Assignment 6

PM522b Introduction to the Theory of Statistics Part 2

Due: March 6, 2018

1. (From lecture) Let $X_1, ..., X_n$ be iid Geometric $(1/\theta)$, $p(x|\theta) = \frac{1}{\theta}(1 - \frac{1}{\theta})^x$, $\theta > 1$
   a. Find a minimal sufficient statistic for θ
   b. Find the MLE for θ
   c. Find the Fisher Information for one observation and for a sample
   d. Is the MLE unbiased?
   e. What does the CRLB imply about the variance of the MLE?
2. Let $X_1, ..., X_n$ be iid with $f(x|\sigma) = \frac{1}{2\sigma^2} exp(-\frac{|x|}{\sigma})$. Find the MLE, determine if it is unbiased and find its MSE.
3. If $X \sim Bin(n, \theta)$, then X/n is an unbiased estimator of θ. The estimate of the variance of X is often $n(X/n)(1 - X/n)$.
   a) Show that $n(X/n)(1 - X/n)$ is a biased estimator of $Var(X)$
   b) Suggest an unbiased estimator of $Var(X)$ by modifying $n(X/n)(1 - X/n)$
4. For a random sample $X_1, X_2, ..., X_n$ from an exponential distribution, $f(x) = \frac{1}{\theta} e^{-x/\theta}$ $(0 < x < \infty)$ where $X_{(1)} = \min(X_1, X_2, ..., X_n)$:
   a) Show $\hat{\theta} = nX_{(1)}$ is an unbiased estimator for θ
   b) Find MSE($\hat{\theta}$)
5. CB 7.16 d)
6. CB 7.19
7. CB 7.20