

# 1 CONSUMER

## 1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + p_t I_t = \pi_t^C + \pi_t^I + K_{t-1}^s r_t + L_t^s W_t \quad \left( \lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left( \lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

## 1.2 Identities

$$Y_t = C_t + p_t I_t \quad (1.4)$$

## 1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu = 0 \quad (L_t^s) \quad (1.7)$$

$$\lambda_t^{\text{CONSUMER}^2} - \lambda_t^{\text{CONSUMER}^1} p_t = 0 \quad (I_t) \quad (1.8)$$

# 2 FIRM C

## 2.1 Optimisation problem

$$\max_{K_t^{\text{Cd}}, L_t^{\text{Cd}}, C_t^s} \pi_t^C = C_t^s - L_t^{\text{Cd}} W_t - r_t K_t^{\text{Cd}} \quad (2.1)$$

s.t. :

$$C_t^s = Z_t K_t^{\text{Cd} \alpha} L_t^{\text{Cd} 1-\alpha} \quad \left( \lambda_t^{\text{FIRM}^{\text{C}^1}} \right) \quad (2.2)$$

## 2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^{\text{C}^1}} Z_t K_t^{\text{C}^{\text{d}}-1+\alpha} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad \left( K_t^{\text{C}^{\text{d}}} \right) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^{\text{C}^1}} Z_t (1 - \alpha) K_t^{\text{C}^{\text{d}}\alpha} L_t^{\text{C}^{\text{d}}-\alpha} = 0 \quad \left( L_t^{\text{C}^{\text{d}}} \right) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^{\text{C}^1}} = 0 \quad (C_t^{\text{s}}) \quad (2.5)$$

## 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{C}^{\text{d}}-1+\alpha} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad \left( K_t^{\text{C}^{\text{d}}} \right) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{C}^{\text{d}}\alpha} L_t^{\text{C}^{\text{d}}-\alpha} = 0 \quad \left( L_t^{\text{C}^{\text{d}}} \right) \quad (2.7)$$

# 3 FIRM I

## 3.1 Optimisation problem

$$\max_{K_t^{\text{I}^{\text{d}}}, L_t^{\text{I}^{\text{d}}}, I_t^{\text{s}}} \pi_t^{\text{I}} = I_t^{\text{s}} - L_t^{\text{I}^{\text{d}}} W_t - r_t K_t^{\text{I}^{\text{d}}} \quad (3.1)$$

s.t. :

$$I_t^{\text{s}} = Z_t K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} \quad \left( \lambda_t^{\text{FIRM}^{\text{I}^1}} \right) \quad (3.2)$$

## 3.2 First order conditions

$$-r_t + \sigma \lambda_t^{\text{FIRM}^{\text{I}^1}} Z_t K_t^{\text{I}^{\text{d}}-1+\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} = 0 \quad \left( K_t^{\text{I}^{\text{d}}} \right) \quad (3.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^{\text{I}^1}} Z_t (1 - \alpha) K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}-\alpha} = 0 \quad \left( L_t^{\text{I}^{\text{d}}} \right) \quad (3.4)$$

$$1 - \lambda_t^{\text{FIRM}^{\text{I}^1}} = 0 \quad (I_t^{\text{s}}) \quad (3.5)$$

## 3.3 First order conditions after reduction

$$-r_t + \sigma Z_t K_t^{\text{I}^{\text{d}}-1+\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} = 0 \quad \left( K_t^{\text{I}^{\text{d}}} \right) \quad (3.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}-\alpha} = 0 \quad \left( L_t^{\text{I}^{\text{d}}} \right) \quad (3.7)$$

## 4 EQUILIBRIUM

### 4.1 Identities

$$K_t^{\text{I}^{\text{d}}} + K_t^{\text{C}^{\text{d}}} = K_{t-1}^{\text{s}} \quad (4.1)$$

$$L_t^{\text{I}^{\text{d}}} + L_t^{\text{C}^{\text{d}}} = L_t^{\text{s}} \quad (4.2)$$

$$C_t = C_t^{\text{s}} \quad (4.3)$$

## 5 EXOG

### 5.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (5.1)$$

## 6 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_t^{\text{C}^{\text{d}}} K_t^{\text{C}^{\text{d}}-1+\alpha} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad (6.1)$$

$$-r_t + \sigma Z_t \left( K_{t-1}^{\text{s}} - K_t^{\text{C}^{\text{d}}} \right)^{-1+\sigma} \left( L_t^{\text{s}} - L_t^{\text{C}^{\text{d}}} \right)^{1-\alpha} = 0 \quad (6.2)$$

$$-C_t + Z_t K_t^{\text{C}^{\text{d}}} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad (6.3)$$

$$-I_t^{\text{s}} + Z_t \left( K_{t-1}^{\text{s}} - K_t^{\text{C}^{\text{d}}} \right)^{\sigma} \left( L_t^{\text{s}} - L_t^{\text{C}^{\text{d}}} \right)^{1-\alpha} = 0 \quad (6.4)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{C}^{\text{d}}} L_t^{\text{C}^{\text{d}}-\alpha} = 0 \quad (6.5)$$

$$-W_t + Z_t (1 - \alpha) \left( K_{t-1}^{\text{s}} - K_t^{\text{C}^{\text{d}}} \right)^{\sigma} \left( L_t^{\text{s}} - L_t^{\text{C}^{\text{d}}} \right)^{-\alpha} = 0 \quad (6.6)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (6.7)$$

$$\beta \left( \mu E_t \left[ r_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left( C_{t+1}^{\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] + \mu (1 - \delta) E_t \left[ p_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left( C_{t+1}^{\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] \right) - \mu p_t C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left( C_t^{\mu} (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = 0 \quad (6.8)$$

$$(-1 + \mu) C_t^{\mu} (1 - L_t^{\text{s}})^{-\mu} \left( C_t^{\mu} (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left( C_t^{\mu} (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = 0 \quad (6.9)$$

$$-C_t + Y_t - p_t I_t = 0 \quad (6.10)$$

$$I_t - K_t^{\text{s}} + K_{t-1}^{\text{s}} (1 - \delta) = 0 \quad (6.11)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.12)$$

$$I_t^s + K_{t-1}^s r_t - p_t I_t - r_t K_t^{C^d} - r_t \left( K_{t-1}^s - K_t^{C^d} \right) + L_t^s W_t - L_t^{C^d} W_t - W_t \left( L_t^s - L_t^{C^d} \right) = 0 \quad (6.13)$$

## 7 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{C^d - 1 + \alpha} L_{ss}^{C^d 1 - \alpha} = 0 \quad (7.1)$$

$$-r_{ss} + \sigma Z_{ss} \left( K_{ss}^s - K_{ss}^{C^d} \right)^{-1 + \sigma} \left( L_{ss}^s - L_{ss}^{C^d} \right)^{1 - \alpha} = 0 \quad (7.2)$$

$$-C_{ss} + Z_{ss} K_{ss}^{C^d \alpha} L_{ss}^{C^d 1 - \alpha} = 0 \quad (7.3)$$

$$-I_{ss}^s + Z_{ss} \left( K_{ss}^s - K_{ss}^{C^d} \right)^\sigma \left( L_{ss}^s - L_{ss}^{C^d} \right)^{1 - \alpha} = 0 \quad (7.4)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{C^d \alpha} L_{ss}^{C^d - \alpha} = 0 \quad (7.5)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) \left( K_{ss}^s - K_{ss}^{C^d} \right)^\sigma \left( L_{ss}^s - L_{ss}^{C^d} \right)^{-\alpha} = 0 \quad (7.6)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (7.7)$$

$$\beta \left( \mu r_{ss} C_{ss}^{-1 + \mu} (1 - L_{ss}^s)^{1 - \mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1 - \mu} \right)^{-\eta} + \mu p_{ss} (1 - \delta) C_{ss}^{-1 + \mu} (1 - L_{ss}^s)^{1 - \mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1 - \mu} \right)^{-\eta} - \mu p_{ss} C_{ss}^{-1 + \mu} (1 - L_{ss}^s)^{1 - \mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1 - \mu} \right)^{-\eta} \right) = 0 \quad (7.8)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1 - \mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1 + \mu} (1 - L_{ss}^s)^{1 - \mu} \left( (1 - L_{ss}^s)^{1 - \mu} C_{ss}^\mu \right)^{-\eta} = 0 \quad (7.9)$$

$$-C_{ss} + Y_{ss} - p_{ss} I_{ss} = 0 \quad (7.10)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (7.11)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1 - \mu} \right)^{1 - \eta} = 0 \quad (7.12)$$

$$I_{ss}^s - p_{ss} I_{ss} + r_{ss} K_{ss}^s - r_{ss} K_{ss}^{C^d} - r_{ss} \left( K_{ss}^s - K_{ss}^{C^d} \right) + L_{ss}^s W_{ss} - L_{ss}^{C^d} W_{ss} - W_{ss} \left( L_{ss}^s - L_{ss}^{C^d} \right) = 0 \quad (7.13)$$

## 8 Parameter settings

$$\alpha = 0.2 \tag{8.1}$$

$$\beta = 0.99 \tag{8.2}$$

$$\delta = 0.025 \tag{8.3}$$

$$\eta = 2 \tag{8.4}$$

$$\mu = 0.3 \tag{8.5}$$

$$\phi = 0.95 \tag{8.6}$$

$$\sigma = 0.4 \tag{8.7}$$

## 9 Steady-state values

	Steady-state values
$p$	1.5318
$r$	0.0538
$C$	0.3374
$I$	0.0439
$I^s$	0.0672
$K^s$	1.7551
$K^{C^d}$	1.2551
$L^s$	0.2914
$L^{C^d}$	0.243
$U$	-176.3002
$W$	1.111
$Y$	0.4046
$Z$	1

## 10 The solution of the perturbation

### 10.1 P

$$\begin{matrix} K_{t-1}^s & Z_{t-1} \\ K^s & \begin{pmatrix} 0.9522 & -0.0054 \\ 0 & 0.95 \end{pmatrix} \\ Z & \end{matrix}$$

### 10.2 Q

$$\begin{matrix} \epsilon^Z \\ K^s & \begin{pmatrix} -0.0056 \\ 1 \end{pmatrix} \\ Z & \end{matrix}$$

### 10.3 R

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ p & \begin{pmatrix} -0.1506 & 1.1645 \\ -1.0646 & 0.95 \\ 0.3338 & 0.95 \\ -0.9139 & -0.2145 \\ -1.0646 & 0.95 \\ 1.3984 & 0 \\ -0.1646 & 0 \\ 0.0677 & 0 \\ -0.0257 & -0.0456 \\ 0.2661 & 0.95 \\ 0.1015 & 0.95 \end{pmatrix} \\ r & \\ C & \\ I & \\ I^s & \\ K^{C^d} & \\ L^s & \\ L^{C^d} & \\ U & \\ W & \\ Y & \end{matrix}$$

## 10.4 S

$$\begin{matrix} p \\ r \\ C \\ I \\ I^s \\ K^{Cd} \\ L^s \\ L^{Cd} \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} \epsilon^Z \\ 1.2258 \\ 1 \\ 1 \\ -0.2258 \\ 1 \\ 0 \\ 0 \\ 0 \\ -0.048 \\ 1 \\ 1 \end{pmatrix}$$

## 11 Statistics of the model

### 11.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
$p$	1.5318	0.113	0.0128	Y
$r$	0.0538	0.0922	0.0085	Y
$C$	0.3374	0.0922	0.0085	Y
$I$	0.0439	0.0208	0.0004	Y
$I^s$	0.0672	0.0922	0.0085	Y
$K^s$	1.7551	0.0018	0	Y
$L^s$	0.2914	0.0003	0	Y
$U$	-176.3002	0.0044	0	Y
$W$	1.111	0.0922	0.0085	Y
$Y$	0.4046	0.0922	0.0085	Y

### 11.2 Correlation matrix

	$p$	$r$	$C$	$I$	$I^s$	$K^s$	$L^s$	$U$	$W$	$Y$
$p$	1	0.9998	1	-0.9966	0.9998	-0.3088	0.0262	-0.9999	1	1
$r$	0.9998	1	0.9996	-0.9948	1	-0.3265	0.0449	-0.9995	0.9997	0.9997
$C$	1	0.9996	1	-0.9973	0.9996	-0.3001	0.0172	-1	1	1
$I$	-0.9966	-0.9948	-0.9973	1	-0.9948	0.229	0.0566	0.9976	-0.9972	-0.9969
$I^s$	0.9998	1	0.9996	-0.9948	1	-0.3265	0.0449	-0.9995	0.9997	0.9997
$K^s$	-0.3088	-0.3265	-0.3001	0.229	-0.3265	1	-0.9589	0.2963	-0.3014	-0.3045
$K^{Cd}$	-0.0262	-0.0449	-0.0172	-0.0566	-0.0449	0.9589	-1	0.0131	-0.0185	-0.0218
$L^s$	0.0262	0.0449	0.0172	0.0566	0.0449	-0.9589	1	-0.0131	0.0185	0.0218
$L^{Cd}$	-0.0262	-0.0449	-0.0172	-0.0566	-0.0449	0.9589	-1	0.0131	-0.0185	-0.0218
$U$	-0.9999	-0.9995	-1	0.9976	-0.9995	0.2963	-0.0131	1	-1	-1
$W$	1	0.9997	1	-0.9972	0.9997	-0.3014	0.0185	-1	1	1
$Y$	1	0.9997	1	-0.9969	0.9997	-0.3045	0.0218	-1	1	1
$Z$	1	0.9998	1	-0.9968	0.9998	-0.3065	0.0238	-0.9999	1	1

### 11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
$p$	0.7135	0.4715	0.2715	0.1102	-0.016
$r$	0.7152	0.4742	0.2746	0.1134	-0.0131
$C$	0.7127	0.4703	0.2701	0.1088	-0.0173
$I$	0.708	0.4629	0.2615	0.1001	-0.0252
$I^s$	0.7152	0.4742	0.2746	0.1134	-0.0131
$K^s$	0.9589	0.8598	0.7228	0.5647	0.3987
$L^s$	0.9589	0.8598	0.7228	0.5647	0.3987
$U$	0.7124	0.4698	0.2695	0.1082	-0.0178
$W$	0.7128	0.4704	0.2703	0.109	-0.0171
$Y$	0.7131	0.4709	0.2708	0.1095	-0.0166

## 12 Statistics of the model

### 12.1 Moments relative to moments of the reference variable

	Steady-state value relative to $Y$	Std. dev. relative to $Y$	Variance relative to $Y$	Loglinear
$p$	3.7858	1.226	1.503	Y
$r$	0.1329	1.0008	1.0015	Y
$C$	0.8339	0.9999	0.9998	Y
$I$	0.1084	0.2261	0.0511	Y
$I^s$	0.1661	1.0008	1.0015	Y
$K^s$	4.3375	0.0199	0.0004	Y
$K^{C^d}$	3.1018	0.0278	0.0008	Y
$L^s$	0.7201	0.0033	0	Y
$L^{C^d}$	0.6005	0.0013	0	Y
$U$	-435.7117	0.048	0.0023	Y
$W$	2.7457	0.9999	0.9999	Y
$Y$	1	1	1	Y
$Z$	2.4714	1	1.0001	Y

### 12.2 Correlations with the reference variable

	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$p$	-0.0189	0.1073	0.2688	0.4693	0.7122	1	0.7144	0.473	0.2734	0.1124	-0.0137
$r$	-0.0284	0.098	0.2604	0.4626	0.7082	0.9997	0.7196	0.4819	0.2845	0.1245	-0.0015
$C$	-0.0143	0.1118	0.2728	0.4725	0.714	1	0.7118	0.4686	0.268	0.1065	-0.0196
$I$	-0.0234	-0.1479	-0.3046	-0.4968	-0.727	-0.9969	-0.6883	-0.4316	-0.2231	-0.0583	0.0677
$I^s$	-0.0284	0.098	0.2604	0.4626	0.7082	0.9997	0.7196	0.4819	0.2845	0.1245	-0.0015
$K^s$	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515	-0.616
$K^{C^d}$	0.5104	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515
$L^s$	-0.5104	-0.491	-0.4367	-0.3392	-0.1896	0.0218	0.3045	0.4925	0.6029	0.6512	0.6515
$L^{C^d}$	0.5104	0.491	0.4367	0.3392	0.1896	-0.0218	-0.3045	-0.4925	-0.6029	-0.6512	-0.6515
$U$	0.0122	-0.1138	-0.2746	-0.4739	-0.7148	-1	-0.7106	-0.4667	-0.2656	-0.1039	0.0223
$W$	-0.0149	0.1111	0.2722	0.472	0.7138	1	0.7121	0.4693	0.2688	0.1074	-0.0188
$Y$	-0.0166	0.1095	0.2708	0.4709	0.7131	1	0.7131	0.4709	0.2708	0.1095	-0.0166
$Z$	-0.0177	0.1085	0.2699	0.4702	0.7127	1	0.7137	0.4718	0.272	0.1108	-0.0153



## 13 Impulse response functions

### 13.1 Shock $\epsilon^Z$

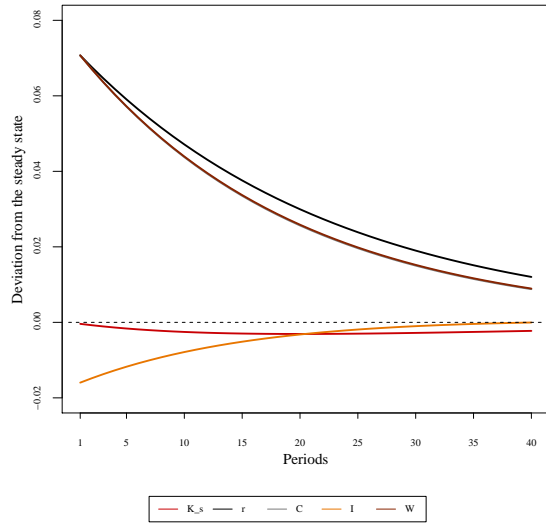


Figure 1: Impulse response function for  $\epsilon^Z$  shock

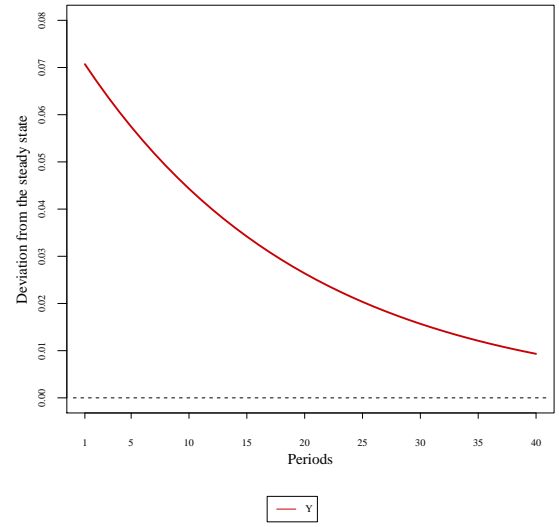


Figure 2: Impulse response function for  $\epsilon^Z$  shock