

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t, C_t, H_t, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - H_t)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t + T_t = \pi_t + TR_t + K_{t-1}r_t + H_t W_t - \psi K_{t-1} (-\delta + K_{t-1}^{-1} I_t)^2 \quad (\lambda_t^c) \quad (1.2)$$

$$K_t = I_t + K_{t-1} (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^c \left(r_{t+1} - \psi (-\delta + K_t^{-1} I_{t+1})^2 + 2\psi K_t^{-1} I_{t+1} (-\delta + K_t^{-1} I_{t+1}) \right) \right] \right) = 0 \quad (K_t) \quad (1.4)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - H_t)^{1-\mu} \left(C_t^\mu (1 - H_t)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - H_t)^{-\mu} \left(C_t^\mu (1 - H_t)^{1-\mu} \right)^{-\eta} = 0 \quad (H_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^2} + \lambda_t^c (-1 - 2\psi (-\delta + K_{t-1}^{-1} I_t)) = 0 \quad (I_t) \quad (1.7)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, H_t^d, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$Y_t = Z_t H_t^{d^{1-\alpha}} K_t^{d^\alpha} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$\pi_t = Y_t - H_t^d W_t - r_t K_t^d \quad \left(\lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^2} r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t H_t^{\text{d}^{1-\alpha}} K_t^{\text{d}^{-1+\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^2} W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) H_t^{\text{d}^{-\alpha}} K_t^{\text{d}^\alpha} = 0 \quad (H_t^{\text{d}}) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^2} = 0 \quad (\pi_t) \quad (2.7)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t H_t^{\text{d}^{1-\alpha}} K_t^{\text{d}^{-1+\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.8)$$

$$-W_t + Z_t (1 - \alpha) H_t^{\text{d}^{-\alpha}} K_t^{\text{d}^\alpha} = 0 \quad (H_t^{\text{d}}) \quad (2.9)$$

3 CONSUMER*

3.1 Optimisation problem

$$\max_{K_t^*, C_t^*, H_t^*, I_t^*} U_t^* = \beta \text{E}_t [U_{t+1}^*] + (1 - \eta)^{-1} \left(C_t^{*\mu} (1 - H_t^*)^{1-\mu} \right)^{1-\eta} \quad (3.1)$$

s.t. :

$$C_t^* + I_t^* + T_t^* = \pi_t^* - TR_t + K_{t-1}^* r_t^* + H_t^* W_t^* - \psi K_{t-1}^* \left(-\delta + K_{t-1}^{*-1} I_t^* \right)^2 \quad \left(\lambda_t^{\text{c}*} \right) \quad (3.2)$$

$$K_t^* = I_t^* + K_{t-1}^* (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^{*2}} \right) \quad (3.3)$$

3.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^{*2}} + \beta \left((1 - \delta) \text{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^{*2}} \right] + \text{E}_t \left[\lambda_{t+1}^{\text{c}*} \left(r_{t+1}^* - \psi \left(-\delta + K_t^{*-1} I_{t+1}^* \right)^2 + 2\psi K_t^{*-1} I_{t+1}^* \left(-\delta + K_t^{*-1} I_{t+1}^* \right) \right) \right] \right) = 0 \quad (K_t^*) \quad (3.4)$$

$$-\lambda_t^{\text{c}*} + \mu C_t^{*-1+\mu} (1 - H_t^*)^{1-\mu} \left(C_t^{*\mu} (1 - H_t^*)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t^*) \quad (3.5)$$

$$\lambda_t^{\text{c}*} W_t^* + (-1 + \mu) C_t^{*\mu} (1 - H_t^*)^{-\mu} \left(C_t^{*\mu} (1 - H_t^*)^{1-\mu} \right)^{-\eta} = 0 \quad (H_t^*) \quad (3.6)$$

$$\lambda_t^{\text{CONSUMER}^{*2}} + \lambda_t^{\text{c}*} \left(-1 - 2\psi \left(-\delta + K_{t-1}^{*-1} I_t^* \right) \right) = 0 \quad (I_t^*) \quad (3.7)$$

4 FIRM*

4.1 Optimisation problem

$$\max_{K_t^{d*}, H_t^{d*}, Y_t^*, \pi_t^*} \Pi_t^* = \pi_t^* \quad (4.1)$$

s.t. :

$$Y_t^* = Z_t^* H_t^{d*1-\alpha} K_t^{d*\alpha} \quad \left(\lambda_t^{\text{FIRM}^*1} \right) \quad (4.2)$$

$$\pi_t^* = Y_t^* - H_t^{d*} W_t^* - r_t^* K_t^{d*} \quad \left(\lambda_t^{\text{FIRM}^*2} \right) \quad (4.3)$$

4.2 First order conditions

$$-\lambda_t^{\text{FIRM}^*2} r_t^* + \alpha \lambda_t^{\text{FIRM}^*1} Z_t^* H_t^{d*1-\alpha} K_t^{d*-1+\alpha} = 0 \quad \left(K_t^{d*} \right) \quad (4.4)$$

$$-\lambda_t^{\text{FIRM}^*2} W_t^* + \lambda_t^{\text{FIRM}^*1} Z_t^* (1-\alpha) H_t^{d*-\alpha} K_t^{d*\alpha} = 0 \quad \left(H_t^{d*} \right) \quad (4.5)$$

$$-\lambda_t^{\text{FIRM}^*1} + \lambda_t^{\text{FIRM}^*2} = 0 \quad (Y_t^*) \quad (4.6)$$

$$1 - \lambda_t^{\text{FIRM}^*2} = 0 \quad (\pi_t^*) \quad (4.7)$$

4.3 First order conditions after reduction

$$-r_t^* + \alpha Z_t^* H_t^{d*1-\alpha} K_t^{d*-1+\alpha} = 0 \quad \left(K_t^{d*} \right) \quad (4.8)$$

$$-W_t^* + Z_t^* (1-\alpha) H_t^{d*-\alpha} K_t^{d*\alpha} = 0 \quad \left(H_t^{d*} \right) \quad (4.9)$$

5 EQUILIBRIUM

5.1 Identities

$$K_t^d = K_{t-1} \quad (5.1)$$

$$H_t^d = H_t \quad (5.2)$$

$$T_t = G_t^d \quad (5.3)$$

$$K_t^{\text{d}*} = K_{t-1}^* \quad (5.4)$$

$$H_t^{\text{d}*} = H_t^* \quad (5.5)$$

$$T_t^* = G_t^{\text{d}*} \quad (5.6)$$

$$\lambda_t^{\text{c}} = \lambda_t^{\text{c}*} \quad (5.7)$$

6 EXOG

6.1 Identities

$$G_t^{\text{d}} = \epsilon_t^{\text{G}} + \phi^{\text{G}} G_{t-1}^{\text{d}} \quad (6.1)$$

$$Z_t = e^{\epsilon_t^{\text{Z}} + \phi^{\text{Z}} \log Z_{t-1}} \quad (6.2)$$

$$G_t^{\text{d}*} = \epsilon_t^{\text{G}*} + \phi^{\text{G}} G_{t-1}^{\text{d}*} \quad (6.3)$$

$$Z_t^* = e^{\epsilon_t^{\text{Z}*} + \phi^{\text{Z}} \log Z_{t-1}^*} \quad (6.4)$$

7 Equilibrium relationships (after reduction)

$$-\lambda_t^{\text{c}} + \mu C_t^{-1+\mu} (1 - H_t)^{1-\mu} \left(C_t^{\mu} (1 - H_t)^{1-\mu} \right)^{-\eta} = 0 \quad (7.1)$$

$$\lambda_t^{\text{c}} - \lambda_t^{\text{c}*} = 0 \quad (7.2)$$

$$-\lambda_t^{\text{c}*} + \mu C_t^{*-1+\mu} (1 - H_t^*)^{1-\mu} \left((1 - H_t^*)^{1-\mu} C_t^{*\mu} \right)^{-\eta} = 0 \quad (7.3)$$

$$-r_t + \alpha Z_t K_{t-1}^{-1+\alpha} H_t^{1-\alpha} = 0 \quad (7.4)$$

$$-r_t^* + \alpha Z_t^* K_{t-1}^{*-1+\alpha} H_t^{*1-\alpha} = 0 \quad (7.5)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\alpha} H_t^{-\alpha} = 0 \quad (7.6)$$

$$-W_t^* + Z_t^* (1 - \alpha) K_{t-1}^{*\alpha} H_t^{*- \alpha} = 0 \quad (7.7)$$

$$-Y_t + Z_t K_{t-1}^\alpha H_t^{1-\alpha} = 0 \quad (7.8)$$

$$-Y_t^* + Z_t^* K_{t-1}^{*\alpha} H_t^{*1-\alpha} = 0 \quad (7.9)$$

$$Z_t - e^{\epsilon_t^Z + \phi^Z \log Z_{t-1}} = 0 \quad (7.10)$$

$$Z_t^* - e^{\epsilon_t^{Z^*} + \phi^Z \log Z_{t-1}^*} = 0 \quad (7.11)$$

$$\beta \left(-(1 - \delta) \mathbb{E}_t \left[\lambda_{t+1}^c (-1 - 2\psi(-\delta + K_t^{-1} I_{t+1})) \right] + \mathbb{E}_t \left[\lambda_{t+1}^c \left(r_{t+1} - \psi(-\delta + K_t^{-1} I_{t+1})^2 + 2\psi K_t^{-1} I_{t+1} (-\delta + K_t^{-1} I_{t+1}) \right) \right] \right) + \lambda_t^c (-1 - 2\psi(-\delta + K_{t-1}^{-1} I_t)) = 0 \quad (7.12)$$

$$\beta \left(-(1 - \delta) \mathbb{E}_t \left[\lambda_{t+1}^{c*} (-1 - 2\psi(-\delta + K_t^{*-1} I_{t+1}^*)) \right] + \mathbb{E}_t \left[\lambda_{t+1}^{c*} \left(r_{t+1}^* - \psi(-\delta + K_t^{*-1} I_{t+1}^*)^2 + 2\psi K_t^{*-1} I_{t+1}^* (-\delta + K_t^{*-1} I_{t+1}^*) \right) \right] \right) + \lambda_t^{c*} (-1 - 2\psi(-\delta + K_{t-1}^{*-1} I_t^*)) = 0 \quad (7.13)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - H_t)^{-\mu} \left(C_t^\mu (1 - H_t)^{1-\mu} \right)^{-\eta} = 0 \quad (7.14)$$

$$\lambda_t^{c*} W_t^* + (-1 + \mu) C_t^{*\mu} (1 - H_t^*)^{-\mu} \left((1 - H_t^*)^{1-\mu} C_t^{*\mu} \right)^{-\eta} = 0 \quad (7.15)$$

$$-\epsilon_t^G + G_t^d - \phi^G G_{t-1}^d = 0 \quad (7.16)$$

$$-\epsilon_t^{G^*} + G_t^{d*} - \phi^G G_{t-1}^{d*} = 0 \quad (7.17)$$

$$I_t - K_t + K_{t-1} (1 - \delta) = 0 \quad (7.18)$$

$$I_t^* - K_t^* + K_{t-1}^* (1 - \delta) = 0 \quad (7.19)$$

$$U_t - \beta \mathbb{E}_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - H_t)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.20)$$

$$U_t^* - \beta E_t [U_{t+1}^*] - (1 - \eta)^{-1} \left(C_t^{*\mu} (1 - H_t^*)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.21)$$

$$-C_t - G_t^d - I_t + TR_t + Y_t - \psi K_{t-1} (-\delta + K_{t-1}^{-1} I_t)^2 = 0 \quad (7.22)$$

$$-C_t^* - G_t^{d*} - I_t^* - TR_t + Y_t^* - \psi K_{t-1}^* (-\delta + K_{t-1}^{*-1} I_t^*)^2 = 0 \quad (7.23)$$

8 Steady state relationships (after reduction)

$$-\lambda_{ss}^c + \mu C_{ss}^{-1+\mu} (1 - H_{ss})^{1-\mu} \left((1 - H_{ss})^{1-\mu} C_{ss}^\mu \right)^{-\eta} = 0 \quad (8.1)$$

$$\lambda_{ss}^c - \lambda_{ss}^{c*} = 0 \quad (8.2)$$

$$-\lambda_{ss}^{c*} + \mu C_{ss}^{*-1+\mu} (1 - H_{ss}^*)^{1-\mu} \left(C_{ss}^{*\mu} (1 - H_{ss}^*)^{1-\mu} \right)^{-\eta} = 0 \quad (8.3)$$

$$-r_{ss} + \alpha Z_{ss} H_{ss}^{1-\alpha} K_{ss}^{-1+\alpha} = 0 \quad (8.4)$$

$$-r_{ss}^* + \alpha Z_{ss}^* H_{ss}^{*1-\alpha} K_{ss}^{*-1+\alpha} = 0 \quad (8.5)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) H_{ss}^{-\alpha} K_{ss}^\alpha = 0 \quad (8.6)$$

$$-W_{ss}^* + Z_{ss}^* (1 - \alpha) H_{ss}^{*-\alpha} K_{ss}^{*\alpha} = 0 \quad (8.7)$$

$$-Y_{ss} + Z_{ss} H_{ss}^{1-\alpha} K_{ss}^\alpha = 0 \quad (8.8)$$

$$-Y_{ss}^* + Z_{ss}^* H_{ss}^{*1-\alpha} K_{ss}^{*\alpha} = 0 \quad (8.9)$$

$$Z_{ss} - e^{\phi^Z \log Z_{ss}} = 0 \quad (8.10)$$

$$Z_{ss}^* - e^{\phi^Z \log Z_{ss}^*} = 0 \quad (8.11)$$

$$\beta \left(\lambda_{ss}^c \left(r_{ss} - \psi (-\delta + I_{ss} K_{ss}^{-1})^2 + 2\psi I_{ss} K_{ss}^{-1} (-\delta + I_{ss} K_{ss}^{-1}) \right) - \lambda_{ss}^c (-1 - 2\psi (-\delta + I_{ss} K_{ss}^{-1})) (1 - \delta) \right) + \lambda_{ss}^c (-1 - 2\psi (-\delta + I_{ss} K_{ss}^{-1})) = 0 \quad (8.12)$$

$$\beta \left(\lambda_{ss}^{c*} \left(r_{ss}^* - \psi \left(-\delta + I_{ss}^* K_{ss}^{*-1} \right)^2 + 2\psi I_{ss}^* K_{ss}^{*-1} \left(-\delta + I_{ss}^* K_{ss}^{*-1} \right) \right) - \lambda_{ss}^{c*} \left(-1 - 2\psi \left(-\delta + I_{ss}^* K_{ss}^{*-1} \right) \right) (1 - \delta) \right) + \lambda_{ss}^{c*} \left(-1 - 2\psi \left(-\delta + I_{ss}^* K_{ss}^{*-1} \right) \right) = 0 \quad (8.13)$$

$$\lambda_{ss}^c W_{ss} + (-1 + \mu) C_{ss}^{\mu} (1 - H_{ss})^{-\mu} \left(C_{ss}^{\mu} (1 - H_{ss})^{1-\mu} \right)^{-\eta} = 0 \quad (8.14)$$

$$\lambda_{ss}^{c*} W_{ss}^* + (-1 + \mu) C_{ss}^{*\mu} (1 - H_{ss}^*)^{-\mu} \left(C_{ss}^{*\mu} (1 - H_{ss}^*)^{1-\mu} \right)^{-\eta} = 0 \quad (8.15)$$

$$G_{ss}^d - \phi^G G_{ss}^d = 0 \quad (8.16)$$

$$G_{ss}^{d*} - \phi^G G_{ss}^{d*} = 0 \quad (8.17)$$

$$I_{ss} - K_{ss} + K_{ss} (1 - \delta) = 0 \quad (8.18)$$

$$I_{ss}^* - K_{ss}^* + K_{ss}^* (1 - \delta) = 0 \quad (8.19)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^{\mu} (1 - H_{ss})^{1-\mu} \right)^{1-\eta} = 0 \quad (8.20)$$

$$U_{ss}^* - \beta U_{ss}^* - (1 - \eta)^{-1} \left((1 - H_{ss}^*)^{1-\mu} C_{ss}^{*\mu} \right)^{1-\eta} = 0 \quad (8.21)$$

$$-C_{ss} - G_{ss}^d - I_{ss} + TR_{ss} + Y_{ss} - \psi K_{ss} \left(-\delta + I_{ss} K_{ss}^{-1} \right)^2 = 0 \quad (8.22)$$

$$-C_{ss}^* - G_{ss}^{d*} - I_{ss}^* - TR_{ss} + Y_{ss}^* - \psi K_{ss}^* \left(-\delta + I_{ss}^* K_{ss}^{*-1} \right)^2 = 0 \quad (8.23)$$

9 Parameter settings

$$\alpha = 0.4 \quad (9.1)$$

$$\beta = 0.99 \quad (9.2)$$

$$\delta = 0.025 \quad (9.3)$$

$$\eta = 2 \tag{9.4}$$

$$\mu = 0.3 \tag{9.5}$$

$$\phi^G = 0.95 \tag{9.6}$$

$$\phi^Z = 0.95 \tag{9.7}$$

$$\psi = 0.8 \tag{9.8}$$

10 Steady-state values

	Steady-state values
λ^c	0.3934
λ^{c*}	0.3934
r	0.0351
r^*	0.0351
C	0.9578
C^*	0.9578
G^d	0
G^{d*}	0
H	0.2645
H^*	0.2645
I	0.3816
I^*	0.3816
K	15.2627
K^*	15.2627
TR	0
U	-125.6048
U^*	-125.6048
W	3.0384
W^*	3.0384
Y	1.3393
Y^*	1.3393
Z	1
Z^*	1

11 The solution of the perturbation

11.1 P

$$\begin{matrix} G^d \\ G^{d*} \\ K \\ K^* \\ Z \\ Z^* \end{matrix} \begin{pmatrix} G_{t-1}^d & G_{t-1}^{d*} & K_{t-1} & K_{t-1}^* & Z_{t-1} & Z_{t-1}^* \\ 0.95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 \\ -0.1542 & -0.1542 & 0.9454 & 0.0244 & 2.2856 & -1.0704 \\ -0.1542 & -0.1542 & 0.0244 & 0.9454 & -1.0704 & 2.2856 \\ 0 & 0 & 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 \end{pmatrix}$$

11.2 Q

$$\begin{matrix} G^d \\ G^{d*} \\ K \\ K^* \\ Z \\ Z^* \end{matrix} \begin{pmatrix} \epsilon^Z & \epsilon^G & \epsilon^{G^*} & \epsilon^{Z^*} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2.4059 & -0.1623 & -0.1623 & -1.1267 \\ -1.1267 & -0.1623 & -0.1623 & 2.4059 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

11.3 R

$$\begin{array}{c}
 \lambda^c \\
 \lambda^{c*} \\
 r \\
 r^* \\
 C \\
 C^* \\
 H \\
 H^* \\
 I \\
 I^* \\
 TR \\
 U \\
 U^* \\
 W \\
 W^* \\
 Y \\
 Y^*
 \end{array}
 \begin{pmatrix}
 G_{t-1}^d & G_{t-1}^{d*} & K_{t-1} & K_{t-1}^* & Z_{t-1} & Z_{t-1}^* \\
 0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
 0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
 0.0044 & 0.0044 & -0.0012 & -0.0004 & 0.0497 & -0.0046 \\
 0.0044 & 0.0044 & -0.0004 & -0.0012 & -0.0046 & 0.0497 \\
 -0.1525 & -0.1525 & 0.0187 & 0.0136 & 0.3448 & 0.1599 \\
 -0.1525 & -0.1525 & 0.0136 & 0.0187 & 0.1599 & 0.3448 \\
 0.0554 & 0.0554 & 0.0023 & -0.0049 & 0.2054 & -0.0581 \\
 0.0554 & 0.0554 & -0.0049 & 0.0023 & -0.0581 & 0.2054 \\
 -0.1542 & -0.1542 & -0.0296 & 0.0244 & 2.2856 & -1.0704 \\
 -0.1542 & -0.1542 & 0.0244 & -0.0296 & -1.0704 & 2.2856 \\
 0.475 & -0.475 & -0.053 & 0.053 & 0.7338 & -0.7338 \\
 -3.1408 & -3.1408 & 0.1608 & 0.2366 & 0.053 & 8.3603 \\
 -3.1408 & -3.1408 & 0.2366 & 0.1608 & 8.3603 & 0.053 \\
 -0.2547 & -0.2547 & 0.0689 & 0.0227 & 1.9424 & 0.2672 \\
 -0.2547 & -0.2547 & 0.0227 & 0.0689 & 0.2672 & 1.9424 \\
 0.1684 & 0.1684 & 0.0422 & -0.015 & 1.8966 & -0.1767 \\
 0.1684 & 0.1684 & -0.015 & 0.0422 & -0.1767 & 1.8966
 \end{pmatrix}$$

11.4 S

$$\begin{array}{c}
 \lambda^c \\
 \lambda^{c*} \\
 r \\
 r^* \\
 C \\
 C^* \\
 H \\
 H^* \\
 I \\
 I^* \\
 TR \\
 U \\
 U^* \\
 W \\
 W^* \\
 Y \\
 Y^*
 \end{array}
 \begin{pmatrix}
 \epsilon^Z & \epsilon^G & \epsilon^{G^*} & \epsilon^{Z^*} \\
 -0.1128 & 0.1075 & 0.1075 & -0.1128 \\
 -0.1128 & 0.1075 & 0.1075 & -0.1128 \\
 0.0523 & 0.0046 & 0.0046 & -0.0049 \\
 -0.0049 & 0.0046 & 0.0046 & 0.0523 \\
 0.3629 & -0.1605 & -0.1605 & 0.1683 \\
 0.1683 & -0.1605 & -0.1605 & 0.3629 \\
 0.2163 & 0.0583 & 0.0583 & -0.0612 \\
 -0.0612 & 0.0583 & 0.0583 & 0.2163 \\
 2.4059 & -0.1623 & -0.1623 & -1.1267 \\
 -1.1267 & -0.1623 & -0.1623 & 2.4059 \\
 0.7724 & 0.5 & -0.5 & -0.7724 \\
 0.0557 & -3.3061 & -3.3061 & 8.8003 \\
 8.8003 & -3.3061 & -3.3061 & 0.0557 \\
 2.0446 & -0.2681 & -0.2681 & 0.2812 \\
 0.2812 & -0.2681 & -0.2681 & 2.0446 \\
 1.9964 & 0.1773 & 0.1773 & -0.186 \\
 -0.186 & 0.1773 & 0.1773 & 1.9964
 \end{pmatrix}$$

12 Statistics of the model

12.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
r	0.0351	0.0051	0	N
C	0.9578	0.0373	0.0014	N
G^d	0	0.0922	0.0085	N
H	0.2645	0.026	0.0007	N
I	0.3816	0.2659	0.0707	N
K	15.2627	0.9072	0.8231	N
TR	0	0.1943	0.0378	N
U	-125.6048	1.065	1.1342	N
W	3.0384	0.1882	0.0354	N
Y	1.3393	0.2048	0.042	N
Z	1	0.0922	0.0085	N

12.2 Correlation matrix

	r	C	G^d	H	I	K	TR	U	W	Y	Z
λ^c	-0.1448	-0.8743	0.2572	0.1061	-0.0803	-0.0047	-0.0187	-0.7069	-0.4894	-0.1301	-0.2886
λ^{c*}	-0.1448	-0.8743	0.2572	0.1061	-0.0803	-0.0047	-0.0187	-0.7069	-0.4894	-0.1301	-0.2886
r	1	0.5686	0.5522	0.8902	0.8158	0.1686	0.5159	-0.2367	0.8651	0.9218	0.9793
r^*	-0.0679	0.2	0.0324	-0.139	-0.6152	-0.1951	-0.7139	0.8085	0.0465	-0.0704	-0.0241
C	0.5686	1	0.0547	0.3898	0.4323	0.2641	0.1824	0.431	0.8511	0.5949	0.7017
C^*	0.0628	0.6885	-0.159	-0.2094	-0.2594	-0.2444	-0.2905	0.9396	0.3136	-0.0074	0.166
G^d	0.5522	0.0547	1	0.6002	0.3541	0.1369	0.4025	-0.2867	0.3765	0.5378	0.5
G^{d*}	0.0645	-0.2751	0	0.1266	-0.3462	-0.1539	-0.5693	0.1362	-0.1009	0.0404	0
H	0.8902	0.3898	0.6002	1	0.7333	0.5322	0.3383	-0.4582	0.8152	0.9721	0.89
H^*	-0.0883	0.0506	0.0674	-0.2437	-0.6045	-0.4681	-0.575	0.764	-0.1071	-0.1999	-0.0989
I	0.8158	0.4323	0.3541	0.7333	1	0.2308	0.8288	-0.5626	0.6899	0.7501	0.7926
I^*	-0.3991	0.0775	-0.2022	-0.447	-0.8206	-0.2188	-0.8136	0.8737	-0.2061	-0.3705	-0.3347
K	0.1686	0.2641	0.1369	0.5322	0.2308	1	-0.1291	-0.3286	0.4695	0.5318	0.2956
K^*	-0.1002	0.0352	-0.0895	-0.4653	-0.1569	-0.8034	0.1682	0.4588	-0.2431	-0.3971	-0.1612
TR	0.5159	0.1824	0.4025	0.3383	0.8288	-0.1291	1	-0.5174	0.3076	0.3418	0.4493
U	-0.2367	0.431	-0.2867	-0.4582	-0.5626	-0.3286	-0.5174	1	0.0099	-0.2901	-0.1453
U^*	0.6951	0.838	0.1218	0.5844	0.8091	0.3957	0.5633	-0.0972	0.8602	0.7235	0.7851
W	0.8651	0.8511	0.3765	0.8152	0.6899	0.4695	0.3076	0.0099	1	0.9283	0.9487
W^*	-0.0133	0.4075	-0.0512	-0.2479	-0.4714	-0.3891	-0.4725	0.9335	0.1149	-0.1126	0.038
Y	0.9218	0.5949	0.5378	0.9721	0.7501	0.5318	0.3418	-0.2901	0.9283	1	0.9555
Y^*	-0.0597	0.197	0.0206	-0.2508	-0.5635	-0.4462	-0.5459	0.8497	-0.0191	-0.1687	-0.0453
Z	0.9793	0.7017	0.5	0.89	0.7926	0.2956	0.4493	-0.1453	0.9487	0.9555	1
Z^*	-0.0479	0.2874	0	-0.1857	-0.5731	-0.277	-0.6354	0.878	0.0749	-0.0889	0

12.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
r	0.7037	0.4562	0.2539	0.0927	-0.0317
C	0.7464	0.5237	0.3324	0.1718	0.0405
G^d	0.7133	0.4711	0.2711	0.1098	-0.0163
H	0.7547	0.5359	0.3455	0.1836	0.0497
I	0.6973	0.4462	0.2424	0.0814	-0.0418
K	0.9563	0.8517	0.7083	0.544	0.3729
TR	0.7199	0.4816	0.2831	0.1217	-0.0057
U	0.7314	0.4998	0.3042	0.1431	0.0138
W	0.7473	0.5247	0.3329	0.1715	0.0394
Y	0.7516	0.5312	0.34	0.1783	0.0449
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

12.4 Variance decomposition

	ϵ^Z	ϵ^G	ϵ^{G^*}	ϵ^{Z^*}
r	0.9794	0.0054	0.0057	0.0095
C	0.5466	0.1185	0.0759	0.2589
G^d	0.25	0.75	0	0
H	0.8104	0.0321	0.0294	0.1281
I	0.6413	0.0024	0.1258	0.2305
K	0.6482	0.0026	0.1239	0.2253
TR	0.2911	0.0422	0.3687	0.298
U	0.0221	0.0614	0.0215	0.895
W	0.9496	0.0133	0.0138	0.0233
Y	0.9443	0.0048	0.0113	0.0396
Z	1	0	0	0

13 Statistics of the model

13.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
λ^c	0.2937	0.0901	0.0081	N
λ^{c*}	0.2937	0.0901	0.0081	N
r	0.0262	0.0249	0.0006	N
r^*	0.0262	0.035	0.0012	N
C	0.7151	0.1821	0.0332	N
C^*	0.7151	0.2229	0.0497	N
G^d	0	0.4499	0.2024	N
G^{d*}	0	0.6363	0.4049	N
H	0.1975	0.1268	0.0161	N
H^*	0.1975	0.1684	0.0283	N
I	0.2849	1.2983	1.6855	N
I^*	0.2849	1.5717	2.4701	N
K	11.3957	4.4289	19.615	N
K^*	11.3957	5.381	28.9555	N
TR	0	0.9485	0.8997	N
U	-93.7814	5.1989	27.0286	N
U^*	-93.7814	4.1109	16.8995	N
W	2.2686	0.9187	0.8439	N
W^*	2.2686	1.281	1.6408	N
Y	1	1	1	N
Y^*	1	1.387	1.9236	N
Z	0.7466	0.4499	0.2024	N
Z^*	0.7466	0.6363	0.4049	N

13.2 Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
λ^c	0.0916	0.0747	0.0466	0.0049	-0.0531	-0.1301	-0.1296	-0.1232	-0.1127	-0.0994	-0.0844
λ^{c*}	0.0916	0.0747	0.0466	0.0049	-0.0531	-0.1301	-0.1296	-0.1232	-0.1127	-0.0994	-0.0844
r	0.1626	0.2765	0.4099	0.5627	0.734	0.9218	0.5737	0.2944	0.0766	-0.0872	-0.2047
r^*	-0.1312	-0.14	-0.1413	-0.1324	-0.1099	-0.0704	0.0026	0.0552	0.0907	0.1122	0.1224
C	-0.0523	0.0267	0.1286	0.256	0.4109	0.5949	0.4729	0.3599	0.2578	0.1678	0.0905
C^*	-0.1138	-0.1178	-0.1126	-0.095	-0.0613	-0.0074	0.0122	0.0279	0.0399	0.0485	0.0539
G^d	0.048	0.1164	0.1992	0.2971	0.4101	0.5378	0.3679	0.2265	0.1117	0.021	-0.0483
G^{d*}	-0.0688	-0.0646	-0.0536	-0.0339	-0.0034	0.0404	0.0468	0.0498	0.0501	0.0482	0.0449
H	0.0668	0.1964	0.352	0.5339	0.7412	0.9721	0.7231	0.5036	0.3144	0.1553	0.0251
H^*	-0.0838	-0.1155	-0.1459	-0.1723	-0.1916	-0.1999	-0.1621	-0.1234	-0.0856	-0.0504	-0.0188
I	0.1968	0.2892	0.3927	0.5058	0.6261	0.7501	0.4389	0.1931	0.0048	-0.1335	-0.2294
I^*	-0.182	-0.2276	-0.2722	-0.3132	-0.3473	-0.3705	-0.1823	-0.0376	0.0693	0.1442	0.1924
K	-0.2171	-0.1269	-0.0086	0.1399	0.3199	0.5318	0.6472	0.6876	0.6718	0.6159	0.5332
K^*	0.0348	-0.0325	-0.1112	-0.1999	-0.2963	-0.3971	-0.4404	-0.4404	-0.4091	-0.3568	-0.2917
TR	0.2346	0.2682	0.2982	0.3223	0.3379	0.3418	0.0736	-0.1192	-0.2494	-0.3285	-0.3669
U	-0.1369	-0.1765	-0.2148	-0.2489	-0.2755	-0.2901	-0.1786	-0.0871	-0.0142	0.0419	0.0832
U^*	0.0647	0.1596	0.2732	0.4055	0.5561	0.7235	0.5187	0.3431	0.196	0.0759	-0.0195
W	0.0052	0.1288	0.2816	0.4654	0.681	0.9283	0.7097	0.5134	0.3413	0.194	0.0712
W^*	-0.1083	-0.1277	-0.1413	-0.146	-0.1379	-0.1126	-0.0813	-0.0516	-0.0245	-0.0006	0.0195
Y	0.0449	0.1783	0.34	0.5312	0.7516	1	0.7516	0.5312	0.34	0.1783	0.0449
Y^*	-0.0956	-0.123	-0.1472	-0.1653	-0.1739	-0.1687	-0.1327	-0.0968	-0.0626	-0.0313	-0.0036
Z	0.1046	0.2262	0.372	0.5424	0.7374	0.9555	0.6479	0.3926	0.1859	0.0232	-0.1003
Z^*	-0.1253	-0.1386	-0.1448	-0.141	-0.1237	-0.0889	-0.0307	0.0141	0.0471	0.0701	0.0845

14 Impulse response functions

14.1 Shock ϵ^Z

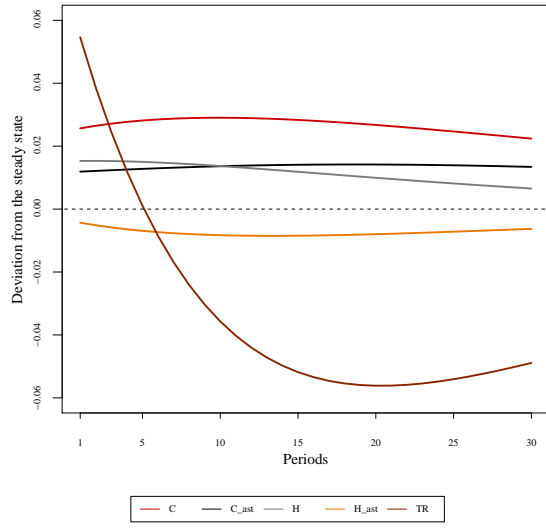


Figure 1: Impulse response function for ϵ^Z shock

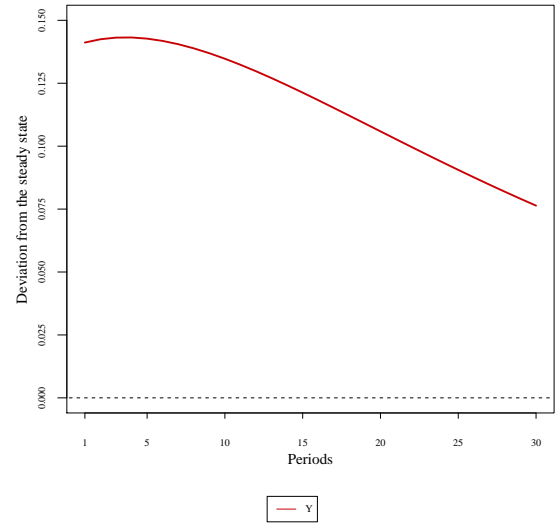


Figure 2: Impulse response function for ϵ^Z shock

14.2 Shock ϵ^{Z^*}

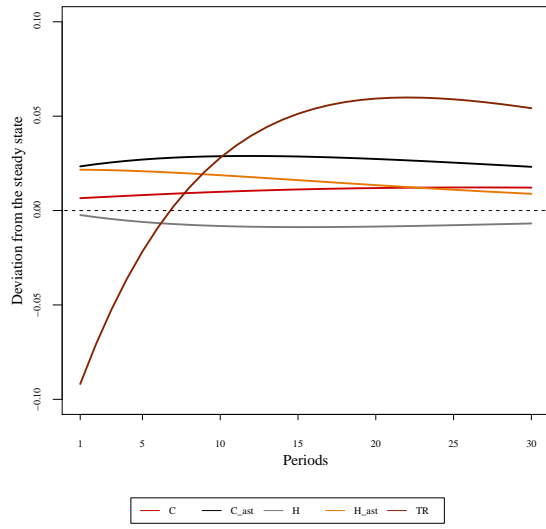


Figure 3: Impulse response function for ϵ^{Z^*} shock

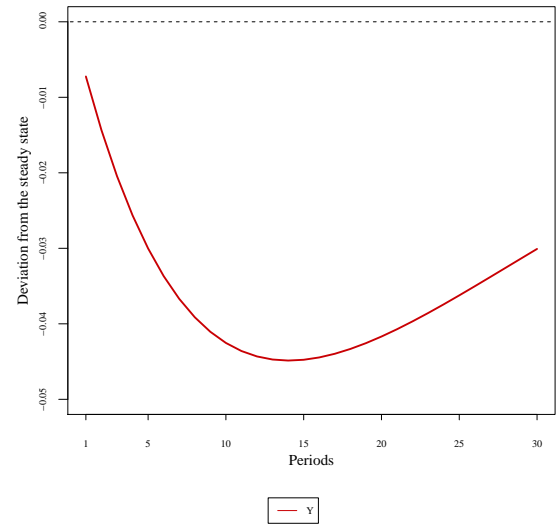


Figure 4: Impulse response function for ϵ^{Z^*} shock