Assignment 10

PM522b Introduction to the Theory of Statistics Part 2

Due: April 17, 2018

1. Let $X_1, X_2, ..., X_n \overset{iid}{\sim} \text{Beta}(\theta, 1)$. Construct a $1-\alpha$ confidence interval for θ using a pivotal quantity.

2. Let $X_1, X_2, ..., X_{n1}$ and $Y_1, Y_2, ..., Y_{n2}$ be two independent random samples from $N(\mu_1, 1)$ and $N(\mu_2, 1)$, respectively. Construct a $1-\alpha$ confidence interval for the difference $\mu_1 - \mu_2$.

3. Let $X_1, X_2, ..., X_{n1}$ and $Y_1, Y_2, ..., Y_{n2}$ be two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Find the $1-\alpha$ confidence inteval for the ratio σ_1^2/σ_2^2.

4. Given a random sample from $f(x|\theta) = \frac{\theta}{(x+1)^{\theta+1}}, x \geq 0$ find a 95% confidence interval using a pivotal quantity.

5. Let $X_1, ..., X_n$ be a random sample from U(θ,1)

   a) Show that $\frac{X_{(1)}-\theta}{1-\theta}$ is a pivotal quantity.
   b) Derive a 1-α confidence interval using this pivotal quantity.
   c) Derive a 1-α confidence interval by pivoting the CDF of $X_{(1)}$.

6. Let $X_1, ..., X_n \overset{iid}{\sim} \text{Exp}(\theta)$ show that both pivoting the CDF and using the pivotal quantity $2\sum_{i=1}^{n} X/\theta$ result in the same interval estimator.