# Work in Progress: The Euler Equation Implied Rate Under Heterogeneous Preferences

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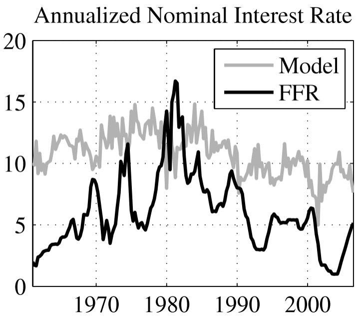
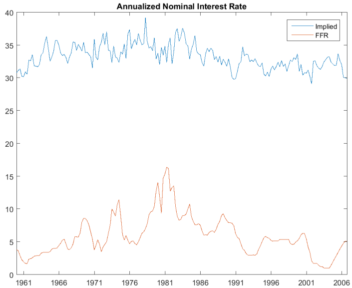
# Last Time

- Literature review
- Cleaned raw data from FRED
- Estimated VAR(4) for consumption, inflation, leisure, FFR, ...
- Computed implied interest rates under CRRA utility
    - Implied real rates corresponded well to Collard and Dellas (2012)
    - Implied nominal rates were very bad...

# Last Time




Collard and Dellas (2012)




Li (2015)

- It turns out this resulted from a matrix indexing error

# Generalized Implied Rates

- As in Collard and Dellas (2012):

$$u(C_t, \ell_t) = \frac{[(C_t / C_{t-1}^{\varphi})^{\nu} \ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

- Discount factor $\beta = 0.9926$
- Coefficient of risk aversion $\alpha = 2$
- **Habit persistence parameter** $\varphi = 0.8$
- **Weight assigned to consumption** $\nu = 0.34$
- When $\varphi = 0$ and $\nu = 1$, this reduces to CRRA utility (last time)

# Generalized Implied Rates

- Euler equation (from first-order conditions)

$$\frac{1}{1+i_t} = \beta \frac{\mathbb{E}_t[C_{t+1}^{\nu(1-\sigma)-1} C_t^{-\varphi\nu(1-\sigma)} \ell_{t+1}^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+2}^{\nu(1-\sigma)} C_{t+1}^{-\varphi\nu(1-\sigma)-1} \ell_{t+2}^{(1-\nu)(1-\sigma)}]/\pi_{t+1}}{\mathbb{E}_t[(C_t^{\nu(1-\sigma)-1} C_{t-1}^{-\varphi\nu(1-\sigma)} \ell_t^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+1}^{\nu(1-\sigma)} C_t^{-\varphi\nu(1-\sigma)-1} \ell_{t+1}^{(1-\nu)(1-\sigma)})]}$$

- Assuming conditional lognormality, nominal interest rate given by

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta\varphi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta\varphi \exp(\chi_{4t})}$$

$$\begin{aligned} \chi_{1t} = {} & (\nu(1-\alpha)-1)\mathrm{E}_t c_{t+1} - \varphi\nu(1-\alpha)c_t + (1-\nu)(1-\alpha)\mathrm{E}_t \ell_{t+1} \\ & - \mathrm{E}_t \pi_{t+1} + \text{constant second-order moments} \\ \chi_{2t} = {} & \ldots \end{aligned}$$

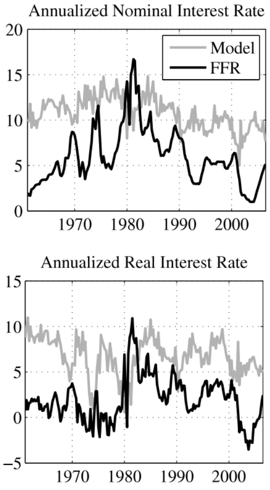
- Real interest rate is same without inflation terms

# Treatments

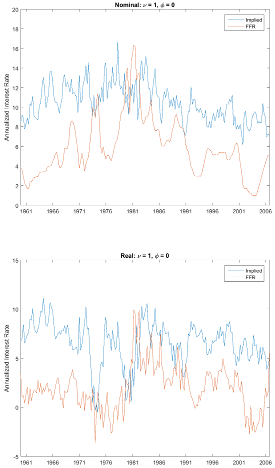
$$u(C_t, \ell_t) = \frac{[(C_t/C_{t-1}^{\varphi})^{\nu} \ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

| | $\phi$ | $\nu$ | Specification |
|---|---|---|---|
| SEP | 0 | 1 | CRRA |
| SEP + HP | 0.8 | 1 | habit persistence |
| NSEP | 0 | 0.34 | nonseparable consumption and leisure |
| NSEP + HP | 0.8 | 0.34 | nonseparable consumption and leisure + habit persistence |

# Results: SEP

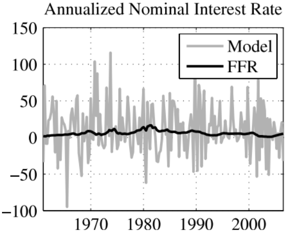


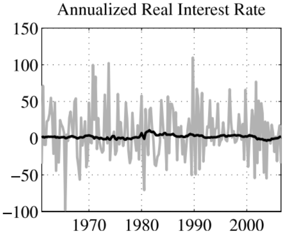

Collard and Dellas (2012)



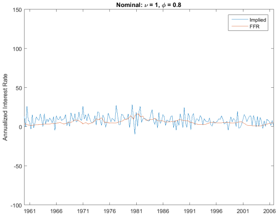

Li (2016)

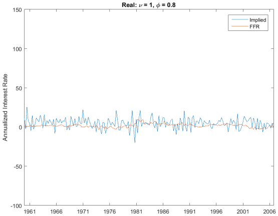
# Results: SEP + HP



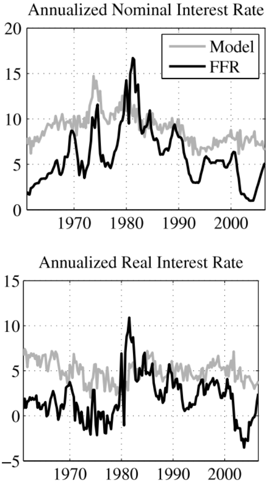

Collard and Dellas (2012)



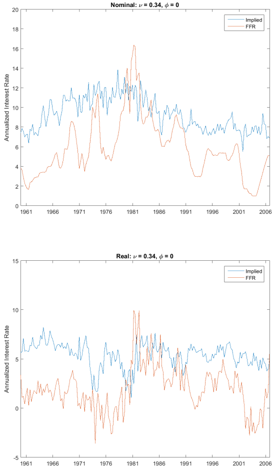




Li (2016)

# Results: NSEP

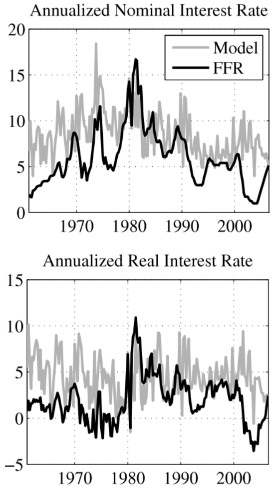



Collard and Dellas (2012)

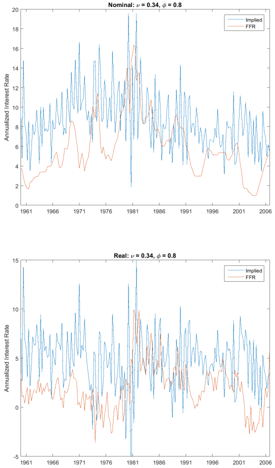



Li (2016)

# Results: NSEP + HP




Collard and Dellas (2012)




Li (2016)

# Impulse Response

- Previously estimated VAR(4)

$$y_t = A_0 + A_1 y_{t-1} + \ldots + A_4 y_{t-4} + \epsilon_t$$

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \Sigma)$$

$$y_t = \begin{bmatrix} \log(\text{real consumption}_t) \\ \text{inflation}_t \\ \text{leisure}_t \\ \log(\text{real disposable income}_t) \\ \log(\text{income less consumption}_t) \\ \text{effective FFR}_t \\ \log(\text{CCI}_t) \end{bmatrix}$$

- Want to observe responses of $y_t$ and implied rate to a $\sigma_{FFR}$ shock to the FFR at $t = 0$ (i.e. $\epsilon_{FFR,0} = \sigma_{FFR}$)

# Impulse Response

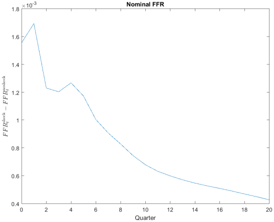
- Generate $y_t^{\text{no shock}}$ for $t = 0, \ldots, 20$ by iterating forward with estimated VAR coefficients from $y_0 = \hat{\mu}$ (sample mean)
- Compute orthogonalized IRF in Stata:

$$\frac{\partial y_t}{\partial \epsilon_{FFR,0}} = (A_1^t + A_2^{t-1} + A_3^{t-2} + A_4^{t-3})\epsilon_0 \qquad \text{(kind of)}$$

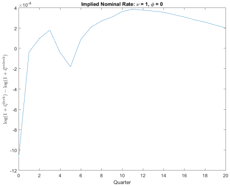
- Generate $y_t^{\text{shock}} = y_t^{\text{no shock}} + \frac{\partial y_t}{\partial \epsilon_{FFR,0}}$
- Compute implied rates and impulse response using $y_t^{\text{no shock}}$ and $y_t^{\text{shock}}$

$$\frac{\partial \log(1 + r_t)}{\partial \epsilon_{FFR,0}} \approx \log(1 + r_t^{\text{shock}}) - \log(1 + r_t^{\text{no shock}})$$

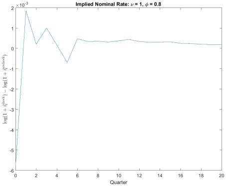
# Results: Nominal Rate



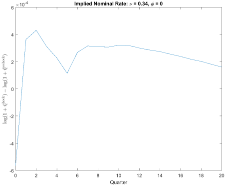

FFR



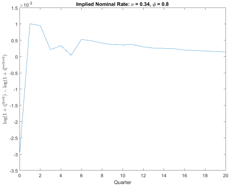

SEP




SEP + HP

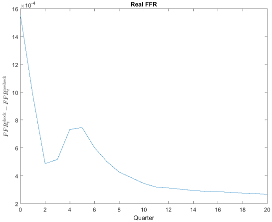



NSEP

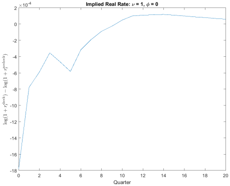



NSEP + HP

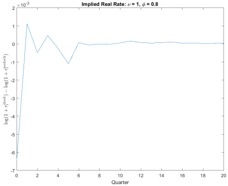
# Results: Real Rate



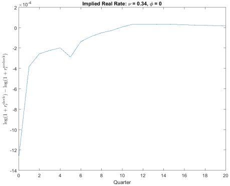

FFR



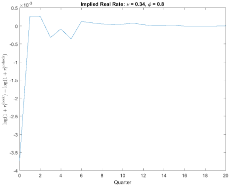

SEP




SEP + HP

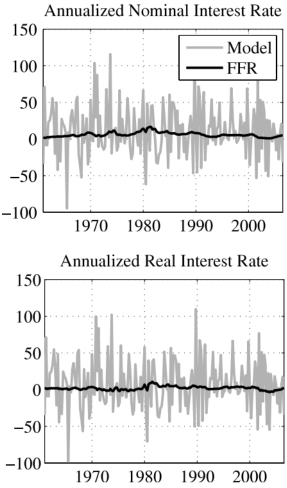



NSEP

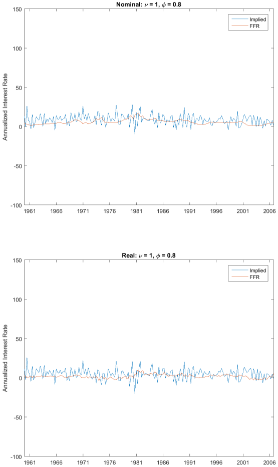



NSEP + HP

# Problems




Collard and Dellas (2012)




Li (2016)

# Problems

- SEP + HP volatility much lower than in Collard and Dellas (2012) and Canzoneri et al. (2007)
  - Implied real rates had standard deviation of 6.64 vs. 33.76 and 31.25
  - SEP + HP still has largest SD of all treatments

# Other Challenges

- 673 lines of code (fewer than I thought) becoming unwieldy
- No way to export impulse response tables from Stata
- Computing impulse responses of implied rates (i.e. as a function of $y_t$ impulse response)

# Next

- Generate confidence intervals for impulse response via Kilian (1998)
- Monte Carlo experiment
- **Heterogeneous preferences** (still)

# References

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba (2007) "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics*.

Collard, Fabrice and Harris Dellas (2012) "Euler equations and monetary policy," *Economics Letters*.

Kilian, Lutz (1998) "Small-Sample Confidence Intervals For Impulse Response Functions," *The Review of Economics and Statistics*.