

LIMITED ASSET MARKET PARTICIPATION  
AND THE EULER EQUATION IMPLIED INTEREST RATE

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# 1 Introduction

Perhaps the main criticism of modern macroeconomic models (in particular, DSGE models) is that the microfoundational assumptions on which they're based often don't actually fit the data very well. Smith (2014) singles out the consumption Euler equation, which expresses intertemporal consumption choice in terms of the real interest rate  $r_t$ . In its typical form:

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[ \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right]$$

Canzoneri et al. (2007) compute the interest rate implied by the consumption Euler equation under several utility specifications. They find that their computed rates are actually negatively correlated with historical money market rates, and furthermore that the spread is correlated with the stance of monetary policy. These results are potentially extremely damaging to the validity of macroeconomic models which assume the Euler equation implied rate and the actual interest rate to be the same – that is, nearly all macro models. Collard and Dellas (2012) repeat this exercise, adding utility nonseparable in consumption and labor, and in fact find the looked-for positive correlation with observed rates.

In this paper, I first attempt to replicate the findings of Canzoneri, Cumby, and Diba (henceforth abbreviated CCD) and Collard and Dellas (2012) using new data up through the second quarter of 2015. This portion includes computing Euler equation implied rates and correlating the spread between implied and observed rates with the stance of monetary policy. The consumption and income data for this section are all national aggregates from the National Income and Product Accounts (NIPA).

The main novel contribution of this paper is the introduction of limited asset market participation to the implied rate framework, inspired by Vissing-Jorgensen (2002). Specifically, I aggregate

household-level data from the Consumer Expenditure Survey (CEX) for bondholders and nonbondholders. I perform the same analyses on the time series of these two groups to test the hypothesis that interest rates implied by bondholders' consumption paths will more resemble observed rates than those from nonbondholders. The intuition for this idea is clear: we expect households with positions in the bond market to adjust their consumption in response to changes in the interest rate, while we don't expect nonbondholders to do so.

## 2 Literature

### 3 Model

We start with the standard household problem from the neoclassical growth model. In period  $t$ , the representative consumer has preferences

$$U_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, C_{s-1}, L_s)$$

where  $\beta$  is her discount rate,  $C_s$  and  $C_{s-1}$  are real consumption today and yesterday, and  $L_s$  is fraction of leisure hours. Each period, she receives labor income with nominal wage  $W_s$  and chooses consumption and nominal holdings  $B_s$  of a risk-free one-period bond. The price of the consumption good is  $P_s$ . This gives the following period budget constraint in nominal units:

$$P_s C_s + (1 + i_{s-1}) B_{s-1} \leq W_s (1 - L_s) + B_s$$

Taking first-order conditions gives the equilibrium nominal interest rate by

$$\frac{1}{1 + i_t} = \mathbb{E}_t \left[ \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[ \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \frac{1}{\Pi_{t+1}} \right] \quad (1)$$

In real units, the period budget constraint is

$$C_s + (1 + r_{s-1}) \frac{B_{s-1}}{P_{s-1}} \leq \frac{W_s}{P_s} (1 - L_s) + \frac{B_s}{P_s}$$

and the real interest rate satisfies

$$\frac{1}{1 + r_t} = \beta \mathbb{E}_t \left[ \frac{\partial U_t / \partial C_{t+1}}{\partial U_t / \partial C_t} \right] \quad (2)$$

To compute the interest rates implied by the Euler equations (1) and (2) requires a few assumptions. We assume that real consumption  $C_t$  and gross inflation  $\Pi_t$  are conditionally lognormal. We use the functional form for utility used by Collard and Dellas (2012):

$$u(C_t, C_{t-1}, L_t) = \frac{[(C_t / C_{t-1}^\phi)^\nu L_t^{1-\nu}]^{1-\alpha}}{1 - \alpha} \quad (3)$$

where  $\alpha$  is the coefficient of relative risk aversion,  $\phi$  is the habit persistence parameter, and  $\nu$  specifies the relative weight of consumption compared to leisure. When  $\phi = 0$  (no habit persistence) and  $\nu = 1$  (utility is separable in consumption and leisure), (3) reduces to the case of constant relative risk aversion:

$$u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha} \quad (4)$$

We'll derive an expression for the implied interest rate in terms of conditional expectations and variances for the CRRA case only and leave the more general case to Collard and Dellas. We denote logs of variables using lowercase letters, i.e.  $c_t := \log C_t$  and  $\pi_t := \log \Pi_t$  (approximately net inflation). From (1), the nominal interest rate under CRRA preferences is given by:

$$\begin{aligned} \frac{1}{1+i_t} &= \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \Pi_{t+1}^{-1} \right] \\ &= \beta \mathbb{E}_t \exp [-\alpha(c_{t+1} - c_t) - \pi_{t+1}] \\ &= \beta \exp \left( \mathbb{E}_t [-\alpha(c_{t+1} - c_t) - \pi_{t+1}] + \frac{1}{2} \text{Var}_t [-\alpha(c_{t+1} - c_t) - \pi_{t+1}] \right) \\ &= \beta \exp \left( -\alpha [\mathbb{E}_t c_{t+1} - c_t] - \mathbb{E}_t \pi_{t+1} + \frac{\alpha^2}{2} \text{Var}_t c_{t+1} + \frac{1}{2} \text{Var}_t \pi_{t+1} + \text{Cov}_t(c_{t+1}, \pi_{t+1}) \right) \end{aligned} \quad (5)$$

where the third equality follows from our assumption of conditional lognormality. The expression for the real interest rate is the same, but without the inflation terms:

$$\frac{1}{1+r_t} = \beta \exp \left( -\alpha [\mathbb{E}_t c_{t+1} - c_t] + \frac{\alpha^2}{2} \text{Var}_t c_{t+1} \right)$$

From Collard and Dellas, the equivalent expression for the implied nominal rate under the more general preferences (3) is

$$\frac{1}{1+i_t} = \beta \frac{\exp(\chi_{1t}) - \beta \phi \exp(\chi_{2t})}{\exp(\chi_{3t}) - \beta \phi \exp(\chi_{4t})}$$

where

$$\begin{aligned}
\chi_{1t} &= (\nu(1-\sigma) - 1)\mathbb{E}_t c_{t+1} - \phi\nu(1-\sigma)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} - \mathbb{E}_t \pi_{t+1} \\
&\quad + \frac{(\nu(1-\sigma) - 1)^2}{2}\text{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\text{Var}_t l_{t+1} + \frac{\text{Var}_t \pi_{t+1}}{2} \\
&\quad - (1-\nu)(1-\sigma)\text{Cov}_t(c_{t+1}, l_{t+1}) + (\nu(1-\sigma) - 1)(1-\nu)(1-\sigma)\text{Cov}_t(\pi_{t+1}, l_{t+1}) \\
&\quad - (\nu(1-\sigma) - 1)\text{Cov}_t(c_{t+1}, \pi_{t+1}) \\
\chi_{2t} &= \nu(1-\sigma)\mathbb{E}_t c_{t+2} - (\phi\nu(1-\sigma) + 1)\mathbb{E}_t c_{t+1} + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+2} - \mathbb{E}_t \pi_{t+1} \\
&\quad + \frac{(\nu(1-\sigma))^2}{2}\text{Var}_t c_{t+2} + \frac{(\phi\nu(1-\sigma) + 1)^2}{2}\text{Var}_t c_{t+1} + \frac{((1-\nu)(1-\sigma))^2}{2}\text{Var}_t l_{t+1} + \frac{\text{Var}_t \pi_{t+1}}{2} \\
&\quad - \nu(1-\sigma)\text{Cov}_t(c_{t+2}, \pi_{t+2}) + (\phi\nu(1-\sigma) + 1)\text{Cov}_t(c_{t+1}, \pi_{t+1}) - (1-\nu)(1-\sigma)\text{Cov}_t(\pi_{t+1}, l_{t+2}) \\
&\quad - \nu(1-\sigma)(\phi\nu(1-\sigma) + 1)\text{Cov}_t(c_{t+1}, c_{t+2}) + \nu(1-\nu)(1-\sigma)^2\text{Cov}_t(c_{t+2}, l_{t+2}) \\
&\quad - (\phi\nu(1-\sigma) + 1)(1-\nu)(1-\sigma)\text{Cov}_t(c_{t+1}, l_{t+2}) \\
\chi_{3t} &= (\nu(1-\sigma) - 1)c_t - \phi\nu(1-\sigma)c_{t-1} + (1-\nu)(1-\sigma)l_t \\
\chi_{4t} &= \nu(1-\sigma)\mathbb{E}_t c_{t+1} - (\phi\nu(1-\sigma) + 1)c_t + (1-\nu)(1-\sigma)\mathbb{E}_t l_{t+1} + \frac{(\nu(1-\sigma))^2}{2}\text{Var}_t c_{t+1} \\
&\quad + \frac{((1-\nu)(1-\sigma))^2}{2}\text{Var}_t l_{t+1} + \nu(1-\nu)(1-\sigma)^2\text{Cov}_t(c_{t+1}, l_{t+1})
\end{aligned}$$

Following Canzoneri et al. (2007), to derive estimates for these conditional moments, we assume that the dynamics of consumption, inflation, and labor can be modeled as the VAR(4) process (written below in companion form)

$$Y_{t+1} = A_0 + A_1 Y_t + u_t, \tag{6}$$

$$u_t \stackrel{\text{iid}}{\sim} N(0, \Sigma)$$



where

$$Y_t = [y_t, y_{t-1}, y_{t-2}, y_{t-3}]'$$

$$y_t = [c_t, \pi_t, l_t, rdi_t, ymc_t, ffr_t, cci_t]'$$

The components of  $y_t$  are log of real consumption, log of gross inflation, leisure fraction (which we'll define more explicitly later), log of real disposable income, log of output less consumption, log of the gross effective federal funds rate, and log of the Thomson Reuters Equal Weight Continuous Commodity Index<sup>1</sup>.

After estimating  $A_0$ ,  $A_1$ , and  $\Sigma$ , we can compute:

$$\begin{aligned} \mathbb{E}_t Y_{t+1} &= A_0 + A_1 Y_t & \text{Var}_t Y_{t+1} &= \Sigma \\ \mathbb{E}_t Y_{t+2} &= A_0 + A_1 A_0 + A_1^2 Y_t & \text{Var}_t Y_{t+2} &= A_1 \Sigma A_1' + \Sigma \\ & & \text{Cov}_t(Y_{t+1}, Y_{t+2}) &= \Sigma A_1' \end{aligned}$$

The conditional moments are then the respective  $(i, j)$  components of these matrices. For example,  $\text{Cov}_t(c_{t+1}, l_{t+2})$  is the  $(1, 3)$  component of  $\text{Cov}_t(Y_{t+1}, Y_{t+2})$ .

Now, given data with which to estimate the vector autoregression (6), we have everything we need to compute the interest rates implied by the Euler equation.

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<sup>1</sup>The CCI is the “old” Thomson Reuters/Jeffries CRB Index, calculated using the same methodology as the CRB Index before it underwent weighting and rebalance changes in 1995.

## 4 Data

### 4.1 Aggregate-level

In the aggregate-level analysis, the endogenous variables making up  $y_t$  in the VAR model are constructed according to Collard and Dellas (2012) whenever possible. Except where mentioned, all the raw time series used are obtained from the St. Louis Fed's Federal Reserve Economic Data (FRED), with variable names in parentheses. Data are at the quarterly level and seasonally adjusted when appropriate, spanning 222 quarters from 1960:I to 2015:II, inclusive.. Real dollar values are in 2009 dollars. All lowercase variables in the vector  $y_t$  denote the natural log of the respective capitalized variable below *except the leisure fraction*  $l_t$ , which is described explicitly below.

*Per capita real consumption*  $C_t$ : Aggregate real consumption is defined as the sum of the chain quantity indices (2009 = 100) for personal consumption expenditures on nondurable goods (DNDGRA3Q086SBEA) and services (DSERRA3Q086SBEA), all multiplied by the sum of nominal non-durables (PCEND) and services (PCESV) consumption in 2009<sup>2</sup>. This amount is divided by the civilian noninstitutional population (CNP160V) to get per capita real consumption.

*Gross quarterly inflation*  $\Pi_t$ : In each quarter, the implicit price deflator  $P_t$  is calculated by dividing aggregate nominal consumption (PCEND + PCESV) by aggregate real consumption (described above). Then gross quarterly inflation is defined as the growth rate of the deflator:  $\Pi_t = \frac{P_t}{P_{t-1}}$ .

*Leisure fraction*  $l_t$ : Labor fraction  $h_t$  is defined as the average weekly hours worked in the nonfarm business sector (PRS85006023), multiplied by the civilian employment-to-population ratio (EMRATIO) and then rescaled so that the mean over all quarters was  $\frac{1}{3}$ . Then the leisure fraction is

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<sup>2</sup>I generate aggregate real consumption from the chain quantity indices because the real consumption variables used by Collard and Dellas, PCNDGC96 and PCESVC96, were not available from FRED for the quarters before 1999:I.

given by  $l_t = 1 - h_t$ .

*Per capita real disposable income  $RDI_t$* : This is computed by dividing real disposable income (DPIC96) by the civilian non-institutional population.

*Per capita real output less consumption  $YMC_t$* : Defined as real gross domestic product (GDPC96) minus aggregate real consumption, again divided by the civilian noninstitutional population.

*Gross quarterly effective federal funds rate  $FFR_t$* : This is computed by raising the gross annualized rate (DFF) to the one-fourth power.

*Continuous Commodity Index  $CCI_t$* : I use the CCI ending price on the first day of each quarter, obtained from Bloomberg. As mentioned, the CCI is the continuation of the CRB Index used by Canzoneri et al. (2007) and Collard and Dellas. What is called the CRB Index today is calculated slightly differently and exists only since 1995.

## 4.2 Household-level

For the comparison of bondholders to nonbondholders, I reuse the inflation, federal funds rate, and CCI variables constructed in the previous section. I generate separate time series for the other four endogenous variables for both bondholders and nonbondholders by aggregating household-level data from the Consumer Expenditure Survey from 1996:I to 2012:IV (68 quarters).

The CEX is a rotating panel of representative “consumer units”<sup>3</sup> in the United States, which are interviewed each quarter for five consecutive quarters. Each observation is a household-quarter. The first interview is for practice, and is not included in the reported survey data. Each quarter,

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<sup>3</sup>I refer to these consumer units informally as households, though the CEX does actually distinguish between the two terms, allowing for multiple consumer units to dwell in the same physical household. However, it is the consumer unit-level at which financial decisions are made and reported to the survey-takers, and hence at which the analysis in this paper is carried out.

20 percent of the households rotate out of the survey after their fifth interview, and a new 20 percent rotate in. Households report their expenditures in very detailed categories each quarter. Demographic and income data are collected in the second and fifth interviews, and asset holdings information is collected only in the fifth interview<sup>4</sup>.

After discarding observations which are flagged by the CEX as being incomplete income respondents ( $\text{RESPSTAT}^5 = 2$ ), I generate the following (nominal) observation-level variables:

*Bondholder status:* I determine whether or not to label each household a bondholder using the criteria set forth by Vissing-Jorgensen (2002). In the fifth interview, the CEX asks each household to estimate its current holdings in a number of asset categories, as well as how those holdings have changed in the preceding year (four quarters). I use a positive response in the asset categories “U.S. Savings Bonds” and “stocks, mutual funds, private bonds, government bonds, or Treasury notes” to determine bondholder status, despite that this definition likely creates some false positives, such as households which hold stocks but not bonds. It is difficult to achieve a more complete separation of households.

Either all observations belonging to a particular household are labeled bondholder observations or none of them are. I do not allow for a household’s bondholder status to change between interviews. A household is defined to be a bondholder if it had positive holdings of at least one of the two asset categories one year before the asset holdings questions are asked in the fifth interview (i.e. at the time of the first interview) — specifically, if one of the following holds:

1. The household reports holding the same amount of the asset as a year ago ( $\text{COMPBND}$  or

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<sup>4</sup>The CEX in fact consists of two separate surveys: the Interview Survey, which I have just described, and the Diary Survey, in which households report weekly expenditures on frequently purchased items. I use the Interview Survey exclusively.

<sup>5</sup>The variable names in the remainder of this section refer to CEX variables unless otherwise specified.

COMPSEC = 1), and reports a positive current holdings amount USBNDX or SECESTX > 0)

2. The household reports lower holdings of the asset than a year ago (COMPBND or COMPSEC = 2)
3. The household reports an increase in holdings in the past year (COMPBND or COMPSEC = 3) by an amount less than the current holdings (COMPBNDX < USBNDX or COMPSECDX < SECESTX)

*Consumption:* Following Heathcote et al. (2010), I define consumption of nondurable goods and services as the sum of the following expenditure categories: food and beverages (FOOD + ALCBEV), clothing (APPAR), gasoline (GASMO), household operation (HOUSOP), public transportation (PUBTRA), medical care excluding health insurance (HEALTH - HEALTHIN), recreation (ENTERT), tobacco (TOBACC), and education (READ + EDUCA).

*Hours worked:* I use the weekly hours worked by the household's reference person (INC\_HRS1). The reference person is the first person mentioned by the survey respondent when asked to "Start with the name of the person or one of the persons who owns or rents the home."

*Disposable income:* I use after-tax income (FINCATAX), as in Hai et al. (2015).

*Output less consumption:* Defined as before-tax income (FINCBTAX) minus consumption (defined above).

Below are summary statistics for bondholders and nonbondholders using the variables defined above. Since bondholders represent a fairly small fraction of the total sample, I include all bondholder observations in the bondholder aggregate but take a random sample of the nonbondholder observations in order to equalize sample size.

## SUMMARY STATS TK

Consumption, disposable income, and output less consumption are each deflated by the unadjusted Consumer Price Index for nondurables for urban consumers (CUUR0000SAN in FRED),

following Vissing-Jorgensen, rescaled to 2009 dollars to correspond with the aggregate-level data. The expenditure categories included in consumption were chosen to allow for the possibility of deflating each category by its own CPI (for example, CPIFABNS from FRED for food and beverages). However, the result of doing so was found to differ only negligibly from using a single CPI.

The CEX provides population weights for each household, which are calibrated so that summing the population weights in a given quarter approximates the number of households in the United States that quarter, while taking the weighted sum of the number of household members approximates the total population. I take the weighted mean of hours worked for each quarter and use it to generate labor fraction  $l_t$  as in the previous section. For each of consumption, disposable income, and output less consumption, I take the weighted sum each quarter and divide it by the population to get per capita variables  $C_t$ ,  $RDI_t$ , and  $YMC_t$ .

Finally, I seasonally adjust log consumption  $c_t$  by regressing it on indicators of the quarters and subtracting off the non-first quarter coefficients.

## 5 Aggregate-Level Replication

In this section, I compute the nominal and real interest rates implied by the Euler equation using the VAR estimated from the full sample (1960:I to 2015:II) of the aggregate series described in subsection 4.1. As in Collard and Dellas (2012), I take the discount rate  $\beta$  to be 0.9926 (so that households discount at an annual rate of 3 percent) and the coefficient of risk aversion  $\alpha$  to be 2. I look at four specifications of utility:

*SEP*: These are standard CRRA preferences (4), in which there is no habit formation ( $\phi = 0$ ) and consumption and leisure are assumed to be additively separable ( $\nu = 1$ ).

*SEP + HP*: Setting  $\phi = 0.8$  gives habit formation as in Fuhrer (2000). Now period utility depends on the ratio of current to previous period consumption, while leisure is still assumed to be separable ( $\nu = 1$ ):

$$u(C_t, C_{t-1}) = \frac{(C_t/C_{t-1})^{1-\alpha}}{1-\alpha}$$

*NSEP*: On the other hand, nonseparability in consumption and leisure is introduced by letting  $\nu = 0.34$ , which implies a work share of 30 percent in the absence of habit formation ( $\phi = 0$ ):

$$u(C_t, l_t) = \frac{(C_t^\nu l_t^{1-\nu})^{1-\alpha}}{1-\alpha}$$

*NSEP + HP*: Finally, letting both  $\phi = 0.8$  and  $\nu = 0.34$  gives the full Collard and Dellas model in (3) with both habit formation and nonseparability in consumption and leisure.

### 5.1 Correlation of implied and observed rates

Summary statistics for the implied rates under each specification are reported below in Table 1, as well as the correlation between each implied rate and the effective federal funds rate.

Table 1: Summary statistics for nominal and real rates (annualized rates)

	Data	SEP	SEP + HP	NSEP	NSEP + HP
Real interest rates					
Mean	1.50	6.19	4.87	5.16	4.72
SD	2.54	2.36	6.97	1.39	3.10
Min	-4.21	-3.21	-22.10	0.03	-8.73
Max	9.95	11.13	25.71	8.24	15.04
Corr	—	0.197	-0.050	0.261	0.033
Nominal interest rates					
Mean	5.11	9.8	8.48	8.77	8.33
SD	3.39	2.61	6.78	2.14	3.19
Min	0.07	-2.43	-21.31	0.25	-3.51
Max	16.37	16.61	28.38	13.84	19.6
Corr	—	0.52	0.142	0.707	0.39

The first thing that stands out is the presence of strong positive correlations overall, but especially for nominal rates in the specifications without habit formation. Notably, the correlation between the nominal rate implied by CRRA preferences (SEP) and the historical nominal effective federal funds rate is 0.52, while adding nonseparability in leisure (NSEP) increases this correlation to 0.707. In the utility models with habit formation (SEP + HP and NSEP + HP), the correlation is less positive for nominal rates and essentially zero for real rates.

These strong positive correlations are noticeably higher than the still-positive correlations found by Collard and Dellas, to say nothing of the strongly negative values found by Canzoneri et al. (2007). As a check, I reestimate the VAR and recompute the implied rates and correlations using only the time period spanned by Collard and Dellas, stopping at 2006:IV instead of 2015:II. The correlations for nominal rates from this restricted sample more closely resemble their results, though the ones for real rates are still rather different. I summarize the restricted sample results in Table 4 in the appendix. In Table 2, I compare the full and restricted sample correlations to those found in the



Table 2: Comparison of correlations between implied rates and effective FFR

	SEP	SEP + HP	NSEP	NSEP + HP	Start	End
Real interest rate correlation						
Full Sample	0.197	-0.050	0.261	0.033	1960:I	2015:II
Restricted Sample	0.020	-0.098	0.065	-0.058	1960:I	2006:IV
Collard and Dellas (2012)	0.05	0.15	0.28	0.27	1960:I	2006:IV
Canzoneri et al. (2007)	-0.37	-0.07	—	—	1966:I	2003:IV
Nominal interest rate correlation						
Full Sample	0.520	0.142	0.707	0.390	1960:I	2015:II
Restricted Sample	0.255	0.030	0.563	-0.225	1960:I	2006:IV
Collard and Dellas (2012)	0.26	0.04	0.63	0.38	1960:I	2006:IV
Canzoneri et al. (2007)	0.20	-0.10	—	—	1966:I	2003:IV

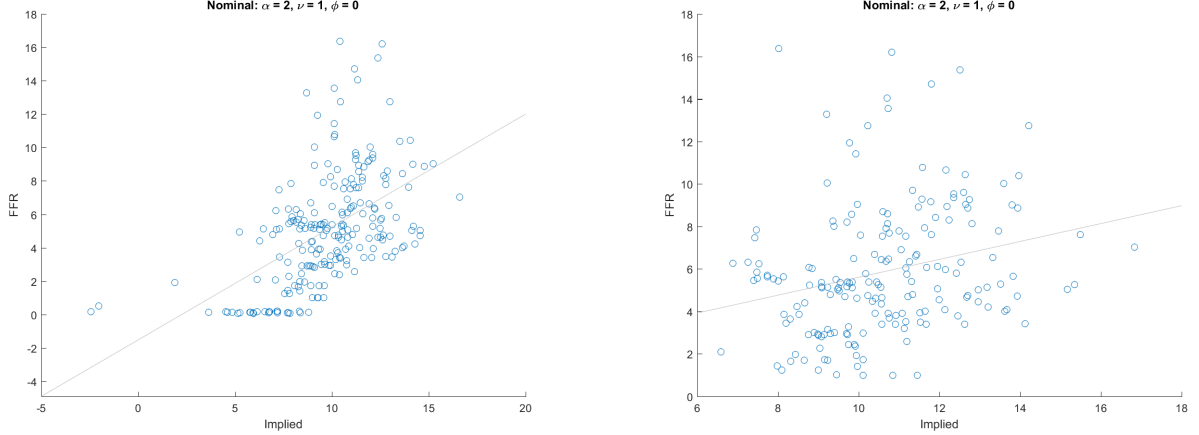
other two papers. Note that the CCD paper examines several utility specifications, including CRRA (SEP) and Fuhrer habit preferences (SEP + HP), but does not include analysis of nonseparability in leisure.

The extreme variation in correlations found suggests two points at which this analysis is not sufficiently robust.

First, comparing the correlations computed from the full sample to those from the restricted sample highlights the impact of the inclusion of the additional quarters from 2007:I to 2015:II. Scatter plots for both samples are shown in Figure 1. (In particular, the data points in the restricted sample plot are not a subset of those in the full sample plot because the implied rates for each were computed using different VAR estimates.) The difference between the two samples is of course the era of near-zero interest rates following the Great Recession in 2008, which can be seen in the full sample plot as the cluster of observations on the  $\text{FFR} = 0$  line. These, along with the outliers in the bottom left (which are also at the zero lower bound), drive the more strongly positive correlation in the full sample.

Figure 1: SEP implied vs. observed nominal rates

Left: full sample,  $\rho = 0.520$ . Right: restricted sample,  $\rho = 0.255$ .



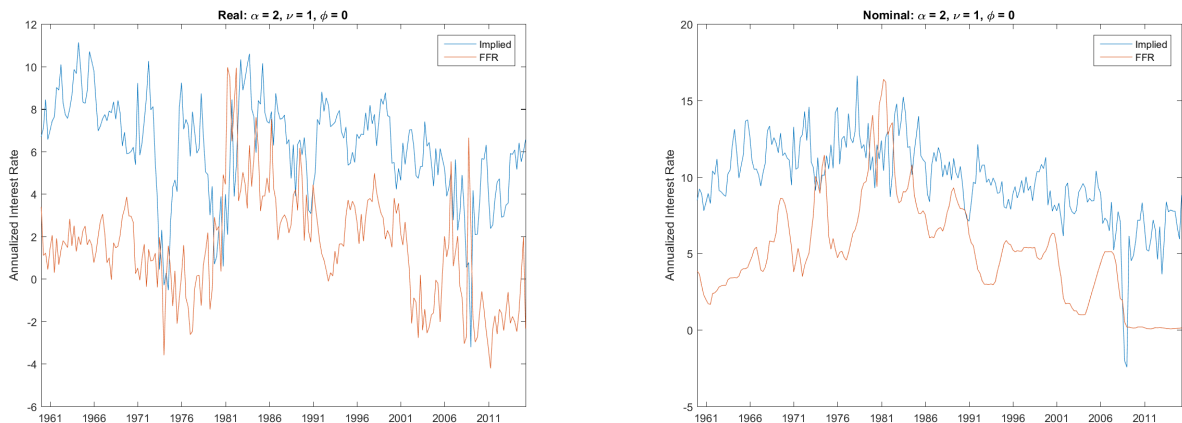
Even within the same time span, comparing the restricted sample correlations to those of Collard and Dellas highlights the fragility of these results with respect to small changes in methodology. I follow their specifications as closely as possible, except where it is not possible or not completely clear what they did. As mentioned in the previous section, due to lack of availability of data, I generate aggregate real consumption from the chain quantity indices scaled by the 2009 nominal consumption, while they use real consumption directly. I also estimate the VARs using log of gross quarterly inflation and interest rates  $\pi_t$  and  $ffr_t$ , while it is possible that Collard and Dellas may have used annualized rates and/or scaled them to units of percentage points. Other possible differences include our choices of base year (2009 in my analysis, versus 2000) and whether we take the natural log of real dollars (as I do) or billions of real dollars.

All of this is to say that the correlation between the implied and observed rates is probably not the most reliable metric by which we should judge the fit of the consumption Euler equation to the data — even though it's arguably the focus of both of these previous papers. This is particularly

evident upon qualitatively examining the paths of the observed and implied rates, which are shown for the SEP case in Figure 2 below. (Plots of the other three utility specifications are found beginning with Figure 4(a) in the appendix.)

Figure 2: SEP implied vs. observed rates

Left: real rates,  $\rho = 0.197$ . Right: nominal rates,  $\rho = 0.520$ .



Despite the positive correlations of 0.197 and 0.520 respectively, both the real and nominal implied rates persist in moving in the opposite direction from the respective ex post rates. The paths of the implied rates look very similar to those found by CCD, who find them to be negatively correlated with the observed rates. A more consistent metric of fit is the correlation of the spread with the stance of monetary policy, which I discuss next.

## 5.2 Response of spread to monetary policy

It is clear that the implied and observed rates differ in level. In Table 1, we see that the mean implied interest rate in each of the four utility specifications is noticeably higher than the mean ex post rate, in both the real and nominal cases. The lowest mean implied rates come from the full

NSEP + HP utility model, with averages of 4.72 and 8.33 for the real and nominal rates respectively, compared to averages of 1.50 and 5.11 observed in the data in the same span.

If the model implied interest rates differed from the observed rates only by a constant factor, this discrepancy would be less worrisome. In practice — for example, in larger macroeconomic models where the interest rate is assumed to be given by the Euler equation — it would be easy to adjust for this constant “measurement error”. However, as I show, and as CCD also find, this spread is in fact not constant, but correlated with the stance of monetary policy. In this aspect, my results strongly resemble those of CCD, as well as what one might expect from visually inspecting the plot of observed and implied rates.

I define the spread as the model implied interest rate less the observed rate. For each of the four utility specifications, I regress the spread on the effective federal funds rate, as well as on four lags of itself. That is, I estimate using least squares the linear model

$$\text{spread}_t = b_0 + b_1 \text{FFR}_t + \sum_{j=1}^4 a_j \text{spread}_{t-j} \quad (7)$$

where all rates are annualized and given in units of percentage points. The estimated coefficients and standard errors for the federal funds rate only are reported below in Table 3. The FFR coefficients  $b_1$  are negative for both real and nominal spreads under all four utility specifications. They are all significant at below the 1 percent level. This suggests that the difference between implied and observed rates tends to increase during periods of monetary expansion (lower FFR) and decrease during periods of monetary tightening.

These  $b_1$  estimates match the negative coefficients estimated by CCD in direction, and are similar in magnitude. I also repeat this regression analysis using VAR estimates and implied interest rates generated using a restricted sample from 1966:I to 2003:IV, the time range used in their paper. I compare the estimates from the full and restricted samples to CCD in Table 5 in the

Table 3: Response of spread to federal funds rate

	SEP	SEP + HP	NSEP	NSEP + HP
Real interest rates				
Coef	-0.307	-0.906	-0.312	-0.623
SE	(0.050)	(0.165)	(0.044)	(0.093)
Nominal interest rates				
Coef	-0.273	-0.954	-0.282	-0.640
SE	(0.043)	(0.161)	(0.036)	(0.089)

appendix.

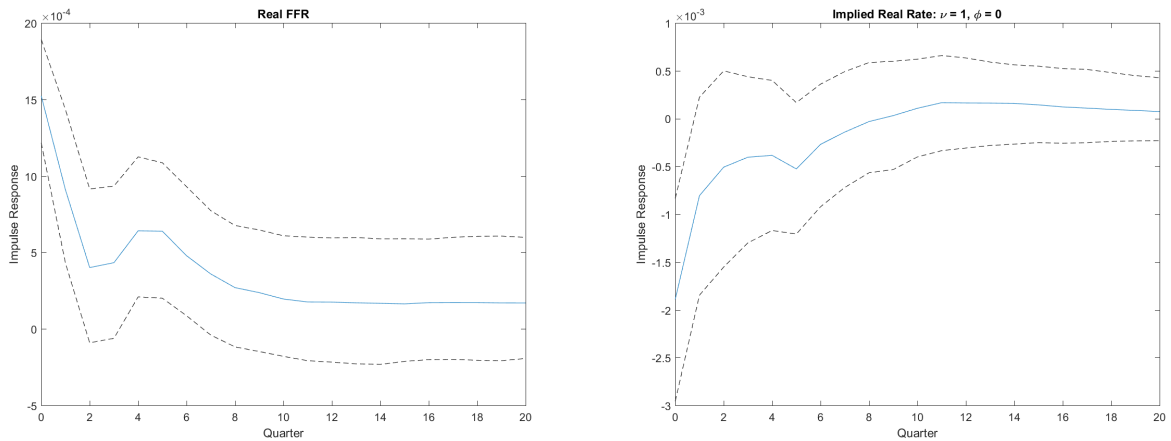
Unlike the correlation between implied and observed rates, the results of regressing spread on the FFR do not change drastically in using this reduced sample, though they do resemble the CCD estimates slightly more than the full sample, as one might expect. This suggests that the response of the implied-observed spread to the FFR is a more robust instrument with which to measure the fit of the implied rate to the ex post rate than the correlation alone. It is also evidence that Collard and Dellas's addition of nonseparability in consumption and leisure to the CRRA model does not in fact improve the fit of the model implied rates to the data. While adding nonseparability does increase the correlation in models both with and without habit formation, it has little discernible effect on the estimated coefficient  $b_1$ .

As CCD discuss, these results indicate that something fundamental is not being captured by the consumption Euler equation, which leads to a difference between implied and observed rates that is not constant but systemically correlated with the stance of monetary policy. They also suggest an explanation for the divergence of the two series: from (5), we see that the Euler equation implied rate increases in expected consumption growth. Expected consumption growth has been shown empirically to increase in response to monetary expansion (lower FFR), causing the implied rate

to increase as well — the opposite direction of movement from the actual interest rate.

I check this hypothesis by computing the orthogonalized impulse response of the model implied rate to a monetary shock. I compare the responses of the SEP implied real interest rate and the real FFR in Figure 3. The dashed lines indicate the 95 percent confidence interval, estimated from 1000 bootstrap simulations using Kilian (1998)’s bootstrap code. The results under the other three specifications were nearly identical, as were all of the nominal rate responses. As anticipated, a negative shock to real FFR causes a positive response in the model implied rate.

Figure 3: Impulse response of observed (left) and SEP implied (right)  
interest rates to a monetary policy shock



Overall, it seems that despite the strong positive correlations between observed and implied rates that I find in each specification of utility, the path of the Euler equation implied interest rate, computed in this way, still deviates remarkably from the path of ex post rates. The addition of nonseparability in consumption and leisure that Collard and Dellas introduce does not change the implied rate’s positive impulse response to the monetary policy shock, so by extension it also doesn’t improve the estimated negative FFR coefficient in the spread regression (7).

## 6 Appendix

Table 4: Summary statistics for nominal and real rates (annualized rates)

Collard and Deltas (2012) sample (1960:I to 2006:IV)

	Data	SEP	SEP + HP	NSEP	NSEP + HP
Real interest rates					
Mean	1.98	6.73	5.21	5.49	4.98
SD	2.30	2.17	7.73	1.29	3.37
Min	-3.59	-0.18	-18.35	1.25	-7.96
Max	9.95	11.92	26.92	8.86	16.69
Corr	—	0.020	-0.098	0.065	-0.058
Nominal interest rates					
Mean	5.87	10.62	9.10	9.39	8.88
SD	3.05	1.85	7.48	1.71	3.33
Min	1.00	6.58	-7.47	5.78	2.63
Max	16.37	16.83	30.87	14.03	21.60
Corr	—	0.255	0.030	0.563	0.225

Figure 4(a): SEP + HP implied vs. observed rates

Left: real rates,  $\rho = -0.050$ . Right: nominal rates,  $\rho = 0.142$ .

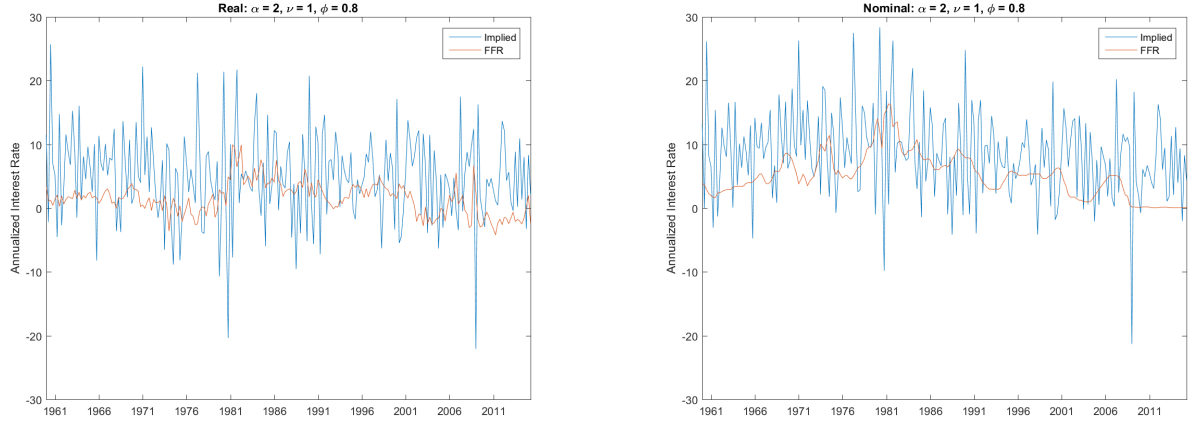


Figure 4(b): NSEP implied vs. observed rates

Left: real rates,  $\rho = 0.261$ . Right: nominal rates,  $\rho = 0.707$ .

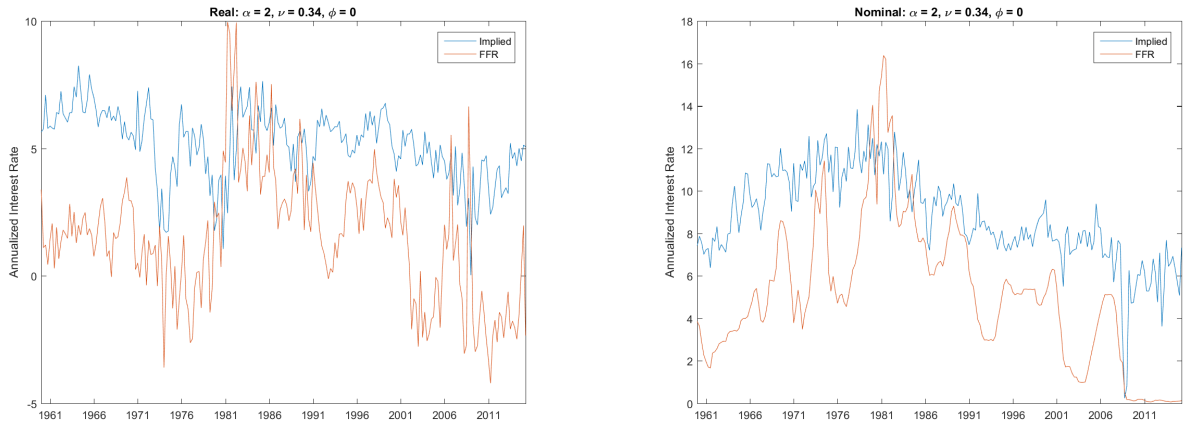




Figure 4(c): NSEP + HP implied vs. observed rates

Left: real rates,  $\rho = 0.033$ . Right: nominal rates,  $\rho = 0.390$ .

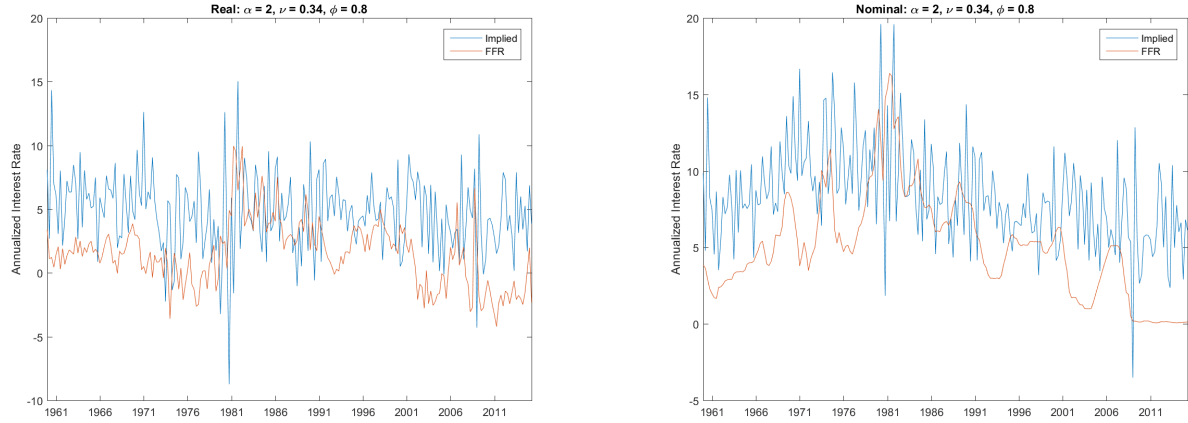


Table 5: Comparison of responses of spread to monetary policy

(Standard errors in parentheses)

	SEP	SEP + HP	NSEP	NSEP + HP	Start	End
Real interest rate spreads						
Full Sample	-0.307 (0.050)	-0.906 (0.165)	-0.312 (0.044)	-0.623 (0.093)	1960:I	2015:II
Restricted Sample	-0.545 (0.069)	-1.054 (0.226)	-0.432 (0.057)	-0.734 (0.130)	1966:I	2003:IV
Canzoneri et al. (2007)	-0.482 (0.064)	-1.215 (0.825)	—	—	1966:I	2003:IV
Nominal interest rate spreads						
Full Sample	-0.273 (0.043)	-0.954 (0.161)	-0.282 (0.036)	-0.640 (0.089)	1960:I	2015:II
Restricted Sample	-0.545 (0.064)	-1.134 (0.223)	-0.433 (0.050)	-0.796 (0.128)	1966:I	2003:IV
Canzoneri et al. (2007)	-0.357 (0.047)	-1.035 (0.826)	—	—	1966:I	2003:IV

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