LIMITED BOND MARKET PARTICIPATION

AND THE EULER EQUATION IMPLIED INTEREST RATE

Pearl Li

AN HONORS THESIS

in

Economics

Presented to the Faculties of the University of Pennsylvania in Partial Fulfillment

of the Requirements for the Degree of Bachelor of Arts with Honors

2016

Jere Behrman, Primary Advisor

Frank Schorfheide, Secondary Advisor

1 Introduction

Perhaps the main criticism of modern macroeconomic models (in particular, DSGE models) is that the microfoundational assumptions on which they're based often don't actually fit the data very well. Smith (2014) singles out the consumption Euler equation, which expresses intertemporal consumption choice in terms of the real interest rate r_t. In its typical form:

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right]$$

Canzoneri et al. (2007) compute the interest rate implied by the consumption Euler equation under several utility specifications. They find that their computed rates are actually negatively correlated with historical money market rates, and furthermore that the spread is correlated with the stance of monetary policy. These results are potentially extremely damaging to the validity of macroeconomic models which assume the Euler equation implied rate and the actual interest rate to be the same – that is, nearly all macro models. Collard and Dellas (2012) repeat this exercise, adding utility nonseparable in consumption and labor, and in fact find the looked-for positive correlation with observed rates.

In this paper, I first attempt to replicate the findings of Canzoneri, Cumby, and Diba (henceforth abbreviated CCD) and Collard and Dellas (2012) using new data up through the second quarter of 2015. This portion includes computing Euler equation implied rates and correlating the spread between implied and observed rates with the stance of monetary policy. The consumption and income data for this section are all national aggregates from the National Income and Product Accounts (NIPA). I find that the implied rates are positively correlated with ex post rates, though I then show that this correlation is in fact not a robust measure of the Euler equation's fit to the data. I also show that the spread between the two rates varies systematically with the nominal federal funds rate due to the implied and observed rates responding in opposite directions to a monetary shock.

The main novel contribution of this paper is the introduction of limited bond market participation to the implied rate framework, inspired by Vissing-Jorgensen (2002). Specifically, I aggregate household-level data from the Consumer Expenditure Survey (CEX) for bondholders and nonbondholders. I perform the same analyses on the time series of these two groups to test the hypothesis that interest rates implied by bondholders' consumption paths will more resemble observed rates than those from nonbondholders. The intuition for this idea is clear: we expect households with positions in the bond market to adjust their consumption in response to changes in the interest rate, while we don't expect nonbondholders to do so. Supporting this hypothesis, I do find that for bondholders, the implied rates are more strongly correlated with observed rates and the spread is more weakly related to the federal funds rate. However, the differences between bondholders and nonbondholders are not statistically significant. The consumption paths that I aggregate for both groups are much more volatile than NIPA consumption, probably due to the relatively small sample sizes. This suggests that more conclusive results may follow from additional data.

2 Literature

There is a substantial body of literature on how the standard representative agent model and its resultant Euler equations fail to fit the data in various ways. Parker (1999) points to several potential explanations for these failures. Among the possibilities: preferences may be nonseparable, aggregation across heterogeneous households may produce bias, or actual asset markets may not be complete. In this paper, I focus on the first two explanations.

2.1 Nonseparability in utility models

Standard preferences feature constant relative risk aversion (CRRA utility), which assume additive separability both across time and between consumption and labor. Relaxing these simplifying assumptions has been shown to explain some results from the empirical literature which initially seemed incompatible with household optimizing behavior.

Fuhrer (2000) develops a utility model with habit formation in order to explain a key feature of aggregate data: the "hump-shaped" responses of consumption and inflation, which he demonstrates aren't seen in impulse response functions computed using the standard CRRA model. In addition to being intuitively reasonable, his addition of habit formation (equivalently, nonseparability across time) allows the model to explain significant delays in response to monetary policy shocks.

Basu and Kimball (2002) observe that assuming additive separability between consumption and labor[1] leads to estimations that the income effect of a permanent wage increase strongly overpower the substitution effect, hence reducing labor supply. This implication is not borne out in empirical comparisons of income versus hours worked. Using aggregate data, they show that the King-

[1] This is traditionally carried out implicitly when utility is made a function of consumption alone and labor only comes into the optimization as a source of income.

Plosser-Rebelo utility model (which is nonseparable in consumption and leisure) leads to more realistic estimates of the intertemporal elasticity of substitution for consumption in the 1980s and 1990s, though not necessarily as well for earlier data.

2.2 Euler equation implied rates

Another particular strand of the literature on the failure of the Euler equation stems from Canzoneri et al. (2007). They computed the interest rates implied by the Euler equation and compare them to observed historical money market rates. The two series turn out to be negatively correlated, a result which is robust to the addition of several different habit formation specifications to the standard CRRA utility model. Furthermore, they find that the spread between the observed rate and the Euler equation implied rate is correlated with the stance of monetary policy, presenting a challenge to modern macroeconomic models which equate the two rates. Finally, they compute impulse response functions of the implied rate to a monetary shock, which move nearly in a mirror image to the observed rate.

This paper is followed by Collard and Dellas (2012), who undertake the same exercise but impose the additional restriction of nonseparability in consumption and leisure, citing the work of Basu and Kimball (2002) discussed earlier. Collard and Dellas find that the enforcement of this nonseparability makes the correlation between observed and Euler equation implied rates strongly positive and the difference between their volatilities smaller. However, the actual path of the Euler implied rate still differs substantially from observed rates.

In a similar vein, Gareis and Mayer (2013) compute the Euler equation implied rate from the Smets and Wouters (2007) DSGE model, which happens to incorporate both habit formation and nonseparability in consumption and leisure. They too find a positive correlation between actual and implied real interest rates. However, their Euler equation implied rates are much more volatile than (about five times the standard deviation of) the actual rates. They also estimate the true distribution of correlations by using the Smets and Wouters model as a data-generating process for a Monte Carlo experiment. They simulate 1000 time series of consumption, inflation, hours worked, and interest rates and correlate the "actual" and Euler implied rates. The resulting distribution of correlations is centered around zero (though the median is slightly positive), in contrast to their nonsimulated findings.

2.3 Heterogeneous agents

Perhaps the most obvious argument against the representative agent model is that in practice, the real population of consumers is not at all homogeneous. Another subset of the literature shows how introducing heterogeneity to the representative agent model can reconcile theory with ostensible empirical irregularities.

Campbell and Mankiw (1989) examine heterogeneity in optimizing behavior. Specifically, on top of the standard optimizing agent who consumes his permanent income, they posit an additional class of "rule of thumb" consumers who do not optimize and consume exactly their income each period. They interpret the coefficient of disposable income on consumption as the fraction of such current-income consumers, which is estimated to be about or above 50%, depending on the specification of controls. They conclude that their model with rule of thumb consumers included better fits some stylized facts found in aggregate data, including consumption's response to income but not real interest rate changes, as well as the "excess smoothness" of consumption compared to what is predicted by the permanent income hypothesis model.

A related approach is taken by Guvenen (2006), who seeks to reconcile macroeconomic assumptions of the elasticity of intertemporal substitution (near 1) with empirical estimates (near 0). He proposes that this inconsistency is a consequence of dissimilarity of high-elasticity stockholders and low-elasticity nonstockholders. Despite making up a relatively small fraction of the overall population, stockholders hold most of the wealth, so analysis of savings and investment typically reflect their dynamics. On the other hand, consumption is much more evenly distributed, so aggregate consumption data reflect the low elasticity of the majority — that is, the poor.

Vissing-Jorgensen (2002) also looks at heterogeneity of asset market participation as a means of reconciling differing estimates of the EIS. Using household-level data from the CEX, she classifies households as being stockholders, bondholders, and nonassetholders. She then estimates the EIS from a log-linearized Euler equation separately for stock- and bondholders using using, respectively, returns on the New York Stock Exchange composite and returns on Treasury bills. She argues that the Euler equation should only hold for a household with a position in that particular asset, and that the inclusion of nonassetholders in Euler equation estimations distorts the results. I take this idea and invert Vissing-Jorgensen's methodology — rather than estimating the EIS from the Euler equation using historical returns, I compute the interest rates implied by the Euler equation assuming some value for the EIS.

3 Model

I start with the standard household problem from the neoclassical growth model. In period t, the representative consumer has preferences

$$U_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, C_{s-1}, l_s)$$

where β is her discount rate, C_s and C_{s-1} are real consumption today and yesterday, and l_s is fraction of leisure hours. Each period, she receives labor income with nominal wage W_s and chooses consumption and nominal holdings B_s of a risk-free one-period bond. The price of the consumption good is P_s. This gives the following period budget constraint in nominal units:

$$P_s C_s + (1 + i_{s-1}) B_{s-1} \leq W_s (1 - l_s) + B_s$$

Taking first-order conditions gives the equilibrium nominal interest rate by

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{1}{\Pi_{t+1}} \right] \tag{1}$$

In real units, the period budget constraint is

$$C_s + (1 + r_{s-1}) \frac{B_{s-1}}{P_{s-1}} \leq \frac{W_s}{P_s} (1 - l_s) + \frac{B_s}{P_s}$$

and the real interest rate satisfies

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right] \tag{2}$$

To compute the interest rates implied by the Euler equations (1) and (2) requires a few assumptions. I assume that real consumption C_t and gross inflation Π_t are conditionally lognormal. I use the functional form for utility used by Collard and Dellas (2012):

$$u(C_t, C_{t-1}, l_t) = \frac{[(C_t / C_{t-1}^{\phi})^{\nu} l_t^{1-\nu}]^{1-\alpha}}{1-\alpha} \tag{3}$$

where α is the coefficient of relative risk aversion, ϕ is the habit persistence parameter, and ν specifies the relative weight of consumption compared to leisure. When $\phi = 0$ (no habit persistence) and $\nu = 1$ (utility is separable in consumption and leisure), (3) reduces to the case of constant relative risk aversion:

$$u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha} \tag{4}$$

With CRRA utility, the elasticity of intertemporal substitution is given by

$$\text{EIS} = -\frac{d\log(C_{t+1}/C_t)}{d\log(u'(C_{t+1})/u'(C_t))} = -\frac{\log(C_{t+1}/C_t)}{-\alpha\log(C_{t+1}/C_t)} = \frac{1}{\alpha}$$

Below, I derive an expression for the implied interest rate in terms of conditional expectations and variances for the CRRA case only, leaving the more general case to Collard and Dellas. Logs of variables are denoting using lowercase letters, i.e. $c_t := \log C_t$ and $\pi_t := \log \Pi_t$ (approximately net inflation). From (1), the nominal interest rate under CRRA preferences is given by:

$$\begin{aligned}
\frac{1}{1+i_t} &= \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\Pi_{t+1}^{-1}\right] \qquad (5)\\
&= \beta\mathbb{E}_t\exp\left[-\alpha(c_{t+1}-c_t)-\pi_{t+1}\right]\\
&= \beta\exp\left(\mathbb{E}_t\left[-\alpha(c_{t+1}-c_t)-\pi_{t+1}\right]+\frac{1}{2}\text{Var}_t\left[-\alpha(c_{t+1}-c_t)-\pi_{t+1}\right]\right)\\
&= \beta\exp\left(-\alpha\left[\mathbb{E}_t c_{t+1}-c_t\right]-\mathbb{E}_t\pi_{t+1}+\frac{\alpha^2}{2}\text{Var}_t c_{t+1}+\frac{1}{2}\text{Var}_t\pi_{t+1}+\text{Cov}_t(c_{t+1},\pi_{t+1})\right)
\end{aligned}$$

where the third equality follows from our assumption of conditional lognormality. The expression for the real interest rate is the same, but without the inflation terms:

$$\frac{1}{1+r_t} = \beta\exp\left(-\alpha\left[\mathbb{E}_t c_{t+1}-c_t\right]+\frac{\alpha^2}{2}\text{Var}_t c_{t+1}\right)$$

From Collard and Dellas, the equivalent expression for the implied nominal rate under the more general preferences (3) is

$$\frac{1}{1+i_t} = \beta\frac{\exp(\chi_{1t})-\beta\phi\exp(\chi_{2t})}{\exp(\chi_{3t})-\beta\phi\exp(\chi_{4t})}$$

where

$$
\begin{aligned}
\chi_{1t} = {} & (\nu(1-\sigma)-1)\mathbb{E}_t c_{t+1} - \phi\nu(1-\sigma)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma)-1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - (1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+1}) + (\nu(1-\sigma)-1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+1}) \\
& - (\nu(1-\sigma)-1)\mathrm{Cov}(c_{t+1}, \pi_{t+1}) \\
\chi_{2t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+2} - (\phi\nu(1-\sigma)+1)\mathbb{E}_t c_{t+1} + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+2} - \mathbb{E}_t \pi_{t+1} \\
& + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+2} + \frac{(\phi\nu(1-\sigma)+1)^2}{2}\mathrm{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \frac{\mathrm{Var}_t \pi_{t+1}}{2} \\
& - \nu(1-\sigma)\mathrm{Cov}_t(c_{t+2}, \pi_{t+2}) + (\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, \pi_{t+1}) - (1-\nu)(1-\sigma)\mathrm{Cov}_t(\pi_{t+1}, l_{t+2}) \\
& - \nu(1-\sigma)(\phi\nu(1-\sigma)+1)\mathrm{Cov}_t(c_{t+1}, c_{t+2}) + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+2}, l_{t+2}) \\
& - (\phi\nu(1-\sigma)+1)(1-\nu)(1-\sigma)\mathrm{Cov}_t(c_{t+1}, l_{t+2}) \\
\chi_{3t} = {} & (\nu(1-\sigma)-1)c_t - \phi\nu(1-\sigma)c_{t-1} + (1-\nu)(1-\sigma)l_t \\
\chi_{4t} = {} & \nu(1-\sigma)\mathbb{E}_t c_{t+1} - (\phi\nu(1-\sigma)+1)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} + \frac{(\nu(1-\sigma))^2}{2}\mathrm{Var}_t c_{t+1} \\
& + \frac{((1-\nu)(1-\sigma))^2}{2}\mathrm{Var}_t l_{t+1} + \nu(1-\nu)(1-\sigma)^2\mathrm{Cov}_t(c_{t+1}, l_{t+1})
\end{aligned}
$$

Following Canzoneri et al. (2007), to derive estimates for these conditional moments, I assume that the dynamics of consumption, inflation, and labor can be modeled as the VAR(4) process (written below in companion form)

$$Y_{t+1} = A_0 + A_1 Y_t + u_t, \tag{6}$$

$$u_t \overset{\text{iid}}{\sim} N(0, \Sigma)$$

where

$$Y_t = [y_t, y_{t-1}, y_{t-2}, y_{t-3}]'$$

$$y_t = [c_t, \pi_t, l_t, rdi_t, ymc_t, ffr_t, cci_t]'$$

The components of y_t are log of real consumption, log of gross inflation, leisure fraction (which I define more explicitly later), log of real disposable income, log of output less consumption, log of the gross effective federal funds rate, and log of the Thomson Reuters Equal Weight Continuous Commodity Index[2,3]

After estimating A_0, A_1, and Σ, I compute:

$$\mathbb{E}_t Y_{t+1} = A_0 + A_1 Y_t \qquad \mathrm{Var}_t Y_{t+1} = \Sigma$$

$$\mathbb{E}_t Y_{t+2} = A_0 + A_1 A_0 + A_1^2 Y_t \qquad \mathrm{Var}_t Y_{t+2} = A_1 \Sigma A_1' + \Sigma$$

$$\mathrm{Cov}_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1'$$

The conditional moments are then the respective (i,j) components of these matrices. For example, $\mathrm{Cov}_t(c_{t+1}, l_{t+2})$ is the $(1,3)$ component of $\mathrm{Cov}_t(Y_{t+1}, Y_{t+2})$.

Now, given data with which to estimate the vector autoregression (6), we have everything we need to compute the interest rates implied by the Euler equation.

---

[2]The CCI is the "old" Thomson Reuters/Jeffries CRB Index, calculated using the same methodology as the CRB Index before it underwent weighting and rebalance changes in 1995.

[3]Since the conditional moments for only consumption, inflation, and leisure are needed to construct the implied interest rate, I also tried a version of my analysis where I estimated the VAR and computed implied rates using only $y = [c_t, \pi_t, l_t]'$. The results were very similar to what I found using the larger model, so in order to maintain consistency with CCD and Collard and Dellas, I proceed using all seven endogenous variables.

4 Data

4.1 Aggregate-level data

In the aggregate-level analysis, the endogenous variables making up y_t in the VAR model are constructed according to Collard and Dellas (2012) whenever possible. Except where mentioned, all the raw time series used are obtained from the St. Louis Fed's Federal Reserve Economic Data (FRED), with variable names in parentheses. Data are at the quarterly level and seasonally adjusted when appropriate, spanning 222 quarters from 1960:I to 2015:II, inclusive. Real dollar values are in 2009 dollars. I describe the construction of these series in more detail in the data appendix. All lowercase variables in the vector y_t denote the natural log of the respective capitalized variable *except the leisure fraction* l_t.

4.2 Household-level data

For the comparison of bondholders to nonbondholders, I reuse the inflation, federal funds rate, and CCI variables constructed in the previous section. I generate separate time series for the other four endogenous variables for both bondholders and nonbondholders by aggregating household-level data from the Consumer Expenditure Survey from 1996:I to 2012:IV (68 quarters). This process is also described in more detail in the data appendix.

The CEX is a rotating panel of representative "consumer units"[4] in the United States, which are interviewed each quarter for five consecutive quarters. Each observation is a household-quarter.

[4] I refer to these consumer units informally as households, though the CEX does actually distinguish between the two terms, allowing for multiple consumer units to dwell in the same physical household. However, it is the consumer unit level at which financial decisions are made and reported to the survey-takers, and hence at which the analysis in this paper is carried out.

The first interview is for practice, and is not included in the reported survey data. Each quarter, 20 percent of the households rotate out of the survey after their fifth interview, and a new 20 percent rotate in. Households report their expenditures in very detailed categories each quarter. Demographic and income data are collected in the second and fifth interviews, and asset holdings information is collected only in the fifth interview[5].

Following standard practice, I discard the 86,403 observations (out of 478,894) which are flagged by the CEX as being incomplete income respondents (`RESPSTAT`[6] = 2). I don't exclude households who are potentially borrowing constrained (indicated by a low wealth-to-income ratio) because I conjecture that credit constraints are a mechanism through which nonbondholders may be less able to optimize their consumption according to the consumption Euler equation.

I determine whether or or not to label each household a bondholder using the criteria set forth by Vissing-Jorgensen (2002). In the fifth interview, the CEX asks each household to estimate its current holdings in a number of asset categories, as well as how those holdings have changed in the preceding year (four quarters). I use a positive response in the asset categories "U.S. Savings Bonds" and "stocks, mutual funds, private bonds, government bonds, or Treasury notes" to determine bondholder status, despite that this definition likely creates some false positives, such as households which hold stocks but not bonds. It is difficult to achieve a more complete separation of households.

[5]The CEX in fact consists of two separate surveys: the Interview Survey, which I have just described, and the Diary Survey, in which households report weekly expenditures on frequently purchased items. I use the Interview Survey exclusively.

[6]The variable names in the remainder of this section and the corresponding section of the data appendix refer to CEX variables unless otherwise specified.

Either all observations belonging to a particular household are labeled bondholder observations or none of them are. I do not allow for a household's bondholder status to change between interviews. A household is defined to be a bondholder if it had positive holdings of at least one of the two asset categories one year before the asset holdings questions are asked in the fifth interview (i.e. at the time of the first interview) — specifically, if one of the following holds:

1. The household reports holding the same amount of the asset as a year ago (`COMPBND` or `COMPSEC` = 1), and reports a positive current holdings amount `USBNDX` or `SECESTX` > 0)

2. The household reports lower holdings of the asset than a year ago (`COMPBND` or `COMPSEC` = 2)

3. The household reports an increase in holdings in the past year (`COMPBND` or `COMPSEC` = 3) by an amount less than the current holdings (`COMPBNDX` < `USBNDX` or `COMPSECX` < `SECESTX`)

Summary statistics for bondholders and nonbondholders are reported in Table 1. Since bondholders represent a fairly small fraction of the total sample, I include all bondholder observations in the bondholder aggregate but take a random sample of the nonbondholder observations in order to equalize sample size.

Table 1: Summary statistics for bondholders and nonbondholders (Per capita, 2009 dollars)

	Bondholders		Nonbondholders	
	Mean	SD	Mean	SD
Consumption	2,326	176	1,624	89.99
Hours worked	41.10	1.19	40.55	0.72
Disposable income	76,789	5,541	51,901	3,958
Output less consumption	80,951	5,154	53,003	3,655
Observations	55,847		336,344	
Households	16,959		125,895	

5 Aggregate-Level Replication

In this section, I compute the nominal and real interest rates implied by the Euler equation using the VAR estimates from the full sample (1960:I to 2015:II) of the aggregate series described in subsection 4.1. As in Collard and Dellas (2012), I take the discount rate β to be 0.9926 (so that households discount at an annual rate of 3 percent) and the coefficient of risk aversion α to be 2. I look at four specifications of Collard and Dellas's utility model (3):

$$u(C_t, C_{t-1}, l_t) = \frac{[(C_t/C_{t-1}^{\phi})^{\nu} l_t^{1-\nu}]^{1-\alpha}}{1-\alpha}$$

where ϕ is the habit persistence parameter and ν is the weight of consumption relative to leisure.

Table 2: Utility specifications

Spec	ϕ	ν	Description
SEP	0	1	CRRA
SEP + HP	0.8	1	Habit formation
NSEP	0	0.34	Nonseparable in consumption and leisure
NSEP + HP	0.8	0.34	Habit formation + nonseparable in consumption and leisure

In Figure 1, I show the paths of the observed and implied rates for the benchmark SEP specification. Plots of the other three utility specifications are found beginning with Figure 6(a) in the appendix. In the SEP case, as in the others, both the real and nominal implied interest rates appear to move in the opposite direction from the respective observed rates. These implied rate paths are generally consistent with those computed by both CCD and Collard and Dellas. Interestingly, though in my results the models with habit formation (SEP + HP and NSEP + HP) do generate the most volatile implied rate paths, the magnitudes of the standard deviations are not as large as those reported by both previous papers.

Real rates ($\rho = 0.197$) Nominal rates ($\rho = 0.520$)

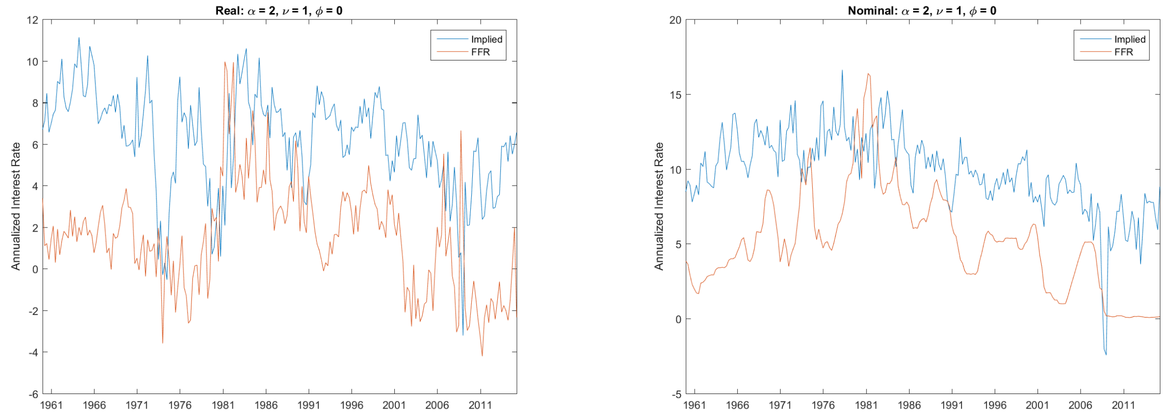


Figure 1: SEP implied vs. observed rates

5.1 Correlation of implied and observed rates

Summary statistics for the implied rates under each specification are reported below in Table 3, as well as the correlation between each implied rate and the effective federal funds rate.

The main result that stands out is the presence of strong positive correlations overall, but especially for nominal rates in the specifications without habit formation. Notably, the correlation between the nominal rate implied by CRRA preferences (SEP) and the historical nominal effective federal funds rate is 0.52, while adding nonseparability in leisure (NSEP) increases this correlation to 0.707. In the utility models with habit formation (SEP + HP and NSEP + HP), the correlation is less positive for nominal rates and essentially zero for real rates. These findings seem very much at odds with the qualitative appearance of the observed and implied rates moving in opposite directions.

These strong positive correlations are noticeably higher than the still-positive correlations found by Collard and Dellas, to say nothing of the strongly negative values found by Canzoneri et al.

Table 3: Summary statistics for nominal and real rates (annualized rates)

	Data	SEP	SEP + HP	NSEP	NSEP + HP
Real interest rates					
Mean	1.50	6.19	4.87	5.16	4.72
SD	2.54	2.36	6.97	1.39	3.10
Min	-4.21	-3.21	-22.10	0.03	-8.73
Max	9.95	11.13	25.71	8.24	15.04
Corr	—	0.197	-0.050	0.261	0.033
Nominal interest rates					
Mean	5.11	9.8	8.48	8.77	8.33
SD	3.39	2.61	6.78	2.14	3.19
Min	0.07	-2.43	-21.31	0.25	-3.51
Max	16.37	16.61	28.38	13.84	19.6
Corr	—	0.52	0.142	0.707	0.39

(2007). As a check, I reestimate the VAR and recompute the implied rates and correlations using only the time period spanned by Collard and Dellas, stopping at 2006:IV instead of 2015:II. The correlations for nominal rates from this restricted sample more closely resemble their results, though the ones for real rates are still rather different. I summarize the restricted sample results in Table 9 in the appendix. In Table 4, I compare the full and restricted sample correlations to those found in the other two papers. Note that the CCD paper examines several utility specifications, including CRRA (SEP) and Fuhrer habit preferences (SEP + HP), but does not include analysis of nonseparability in leisure.

The extreme variation in correlations found suggests two points at which this analysis is not sufficiently robust.

First, comparing the correlations computed from the full sample to those from the restricted sample highlights the impact of the inclusion of the additional quarters from 2007:I to 2015:II. Scatter plots for both samples are shown in Figure 2. (In particular, the data points in the

Table 4: Comparison of correlations between implied rates and effective FFR

	SEP	SEP + HP	NSEP	NSEP + HP	Start	End
Real interest rate correlation						
Full Sample	0.197	-0.050	0.261	0.033	1960:I	2015:II
Restricted Sample	0.020	-0.098	0.065	-0.058	1960:I	2006:IV
Collard and Dellas (2012)	0.05	0.15	0.28	0.27	1960:I	2006:IV
Canzoneri et al. (2007)	-0.37	-0.07	—	—	1966:I	2003:IV
Nominal interest rate correlation						
Full Sample	0.520	0.142	0.707	0.390	1960:I	2015:II
Restricted Sample	0.255	0.030	0.563	-0.225	1960:I	2006:IV
Collard and Dellas (2012)	0.26	0.04	0.63	0.38	1960:I	2006:IV
Canzoneri et al. (2007)	0.20	-0.10	—	—	1966:I	2003:IV

restricted sample plot are not a subset of those in the full sample plot because the implied rates for each were computed using different VAR estimates.) The difference between the two samples is of course the era of near-zero interest rates following the Great Recession in 2008, which can be seen in the full sample plot as the cluster of observations on the FFR = 0 line. These, along with the outliers in the bottom left (which are also at the zero lower bound), drive the more strongly positive correlation in the full sample.

Even within the same time span, comparing the restricted sample correlations to those of Collard and Dellas highlights the fragility of these results with respect to small changes in methodology. I follow their specifications as closely as possible, except where it is not possible or not completely clear what they did. As mentioned in the previous section, due to lack of availability of data, I generate aggregate real consumption from the chain quantity indices scaled by the 2009 nominal consumption, while they use real consumption directly. I also estimate the VARs using log of gross quarterly inflation and interest rates π_t and ffr_t, while it is possible that Collard and Dellas may have used annualized rates and/or scaled them to units of percentage points. Other possible

Full sample ($\rho = 0.520$) Restricted sample ($\rho = 0.255$)

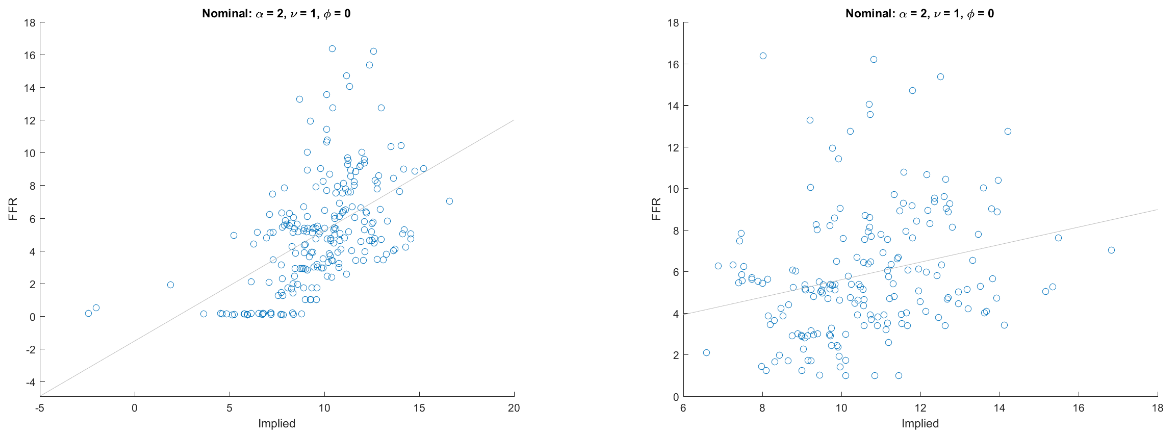


Figure 2: SEP implied vs. observed nominal rates

differences include our choices of base year (2009 in my analysis, versus 2000) and whether we take the natural log of real dollars (as I do) or billions of real dollars.

All of this is to say that the correlation between the implied and observed rates is probably not the most reliable metric by which we should judge the fit of the consumption Euler equation to the data — even though it's arguably the focus of both of these previous papers. A more consistent metric of fit is the correlation of the spread with the stance of monetary policy, which I discuss next.

5.2 Response of spread to monetary policy

It is clear that the implied and observed rates differ in level. In Table 3, we see that the mean implied interest rate in each of the four utility specifications is noticeably higher than the mean ex post rate, in both the real and nominal cases. The lowest mean implied rates come from the full NSEP + HP utility model, with averages of 4.72 and 8.33 for the real and nominal rates respectively, compared to averages of 1.50 and 5.11 observed in the data in the same span.

If the model implied interest rates differed from the observed rates only by a constant factor, this discrepancy would be less worrisome. In practice — for example, in larger macroeconomic models where the interest rate is assumed to be given by the Euler equation — it would be easy to adjust for this constant "measurement error". However, as I show, and as CCD also find, this spread is in fact not constant, but correlated with the stance of monetary policy. In this aspect, my results strongly resemble those of CCD, as well as what one might expect from visually inspecting the plot of observed and implied rates.

I define the spread as the model implied interest rate less the observed rate. For each of the four utility specifications, I regress the spread on the effective (nominal) federal funds rate, as well as on four lags of itself. That is, I estimate using least squares the linear model

$$\text{spread}_t = b_0 + b_1 \text{FFR}_t + \sum_{j=1}^{4} a_j \text{spread}_{t-j} \tag{7}$$

where all rates are annualized and given in units of percentage points. The estimated coefficients and standard errors for the federal funds rate only are reported below in Table 5. The FFR coefficients b_1 are negative for both real and nominal spreads under all four utility specifications. They are all significant at below the 1 percent level. This suggests that the difference between implied and observed rates tends to increase during periods of monetary expansion (lower FFR) and decrease during periods of monetary tightening.

These b_1 estimates match the negative coefficients estimated by CCD in direction, and are similar in magnitude. I also repeat this regression analysis using VAR estimates and implied interest rates generated using a restricted sample from 1966:I to 2003:IV, the time range used in their paper. I compare the estimates from the full and restricted samples to CCD in Table 10 in the appendix.

Unlike the correlation between implied and observed rates, the results of regressing spread on

Table 5: Response of spread to nominal FFR

	SEP	SEP + HP	NSEP	NSEP + HP
Real interest rates				
Coef	-0.307	-0.906	-0.312	-0.623
SE	(0.050)	(0.165)	(0.044)	(0.093)
Nominal interest rates				
Coef	-0.273	-0.954	-0.282	-0.640
SE	(0.043)	(0.161)	(0.036)	(0.089)

the FFR do not change drastically in using this reduced sample, though they do resemble the CCD estimates slightly more than the full sample, as one might expect. This suggests that the response of the implied-observed spread to the FFR is a more robust instrument with which to measure the fit of the implied rate to the ex post rate than the correlation alone. It is also evidence that Collard and Dellas's addition of nonseparability in consumption and leisure to the CRRA model does not in fact improve the fit of the model implied rates to the data. While adding nonseparability does increase the correlation in models both with and without habit formation, it has little discernible effect on the estimated coefficient b_1.

As CCD discuss, these results indicate that something fundamental is not being captured by the consumption Euler equation, which leads to a difference between implied and observed rates that is not constant but systematically correlated with the stance of monetary policy. They also suggest an explanation for the divergence of the two series: from (5), we see that the Euler equation implied rate increases in expected consumption growth. Expected consumption growth has been shown empirically to increase in response to monetary expansion (lower FFR), causing the implied rate to increase as well — the opposite direction of movement from the actual interest rate.

I check this hypothesis by computing the orthogonalized impulse response of the model implied rate to a monetary shock. I compare the responses of the SEP implied real interest rate and the real FFR in Figure 3. The dashed lines indicate the 95 percent confidence interval, estimated from 1000 bootstrap simulations using Kilian (1998)'s bootstrap code. The results under the other three specifications were nearly identical, as were all of the nominal rate responses. As anticipated, a negative shock to real FFR causes a positive response in the model implied rate.

Real FFR | SEP Implied Real Rate

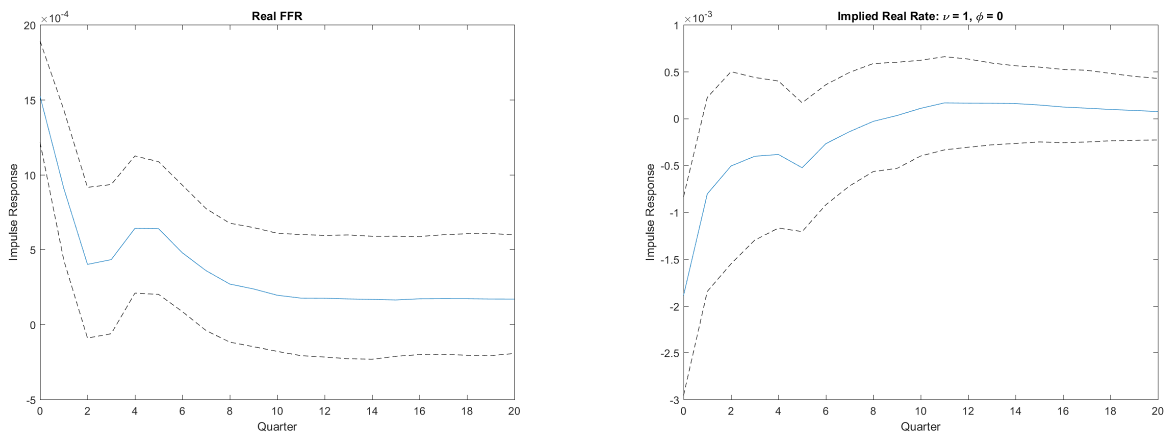


Figure 3: Impulse response to a monetary policy shock

Overall, it seems that despite the strong positive correlations between observed and implied rates that I find in each specification of utility, the path of the Euler equation implied interest rate, computed in this way, still deviates remarkably from the path of ex post rates. The addition of nonseparability in consumption and leisure that Collard and Dellas introduce does not change the implied rate's positive impulse response to the monetary policy shock, so by extension it also doesn't improve the estimated negative FFR coefficient in the spread regression (7).

6 Effect of Bond Market Participation

Next, I follow the same framework in computing Euler equation implied interest rates for both bondholders and nonbondholders. I estimate two VARs using the time series aggregated from the two groups as described in subsection 4.2, using CEX data from 1996:I to 2012:IV (68 quarters). As before, I set the discount factor β to be 0.9926.

In this section, I only compute implied interest rates using CRRA utility — that is, $\phi = 0$ and $\nu = 1$. The four utility specifications I evaluated in the previous section produced results that were qualitatively very similar: positive correlations between implied and observed rates, but spreads negatively correlated with the stance of monetary policy. I choose to single out CRRA utility (SEP) because it is the simplest model, which nonetheless gives the least negative FFR coefficient in the spread regression (see Table 5). This suggests that among the four utility models, the spread between observed and implied rates is least systematically related to monetary policy under the assumption of CRRA utility.

In the previous section, following Canzoneri et al. (2007) and Collard and Dellas (2012), I had $\alpha = 2$. Here, in contrast, I set the coefficient of relative risk aversion α to be 0.2. The consumption, income, and leisure paths that I aggregate from household-level CEX data are much more volatile

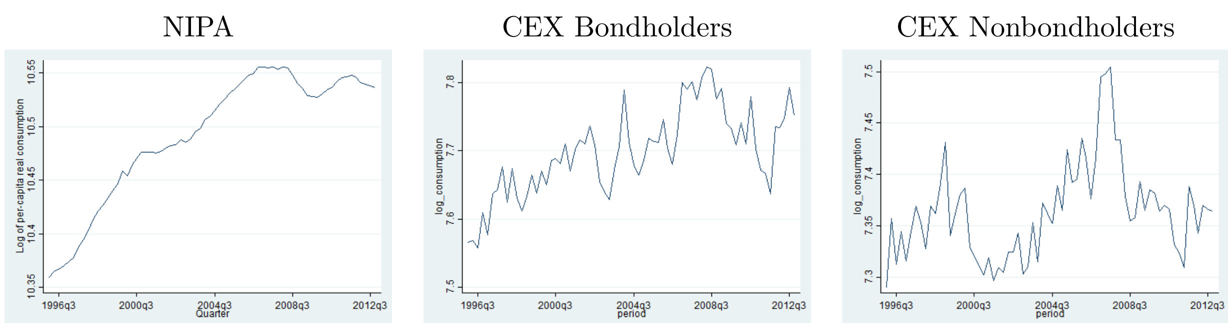


Figure 4: Comparison of log of real consumption c_t from 1996:I to 2012:IV

than the nationwide aggregates from NIPA that I use in the replication. In Figure 4, I compare the log of real consumption c_t generated from NIPA to those of CEX bondholders and nonbondholders. The paths of the other series (leisure fraction, log of real disposable income, and log of output less consumption) are similarly more volatile when aggregated from household-level data.

I argued previously that the Euler equation implied rate is proportional to expected consumption growth. From (5), it also increases in the risk aversion coefficient α:

$$\frac{1}{1+r_t} = \beta \exp\left(-\alpha\left[\mathbb{E}_t c_{t+1} - c_t\right] + \text{constant terms}\right)$$
$$\frac{1}{1+i_t} = \beta \exp\left(-\alpha\left[\mathbb{E}_t c_{t+1} - c_t\right] - \mathbb{E}_t \pi_{t+1} + \text{constant terms}\right)$$

In order to achieve implied interest rates approximately in the range we see in real life, a much lower risk aversion is needed to offset the large consumption growth fluctuations in the aggregated CEX data. (Net consumption growth is approximately $\log\left(\frac{C_t}{C_{t-1}}\right) = c_t - c_{t-1}$.) Since the CEX consumption growth fluctuations are about ten times larger than those seen in the NIPA data in the same time span, I choose α to be ten times smaller than in the previous section, giving $\alpha = 0.2$.

Comparing the interest rate paths qualitatively in Figure 5(a) and Figure 5(b) shows how noisy the implied rates are in this specification. The risk aversion coefficient $\alpha = 0.2$ implies an elasticity of intertemporal substitution of $\frac{1}{\alpha} = 5$, which is extremely high and well outside the range of estimated elasticities. Guvenen (2006) cites elasticity values in the literature in the range of 1 for stockholders and 0.1 for nonstockholders. This suggests that the volatility of the consumption paths aggregated from the CEX is mostly the result of the small sample size and is not necessarily representative of the actual variation in households' consumption choices.

Bondholders ($\rho = 0.235$) Nonbondholders ($\rho = 0.133$)

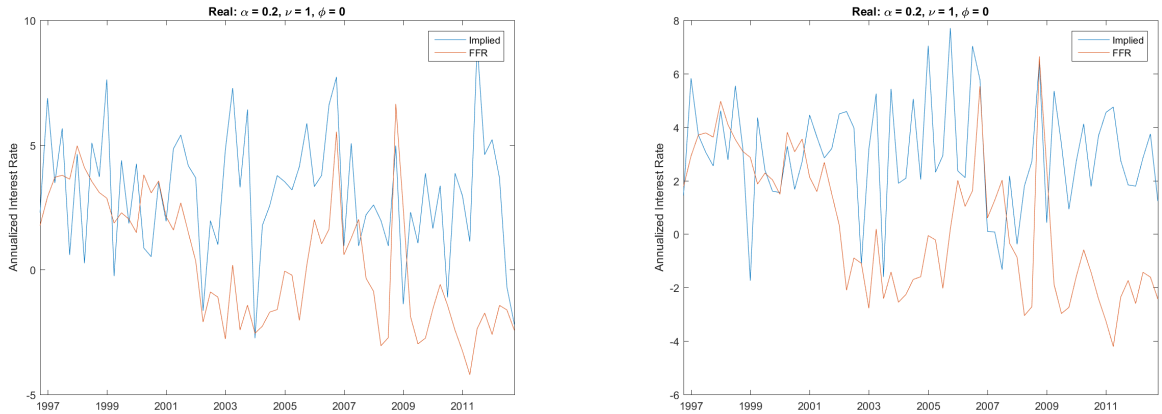


Figure 5(a): Implied vs. observed real rates

Bondholders ($\rho = 0.193$) Nonbondholders ($\rho = 0.028$)

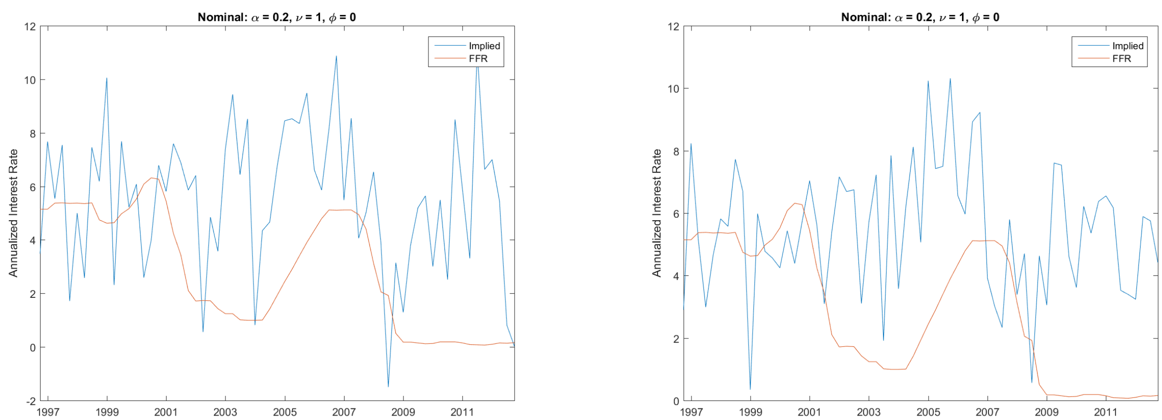


Figure 5(b): Implied vs. observed nominal rates

6.1 Correlation of implied and observed rates

In Table 6, I report summary statistics for the interest rates implied by the consumption, income, and leisure paths of bondholders and nonbondholders. As in the aggregate analysis, the standard deviations of the implied rates are similar to those seen in the data, while the levels are higher

Table 6: Summary statistics for nominal and real rates (annualized rates)

	Data	CEX Bondholders	CEX Nonbondholders
Real interest rates			
Mean	0.35	3.06	2.96
SD	2.58	2.50	2.06
Min	-4.21	-2.74	-1.74
Max	6.64	9.05	7.70
Corr	—	0.235	0.133
Nominal interest rates			
Mean	2.83	5.52	5.44
SD	2.19	2.69	2.08
Min	0.07	-1.50	0.35
Max	6.32	11.05	10.32
Corr	—	0.193	0.028

overall. The correlation between implied and ex post rates is higher in both the real and nominal cases for bondholders (0.235 and 0.193 respectively) than for nonbondholders (0.133 and 0.028). This could suggest that bondholders — households who actually have a position in the bond market — are more likely to adjust their consumption paths in response to changes in the interest rate than are nonbondholders. If this were the case, the interest rates implied by their consumption paths would more resemble observed rates.

To test whether the correlation is significantly higher for bondholders than for nonbondholders, I use Fisher's transformation. First, I transform each correlation ρ by

$$\rho' = \frac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right)$$

Then the difference between the transformed correlations, $\rho'_{BH} - \rho'_{NBH}$, is normally distributed with standard error $\sqrt{\frac{1}{n_{BH}-3} + \frac{1}{n_{NBH}-3}}$. I report these estimates in Table 7. The difference between correlations for bondholders and nonbondholders is not significant for both real and nominal interest

Table 7: Test of difference in correlations between bondholders and nonbondholders

	ρ_{BH}	ρ_{NBH}	$\rho'_{BH} - \rho'_{NBH}$	SE
Real interest rates	0.235	0.133	0.106	0.180
Nominal interest rates	0.193	0.028	0.168	0.180

rates.

One explanation for the lack of significance may be the “fuzziness” inherent in determining bondholder status. As I discussed in subsection 4.2 and as Vissing-Jorgensen (2002) also addresses, I am limited by the questions asked of CEX households. The households that I classify as bondholders may, for example, erroneously include stockholders who are not also bondholders, while excluding households that hold bonds through their pension plans. This problem may be resolved using data that makes a clearer distinction between bondholders and nonbondholders.

6.2 Response of spread to monetary policy

So much for the correlations between the implied and observed rates. Since I argued that the correlation is not necessarily the most reliable indicator of fit, it is reasonable to again check how the spread between implied and observed rates depends on the stance of monetary policy. To test whether the FFR coefficient on spread is significantly different for bondholders and nonbondholders, I estimate the linear model

$$\text{spread}_t = b_0 + b_1\text{FFR}_t + b_2\text{bondholder} + b_3\text{bondholder} \times \text{FFR}_t + \sum_{j=1}^{4} a_j \text{spread}_{t-j} \qquad (8)$$

The estimated coefficients and standard errors for the non-lag terms are reported in Table 8. The FFR coefficients for real spreads are $b_1 = -0.985$ for nonbondholders and $b_1 + b_3 = -0.885$ for bondholders; for nominal spreads they are -1.031 for nonbondholders and -0.862 for nonbondholders. The difference in slopes b_3 is not significant for either real or nominal spreads.

Table 8: Response of spread to nominal FFR for bondholders and nonbondholders

	FFR	Bondholder	Bondholder × FFR
		Real spreads	
Coef	-0.985	-0.412	0.188
SE	(0.235)	(0.814)	(0.222)
		Nominal spreads	
Coef	-1.031	-0.359	0.169
SE	(0.220)	(0.735)	(0.201)

As in the aggregate analysis, the negative FFR coefficients themselves are significant at below the 1 percent level. This is further evidence that the consumption Euler equation (in the form that I have examined) does not adequately describe the relationship between expected consumption growth and the interest rate. Separating out bondholders, who are perhaps most likely to optimize according to the Euler equation, does not improve the negative relationship between the spread and the federal funds rate.

7 Conclusion

My findings at the aggregate level are mostly consistent with the analyses of Canzoneri et al. (2007) and Collard and Dellas (2012). I show that the paths of the observed and Euler equation implied interest rates diverge following a monetary policy shock, which causes the spread between the two to be systematically related to the stance of monetary policy. Seemingly at odds with this result, I do find strong positive correlations between the implied and observed rates. However, I demonstrate that these correlations are not a robust measurement of the fit of the consumption Euler equation to the data. They are extremely sensitive to the choice of time period, as well as other relatively small methodological changes. Unlike Collard and Dellas, I do not conclude that introducing nonseparability in consumption and leisure to the utility model substantially improves the fit: though the implied rates under nonseparability are correlated more positively with observed rates, there is virtually no effect on the more reliable measure, the response of the spreads to the federal funds rate.

One possible weakness in this methodology is the assumption that the dynamics of the system (including consumption, inflation, income, and the federal funds rate) can be modeled by a vector autoregression. VARs are favored for being simple and model-agnostic, but they may not be sophisticated enough to accurately estimate the conditional moments used in computing the implied rates. A future researcher could try estimating the conditional moments from a larger-scale model such as the Smets and Wouters (2007) DSGE model (though this presents an endogeneity problem, as the Smets and Wouters model critically assumes a version of the consumption Euler equation).

At the household level, distinguishing between bondholders and nonbondholders in computing implied rates gives results that qualitatively resemble what one might expect: that bondholders optimize their consumption in response to interest rate changes as the Euler equation predicts, while nonbondholders' behavior is less well described by the Euler equation. Compared to nonbondholders, I find that the interest rates implied by bondholders' consumption paths are more strongly correlated with ex post rates, and the effect of the federal funds rate on the spread, while still significant and negative, is weaker. However, neither the difference in correlations nor the difference in coefficients between bondholders and nonbondholders is statistically significant. This may result in part from the imperfect determination of which households constitute bondholders, which is limited by the questions asked in the CEX. Another potential explanation is the high volatility of consumption growth in the data aggregated from the household level, which may reflect the relatively small sample size for bondholders more than the actual variation experienced by consumers. Further investigation is needed, possibly involving examining additional data or bootstrapping the sample of bondholders to achieve more observations.

These findings continue to pose a challenge to the Euler equation's privileged role in macroeconomic estimation and forecasting. While it is a useful and elegant abstraction, a wealth of empirical evidence has accumulated against its predictions. In particular, my conclusion that the spread between implied and historical rates varies systematically with the federal funds rate is consistent across all utility models, and holds for data aggregated from both bondholders and nonbondholders. However, though the results from this paper are not conclusive, it is possible that with better data, we may be able to explain part of this discrepancy as resulting from heterogeneity in bond market participation.

8 Data appendix

8.1 Aggregate-level data

First, I describe the construction of the endogenous variables (prior to taking logs) used in the aggregate-level replication.

Per capita real consumption C_t: Aggregate real consumption is defined as the sum of the chain quantity indices (2009 = 100) for personal consumption expenditures on nondurable goods (`DNDGRA3Q086SBEA`) and services (`DSERRA3Q086SBEA`), all multiplied by the sum of nominal nondurables (`PCEND`) and services (`PCESV`) consumption in 2009[7]. This amount is divided by the civilian noninstitutional population (`CNP16OV`) to get per capita real consumption.

Gross quarterly inflation Π_t: In each quarter, the implicit price deflator P_t is calculated by dividing aggregate nominal consumption (`PCEND + PCESV`) by aggregate real consumption (described above). Then gross quarterly inflation is defined as the growth rate of the deflator: $\Pi_t = \frac{P_t}{P_{t-1}}$.

Leisure fraction l_t: Labor fraction h_t is defined as the average weekly hours worked in the nonfarm business sector (`PRS85006023`), multiplied by the civilian employment-to-population ratio (`EMRATIO`). Following Collard and Dellas, I then rescale so that the mean over all quarters is $\frac{1}{3}$, corresponding to an average of 8 hours worked per weekday. Then the leisure fraction is given by $l_t = 1 - h_t$.

Per capita real disposable income RDI_t: This is computed by dividing real disposable income (`DPIC96`) by the civilian non-institutional population.

Per capita real output less consumption YMC_t: Defined as real gross domestic product (`GDPC96`)

---

[7] I generate aggregate real consumption from the chain quantity indices because the real consumption variables used by Collard and Dellas, `PCNDGC96` and `PCESVC96`, were not available from FRED for the quarters before 1999:I.

minus aggregate real consumption, again divided by the civilian noninstitutional population.

Gross quarterly effective federal funds rate FFR_t: This is computed by raising the gross annualized rate (`DFF`) to the one-fourth power.

Continuous Commodity Index CCI_t: I use the CCI ending price on the first day of each quarter, obtained from Bloomberg. As mentioned, the CCI is the continuation of the CRB Index used by Canzoneri et al. (2007) and Collard and Dellas. What is called the CRB Index today is calculated slightly differently and exists only since 1995.

8.2 Household-level data

To construct equivalent series from the CEX data, I first generate the following observation-level (nominal) variables:

Consumption: Following Heathcote et al. (2010), I define consumption of nondurable goods and services as the sum of the following expenditure categories: food and beverages (`FOOD` + `ALCBEV`), clothing (`APPAR`), gasoline (`GASMO`), household operation (`HOUSOP`), public transportation (`PUBTRA`), medical care excluding health insurance (`HEALTH` − `HEALTHIN`), recreation (`ENTERT`), tobacco (`TOBACC`), and education (`READ` + `EDUCA`).

Hours worked: I use the weekly hours worked by the household's reference person (`INC_HRS1`). The reference person is the first person mentioned by the survey respondent when asked to "Start with the name of the person or one of the persons who owns or rents the home."

Disposable income: I use after-tax income (`FINCATAX`), as in Hai et al. (2015).

Output less consumption: Defined as before-tax income (`FINCBTAX`) minus consumption (defined above).

Consumption, disposable income, and output less consumption are each deflated by the unadjusted Consumer Price Index for nondurables for urban consumers (`CUUR0000SAN` in FRED), following Vissing-Jorgensen, rescaled to 2009 dollars to correspond with the aggregate-level data. The expenditure categories included in consumption were chosen to allow for the possibility of deflating each category by its own CPI (for example, `CPIFABNS` from FRED for food and beverages). However, the result of doing so was found to differ only negligibly from using a single CPI.

The CEX provides population weights for each household, which are calibrated so that summing the population weights in a given quarter approximates the number of households in the United States that quarter, while taking the weighted sum of the number of household members approximates the total population. I take the weighted mean of hours worked for each quarter and use it to generate labor fraction l_t as in the previous section. For each of consumption, disposable income, and output less consumption, I take the weighted sum each quarter and divide it by the population to get per capita variables C_t, RDI_t, and YMC_t.

Finally, I seasonally adjust log consumption c_t by regressing it on indicators of the quarters and subtracting off the non-first quarter coefficients.

9 Additional Tables and Figures

Table 9: Summary statistics for nominal and real rates (annualized rates)

Collard and Dellas (2012) sample (1960:I to 2006:IV)

	Data	SEP	SEP + HP	NSEP	NSEP + HP
			Real interest rates		
Mean	1.98	6.73	5.21	5.49	4.98
SD	2.30	2.17	7.73	1.29	3.37
Min	-3.59	-0.18	-18.35	1.25	-7.96
Max	9.95	11.92	26.92	8.86	16.69
Corr	—	0.020	-0.098	0.065	-0.058
			Nominal interest rates		
Mean	5.87	10.62	9.10	9.39	8.88
SD	3.05	1.85	7.48	1.71	3.33
Min	1.00	6.58	-7.47	5.78	2.63
Max	16.37	16.83	30.87	14.03	21.60
Corr	—	0.255	0.030	0.563	0.225

Table 10: Comparison of responses of spread to monetary policy

(Standard errors in parentheses)

	SEP	SEP + HP	NSEP	NSEP + HP	Start	End
			Real interest rate spreads			
Full Sample	-0.307	-0.906	-0.312	-0.623	1960:I	2015:II
	(0.050)	(0.165)	(0.044)	(0.093)		
Restricted Sample	-0.545	-1.054	-0.432	-0.734	1966:I	2003:IV
	(0.069)	(0.226)	(0.057)	(0.130)		
Canzoneri et al. (2007)	-0.482	-1.215	—	—	1966:I	2003:IV
	(0.064)	(0.825)				
			Nominal interest rate spreads			
Full Sample	-0.273	-0.954	-0.282	-0.640	1960:I	2015:II
	(0.043)	(0.161)	(0.036)	(0.089)		
Restricted Sample	-0.545	-1.134	-0.433	-0.796	1966:I	2003:IV
	(0.064)	(0.223)	(0.050)	(0.128)		
Canzoneri et al. (2007)	-0.357	-1.035	—	—	1966:I	2003:IV
	(0.047)	(0.826)				

Real rates ($\rho = -0.050$)

Nominal rates ($\rho = 0.142$)

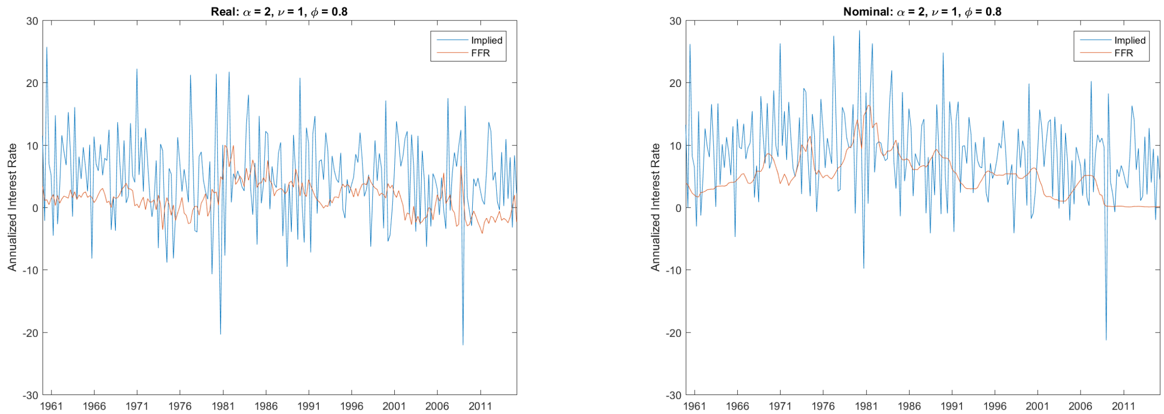


Figure 6(a): SEP + HP implied vs. observed rates

Real rates ($\rho = 0.261$)

Nominal rates ($\rho = 0.707$)

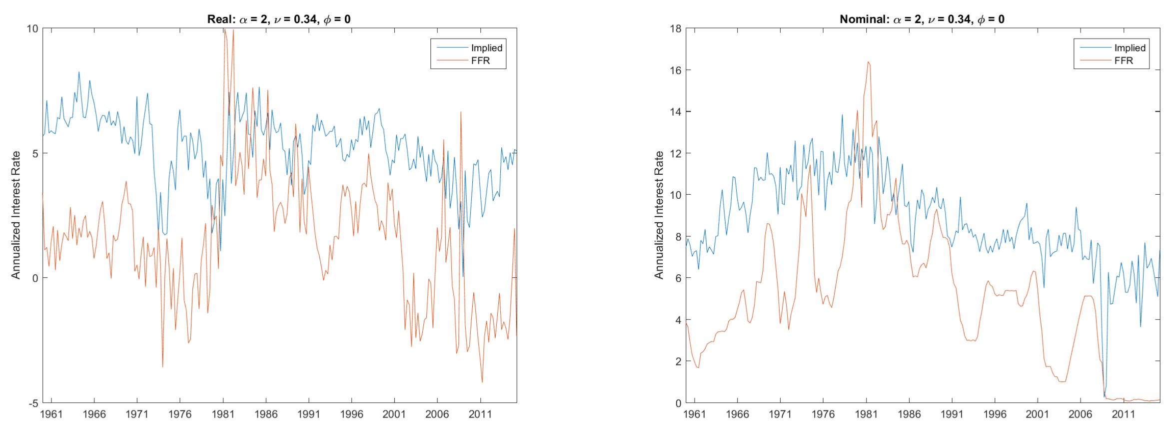
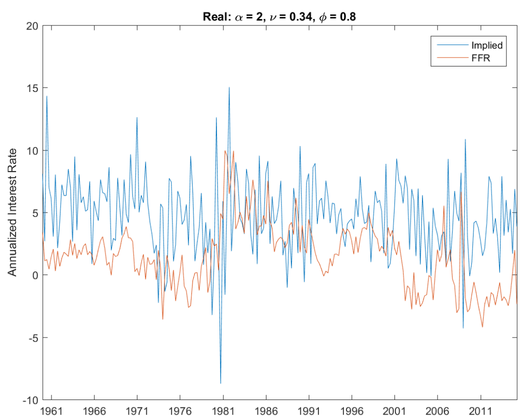


Figure 6(b): NSEP implied vs. observed rates

Real rates ($\rho = 0.033$)



Nominal rates ($\rho = 0.390$)

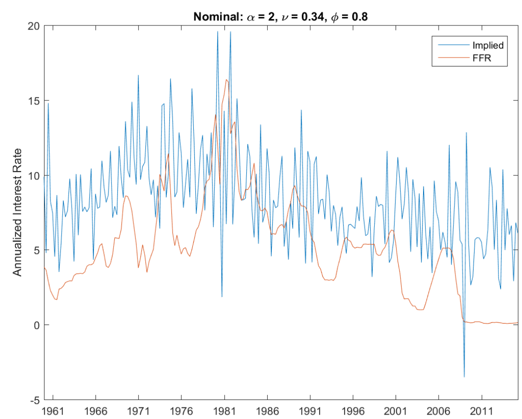


Figure 6(c): NSEP + HP implied vs. observed rates

10 References

Basu, Susanto and Miles S. Kimball (2002) "Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption," `http://www-personal.umich.edu/~mkimball/pdf/cee_oct02-3.pdf`.

Campbell, John Y. and N. Gregory Mankiw (1989) "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual*.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba (2007) "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics*.

Collard, Fabrice and Harris Dellas (2012) "Euler Equations and Monetary Policy," *Economics Letters*.

Fuhrer, Jeffrey C. (2000) "Habit Formation in Consumption and Its Implications for Monetary Policy Models," *American Economic Review*.

Gareis, Johannes and Eric Mayer (2013) "Euler Equations and Money Market Interest Rates: The Role of Monetary Policy and Risk Premium Shocks," *Economics Letters*.

Guvenen, Fatih (2006) "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics*.

Hai, Rong, Dirk Krueger, and Andrew Postlewaite (2015) "On the Welfare Cost of Consumption Fluctuations in the Presence of Memorable Goods," `http://economics.sas.upenn.edu/~dkrueger/research/Memories.pdf`.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante (2010) "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*.

Kilian, Lutz (1998) "Small-Sample Confidence Intervals For Impulse Response Functions," *The Review of Economics and Statistics*.

Parker, Jonathan A. (1999) "The Consumption Function Re-Estimated," `http://japarker.scripts.mit.edu/docs/cfunc1199.pdf`.

Smets, Frank and Rafael Wouters (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*.

Smith, Noah (2014) "The Equation at the Core of Modern Macro," `http://noahpinionblog.blogspot.com/2014/01/the-equation-at-core-of-modern-macro.html`.

Vissing-Jorgensen, Annette (2002) "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*.