

8. Derive the optimality conditions associated with the household's problem.
9. Show whether the equilibrium level of consumption in period 1 is greater than, less than, or equal to the one that arises when  $\sigma = 0$ .
10. Find the sign of the trade balance in equilibrium. Compare your answer to the one for the case  $\sigma = 0$  and provide intuition. In particular, discuss why a mean preserving increase in interest-rate uncertainty affects the trade balance in period 1 the way it does.
11. Are the results obtained above due to the particular (logarithmic) preference specification considered? To address this question, show that all of the results obtained above continue to obtain under a more general class of preferences, namely, the class of CRRA preferences

$$\frac{C_1^{1-\gamma} - 1}{1-\gamma} + E_1 \frac{C_2^{1-\gamma} - 1}{1-\gamma},$$

for  $\gamma > 0$ , which encompasses the log specification as a special case when  $\gamma \rightarrow 1$ .

12. Finally, show that interest rate uncertainty does have real effects when the desired asset position in the absence of uncertainty is nonzero. To this end, return to the log preference specification and assume that the endowment in period 1 is zero and that the endowment in period 2 is  $Q > 0$ . How does the trade balance in period 1 compare under no uncertainty ( $\sigma = 0$ ) and under uncertainty ( $\sigma > 0$ )?

**Exercise 6.5 (Inducing Stationarity and Interest-Rate Shocks)** Chapter 4 shows that the business cycle implied by the SOE-RBC model is not affected by the method used to induce stationarity. This result, however, was derived in the context of a model driven by technology shocks. The present exercise aims to establish whether this finding is robust to assuming that business cycles are driven by world-interest-rate shocks.

1. Consider the external debt-elastic interest-rate (EDEIR) model of section 4.1.1 of chapter 4. Shut down the productivity shock by setting  $\tilde{\eta} = 0$ . Replace equation (4.14) with

$$r_t = r_t^* + p(\tilde{d}_t),$$

and

$$r_t^* = r^* + \xi(r_{t-1}^* - r^*) + \mu_t,$$

where  $\mu_t \sim N(0, \sigma_\mu^2)$ . Set  $\xi = 0.8$  and  $\sigma_\mu = 0.012$ . Calibrate all other parameters of the model at the values given in table 4.1. Using this version of the EDEIR model, compute the statistics considered in table 4.4 and make a table. Make a figure showing impulse responses of output, consumption, hours, investment, the trade-balance-to-output ratio, and the current-account-to-output ratio implied by the EDEIR model driven by interest-rate shocks. Provide intuition for these results.

2. Now consider the internal discount factor (IDF) model of section 4.10.4. Again, set  $\tilde{\eta} = 0$ . Replace the assumption that  $r_t = r^*$  with

$$r_t = r^* + \xi(r_{t-1} - r^*) + \mu_t.$$

Calibrate  $\xi$ ,  $\sigma_\mu$ , and all common parameters as in the previous question. Calibrate  $\psi_3$  as in section 4.10.4. Use the resulting calibrated model to compute unconditional second moments and impulse responses. Provide intuition for your results. To facilitate comparison, place the information generated here in the same table and figure produced in the previous question.

3. Compare the predictions of the EDEIR and IDF models driven by interest rate shocks. Does the stationarity-inducing mechanism make any difference for the business-cycles implied by

*the SOE model driven by interest-rate shocks?*