Work in Progress: The Euler Equation Implied Rate Under Heterogeneous Preferences

Pearl Li

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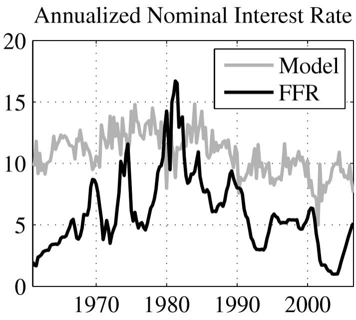
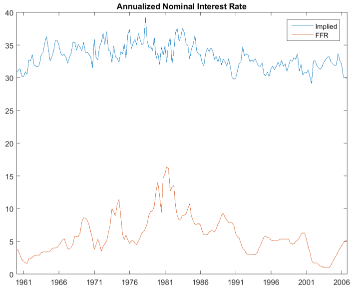
Last Time

- Literature review
- Cleaned raw data from FRED
- Estimated VAR(4) for consumption, inflation, leisure, FFR, ...
- Computed implied interest rates under CRRA utility
  - Implied real rates corresponded well to Collard and Dellas (2012)
  - Implied nominal rates were very bad...

Last Time



Collard and Dellas (2012)



Li (2015)

- It turns out this resulted from a matrix indexing error

Generalized Implied Rates

- As in Collard and Dellas (2012):

$$u(C_t, \ell_t) = \frac{[(C_t / C_{t-1}^{\varphi})^{\nu} \ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

- Discount factor $\beta = 0.9926$
- Coefficient of risk aversion $\alpha = 2$
- **Habit persistence parameter** $\varphi = 0.8$
- **Weight assigned to consumption** $\nu = 0.34$
- When $\varphi = 0$ and $\nu = 1$, this reduces to CRRA utility (last time)

Generalized Implied Rates

- Euler equation (from first-order conditions)

$$\frac{1}{1+i_t} = \beta \frac{\mathbb{E}_t[C_{t+1}^{\nu(1-\sigma)-1} C_t^{-\varphi\nu(1-\sigma)} \ell_{t+1}^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+2}^{\nu(1-\sigma)} C_{t+1}^{-\varphi\nu(1-\sigma)-1} \ell_{t+2}^{(1-\nu)(1-\sigma)}]/\pi_{t+1}}{\mathbb{E}_t[(C_t^{\nu(1-\sigma)-1} C_{t-1}^{-\varphi\nu(1-\sigma)} \ell_t^{(1-\nu)(1-\sigma)} - \beta\varphi C_{t+1}^{\nu(1-\sigma)} C_t^{-\varphi\nu(1-\sigma)-1} \ell_{t+1}^{(1-\nu)(1-\sigma)})]}$$

- Assuming conditional lognormality, nominal interest rate given by

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta\varphi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta\varphi \exp(\chi_{4t})}$$

$$\begin{aligned} \chi_{1t} = {} & (\nu(1-\alpha)-1)\mathrm{E}_t c_{t+1} - \varphi\nu(1-\alpha)c_t + (1-\nu)(1-\alpha)\mathrm{E}_t \ell_{t+1} \\ & - \mathrm{E}_t \pi_{t+1} + \text{constant second-order moments} \\ \chi_{2t} = {} & \dots \end{aligned}$$

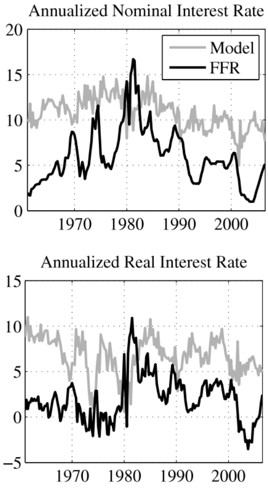
- Real interest rate is same without inflation terms

Treatments

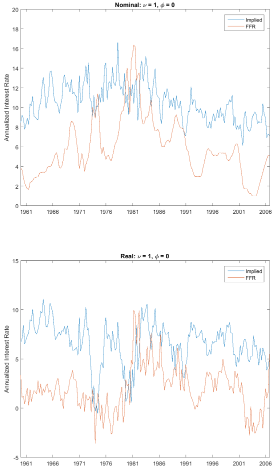
$$u(C_t, \ell_t) = \frac{[(C_t/C_{t-1}^{\varphi})^{\nu}\ell_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

- $\varphi = 0, \nu = 1$: CRRA (SEP)
- $\varphi = 0.8, \nu = 1$: habit persistence (SEP + HP)
- $\varphi = 0, \nu = 0.34$: non-separable consumption and leisure (NSEP)
- $\varphi = 0.8, \nu = 0.34$: non-separable consumption and leisure + habit persistence (NSEP + HP)

Results: SEP

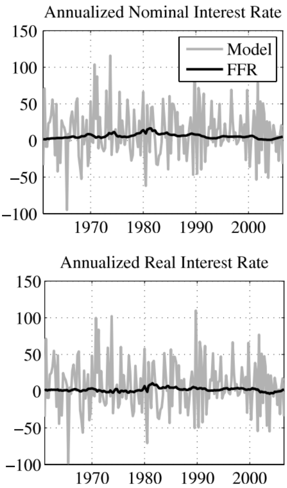


Collard and Dellas (2012)

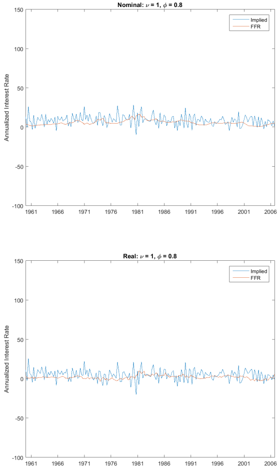


Li (2016)

Results: SEP + HP

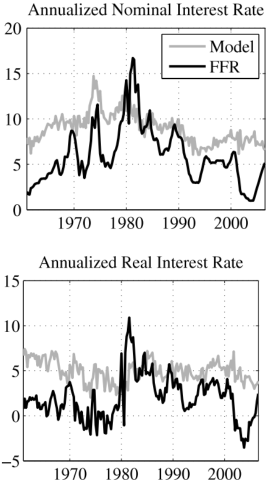


Collard and Dellas (2012)

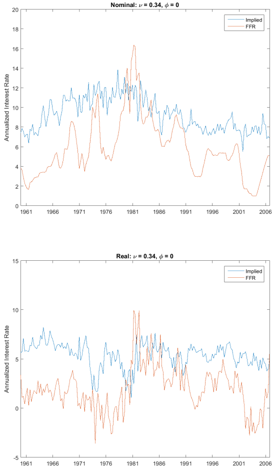


Li (2016)

Results: NSEP

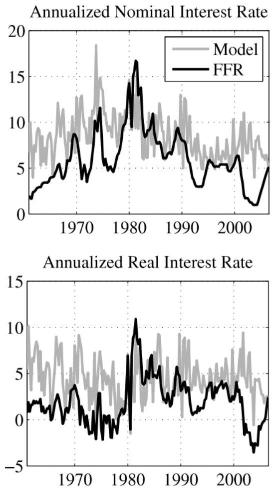


Collard and Dellas (2012)

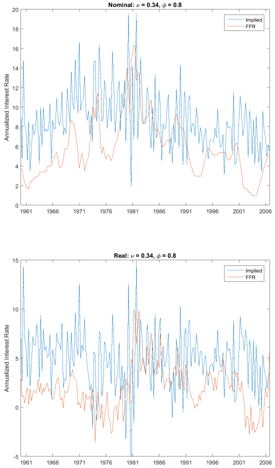


Li (2016)

Results: NSEP + HP



Collard and Dellas (2012)



Li (2016)

Impulse Response

- Previously estimated VAR(4)

$$y_t = A_0 + A_1 y_{t-1} + \ldots + A_4 y_{t-4} + \epsilon_t$$

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \Sigma)$$

$$y_t = \begin{bmatrix} \log(\text{real consumption}_t) \\ \text{inflation}_t \\ \text{leisure}_t \\ \log(\text{real disposable income}_t) \\ \log(\text{income less consumption}_t) \\ \text{effective FFR}_t \\ \log(\text{CCI}_t) \end{bmatrix}$$

Impulse Response

- Want to observe response of y_t to a σ_{FFR} shock to the FFR at $t = 0$ (i.e. $\epsilon_{FFR,0} = \sigma_{FFR}$)
- No other shocks occur: $\epsilon_{i,0} = 0$ for all other covariates, and $\epsilon_t = 0$ for $t > 0$
- Compute by iterating forward:

$$\Delta y_0 = \begin{bmatrix} \epsilon_{c,0} \\ \epsilon_{\pi,0} \\ \epsilon_{\ell,0} \\ \epsilon_{RDI,0} \\ \epsilon_{YMC,0} \\ \epsilon_{FFR,0} \\ \epsilon_{CCI,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_{FFR} \\ 0 \end{bmatrix}, \qquad \begin{aligned} \Delta y_1 &= A_1(\Delta y_0), \\ \Delta y_2 &= A_1(\Delta y_1) + A_2(\Delta y_0), \\ &\ldots \end{aligned}$$

where $\Delta y_t = y_t^{\text{shock}} - y_t^{\text{no shock}}$

Problems

Next

- Get confidence intervals for impulse response via Kilian (1998)
- Monte Carlo experiment
- **Heterogeneous preferences**

References

Collard, Fabrice and Harris Dellas (2012) "Euler equations and monetary policy," *Economics Letters*.

Kilian, Lutz (1998) "Small-Sample Confidence Intervals For Impulse Response Functions," *The Review of Economics and Statistics*.