529								
Adj. R-Square			0.422		0.407		0.422		0.431	
Sample size	2655	2654								

Figure 3: Fig

OB decomposition

Reference Group:	(1) Using Male Coef. from col. 2, Table 2		(2) Using Male Coef. from col. 4, Table 2		(3) Using Female Coef.		(4) Using Weighted Sum		(5) Using Pooled from col. 5, Table 2	
Unadjusted mean log wage gap :										
$E[\ln(w_m)]-E[\ln(w_f)]$	0.233	(0.015)	0.233	(0.015)	0.233	(0.015)	0.233	(0.015)	0.233	(0.015)
Composition effects attributable to										
Age, race, region, etc.	0.012	(0.003)	0.012	(0.003)	0.009	(0.003)	0.011	(0.003)	0.010	(0.003)
Education	-0.012	(0.006)	-0.012	(0.006)	-0.008	(0.004)	-0.010	(0.005)	-0.010	(0.005)
AFQT	0.011	(0.003)	0.011	(0.003)	0.011	(0.003)	0.011	(0.003)	0.011	(0.003)
L.T. withdrawal due to family	0.033	(0.011)	0.033	(0.011)	0.035	(0.008)	0.034	(0.007)	0.028	(0.007)
Life-time work experience	0.137	(0.011)	0.137	(0.011)	0.087	(0.01)	0.112	(0.008)	0.092	(0.007)
Industrial sectors	0.017	(0.006)	0.017	(0.006)	0.003	(0.005)	0.010	(0.004)	0.009	(0.004)
Total explained by model	0.197	(0.018)	0.197	(0.018)	0.136	(0.014)	0.167	(0.013)	0.142	(0.012)
Wage structure effects attributable to										
Age, race, region, etc.	-0.098	(0.234)	-0.098	(0.234)	-0.096	(0.232)	-0.097	(0.233)	-0.097	(0.24)
Education	0.045	(0.034)	0.045	(0.034)	0.041	(0.033)	0.043	(0.034)	0.043	(0.031)
AFQT	0.003	(0.023)	0.003	(0.023)	0.003	(0.025)	0.003	(0.024)	0.002	(0.025)
L.T. withdrawal due to family	0.003	(0.017)	0.003	(0.017)	0.001	(0.004)	0.002	(0.011)	0.007	(0.01)
Life-time work experience	0.048	(0.062)	0.048	(0.062)	0.098	(0.067)	0.073	(0.064)	0.092	(0.065)
Industrial sectors	-0.092	(0.033)	0.014	(0.028)	-0.077	(0.029)	-0.085	(0.031)	-0.084	(0.032)
Constant	0.128	(0.213)	0.022	(0.212)	0.193	(0.211)	0.128	(0.213)	0.128	(0.216)
Total wage structure - Unexplained log wage gap	0.036	(0.019)	0.036	(0.019)	0.097	(0.016)	0.066	(0.015)	0.092	(0.014)

Figure 4: Fig