LIMITED ASSET MARKET PARTICIPATION
AND THE EULER EQUATION IMPLIED INTEREST RATE

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Contents

1 Introduction

Perhaps the main criticism of modern macroeconomic models (in particular, DSGE models) is that the microfoundational assumptions on which they're based often don't actually fit the data very well. Smith (2014) singles out the consumption Euler equation, which expresses intertemporal consumption choice in terms of the real interest rate r_t. In its typical form:

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right]$$

Canzoneri et al. (2007) compute the interest rate implied by the consumption Euler equation under several utility specifications. They find that their computed rates are actually negatively correlated with historical money market rates, and furthermore that the spread is correlated with the stance of monetary policy. These results are potentially extremely damaging to the validity of macroeconomic models which assume the Euler equation implied rate and the actual interest rate to be the same – that is, nearly all macro models. Collard and Dellas (2012) repeat this exercise, adding utility nonseparable in consumption and labor, and in fact find the looked-for positive correlation with observed rates.

In this paper, I first attempt to replicate the findings of Canzoneri et al. (2007) and Collard and Dellas (2012) using new data up through the second quarter of 2015. This portion includes computing Euler equation implied rates and correlating the spread between implied and observed rates with the stance of monetary policy. The consumption and income data for this section are all national aggregates from the National Income and Product Accounts (NIPA).

The main novel contribution of this paper is the introduction of limited asset market participation to the implied rate framework, inspired by Vissing-Jorgensen (2002). Specifically, I aggregate household-level data from the Consumer Expenditure Survey (CEX) for bondholders and nonbondholders. I perform the same analyses on the time series of these two groups to test the hypothesis that interest rates implied by bondholders' consumption paths will more resemble observed rates than those from nonbondholders. The intuition for this idea is clear: we expect households with positions in the bond market to adjust their consumption in response to changes in the interest rate, while we don't expect nonbondholders to do so.

2 Literature

3 Model

We start with the standard household problem from the neoclassical growth model. In period t, the representative consumer has preferences

$$U_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, C_{s-1}, L_s)$$

where β is her discount rate, C_s and C_{s-1} are real consumption today and yesterday, and L_s is fraction of leisure hours. Each period, she receives labor income with nominal wage W_s and chooses consumption and nominal holdings B_s of a risk-free one-period bond. The price of the consumption good is P_s. This gives the following period budget constraint in nominal units:

$$P_s C_s + (1 + i_{s-1}) B_{s-1} \leq W_s (1 - L_s) + B_s$$

Taking first-order conditions gives the equilibrium nominal interest rate by

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{1}{\Pi_{t+1}} \right] \tag{1}$$

In real units, the period budget constraint is

$$C_s + (1 + r_{s-1}) \frac{B_{s-1}}{P_{s-1}} \leq \frac{W_s}{P_s} (1 - L_s) + \frac{B_s}{P_s}$$

and the real interest rate satisfies

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right] \tag{2}$$

To compute the interest rates implied by the Euler equations (1) and (2) requires a few assumptions. We assume that real consumption C_t and gross inflation Π_t are conditionally lognormal. We use the functional form for utility used by Collard and Dellas (2012):

$$u(C_t, C_{t-1}, L_t) = \frac{[(C_t / C_{t-1}^{\phi})^{\nu} L_t^{1-\nu}]^{1-\alpha}}{1-\alpha} \tag{3}$$

where α is the coefficient of relative risk aversion, ϕ is the habit persistence parameter, and ν specifies the relative weight of consumption compared to leisure. When $\phi = 0$ (no habit persistence) and $\nu = 1$ (utility is separable in consumption and leisure), (3) reduces to the case of CRRA utility:

$$u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha}$$

We'll derive an expression for the implied interest rate in terms of conditional expectations and variances for the CRRA case only and leave the more general case to Collard and Dellas (2012). We denote logs of variables using lowercase letters, i.e. $c_t := \log C_t$ and $\pi_t := \log \Pi_t$ (approximately net inflation). From (1), the nominal interest rate under CRRA preferences is given by:

$$\begin{aligned}
\frac{1}{1+i_t} &= \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \Pi_{t+1}^{-1} \right] \\
&= \beta \mathbb{E}_t \exp\left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] \\
&= \beta \exp\left(\mathbb{E}_t \left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t \left[-\alpha(c_{t+1} - c_t) - \pi_{t+1}\right] \right) \\
&= \beta \exp\left(-\alpha \left[\mathbb{E}_t c_{t+1} - c_t\right] - \mathbb{E}_t \pi_{t+1} + \frac{\alpha^2}{2}\mathrm{Var}_t c_{t+1} + \frac{1}{2}\mathrm{Var}_t \pi_{t+1} + \mathrm{Cov}_t(c_{t+1}, \pi_{t+1}) \right)
\end{aligned}$$

where the third equality follows from our assumption of conditional lognormality. The expression for the real interest rate is the same, but without the inflation terms:

$$\frac{1}{1+r_t} = \beta \exp\left(-\alpha \left[\mathbb{E}_t c_{t+1} - c_t\right] + \frac{\alpha^2}{2}\mathrm{Var}_t c_{t+1} \right)$$

From Collard and Dellas (2012), the equivalent expression for the implied nominal rate under the more general preferences (3) is

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta\phi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta\phi \exp(\chi_{4t})}$$

where

$$
\begin{aligned}
\chi_{1t} = {} & (\nu(1-\sigma)-1)\mathbb{E}_t c_{t+1} - \phi\nu(1-\sigma)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma)-1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - (1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+1}) + (\nu(1-\sigma)-1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+1}) \\
& - (\nu(1-\sigma)-1)\mathrm{Cov}(c_{t+1}, \pi_{t+1}) \\
\chi_{2t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+2} - (\phi\nu(1-\sigma)+1)\mathbb{E}_t c_{t+1} + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+2} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+2} + \frac{(\phi\nu(1-\sigma)+1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - \nu(1-\sigma)\mathrm{Cov}_t(c_{t+2}, \pi_{t+2}) + (\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, \pi_{t+1}) - (1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+2}) \\
& - \nu(1-\sigma)(\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, c_{t+2}) + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+2}, l_{t+2}) \\
& - (\phi\nu(1-\sigma)+1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+2}) \\
\chi_{3t} = {} & (\nu(1-\sigma)-1)c_t - \phi\nu(1-\sigma)c_{t-1} + (1-\nu)(1-\sigma)l_t \\
\chi_{4t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+1} - (\phi\nu(1-\sigma)+1)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+1} \\
& + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+1}, l_{t+1})
\end{aligned}
$$

Following Canzoneri et al. (2007), to derive estimates for these conditional moments, we assume that the dynamics of consumption, inflation, and labor can be modeled as the VAR(4) process (written below in companion form)

$$
Y_{t+1} = A_0 + A_1 Y_t + u_t, \tag{4}
$$

$$
u_t \overset{\text{iid}}{\sim} N(0, \Sigma)
$$

where

$$Y_t = [y_t, y_{t-1}, y_{t-2}, y_{t-3}]'$$

$$y_t = [c_t, \pi_t, l_t, rdi_t, ymc_t, ffr_t, cci_t]'$$

The components of y_t are log of real consumption, log of gross inflation, leisure fraction (which we'll define more explicitly later), log of real disposable income, log of output less consumption, log of the gross effective federal funds rate, and log of the Thomson Reuters Equal Weight Continuous Commodity Index[1].

After estimating A_0, A_1, and Σ, we can compute:

$$\mathbb{E}_t Y_{t+1} = A_0 + A_1 Y_t$$

$$\mathbb{E}_t Y_{t+2} = A_0 + A_1 A_0 + A_1^2 Y_t$$

$$\mathrm{Var}_t Y_{t+1} = \Sigma$$

$$\mathrm{Var}_t Y_{t+2} = A_1 \Sigma A_1' + \Sigma$$

$$\mathrm{Cov}_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1'$$

The conditional moments are then the respective (i, j) components of these matrices. For example, $\mathrm{Cov}_t(c_{t+1}, l_{t+2})$ is the $(1, 3)$ component of $\mathrm{Cov}_t(Y_{t+1}, Y_{t+2})$.

Now, given data with which to estimate the vector autoregression (4), we have everything we need to compute the interest rates implied by the Euler equation.

[1]The CCI is the "old" Thomson Reuters/Jeffries CRB Index, calculated using the same methodology as the CRB Index before it underwent weighting and rebalance changes in 1995.

4 Data

4.1 Aggregate-Level

In the aggregate-level analysis, the endogenous variables making up y_t in the VAR model are constructed mostly according to Collard and Dellas (2012). Except where mentioned, all the raw time series used are obtained from the St. Louis Fed's Federal Reserve Economic Data (FRED), with variable names in parentheses. Data are at the quarterly level and seasonally adjusted when appropriate, spanning 222 quarters from 1960.I to 2015.II, inclusive.. Real dollar values are in 2009 dollars. All lowercase variables in the vector y_t denote the natural log of the respective capitalized variable below *except the leisure fraction* l_t, which is described explicitly below.

Per capita real consumption C_t: Aggregate real consumption is defined as the sum of the chain quantity indices (2009 = 100) for personal consumption expenditures on nondurable goods (`DNDGRA3Q086SBEA`) and services (`DSERRA3Q086SBEA`), all multiplied by the sum of nominal nondurables (`PCEND`) and services (`PCESV`) consumption in 2009. This amount is divided by the civilian noninstitutional population (`CNP16OV`) to get per capita real consumption.

Gross quarterly inflation Π_t: In each quarter, the implicit price deflator P_t is calculated by dividing aggregate nominal consumption (`PCEND + PCESV`) by aggregate real consumption (described above). Then gross quarterly inflation is defined as the growth rate of the deflator: $\Pi_t = \frac{P_t}{P_{t-1}}$.

Leisure fraction l_t: Labor fraction h_t is defined as the average weekly hours worked in the nonfarm business sector (`PRS85006023`), multiplied by the civilian employment-to-population ratio (`EMRATIO`) and then rescaled so that the mean over all quarters was $\frac{1}{3}$. Then the leisure fraction is given by $l_t = 1 - h_t$.

Per capita real disposable income RDI_t: This is computed by dividing real disposable income

(DPIC96) by the civilian non-institutional population.

Per capita real output less consumption YMC_t: Defined as real gross domestic product (GDPC96) minus aggregate real consumption, again divided by the civilian noninstitutional population.

Gross quarterly effective federal funds rate FFR_t: This is computed by raising the gross annualized rate (DFF) to the one-fourth power.

Continuous Commodity Index CCI_t: I use the CCI ending price on the first day of each quarter, obtained from Bloomberg. As mentioned, the CCI is the continuation of the CRB Index used by Canzoneri et al. (2007) and Collard and Dellas (2012). What is called the CRB Index today is calculated slightly differently and exists only since 1995.

4.2 Household-Level

For the comparison of bondholders to nonbondholders, I reuse the inflation, federal funds rate, and CCI variables constructed in the previous section. I generate separate time series for the other four endogenous variables for both bondholders and nonbondholders by aggregating household-level data from the Consumer Expenditure Survey from 1996.I to 2012.IV (68 quarters).

The CEX is a rotating panel of representative "consumer units"[2] in the United States, which are interviewed each quarter for five consecutive quarters. Each observation is a household-quarter. The first interview is for practice, and is not included in the reported survey data. Each quarter, 20 percent of the households rotate out of the survey after their fifth interview, and a new 20 percent rotate in. Households report their expenditures in very detailed categories each quarter.

[2] I refer to these consumer units informally as households, though the CEX does actually distinguish between the two terms, allowing for multiple consumer units to dwell in the same physical household. However, it is the consumer unit-level at which financial decisions are made and reported to the survey-takers, and hence at which the analysis in this paper is carried out.

Demographic and income data are collected in the second and fifth interviews, and asset holdings information is collected only in the fifth interview[3].

After discarding observations which are flagged by the CEX as being incomplete income respondents (`RESPSTAT`[4] $= 2$), I generate the following (nominal) observation-level variables:

Bondholder status: I determine whether or or not to label each household a bondholder using the criteria set forth in Vissing-Jorgensen (2002). In the fifth interview, the CEX asks each household to estimate its current holdings in a number of asset categories, as well as how those holdings have changed in the preceding year (four quarters). I use a positive response in the asset categories "U.S. Savings Bonds" and "stocks, mutual funds, private bonds, government bonds, or Treasury notes" to determine bondholder status, despite that this definition likely creates some false positives, such as households which hold stocks but not bonds. It is difficult to achieve a more complete separation of households.

Either all observations belonging to a particular household are labeled bondholder observations or none of them are. I do not allow for a household's bondholder status to change between interviews. A household is defined to be a bondholder if it had positive holdings of at least one of the two asset categories one year before the asset holdings questions are asked in the fifth interview (i.e. at the time of the first interview) — specifically, if one of the following holds:

1. The household reports holding the same amount of the asset as a year ago (`COMPBND` or `COMPSEC` $= 1$), and reports a positive current holdings amount `USBNDX` or `SECESTX` > 0)

2. The household reports lower holdings of the asset than a year ago (`COMPBND` or `COMPSEC` $= 2$)

[3]The CEX in fact consists of two separate surveys: the Interview Survey, which I have just described, and the Diary Survey, in which households report weekly expenditures on frequently purchased items. I use the Interview Survey exclusively.

[4]The variable names in the remainder of this section refer to CEX variables unless otherwise specified.

3. The household reports an increase in holdings in the past year (`COMPBND` or `COMPSEC` $= 3$) by an amount less than the current holdings (`COMPBNDX` $<$ `USBNDX` or `COMPSECX` $<$ `SECESTX`)

Consumption: Following Heathcote et al. (2010), I define consumption of nondurable goods and services as the sum of the following expenditure categories: food and beverages (`FOOD` + `ALCBEV`), clothing (`APPAR`), gasoline (`GASMO`), household operation (`HOUSOP`), public transportation (`PUBTRA`), medical care excluding health insurance (`HEALTH` − `HEALTHIN`), recreation (`ENTERT`), tobacco (`TOBACC`), and education (`READ` + `EDUCA`).

Hours worked: I use the weekly hours worked by the household's reference person (`INC_HRS1`). The reference person is the first person mentioned by the survey respondent when asked to "Start with the name of the person or one of the persons who owns or rents the home."

Disposable income: I use after-tax income (`FINCATAX`), as in Hai et al. (2015).

Output less consumption: Defined as before-tax income (`FINCBTAX`) minus consumption (defined above).

Below are summary statistics for bondholders and nonbondholders using the variables defined above. Since bondholders represent a fairly small fraction of the total sample, I include all bondholder observations in the bondholder aggregate but take a random sample of the nonbondholder observations in order to equalize sample size.

SUMMARY STATS TK

Consumption, disposable income, and output less consumption are each deflated by the unadjusted Consumer Price Index for nondurables for urban consumers (`CUUR0000SAN` in FRED) as in Vissing-Jorgensen (2002), rescaled to 2009 dollars to correspond with the aggregate-level data. The expenditure categories included in consumption were chosen to allow for the possibility of deflating each category by its own CPI (for example, `CPIFABNS` from FRED for food and beverages). However, the result of doing so was found to differ only negligibly from using a single CPI.

The CEX provides population weights for each household, which are calibrated so that summing the population weights in a given quarter approximates the number of households in the United States that quarter, while taking the weighted sum of the number of household members approximates the total population. I take the weighted mean of hours worked for each quarter and use it to generate labor fraction l_t as in the previous section. For each of consumption, disposable income, and output less consumption, I take the weighted sum each quarter and divide it by the population to get per capita variables C_t, RDI_t, and YMC_t.

Finally, I seasonally adjust log consumption c_t by regressing it on indicators of the quarters and subtracting off the non-first quarter coefficients.

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