Problem set 4

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Consider the Aiyagari model described in the lecture. The log of labor income, y, follows an AR(1) process with standard deviation of innovation equal to 0.1, *and autocorrelation* 0.9. *Discretize it through a 7 -state Markov chain. The borrowing limit is* $\underline{a} = 0$. *Period utility is CRRA with risk aversion parameter* $\sigma = 2$, *and the discount factor is* $\beta = 0.95$. *Production technology is Cobb-Douglas with capital share equal to* 0.33, *and depreciation rate equal to* 0.1.

Problem 1 (required) - Partial Equilibrium

a. *Set the interest rate to* 0.04, *the wage to* 1, *and solve the household's income-fluctuation problem. Use both piecewise-linear interpolation and the endogenous-grid method for policy-function iteration. (*35 *points)*

Solution. For this first item, we obtained the policy function for consumption and capital, that are represented in Figures 1 and 2, respectively.

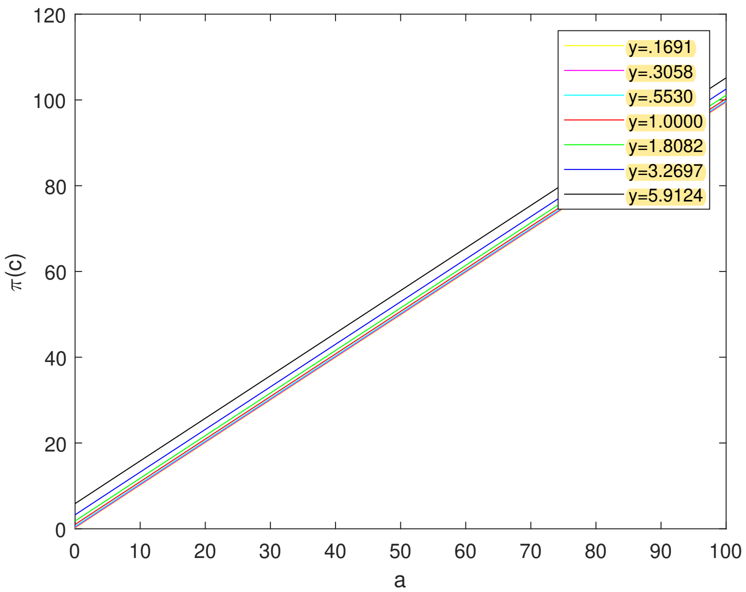

Figure 1 – Policy function for consumption for different values of productivity in the Partial Equilibrium.

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Differently from the consumption policy function, we see that the asset policy is almost indistinguishable for different values of the productivity y, causing the lines in Figure 2 to be almost overlapping completely.

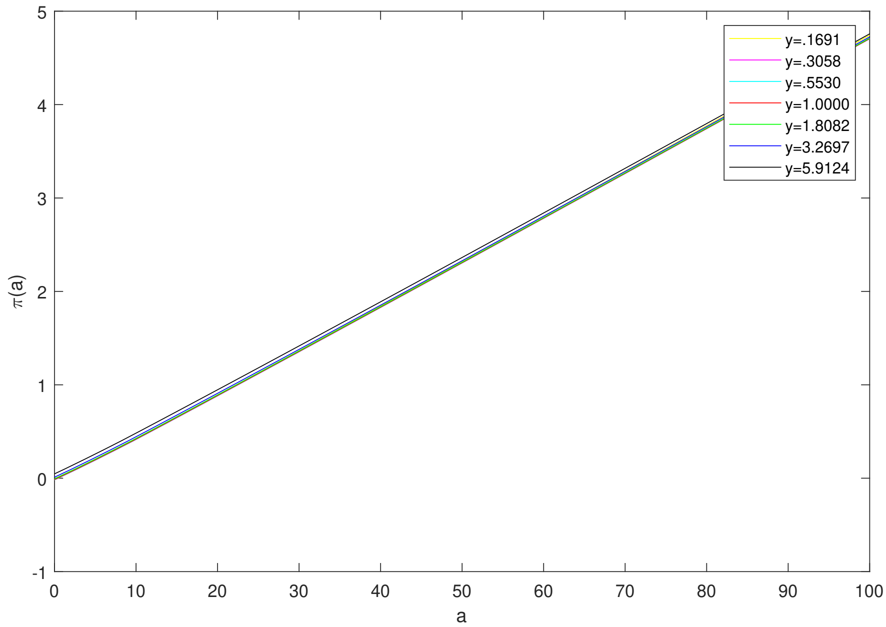

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Figure 2 – Policy function for the assets for different values of productivity in the Partial Equilibrium.

□

b. *Compute the steady state of this economy. Use an approximation to the density or a Monte-Carlo simulation (or both and compare if you have time). (30 points)*

The steady state for the partial equilibrium is characterized by the policy functions that result from the household optimization problem, which includes to take into account the borrowing constraint (which we imposed by choosing the maximum between $a'(a,y)$ and $\underline{a}$ to compute the Euler equation). Then, we need to find the transition function Q, based on the transition probabilities for the productivity (which is obtained in our case using Rouwenhorst's algorithm). Our attempt can be seen here: but it is not working. Our criteria is never met hence the Q function that is returned is only zeros.

The next element necessary to have the stationary partial equilibrium is the stationary distribution $\lambda(\mathcal{A}, \mathcal{Y})$, which is obtained by integrating the transition function Q. Since our function Q didn't work, we were not able to find $\lambda(\mathcal{A}, \mathcal{Y})$. Still, we made an implementation based on the idea that the stationary distribution should be invariant under the dot product, i.e., if we repeat iteratively a dot product of the matrix with a vector, we should be able to converge to λ. Our attempt can be seen here.

What we furthermore tried is calculating the invariant distribution with piecewise-linear approximation. Before using the actual policy function, we used $g_a(a_k, y_j) = (1 + r_0) * a_k + y_j$ to see whether our algorithm works: After adjusting the distribution $\lambda(a, y)$ in 57 iterations, we can find an aggregate level of assets $A = 0.0518$. The convergence is faster when using π, the direct measure for the stationary distribution of the Markov process which we obtained with the Rouwenhorst discretisation: We only need 50 iterations to arrive at $A = 0.0884$. We are unfortunately aware, that someting in our code is not working as the equilibrium labour supply is negative with -0.0780.

To find the labour demand given the interest rate r_0, we know from the optimality condition of the firm:

$$r_0 = MPK = \alpha K^{\alpha-1}L^{1-\alpha} = 0.33K^{-0.67}L^{0.67} = 0.04$$
$$w = MPL = (1-\alpha)K^{\alpha}L^{-\alpha} = 0.67K^{0.33}L^{-0.33} = \left(\frac{K}{L}\right)^{0.33}$$
$$(0.33/0.04)^{(1/0.67)} = \left(\frac{K}{L}\right)$$
$$w(r_0) = 0.67\left((0.33/0.04)^{(1/0.67)}\right)^{0.33} = 1.8944$$

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With a wage level of $w = 1$, we therefore have more labour demand than labour supply (firms would be willing to pay more than current wage).

c. *Compute and plot the Lorenz curve for earnings. (*10 *bonus points)*

Problem 2 (required) General Equilibrium and Calibration

a. *Compute the interest rate and wage that constitute a stationary general equilibrium. (35 points)*

Unlike question 1, we now will compute the model considering that both r and w are not given. In practice, this means that we solve the household optimization problem, and find the stationary equilibrium for a given interest rate level r_0. We compare this value with the one that is obtained from the stationary levels of aggregate capital and labor supply.If the asset demand (firm side) is higher than asset supply (household side), we can increase the interest rate such that households have a higher incentive to lend to firms. We run our algorithm again, get a new stationary distribution for $a'(a, y)$ as the policy function for asset has changed; with this new distribution, we can again calculate the partial equilibrium asset supply by households and check whether the difference in asset supply and demand is lower. We iterate this procedure as long as the difference is larger than a stopping condition $\epsilon > 0$.

Our implementation can be seen here. We had technical difficulties in imposing the borrowing constraint, which causes the consumption level to be negative in some entries. This makes the policy function search to not be feasible. This algorithm also uses a Monte-Carlo simulation instead of the piecewise linear approximation of the invariant distribution; it is way slower than than our procedure in 1b), but seems to give more reasonable results.

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b. *Find the discount factor, and a total factor productivity that yield an equilibrium with a wealth-to-output ratio of* 2.5*, and output equal to* 1*. Otherwise use the same parameters described above. (*5 *bonus points)*