Assignment 11

PM522b Introduction to the Theory of Statistics Part 2

Due: April 26, 2018

1. CB 10.1
2. Using the definitions in the course slides, develop a consistent estimator for the binomial parameter p.
3. Are the methods of moments estimators for $N(\mu, \sigma^2)$ (both parameters) consistent?
4. CB 10.3
5. From class, $X \sim \text{Gamma}(4, \theta)$, show that the MLE is efficient for θ, and what is the asymptotic distribution of $\sqrt{n}(\hat{\theta}_{mle} - \theta)$?
6. Let $X_1, ...X_n$ be a random sample from $N(\mu, \sigma^2)$, both parameters unknown. Given $S^{*2} = \frac{1}{n}\sum\limits_{i=1}^{n}(X_i - \bar{X})^2 = \frac{n-1}{n}S^2$:
   1. Show S^{*2} is asymptotically unbiased.
   2. Find the asymptotic relative efficiency of S^{*2} and S^2.
   3. Compare the MSE of the two estimators and their asymptotic behaviour.
7. Suppose $X_1, ..., X_m$ and $Y_1, ..., Y_{2m} \sim \text{Exp}(1)$. Define $\bar{X}_m = \sum_{i=1}^{m} X_i/m$ and $\bar{Y}_{2m} = \sum_{i=1}^{2m} Y_i/2m$. Let
$$B_m = \frac{\bar{X}_m}{\bar{X}_m + 2\bar{Y}_{2m}}$$
   a. Show that B_m converges in probability to $1/3$ as $m \to \infty$.
   b. Derive the moment generating function of $\bar{X}_m - \bar{Y}_{2m}$.
   c. Show that $\sqrt{m}(\bar{X}_m - \bar{Y}_{2m})$ converges in distribution to a normal random variable with mean 0 and variance $3/2$.
   d. Derive the asymptotic distribution of $\sqrt{m}(B_m - 1/3)$.