Optimal monetary policy with interest variances in the objective function

March 1, 2019

1 Optimal Monetary Policy

Similar to Boehm and House (2014), Evans and Honkapohja (2003) and Woodford (2003), we assume that the central bank wishes to minimize an expected discounted sum of weighted squared inflation and output gap

$$(1-\vartheta)E\Big[\Sigma_{t=0}^{\infty}\vartheta^t[\pi_t^2+\omega_y y_t^2+\omega_r r_t^2]\Big]=\omega\sigma_\pi^2+\omega_y\sigma_y^2+\omega_r\sigma_r^2, \tag{1.1}$$

where ω is the relative weight that the central bank places on inflation. Based on our calculation

$$\sigma_y^2 = \frac{\widetilde{g}_1}{(1+\gamma\varphi\phi_\pi+\varphi\phi_y)^2(1-\rho^2)(1-\rho\lambda_1)(1-\rho\lambda_2)(1-\lambda_1^2)(1-\lambda_2^2)(1-\lambda_1\lambda_2)}, \tag{1.2}$$

$$\sigma_\pi^2 = \frac{\widetilde{g}_2}{(1+\gamma\varphi\phi_\pi+\varphi\phi_y)^2(1-\rho^2)(1-\rho\lambda_1)(1-\rho\lambda_2)(1-\lambda_1^2)(1-\lambda_2^2)(1-\lambda_1\lambda_2)}, \tag{1.3}$$

$$\sigma_r^2 = \phi_y^2\sigma_y^2+\phi_\pi^2\sigma_\pi^2+2\phi_y\phi_\pi E(y\pi), \tag{1.4}$$

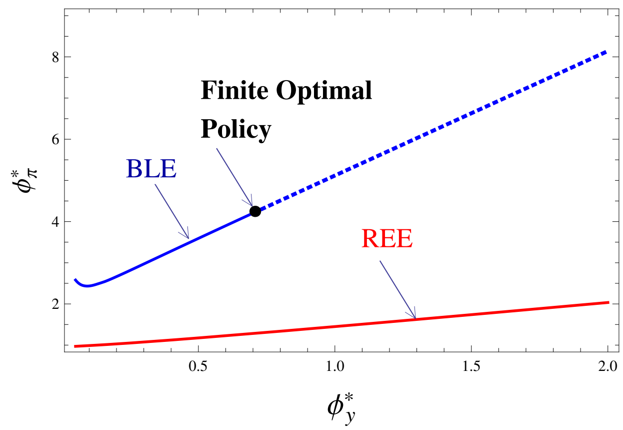


Figure 1: Optimal policies at the BLE and at the REE. Parameters are: $\lambda = 0.99, \varphi = 1, \rho = 0.5, \gamma = 0.04, \sigma_1 = 1, \sigma_2 = 0.5$ and $\omega_y = 0.1, \omega_r = 0.05$.

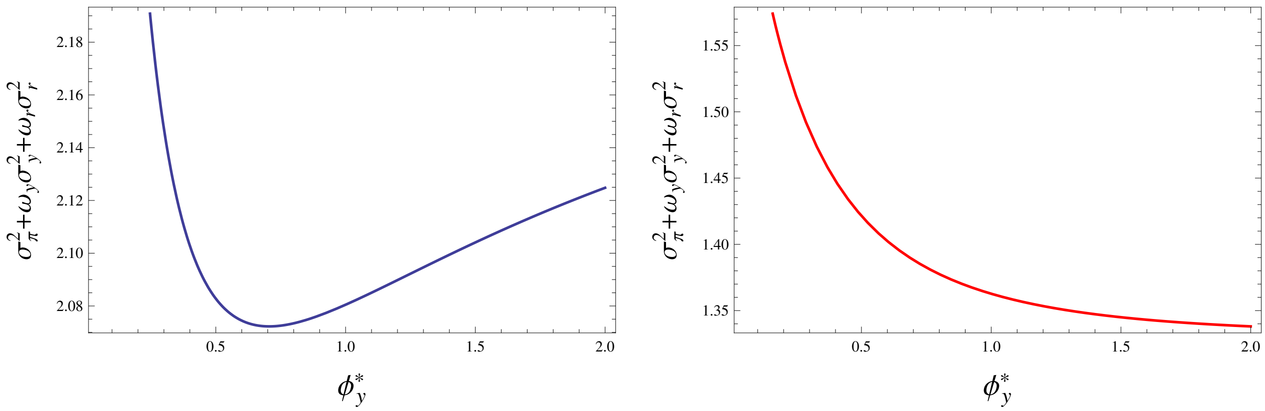
where $\widetilde{g}_1$, $\widetilde{g}_2$, λ_1, λ_2 are given by the equations (**??**), (**??**), (**??**) and (**??**), and

$$\begin{aligned}
E(y\pi) \;=\; & \Big(-\sigma_1^2\gamma\big(-(1+\gamma\varphi\phi_\pi+\varphi\phi_y)(1+\gamma\varphi\phi_\pi+\varphi\phi_y+\beta_1^2\rho)+\beta_2^4\lambda(1+\gamma\varphi\phi_\pi \\
& +\varphi\phi_y+\beta_1^2\rho)(\lambda+\gamma\varphi+\lambda\varphi\phi_y)+\beta_2^2\rho[\beta_1^4\lambda+\beta_1^2\lambda\rho(1+\gamma\varphi\phi_\pi+\varphi\phi_y)+\gamma\varphi \\
& (-1+\lambda\phi_\pi)(1+\gamma\varphi\phi_\pi+\varphi\phi_y)]-\beta_1^2\beta_2^6\lambda^2\rho(\beta_1^2\lambda+\rho(\lambda+\gamma\varphi+\lambda\varphi\phi_y))\big)+\sigma_2^2\varphi \\
& \big(-\phi_\pi(1+\gamma\varphi\phi_\pi+\varphi\phi_y)[-\beta_1^4-\beta_1^2\gamma\rho\varphi\phi_\pi+(1+\varphi\phi_y)(1+\gamma\varphi\phi_\pi+\varphi\phi_y)]+\beta_1^2\beta_2^6\lambda\rho \\
& [-\gamma\varphi(-\beta_1^2+\rho+\rho\varphi\phi_y)+\lambda\rho(\beta_1^4-(1+\varphi\phi_y)^2)]+\beta_2^4\big(\gamma\varphi(1-\beta_1^2\rho+\varphi\phi_y)(1+\gamma\varphi\phi_\pi \\
& +\varphi\phi_y)+\lambda(-1+\beta_1^2\rho-\varphi(\gamma\phi_\pi+\phi_y))(\beta_1^4-(1+\varphi\phi_y)^2)+\beta_1^2\lambda^2\rho\phi_\pi(-\beta_1^4+ \\
& (1+\varphi\phi_y)^2)\big)+\beta_2^2\rho[-\beta_1^6\lambda\rho\phi_\pi+\beta_1^2\lambda\rho\phi_\pi(1+\varphi\phi_y)(1+\gamma\varphi\phi_\pi+\varphi\phi_y)-(-1+\lambda\phi_\pi) \\
& (1+\varphi\phi_y)^2(1+\gamma\varphi\phi_\pi+\varphi\phi_y)+\beta_1^4(-1-\varphi(\gamma\phi_\pi+\varphi_y)+\lambda(\phi_\pi+\varphi\phi_\pi\phi_y))]\big)\Big)\Big/ \\
& \Big((-1+\rho^2)(-1+\beta_1^2\beta_2^2\lambda-\varphi(\gamma\phi_\pi+\phi_y))(1+\beta_1^2\rho(-1+\beta_2^2\lambda\rho)+\gamma\varphi\phi_\pi+\varphi\phi_y \\
& -\beta_2^2\rho(\lambda+\gamma\varphi+\lambda\varphi\phi_y))\big(\beta_1^4(-1+\beta_2^4\lambda^2)+2\beta_1^2\beta_2^2\gamma\varphi(-1+\lambda\phi_\pi)+(1+\gamma\varphi\phi_\pi+\varphi\phi_y)^2 \\
& -\beta_2^4(\lambda+\gamma\varphi+\lambda\varphi\phi_y)^2\big)\Big).
\end{aligned}$$

In the following we study the optimal values (ϕ_y^*, ϕ_π^*) that minimize the central bank's loss function (1.1) at the BLE, where β_1^* and β_2^* are at the behavioral learning equilibria.

As before in the benchmark we consider the parameters $\lambda = 0.99, \varphi = 1, \gamma = 0.04, \rho = 0.5, \sigma_1 = 1, \sigma_2 = 0.5$. We first consider the case $\omega_y = 0.1, \omega_r = 0.05$, that is, the central bank places relatively large weight on inflation. Interestingly, we find that the optimal Taylor rule coefficients (ϕ_y^*, ϕ_π^*) are finite in this case[1]. As shown in Figure 1a, the cor-

[1] We first select a policy parameter domain (e.g. $[0, 100] \times [1, 100]$) and define a lattice with some small



(a) At the BLE

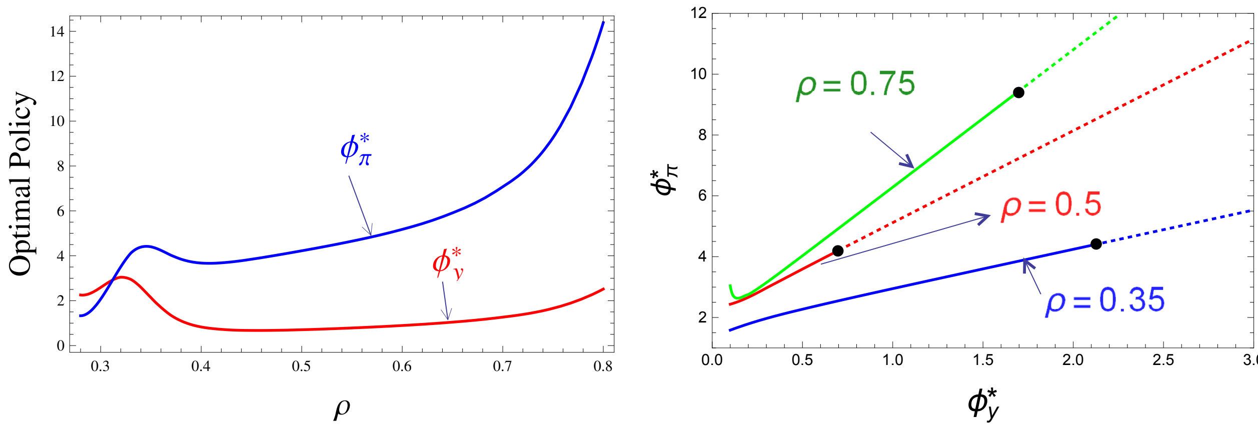
(b) At the REE

Figure 2: Loss function along the optimal paths (ϕ_y^*, ϕ_π^*) in Figure 1 at the BLE (a) and REE (b). Parameters are: $\lambda = 0.99, \varphi = 1, \rho = 0.5, \gamma = 0.04, \sigma_1 = 1, \sigma_2 = 0.5$ and $\omega_y = 0.1, \omega_r = 0.05$.

responding optimal policy $(\phi_y^*, \phi_\pi^*) = (0.7058, 4.2228)$. This is different from REE, where there is no finite optimal policy here. In fact, from Figure 1 it can be seen that in the case ϕ_y^* is small enough (i.e. < 0.7058) the coefficients ϕ_y^* and ϕ_π^* lie on a manifold and the loss function (1.1) decreases gradually along the manifold within the region, which is similar to REE but with higher ϕ_π^*. However, differently in the case $\phi_y^* > 0.7058$, the loss function (1.1) starts to increase, while in the REE the loss function (1.1) still decreases as shown in Figure 2. That is to say, there exist finite optimal Taylor rule coefficients at the BLE, but not at the REE. This is mainly because at the BLE the actual law of motion has higher volatility (especially for inflation) than at the REE in most cases and minimizing the loss function, i.e. minimizing the weighted variances of output gap and inflation, requires balancing the different responses in terms of policy parameters (ϕ_y, ϕ_π).

Are the finite optimal policies more aggressive in response to more persistent underlying shocks? At the REE with measurement error the finite coefficients ϕ_y^* and ϕ_π^* increase as the persistence of shocks grows within some range, see Boehm and House (2014). But at the BLE, we find that for relatively large ρ the finite coefficients ϕ_y^* and ϕ_π^* increase

step (*e.g.* 0.01). Then for each lattice point (ϕ_y, ϕ_π), we find the BLE $(\beta_1^*(\phi_y, \phi_\pi), \beta_2^*(\phi_y, \phi_\pi))$ and the corresponding central bank's loss function $\omega\sigma_\pi^2 + \omega_y\sigma_y^2 + \omega_r\sigma_r^2$ at the BLE. Finally we interpolate the loss function with respect to (ϕ_y, ϕ_π) to find the finite optimal values. It is easy to get analytic expressions of REE and the corresponding variances. In contrast, it is impossible to obtain analytic expressions of the optimal policy parameters under BLE and therefore we have to rely on numerical approximations.



(a) Optimal policy

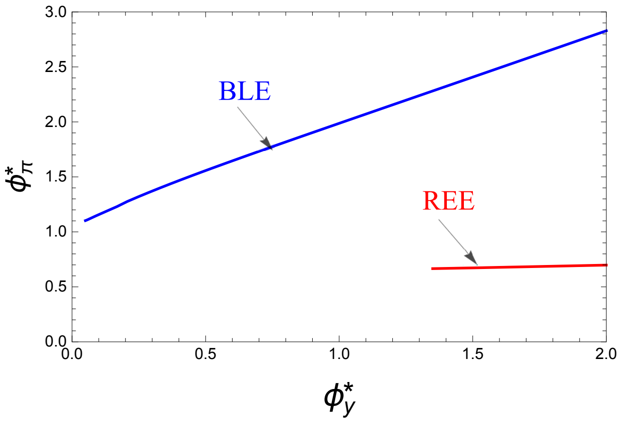
(b) Optimal manifold

Figure 3: Optimal policies at the BLE with respect to ρ (a) and corresponding optimal manifolds for three different ρ (connection points of solid and dotted curves corresponding to finite optimal policies) (b). Parameters are: $\lambda = 0.99, \varphi = 1, \gamma = 0.04, \sigma_1 = 1, \sigma_2 = 0.5$ and $\omega_y = 0.1, \omega_r = 0.05$.

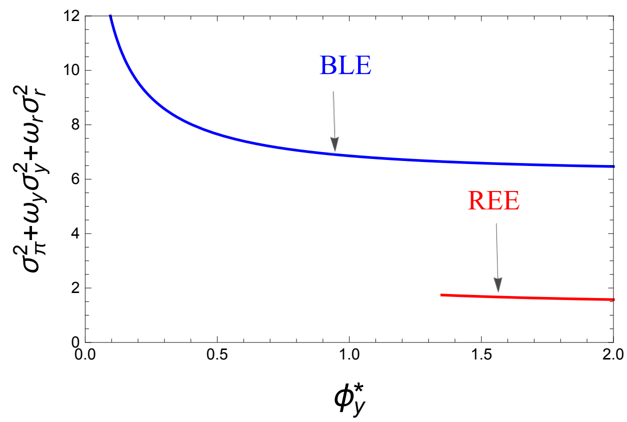
as the persistence ρ of shocks grows, while for some smaller range of ρ the finite coefficients ϕ_y^* and ϕ_π^* first increases and then decrease as the persistence ρ of shocks grows, as shown in Figure 3a. Furthermore, Figure 3b suggests that the optimal manifold always moves up as the persistence of shocks ρ grows. The finite optimal policy lies at the point in the optimal manifold connecting the solid and dotted lines in Figure 3b. The location of the optimal point corresponding to finite optimal policies depends on the relative values of variances of output gap and inflation. In the case ρ is large enough, the loss function is mainly dominated by the variance of inflation and hence the optimal policy ϕ_π^* grows quickly converging to ∞ and the slope of $\frac{\phi_\pi^*}{\phi_y^*}$ converging to a relatively large constant. For relatively small ρ, the effect of ρ with interest variances in the objective function is different from the traditional case without interest variance in the objective function. In any case, the corresponding loss function at the optimal policies at the BLE increases with respect to ρ, for example $= \omega\sigma_\pi^2 + \omega_y\sigma_y^2 + \omega_r\sigma_r^2 = 0.5855, 2.0723, 12.8143$ for $\rho = 0.35, 0.5, 0.75$, respectively.

Now consider the case $\omega_y = 1, \omega_r = 0.05$ with all the other parameters as before. In this case, we find that the optimal monetary policies at the BLE are such that that $\phi_y^* \to \infty$ and ϕ_π^* lies at a manifold as shown in Figure 4a[2]. However, the loss function (1.1)

[2]Although the figure only shows the range of ϕ_y^* within $[0, 2]$, we checked for a larger range $\phi_y^* \in [0, 100]$ and still can not find a finite optimal policy. The corresponding loss function (1.1) keeps decreasing even



(a) Optimal manifold



(b) Loss function along optimal manifold

Figure 4: Optimal manifolds at the BLE and REE given each ϕ^*_y (a) and the corresponding loss function along the optimal paths (b) with the contemporaneous interest rate rule. Parameters are: $\lambda = 0.99, \varphi = 1, \gamma = 0.04, \sigma_1 = 1, \sigma_2 = 0.5$ and $\omega_y = 1, \omega_r = 0.05$.

along the optimal line varies only little after $\phi^*_y > 0.5$, as shown in Figure 4b. In fact, if the policy $\phi^*_y = 0.5$, then the corresponding $\phi^*_\pi \approx 1.5$, which suggests that the traditional choice of $(\phi_y, \phi_\pi) = (0.5, 1.5)$ is reasonable and nearly optimal under the BLE framework when the weights on inflation and output gap are equal and the weight on interest rate is small enough. In addition, compared with the REE, all (ϕ^*_y, ϕ^*_π) in the optimal manifold at the BLE satisfy the determinacy condition, which is also a sufficient condition for the existence of BLE, while only for sufficient large $\phi^*_y (> 1.35)$ does the corresponding (ϕ^*_y, ϕ^*_π) in the optimal manifold at the REE satisfy the determinacy condition. Furthermore it is easy to see that given $\phi^*_y > 1.35$, ϕ^*_π in the manifold at the BLE is much greater than at the REE. Therefore, although the optimal policy at the BLE with $\omega_y = 1, \omega_r = 0.05$ is also $\phi^*_y \to \infty$ and ϕ^*_π lies at a manifold, the optimal manifold at the BLE is much higher than at the REE, which is consistent with higher persistence and higher volatility of inflation compared to the output gap at the BLE.

if the decreasing speed is very slow for large ϕ^*_y.