Paper presentation:

"Financial frictions and the wealth distribution"

by Fernandez-Villaverde, Hurtado and Nuño (2020)

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Roadmap

Contents of this presentation

- **Overview**
  - Motivation
  - Economic model
- **Model solution**: neural network
- **Estimation**: likelihood function + inference with diffusions
- **Main takeaways**

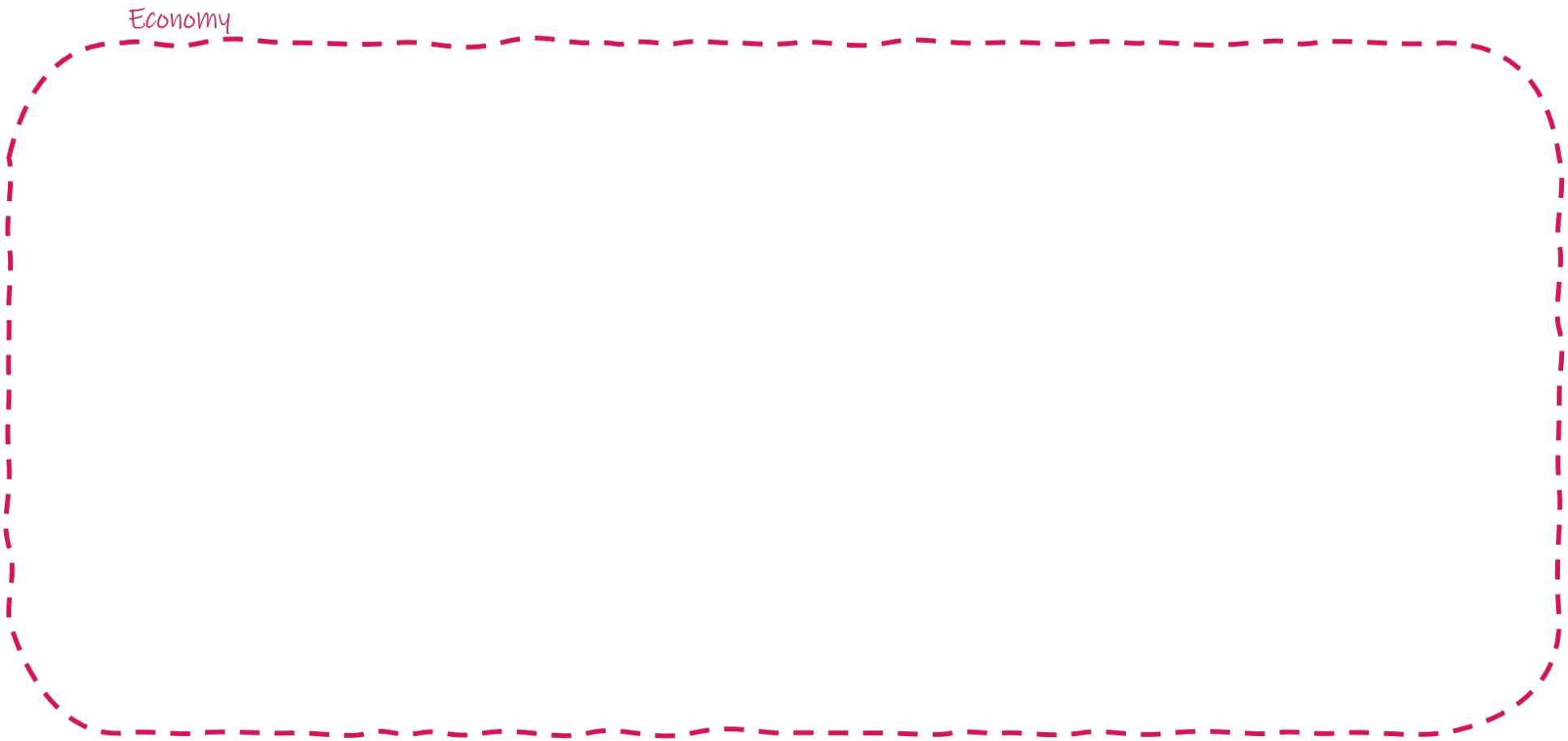
How financial frictions can led to wealth inequality?

What motivates this paper

- Empirical evidence of a **non-linear** relation between the level of leverage in the economy with **fluctuations in** macro aggregates
- **Proposal:** to build a nonlinear DSGE model with a financial sector that can reproduce the empirical observations

The economic model

Main characteristics



Source: Own elaboration based on VHN (2020).

The economic model

Main characteristics

Economy

Households

- Continuum of hh that live forever
- Differ in wealth and labor supply
- Are subject to (uninsurable) idiosyncratic labor productivity shocks (2 states, a la Hugget)
- Can only save (can't short) using the riskless debt
- More impatient than the expert

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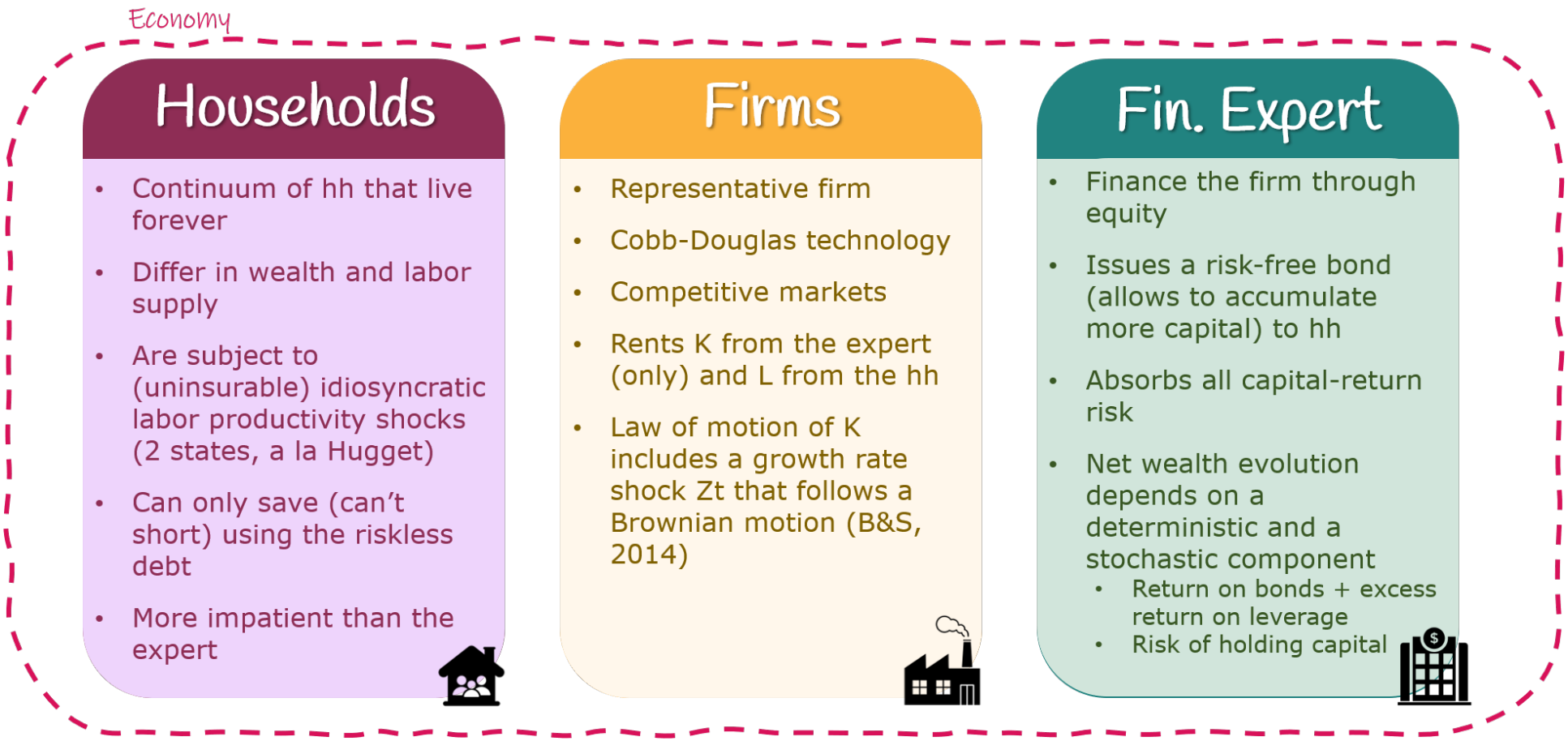
Firms

- Representative firm
- Cobb-Douglas technology
- Competitive markets
- Rents K from the expert (only) and L from the hh
- Law of motion of K includes a growth rate shock Zt that follows a Brownian motion (B&S, 2014)

Source: Own elaboration based on VHN (2020).

The economic model

Main characteristics



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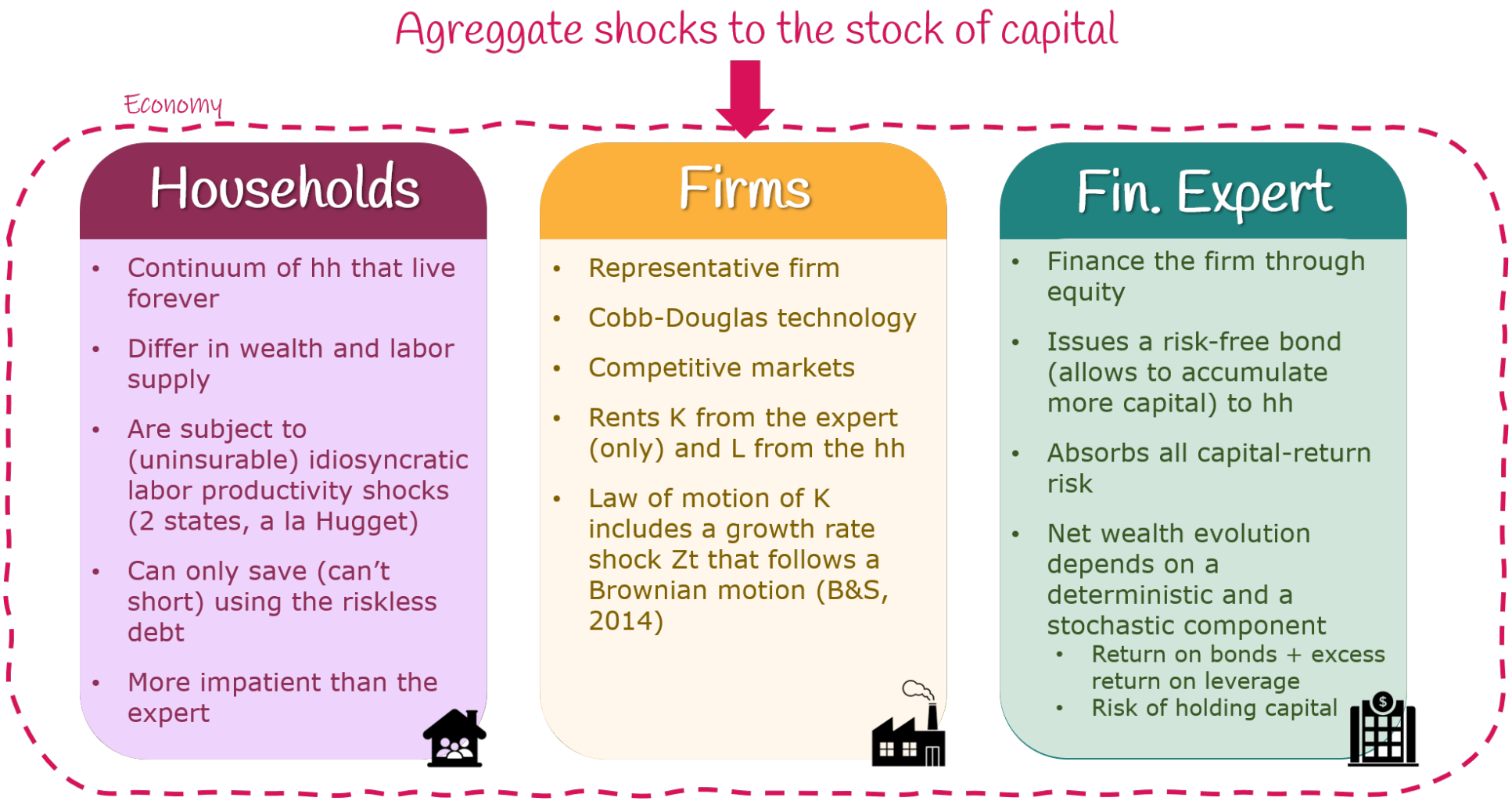

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The economic model

Main characteristics (cont.)

- The inclusion of the **financial sector** is made to capture the evolution of debt and leverage,
- which in combination to **heterogeneous households** will led to *endogenous aggregate risk*...
- resulting in a *regime switching process* for output, risk-free rate, excess returns, debt and leverage.
- The model presents **multimodal distributions** to the aggregates above, with **time-varying levels of volatility and skewness** and supercycles of borrowing and deleveraging

The economic model

Main characteristics (cont.)

- Using parameters to match the US economy and maximizing the likelihood function, the model has one deterministic steady state (DSS) and multiple stochastic steady states (SSS)
  - A high leverage SSS
  - A low leverage SSS

HL-SSS

- Endogenous aggregate risk is high
- After a negative aggregate shock, economy stays in a recession
- HH accumulate bonds
  - ↑ wealth inequality
  - ↓ risk-free rate
- Expert leverages more
- Wages are low
- Supply of credit is low
- Demand for credit is low

Source: Own elaboration based on VHN (2020).

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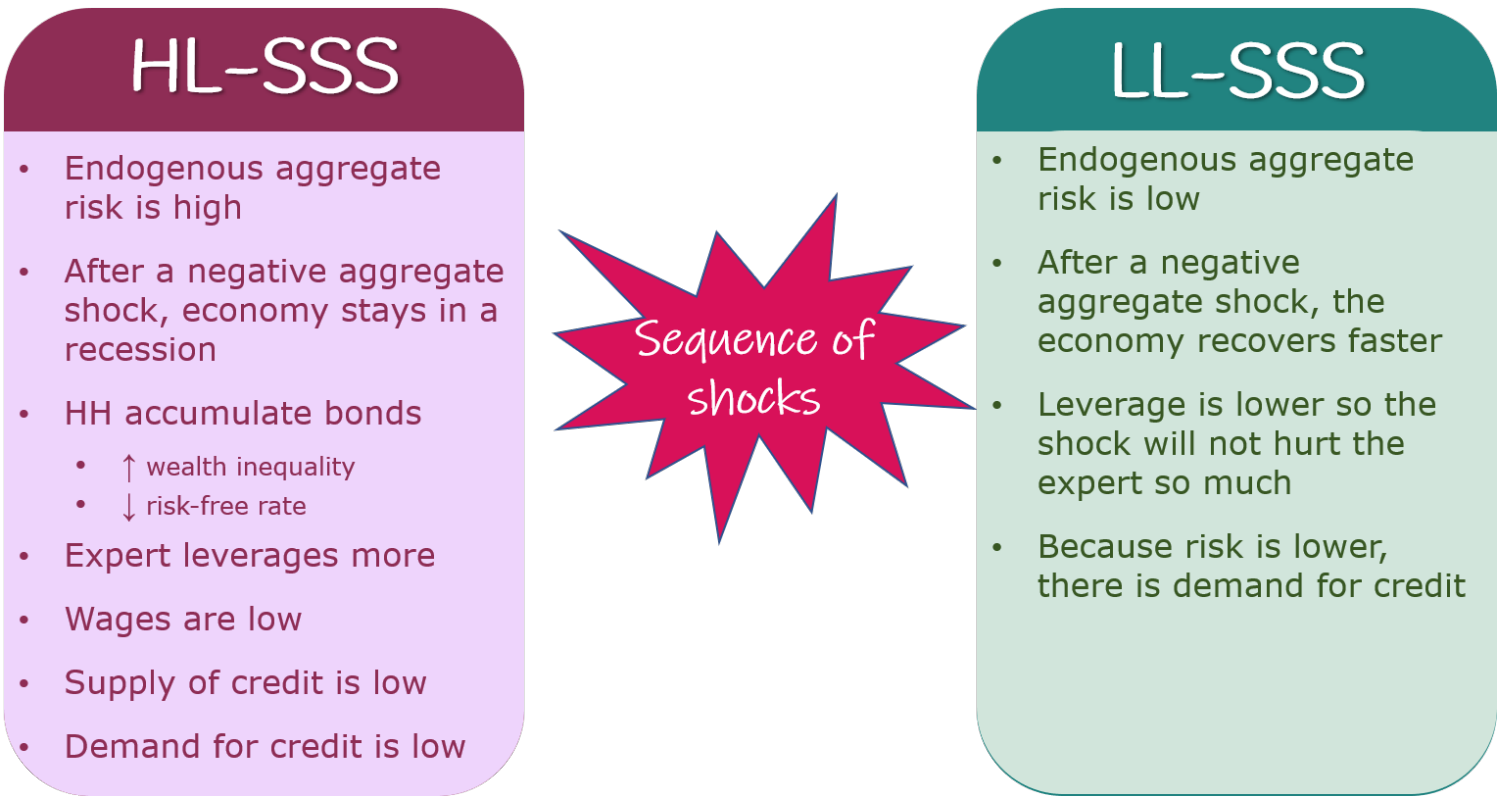

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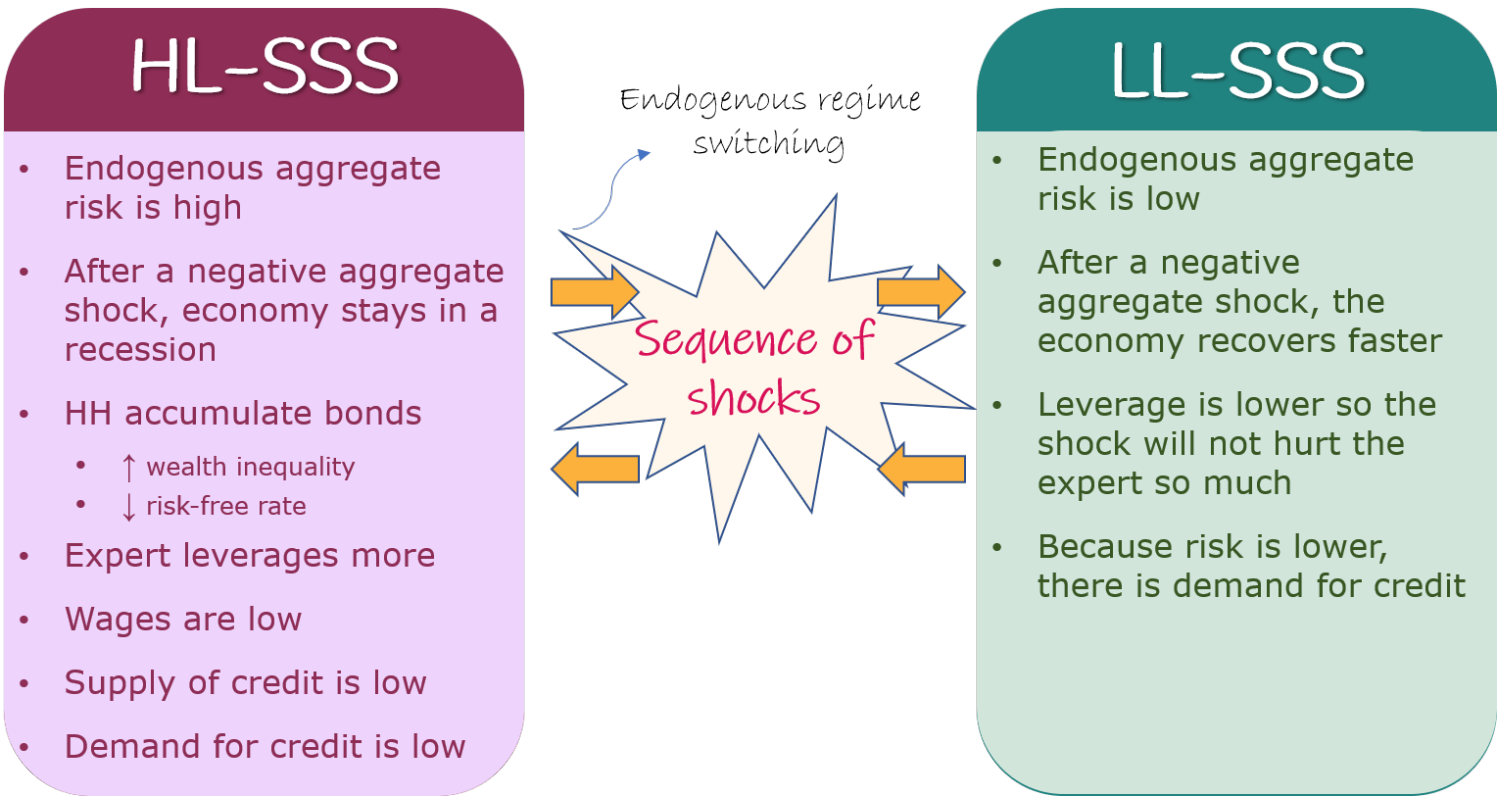

Source: Own elaboration based on VHN (2020).

Model solution

Main idea

- Wealth distribution has infinite dimensions
  - Standard dynamic programming procedures will not work
- In the paper, they propose a new set of tools for global, nonlinear solution and structural estimation of heterogeneous agent models with aggregate shocks
- To get an approximation for the perceived law of motion (PLM) of the cross-sectional distribution of assets, they use a neural network algorithm

Model solution

Comparison to the KS method

Is it possible to approximate the PLM in this model using the moments obtained from the KS algorithm?

Source: Own elaboration based on VHN (2020).

Model solution

Comparison to the KS method

Is it possible to approximate the PLM in this model using the moments obtained from the KS algorithm?

Yes

- Consumption decision rule of hh is almost linear wrt to hh state variables (except in the left tail)

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Model solution

Comparison to the KS method

Is it possible to approximate the PLM in this model using the moments obtained from the KS algorithm?

Yes

- Consumption decision rule of hh is almost linear wrt to hh state variables (except in the left tail)

No

- Consumption decision rule of hh is NOT linear wrt to aggregate states (equity and debt)

Source: Own elaboration based on VHN (2020).

Model solution

In more detail

- To find the equilibrium in this model, one needs:
  - C_t: total consumption (households) and
  - tracking the household's density on assets $\{g_{it}(\cdot)\}$, $i \in \{1, 2\}$, where i is the labor productivity state
- $\{g_{it}(\cdot)\}$'s evolution depends on the optimal consumption-saving decision from the household *and* the states of employment/unemployment
- How to find C_t given the structural parameter values $\Psi\{\alpha, \delta, \sigma, \hat{\rho}, \rho, \gamma, z_1, z_2, \lambda_1, \lambda_2\}$?

Model solution

In more detail (cont.)

- Assumption: households will use a finite set of moments of the cross-sectional distribution of assets (and not the whole distribution)
  - In [Krusell and Smith, 1998], the only endogenous state variable is the income-wealth distribution;
  - in this paper, the net wealth of the expert also is an endogenous state variable (and there is no exogenous state variable)
- Households do not observe/acknowledge the exact law of motion for aggregate debt,

$$dB_t = \left((1-\alpha)K_t^{\alpha} + \left(\alpha K_t^{\alpha-1} - \delta - \sigma^2 \frac{K_t}{N_t}\right) B_t - C_t\right) dt,$$

- instead, they have a *Perceived Law of Motion* (PLM) of aggregate debt:

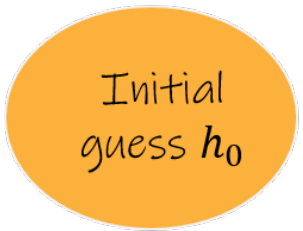
$$dB_t = h(B_t, N_t)dt$$

  - $h(B, N)$ represents the conditional expectation of dB_t given the information available (B_t, N_t)
- Given the PLM, the hh's problem has an associated Hamilton-Jacobi-Bellman equation.

Model solution

Algorithm steps

How to find $h(B, N)$?



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Model solution

Algorithm steps

How to find $h(B, N)$?

Initial guess h_0

- Use the hh's HJB equation to solve for c
  - May use an upwind finite difference scheme (or other numerical algorithms)

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Model solution

Algorithm steps

How to find $h(B, N)$?

Initial guess h_0

- Use the hh's HJB equation to solve for c
  - May use an upwind finite difference scheme (or other numerical algorithms)

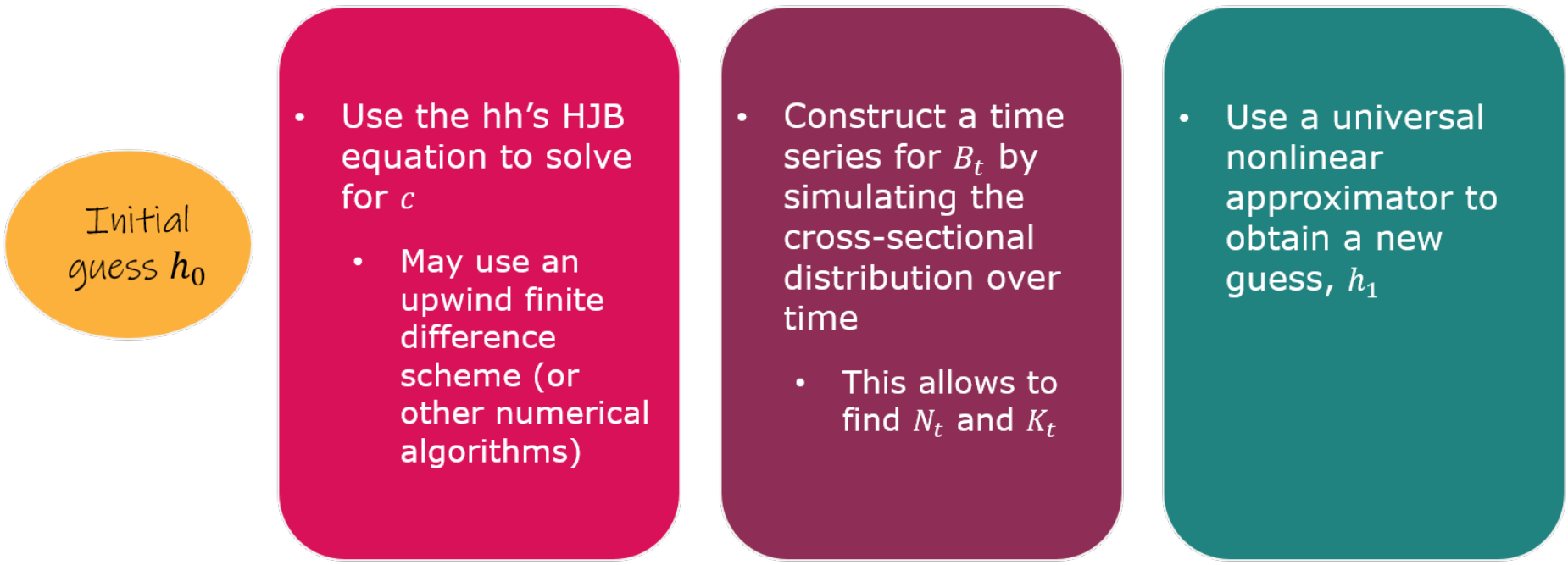
- Construct a time series for B_t by simulating the cross-sectional distribution over time
  - This allows to find N_t and K_t

Source: Own elaboration based on VHN (2020).

Model solution

Algorithm steps

How to find $h(B, N)$?



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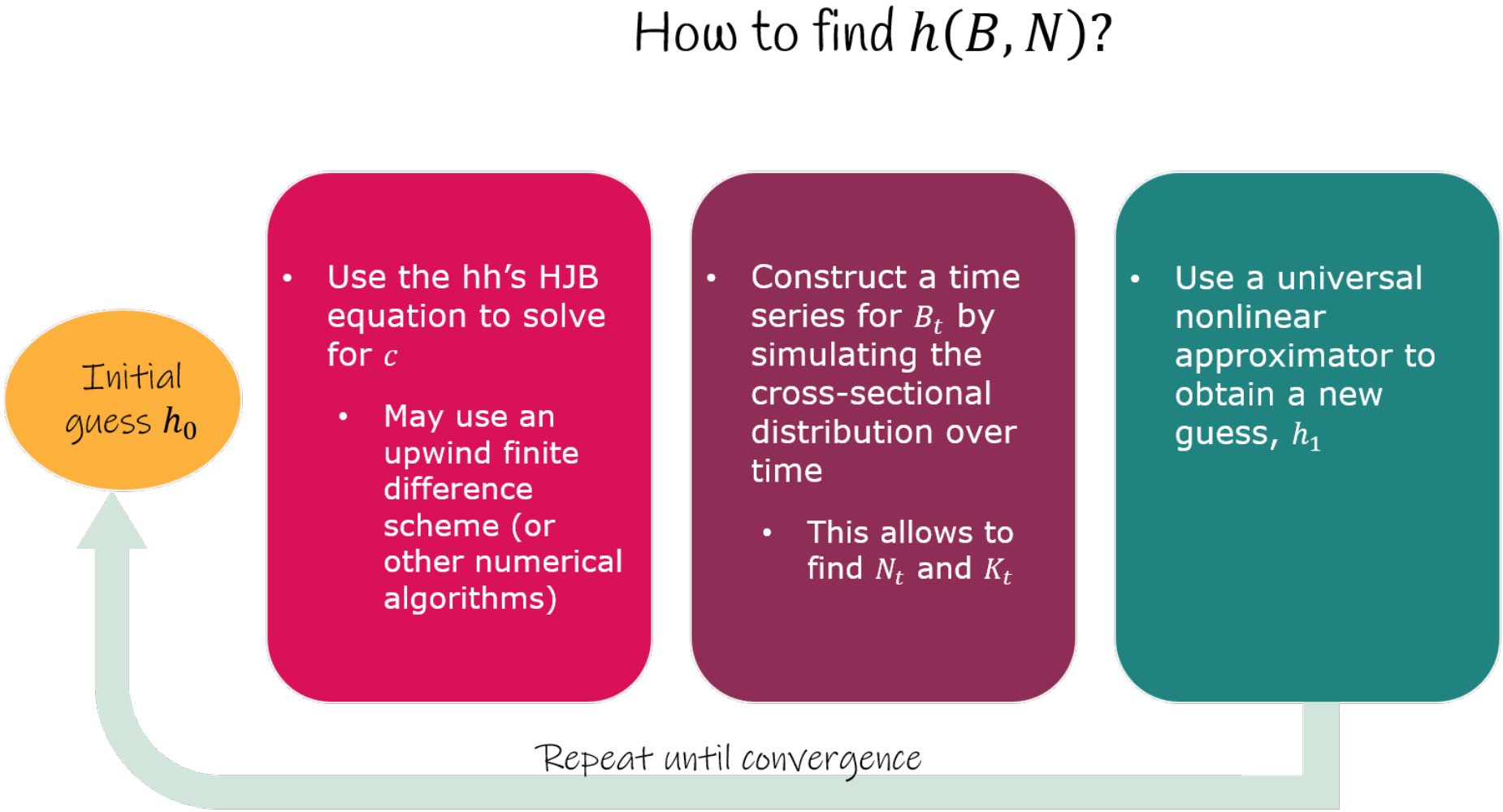
Model solution

Algorithm steps



Source: Own elaboration based on VHN (2020).

Model solution

Differences to KS algorithm

- **Continuous time**:
  - Sparcity in the transition probabilities' matrices will prevent "jumps" (hh reach only states that are neighbor to current state);
  - Characterization of optimal consumption has a simpler (equation) structure;
  - Easier to capture occasionally binding constraints
  - It is more efficient to simulate the time series of the cross-sectional distribution (which can can bring you memories from HW5)

Model solution

Differences to KS algorithm

- **Continuous time**;
- **Universal nonlinear approximator** (neural network)
  - In KS: log-linear equation connecting mean of capital tomorrow and today with coefficients that depend on the aggregate shock;
    - Very accurate because in their model there is near-loglinearity around the DSS (and SSS almost coincides)
    - Not the case here.
  - Two problems arise from the non-linear structure:
    1. Approximation: the algorithm needs to search for an unknown nonlinear function
    2. Extrapolation: it is not possible to explore the whole domain of B_t and N_t, but you need a good extrapolation to what happen in regions not visited by the algorithm

Model solution

A non-linear approximation technique

- They claim their NN approach has 4 strengths
  1. Can approximate any Borel measurable function relatively well (even with kinks and constraints)
  2. The coefficients can be efficiently estimated using <gradient descent methods> and <back-propagation>
  3. Errors don't increase with dimension as in other methods (polynomials, splines, trigonometric expansions)
  4. Extrapolation (outside training areas) in NN is far superior than with other methods (for example using Chebyshev polynomials)

Model solution

NN approximator for the PLM

$$h(\boldsymbol{s};\theta) = \theta_0^2 + \sum_{q=1}^{Q} \theta_q^2 \phi \left(\theta_{0,q}^1 + \sum_{i=1}^{2} \theta_{i,q}^1 s^i \right)$$

Source: Own elaboration based on VHN (2020).

Model solution

NN approximator for the PLM

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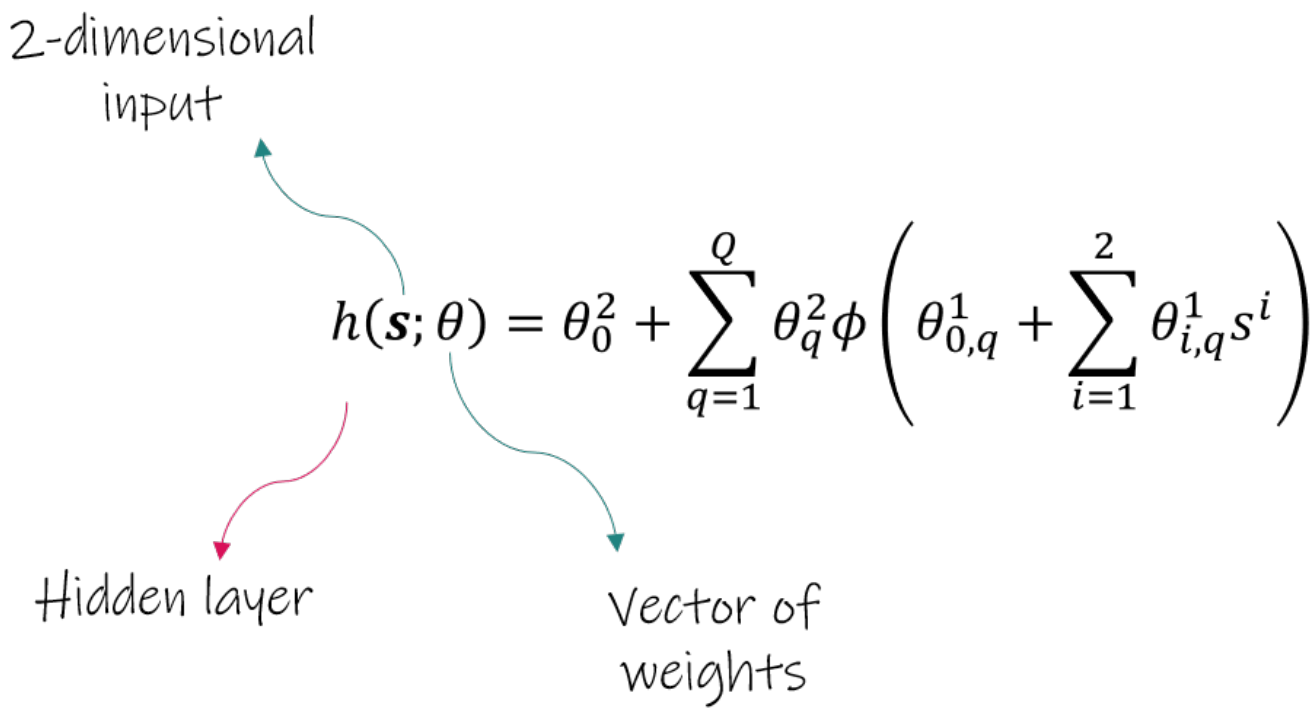
Hidden layer

Source: Own elaboration based on VHN (2020).

Model solution

NN approximator for the PLM



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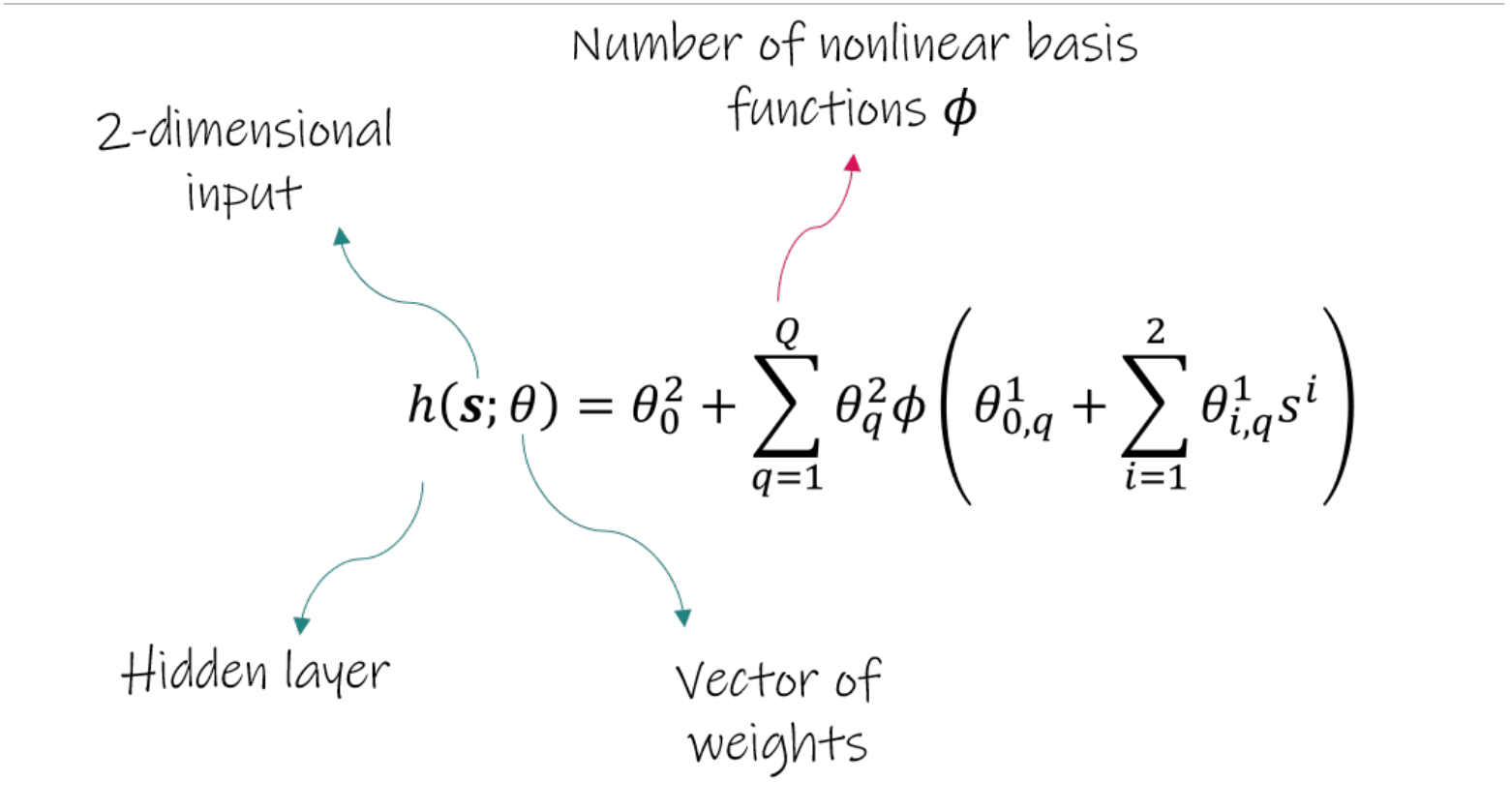
Model solution

NN approximator for the PLM



Source: Own elaboration based on VHN (2020).

Model estimation

Quick tour

- After solving the model, you put together with data to estimate Ψ
- This is done in two steps:
  - Building the likelihood
    - Likelihood function is derived for output
    - You use the joint density of output and debt and write the likelihood for output at time *t* conditional on the previous observations
    - Some stochastic calculus is used (diffusions)
  - Maximizing it (using some calibrated parameters)

Main takeaways

Wrapping up

- This new proposed model can account for some stylized facts:
  - Recent heightened fragility of the advanced economies to adverse shocks
  - Rise in wealth inequality that preceded the financial crisis
  - The increase in debt and leverage (also before the FC)
  - Low risk-free interest rates environment from the last decades
- It presents endogenous regime-switching caused by the endogenous aggregate risk
- Multiple SSS
- Results strongly depend on the heterogeneity of hh.

Source: This <set of slides> from the authors.

References


Fernández-Villaverde, J., Hurtado, S., and Nuño, G. (2020).
Financial frictions and the wealth distribution.
SSRN Working Paper no. 3615695.


Krusell, P. and Smith, Jr, A. A. (1998).
Income and wealth heterogeneity in the macroeconomy.
Journal of political Economy, 106(5):867–896.

“Financial frictions and the Wealth Distribution”

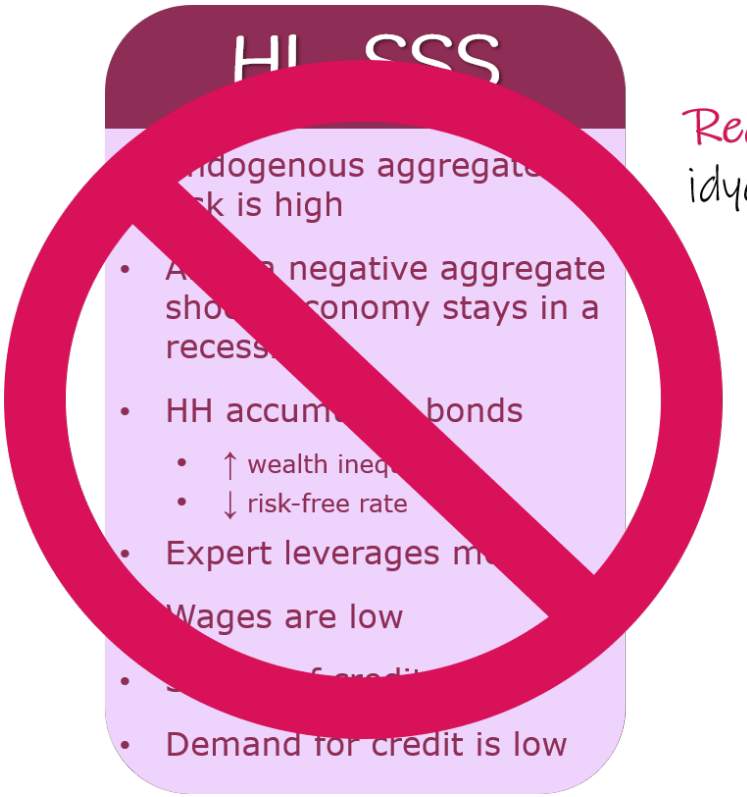
The end

Thank you!

- Questions?

The economic model

Comparison to the RA model



Reduction in the idyosincratic risk

LL-SSS

- Endogenous aggregate risk is low
- After a negative aggregate shock, the economy recovers faster
- Leverage is lower so the shock will not hurt the expert so much
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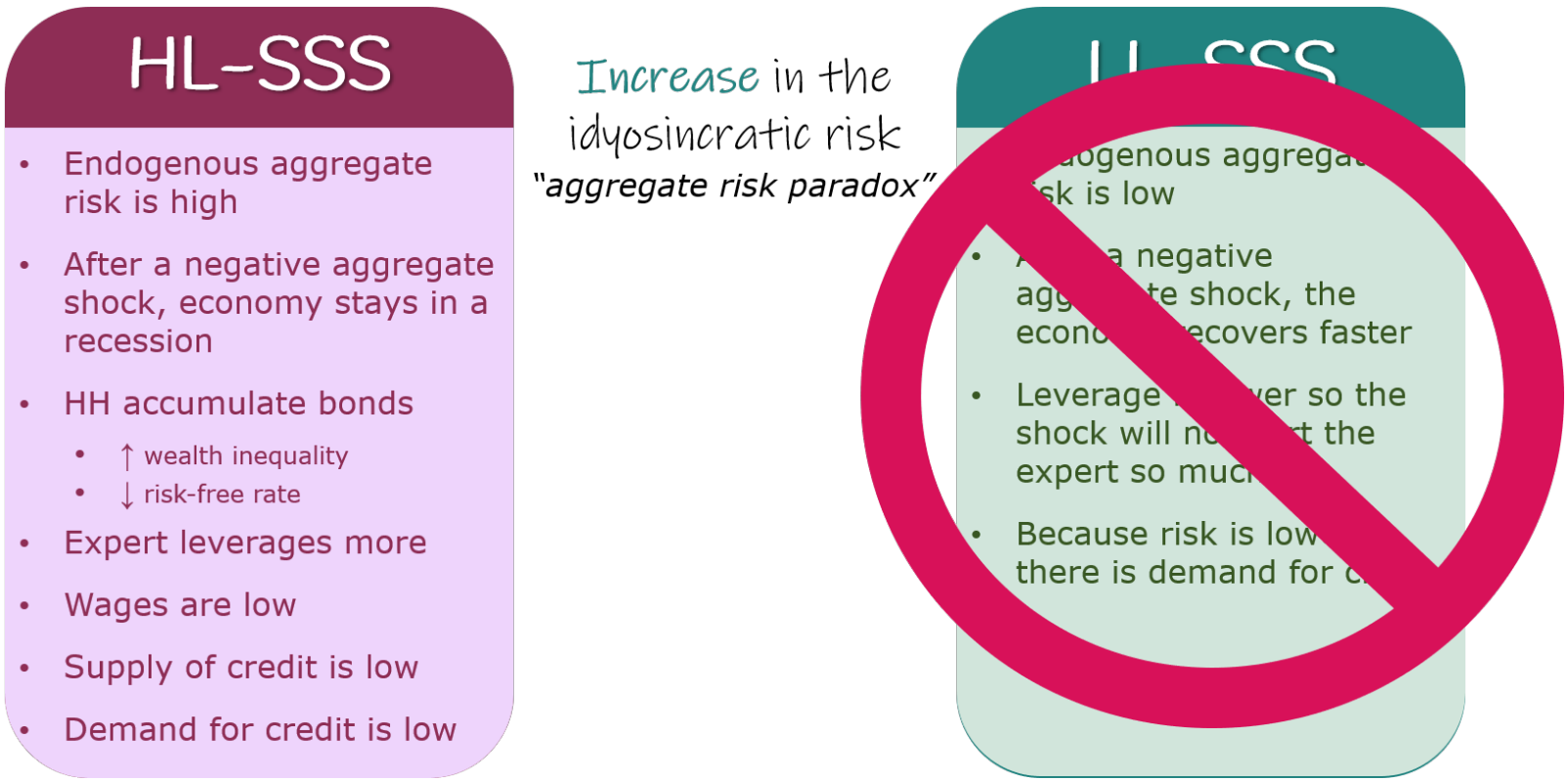
The economic model

Comparison to the RA model (cont.)



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