LIMITED ASSET MARKET PARTICIPATION

AND THE EULER EQUATION IMPLIED INTEREST RATE

Pearl Li

AN HONORS THESIS

in

Economics

Presented to the Faculties of the University of Pennsylvania in Partial Fulfillment

of the Requirements for the Degree of Bachelor of Arts with Honors

2016

Contents

1 Introduction

Perhaps the main criticism of modern macroeconomic models (in particular, DSGE models) is that the microfoundational assumptions on which they're based often don't actually fit the data very well. Smith (2014) singles out the consumption Euler equation, which expresses intertemporal consumption choice in terms of the real interest rate r_t. In its typical form:

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right]$$

Canzoneri et al. (2007) computed the interest rate implied by the consumption Euler equation under several utility specifications. They found that their computed rates were actually negatively correlated with historical money market rates, and furthermore that the spread is correlated with the stance of monetary policy. These results are potentially extremely damaging to the validity of macroeconomic models which assume the Euler equation implied rate and the actual interest rate to be the same – that is, nearly all macro models. Collard and Dellas (2012) repeated this exercise, adding utility nonseparable in consumption and labor, and in fact found the looked-for positive correlation with observed rates.

In this paper, I first attempt to replicate the findings of Canzoneri et al. (2007) and Collard and Dellas (2012) using new data up through the second quarter of 2015. This portion includes computing Euler equation implied rates and correlating the spread between implied and observed rates with the stance of monetary policy. The consumption and income data for this section are all national aggregates from the National Income and Product Accounts (NIPA).

The main novel contribution of this paper will be the introduction of limited asset market participation to the implied rate framework, inspired by Vissing-Jorgensen (2002). Specifically, I'll aggregate household-level data from the Consumer Expenditure Survey (CEX) for bondholders and nonbondholders. I'll perform the same analyses on the time series of these two groups to test the hypothesis that interest rates implied by bondholders' consumption paths will more resemble observed rates than those from nonbondholders. The intuition for this idea is clear: we expect households with positions in the bond market to adjust their consumption in response to changes in the interest rate, while we don't expect nonbondholders to do so.

2 Literature

3 Model

We start with the standard household problem from the neoclassical growth model. In period t, the representative consumer has preferences

$$U_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, C_{s-1}, L_s)$$

where β is her discount rate, C_s and C_{s-1} are real consumption today and yesterday, and L_s is fraction of leisure hours. Each period, she receives labor income with nominal wage W_s and chooses consumption and nominal holdings B_s of a risk-free one-period bond. The price of the consumption good is P_s. This gives the following period budget constraint in nominal units:

$$P_s C_s + (1 + i_{s-1}) B_{s-1} \leq W_s (1 - L_s) + B_s$$

Taking first-order conditions gives the equilibrium nominal interest rate by

$$\frac{1}{1 + i_t} = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{1}{\Pi_{t+1}} \right] \tag{1}$$

In real units, the period budget constraint is

$$C_s + (1 + r_{s-1}) \frac{B_{s-1}}{P_{s-1}} \leq \frac{W_s}{P_s} (1 - L_s) + \frac{B_s}{P_s}$$

and the real interest rate satisfies

$$\frac{1}{1 + r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right] \tag{2}$$

To compute the interest rates implied by the Euler equations (1) and (2) requires a few assumptions. We assume that real consumption C_t and gross inflation Π_t are conditionally lognormal. We use the functional form for utility used by Collard and Dellas (2012):

$$u(C_t, C_{t-1}, L_t) = \frac{[(C_t / C_{t-1}^{\phi})^{\nu} L_t^{1-\nu}]^{1-\alpha}}{1 - \alpha} \tag{3}$$

where α is the coefficient of relative risk aversion, ϕ is the habit persistence parameter, and ν specifies the relative weight of consumption compared to leisure. When $\phi = 0$ (no habit persistence) and $\nu = 1$ (utility is separable in consumption and leisure), (3) reduces to the case of CRRA utility:

$$u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha}$$

We'll derive an expression for the implied interest rate in terms of conditional expectations and variances for the CRRA case only and leave the more general case to Collard and Dellas (2012). We denote logs of variables using lowercase letters, i.e. $c_t := \log C_t$ and $\pi_t := \log \Pi_t$ (approximately net inflation). From (1), the nominal interest rate under CRRA preferences is given by:

$$\begin{aligned}
\frac{1}{1+i_t} &= \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \Pi_{t+1}^{-1} \right] \\
&= \beta \mathbb{E}_t \exp\left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] \\
&= \beta \exp\left(\mathbb{E}_t \left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t \left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] \right) \\
&= \beta \exp\left(-\alpha \left[\mathbb{E}_t c_{t+1} - c_t\right] - \mathbb{E}_t \pi_{t+1} + \frac{\alpha^2}{2}\mathrm{Var}_t c_{t+1} + \frac{1}{2}\mathrm{Var}_t \pi_{t+1} + \mathrm{Cov}_t(c_{t+1}, \pi_{t+1}) \right)
\end{aligned}$$

where the third equality follows from our assumption of conditional lognormality. The expression for the real interest rate is the same, but without the inflation terms:

$$\frac{1}{1+r_t} = \beta \exp\left(-\alpha \left[\mathbb{E}_t c_{t+1} - c_t\right] + \frac{\alpha^2}{2}\mathrm{Var}_t c_{t+1} \right)$$

From Collard and Dellas (2012), the equivalent expression for the implied nominal rate under the more general preferences (3) is

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta\phi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta\phi \exp(\chi_{4t})}$$

where

$$
\begin{aligned}
\chi_{1t} = {} & (\nu(1-\sigma)-1)\mathbb{E}_t c_{t+1} - \phi\nu(1-\sigma)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma)-1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - (1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+1}) + (\nu(1-\sigma)-1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+1}) \\
& - (\nu(1-\sigma)-1)\mathrm{Cov}(c_{t+1}, \pi_{t+1}) \\
\chi_{2t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+2} - (\phi\nu(1-\sigma)+1)\mathbb{E}_t c_{t+1} + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+2} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+2} + \frac{(\phi\nu(1-\sigma)+1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - \nu(1-\sigma)\mathrm{Cov}_t(c_{t+2}, \pi_{t+2}) + (\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, \pi_{t+1}) - (1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+2}) \\
& - \nu(1-\sigma)(\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, c_{t+2}) + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+2}, l_{t+2}) \\
& - (\phi\nu(1-\sigma)+1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+2}) \\
\chi_{3t} = {} & (\nu(1-\sigma)-1)c_t - \phi\nu(1-\sigma)c_{t-1} + (1-\nu)(1-\sigma)l_t \\
\chi_{4t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+1} - (\phi\nu(1-\sigma)+1)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+1} \\
& + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+1}, l_{t+1})
\end{aligned}
$$

Following Canzoneri et al. (2007), to derive estimates for these conditional moments, we assume that the dynamics of consumption, inflation, and labor can be modeled as the VAR(4) process (written below in companion form)

$$
Y_{t+1} = A_0 + A_1 Y_t + u_t, \tag{4}
$$

$$
u_t \overset{\text{iid}}{\sim} N(0, \Sigma)
$$

where

$$Y_t = [y_t, y_{t-1}, y_{t-2}, y_{t-3}]'$$

$$y_t = [c_t, \pi_t, l_t, rdi_t, ymc_t, ffr_t, cci_t]'$$

The components of y_t are log of real consumption, log of gross inflation, leisure fraction (which we'll define more explicitly later), log of real disposable income, log of output less consumption, log of the gross effective federal funds rate, and log of the Thomson Reuters Equal Weight Continuous Commodity Index [1].

After estimating A_0, A_1, and Σ, we can compute:

$$\mathrm{Var}_t Y_{t+1} = \Sigma$$

$$\mathbb{E}_t Y_{t+1} = A_0 + A_1 Y_t$$

$$\mathrm{Var}_t Y_{t+2} = A_1 \Sigma A_1' + \Sigma$$

$$\mathbb{E}_t Y_{t+2} = A_0 + A_1 A_0 + A_1^2 Y_t$$

$$\mathrm{Cov}_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1'$$

The conditional moments are then the respective (i, j) components of these matrices. For example, $\mathrm{Cov}_t(c_{t+1}, l_{t+2})$ is the $(1, 3)$ component of $\mathrm{Cov}_t(Y_{t+1}, Y_{t+2})$.

Now, given data with which to estimate the vector autoregression (4), we have everything we need to compute the interest rates implied by the Euler equation.

[1] The CCI is the "old" Thomson Reuters/Jeffries CRB Index, calculated using the same methodology as the CRB Index before it underwent weighting and rebalance changes in 1995.

4 References

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba (2007) "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics*.

Collard, Fabrice and Harris Dellas (2012) "Euler Equations and Monetary Policy," *Economics Letters*.

Smith, Noah (2014) "The Equation at the Core of Modern Macro," `http://noahpinionblog.blogspot.com/2014/01/the-equation-at-core-of-modern-macro.html`.

Vissing-Jorgensen, Annette (2002) "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*.