Advanced Topics in Macro 1
Problem Set 2

Due date: November 10, 2020, at 12.00 noon
Total points: 100 (+10 bonus)

Problem 1 (required). Consider the AR(1) process

$$z_{t+1} = \rho z_t + (1 - \rho^2)\epsilon_{t+1},$$

where ϵ_{t+1} is a standard normal.

(a) Compute (analytically) the stationary distribution of this Markov process for $|\rho| < 1$. *[5 points]*

(b) Assume $\rho = 0.7$. Implement Tauchen's method with equidistant grid points with $m = 3$, $N = 5$ and importance sampling with $N = 5$. Simulate the process for $T = 2000$ periods and throw away the first 500 periods to limit the impact of your initial value. Compare the mean, standard deviation and autocorrelation resulting from the simulated data with the true values of the stationary distribution for both the equidistant grid and importance sampling. *[10 points]*

(c) Assume $\rho = 0.7$. Implement Rouwenhorst's method with $N = 5$. Simulate the process for $T = 2000$ periods and throw away the first 500 periods to limit the impact of your initial value. Compare the mean, standard deviation and autocorrelation resulting from the simulated data with your results from (c). *[15 points]*
Hint: Make this code generic and reuse it in Problem 2.

(d) Repeat (b) and (c) for $\rho = 0.99$. Interpret your results. *[10 points]*

Problem 2 (required) Consider the stochastic growth model described in class with $u(c) = \log(c)$ and $f(k) = k^\alpha$, where the process z is defined by

$$\begin{aligned} z_t &= \exp(y_t) \\ y_t &= \rho y_{t-1} + \epsilon_t \\ \epsilon_t &\sim \mathcal{N}(0, \sigma^2). \end{aligned}$$

(a) Use Rouwenhorst's method with $N = 3$ to discretize the Markov process. *[15 points]*

(b) Implement value function and policy function iteration and compare their performance in terms of speed and Euler equation errors (for a given number of grid points). *[45 points]*
Hint: You may reuse your code from Week 1.

Problem 3 (optional). Consider the AR(1) process from Problem 1

$$z_{t+1} = \rho z_t + (1 - \rho^2)\epsilon_{t+1},$$

where ϵ_{t+1} is a standard normal.

(a) Assume $\rho = 0.7$. Implement Tauchen's method with importance sampling using $N = 5$. Simulate the process for $T = 2000$ periods and throw away the first 500 periods to limit the impact of your initial value. Compare the mean, standard deviation and autocorrelation resulting from the simulated data with the true values of the stationary distribution for both the equidistant grid and importance sampling. *[2 bonus points]*

(d) Repeat (a) for $\rho = 0.99$ and compare to your results in Problem 1 (d). *[1 bonus points]*

Problem 4 (optional) Consider an economy which consists of measure one of young adults, who can live alone or co-reside with their parents. Each individual is subject to the individual productivity shock $\varepsilon \sim F(\cdot)$ and disutility shock $\eta \sim G(\cdot)$ from living with their parents. Young who live alone solve the following problem

$$\max_{c_A, h_A} \frac{c_A^{1-\sigma}}{1-\sigma} - \psi \frac{h_A^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$
$$\text{s.t. } c_A = w\varepsilon h_A$$

where w is the wage rate. Young who choose to live their parents solve

$$\max_{c_T, h_T} \frac{(c_T+\zeta)^{1-\sigma}}{1-\sigma} - \psi \frac{h_T^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \eta$$
$$\text{s.t. } c_T = w\varepsilon h_T$$

where $\zeta > 0$ is a consumption flow young adult receives from parents, while living with them (free housing, free food etc.). The indifference condition equalizing utility from living alone with the utility from living with parents defines the threshold $\eta^*(\varepsilon)$ splitting the population into two groups.

(a) Set $\sigma = 0.436$, $\nu = 1.389$, $\psi = 4.193$, $w = 0.006$ and $\zeta = 0.035$. Solve for the labor supply and consumption conditional on the living arrangement (for young living alone and for young living with parents). Solve for the labor supply of young living with parents h_T using Taylor approximation around $\log(h_A)$, which you can get in a closed form.
[2 bonus points]

(b) Let F have the log-normal distribution with mean $\mu_F = 4.915$ and standard deviation $\sigma_F = 0.849$ and let G have the normal distribution with mean $\mu_G = 0.084$ and standard deviation $\sigma_G = 0.135$. Compute the fraction of the young adults living with their parents x given by the equation

$$x = \int_0^\infty \int_{-\infty}^{\eta^*(\varepsilon)} dGdF \tag{1}$$

Analogously compute the total hours of young alone and young living with parents given by

$$H_T = \int_0^\infty \int_{-\infty}^{\eta^*(\varepsilon)} h_T(\varepsilon)\; dGdF \tag{2}$$
$$H_A = \int_0^\infty \int_{\eta^*(\varepsilon)}^{\infty} h_A(\varepsilon)\; dGdF \tag{3}$$

To compute integrals use Gauss-Hermitte quadrature with 15 nodes. Plot the threshold $\eta^*(\varepsilon)$. Comment on economic forces behind the selection of the young into the living arrangements, i.e. justify who lives where and why? *[2 bonus points]*

(c) Now, assume that F is distributed according to the Gamma distribution with parameters $k = 15.596$ (shape) and $\theta = 4.274$ (scale) and let G have the normal distribution with mean $\mu_G = 0.371$ and standard deviation $\sigma_G = 0.254$. Moreover, let $\sigma = 0.430$, $\nu = 1.131$, $\psi = 4.483$, $w = 0.016$ and $\zeta = 0.222$. Redo part (a) and part (b), use Gauss-Laguerre quadrature with 15 nodes to compute integrals. *[3 bonus points]*