

# Assignment 3

PM522b Introduction to the Theory of Statistics Part 2

Due: February 6, 2018

1. Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\sigma, \sigma)$  where  $\sigma > 0$ . a) Find a minimal sufficient statistic for  $\sigma$ . b) Show that  $(\bar{X}, S^2)$  is a sufficient statistic but not complete for  $\sigma$ .
2. Let  $X_1, \dots, X_n$  be a random sample with pdf  $f(x) = e^{-(x-\theta)}, x > \theta$ . Show that  $X_{(1)}$  is a complete sufficient statistic for  $\theta$  and  $X_{(n)} - X_{(1)}$  is an ancillary statistic.
3. CB 6.21
4. Using R, create a likelihood plot (similar to that shown in Slides 3.pdf) for  $\mu$  and  $\sigma^2$  from a normal distribution. Choose your own experiment, but please state all of your assumptions.
5. Let  $X_1, \dots, X_n$  be a random sample with pdf  $f(x|\theta) = \theta x^{\theta-1}$  where  $\theta > 0$  and  $0 < x < 1$ , find the maximum likelihood estimator for  $\theta$ .
6. Let  $X_1, \dots, X_n$  be a random sample with pdf  $f(x|\theta) = (\theta + 1)x^\theta$  for  $0 < x < 1$  and  $\theta > -1$ , find the MLE for  $\theta$ .