

1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L_t^s} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t P_t^{\text{FIN}} = \pi_t + \pi_t^{\text{ps}} + L_t^s W_t \quad (\lambda_t^c) \quad (1.2)$$

1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.3)$$

$$-\lambda_t^c P_t^{\text{FIN}} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.4)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.5)$$

2 INTERMEDIATE FIRM

2.1 Optimisation problem

$$\max_{K_t, L_t^d, Y_t, I_t, \pi_t} \Pi_t = \pi_t + \lambda_t^{c-1} E_t [\lambda_{t+1}^U \lambda_{t+1}^c \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$\pi_t = -I_t - L_t^d W_t + P_t Y_t \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} \right) \quad (2.2)$$

$$Y_t = K_{t-1}^\alpha (L_t^d Z_t)^{1-\alpha} \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} \right) \quad (2.3)$$

$$K_t = I_t + K_{t-1} (1 - \delta) \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^3}} \right) \quad (2.4)$$

2.2 First order conditions

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + \lambda_{t-1}^{\text{c}}^{-1} \lambda_t^{\text{U}} \lambda_t^{\text{c}} = 0 \quad (\Pi_t) \quad (2.5)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^3}} + \text{E}_t \left[\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} \left(\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^3}} (1 - \delta) + \alpha \lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^2}} K_t^{-1+\alpha} (L_{t+1}^{\text{d}} Z_{t+1})^{1-\alpha} \right) \right] = 0 \quad (K_t) \quad (2.6)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} W_t + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} Z_t (1 - \alpha) K_{t-1}^{\alpha} (L_t^{\text{d}} Z_t)^{-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} P_t = 0 \quad (Y_t) \quad (2.8)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^3}} = 0 \quad (I_t) \quad (2.9)$$

$$1 - \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} = 0 \quad (\pi_t) \quad (2.10)$$

2.3 First order conditions after reduction

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + \lambda_{t-1}^{\text{c}}^{-1} \lambda_t^{\text{U}} \lambda_t^{\text{c}} = 0 \quad (\Pi_t) \quad (2.11)$$

$$-1 + \text{E}_t \left[\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} \left(1 - \delta + \alpha \lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^2}} K_t^{-1+\alpha} (L_{t+1}^{\text{d}} Z_{t+1})^{1-\alpha} \right) \right] = 0 \quad (K_t) \quad (2.12)$$

$$-W_t + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} Z_t (1 - \alpha) K_{t-1}^{\alpha} (L_t^{\text{d}} Z_t)^{-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.13)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} + P_t = 0 \quad (Y_t) \quad (2.14)$$

3 PRICE SETTING

3.1 Optimisation problem

$$\max_{\pi_t^{\text{ps}}, Y_t^{\text{MON}}, P_t^{\text{MON}}} \Pi_t^{\text{PS}} = \pi_t^{\text{ps}} \quad (3.1)$$

s.t. :

$$\pi_t^{\text{ps}} = Y_t^{\text{MON}} (-P_t + P_t^{\text{MON}}) \quad \left(\lambda_t^{\text{PRICE}^{\text{SETTING}^1}} \right) \quad (3.2)$$

$$Y_t^{\text{MON}} = Y_t^{\text{FIN}} \left(P_t^{\text{FIN}^{-1}} P_t^{\text{MON}} \right)^{-\rho} \quad \left(\lambda_t^{\text{PRICE}^{\text{SETTING}^2}} \right) \quad (3.3)$$

3.2 First order conditions

$$1 - \lambda_t^{\text{PRICESETTING}^1} = 0 \quad (\pi_t^{\text{PS}}) \quad (3.4)$$

$$-\lambda_t^{\text{PRICESETTING}^2} + \lambda_t^{\text{PRICESETTING}^1} (-P_t + P_t^{\text{MON}}) = 0 \quad (Y_t^{\text{MON}}) \quad (3.5)$$

$$\lambda_t^{\text{PRICESETTING}^1} Y_t^{\text{MON}} - \rho \lambda_t^{\text{PRICESETTING}^2} P_t^{\text{FIN}-1} Y_t^{\text{FIN}} (P_t^{\text{FIN}-1} P_t^{\text{MON}})^{-1-\rho} = 0 \quad (P_t^{\text{MON}}) \quad (3.6)$$

3.3 First order conditions after reduction

$$Y_t^{\text{MON}} - \rho P_t^{\text{FIN}-1} Y_t^{\text{FIN}} (-P_t + P_t^{\text{MON}}) (P_t^{\text{FIN}-1} P_t^{\text{MON}})^{-1-\rho} = 0 \quad (P_t^{\text{MON}}) \quad (3.7)$$

4 FINAL FIRM

4.1 Identities

$$Y_t^{\text{FIN}} = Y_t^{\text{MON}} \quad (4.1)$$

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5 EQUILIBRIUM

5.1 Identities

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (5.1)$$

$$P_t^{\text{FIN}} = 1 \quad (5.2)$$

$$Y_t^{\text{MON}} = Y_t \quad (5.3)$$

6 EXOG

6.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (6.1)$$

7 Equilibrium relationships (after reduction)

$$-1+\beta \left(C_t^{-1+\mu}\right)^{-1} \left(1-L_t^s\right)^{1-\mu} \left(\left(C_t^\mu(1-L_t^s)^{1-\mu}\right)^{-\eta}\right)^{-1} \text{E}_t \left[\left(1-\delta+\alpha P_{t+1} K_t^{-1+\alpha} \left(L_{t+1}^s Z_{t+1}\right)^{1-\alpha}\right) C_{t+1}^{-1+\mu} \left(1-L_{t+1}^s\right)^{1-\mu} \left(C_{t+1}^\mu \left(1-L_{t+1}^s\right)^{1-\mu}\right)^{-\eta}\right] = 0 \quad (7.1)$$

$$-\pi_t^{\text{ps}} + \Pi_t^{\text{ps}} = 0 \quad (7.2)$$

$$-\pi_t^{\text{ps}} + Y_t \left(-P_t + P_t^{\text{MON}}\right) = 0 \quad (7.3)$$

$$-W_t + P_t Z_t (1-\alpha) K_{t-1}^\alpha \left(L_t^s Z_t\right)^{-\alpha} = 0 \quad (7.4)$$

$$-Y_t + Y_t P_t^{\text{MON}^{-\rho}} = 0 \quad (7.5)$$

$$-Y_t + K_{t-1}^\alpha \left(L_t^s Z_t\right)^{1-\alpha} = 0 \quad (7.6)$$

$$Y_t - \rho Y_t \left(-P_t + P_t^{\text{MON}}\right) P_t^{\text{MON}^{-1-\rho}} = 0 \quad (7.7)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (7.8)$$

$$(-1+\mu) C_t^\mu (1-L_t^s)^{-\mu} \left(C_t^\mu (1-L_t^s)^{1-\mu}\right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1-L_t^s)^{1-\mu} \left(C_t^\mu (1-L_t^s)^{1-\mu}\right)^{-\eta} = 0 \quad (7.9)$$

$$-\pi_t + \Pi_t - \beta \left(C_t^{-1+\mu}\right)^{-1} \left(1-L_t^s\right)^{1-\mu} \left(\left(C_t^\mu(1-L_t^s)^{1-\mu}\right)^{-\eta}\right)^{-1} \text{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} \left(1-L_{t+1}^s\right)^{1-\mu} \left(C_{t+1}^\mu \left(1-L_{t+1}^s\right)^{1-\mu}\right)^{-\eta}\right] = 0 \quad (7.10)$$

$$I_t - K_t + K_{t-1} (1-\delta) = 0 \quad (7.11)$$

$$U_t - \beta \text{E}_t [U_{t+1}] - (1-\eta)^{-1} \left(C_t^\mu (1-L_t^s)^{1-\mu}\right)^{1-\eta} = 0 \quad (7.12)$$

$$-\pi_t - I_t - L_t^s W_t + P_t Y_t = 0 \quad (7.13)$$

$$\pi_t + \pi_t^{\text{ps}} - C_t + L_t^s W_t = 0 \quad (7.14)$$

8 Steady state relationships (after reduction)

$$-1 + \beta \left(1 - \delta + \alpha P_{ss} K_{ss}^{-1+\alpha} (L_{ss}^s Z_{ss})^{1-\alpha} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} (1 - L_{ss}^s)^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} = 0 \quad (8.1)$$

$$-\pi_{ss}^{\text{ps}} + \Pi_{ss}^{\text{PS}} = 0 \quad (8.2)$$

$$-\pi_{ss}^{\text{ps}} + Y_{ss} (-P_{ss} + P_{ss}^{\text{MON}}) = 0 \quad (8.3)$$

$$-W_{ss} + P_{ss} Z_{ss} (1 - \alpha) K_{ss}^{-\alpha} (L_{ss}^s Z_{ss})^{-\alpha} = 0 \quad (8.4)$$

$$-Y_{ss} + Y_{ss} P_{ss}^{\text{MON}^{-\rho}} = 0 \quad (8.5)$$

$$-Y_{ss} + K_{ss}^{-\alpha} (L_{ss}^s Z_{ss})^{1-\alpha} = 0 \quad (8.6)$$

$$Y_{ss} - \rho Y_{ss} (-P_{ss} + P_{ss}^{\text{MON}}) P_{ss}^{\text{MON}^{-1-\rho}} = 0 \quad (8.7)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (8.8)$$

$$(-1 + \mu) C_{ss}^{\mu} (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (8.9)$$

$$-\pi_{ss} + \Pi_{ss} - \beta \Pi_{ss} (1 - L_{ss}^s)^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} = 0 \quad (8.10)$$

$$I_{ss} - K_{ss} + K_{ss} (1 - \delta) = 0 \quad (8.11)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (8.12)$$

$$-\pi_{ss} - I_{ss} - L_{ss}^s W_{ss} + P_{ss} Y_{ss} = 0 \quad (8.13)$$

$$\pi_{ss} + \pi_{ss}^{\text{ps}} - C_{ss} + L_{ss}^s W_{ss} = 0 \quad (8.14)$$

9 Parameter settings

$\alpha = 0.33$ (9.1)

$\beta = 0.99$ (9.2)

$\delta = 0.025$ (9.3)

$\eta = 2$ (9.4)

$\mu = 0.3$ (9.5)

$\phi = 0.95$ (9.6)

$\rho = 11$ (9.7)

10 Steady-state values

	Steady-state values
π	0.0619
π^{ps}	0.0652
C	0.5638
I	0.1532
K	6.1285
L^s	0.2492
P	0.9091
P^{MON}	1
Π	6.1904
Π^{ps}	0.0652
U	-145.144
W	1.7524
Y	0.7171
Z	1

11 The solution of the perturbation

11.1 P

$$\begin{matrix} K_{t-1} & Z_{t-1} \\ K & \begin{pmatrix} 0.958 & 0.0744 \\ 0 & 0.95 \end{pmatrix} \\ Z & \end{matrix}$$

11.2 Q

$$\begin{matrix} \epsilon^Z \\ K & \begin{pmatrix} 0.0783 \\ 1 \end{pmatrix} \\ Z & \end{matrix}$$

11.3 R

$$\begin{matrix} & K_{t-1} & Z_{t-1} \\ \pi & \begin{pmatrix} 2.4086 & -4.1777 \\ 0.2085 & 0.9179 \\ 0.45 & 0.3585 \\ -0.6804 & 2.9768 \\ -0.1813 & 0.42 \\ 0 & 0 \\ 0 & 0 \\ 0.9725 & 0.0319 \\ 0.2085 & 0.9179 \\ -0.0343 & -0.0442 \\ 0.3898 & 0.4979 \\ 0.2085 & 0.9179 \end{pmatrix} \\ \pi^{\text{ps}} & \\ C & \\ I & \\ L^s & \\ P & \\ P^{\text{MON}} & \\ \Pi & \\ \Pi^{\text{ps}} & \\ U & \\ W & \\ Y & \end{matrix}$$

11.4 S

$$\begin{array}{c} \pi \\ \pi^{\text{ps}} \\ C \\ I \\ L^s \\ P \\ P^{\text{MON}} \\ \Pi \\ \Pi^{\text{ps}} \\ U \\ W \\ Y \end{array} \begin{pmatrix} \epsilon^Z \\ -4.3976 \\ 0.9662 \\ 0.3773 \\ 3.1334 \\ 0.4421 \\ 0 \\ 0 \\ 0.0336 \\ 0.9662 \\ -0.0465 \\ 0.5241 \\ 0.9662 \end{pmatrix}$$

12 Statistics of the model

12.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
C	0.5638	0.5188	0.2691	Y
I	0.1532	4.0905	16.732	Y
K	6.1285	0.3617	0.1308	Y
L^s	0.2492	0.5797	0.336	Y
U	-145.144	0.0619	0.0038	Y
W	1.7524	0.6982	0.4875	Y
Y	0.7171	1.262	1.5927	Y
Z	1	1.3034	1.699	Y

12.2 Correlation matrix

	C	I	K	L^s	U	W	Y	Z
π	-0.8915	-0.9959	-0.1395	-0.9993	0.9384	-0.9379	-0.9779	-0.9886
π^{ps}	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
C	1	0.9289	0.5729	0.9079	-0.9931	0.9933	0.9666	0.9495
I	0.9289	1	0.2287	0.9986	-0.9659	0.9655	0.9928	0.9982
K	0.5729	0.2287	1	0.1766	-0.4731	0.4744	0.3436	0.2868
L^s	0.9079	0.9986	0.1766	1	-0.9507	0.9503	0.985	0.9936
P	0	0	0	0	0	0	0	0
P^{MON}	0	0	0	0	0	0	0	0
Π	0.4328	0.0682	0.9868	0.0151	-0.3243	0.3257	0.1871	0.1281
Π^{ps}	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
U	-0.9931	-0.9659	-0.4731	-0.9507	1	-1	-0.9899	-0.9797
W	0.9933	0.9655	0.4744	0.9503	-1	1	0.9897	0.9794
Y	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
Z	0.9495	0.9982	0.2868	0.9936	-0.9797	0.9794	0.9982	1

12.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
C	0.7598	0.5448	0.3569	0.1964	0.0629
I	0.7097	0.4654	0.2644	0.1031	-0.0225
K	0.9593	0.8611	0.7252	0.5682	0.403
L^s	0.708	0.4628	0.2614	0.1	-0.0253
U	0.7378	0.5101	0.3164	0.1556	0.0256
W	0.7381	0.5105	0.3169	0.1561	0.026
Y	0.7186	0.4796	0.2809	0.1198	-0.0072
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

13 Statistics of the model

13.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
C	0.5638	0.5188	0.2691	Y
I	0.1532	4.0905	16.732	Y
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U	-145.144	0.0619	0.0038	Y
W	1.7524	0.6982	0.4875	Y
Y	0.7171	1.262	1.5927	Y
Z	1	1.3034	1.699	Y

13.2 Correlation matrix

	C	I	K	L^s	U	W	Y	Z
π	-0.8915	-0.9959	-0.1395	-0.9993	0.9384	-0.9379	-0.9779	-0.9886
π^{PS}	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
C	1	0.9289	0.5729	0.9079	-0.9931	0.9933	0.9666	0.9495
I	0.9289	1	0.2287	0.9986	-0.9659	0.9655	0.9928	0.9982
K	0.5729	0.2287	1	0.1766	-0.4731	0.4744	0.3436	0.2868
L^s	0.9079	0.9986	0.1766	1	-0.9507	0.9503	0.985	0.9936
P	0	0	0	0	0	0	0	0
P^{MON}	0	0	0	0	0	0	0	0
Π	0.4328	0.0682	0.9868	0.0151	-0.3243	0.3257	0.1871	0.1281
Π^{PS}	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
U	-0.9931	-0.9659	-0.4731	-0.9507	1	-1	-0.9899	-0.9797
W	0.9933	0.9655	0.4744	0.9503	-1	1	0.9897	0.9794
Y	0.9666	0.9928	0.3436	0.985	-0.9899	0.9897	1	0.9982
Z	0.9495	0.9982	0.2868	0.9936	-0.9797	0.9794	0.9982	1

13.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
C	0.7598	0.5448	0.3569	0.1964	0.0629
I	0.7097	0.4654	0.2644	0.1031	-0.0225
K	0.9593	0.8611	0.7252	0.5682	0.403
L^s	0.708	0.4628	0.2614	0.1	-0.0253
U	0.7378	0.5101	0.3164	0.1556	0.0256
W	0.7381	0.5105	0.3169	0.1561	0.026
Y	0.7186	0.4796	0.2809	0.1198	-0.0072
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

14 Impulse response functions

14.1 Shock ϵ^Z

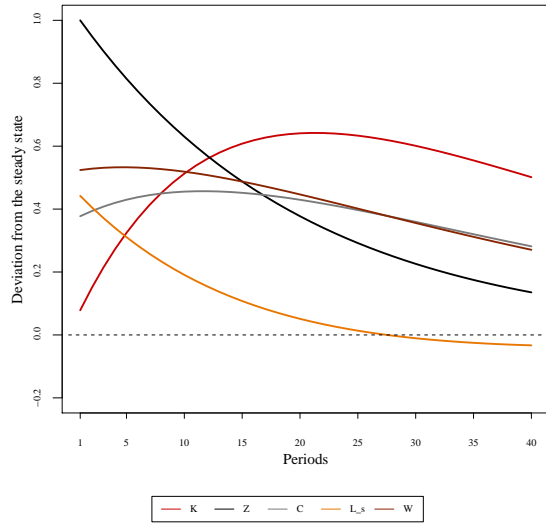


Figure 1: Impulse response function for ϵ^Z shock

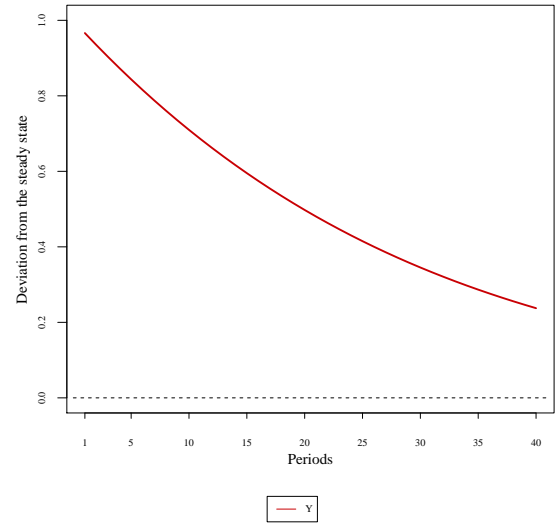


Figure 2: Impulse response function for ϵ^Z shock

15 Random path simulation

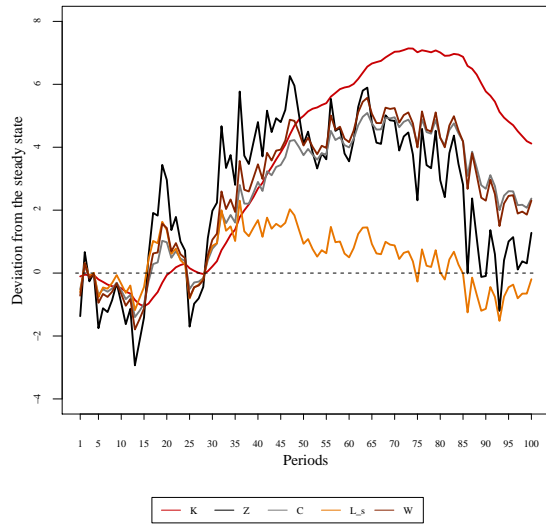


Figure 3: Random path simulation for 100 periods

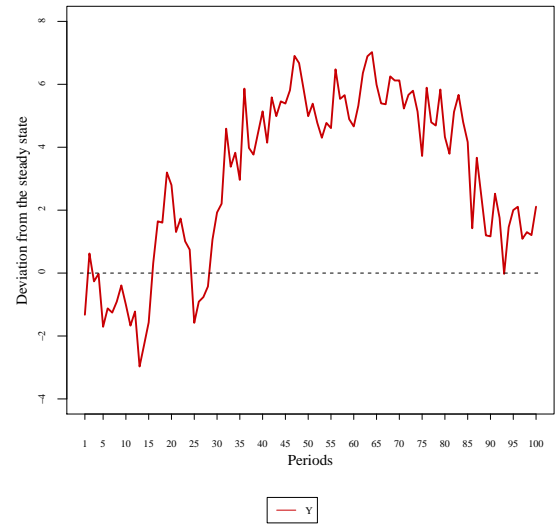


Figure 4: Random path simulation for 100 periods