

# Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models

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# Overview

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- Data
- Empirical Analysis
  - Model
  - Utility Specifications
- Results
- Thesis Connection

# Questions

- How do the interest rates implied by the consumption Euler equation compare to observed historical interest rates?
- Is the spread between these two rates correlated with the stance of monetary policy?

# Background

- Consumption Euler equation expresses intertemporal first-order condition
- Typical form  $u'(C_t) = \beta(1 + r_t)E_t u'(C_{t+1})$
- Equivalently,  $\frac{\beta E_t u'(C_{t+1})}{u'(C_t)} = \frac{1}{1+r_t}$  (i.e. MRS = MRT)
- Households delay consumption when interest rates are high
- **Assumption of nearly all standard macroeconomic models**

# Data

- U.S. data from 1966-2004 (source unknown)
- Quarterly time series for:
  1. Money market interest rates
  2. Per capita consumption
  3. Inflation (log change in the deflator for nondurables and services consumption)
  4. Journal of Commerce industrial materials commodity price index
  5. Per capita real disposable income
  6. Federal funds rate
  7. Per capita real nonconsumption GDP

# Empirical Analysis

- Compute implied interest rates under 5 utility specifications
- Regress spread (model rate - observed rate) on 2 measures of monetary policy
- Estimate VAR of variables 2-7 on previous slide

$$Y_t = A_0 + A_1 Y_{t-1} + \nu_t$$

- Compute response of model rate to monetary policy shock under 2 sets of preferences and compare to impulse response of FFR

# Model

- Representative agent maximizes lifetime utility

$$u_t = \sum_{s=1}^{\infty} \beta^{s-t} E_t u(C_s, Z_s)$$

where  $Z_s$  is the habit or reference level of consumption in period  $s$

- Real interest rate  $r_t$  given by

$$\frac{1}{1 + r_t} = \frac{\beta E_t u'(c_{t+1})}{u'(c_t)}$$

# Utility Specifications

1. Standard, additively separable, constant relative risk aversion (CRRA):  $u(C_t) = \frac{1}{1-\alpha} C_t^{1-\alpha}$
2. Fuhrer (2000): utility depends on ratio  $\frac{C_t}{C_{t-1}}$
3. Christiano, Eichenbaum, & Evans (2005): utility depends on difference  $C_t - bC_{t-1}$
4. Abel (1990, 1999): utility depends on  $\frac{C_t}{Z_t}$ , where reference level  $Z_t$  is aggregate consumption last period (external)
5. Campbell & Cochrane (1999): utility depends on  $C_t - Z_t$ , where  $Z_t$  is again external



# Empirical Analysis

- Compute implied interest rates under 5 utility specifications

# Results

## Computed Implied Rates

	Observed	CRRA	Fuhrer	Christiano- Eichenbaum- Evans	Abel	Campbell- Cochrane
Mean	2.32	7.08	5.66	2.10	8.34	2.20
Std Dev	2.39	1.66	31.25	7.39	26.55	1.64
Correlation		-0.37	-0.07	-0.09	-0.36	-0.37

- Interest rates implied by Euler equation negatively correlated with observed historical rates
- Negative correlation robust to specification of different utility functions
- Effect is weaker for preferences that imply the Euler equation is extremely volatile

# Results

Figure 1: Real Interest Rates  
Ex Post and CRRA Euler Equation

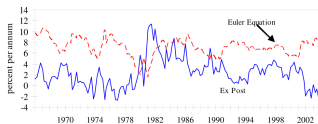


Figure 2: Real Interest Rates  
Ex Post and Fuhrer Euler Equation

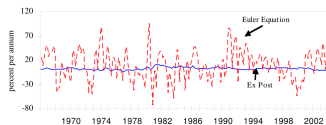


Figure 3: Real Interest Rates  
Ex Post and C-E-E Euler Equation



Figure 4B: Real Interest Rates  
Ex Post and Abel Model

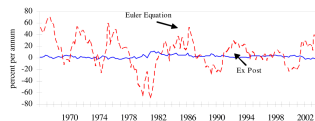
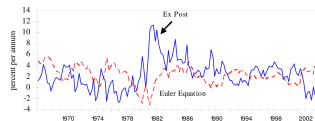


Figure 5: Real Interest Rates  
Ex Post and Campbell-Cochrane Model



# Empirical Analysis

- Compute implied interest rates under 5 utility specifications
- Regress spread (model rate - observed rate) on 2 measures of monetary policy

# Results

## Response of Interest Rate Spreads to Monetary Policy (standard errors in parentheses)

	CRRA	Fuhrer	Christiano- Eichenbaum- Evans	Abel	Campbell- Cochrane
FFR	-0.482 (0.064)	-1.215 (0.825)	-0.586 (0.214)	-1.714 (0.103)	-0.482 (0.062)
S-Ratio	0.062 (0.015)	0.263 (0.214)	0.079 (0.052)	0.346 (0.103)	0.015 (0.015)

- $S\text{-ratio} = \frac{\text{non-borrowed reserves} + \text{extended credit}}{\text{total reserves}}$
- Expansionary monetary policy (lower FFR, higher S-ratio) associated with significantly greater spreads

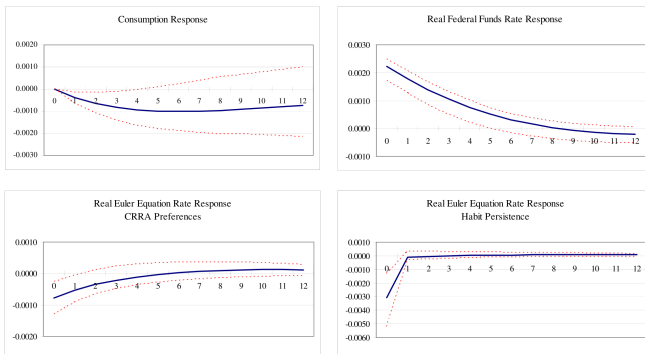
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## Impulse Response Functions for Federal Funds Rate Shock



- Euler equation rates and FFR move in opposite directions
- Monetary tightening (raising FFR) decreases expected consumption growth, which is associated with decreasing real interest rates

# Thesis Connection

- Want to test DSGE microfoundations, including consumption Euler equation
- Rather than estimating system evolution with time series, will use closed-form solution of small-scale model