

## 1 CONSUMER

### 1.1 Optimization problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + p_t I_t = \pi_t^C + \pi_t^I + K_{t-1}^s r_t + L_t^s W_t \quad (\lambda_t^{\text{CONSUMER}^1}) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad (\lambda_t^{\text{CONSUMER}^2}) \quad (1.3)$$

### 1.2 Identities

$$Y_t = C_t + p_t I_t \quad (1.4)$$

### 1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) E_t [\lambda_{t+1}^{\text{CONSUMER}^2}] + E_t [\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1}] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu = 0 \quad (L_t^s) \quad (1.7)$$

$$\lambda_t^{\text{CONSUMER}^2} - \lambda_t^{\text{CONSUMER}^1} p_t = 0 \quad (I_t) \quad (1.8)$$

## 2 FIRM C

### 2.1 Optimization problem

$$\max_{K_t^{C^d}, L_t^{C^d}, C_t^s} \pi_t^C = C_t^s - L_t^{C^d} W_t - r_t K_t^{C^d} \quad (2.1)$$

s.t. :

$$C_t^s = Z_t K_t^{C^d \alpha} L_t^{C^d 1-\alpha} \quad (\lambda_t^{\text{FIRM}^{C^1}}) \quad (2.2)$$

### 2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^{C^1}} Z_t K_t^{C^d-1+\alpha} L_t^{C^d 1-\alpha} = 0 \quad (K_t^{C^d}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^{C^1}} Z_t (1 - \alpha) K_t^{C^d \alpha} L_t^{C^d-\alpha} = 0 \quad (L_t^{C^d}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^{C^1}} = 0 \quad (C_t^s) \quad (2.5)$$

### 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{C^d-1+\alpha} L_t^{C^d 1-\alpha} = 0 \quad (K_t^{C^d}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{C^d \alpha} L_t^{C^d-\alpha} = 0 \quad (L_t^{C^d}) \quad (2.7)$$

### 3 FIRM I

#### 3.1 Optimization problem

$$\max_{K_t^{I^d}, L_t^{I^d}, I_t^s} \pi_t^I = I_t^s - L_t^{I^d} W_t - r_t K_t^{I^d} \quad (3.1)$$

s.t. :

$$I_t^s = Z_t K_t^{I^d \sigma} L_t^{I^d 1-\alpha} (\lambda_t^{\text{FIRM}^I}) \quad (3.2)$$

#### 3.2 First order conditions

$$-r_t + \sigma \lambda_t^{\text{FIRM}^I} Z_t K_t^{I^d -1 + \sigma} L_t^{I^d 1-\alpha} = 0 \quad (K_t^{I^d}) \quad (3.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^I} Z_t (1 - \alpha) K_t^{I^d \sigma} L_t^{I^d -\alpha} = 0 \quad (L_t^{I^d}) \quad (3.4)$$

$$1 - \lambda_t^{\text{FIRM}^I} = 0 \quad (I_t^s) \quad (3.5)$$

#### 3.3 First order conditions after reduction

$$-r_t + \sigma Z_t K_t^{I^d -1 + \sigma} L_t^{I^d 1-\alpha} = 0 \quad (K_t^{I^d}) \quad (3.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{I^d \sigma} L_t^{I^d -\alpha} = 0 \quad (L_t^{I^d}) \quad (3.7)$$

## 4 EQUILIBRIUM

#### 4.1 Identities

$$K_t^{C^d} + K_t^{I^d} = K_{t-1}^s \quad (4.1)$$

$$L_t^{C^d} + L_t^{I^d} = L_t^s \quad (4.2)$$

$$C_t = C_t^s \quad (4.3)$$

## 5 EXOG

#### 5.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (5.1)$$

## 6 Equilibrium relationships

$$-r_t + \alpha Z_t K_t^{C^d -1 + \alpha} L_t^{C^d 1-\alpha} = 0 \quad (6.1)$$

$$-r_t + \sigma Z_t K_t^{I^d -1 + \sigma} L_t^{I^d 1-\alpha} = 0 \quad (6.2)$$

$$-C_t + Z_t K_t^{C^d \alpha} L_t^{C^d 1-\alpha} = 0 \quad (6.3)$$

$$-I_t^s + Z_t K_t^{I^d \sigma} L_t^{I^d 1-\alpha} = 0 \quad (6.4)$$

$$-W_t + Z_t (1 - \alpha) K_t^{C^d \alpha} L_t^{C^d -\alpha} = 0 \quad (6.5)$$

$$-W_t + Z_t (1 - \alpha) K_t^{I^d \sigma} L_t^{I^d -\alpha} = 0 \quad (6.6)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (6.7)$$

$$\beta \left( \mu \mathbb{E}_t \left[ r_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] + \mu (1 - \delta) \mathbb{E}_t \left[ p_{t+1} (1 - L_{t+1}^s)^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} C_{t+1} \right] \right. \\ \left. (-1 + \mu) (1 - L_t^s)^{-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \right. \quad (6.8)$$

$$(-1 + \mu) (1 - L_t^s)^{-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (6.9)$$

$$-K_{t-1}^s + K_t^{C^d} + K_t^{I^d} = 0 \quad (6.10)$$

$$-C_t + Y_t - p_t I_t = 0 \quad (6.11)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (6.12)$$

$$-L_t^s + L_t^{C^d} + L_t^{I^d} = 0 \quad (6.13)$$

$$U_t - \beta \mathbb{E}_t [U_{t+1}] - (1 - \eta)^{-1} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} = 0 \quad (6.14)$$

$$I_t^s + K_{t-1}^s r_t - p_t I_t - r_t K_t^{C^d} - r_t K_t^{I^d} + L_t^s W_t - L_t^{C^d} W_t - L_t^{I^d} W_t = 0 \quad (6.15)$$

## 7 Steady state relationships

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{C^d-1+\alpha} L_{ss}^{C^d 1-\alpha} = 0 \quad (7.1)$$

$$-r_{ss} + \sigma Z_{ss} K_{ss}^{I^d-1+\sigma} L_{ss}^{I^d 1-\alpha} = 0 \quad (7.2)$$

$$-C_{ss} + Z_{ss} K_{ss}^{C^d \alpha} L_{ss}^{C^d 1-\alpha} = 0 \quad (7.3)$$

$$-I_{ss}^s + Z_{ss} K_{ss}^{I^d \sigma} L_{ss}^{I^d 1-\alpha} = 0 \quad (7.4)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{C^d \alpha} L_{ss}^{C^d -\alpha} = 0 \quad (7.5)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{I^d \sigma} L_{ss}^{I^d -\alpha} = 0 \quad (7.6)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (7.7)$$

$$\beta \left( \mu r_{ss} (1 - L_{ss}^s)^{1-\mu} \left( (1 - L_{ss}^s)^{1-\mu} C_{ss}^\mu \right)^{-\eta} C_{ss}^{-1+\mu} + \mu p_{ss} (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) - \mu p_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \\ (-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (7.8)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (7.9)$$

$$-K_{ss}^s + K_{ss}^{C^d} + K_{ss}^{I^d} = 0 \quad (7.10)$$

$$-C_{ss} + Y_{ss} - p_{ss} I_{ss} = 0 \quad (7.11)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (7.12)$$

$$-L_{ss}^s + L_{ss}^{C^d} + L_{ss}^{I^d} = 0 \quad (7.13)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.14)$$

$$I_{ss}^s - p_{ss} I_{ss} + r_{ss} K_{ss}^s - r_{ss} K_{ss}^{C^d} - r_{ss} K_{ss}^{I^d} + L_{ss}^s W_{ss} - L_{ss}^{C^d} W_{ss} - L_{ss}^{I^d} W_{ss} = 0 \quad (7.15)$$

## 8 Parameter settings

$$\alpha = 0.2 \quad (8.1)$$

$$\beta = 0.99 \quad (8.2)$$

$$\delta = 0.025 \quad (8.3)$$

$$\eta = 2 \quad (8.4)$$

$$\mu = 0.3 \quad (8.5)$$

$$\phi = 0.95 \quad (8.6)$$

$$\sigma = 0.4 \quad (8.7)$$

## 9 Steady state values

	Steady state values
$p$	1.5318
$r$	0.0538
$C$	0.3374
$I$	0.0439
$I^s$	0.0672
$K^s$	1.7551
$K^{C^d}$	1.2551
$K^{I^d}$	0.5
$L^s$	0.2914
$L^{C^d}$	0.243
$L^{I^d}$	0.0484
$U$	-176.3002
$W$	1.111
$Y$	0.4046
$Z$	1

## 10 The solution of the perturbation

### 10.1 P

$$\begin{matrix} K^s & Z_{t-1} \\ Z & \begin{pmatrix} 0.9522 & -0.0054 \\ 0 & 0.95 \end{pmatrix} \end{matrix}$$

### 10.2 Q

$$\begin{matrix} \epsilon^Z \\ K^s & \\ Z & \begin{pmatrix} -0.0056 \\ 1 \end{pmatrix} \end{matrix}$$

### 10.3 R

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ \begin{matrix} p \\ r \\ C \\ I \\ I^s \\ K^{C^d} \\ K^{I^d} \\ L^s \\ L^{C^d} \\ L^{I^d} \\ U \\ W \\ Y \end{matrix} & \begin{pmatrix} -0.1506 & 1.1645 \\ -1.0646 & 0.95 \\ 0.3338 & 0.95 \\ -0.9139 & -0.2145 \\ -1.0646 & 0.95 \\ 1.3984 & 0 \\ 0 & 0 \\ -0.1646 & 0 \\ 0.0677 & 0 \\ -1.3307 & 0 \\ -0.0257 & -0.0456 \\ 0.2661 & 0.95 \\ 0.1015 & 0.95 \end{pmatrix} \end{matrix}$$

### 10.4 S

$$\begin{matrix} & \epsilon^Z \\ \begin{matrix} p \\ r \\ C \\ I \\ I^s \\ K^{C^d} \\ K^{I^d} \\ L^s \\ L^{C^d} \\ L^{I^d} \\ U \\ W \\ Y \end{matrix} & \begin{pmatrix} 1.2258 \\ 1 \\ 1 \\ -0.2258 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.048 \\ 1 \\ 1 \end{pmatrix} \end{matrix}$$

## 11 Statistics of the model

### 11.1 Moments

	Steady state value	Std. dev.	Variance	Loglinear
$p$	1.5318	0.113	0.0128	Y
$r$	0.0538	0.0922	0.0085	Y
$C$	0.3374	0.0922	0.0085	Y
$I$	0.0439	0.0208	0.0004	Y
$I^s$	0.0672	0.0922	0.0085	Y
$K^s$	1.7551	0.0018	0	Y
$L^s$	0.2914	0.0003	0	Y
$U$	-176.3002	0.0044	0	Y
$W$	1.111	0.0922	0.0085	Y
$Y$	0.4046	0.0922	0.0085	Y

## 11.2 Correlation matrix

	$p$	$r$	$C$	$I$	$I^s$	$K^s$	$L^s$	$U$	$W$	$Y$
$p$	1	0.9998	1	-0.9966	0.9998	-0.3088	0.0262	-0.9999	1	1
$r$	0.9998	1	0.9996	-0.9948	1	-0.3265	0.0449	-0.9995	0.9997	0.9997
$C$	1	0.9996	1	-0.9973	0.9996	-0.3001	0.0172	-1	1	1
$I$	-0.9966	-0.9948	-0.9973	1	-0.9948	0.229	0.0566	0.9976	-0.9972	-0.9969
$I^s$	0.9998	1	0.9996	-0.9948	1	-0.3265	0.0449	-0.9995	0.9997	0.9997
$K^s$	-0.3088	-0.3265	-0.3001	0.229	-0.3265	1	-0.9589	0.2963	-0.3014	-0.3045
$K^{C^d}$	-0.0262	-0.0449	-0.0172	-0.0566	-0.0449	0.9589	-1	0.0131	-0.0185	-0.0218
$K^{I^d}$	0	0	0	0	0	0	0	0	0	0
$L^s$	0.0262	0.0449	0.0172	0.0566	0.0449	-0.9589	1	-0.0131	0.0185	0.0218
$L^{C^d}$	-0.0262	-0.0449	-0.0172	-0.0566	-0.0449	0.9589	-1	0.0131	-0.0185	-0.0218
$L^{I^d}$	0.0262	0.0449	0.0172	0.0566	0.0449	-0.9589	1	-0.0131	0.0185	0.0218
$U$	-0.9999	-0.9995	-1	0.9976	-0.9995	0.2963	-0.0131	1	-1	-1
$W$	1	0.9997	1	-0.9972	0.9997	-0.3014	0.0185	-1	1	1
$Y$	1	0.9997	1	-0.9969	0.9997	-0.3045	0.0218	-1	1	1
$Z$	1	0.9998	1	-0.9968	0.9998	-0.3065	0.0238	-0.9999	1	1

## 11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
$p$	0.7135	0.4715	0.2715	0.1102	-0.016
$r$	0.7152	0.4742	0.2746	0.1134	-0.0131
$C$	0.7127	0.4703	0.2701	0.1088	-0.0173
$I$	0.708	0.4629	0.2615	0.1001	-0.0252
$I^s$	0.7152	0.4742	0.2746	0.1134	-0.0131
$K^s$	0.9589	0.8598	0.7228	0.5647	0.3987
$L^s$	0.9589	0.8598	0.7228	0.5647	0.3987
$U$	0.7124	0.4698	0.2695	0.1082	-0.0178
$W$	0.7128	0.4704	0.2703	0.109	-0.0171
$Y$	0.7131	0.4709	0.2708	0.1095	-0.0166

## 12 Statistics of the model

### 12.1 Moments relative to moments of the reference variable

	Steady state value relative to $Y$	Std. dev. relative to $Y$	Variance relative to $Y$	Loglinear
$p$	3.7858	1.226	1.503	Y
$r$	0.1329	1.0008	1.0015	Y
$C$	0.8339	0.9999	0.9998	Y
$I$	0.1084	0.2261	0.0511	Y
$I^s$	0.1661	1.0008	1.0015	Y
$K^s$	4.3375	0.0199	0.0004	Y
$K^{C^d}$	3.1018	0.0278	0.0008	Y
$K^{I^d}$	1.2357	0	0	Y
$L^s$	0.7201	0.0033	0	Y
$L^{C^d}$	0.6005	0.0013	0	Y
$L^{I^d}$	0.1196	0.0265	0.0007	Y
$U$	-435.7117	0.048	0.0023	Y
$W$	2.7457	0.9999	0.9999	Y
$Y$	1	1	1	Y
$Z$	2.4714	1	1.0001	Y

## 12.2 Correlations with the reference variable

	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$p$	-0.0189	0.1073	0.2688	0.4693	0.7122	1	0.7144	0.473	0.2734	0.1124	-0.0137
$r$	-0.0284	0.098	0.2604	0.4626	0.7082	0.9997	0.7196	0.4819	0.2845	0.1245	-0.0015
$C$	-0.0143	0.1118	0.2728	0.4725	0.714	1	0.7118	0.4686	0.268	0.1065	-0.0196
$I$	-0.0234	-0.1479	-0.3046	-0.4968	-0.727	-0.9969	-0.6883	-0.4316	-0.2231	-0.0583	0.0677
$I^s$	-0.0284	0.098	0.2604	0.4626	0.7082	0.9997	0.7196	0.4819	0.2845	0.1245	-0.0015
$K^s$	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515	-0.616
$K^{C^d}$	0.5104	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515
$K^{I^d}$	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
$L^s$	-0.5104	-0.491	-0.4367	-0.3392	-0.1896	0.0218	0.3045	0.4925	0.6029	0.6512	0.6515
$L^{C^d}$	0.5104	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515
$L^{I^d}$	-0.5104	-0.491	-0.4367	-0.3392	-0.1896	0.0218	0.3045	0.4925	0.6029	0.6512	0.6515
$U$	0.0122	-0.1138	-0.2746	-0.4739	-0.7148	-1	-0.7106	-0.4667	-0.2656	-0.1039	0.0223
$W$	-0.0149	0.1111	0.2722	0.472	0.7138	1	0.7121	0.4693	0.2688	0.1074	-0.0188
$Y$	-0.0166	0.1095	0.2708	0.4709	0.7131	1	0.7131	0.4709	0.2708	0.1095	-0.0166
$Z$	-0.0177	0.1085	0.2699	0.4702	0.7127	1	0.7137	0.4718	0.272	0.1108	-0.0153

# 13 Impulse response functions

## 13.1 Shock $\epsilon^Z$

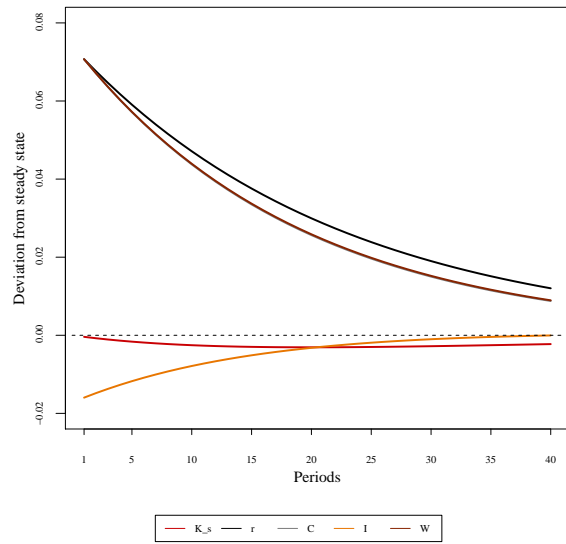


Figure 1: Impulse response function for  $\epsilon^Z$  shock

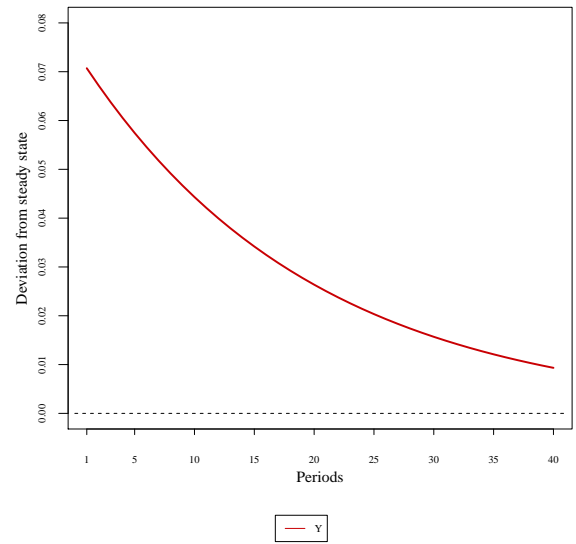


Figure 2: Impulse response function for  $\epsilon^Z$  shock