

Index sets

$$HH = \{l, s\}$$

$$SEC = \{A, B, C\}$$

1 CONSUMER $h \in HH$

1.1 Optimisation problem

$$\max_{(D^{\langle s, h \rangle})_{s \in SEC}} U^{\langle h \rangle} = \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} \quad (1.1)$$

s.t. :

$$INC^{\langle h \rangle} = \sum_{s \in SEC} p^{\langle s \rangle} D^{\langle s, h \rangle} \quad \left(\lambda^{\text{CONSUMER}^1 \langle h \rangle} \right) \quad (1.2)$$

1.2 Identities

$$INC^{\langle h \rangle} = CAP^{\langle h \rangle} + p^k K^{\langle h \rangle} + p^l L^{\langle h \rangle} \quad (1.3)$$

1.3 First order conditions

$$s \in SEC: \quad \lambda^{\text{CONSUMER}^1 \langle h \rangle} p^{\langle s \rangle} + \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle -1+\omega^{-1}(-1+\omega)} \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad \left(D^{\langle s, h \rangle} \right) \quad (1.4)$$

2 SECTOR $s \in SEC$

2.1 Optimisation problem

$$\max_{Y^{\langle s \rangle}, K^{\langle s \rangle}, L^{\langle s \rangle}, Y^{\text{VA} \langle s \rangle}, Y^{\text{INT} \langle s \rangle}} \pi^{\langle s \rangle} = -p^{\text{kc}} K^{\langle s \rangle} - p^{\text{lc}} L^{\langle s \rangle} + p^{\langle s \rangle} Y^{\langle s \rangle} - Y^{\text{INT} \langle s \rangle} \left(\sum_{\mathbf{s}i \in SEC} \beta^{\mathbf{x} \langle \mathbf{s}i, s \rangle - 1} p^{\langle \mathbf{s}i \rangle} \right) \quad (2.1)$$

s.t. :

$$Y^{\langle s \rangle} = Y^{\text{VA} \langle s \rangle} \left(\lambda^{\text{SECTOR}^1 \langle s \rangle} \right) \quad (2.2)$$

$$Y^{\langle s \rangle} = Y^{\text{INT} \langle s \rangle} \left(\lambda^{\text{SECTOR}^2 \langle s \rangle} \right) \quad (2.3)$$

$$Y^{\text{VA} \langle s \rangle} = \gamma^{\text{yva} \langle s \rangle} K^{\langle s \rangle \beta^{\text{k} \langle s \rangle}} L^{\langle s \rangle \beta^{\text{l} \langle s \rangle}} \left(\lambda^{\text{SECTOR}^3 \langle s \rangle} \right) \quad (2.4)$$

2.2 Identities

$$\mathbf{s}i \in SEC: \quad X^{\langle \mathbf{s}i, s \rangle} = \beta^{\mathbf{x} \langle \mathbf{s}i, s \rangle - 1} Y^{\text{INT} \langle s \rangle} \quad (2.5)$$

2.3 First order conditions

$$-\lambda^{\text{SECTOR}^1 \langle s \rangle} - \lambda^{\text{SECTOR}^2 \langle s \rangle} + p^{\langle s \rangle} = 0 \quad \left(Y^{\langle s \rangle} \right) \quad (2.6)$$

$$-p^{\text{kc}} + \beta^{\text{k} \langle s \rangle} \gamma^{\text{yva} \langle s \rangle} \lambda^{\text{SECTOR}^3 \langle s \rangle} K^{\langle s \rangle - 1 + \beta^{\text{k} \langle s \rangle}} L^{\langle s \rangle \beta^{\text{l} \langle s \rangle}} = 0 \quad \left(K^{\langle s \rangle} \right) \quad (2.7)$$

$$-p^{\text{lc}} + \beta^{\text{l} \langle s \rangle} \gamma^{\text{yva} \langle s \rangle} \lambda^{\text{SECTOR}^3 \langle s \rangle} K^{\langle s \rangle \beta^{\text{k} \langle s \rangle}} L^{\langle s \rangle - 1 + \beta^{\text{l} \langle s \rangle}} = 0 \quad \left(L^{\langle s \rangle} \right) \quad (2.8)$$

$$\lambda^{\text{SECTOR}^1 \langle s \rangle} - \lambda^{\text{SECTOR}^3 \langle s \rangle} = 0 \quad \left(Y^{\text{VA} \langle s \rangle} \right) \quad (2.9)$$

$$\lambda^{\text{SECTOR}^2 \langle s \rangle} - \sum_{\mathbf{s}i \in SEC} \beta^{\mathbf{x} \langle \mathbf{s}i, s \rangle - 1} p^{\langle \mathbf{s}i \rangle} = 0 \quad \left(Y^{\text{INT} \langle s \rangle} \right) \quad (2.10)$$

2.4 First order conditions after reduction

$$-p^{\text{kc}} + \beta^{\text{k}\langle s \rangle} \gamma^{\text{yva}\langle s \rangle} \left(p^{\langle s \rangle} - \sum_{\text{si} \in \text{SEC}} \beta^{\text{x}\langle \text{si}, s \rangle - 1} p^{\langle \text{si} \rangle} \right) K^{\langle s \rangle - 1 + \beta^{\text{k}\langle s \rangle}} L^{\langle s \rangle \beta^{1\langle s \rangle}} = 0 \quad \left(K^{\langle s \rangle} \right) \quad (2.11)$$

$$-p^{\text{lc}} + \beta^{1\langle s \rangle} \gamma^{\text{yva}\langle s \rangle} \left(p^{\langle s \rangle} - \sum_{\text{si} \in \text{SEC}} \beta^{\text{x}\langle \text{si}, s \rangle - 1} p^{\langle \text{si} \rangle} \right) K^{\langle s \rangle \beta^{\text{k}\langle s \rangle}} L^{\langle s \rangle - 1 + \beta^{1\langle s \rangle}} = 0 \quad \left(L^{\langle s \rangle} \right) \quad (2.12)$$

3 EQUILIBRIUM

3.1 Identities

$$p^1 = 1 \quad (3.1)$$

$$s \in \text{SEC}: \quad p^{\langle s \rangle} = 1 \quad (3.2)$$

$$KS = \sum_{s \in \text{SEC}} K^{\langle s \rangle} \quad (3.3)$$

$$KS = ks^{\text{data}} \quad (3.4)$$

$$h \in HH: \quad K^{\langle h \rangle} = \alpha w^{\langle h \rangle} \text{scale}^{\langle h \rangle - 1} KS \quad (3.5)$$

$$LS = ls^{\text{data}} \quad (3.6)$$

$$h \in HH: \quad L^{\langle h \rangle} = \alpha w^{\langle h \rangle} \text{scale}^{\langle h \rangle - 1} LS \quad (3.7)$$

$$p^{\text{kc}} = p^{\text{k}} \quad (3.8)$$

$$p^{\text{lc}} = p^1 \quad (3.9)$$

$$\Pi = \sum_{s \in \text{SEC}} \pi^{\langle s \rangle} \quad (3.10)$$

$$K^{\text{f}} = \Pi + \alpha w^{\text{f}} KS \quad (3.11)$$

$$h \in HH: \quad CAP^{\langle h \rangle} = \alpha w^{\text{f}\langle h \rangle} \text{scale}^{\langle h \rangle - 1} K^{\text{f}} \quad (3.12)$$

4 Equilibrium relationships (before expansion and reduction)

$$1 - p^l = 0 \quad (4.1)$$

$$k_s^{\text{data}} - KS = 0 \quad (4.2)$$

$$l_s^{\text{data}} - LS = 0 \quad (4.3)$$

$$p^k - p^{kc} = 0 \quad (4.4)$$

$$p^l - p^{lc} = 0 \quad (4.5)$$

$$-KS + \sum_{s \in SEC} K^{\langle s \rangle} = 0 \quad (4.6)$$

$$-\Pi + \sum_{s \in SEC} \pi^{\langle s \rangle} = 0 \quad (4.7)$$

$$-K^f + \Pi + \alpha w^f KS = 0 \quad (4.8)$$

$$h \in HH: \quad -CAP^{\langle h \rangle} + \alpha w f^{\langle h \rangle} scale^{\langle h \rangle - 1} K^f = 0 \quad (4.9)$$

$$h \in HH: \quad -INC^{\langle h \rangle} + \sum_{s \in SEC} p^{\langle s \rangle} D^{\langle s, h \rangle} = 0 \quad (4.10)$$

$$h \in HH: \quad -K^{\langle h \rangle} + \alpha w^{\langle h \rangle} scale^{\langle h \rangle - 1} KS = 0 \quad (4.11)$$

$$h \in HH: \quad -L^{\langle h \rangle} + \alpha w w^{\langle h \rangle} scale^{\langle h \rangle - 1} LS = 0 \quad (4.12)$$

$$h \in HH: \quad U^{\langle h \rangle} - \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (4.13)$$

$$h \in HH: \quad CAP^{\langle h \rangle} - INC^{\langle h \rangle} + p^k K^{\langle h \rangle} + p^l L^{\langle h \rangle} = 0 \quad (4.14)$$

$$h \in HH: \quad s \in SEC: \quad \lambda^{\text{CONSUMER}^1 \langle h \rangle} p^{\langle s \rangle} + \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle - 1 + \omega^{-1}(-1+\omega)} \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{-1 + \omega(-1+\omega)^{-1}} = 0 \quad (4.15)$$

$$s \in SEC: \quad 1 - p^{\langle s \rangle} = 0 \quad (4.16)$$

$$s \in SEC: \quad -p^{\text{kc}} + \beta^{\text{k}\langle s \rangle} \gamma^{\text{yva}\langle s \rangle} \left(p^{\langle s \rangle} - \sum_{\vec{s}i \in SEC} \beta^{\text{x}\langle \vec{s}i, s \rangle - 1} p^{\langle \vec{s}i \rangle} \right) K^{\langle s \rangle - 1 + \beta^{\text{k}\langle s \rangle}} L^{\langle s \rangle \beta^{\text{l}\langle s \rangle}} = 0 \quad (4.17)$$

$$s \in SEC: \quad -p^{\text{lc}} + \beta^{\text{l}\langle s \rangle} \gamma^{\text{yva}\langle s \rangle} \left(p^{\langle s \rangle} - \sum_{\vec{s}i \in SEC} \beta^{\text{x}\langle \vec{s}i, s \rangle - 1} p^{\langle \vec{s}i \rangle} \right) K^{\langle s \rangle \beta^{\text{k}\langle s \rangle}} L^{\langle s \rangle - 1 + \beta^{\text{l}\langle s \rangle}} = 0 \quad (4.18)$$

$$s \in SEC: \quad -Y^{\langle s \rangle} + Y^{\text{VA}\langle s \rangle} = 0 \quad (4.19)$$

$$s \in SEC: \quad -Y^{\langle s \rangle} + Y^{\text{INT}\langle s \rangle} = 0 \quad (4.20)$$

$$s \in SEC: \quad -Y^{\text{VA}\langle s \rangle} + \gamma^{\text{yva}\langle s \rangle} K^{\langle s \rangle \beta^{\text{k}\langle s \rangle}} L^{\langle s \rangle \beta^{\text{l}\langle s \rangle}} = 0 \quad (4.21)$$

$$s \in SEC: \quad \pi^{\langle s \rangle} + p^{\text{kc}} K^{\langle s \rangle} + p^{\text{lc}} L^{\langle s \rangle} - p^{\langle s \rangle} Y^{\langle s \rangle} + Y^{\text{INT}\langle s \rangle} \left(\sum_{\vec{s}i \in SEC} \beta^{\text{x}\langle \vec{s}i, s \rangle - 1} p^{\langle \vec{s}i \rangle} \right) = 0 \quad (4.22)$$

$$s \in SEC: \quad \vec{s}i \in SEC: \quad -X^{\langle \vec{s}i, s \rangle} + \beta^{\text{x}\langle \vec{s}i, s \rangle - 1} Y^{\text{INT}\langle s \rangle} = 0 \quad (4.23)$$

5 Equilibrium relationships (after expansion and reduction)

$$1 - p^{\text{l}} = 0 \quad (5.1)$$

$$1 - p^{\langle \text{A} \rangle} = 0 \quad (5.2)$$

$$1 - p^{\langle \text{B} \rangle} = 0 \quad (5.3)$$

$$1 - p^{\langle \text{C} \rangle} = 0 \quad (5.4)$$

$$k^{\text{S}}_{\text{S}}^{\text{data}} - KS = 0 \quad (5.5)$$

$$l^{\text{S}}_{\text{S}}^{\text{data}} - LS = 0 \quad (5.6)$$

$$p^k - p^{kc} = 0 \quad (5.7)$$

$$p^l - p^{lc} = 0 \quad (5.8)$$

$$-p^{kc} + \beta^{k\langle A \rangle} \gamma^{yva\langle A \rangle} \left(p^{\langle A \rangle} - \beta^{x\langle A, A \rangle} p^{\langle A \rangle} - \beta^{x\langle B, A \rangle} p^{\langle B \rangle} - \beta^{x\langle C, A \rangle} p^{\langle C \rangle} \right) K^{\langle A \rangle - 1 + \beta^{k\langle A \rangle}} L^{\langle A \rangle \beta^{1\langle A \rangle}} = 0 \quad (5.9)$$

$$-p^{kc} + \beta^{k\langle B \rangle} \gamma^{yva\langle B \rangle} \left(p^{\langle B \rangle} - \beta^{x\langle A, B \rangle} p^{\langle A \rangle} - \beta^{x\langle B, B \rangle} p^{\langle B \rangle} - \beta^{x\langle C, B \rangle} p^{\langle C \rangle} \right) K^{\langle B \rangle - 1 + \beta^{k\langle B \rangle}} L^{\langle B \rangle \beta^{1\langle B \rangle}} = 0 \quad (5.10)$$

$$-p^{kc} + \beta^{k\langle C \rangle} \gamma^{yva\langle C \rangle} \left(p^{\langle C \rangle} - \beta^{x\langle A, C \rangle} p^{\langle A \rangle} - \beta^{x\langle B, C \rangle} p^{\langle B \rangle} - \beta^{x\langle C, C \rangle} p^{\langle C \rangle} \right) K^{\langle C \rangle - 1 + \beta^{k\langle C \rangle}} L^{\langle C \rangle \beta^{1\langle C \rangle}} = 0 \quad (5.11)$$

$$-p^{lc} + \beta^{l\langle A \rangle} \gamma^{yva\langle A \rangle} \left(p^{\langle A \rangle} - \beta^{x\langle A, A \rangle} p^{\langle A \rangle} - \beta^{x\langle B, A \rangle} p^{\langle B \rangle} - \beta^{x\langle C, A \rangle} p^{\langle C \rangle} \right) K^{\langle A \rangle \beta^{k\langle A \rangle}} L^{\langle A \rangle - 1 + \beta^{1\langle A \rangle}} = 0 \quad (5.12)$$

$$-p^{lc} + \beta^{l\langle B \rangle} \gamma^{yva\langle B \rangle} \left(p^{\langle B \rangle} - \beta^{x\langle A, B \rangle} p^{\langle A \rangle} - \beta^{x\langle B, B \rangle} p^{\langle B \rangle} - \beta^{x\langle C, B \rangle} p^{\langle C \rangle} \right) K^{\langle B \rangle \beta^{k\langle B \rangle}} L^{\langle B \rangle - 1 + \beta^{1\langle B \rangle}} = 0 \quad (5.13)$$

$$-p^{lc} + \beta^{l\langle C \rangle} \gamma^{yva\langle C \rangle} \left(p^{\langle C \rangle} - \beta^{x\langle A, C \rangle} p^{\langle A \rangle} - \beta^{x\langle B, C \rangle} p^{\langle B \rangle} - \beta^{x\langle C, C \rangle} p^{\langle C \rangle} \right) K^{\langle C \rangle \beta^{k\langle C \rangle}} L^{\langle C \rangle - 1 + \beta^{1\langle C \rangle}} = 0 \quad (5.14)$$

$$-CAP^{\langle l \rangle} + \alpha w f^{\langle l \rangle} scale^{\langle l \rangle - 1} K^f = 0 \quad (5.15)$$

$$-CAP^{\langle s \rangle} + \alpha w f^{\langle s \rangle} scale^{\langle s \rangle - 1} K^f = 0 \quad (5.16)$$

$$-K^{\langle l \rangle} + \alpha w c^{\langle l \rangle} scale^{\langle l \rangle - 1} KS = 0 \quad (5.17)$$

$$-K^{\langle s \rangle} + \alpha w c^{\langle s \rangle} scale^{\langle s \rangle - 1} KS = 0 \quad (5.18)$$

$$-L^{\langle l \rangle} + \alpha w w^{\langle l \rangle} scale^{\langle l \rangle - 1} LS = 0 \quad (5.19)$$

$$-L^{\langle s \rangle} + \alpha w w^{\langle s \rangle} scale^{\langle s \rangle - 1} LS = 0 \quad (5.20)$$

$$U^{\langle l \rangle} - \left(\alpha^{\langle A, l \rangle} D^{\langle A, l \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle B, l \rangle} D^{\langle B, l \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle C, l \rangle} D^{\langle C, l \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.21)$$

$$U^{\langle s \rangle} - \left(\alpha^{\langle A, s \rangle} D^{\langle A, s \rangle} \omega^{-1(-1+\omega)} + \alpha^{\langle B, s \rangle} D^{\langle B, s \rangle} \omega^{-1(-1+\omega)} + \alpha^{\langle C, s \rangle} D^{\langle C, s \rangle} \omega^{-1(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.22)$$

$$-X^{\langle A, A \rangle} + \beta^{\mathbf{x}\langle A, A \rangle} Y^{\text{INT}\langle A \rangle} = 0 \quad (5.23)$$

$$-X^{\langle A, B \rangle} + \beta^{\mathbf{x}\langle A, B \rangle} Y^{\text{INT}\langle B \rangle} = 0 \quad (5.24)$$

$$-X^{\langle A, C \rangle} + \beta^{\mathbf{x}\langle A, C \rangle} Y^{\text{INT}\langle C \rangle} = 0 \quad (5.25)$$

$$-X^{\langle B, A \rangle} + \beta^{\mathbf{x}\langle B, A \rangle} Y^{\text{INT}\langle A \rangle} = 0 \quad (5.26)$$

$$-X^{\langle B, B \rangle} + \beta^{\mathbf{x}\langle B, B \rangle} Y^{\text{INT}\langle B \rangle} = 0 \quad (5.27)$$

$$-X^{\langle B, C \rangle} + \beta^{\mathbf{x}\langle B, C \rangle} Y^{\text{INT}\langle C \rangle} = 0 \quad (5.28)$$

$$-X^{\langle C, A \rangle} + \beta^{\mathbf{x}\langle C, A \rangle} Y^{\text{INT}\langle A \rangle} = 0 \quad (5.29)$$

$$-X^{\langle C, B \rangle} + \beta^{\mathbf{x}\langle C, B \rangle} Y^{\text{INT}\langle B \rangle} = 0 \quad (5.30)$$

$$-X^{\langle C, C \rangle} + \beta^{\mathbf{x}\langle C, C \rangle} Y^{\text{INT}\langle C \rangle} = 0 \quad (5.31)$$

$$-Y^{\langle A \rangle} + Y^{\text{VA}\langle A \rangle} = 0 \quad (5.32)$$

$$-Y^{\langle A \rangle} + Y^{\text{INT}\langle A \rangle} = 0 \quad (5.33)$$

$$-Y^{\langle B \rangle} + Y^{\text{VA}\langle B \rangle} = 0 \quad (5.34)$$

$$-Y^{\langle B \rangle} + Y^{\text{INT}\langle B \rangle} = 0 \quad (5.35)$$

$$-Y^{\langle C \rangle} + Y^{\text{VA}\langle C \rangle} = 0 \quad (5.36)$$

$$-Y^{\langle C \rangle} + Y^{\text{INT}\langle C \rangle} = 0 \quad (5.37)$$

$$-Y^{\text{VA}\langle\text{A}\rangle} + \gamma^{\text{yva}\langle\text{A}\rangle} K^{\langle\text{A}\rangle\beta^{\text{k}\langle\text{A}\rangle}} L^{\langle\text{A}\rangle\beta^{\text{l}\langle\text{A}\rangle}} = 0 \quad (5.38)$$

$$-Y^{\text{VA}\langle\text{B}\rangle} + \gamma^{\text{yva}\langle\text{B}\rangle} K^{\langle\text{B}\rangle\beta^{\text{k}\langle\text{B}\rangle}} L^{\langle\text{B}\rangle\beta^{\text{l}\langle\text{B}\rangle}} = 0 \quad (5.39)$$

$$-Y^{\text{VA}\langle\text{C}\rangle} + \gamma^{\text{yva}\langle\text{C}\rangle} K^{\langle\text{C}\rangle\beta^{\text{k}\langle\text{C}\rangle}} L^{\langle\text{C}\rangle\beta^{\text{l}\langle\text{C}\rangle}} = 0 \quad (5.40)$$

$$\lambda^{\text{CONSUMER}^1\langle\text{l}\rangle} p^{\langle\text{A}\rangle} + \alpha^{\langle\text{A,l}\rangle} D^{\langle\text{A,l}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,l}\rangle} D^{\langle\text{A,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,l}\rangle} D^{\langle\text{B,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,l}\rangle} D^{\langle\text{C,l}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.41)$$

$$\lambda^{\text{CONSUMER}^1\langle\text{l}\rangle} p^{\langle\text{B}\rangle} + \alpha^{\langle\text{B,l}\rangle} D^{\langle\text{B,l}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,l}\rangle} D^{\langle\text{A,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,l}\rangle} D^{\langle\text{B,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,l}\rangle} D^{\langle\text{C,l}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.42)$$

$$\lambda^{\text{CONSUMER}^1\langle\text{l}\rangle} p^{\langle\text{C}\rangle} + \alpha^{\langle\text{C,l}\rangle} D^{\langle\text{C,l}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,l}\rangle} D^{\langle\text{A,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,l}\rangle} D^{\langle\text{B,l}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,l}\rangle} D^{\langle\text{C,l}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.43)$$

$$\infty \quad \lambda^{\text{CONSUMER}^1\langle\text{s}\rangle} p^{\langle\text{A}\rangle} + \alpha^{\langle\text{A,s}\rangle} D^{\langle\text{A,s}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,s}\rangle} D^{\langle\text{A,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,s}\rangle} D^{\langle\text{B,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,s}\rangle} D^{\langle\text{C,s}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.44)$$

$$\lambda^{\text{CONSUMER}^1\langle\text{s}\rangle} p^{\langle\text{B}\rangle} + \alpha^{\langle\text{B,s}\rangle} D^{\langle\text{B,s}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,s}\rangle} D^{\langle\text{A,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,s}\rangle} D^{\langle\text{B,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,s}\rangle} D^{\langle\text{C,s}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.45)$$

$$\lambda^{\text{CONSUMER}^1\langle\text{s}\rangle} p^{\langle\text{C}\rangle} + \alpha^{\langle\text{C,s}\rangle} D^{\langle\text{C,s}\rangle-1+\omega^{-1}(-1+\omega)} \left(\alpha^{\langle\text{A,s}\rangle} D^{\langle\text{A,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{B,s}\rangle} D^{\langle\text{B,s}\rangle\omega^{-1}(-1+\omega)} + \alpha^{\langle\text{C,s}\rangle} D^{\langle\text{C,s}\rangle\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.46)$$

$$-K^{\text{f}} + \Pi + \alpha w^{\text{f}} KS = 0 \quad (5.47)$$

$$-KS + K^{\langle\text{A}\rangle} + K^{\langle\text{B}\rangle} + K^{\langle\text{C}\rangle} = 0 \quad (5.48)$$

$$-\Pi + \pi^{\langle\text{A}\rangle} + \pi^{\langle\text{B}\rangle} + \pi^{\langle\text{C}\rangle} = 0 \quad (5.49)$$

$$CAP^{\langle\text{l}\rangle} - INC^{\langle\text{l}\rangle} + p^{\text{k}} K^{\langle\text{l}\rangle} + p^{\text{l}} L^{\langle\text{l}\rangle} = 0 \quad (5.50)$$

$$CAP^{\langle\text{s}\rangle} - INC^{\langle\text{s}\rangle} + p^{\text{k}} K^{\langle\text{s}\rangle} + p^{\text{l}} L^{\langle\text{s}\rangle} = 0 \quad (5.51)$$

$$-INC^{\langle l \rangle} + p^{\langle A \rangle} D^{\langle A, l \rangle} + p^{\langle B \rangle} D^{\langle B, l \rangle} + p^{\langle C \rangle} D^{\langle C, l \rangle} = 0 \quad (5.52)$$

$$-INC^{\langle s \rangle} + p^{\langle A \rangle} D^{\langle A, s \rangle} + p^{\langle B \rangle} D^{\langle B, s \rangle} + p^{\langle C \rangle} D^{\langle C, s \rangle} = 0 \quad (5.53)$$

$$\pi^{\langle A \rangle} + p^{kc} K^{\langle A \rangle} + p^{lc} L^{\langle A \rangle} - p^{\langle A \rangle} Y^{\langle A \rangle} + Y^{\text{INT}\langle A \rangle} \left(\beta^{x\langle A, A \rangle -1} p^{\langle A \rangle} + \beta^{x\langle B, A \rangle -1} p^{\langle B \rangle} + \beta^{x\langle C, A \rangle -1} p^{\langle C \rangle} \right) = 0 \quad (5.54)$$

$$\pi^{\langle B \rangle} + p^{kc} K^{\langle B \rangle} + p^{lc} L^{\langle B \rangle} - p^{\langle B \rangle} Y^{\langle B \rangle} + Y^{\text{INT}\langle B \rangle} \left(\beta^{x\langle A, B \rangle -1} p^{\langle A \rangle} + \beta^{x\langle B, B \rangle -1} p^{\langle B \rangle} + \beta^{x\langle C, B \rangle -1} p^{\langle C \rangle} \right) = 0 \quad (5.55)$$

$$\pi^{\langle C \rangle} + p^{kc} K^{\langle C \rangle} + p^{lc} L^{\langle C \rangle} - p^{\langle C \rangle} Y^{\langle C \rangle} + Y^{\text{INT}\langle C \rangle} \left(\beta^{x\langle A, C \rangle -1} p^{\langle A \rangle} + \beta^{x\langle B, C \rangle -1} p^{\langle B \rangle} + \beta^{x\langle C, C \rangle -1} p^{\langle C \rangle} \right) = 0 \quad (5.56)$$

6 Calibrating equations

$$-k^{\text{fdata}} + K^{\text{f}} = 0 \quad (6.1)$$

$$-l^{\text{data}\langle A \rangle} + L^{\langle A \rangle} = 0 \quad (6.2)$$

$$-l^{\text{data}\langle B \rangle} + L^{\langle B \rangle} = 0 \quad (6.3)$$

$$-l^{\text{data}\langle C \rangle} + L^{\langle C \rangle} = 0 \quad (6.4)$$

$$-x^{\text{data}\langle A, A \rangle} + X^{\langle A, A \rangle} = 0 \quad (6.5)$$

$$-x^{\text{data}\langle A, B \rangle} + X^{\langle A, B \rangle} = 0 \quad (6.6)$$

$$-x^{\text{data}\langle A, C \rangle} + X^{\langle A, C \rangle} = 0 \quad (6.7)$$

$$-x^{\text{data}\langle B, A \rangle} + X^{\langle B, A \rangle} = 0 \quad (6.8)$$

$$-x^{\text{data}\langle B, B \rangle} + X^{\langle B, B \rangle} = 0 \quad (6.9)$$

$$-x^{\text{data}\langle B, C \rangle} + X^{\langle B, C \rangle} = 0 \quad (6.10)$$

$$-x^{\text{data}\langle\text{C,A}\rangle} + X^{\langle\text{C,A}\rangle} = 0 \quad (6.11)$$

$$-x^{\text{data}\langle\text{C,B}\rangle} + X^{\langle\text{C,B}\rangle} = 0 \quad (6.12)$$

$$-x^{\text{data}\langle\text{C,C}\rangle} + X^{\langle\text{C,C}\rangle} = 0 \quad (6.13)$$

$$-y^{\text{data}\langle\text{A}\rangle} + Y^{\text{VA}\langle\text{A}\rangle} = 0 \quad (6.14)$$

$$-y^{\text{data}\langle\text{B}\rangle} + Y^{\text{VA}\langle\text{B}\rangle} = 0 \quad (6.15)$$

$$-y^{\text{data}\langle\text{C}\rangle} + Y^{\text{VA}\langle\text{C}\rangle} = 0 \quad (6.16)$$

$$CAP^{\langle\text{l}\rangle} - \text{cap}^{\text{data}\langle\text{l}\rangle} \text{scale}^{\langle\text{l}\rangle-1} = 0 \quad (6.17)$$

$$D^{\langle\text{B,l}\rangle} - d^{\text{data}\langle\text{B,l}\rangle} \text{scale}^{\langle\text{l}\rangle-1} = 0 \quad (6.18)$$

$$D^{\langle\text{B,s}\rangle} - d^{\text{data}\langle\text{B,s}\rangle} \text{scale}^{\langle\text{s}\rangle-1} = 0 \quad (6.19)$$

$$D^{\langle\text{C,l}\rangle} - d^{\text{data}\langle\text{C,l}\rangle} \text{scale}^{\langle\text{l}\rangle-1} = 0 \quad (6.20)$$

$$D^{\langle\text{C,s}\rangle} - d^{\text{data}\langle\text{C,s}\rangle} \text{scale}^{\langle\text{s}\rangle-1} = 0 \quad (6.21)$$

$$K^{\langle\text{l}\rangle} - k^{\text{data}\langle\text{l}\rangle} \text{scale}^{\langle\text{l}\rangle-1} = 0 \quad (6.22)$$

$$K^{\langle\text{s}\rangle} - k^{\text{data}\langle\text{s}\rangle} \text{scale}^{\langle\text{s}\rangle-1} = 0 \quad (6.23)$$

$$L^{\langle\text{l}\rangle} - l^{\text{data}\langle\text{l}\rangle} \text{scale}^{\langle\text{l}\rangle-1} = 0 \quad (6.24)$$

$$-1 + \beta^{\text{k}\langle\text{A}\rangle} + \beta^{\text{l}\langle\text{A}\rangle} = 0 \quad (6.25)$$

$$-1 + \beta^{\text{k}\langle\text{B}\rangle} + \beta^{\text{l}\langle\text{B}\rangle} = 0 \quad (6.26)$$

$$-1 + \beta^{\mathbf{k}\langle\mathbf{C}\rangle} + \beta^{\mathbf{l}\langle\mathbf{C}\rangle} = 0 \quad (6.27)$$

$$-1 + \alpha w^{\langle\mathbf{l}\rangle} + \alpha w^{\langle\mathbf{s}\rangle} = 0 \quad (6.28)$$

$$-1 + \alpha f^{\langle\mathbf{l}\rangle} + \alpha f^{\langle\mathbf{s}\rangle} = 0 \quad (6.29)$$

$$-1 + \alpha^{\langle\mathbf{A},\mathbf{l}\rangle\omega} + \alpha^{\langle\mathbf{B},\mathbf{l}\rangle\omega} + \alpha^{\langle\mathbf{C},\mathbf{l}\rangle\omega} = 0 \quad (6.30)$$

$$-1 + \alpha^{\langle\mathbf{A},\mathbf{s}\rangle\omega} + \alpha^{\langle\mathbf{B},\mathbf{s}\rangle\omega} + \alpha^{\langle\mathbf{C},\mathbf{s}\rangle\omega} = 0 \quad (6.31)$$

7 Equilibrium values

	Equilibrium value
p^k	1
p^l	1
p^{kc}	1
p^{lc}	1
K^f	80.9217
KS	163.4388
LS	88.233
Π	0
$\lambda^{\text{CONSUMER}^{(l)}}$	-1
$\lambda^{\text{CONSUMER}^{(s)}}$	-1
$p^{(A)}$	1
$p^{(B)}$	1
$p^{(C)}$	1
$\pi^{(A)}$	0
$\pi^{(B)}$	0
$\pi^{(C)}$	0
$CAP^{(l)}$	5.1044
$CAP^{(s)}$	6.0504
$D^{(A,l)}$	9.9136
$D^{(A,s)}$	4.044
$D^{(B,l)}$	6.6431
$D^{(B,s)}$	3.1495
$D^{(C,l)}$	9.9581
$D^{(C,s)}$	7.3678
$INC^{(l)}$	26.5147
$INC^{(s)}$	14.5613
$K^{(l)}$	10.5158
$K^{(s)}$	4.0454
$K^{(A)}$	38.0989
$K^{(B)}$	57.5316
$K^{(C)}$	67.8083
$L^{(l)}$	10.8945
$L^{(s)}$	4.4655
$L^{(A)}$	9.4287
$L^{(B)}$	39.9867
$L^{(C)}$	38.8177
$U^{(l)}$	26.5147
$U^{(s)}$	14.5613
$X^{(A,A)}$	73.3234
$X^{(A,B)}$	43.0305
$X^{(A,C)}$	31.0471
$X^{(B,A)}$	61.3203
$X^{(B,B)}$	119.9318
$X^{(B,C)}$	105.2489
$X^{(C,A)}$	45.3241
$X^{(C,B)}$	84.0875
$X^{(C,C)}$	228.0633
$Y^{(A)}$	227.4954
$Y^{(B)}$	344.5681
$Y^{(C)}$	470.9853
$Y^{\text{VA}^{(A)}}$	227.4954
$Y^{\text{VA}^{(B)}}$	344.5681
$Y^{\text{VA}^{(C)}}$	470.9853
$Y^{\text{INT}^{(A)}}$	227.4954
$Y^{\text{INT}^{(B)}}$	344.5681
$Y^{\text{INT}^{(C)}}$	470.9853

8 Model parameters

	Value
k^{fdata}	80.9217
ks^{data}	163.4388
ls^{data}	88.233
ω	2
αx^{f}	0.4951
$\alpha^{\langle \text{A}, \text{l} \rangle}$	0.6115
$\alpha^{\langle \text{A}, \text{s} \rangle}$	0.527
$\alpha^{\langle \text{B}, \text{l} \rangle}$	0.5005
$\alpha^{\langle \text{B}, \text{s} \rangle}$	0.4651
$\alpha^{\langle \text{C}, \text{l} \rangle}$	0.6128
$\alpha^{\langle \text{C}, \text{s} \rangle}$	0.7113
$\beta^{\text{k} \langle \text{A} \rangle}$	0.8016
$\beta^{\text{k} \langle \text{B} \rangle}$	0.59
$\beta^{\text{k} \langle \text{C} \rangle}$	0.6359
$\beta^{\text{l} \langle \text{A} \rangle}$	0.1984
$\beta^{\text{l} \langle \text{B} \rangle}$	0.41
$\beta^{\text{l} \langle \text{C} \rangle}$	0.3641
$\beta^{\text{x} \langle \text{A}, \text{A} \rangle}$	3.1026
$\beta^{\text{x} \langle \text{A}, \text{B} \rangle}$	8.0075
$\beta^{\text{x} \langle \text{A}, \text{C} \rangle}$	15.17
$\beta^{\text{x} \langle \text{B}, \text{A} \rangle}$	3.7099
$\beta^{\text{x} \langle \text{B}, \text{B} \rangle}$	2.873
$\beta^{\text{x} \langle \text{B}, \text{C} \rangle}$	4.475
$\beta^{\text{x} \langle \text{C}, \text{A} \rangle}$	5.0193
$\beta^{\text{x} \langle \text{C}, \text{B} \rangle}$	4.0977
$\beta^{\text{x} \langle \text{C}, \text{C} \rangle}$	2.0652
$\alpha p^{\text{data} \langle \text{l} \rangle}$	20.4174
$d^{\text{data} \langle \text{B}, \text{l} \rangle}$	26.5723
$d^{\text{data} \langle \text{B}, \text{s} \rangle}$	31.4947
$d^{\text{data} \langle \text{C}, \text{l} \rangle}$	39.8322
$d^{\text{data} \langle \text{C}, \text{s} \rangle}$	73.6782
$\gamma^{\text{yva} \langle \text{A} \rangle}$	7.8772
$\gamma^{\text{yva} \langle \text{B} \rangle}$	6.9527
$\gamma^{\text{yva} \langle \text{C} \rangle}$	8.5098
$k^{\text{data} \langle \text{l} \rangle}$	42.0633
$k^{\text{data} \langle \text{s} \rangle}$	40.4538
$l^{\text{data} \langle \text{l} \rangle}$	43.5782
$l^{\text{data} \langle \text{A} \rangle}$	9.4287
$l^{\text{data} \langle \text{B} \rangle}$	39.9867
$l^{\text{data} \langle \text{C} \rangle}$	38.8177
$\alpha w^{\langle \text{l} \rangle}$	0.2574
$\alpha w^{\langle \text{s} \rangle}$	0.2475
$\alpha w w^{\langle \text{l} \rangle}$	0.4939
$\alpha w w^{\langle \text{s} \rangle}$	0.5061
$\alpha w f^{\langle \text{l} \rangle}$	0.2523
$\alpha w f^{\langle \text{s} \rangle}$	0.7477
$scale^{\langle \text{l} \rangle}$	4
$scale^{\langle \text{s} \rangle}$	10
$x^{\text{data} \langle \text{A}, \text{A} \rangle}$	73.3234
$x^{\text{data} \langle \text{A}, \text{B} \rangle}$	43.0305
$x^{\text{data} \langle \text{A}, \text{C} \rangle}$	31.0471
$x^{\text{data} \langle \text{B}, \text{A} \rangle}$	61.3203
$x^{\text{data} \langle \text{B}, \text{B} \rangle}$	119.9318
$x^{\text{data} \langle \text{B}, \text{C} \rangle}$	105.2489