

1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L_t^s} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t = \pi_t + L_t^s W_t \quad (\lambda_t^c) \quad (1.2)$$

1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.3)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.4)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.5)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t, L_t^d, Y_t, I_t, \pi_t, CqU_t} \Pi_t = \pi_t + \lambda_t^{c-1} E_t [\lambda_{t+1}^U \lambda_{t+1}^c \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$Y_t = L_t^{d^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$K_t = I_t + K_{t-1} (1 - \delta CqU_t)^\omega \quad \left(\lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

$$\pi_t = -I_t - L_t^d W_t + P_t Y_t \quad \left(\lambda_t^{\text{FIRM}^3} \right) \quad (2.4)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c{}^{-1} \lambda_t^U \lambda_t^c = 0 \quad (\Pi_t) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^2} + \text{E}_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(\lambda_{t+1}^{\text{FIRM}^2} (1 - \delta CqU_{t+1}^\omega) + \alpha \lambda_{t+1}^{\text{FIRM}^1} CqU_{t+1} L_{t+1}^d{}^{1-\alpha} Z_{t+1}^{1-\alpha} (K_t CqU_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.6)$$

$$-\lambda_t^{\text{FIRM}^3} W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{d-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (L_t^d) \quad (2.7)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^3} P_t = 0 \quad (Y_t) \quad (2.8)$$

$$\lambda_t^{\text{FIRM}^2} - \lambda_t^{\text{FIRM}^3} = 0 \quad (I_t) \quad (2.9)$$

$$1 - \lambda_t^{\text{FIRM}^3} = 0 \quad (\pi_t) \quad (2.10)$$

$$-\delta \omega K_{t-1} \lambda_t^{\text{FIRM}^2} CqU_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{d1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^{-1+\alpha} = 0 \quad (CqU_t) \quad (2.11)$$

2.3 First order conditions after reduction

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c{}^{-1} \lambda_t^U \lambda_t^c = 0 \quad (\Pi_t) \quad (2.12)$$

$$-1 + \text{E}_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(1 - \delta CqU_{t+1}^\omega + \alpha \lambda_{t+1}^{\text{FIRM}^1} CqU_{t+1} L_{t+1}^d{}^{1-\alpha} Z_{t+1}^{1-\alpha} (K_t CqU_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.13)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{d-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (L_t^d) \quad (2.14)$$

$$-\lambda_t^{\text{FIRM}^1} + P_t = 0 \quad (Y_t) \quad (2.15)$$

$$-\delta \omega K_{t-1} CqU_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{d1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^{-1+\alpha} = 0 \quad (CqU_t) \quad (2.16)$$

3 EQUILIBRIUM

3.1 Identities

$$P_t = 1 \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$-1 + \beta C_t^{1-\mu} (1 - L_t^s)^{-1+\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^\eta \text{E}_t \left[\left(1 - \delta C_q U_{t+1}^\omega + \alpha C_q U_{t+1} L_{t+1}^s {}^{1-\alpha} Z_{t+1} {}^{1-\alpha} (K_t C_q U_{t+1})^{-1+\alpha} \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.1)$$

$$-W_t + (1 - \alpha) L_t^{s-1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_q U_t)^\alpha = 0 \quad (5.2)$$

$$-Y_t + L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_q U_t)^\alpha = 0 \quad (5.3)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$-\delta \omega K_{t-1} C_q U_t^{-1+\omega} + \alpha K_{t-1} L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_q U_t)^{-1+\alpha} = 0 \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$I_t - K_t + K_{t-1} (1 - \delta C_q U_t^\omega) = 0 \quad (5.7)$$

$$U_t - \beta \text{E}_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.8)$$

$$-C_t + \Pi_t + L_t^s W_t - \beta (C_t^{-1+\mu})^{-1} \left((1 - L_t^s)^{1-\mu} \right)^{-1} \left(\left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} \right)^{-1} \text{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.9)$$

$$-I_t - \Pi_t + Y_t - L_t^s W_t + \beta (C_t^{-1+\mu})^{-1} \left((1 - L_t^s)^{1-\mu} \right)^{-1} \left(\left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} \right)^{-1} \text{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.10)$$

6 Steady state relationships (after reduction)

$$-1 + \beta \left(1 - \delta C_{\mathcal{Q}} U_{\text{ss}}^{\omega} + \alpha C_{\mathcal{Q}} U_{\text{ss}} L_{\text{ss}}^s{}^{1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\mathcal{Q}} U_{\text{ss}} K_{\text{ss}})^{-1+\alpha} \right) C_{\text{ss}}^{-1+\mu} C_{\text{ss}}^{1-\mu} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.1)$$

$$-W_{\text{ss}} + (1 - \alpha) L_{\text{ss}}^s{}^{-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\mathcal{Q}} U_{\text{ss}} K_{\text{ss}})^{\alpha} = 0 \quad (6.2)$$

$$-Y_{\text{ss}} + L_{\text{ss}}^s{}^{1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\mathcal{Q}} U_{\text{ss}} K_{\text{ss}})^{\alpha} = 0 \quad (6.3)$$

$$Z_{\text{ss}} - e^{\phi \log Z_{\text{ss}}} = 0 \quad (6.4)$$

$$-\delta \omega K_{\text{ss}} C_{\mathcal{Q}} U_{\text{ss}}^{-1+\omega} + \alpha K_{\text{ss}} L_{\text{ss}}^s{}^{1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\mathcal{Q}} U_{\text{ss}} K_{\text{ss}})^{-1+\alpha} = 0 \quad (6.5)$$

$$(-1 + \mu) C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{-\mu} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{-\eta} + \mu W_{\text{ss}} C_{\text{ss}}^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$I_{\text{ss}} - K_{\text{ss}} + K_{\text{ss}} (1 - \delta C_{\mathcal{Q}} U_{\text{ss}}^{\omega}) = 0 \quad (6.7)$$

$$U_{\text{ss}} - \beta U_{\text{ss}} - (1 - \eta)^{-1} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.8)$$

$$-C_{\text{ss}} + \Pi_{\text{ss}} + L_{\text{ss}}^s W_{\text{ss}} - \beta \Pi_{\text{ss}} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.9)$$

$$-I_{\text{ss}} - \Pi_{\text{ss}} + Y_{\text{ss}} - L_{\text{ss}}^s W_{\text{ss}} + \beta \Pi_{\text{ss}} C_{\text{ss}}^{-1+\mu} C_{\text{ss}}^{1-\mu} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.10)$$

7 Parameter settings

$$\alpha = 0.36 \quad (7.1)$$

$$\beta = 0.99 \quad (7.2)$$

$$\delta = 0.025 \quad (7.3)$$

$$\eta = 2 \quad (7.4)$$

$$\mu = 0.3 \quad (7.5)$$

$$\omega = 1.45 \tag{7.6}$$

$$\phi = 0.95 \tag{7.7}$$

8 Steady-state values

	Steady-state values
C	0.7449
$CqUt$	0.9284
I	0.246
K	10.96
L^s	0.2673
Π	11.0707
U	-135.8123
W	2.3722
Y	0.9909
Z	1

9 The solution of the perturbation

9.1 P

$$\begin{matrix} K & Z \\ Z \end{matrix} \begin{pmatrix} K_{t-1} & Z_{t-1} \\ 0.9758 & 0.0705 \\ 0 & 0.95 \end{pmatrix}$$

9.2 Q

$$\begin{matrix} K \\ Z \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.0742 \\ 1 \end{pmatrix}$$

9.3 R

$$\begin{matrix} C \\ CqUt \\ I \\ L^s \\ \Pi \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} K_{t-1} & Z_{t-1} \\ 0.2823 & 0.4185 \\ -0.74 & 1.0041 \\ -1.1491 & 4.5972 \\ -0.2604 & 0.7601 \\ 0.9893 & 0.0146 \\ -0.0446 & -0.0408 \\ 0.1873 & 0.6958 \\ -0.0731 & 1.456 \end{pmatrix}$$

9.4 S

$$\begin{matrix} C \\ CqUt \\ I \\ L^s \\ \Pi \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.4405 \\ 1.057 \\ 4.8392 \\ 0.8001 \\ 0.0153 \\ -0.0429 \\ 0.7325 \\ 1.5326 \end{pmatrix}$$

10 Statistics of the model

10.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
C	0.7449	0.0408	0.0017	Y
$CqUt$	0.9284	0.1001	0.01	Y
I	0.246	0.4485	0.2011	Y
K	10.96	0.0245	0.0006	Y
L^s	0.2673	0.0744	0.0055	Y
Π	11.0707	0.0242	0.0006	Y
U	-135.8123	0.004	0	Y
W	2.3722	0.0674	0.0045	Y
Y	0.9909	0.1414	0.02	Y
Z	1	0.0922	0.0085	Y

10.2 Correlation matrix

	C	$CqUt$	I	K	L^s	Π	U	W	Y	Z
C	1	0.9387	0.973	0.3867	0.9675	0.1719	-0.9947	0.9948	0.9834	0.9856
$CqUt$	0.9387	1	0.9929	0.0452	0.9954	-0.1781	-0.8984	0.9689	0.9857	0.9835
I	0.973	0.9929	1	0.1636	0.9997	-0.0599	-0.9442	0.9915	0.9987	0.998
K	0.3867	0.0452	0.1636	1	0.1408	0.975	-0.4794	0.2908	0.2128	0.2251
L^s	0.9675	0.9954	0.9997	0.1408	1	-0.0829	-0.9364	0.9882	0.9973	0.9963
Π	0.1719	-0.1781	-0.0599	0.975	-0.0829	1	-0.2722	0.0707	-0.0099	0.0028
U	-0.9947	-0.8984	-0.9442	-0.4794	-0.9364	-0.2722	1	-0.9791	-0.9595	-0.963
W	0.9948	0.9689	0.9915	0.2908	0.9882	0.0707	-0.9791	1	0.9967	0.9977
Y	0.9834	0.9857	0.9987	0.2128	0.9973	-0.0099	-0.9595	0.9967	1	0.9999
Z	0.9856	0.9835	0.998	0.2251	0.9963	0.0028	-0.963	0.9977	0.9999	1

10.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
C	0.7277	0.494	0.2979	0.137	0.0087
$CqUt$	0.7137	0.4719	0.272	0.1107	-0.0155
I	0.7115	0.4684	0.2679	0.1066	-0.0193
K	0.9603	0.8642	0.731	0.5765	0.4135
L^s	0.7114	0.4682	0.2676	0.1063	-0.0196
Π	0.9636	0.8695	0.7371	0.5827	0.4193
U	0.7427	0.518	0.3259	0.1654	0.0349
W	0.7174	0.4777	0.2787	0.1176	-0.0092
Y	0.7128	0.4703	0.2701	0.1089	-0.0172
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

11 Statistics of the model

11.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
C	0.7517	0.2886	0.0833	Y
$CqUt$	0.9369	0.7079	0.5011	Y
I	0.2483	3.1723	10.0637	Y
K	11.0607	0.1733	0.03	Y
L^s	0.2698	0.5261	0.2768	Y
Π	11.1724	0.1711	0.0293	Y
U	-137.0599	0.0286	0.0008	Y
W	2.394	0.4768	0.2274	Y
Y	1	1	1	Y
Z	1.0092	0.652	0.4251	Y

11.2 Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
C	-0.1167	0.01	0.1782	0.3932	0.66	0.9834	0.7485	0.5426	0.3656	0.2168	0.0945
$CqUt$	0.0755	0.1973	0.3474	0.5279	0.7405	0.9857	0.6585	0.3894	0.1736	0.0056	-0.1203
I	0.0103	0.1355	0.2939	0.4889	0.7231	0.9987	0.6988	0.4477	0.2423	0.0785	-0.0479
K	-0.5404	-0.4988	-0.413	-0.273	-0.0682	0.2128	0.4076	0.5307	0.5952	0.6134	0.5959
L^s	0.0229	0.1476	0.3046	0.497	0.7273	0.9973	0.6917	0.4369	0.2292	0.0645	-0.062
Π	-0.5491	-0.5352	-0.4842	-0.3865	-0.232	-0.0099	0.2549	0.436	0.5477	0.6029	0.6138
U	0.1712	0.046	-0.1236	-0.3437	-0.6205	-0.9595	-0.7577	-0.5754	-0.4142	-0.2746	-0.1563
W	-0.0614	0.0655	0.2305	0.438	0.6923	0.9967	0.7315	0.5043	0.3136	0.1572	0.0323
Y	-0.0172	0.1089	0.2701	0.4703	0.7128	1	0.7128	0.4703	0.2701	0.1089	-0.0172
Z	-0.0242	0.1021	0.264	0.4655	0.7099	0.9999	0.716	0.4759	0.2771	0.1165	-0.0095

12 Impulse response functions

12.1 Shock ϵ^Z

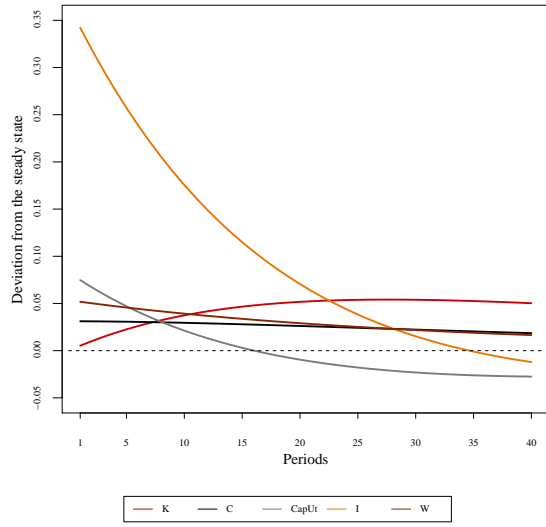


Figure 1: Impulse response function for ϵ^Z shock

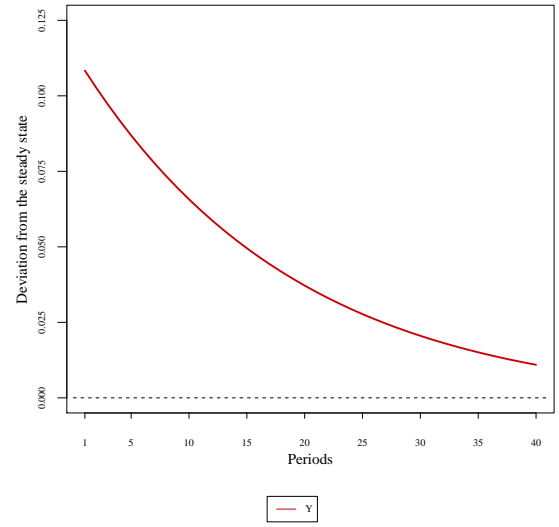


Figure 2: Impulse response function for ϵ^Z shock