5.8 Exercises

Exercise 5.1 (Explaining the Serial Correlation of Investment Growth) *The version of the GPU model estimated in this chapter predicts a negative first-order serial correlation of investment growth of -0.098 (see table 5.5). By contrast, the empirical counterpart is positive and significant, with a point estimate of 0.32. This empirical fact is also observed in other emerging countries over long horizons. For example, Miyamoto and Nguyen (2013) report serial correlations of investment growth greater than or equal to 0.2 for Brazil, Mexico, Peru, Turkey, and Venezuela using annual data covering the period 1900 to 2006.*

1. *Think of a possible modification of the theoretical model that would result in an improvement of the model's prediction along this dimension. Provide intuition.*

2. *Implement your suggestion. Show the complete set of equilibrium conditions.*

3. *Reestimate your model using the data set for Argentina on which the GPU model of this chapter was estimated.*

4. *Summarize your results by expanding table 5.5 with appropriate lines containing the predictions of your model.*

5. *Compare the performance of your model with the data and with the predictions of the version of the GPU model analyzed in this chapter.*

Exercise 5.2 (The Effect of Nonstationary Productivity Shocks on the Trade Balance in the Presence of Nontraded Goods)

This exercise introduces nontradable goods a subject that we will take up in detail in chapter 8. You may choose to postpone working on this exercise until you have read that chapter. However, the tools developed in the current chapter should be sufficient to solve this exercise. Consider an *economy populated by a large number of identical households with preferences described by the utility function*

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^{\gamma}(1-h_t)^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma},$$

where C_t denotes consumption and h_t denotes hours worked.

Let A_t denote domestic absorption. That is, A_t satisfies the identity

$$A_t = C_t + I_t,$$

where I_t denotes gross domestic investment. Assume that A_t is a composite of tradable and non-tradable goods produced via the following aggregator function:

$$A_t = \left[\eta (A_t^T)^{1-1/\mu} + (1-\eta)(A_t^N)^{1-1/\mu}\right]^{1/(1-1/\mu)},$$

where A_t^T and A_t^N denote, respectively, domestic absorptions of tradable and nontradable goods. Tradable and nontradable goods, denoted, respectively, Y_t^T and Y_t^N, are produced under constant returns to scale using capital and labor as inputs:

$$Y_t^T = z_t (K_t^T)^{1-\alpha_T} (X_t h_t^T)^{\alpha_T}$$

and

$$Y_t^N = z_t (K_t^N)^{1-\alpha_N} (X_t h_t^N)^{\alpha_N},$$

where K_t^i and h_t^i denote, respectively, capital and labor services employed in sector i, for $i = T, N$. The variable z_t represents an exogenous, stationary, stochastic productivity shock and X_t represents an exogenous, stochastic, nonstationary productivity shock. The capital stocks evolve according to *the following laws of motion:*

$$K_{t+1}^T = (1-\delta)K_t^T + I_t^T - \frac{\phi}{2}\left(\frac{K_{t+1}^T}{K_t^T} - g\right)^2 K_t^T$$

and

$$K_{t+1}^N = (1-\delta)K_t^N + I_t^N - \frac{\phi}{2}\left(\frac{K_{t+1}^N}{K_t^N} - g\right)^2 K_t^N,$$

where I_t^i denotes investment in sector $i = T, N$ and satisfies

$$I_t = I_t^T + I_t^N.$$

Market clearing in the nontraded sector requires that domestic absorption equal production:

$$Y_t^N = A_t^N.$$

Also, market clearing in the labor market requires that

$$h_t = h_t^T + h_t^N.$$

Assume that the country has access to a single, one-period, internationally traded bond that pays the debt-elastic gross interest rate R_t when held between periods t and $t+1$. The evolution of the household's net foreign debt position, D_t, is given by

$$\frac{D_{t+1}}{R_t} = D_t + A_t^T - Y_t^T,$$

with

$$R_t = R^* + \psi\left[e^{\tilde{D}_{t+1}/X_t - \overline{d}} - 1\right],$$

where $\tilde{D}_t$ denotes average external debt per capita due in period t. Because households are homogeneous, we have that in equilibrium the following condition must hold:

$$\tilde{D}_t = D_t.$$

Finally, assume that the stationary productivity shock z_t follows an AR(1) process of the form

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_t^z.$$

And the nonstationary productivity shock evolves according to

$$\ln(g_t/g) = \rho_g \ln(g_{t-1}/g) + \sigma_g \epsilon_t^g,$$

where

$$g_t \equiv \frac{X_t}{X_{t-1}},$$

$\rho_z, \rho_g \in (-1, 1)$, $\sigma_z, \sigma_g > 0$, *and ϵ_t^z and ϵ_t^g are independent white noises distributed $N(0, 1)$.*

Assume that the time unit is one quarter. Use the following table to calibrate the model:

β	γ	$\overline{d}$	ψ	α_T	α_N	σ	δ	σ_g	σ_z	ρ_g	ρ_z	g	ϕ	μ
0.98	0.36	0.10	0.001	0.40	0.80	2	0.05	0.0213	0.0053	0.00	0.95	1.0066	1.37	0.44

Guidelines for calibrating the parameter η are given below.

In addressing the numerical portions of the following questions, write your answers to 4 decimal places.

1. *Write down the complete set of equilibrium conditions. Count the number of variables and equations.*

2. *Let p_t^N and p_t be the (shadow) relative prices of the nontradable good and the composite good*

in terms of tradable goods, respectively. Write down expressions for p_t^N *and* p_t *in terms of* A_t^T *and* A_t^N*. Are* p_t^N *and* p_t *stationary variables? Why?*

3. *Write down the complete set of equilibrium conditions in stationary form along a balanced-growth path.*

4. *Derive a restriction on* R^* *that guarantees that the steady-state value of* $d_{t+1} \equiv D_{t+1}/X_t$ *along the balanced-growth path equals* $\overline{d}$*. Compute the steady state of the model. Set* η *to ensure that the share of nontraded output in total output, defined as* $p_t^N Y_t^N/(p_t^N Y_t^N + Y_t^T)$ *equals 0.65 in the steady state. Report the numerical value of* η *and the steady-state values of all endogenous variables of the model.*

5. *Report the numerical values of the eigenvalues of the matrix* h_x *defining the linearized equilibrium law of motion of the state vector.*

6. *Define GDP as* $Y_t = (Y_t^T + p_t^N Y_t^N)/p_t$*. Define the trade balance,* TB_t*, as* $TB_t = Y_t - C_t - I_t$*. Compute the unconditional standard deviations of the growth rates of consumption and output, defined, respectively, as* $\Delta C_t \equiv \log(C_t/C_{t-1})$ *and* $\Delta Y_t \equiv \log(Y_t/Y_{t-1})$*. Compute the correlation between* ΔY_t *and the trade-balance-to-output ratio, defined as* $tby_t \equiv TB_t/Y_t$*.*

7. *Compute other second moments of your choice and discuss whether they are in line with empirical regularities in emerging economies.*

8. *Explain how you would reparameterize the model to make it coincide with the one-sector model studied in section 5.2. Answer the previous two questions under your proposed parameterization.*

9. *Based on your answers to the previous three questions, evaluate the ability of the present traded-nontraded model to explain key stylized facts in emerging countries. Provide intuition*

making sure to emphasize the differences you can identify between the dynamics implied by the one-good SOE RBC model and the present two-good SOE RBC model.

Exercise 5.3 (Slow Diffusion of Technology Shocks to the Country Premium, Household Production, and Government Spending)

The model presented in section 5.3 assumes that permanent productivity shocks affect not only the productivity of labor and capital in producing market goods, but also the country premium, home production, and government spending. For instance, the assumption that the country interest rate depends on $\widetilde{D}_{t+1}/X_t$, implies that a positive innovation in X_t in period t, causes, all other things equal, a fall in the country premium. In this exercise, we attenuate this type of effect by reformulating the model. Let

$$\widetilde{X}_t = \widetilde{X}_{t-1}^{\zeta} X_t^{1-\zeta},$$

with $\zeta \in [0, 1)$. Note that the original formulation obtains when $\zeta = 0$. Replace equations (5.3), (5.5), and (5.10), respectively, with

$$E_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{\left[C_t - \omega^{-1} \widetilde{X}_{t-1} h_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma},$$

$$s_t = \frac{S_t}{\widetilde{X}_{t-1}},$$

and

$$r_t = r^* + \psi \left(e^{(\widetilde{D}_{t+1}/\widetilde{X}_t - \overline{d})/\overline{y}} - 1 \right) + e^{\mu_t - 1} - 1.$$

Keep all other features of the model as presented in section 5.3.

1. *Present the equilibrium conditions of the model in stationary form.*

2. *Using Bayesian techniques, reestimate the model adding ζ to the vector of estimated parame-*