Theoretical Exercise

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We consider the set up of Chari, Christiano and Kehoe (1994) and take the results seen in class as given.

Set-up and First Best

Household's preferences and the feasibility constraint are respectively given by:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta t \pi_t(s^t) u(c_t(s^t), 1 - n_t(s^t)) \tag{1}$$

$$c_t(s^t) + g_t(s^t) + k_{t+1}(s^t) = a_t(s^t) F(k_t(s^{t-1}), n_t(s^t)) + (1 - \delta) k_t(s^t) \tag{2}$$

The first best allocation is given by the maximization of households preferences with respect to the feasibility constraint.

Competitive Equilibrium

In this exercise we sketch the first steps of the resolution of the Ramsey problem in the case of complete markets with both labor, capital and lump sum taxes in order to compare the competitive equilibrium with lump sum taxes to the first best. Our first step is to solve for the competitive equilibrium. The problem is identical to the one seen in class, except for the household's and government's budget constraint which, respectively, become:

$$\begin{aligned} c_t + k_{t+1} + T_t + \sum_{s_{t+1}} p_{t+1}(s_{t+1}/s^t) =& b_t(s^t) + (1 - \tau_t^n)\omega_t n_t + (1 - \tau_t^k) r_t k_t \\ &+ (1 - \delta) k_t \end{aligned} \tag{3}$$

$$g_t + b_t + T_t = \tau_t^k r_t k_t + \tau_t^n)\omega_t n_t + \sum_{s_{t+1}} p_{t+1}(s_{t+1}/s^t) = b_t(s^t) \tag{4}$$

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The household maximization problem and the firm's one give - under these different constraints - the same first order conditions than the ones in the case without lump-sum taxes, which are:

$$\frac{u_x(s^t)}{u_c(s^t)} = (1 - \tau_t^n(st))\omega_t(s^t) \tag{5}$$

$$p_{t+1}(s_{t+1}|s_t) = \beta\pi_{t+1}(s_{t+1}|s_t)\frac{u_c(s^{t+1})}{u_c(s^t)} \tag{6}$$

$$u_c(s^t) = \beta E_t u_c(s^{t+1})[1 - \delta + r_{t+1}(s^{t+1})(1 - \tau_{t+1}^k(s^{t+1}))] \tag{7}$$

$$r_t = Fk, t \tag{8}$$

$$\omega_t = F_{n,t} \tag{9}$$

Where p_{t+1} is the price in time t - history g^t - for one unit in period $t+1$.

The Ramsey Plan

We can now build the Ramsey plan. The first step is to get the implementability constraint, either from the government's budget constraint or from the household's one. Here we choose the household's one. Its time-zero formulation is:

$$\sum_{t=0}^{\infty}\sum_{s_t} q_t^0(s^t)[c_t + T_t - (1 - \tau_t^n)\omega_t\eta_t] = b_0 + [(1 - \tau_0^k)r_0 + 1 - \delta]k_0 \tag{10}$$

We then replace prices in it using the household's and the firm's first order conditions (equations (5) to (9)) to get the implementability constraint. The constraint becomes:

$$\sum_{t=0}^{\infty}\sum_{s_t} \pi_t\beta^t[u_{c,t}T_t + u_{c,t}c_t - u_{x,t}(1 - x_t)] = u_{c,0}b_0 + [(1 - \tau_0^k)F_{k0} + 1 - \delta]u_{c,0}k_0 \tag{11}$$

We can now rewrite the Ramsey problem as the maximization of household's discounted utility subject to the feasibility and the implementability constraints. Its Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t u(c_t, x_t) + \phi[\sum_{t=0}^{\infty}\beta^t[u_{c,t}T_t + u_{c,t}c_t - u_{x,t}(1 - x_t) - A] + \theta(a_tF(k_{,t}n_t) + (1 - \delta)k_t - c_t - g_t - k_{t+1}) \tag{12}$$

where A $= u_{c,0}b_0 + [(1 - \tau_0^k)F_{k0} + 1 - \delta]u_{c,0}k_0$.

Main Result

It is straightforward that deriving the Lagrangian with respect to the lump sum tax T_t leads to:

$$\beta^t \phi u_{c,t} = 0 \tag{13}$$

Since marginal utility is normally strictly positive, we can hence conclude that $\phi = 0$.

With this last result we can rewrite the Ramsey problem as the maximization of the household's discounted utility only subject to the feasibility constraint. This problem is the same than the one we derive to obtain the first best. Therefore, with lump sum taxes, the optimal Ramsey plan reaches the first-best allocation.