

$$\begin{aligned}
\text{In}[7]:= & \mathbf{f1} = \mathbf{A_t} - \mathbf{C_t} + \left(\mathbf{KT_{t+1}} - (1 - \delta) \mathbf{KT_t} + \frac{\phi}{2} \left(\frac{\mathbf{KT_{t+1}}}{\mathbf{KT_t}} - g \right)^2 \mathbf{KT_t} \right) + \left(\mathbf{KN_{t+1}} - (1 - \delta) \mathbf{KN_t} + \frac{\phi}{2} \left(\frac{\mathbf{KN_{t+1}}}{\mathbf{KN_t}} - g \right)^2 \mathbf{KN_t} \right) \\
& \mathbf{f2} = \left(\eta \left(\mathbf{AT_t} \right)^{1-1/\mu} + (1 - \eta) \left(\mathbf{AN_t} \right)^{1-1/\mu} \right)^{\frac{\mu}{\mu-1}} - \mathbf{A_t} \\
& \mathbf{f3} = \mathbf{z_t} \left(\mathbf{KN_t} \right)^{1-\alpha N} \left(\mathbf{X_t} \mathbf{hN_t} \right)^{\alpha N} - \mathbf{AN_t} \\
& \mathbf{f4} = \mathbf{z_t} \left(\mathbf{KT_t} \right)^{1-\alpha T} \left(\mathbf{X_t} \mathbf{hT_t} \right)^{\alpha T} - \mathbf{AT_t} + \frac{\mathbf{D_{t+1}}}{\mathbf{R_t}} - \mathbf{D_t} \\
& \mathbf{f5} = \mathbf{h_t} - \mathbf{hT_t} - \mathbf{hN_t} \\
\text{Out}[7]= & \mathbf{A_t} - \mathbf{C_t} - (1 - \delta) \mathbf{KN_t} + \mathbf{KN_{1+t}} + \frac{1}{2} \phi \mathbf{KN_t} \left(-g + \frac{\mathbf{KN_{1+t}}}{\mathbf{KN_t}} \right)^2 - (1 - \delta) \mathbf{KT_t} + \mathbf{KT_{1+t}} + \frac{1}{2} \phi \mathbf{KT_t} \left(-g + \frac{\mathbf{KT_{1+t}}}{\mathbf{KT_t}} \right)^2 \\
\text{Out}[8]= & -\mathbf{A_t} + \left((1 - \eta) \mathbf{AN_t}^{1-\frac{1}{\mu}} + \eta \mathbf{AT_t}^{1-\frac{1}{\mu}} \right)^{\frac{\mu}{-1+\mu}} \\
\text{Out}[9]= & -\mathbf{AN_t} + \mathbf{KN_t}^{1-\alpha N} \left(\mathbf{hN_t} \mathbf{X_t} \right)^{\alpha N} \mathbf{z_t} \\
\text{Out}[10]= & -\mathbf{AT_t} - \mathbf{D_t} + \frac{\mathbf{D_{1+t}}}{\mathbf{R_t}} + \mathbf{KT_t}^{1-\alpha T} \left(\mathbf{hT_t} \mathbf{X_t} \right)^{\alpha T} \mathbf{z_t} \\
\text{Out}[11]= & \mathbf{h_t} - \mathbf{hN_t} - \mathbf{hT_t} \\
\text{In}[31]:= & \mathbf{U} = \frac{\left(\mathbf{C_t}^\gamma \left(1 - \mathbf{h_t} \right)^{1-\gamma} \right)^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{\left(\mathbf{C_{t+1}}^\gamma \left(1 - \mathbf{h_{t+1}} \right)^{1-\gamma} \right)^{1-\sigma} - 1}{1 - \sigma} + \lambda_{1t} \mathbf{f1} + \lambda_{2t} \mathbf{f2} + \lambda_{3t} \mathbf{f3} + \\
& \lambda_{4t} \mathbf{f4} + \lambda_{5t} \mathbf{f5} + \beta \left(\left(\lambda_{1t} \mathbf{f1} + \lambda_{2t} \mathbf{f2} + \lambda_{3t} \mathbf{f3} + \lambda_{4t} \mathbf{f4} + \lambda_{5t} \mathbf{f5} \right) / \cdot \{ \mathbf{t} \rightarrow (\mathbf{t} + 1) \} \right) \\
\text{Out}[31]= & \frac{-1 + \left(\mathbf{C_t}^\gamma \left(1 - \mathbf{h_t} \right)^{1-\gamma} \right)^{1-\sigma}}{1 - \sigma} + \frac{\beta \left(-1 + \left(\mathbf{C_{1+t}}^\gamma \left(1 - \mathbf{h_{1+t}} \right)^{1-\gamma} \right)^{1-\sigma} \right)}{1 - \sigma} + \\
& \left(\mathbf{A_t} - \mathbf{C_t} - (1 - \delta) \mathbf{KN_t} + \mathbf{KN_{1+t}} + \frac{1}{2} \phi \mathbf{KN_t} \left(-g + \frac{\mathbf{KN_{1+t}}}{\mathbf{KN_t}} \right)^2 - (1 - \delta) \mathbf{KT_t} + \mathbf{KT_{1+t}} + \frac{1}{2} \phi \mathbf{KT_t} \left(-g + \frac{\mathbf{KT_{1+t}}}{\mathbf{KT_t}} \right)^2 \right) \\
& \lambda_{1t} + \left(-\mathbf{A_t} + \left((1 - \eta) \mathbf{AN_t}^{1-\frac{1}{\mu}} + \eta \mathbf{AT_t}^{1-\frac{1}{\mu}} \right)^{\frac{\mu}{-1+\mu}} \right) \lambda_{2t} + \left(-\mathbf{AN_t} + \mathbf{KN_t}^{1-\alpha N} \left(\mathbf{hN_t} \mathbf{X_t} \right)^{\alpha N} \mathbf{z_t} \right) \lambda_{3t} + \\
& \left(-\mathbf{AT_t} - \mathbf{D_t} + \frac{\mathbf{D_{1+t}}}{\mathbf{R_t}} + \mathbf{KT_t}^{1-\alpha T} \left(\mathbf{hT_t} \mathbf{X_t} \right)^{\alpha T} \mathbf{z_t} \right) \lambda_{4t} + \left(\mathbf{h_t} - \mathbf{hN_t} - \mathbf{hT_t} \right) \lambda_{5t} + \\
& \beta \left(\left(\mathbf{A_{1+t}} - \mathbf{C_{1+t}} - (1 - \delta) \mathbf{KN_{1+t}} + \mathbf{KN_{2+t}} + \frac{1}{2} \phi \mathbf{KN_{1+t}} \left(-g + \frac{\mathbf{KN_{2+t}}}{\mathbf{KN_{1+t}}} \right)^2 - \right. \right. \\
& \left. \left. (1 - \delta) \mathbf{KT_{1+t}} + \mathbf{KT_{2+t}} + \frac{1}{2} \phi \mathbf{KT_{1+t}} \left(-g + \frac{\mathbf{KT_{2+t}}}{\mathbf{KT_{1+t}}} \right)^2 \right) \lambda_{1_{1+t}} + \right. \\
& \left. \left(-\mathbf{A_{1+t}} + \left((1 - \eta) \mathbf{AN_{1+t}}^{1-\frac{1}{\mu}} + \eta \mathbf{AT_{1+t}}^{1-\frac{1}{\mu}} \right)^{\frac{\mu}{-1+\mu}} \right) \lambda_{2_{1+t}} + \left(-\mathbf{AN_{1+t}} + \mathbf{KN_{1+t}}^{1-\alpha N} \left(\mathbf{hN_{1+t}} \mathbf{X_{1+t}} \right)^{\alpha N} \mathbf{z_{1+t}} \right) \lambda_{3_{1+t}} + \right. \\
& \left. \left(-\mathbf{AT_{1+t}} - \mathbf{D_{1+t}} + \frac{\mathbf{D_{2+t}}}{\mathbf{R_{1+t}}} + \mathbf{KT_{1+t}}^{1-\alpha T} \left(\mathbf{hT_{1+t}} \mathbf{X_{1+t}} \right)^{\alpha T} \mathbf{z_{1+t}} \right) \lambda_{4_{1+t}} + \left(\mathbf{h_{1+t}} - \mathbf{hN_{1+t}} - \mathbf{hT_{1+t}} \right) \lambda_{5_{1+t}} \right)
\end{aligned}$$

In[63]:= **Simplify**[D[U, hN_t]]
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Simplify[D[U, hT_t]]
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Simplify[D[U, h_t]]
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$$\text{Out[63]} = \frac{\alpha N K N_t^{1-\alpha N} (h N_t X_t)^{\alpha N} z_t \lambda 3_t}{h N_t} - \lambda 5_t$$

$$\text{Out[64]} = \frac{\alpha T K T_t^{1-\alpha T} (h T_t X_t)^{\alpha T} z_t \lambda 4_t}{h T_t} - \lambda 5_t$$

$$\text{Out[65]} = \frac{(-1 + \gamma) \left(C_t^\gamma (1 - h_t)^{1-\gamma} \right)^{1-\sigma}}{1 - h_t} + \lambda 5_t$$

In[70]:= **Simplify**[D[U, A_t]]
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Simplify[D[U, AT_t]]
 化简 偏导

Simplify[D[U, AN_t]]
 化简 偏导

$$\text{Out[70]} = \lambda 1_t - \lambda 2_t$$

$$\text{Out[71]} = \eta A T_t^{-1/\mu} \left(-(-1 + \eta) A N_t^{\frac{-1+\mu}{\mu}} + \eta A T_t^{\frac{-1+\mu}{\mu}} \right)^{\frac{1}{-1+\mu}} \lambda 2_t - \lambda 4_t$$

$$\text{Out[72]} = -(-1 + \eta) A N_t^{-1/\mu} \left(-(-1 + \eta) A N_t^{\frac{-1+\mu}{\mu}} + \eta A T_t^{\frac{-1+\mu}{\mu}} \right)^{\frac{1}{-1+\mu}} \lambda 2_t - \lambda 3_t$$

In[80]:= **Simplify**[**D**[**U**, **KN**_{t+1}] == 0]
 [化简] [偏导]

Simplify[**D**[**U**, **KT**_{t+1}] == 0]
 [化简] [偏导]

Simplify[**D**[**U**, **C**_t] == 0]
 [化简] [偏导]

Simplify[**D**[**U**, **D**_{t+1}] == 0]
 [化简] [偏导]

$$\text{Out[80]} = \left(1 - g \phi + \frac{\phi \text{KN}_{1+t}}{\text{KN}_t} \right) \lambda 1_t + \frac{1}{2} \beta \left(-2 + 2 \delta + g^2 \phi - \frac{\phi \text{KN}_{2+t}^2}{\text{KN}_{1+t}^2} \right) \lambda 1_{1+t} =$$

$$(-1 + \alpha N) \beta \text{KN}_{1+t}^{-\alpha N} (h N_{1+t} X_{1+t})^{\alpha N} z_{1+t} \lambda 3_{1+t}$$

$$\text{Out[81]} = \left(1 - g \phi + \frac{\phi \text{KT}_{1+t}}{\text{KT}_t} \right) \lambda 1_t + \frac{1}{2} \beta \left(-2 + 2 \delta + g^2 \phi - \frac{\phi \text{KT}_{2+t}^2}{\text{KT}_{1+t}^2} \right) \lambda 1_{1+t} =$$

$$(-1 + \alpha T) \beta \text{KT}_{1+t}^{-\alpha T} (h T_{1+t} X_{1+t})^{\alpha T} z_{1+t} \lambda 4_{1+t}$$

$$\text{Out[82]} = \lambda 1_t = \frac{\gamma \left(C_t^\gamma (1 - h_t)^{1-\gamma} \right)^{1-\sigma}}{C_t}$$

$$\text{Out[83]} = \beta \lambda 4_{1+t} = \frac{\lambda 4_t}{R_t}$$