Problem Set

Deadline: Dec 15. You can work in teams of 2 students.

Complete Markets: Replication

In this first part, you are asked to replicate some results of Lucas Stokey (83).

Environment Assume the following form for the utility function

$$U(c,n) = \frac{c^{1-\rho}-1}{1-\rho} - \chi\frac{n^{1+\psi}}{1+\psi},$$

where $\chi = 1.5$, $\psi = 1$, and $\rho = 1.01$.

Assume government spending follow a two-state Markov chain, with $g_L = .1$, $g_H = .2$, $g_0 = g_L$, and the transition matrix Π is defined as

$$\Pi(1,:) = [.9\ .1]$$
$$\Pi(2,:) = [.5\ .5]$$

The goal is to solve for the optimal fiscal plan – state-contingent debt and labor taxes, and allocations – using the time-zero formulation and the primal approach.

1. We first focus on $t \geq 1$.
   (a) Plot the history-independent and time-invariant tax $\tau(\Phi)$. (Hint: Build a grid on $\Phi = [0,1]$.)
   (b) Plot the history-independent and time-invariant allocations $\{c(\Phi,g), n(\Phi,g)\}\ \forall g$, as a function of Φ.
   (c) Write the implementability constraints recursively to find $\{b(\Phi,g)\}$, and plot debt policies as a function of Φ.
   (d) What can you say about the relationship between Φ and τ? Between Φ and c?
2. Now we turn to the initial period.
   (a) First, compute allocations $\{c_0, n_0\}$, as a function of b_0 and Φ. (Hint: Build a grid on $b_0 = [-0.1, 0.1]$).

(b) Then, use a bisection method to find Φ as a function of b_0. Plot Φ as a function of b_0.

(c) What can you say about the relation between b_0 and Φ?

3. Finally, we simulate the economy.

(a) Assume $b_0 = 0$. Simulate the economy for 100 periods. Plot the simulated government spending, allocations, tax and debt-to-output ratios. What can you say about τ_0?

(b) Now assume $b_0 = 0.1$. Repeat the exercise.

(c) Now assume $b_0 = -0.1$. Repeat the exercise.

Complete Markets: Theoretical Exercise

In the model with complete markets and capital, prove that if the government can use lump-sum taxes: (1) the optimal Ramsey plan reaches the first-best allocation; (2) the Lagrange multiplier of the implementability constraint Φ is equal to zero.

Note: You must handle one Matlab file, where I just have to press the 'run' button to run the whole code and generate all the graphs, and one PDF to describe the algorithm that you are using step by step and to answer all questions.