Work in Progress: The Euler Equation Implied Rate Under Heterogeneous Preferences

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Literature

- Hara (2009): aggregating heterogeneous discount rates and risk attitudes into a representative agent
- Tesfatsion (2006): agent-based computational modeling

Raw Data

- Quarterly data from 1960:Q1 to 2015:Q2
- FRED (Federal Reserve Economic Data from the St. Louis Fed)
  - Civilian noninstitutional population
  - Effective fed funds rate
  - Employment ratio
  - Weekly hours worked (non-farm)
  - Nominal consumption of nondurable goods
  - Nominal consumption of services
  - Real disposable income
  - Real GDP
  - Chain quantity index: nondurable goods
  - Chain quantity index: services
- Continuous Commodity Index

Generated Series

- Nominal consumption$_t$ = nondurable goods$_t$ + services$_t$
- Real consumption$_t$ (in chained 2009 dollars) = 2009 nominal consumption $\times$ chain quantity index$_t$
- Consumption deflator$_t$ = $\frac{\text{nominal consumption}_t}{\text{real consumption}_t}$
- Gross inflation$_t$ = $\frac{\text{consumption deflator}_t}{\text{consumption deflator}_{t-1}}$
- Labor fraction$_t$ = hours worked$_t$ $\times$ employment ratio$_t$, rescaled to have mean $\frac{1}{3}$
- Leisure fraction$_t$ = $1 -$ labor fraction$_t$

- Most series following Collard and Dellas (2012)

VAR Model

- Estimate VAR(4) (written below in companion form)
- Lag order 4 chosen for optimal likelihood ratio — as in Fuhrer (2000), Collard and Dellas (2012)

$$y_t = A_0 + A_1 Y_{t-1} + \nu_t$$

$$\nu_t \overset{\text{iid}}{\sim} N(0, \Sigma)$$

$$y_t = \begin{bmatrix} \log(\text{real consumption}_t) \\ \text{net inflation}_t \\ \text{leisure fraction}_t \\ \log(\text{real disposable income}_t) \\ \log(\text{income less consumption}_t) \\ \text{effective FFR}_t \\ \text{CCI}_t \end{bmatrix} \qquad Y_t = \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \end{bmatrix}$$

VAR Estimates

- Sample: 1960:Q2 to 2015:Q2 (221 observations)
- Log likelihood: 2282.281
- Ljung-Box statistic: 4544.5290 (p-value 0.0000)

- Conditional moments:
    - $\mathrm{E}_t Y_{t+1} = A_0 + A_1 Y_t$
    - $\mathrm{Cov}_t Y_{t+1} = \hat{\Sigma}$

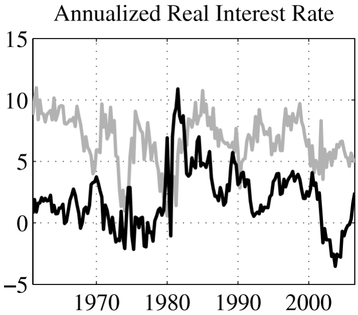
CRRA Implied Rates

- CRRA utility $u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha}$
- Euler equation $\frac{1}{1+r_t} = \beta \frac{E_t u'(C_{t+1})}{u'(C_t)} = \beta \left(\frac{E_t C_{t+1}}{C_t} \right)^{-\alpha}$
- From Canzoneri et al. (2007): assuming consumption is conditionally lognormal, real interest rate given by

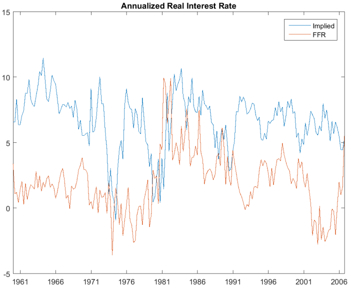
$$\frac{1}{1+r_t} = \beta \exp \left[-\alpha(E_t c_{t+1} - c_t) + \frac{\alpha^2}{2} \mathrm{Var}_t c_{t+1} \right]$$

  where c_t is log of real consumption C_t
- $\beta = 0.9926$, $\alpha = 2$ as in Collard and Dellas (2012)

CRRA Implied Rates



Collard and Dellas (2012)
(gray = implied, black = FFR)



Li (2015)

CRRA Implied Rates

	Collard and Dellas (2012)	Li (2015)
Mean implied rate	6.70	6.66
SD implied rate	2.09	2.13
Mean FFR	1.99	1.98
SD implied rate	2.51	2.30
Correlation	0.05	0.0146

References

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba (2007) "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics*.

Collard, Fabrice and Harris Dellas (2012) "Euler equations and monetary policy," *Economics Letters*.

Fuhrer, Jeffrey C. (2000) "Habit Formation in Consumption and Its Implications for Monetary Policy Models," *American Economic Review*.

Hara, Chiaki (2009) "Heterogeneous Impatience in a Continuous-Time Model," `http://www.kier.kyoto-u.ac.jp/DP/DP665.pdf`.

Tesfatsion, Leigh (2006) "Agent-Based Computational Modeling and Macroeconomics," in *Post Walrasian Macroeconomics: Beyond the Dynamic Stochastic General Equilibrium Model*: Cambridge University Press.