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Index sets

$$IND = \{\mathrm{H}, \mathrm{M}\}$$

1 CONSUMER

1.1 Optimisation problem

$$\max_{\left(K_t^{\langle i\rangle}\right)_{i\in IND},\left(C_t^{\langle i\rangle}\right)_{i\in IND},\left(N_t^{\langle i\rangle}\right)_{i\in IND},\left(I_t^{\langle i\rangle}\right)_{i\in IND}} U_t = \beta \mathrm{E}_t\left[U_{t+1}\right] + \log\left(1 - \sum_{i\in IND} N_t^{\langle i\rangle}\right)(1-b) + be^{-1}\log\left(aC_t^{\langle \mathrm{M}\rangle\, e} + (1-a)\, C_t^{\langle \mathrm{H}\rangle\, e}\right) \tag{1.1}$$

s.t. :

$$C_t^{\langle \mathrm{M}\rangle} + \sum_{i\in IND} I_t^{\langle i\rangle} = \pi_t + r_t K_{t-1}^{\langle \mathrm{M}\rangle} + W_t N_t^{\langle \mathrm{M}\rangle} \quad \left(\lambda_t^{\mathrm{CONSUMER}^1}\right) \tag{1.2}$$

$$i \in IND\colon \quad K_t^{\langle i\rangle} = I_t^{\langle i\rangle} + K_{t-1}^{\langle i\rangle}(1-\delta) \quad \left(\lambda_t^{\mathrm{CONSUMER}^{2\,\langle i\rangle}}\right) \tag{1.3}$$

$$C_t^{\langle \mathrm{H}\rangle} = \Gamma Z_t^{\langle \mathrm{H}\rangle} K_{t-1}^{\langle \mathrm{H}\rangle\,\theta} N_t^{\langle \mathrm{H}\rangle\, 1-\theta} \quad \left(\lambda_t^{\mathrm{CONSUMER}^3}\right) \tag{1.4}$$

1.2 Identities

$$K_t = \sum_{i\in IND} K_t^{\langle i\rangle} \tag{1.5}$$

$$I_t = \sum_{i\in IND} I_t^{\langle i\rangle} \tag{1.6}$$

$$N_t = \sum_{i\in IND} N_t^{\langle i\rangle} \tag{1.7}$$

1.3 First order conditions

$$i \in IND\colon \quad -\lambda^{\mathrm{CONSUMER}^{2\,\langle i\rangle}}_{t} + \beta\left(\delta^{\langle \mathrm{M},i\rangle}\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}\right] + (1-\delta)\,\mathrm{E}_t\left[\lambda^{\mathrm{CONSUMER}^{2\,\langle i\rangle}}_{t+1}\right] + \delta^{\langle \mathrm{H},i\rangle}\theta\Gamma K_t^{\langle \mathrm{H}\rangle\,-1+\theta}\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^3} Z_{t+1}^{\langle \mathrm{H}\rangle} N_{t+1}^{\langle \mathrm{H}\rangle\, 1-\theta}\right]\right) = 0 \quad \left(K_t^{\langle i\rangle}\right) \tag{1.8}$$

$$i \in IND: \quad -\delta^{\langle \mathrm{M},i\rangle}\lambda_t^{\mathrm{CONSUMER}^1} - \delta^{\langle \mathrm{H},i\rangle}\lambda_t^{\mathrm{CONSUMER}^3} + be^{-1}\left(aC_t^{\langle \mathrm{M}\rangle\, e} + (1-a)\, C_t^{\langle \mathrm{H}\rangle\, e}\right)^{-1}\left(\delta^{\langle \mathrm{M},i\rangle} aeC_t^{\langle \mathrm{M}\rangle\, -1+e} + \delta^{\langle \mathrm{H},i\rangle} e\,(1-a)\, C_t^{\langle \mathrm{H}\rangle\, -1+e}\right) = 0 \quad \left(C_t^{\langle i\rangle}\right) \tag{1.9}$$

$$i \in IND: \quad -(1-b)\left(1 - \sum_{i\in IND} N_t^{\langle i\rangle}\right)^{-1} + \delta^{\langle \mathrm{M},i\rangle}\lambda_t^{\mathrm{CONSUMER}^1} W_t + \delta^{\langle \mathrm{H},i\rangle}\Gamma\lambda_t^{\mathrm{CONSUMER}^3} Z_t^{\langle \mathrm{H}\rangle}\,(1-\theta)\, K_{t-1}^{\langle \mathrm{H}\rangle\,\theta} N_t^{\langle \mathrm{H}\rangle\, -\theta} = 0 \quad \left(N_t^{\langle i\rangle}\right) \tag{1.10}$$

$$i \in IND: \quad -\lambda_t^{\mathrm{CONSUMER}^1} + \lambda^{\mathrm{CONSUMER}^2}{}_t^{\langle i\rangle} = 0 \quad \left(I_t^{\langle i\rangle}\right) \tag{1.11}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\mathrm{m^d}}, N_t^{\mathrm{m^d}}, Y_t, \pi_t} \Pi_t = \pi_t \tag{2.1}$$

s.t. :

$$\pi_t = Y_t - N_t^{\mathrm{m^d}} W_t - r_t K_t^{\mathrm{m^d}} \quad \left(\lambda_t^{\mathrm{FIRM}^1}\right) \tag{2.2}$$

$$Y_t = \Gamma Z_t^{\langle \mathrm{M}\rangle} K_t^{\mathrm{m^d}\,\alpha} N_t^{\mathrm{m^d}\,1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM}^2}\right) \tag{2.3}$$

2.2 First order conditions

$$-\lambda_t^{\mathrm{FIRM}^1} r_t + \alpha\Gamma\lambda_t^{\mathrm{FIRM}^2} Z_t^{\langle \mathrm{M}\rangle} K_t^{\mathrm{m^d}\,-1+\alpha} N_t^{\mathrm{m^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{m^d}}\right) \tag{2.4}$$

$$-\lambda_t^{\mathrm{FIRM}^1} W_t + \Gamma\lambda_t^{\mathrm{FIRM}^2} Z_t^{\langle \mathrm{M}\rangle}\,(1-\alpha)\, K_t^{\mathrm{m^d}\,\alpha} N_t^{\mathrm{m^d}\,-\alpha} = 0 \quad \left(N_t^{\mathrm{m^d}}\right) \tag{2.5}$$

$$\lambda_t^{\mathrm{FIRM}^1} - \lambda_t^{\mathrm{FIRM}^2} = 0 \quad (Y_t) \tag{2.6}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad (\pi_t) \tag{2.7}$$

2.3 First order conditions after reduction

$$-r_t + \alpha\Gamma Z_t^{\langle \mathrm{M}\rangle} K_t^{\mathrm{m^d}\,-1+\alpha} N_t^{\mathrm{m^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{m^d}}\right) \tag{2.8}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M}\rangle}\,(1-\alpha)\, K_t^{\mathrm{m^d}\,\alpha} N_t^{\mathrm{m^d}\,-\alpha} = 0 \quad \left(N_t^{\mathrm{m^d}}\right) \tag{2.9}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{m^d}} = K_{t-1}^{\langle \mathrm{M} \rangle} \tag{3.1}$$

$$N_t^{\mathrm{m^d}} = N_t^{\langle \mathrm{M} \rangle} \tag{3.2}$$

4 EXOG

4.1 Identities

$$i \in IND: \quad Z_t^{\langle i \rangle} = e^{\epsilon_t^{\langle i \rangle} + \psi^{\langle i \rangle} \log Z_{t-1}^{\langle i \rangle}} \tag{4.1}$$

5 Equilibrium relationships (before expansion and reduction)

$$-\pi_t + \Pi_t = 0 \tag{5.1}$$

$$-r_t + \alpha \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_t^{\mathrm{m^d}}}^{-1+\alpha} {N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{5.2}$$

$$I_t - \sum_{i \in IND} I_t^{\langle i \rangle} = 0 \tag{5.3}$$

$$K_t - \sum_{i \in IND} K_t^{\langle i \rangle} = 0 \tag{5.4}$$

$$K_t^{\mathrm{m^d}} - K_{t-1}^{\langle \mathrm{M} \rangle} = 0 \tag{5.5}$$

$$N_t - \sum_{i \in IND} N_t^{\langle i \rangle} = 0 \tag{5.6}$$

$$N_t^{\mathrm{m^d}} - N_t^{\langle \mathrm{M} \rangle} = 0 \tag{5.7}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} (1-\alpha) {K_t^{\mathrm{m^d}}}^{\alpha} {N_t^{\mathrm{m^d}}}^{-\alpha} = 0 \tag{5.8}$$

$$-Y_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_t^{\mathrm{m^d}}}^{\alpha} {N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{5.9}$$

$$-C_t^{\langle \mathrm{H} \rangle} + \Gamma Z_t^{\langle \mathrm{H} \rangle} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {N_t^{\langle \mathrm{H} \rangle}}^{1-\theta} = 0 \tag{5.10}$$

$$-\pi_t + Y_t - r_t K_t^{\mathrm{m^d}} - N_t^{\mathrm{m^d}} W_t = 0 \tag{5.11}$$

$$U_t - \beta \mathrm{E}_t \left[U_{t+1} \right] - \log \left(1 - \sum_{i \in IND} N_t^{\langle i \rangle} \right) (1-b) - b e^{-1} \log \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right) = 0 \tag{5.12}$$

$$\pi_t - C_t^{\langle \mathrm{M} \rangle} + r_t K_{t-1}^{\langle \mathrm{M} \rangle} + W_t N_t^{\langle \mathrm{M} \rangle} - \sum_{i \in IND} I_t^{\langle i \rangle} = 0 \tag{5.13}$$

$$i \in IND: \quad - \lambda_t^{\mathrm{CONSUMER^1}} + {\lambda^{\mathrm{CONSUMER^2}}}_t^{\langle i \rangle} = 0 \tag{5.14}$$

$$i \in IND: \quad - {\lambda^{\mathrm{CONSUMER^2}}}_t^{\langle i \rangle} + \beta \left(\delta^{\langle \mathrm{M},i \rangle} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER^1}} r_{t+1} \right] + (1-\delta) \mathrm{E}_t \left[{\lambda^{\mathrm{CONSUMER^2}}}_{t+1}^{\langle i \rangle} \right] + \delta^{\langle \mathrm{H},i \rangle} \theta \Gamma {K_t^{\langle \mathrm{H} \rangle}}^{-1+\theta} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER^3}} Z_{t+1}^{\langle \mathrm{H} \rangle} {N_{t+1}^{\langle \mathrm{H} \rangle}}^{1-\theta} \right] \right) = 0 \tag{5.15}$$

$$i \in IND: \quad Z_t^{\langle i \rangle} - e^{\epsilon_t^{\langle i \rangle} + \psi^{\langle i \rangle} \log Z_{t-1}^{\langle i \rangle}} = 0 \tag{5.16}$$

$$i \in IND: \quad I_t^{\langle i \rangle} - K_t^{\langle i \rangle} + K_{t-1}^{\langle i \rangle} (1-\delta) = 0 \tag{5.17}$$

$$i \in IND: \quad - \delta^{\langle \mathrm{M},i \rangle} \lambda_t^{\mathrm{CONSUMER^1}} - \delta^{\langle \mathrm{H},i \rangle} \lambda_t^{\mathrm{CONSUMER^3}} + b e^{-1} \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} \left(\delta^{\langle \mathrm{M},i \rangle} a e {C_t^{\langle \mathrm{M} \rangle}}^{-1+e} + \delta^{\langle \mathrm{H},i \rangle} e (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{-1+e} \right) = 0 \tag{5.18}$$

$$i \in IND: \quad - (1-b) \left(1 - \sum_{i \in IND} N_t^{\langle i \rangle} \right)^{-1} + \delta^{\langle \mathrm{M},i \rangle} \lambda_t^{\mathrm{CONSUMER^1}} W_t + \delta^{\langle \mathrm{H},i \rangle} \Gamma \lambda_t^{\mathrm{CONSUMER^3}} Z_t^{\langle \mathrm{H} \rangle} (1-\theta) {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {N_t^{\langle \mathrm{H} \rangle}}^{-\theta} = 0 \tag{5.19}$$

6 Equilibrium relationships (after expansion and reduction)

$$-r_t + \alpha \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_{t-1}^{\langle \mathrm{M} \rangle}}^{-1+\alpha} {N_t^{\langle \mathrm{M} \rangle}}^{1-\alpha} = 0 \tag{6.1}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} (1-\alpha) {K_{t-1}^{\langle \mathrm{M} \rangle}}^{\alpha} {N_t^{\langle \mathrm{M} \rangle}}^{-\alpha} = 0 \tag{6.2}$$

$$-Y_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_{t-1}^{\langle \mathrm{M} \rangle}}^{\alpha} {N_t^{\langle \mathrm{M} \rangle}}^{1-\alpha} = 0 \tag{6.3}$$

$$-C_t^{\langle \mathrm{H} \rangle} + \Gamma Z_t^{\langle \mathrm{H} \rangle} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {N_t^{\langle \mathrm{H} \rangle}}^{1-\theta} = 0 \tag{6.4}$$

$$Z_t^{\langle \mathrm{H} \rangle} - e^{\epsilon_t^{\langle \mathrm{H} \rangle} + \psi^{\langle \mathrm{H} \rangle} \log Z_{t-1}^{\langle \mathrm{H} \rangle}} = 0 \tag{6.5}$$

$$Z_t^{\langle \mathrm{M} \rangle} - e^{\epsilon_t^{\langle \mathrm{M} \rangle} + \psi^{\langle \mathrm{M} \rangle} \log Z_{t-1}^{\langle \mathrm{M} \rangle}} = 0 \tag{6.6}$$

$$\beta \left(ab \mathrm{E}_t \left[r_{t+1} \left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] + ab(1-\delta) \mathrm{E}_t \left[\left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] \right) - ab \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_t^{\langle \mathrm{M} \rangle}}^{-1+e} = 0 \tag{6.7}$$

$$\beta \left(ab(1-\delta) \mathrm{E}_t \left[\left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] + b\theta\Gamma(1-a) {K_t^{\langle \mathrm{H} \rangle}}^{-1+\theta} \mathrm{E}_t \left[Z_{t+1}^{\langle \mathrm{H} \rangle} \left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{H} \rangle}}^{-1+e} {N_{t+1}^{\langle \mathrm{H} \rangle}}^{1-\theta} \right] \right) - ab \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_t^{\langle \mathrm{M} \rangle}} \tag{6.8}$$

$$-(1-b) \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right)^{-1} + abW_t \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_t^{\langle \mathrm{M} \rangle}}^{-1+e} = 0 \tag{6.9}$$

$$-(1-b) \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right)^{-1} + b\Gamma Z_t^{\langle \mathrm{H} \rangle} (1-a)(1-\theta) \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {C_t^{\langle \mathrm{H} \rangle}}^{-1+e} {N_t^{\langle \mathrm{H} \rangle}}^{-\theta} = 0 \tag{6.10}$$

$$I_t - I_t^{\langle \mathrm{H} \rangle} - I_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.11}$$

$$K_t - K_t^{\langle \mathrm{H} \rangle} - K_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.12}$$

$$N_t - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.13}$$

$$I_t^{\langle \mathrm{H} \rangle} - K_t^{\langle \mathrm{H} \rangle} + K_{t-1}^{\langle \mathrm{H} \rangle} (1-\delta) = 0 \tag{6.14}$$

$$I_t^{\langle \mathrm{M} \rangle} - K_t^{\langle \mathrm{M} \rangle} + K_{t-1}^{\langle \mathrm{M} \rangle} (1-\delta) = 0 \tag{6.15}$$

$$U_t - \beta \mathrm{E}_t \left[U_{t+1} \right] - \log \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right) (1-b) - be^{-1} \log \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right) = 0 \tag{6.16}$$

$$Y_t - C_t^{\langle \mathrm{M} \rangle} - I_t^{\langle \mathrm{H} \rangle} - I_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.17}$$

7 Steady state relationships (before expansion and reduction)

$$-\pi_{\mathrm{ss}} + \Pi_{\mathrm{ss}} = 0 \tag{7.1}$$

$$-r_{\mathrm{ss}} + \alpha \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} {K_{\mathrm{ss}}^{\mathrm{m^d}}}^{-1+\alpha} {N_{\mathrm{ss}}^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{7.2}$$

$$I_{\mathrm{ss}} - \sum_{i \in IND} I_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.3}$$

$$K_{\mathrm{ss}} - \sum_{i \in IND} K_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.4}$$

$$K_{\mathrm{ss}}^{\mathrm{m^d}} - K_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} = 0 \tag{7.5}$$

$$N_{\mathrm{ss}} - \sum_{i \in IND} N_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.6}$$

$$N_{\mathrm{ss}}^{\mathrm{m^d}} - N_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} = 0 \tag{7.7}$$

$$-W_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} (1-\alpha) {K_{\mathrm{ss}}^{\mathrm{m^d}}}^{\alpha} {N_{\mathrm{ss}}^{\mathrm{m^d}}}^{-\alpha} = 0 \tag{7.8}$$

$$-Y_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} {K_{\mathrm{ss}}^{\mathrm{m^d}}}^{\alpha} {N_{\mathrm{ss}}^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{7.9}$$

$$-C_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} {K_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{\theta} {N_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{1-\theta} = 0 \tag{7.10}$$

$$-\pi_{\mathrm{ss}} + Y_{\mathrm{ss}} - r_{\mathrm{ss}} K_{\mathrm{ss}}^{\mathrm{m^d}} - N_{\mathrm{ss}}^{\mathrm{m^d}} W_{\mathrm{ss}} = 0 \tag{7.11}$$

$$U_{\mathrm{ss}} - \beta U_{\mathrm{ss}} - \log\left(1 - \sum_{i \in IND} N_{\mathrm{ss}}^{\langle i \rangle}\right)(1-b) - b e^{-1} \log\left(a {C_{\mathrm{ss}}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{e}\right) = 0 \tag{7.12}$$

$$\pi_{\mathrm{ss}} - C_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} + r_{\mathrm{ss}} K_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} + W_{\mathrm{ss}} N_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} - \sum_{i \in IND} I_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.13}$$

$$i \in IND: \quad -\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} + {\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.14}$$

$$i \in IND: \quad -{\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle} + \beta\left({\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle}(1-\delta) + \delta^{\langle \mathrm{M},i \rangle} \lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} r_{\mathrm{ss}} + \delta^{\langle \mathrm{H},i \rangle} \theta \Gamma \lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^3} Z_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} {K_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{-1+\theta} {N_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{1-\theta}\right) = 0 \tag{7.15}$$

$$i \in IND: \quad Z_{\text{ss}}^{\langle i \rangle} - e^{\epsilon_{\text{ss}}^{\langle i \rangle} + \psi^{\langle i \rangle} \log Z_{\text{ss}}^{\langle i \rangle}} = 0 \tag{7.16}$$

$$i \in IND: \quad I_{\text{ss}}^{\langle i \rangle} - K_{\text{ss}}^{\langle i \rangle} + K_{\text{ss}}^{\langle i \rangle} (1 - \delta) = 0 \tag{7.17}$$

$$i \in IND: \quad - \delta^{\langle \text{M},i \rangle} \lambda_{\text{ss}}^{\text{CONSUMER}^1} - \delta^{\langle \text{H},i \rangle} \lambda_{\text{ss}}^{\text{CONSUMER}^3} + be^{-1} \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} \left(\delta^{\langle \text{M},i \rangle} aeC_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} + \delta^{\langle \text{H},i \rangle} e (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^{-1+e}} \right) = 0 \tag{7.18}$$

$$i \in IND: \quad - (1 - b) \left(1 - \sum_{i \in IND} N_{\text{ss}}^{\langle i \rangle} \right)^{-1} + \delta^{\langle \text{M},i \rangle} \lambda_{\text{ss}}^{\text{CONSUMER}^1} W_{\text{ss}} + \delta^{\langle \text{H},i \rangle} \Gamma \lambda_{\text{ss}}^{\text{CONSUMER}^3} Z_{\text{ss}}^{\langle \text{H} \rangle} (1 - \theta) K_{\text{ss}}^{\langle \text{H} \rangle^\theta} N_{\text{ss}}^{\langle \text{H} \rangle^{-\theta}} = 0 \tag{7.19}$$

8 Steady state relationships (after expansion and reduction)

$$-r_{\text{ss}} + \alpha \Gamma Z_{\text{ss}}^{\langle \text{M} \rangle} K_{\text{ss}}^{\langle \text{M} \rangle^{-1+\alpha}} N_{\text{ss}}^{\langle \text{M} \rangle^{1-\alpha}} = 0 \tag{8.1}$$

$$-W_{\text{ss}} + \Gamma Z_{\text{ss}}^{\langle \text{M} \rangle} (1 - \alpha) K_{\text{ss}}^{\langle \text{M} \rangle^\alpha} N_{\text{ss}}^{\langle \text{M} \rangle^{-\alpha}} = 0 \tag{8.2}$$

$$-Y_{\text{ss}} + \Gamma Z_{\text{ss}}^{\langle \text{M} \rangle} K_{\text{ss}}^{\langle \text{M} \rangle^\alpha} N_{\text{ss}}^{\langle \text{M} \rangle^{1-\alpha}} = 0 \tag{8.3}$$

$$-C_{\text{ss}}^{\langle \text{H} \rangle} + \Gamma Z_{\text{ss}}^{\langle \text{H} \rangle} K_{\text{ss}}^{\langle \text{H} \rangle^\theta} N_{\text{ss}}^{\langle \text{H} \rangle^{1-\theta}} = 0 \tag{8.4}$$

$$Z_{\text{ss}}^{\langle \text{H} \rangle} - e^{\psi^{\langle \text{H} \rangle} \log Z_{\text{ss}}^{\langle \text{H} \rangle}} = 0 \tag{8.5}$$

$$Z_{\text{ss}}^{\langle \text{M} \rangle} - e^{\psi^{\langle \text{M} \rangle} \log Z_{\text{ss}}^{\langle \text{M} \rangle}} = 0 \tag{8.6}$$

$$\beta \left(abr_{\text{ss}} \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} + ab (1 - \delta) \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} \right) - ab \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} = 0 \tag{8.7}$$

$$\beta \left(ab (1 - \delta) \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} + b\theta \Gamma Z_{\text{ss}}^{\langle \text{H} \rangle} (1 - a) \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{H} \rangle^{-1+e}} K_{\text{ss}}^{\langle \text{H} \rangle^{-1+\theta}} N_{\text{ss}}^{\langle \text{H} \rangle^{1-\theta}} \right) - ab \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} = 0 \tag{8.8}$$

$$- (1 - b) \left(1 - N_{\text{ss}}^{\langle \text{H} \rangle} - N_{\text{ss}}^{\langle \text{M} \rangle} \right)^{-1} + abW_{\text{ss}} \left(aC_{\text{ss}}^{\langle \text{M} \rangle^e} + (1 - a) C_{\text{ss}}^{\langle \text{H} \rangle^e} \right)^{-1} C_{\text{ss}}^{\langle \text{M} \rangle^{-1+e}} = 0 \tag{8.9}$$

$$-(1-b)\left(1-N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}\right)^{-1}+b\Gamma Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}(1-a)(1-\theta)\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\, e}+(1-a)\,C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle -1+e}K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle \theta}N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle -\theta}=0 \tag{8.10}$$

$$I_{\mathrm{ss}}-I_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-I_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}=0 \tag{8.11}$$

$$K_{\mathrm{ss}}-K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}=0 \tag{8.12}$$

$$N_{\mathrm{ss}}-N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}=0 \tag{8.13}$$

$$I_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}+K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}(1-\delta)=0 \tag{8.14}$$

$$I_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}-K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}+K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}(1-\delta)=0 \tag{8.15}$$

$$U_{\mathrm{ss}}-\beta U_{\mathrm{ss}}-\log\left(1-N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}\right)(1-b)-be^{-1}\log\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\, e}+(1-a)\,C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle e}\right)=0 \tag{8.16}$$

$$Y_{\mathrm{ss}}-C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}-I_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}-I_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}=0 \tag{8.17}$$

∞

9 Parameter settings

$$a=0.337 \tag{9.1}$$

$$\alpha=0.36 \tag{9.2}$$

$$b=0.63 \tag{9.3}$$

$$\beta=0.99 \tag{9.4}$$

$$\delta=0.025 \tag{9.5}$$

$$e=0.8 \tag{9.6}$$

$$\theta=0.08 \tag{9.7}$$

$$\Gamma = 1 \tag{9.8}$$

$$\psi^{\langle \mathrm{H} \rangle} = 0.95 \tag{9.9}$$

$$\psi^{\langle \mathrm{M} \rangle} = 0.95 \tag{9.10}$$

10 Steady-state values

	Steady-state values
r	0.0351
I	0.3143
K	12.5726
N	0.6102
U	-79.6929
W	2.3706
Y	1.0367
C^{H}	0.3805
C^{M}	0.7224
I^{H}	0.0485
I^{M}	0.2658
K^{H}	1.9397
K^{M}	10.6329
N^{H}	0.3303
N^{M}	0.2799
Z^{H}	1
Z^{M}	1

11 The solution of the perturbation

11.1 P

$$
\begin{array}{c} \\ K^{\mathrm{H}} \\ K^{\mathrm{M}} \\ Z^{\mathrm{H}} \\ Z^{\mathrm{M}} \end{array}
\begin{array}{c}
\begin{array}{cccc} K^{\mathrm{H}}_{t-1} & K^{\mathrm{M}}_{t-1} & Z^{\mathrm{H}}_{t-1} & Z^{\mathrm{M}}_{t-1} \end{array} \\
\begin{pmatrix} 0.0826 & 0.4683 & 2.0323 & -2.6403 \\ 0.1545 & 0.8762 & -0.3729 & 0.6255 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.95 \end{pmatrix}
\end{array}
$$

11.2 Q

$$
\begin{array}{c} \\ K^{\mathrm{H}} \\ K^{\mathrm{M}} \\ Z^{\mathrm{H}} \\ Z^{\mathrm{M}} \end{array}
\begin{array}{c}
\begin{array}{cc} \epsilon^{\mathrm{H}} & \epsilon^{\mathrm{M}} \end{array} \\
\begin{pmatrix} 2.1393 & -2.7792 \\ -0.3926 & 0.6584 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}
\end{array}
$$

11.3 R

$$
\begin{array}{c} \\ r \\ I \\ K \\ N \\ U \\ W \\ Y \\ C^{\mathrm{H}} \\ C^{\mathrm{M}} \\ I^{\mathrm{H}} \\ I^{\mathrm{M}} \\ N^{\mathrm{H}} \\ N^{\mathrm{M}} \end{array}
\begin{array}{c}
\begin{array}{cccc} K^{\mathrm{H}}_{t-1} & K^{\mathrm{M}}_{t-1} & Z^{\mathrm{H}}_{t-1} & Z^{\mathrm{M}}_{t-1} \end{array} \\
\begin{pmatrix} -0.08 & -0.4894 & -0.6218 & 1.96 \\ -0.2798 & -0.4533 & -0.0746 & 4.867 \\ 0.1434 & 0.8132 & -0.0019 & 0.1217 \\ -0.0155 & -0.0751 & 0.0429 & 0.226 \\ -0.0098 & -0.054 & -0.0683 & -0.0832 \\ 0.045 & 0.2753 & 0.3497 & 0.3819 \\ -0.08 & 0.5106 & -0.6218 & 1.96 \\ 0.1511 & -0.3112 & 1.7804 & -0.8463 \\ 0.0069 & 0.93 & -0.8599 & 0.6952 \\ -35.696 & 18.734 & 81.2939 & -105.6101 \\ 6.1809 & -3.9534 & -14.918 & 25.0205 \\ 0.0772 & -0.3382 & 0.9026 & -0.9199 \\ -0.125 & 0.2353 & -0.9715 & 1.5781 \end{pmatrix}
\end{array}
$$

11.4 S

$$
\begin{array}{c} \\ r \\ I \\ K \\ N \\ U \\ W \\ Y \\ C^{\mathrm{H}} \\ C^{\mathrm{M}} \\ I^{\mathrm{H}} \\ I^{\mathrm{M}} \\ N^{\mathrm{H}} \\ N^{\mathrm{M}} \end{array}
\begin{array}{c}
\begin{array}{cc} \epsilon^{\mathrm{H}} & \epsilon^{\mathrm{M}} \end{array} \\
\left(\begin{array}{cc}
-0.6545 & 2.0631 \\
-0.0785 & 5.1231 \\
-0.002 & 0.1281 \\
0.0452 & 0.2379 \\
-0.0719 & -0.0875 \\
0.3682 & 0.402 \\
-0.6545 & 2.0631 \\
1.8741 & -0.8908 \\
-0.9051 & 0.7318 \\
85.5725 & -111.1686 \\
-15.7031 & 26.3373 \\
0.9501 & -0.9683 \\
-1.0227 & 1.6612
\end{array} \right)
\end{array}
$$

12 Statistics of the model

12.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
r	0.0351	1.5572	2.4248	Y
I	0.3143	4.828	23.3092	Y
K	12.5726	0.4326	0.1871	Y
N	0.6102	0.2513	0.0632	Y
W	2.3706	0.6601	0.4358	Y
Y	1.0367	1.7868	3.1926	Y
C^{M}	0.7224	0.9767	0.954	Y
I^{H}	0.0485	57.2629	3279.0405	Y
I^{M}	0.2658	12.888	166.0993	Y
N^{H}	0.3303	0.9286	0.8623	Y
N^{M}	0.2799	1.3761	1.8936	Y

12.2 Correlation matrix

	r	I	K	N	W	Y	C^{M}	I^{H}	I^{M}	N^{H}	N^{M}
r	1	0.9644	0.1003	0.9227	0.7172	0.9373	0.3865	-0.3461	0.7077	-0.635	0.873
I	0.9644	1	0.228	0.9793	0.8717	0.9314	0.2945	-0.2045	0.6087	-0.5039	0.7912
K	0.1003	0.228	1	0.118	0.4488	0.4021	0.5653	0.1935	-0.0558	-0.3263	0.3068
N	0.9227	0.9793	0.118	1	0.8964	0.8384	0.0949	-0.2173	0.6099	-0.327	0.6585
U	-0.7291	-0.8795	-0.4398	-0.9038	-0.9998	-0.7428	-0.0588	0.0451	-0.4261	0.157	-0.4849
W	0.7172	0.8717	0.4488	0.8964	1	0.7353	0.0558	-0.028	0.4088	-0.1484	0.4751
Y	0.9373	0.9314	0.4021	0.8384	0.7353	1	0.6222	-0.1597	0.542	-0.7684	0.9457
C^{H}	-0.1576	0.0411	-0.0964	0.2217	0.4206	-0.2875	-0.8433	0.0844	-0.0502	0.8331	-0.5751
C^{M}	0.3865	0.2945	0.5653	0.0949	0.0558	0.6222	1	0.0207	0.1137	-0.9335	0.7811
I^{H}	-0.3461	-0.2045	0.1935	-0.2173	-0.028	-0.1597	0.0207	1	-0.9011	0.1348	-0.1939
I^{M}	0.7077	0.6087	-0.0558	0.6099	0.4088	0.542	0.1137	-0.9011	1	-0.3325	0.5076
K^{H}	-0.8383	-0.6843	-0.0538	-0.5635	-0.2714	-0.8286	-0.7036	0.4086	-0.6343	0.9058	-0.9457
K^{M}	0.5408	0.5593	0.8367	0.4043	0.5111	0.779	0.8422	-0.0679	0.3028	-0.7602	0.7663
N^{H}	-0.635	-0.5039	-0.3263	-0.327	-0.1484	-0.7684	-0.9335	0.1348	-0.3325	1	-0.9265
N^{M}	0.873	0.7912	0.3068	0.6585	0.4751	0.9457	0.7811	-0.1939	0.5076	-0.9265	1
Z^{H}	0.431	0.6312	0.1791	0.7441	0.8825	0.3619	-0.4075	0.0732	0.2203	0.3136	0.0465
Z^{M}	0.9465	0.9962	0.2694	0.982	0.9037	0.9164	0.2633	-0.2256	0.6241	-0.4588	0.7563

12.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
r	0.7052	0.4611	0.2605	0.0998	-0.025
I	0.7333	0.4818	0.2747	0.1084	-0.0212
K	0.9604	0.8629	0.7274	0.5705	0.4053
N	0.7125	0.466	0.2634	0.1011	-0.025
W	0.7395	0.5114	0.3175	0.1563	0.026
Y	0.7874	0.5333	0.3205	0.1464	0.0079
C^{M}	0.8516	0.6216	0.4176	0.2412	0.0924
I^{H}	-0.078	-0.0735	-0.0678	-0.0612	-0.0543
I^{M}	0.0756	0.0316	-0.0029	-0.0291	-0.048
N^{H}	0.8197	0.555	0.3334	0.1522	0.0083
N^{M}	0.8032	0.5369	0.3157	0.1364	-0.0048

12.4 Variance decomposition

	ϵ^{H}	ϵ^{M}
r	0.1955	0.8045
I	0.3999	0.6001
K	0.3874	0.6126
N	0.563	0.437
W	0.796	0.204
Y	0.1361	0.8639
C^{M}	0.2383	0.7617
I^{H}	0.0146	0.9854
I^{M}	0.0535	0.9465
N^{H}	0.1058	0.8942
N^{M}	0.0032	0.9968

13 Statistics of the model

13.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
r	0.0339	0.8715	0.7595	Y
I	0.3032	2.7021	7.3011	Y
K	12.1271	0.2421	0.0586	Y
N	0.5886	0.1407	0.0198	Y
U	-76.8688	0.0761	0.0058	Y
W	2.2866	0.3695	0.1365	Y
Y	1	1	1	Y
C^{H}	0.3671	0.8507	0.7237	Y
C^{M}	0.6968	0.5466	0.2988	Y
I^{H}	0.0468	32.0482	1027.0892	Y
I^{M}	0.2564	7.213	52.027	Y
K^{H}	1.8709	1.0662	1.1369	Y
K^{M}	10.2561	0.3547	0.1258	Y
N^{H}	0.3186	0.5197	0.2701	Y
N^{M}	0.27	0.7702	0.5931	Y
Z^{H}	0.9646	0.5106	0.2608	Y
Z^{M}	0.9646	0.5106	0.2608	Y

13.2 Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r	0.1526	0.278	0.4282	0.6043	0.8064	0.9373	0.5864	0.3062	0.0862	-0.0805	-0.2011
I	0.0997	0.2242	0.3758	0.556	0.7659	0.9314	0.6451	0.3796	0.1665	0.0006	-0.1238
K	-0.4168	-0.3439	-0.2304	-0.0695	0.1459	0.4021	0.5721	0.6637	0.6935	0.6763	0.6249
N	0.1448	0.2541	0.3843	0.5358	0.7088	0.8384	0.5281	0.2718	0.0712	-0.0804	-0.1896
U	0.0268	-0.0751	-0.2046	-0.3643	-0.5562	-0.7428	-0.5653	-0.4095	-0.274	-0.1588	-0.0632
W	-0.0324	0.0684	0.1969	0.3556	0.5466	0.7353	0.5683	0.4149	0.2809	0.1664	0.071
Y	0.0079	0.1464	0.3205	0.5333	0.7874	1	0.7874	0.5333	0.3205	0.1464	0.0079
C^{H}	0.0441	-0.0009	-0.0602	-0.1357	-0.229	-0.2875	-0.318	-0.238	-0.1678	-0.1076	-0.057
C^{M}	-0.1938	-0.0979	0.0332	0.2043	0.4199	0.6222	0.6796	0.5837	0.4832	0.383	0.2869
I^{H}	-0.1238	-0.1565	-0.1921	-0.2303	-0.2702	-0.1597	0.3725	0.3066	0.2462	0.1919	0.144
I^{M}	0.1445	0.2261	0.3222	0.433	0.5582	0.542	-0.0161	-0.0804	-0.1258	-0.1553	-0.1716
K^{H}	-0.1204	-0.2349	-0.3735	-0.5372	-0.7268	-0.8286	-0.528	-0.2844	-0.0923	0.0542	0.1611
K^{M}	-0.2704	-0.1487	0.0188	0.2385	0.5164	0.779	0.7513	0.6917	0.6104	0.5162	0.4161
N^{H}	0.04	-0.0704	-0.2118	-0.3875	-0.6001	-0.7684	-0.6774	-0.4837	-0.3177	-0.1786	-0.0648
N^{M}	0.0258	0.1572	0.3216	0.5219	0.7601	0.9457	0.7497	0.4934	0.2814	0.1102	-0.0239
Z^{H}	0.0406	0.0846	0.1373	0.1992	0.2702	0.3619	0.243	0.1447	0.0655	0.0035	-0.0434
Z^{M}	0.0787	0.2025	0.3543	0.5359	0.7486	0.9164	0.6234	0.3802	0.183	0.0276	-0.0906

14 Impulse response functions

14.1 Shock ϵ^{H}

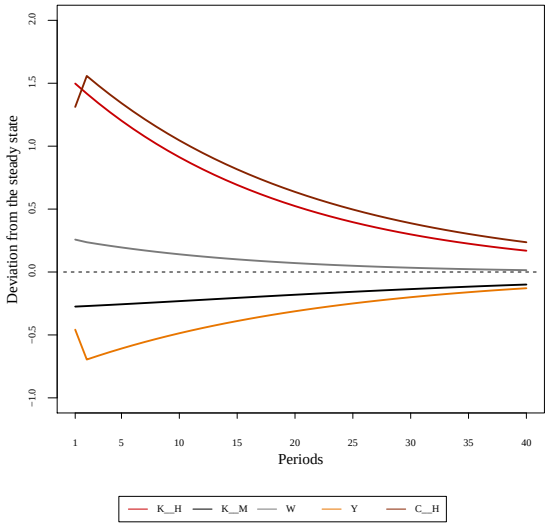

Figure 1: Impulse response function for ϵ^{H} shock

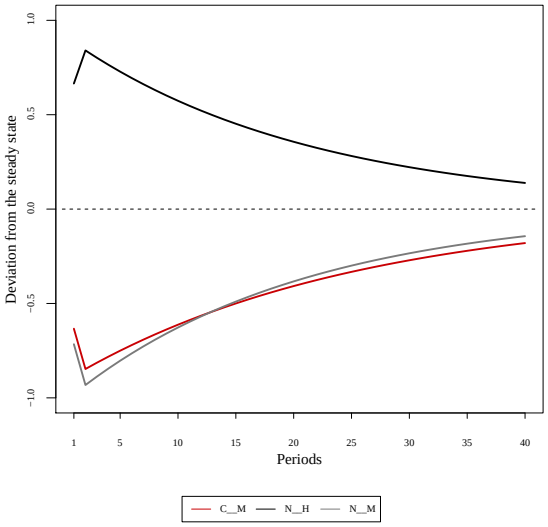

Figure 2: Impulse response function for ϵ^{H} shock

14.2 Shock ϵ^{M}

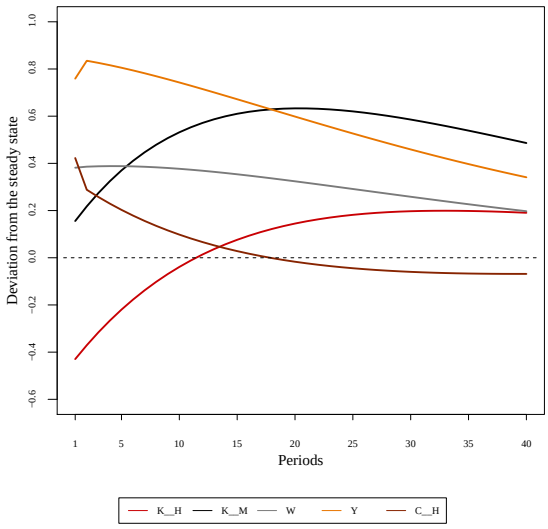

Figure 3: Impulse response function for ϵ^{M} shock

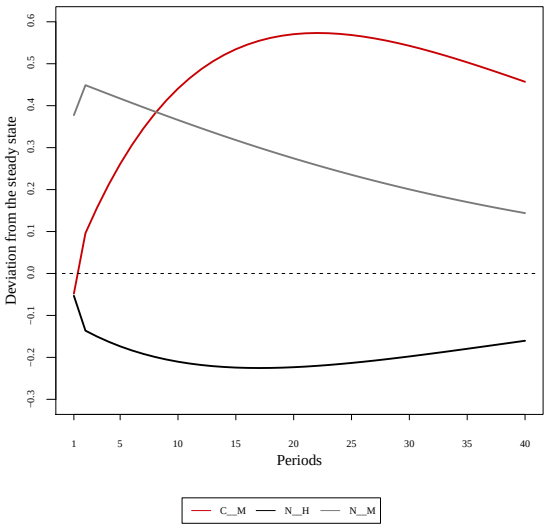

Figure 4: Impulse response function for ϵ^{M} shock