Work in Progress: The Euler Equation Implied Rate Under Heterogeneous Preferences

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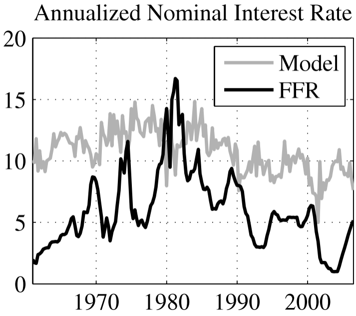
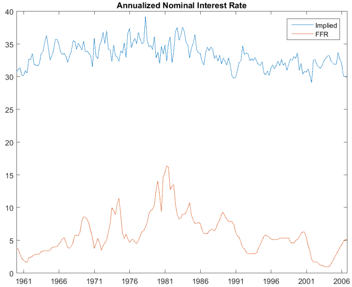
Last Time

- Literature review
- Cleaned raw data from FRED
- Estimated VAR(4) for consumption, inflation, leisure, FFR, ...
- Computed implied interest rates under CRRA utility
  - Implied real rates corresponded well to Collard and Dellas (2012)
  - Implied nominal rates were very bad...

Last Time



Collard and Dellas (2012)



Li (2015)

- It turns out this resulted from a matrix indexing error

Generalized Implied Rates

- As in Collard and Dellas (2012):

$$u(C_t, \ell_t) = \frac{[(C_t/C_{t-1}^{\varphi})^{\nu}\ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

- Discount factor $\beta = 0.9926$
- Coefficient of risk aversion $\alpha = 2$
- **Habit persistence parameter** $\varphi = 0.8$
- **Weight assigned to consumption** $\nu = 0.34$
- When $\varphi = 0$ and $\nu = 1$, this reduces to CRRA utility (last time)

Generalized Implied Rates

- Euler equation (from first-order conditions)

$$\frac{1}{1+i_t} = \beta \frac{\mathbb{E}_t[C_{t+1}^{\nu(1-\sigma)-1} C_t^{-\varphi\nu(1-\sigma)} \ell_{t+1}^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+2}^{\nu(1-\sigma)} C_{t+1}^{-\varphi\nu(1-\sigma)-1} \ell_{t+2}^{(1-\nu)(1-\sigma)}]/\pi_{t+1}}{\mathbb{E}_t[(C_t^{\nu(1-\sigma)-1} C_{t-1}^{-\varphi\nu(1-\sigma)} \ell_t^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+1}^{\nu(1-\sigma)} C_t^{-\varphi\nu(1-\sigma)-1} \ell_{t+1}^{(1-\nu)(1-\sigma)})]}$$

- Assuming conditional lognormality, nominal interest rate given by

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta\varphi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta\varphi \exp(\chi_{4t})}$$

$$\begin{aligned}\chi_{1t} = {} & (\nu(1-\alpha)-1)\mathrm{E}_t c_{t+1} - \varphi\nu(1-\alpha)c_t + (1-\nu)(1-\alpha)\mathrm{E}_t \ell_{t+1} \\ & - \mathrm{E}_t \pi_{t+1} + \text{constant second-order moments} \\ \chi_{2t} = {} & \dots\end{aligned}$$

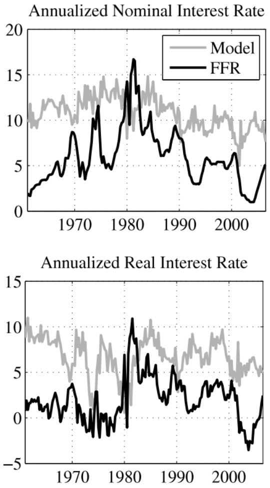
- Real interest rate is same without inflation terms

Treatments

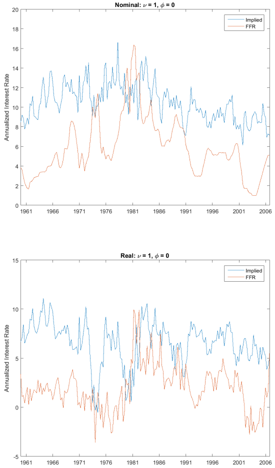
$$u(C_t, \ell_t) = \frac{[(C_t / C_{t-1}^{\varphi})^{\nu} \ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

	φ	ν	Specification
SEP	0	1	CRRA
SEP + HP	0.8	1	habit persistence
NSEP	0	0.34	nonseparable consumption and leisure
NSEP + HP	0.8	0.34	nonseparable consumption and leisure + habit persistence

Results: SEP

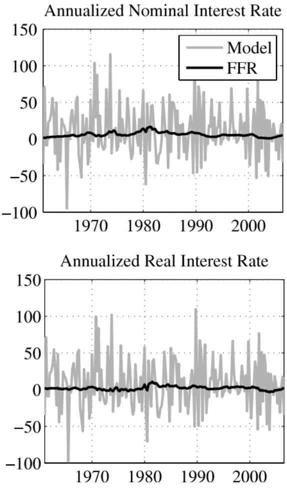


Collard and Dellas (2012)

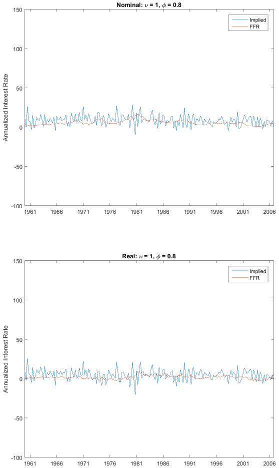


Li (2016)

Results: SEP + HP

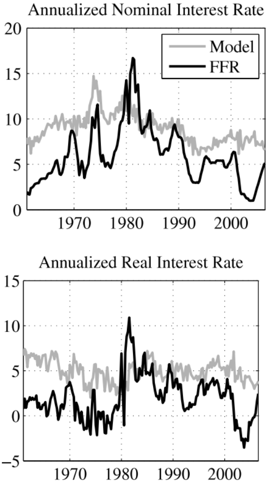


Collard and Dellas (2012)

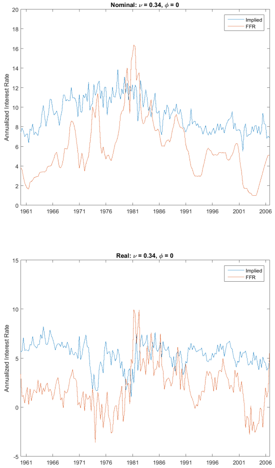


Li (2016)

Results: NSEP

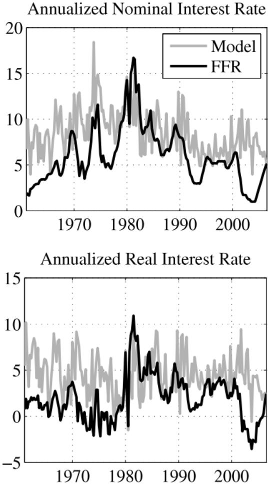


Collard and Dellas (2012)

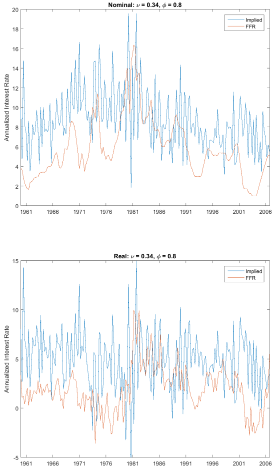


Li (2016)

Results: NSEP + HP



Collard and Dellas (2012)



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Impulse Response

- Previously estimated VAR(4)

$$y_t = A_0 + A_1 y_{t-1} + \ldots + A_4 y_{t-4} + \epsilon_t$$

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \Sigma)$$

$$y_t = \begin{bmatrix} \log(\text{real consumption}_t) \\ \text{inflation}_t \\ \text{leisure}_t \\ \log(\text{real disposable income}_t) \\ \log(\text{income less consumption}_t) \\ \text{effective FFR}_t \\ \log(\text{CCI}_t) \end{bmatrix}$$

Impulse Response

- Want to observe responses of y_t and implied rate to a $\epsilon_{FFR,0} = 1$ shock to the FFR at $t = 0$
- Impulse response:

$$\frac{\partial y_t}{\partial \epsilon_{FFR,0}} = (A_1^t + A_2^{t-1} + A_3^{t-2} + A_4^{t-3})\epsilon_0$$

- Orthogonalized IRF:
  - When error terms $\epsilon_{j,t}$ are correlated, exogenous shock to FFR will be correlated with shock to other covariates
  - Cholesky decomposition $\Sigma = PP'$ (P lower triangular)
  - New error terms $u_t = P^{-1}\epsilon_t \sim N(0, I)$
  - Orthogonalized shock $\frac{\partial y_t}{\partial u_{FFR,0}}$

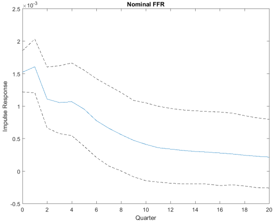
Impulse Response

- Generate $y_t^{\text{no shock}}$ for $t = 0, \ldots, 20$ by iterating forward with estimated VAR coefficients from $y_0 = \hat{\mu}$ (sample mean)
- Using Kilian (1998), for each treatment, for 1000 simulations:
  1. Simulate data series by choosing random start point y_0 and iterating forward with random errors
  2. Compute $\frac{\partial y_t}{\partial u_{FFR,0}}$ for simulated series
  3. Generate $y_t^{\text{shock}} = y_t^{\text{no shock}} + \frac{\partial y_t}{\partial u_{FFR,0}}$
  4. Compute implied rates and impulse response using $y_t^{\text{no shock}}$ and y_t^{shock}:

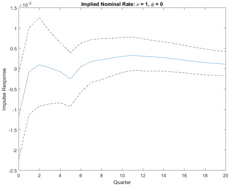
$$\frac{\partial \log(1 + r_t)}{\partial u_{FFR,0}} \approx \log(1 + r_t^{\text{shock}}) - \log(1 + r_t^{\text{no shock}})$$

- Plot point estimate and 95% confidence intervals for each treatment

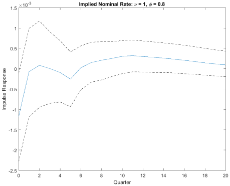
Results: Nominal Rate



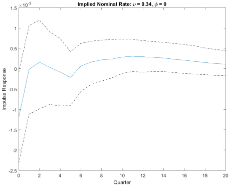
FFR



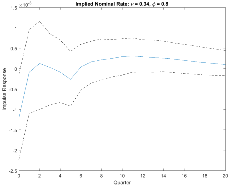
SEP



SEP + HP

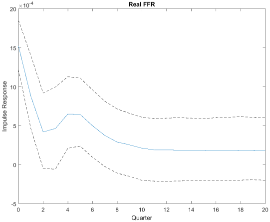


NSEP

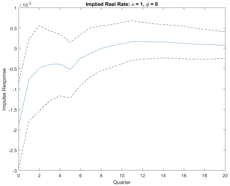


NSEP + HP

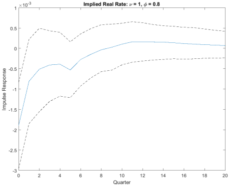
Results: Real Rate



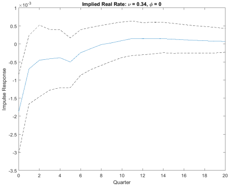
FFR



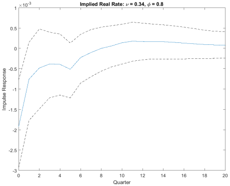
SEP



SEP + HP

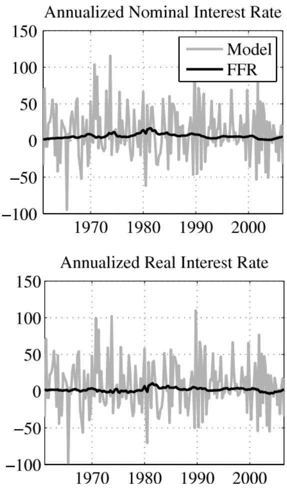


NSEP

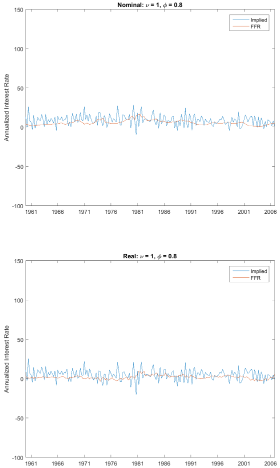


NSEP + HP

Problems: SEP + HP Implied Rates



Collard and Dellas (2012)



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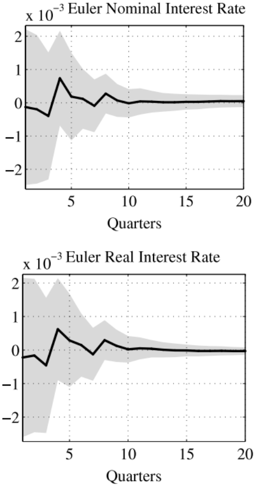
Problems: SEP + HP Implied Rates

- SEP + HP volatility much lower than in Collard and Dellas (2012) and Canzoneri et al. (2007)
  - Implied real rates had standard deviation of 6.64 vs. 33.76 and 31.25
  - SEP + HP still has largest SD of all treatments

Problems: NSEP + HP Impulse Response



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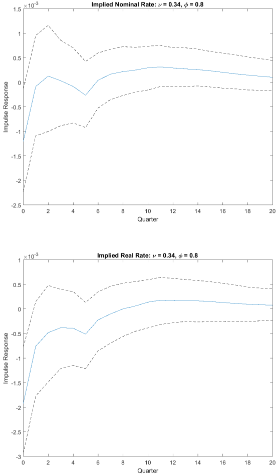

Li (2016)

Other Challenges

- Maintaining code modularity and organization
- No way to export impulse response tables from Stata
- Computing impulse responses of implied rates (i.e. as a function of y_t impulse response)
- Kilian (1998)'s 18-year-old MATLAB code

Next

- Monte Carlo experiment
- **Heterogeneous preferences** (still)

References

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba (2007) "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics*.

Collard, Fabrice and Harris Dellas (2012) "Euler equations and monetary policy," *Economics Letters*.

Kilian, Lutz (1998) "Small-Sample Confidence Intervals For Impulse Response Functions," *The Review of Economics and Statistics*.