1 CONSUMER

1.1 Optimization problem

$$\max_{K_t^s, C_t, I_t} U_t = \beta \left(\mathrm{E}_t \left[U_{t+1}^{1-\theta^{\mathrm{EZ}}} \right] \right)^{\left(1-\theta^{\mathrm{EZ}}\right)^{-1}} + \left(-1 + C_t^{1-\eta} \right) \left(1 - \eta \right)^{-1} \tag{1.1}$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad (\lambda_t^{\mathrm{CONSUMER}^1}) \tag{1.2}$$

$$K_t^s = I_t + K_{t-1}^s \left(1 - \delta \right) \quad (\lambda_t^{\mathrm{CONSUMER}^2}) \tag{1.3}$$

1.2 Identities

$$L_t^s = 1 \tag{1.4}$$

1.3 First order conditions

$$-\lambda_t^{\mathrm{CONSUMER}^U} + \beta q_{t-1}^{\mathrm{CONSUMER}^1 \,-1+\left(1-\theta^{\mathrm{EZ}}\right)^{-1}} U_t^{-\theta^{\mathrm{EZ}}} = 0 \quad (U_t) \tag{1.5}$$

$$-\lambda_t^{\mathrm{CONSUMER}^2} + \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^U} \left(\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1} + \lambda_{t+1}^{\mathrm{CONSUMER}^2} \left(1 - \delta \right) \right) \right] = 0 \quad (K_t^s) \tag{1.6}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + C_t^{-\eta} = 0 \quad (C_t) \tag{1.7}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + \lambda_t^{\mathrm{CONSUMER}^2} = 0 \quad (I_t) \tag{1.8}$$

2 FIRM

2.1 Optimization problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \tag{2.1}$$

s.t. :

$$Y_t = Z_t L_t^{d\,1-\alpha} K_t^{d\,\alpha} \quad (\lambda_t^{\mathrm{FIRM}^1}) \tag{2.2}$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\mathrm{FIRM}^1} Z_t K_t^{d\,-1+\alpha} L_t^{d\,1-\alpha} = 0 \quad (K_t^d) \tag{2.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1} Z_t \left(1 - \alpha \right) L_t^{d\,-\alpha} K_t^{d\,\alpha} = 0 \quad (L_t^d) \tag{2.4}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad (Y_t) \tag{2.5}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{d\,-1+\alpha} L_t^{d\,1-\alpha} = 0 \quad (K_t^d) \tag{2.6}$$

$$-W_t + Z_t \left(1 - \alpha \right) L_t^{d\,-\alpha} K_t^{d\,\alpha} = 0 \quad (L_t^d) \tag{2.7}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^d = K_{t-1}^s \tag{3.1}$$

$$L_t^d = L_t^s \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships

$$-1 + L_t^d = 0 \tag{5.1}$$

$$q_t^{\text{CONSUMER}^1} - \mathrm{E}_t\left[U_{t+1}^{1-\theta^{\text{EZ}}}\right] = 0 \tag{5.2}$$

$$-r_t + \alpha Z_t K_{t-1}^{s}{}^{-1+\alpha} L_t^{d\,1-\alpha} = 0 \tag{5.3}$$

$$-W_t + Z_t\left(1-\alpha\right) K_{t-1}^{s}{}^{\alpha} L_t^{d\,-\alpha} = 0 \tag{5.4}$$

$$-Y_t + Z_t K_{t-1}^{s}{}^{\alpha} L_t^{d\,1-\alpha} = 0 \tag{5.5}$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \tag{5.6}$$

$$\beta q_t^{\text{CONSUMER}^1\,-1+\left(1-\theta^{\text{EZ}}\right)^{-1}} \mathrm{E}_t\left[\left(r_{t+1} C_{t+1}^{-\eta} + \left(1-\delta\right) C_{t+1}^{-\eta}\right) U_{t+1}^{-\theta^{\text{EZ}}}\right] - C_t^{-\eta} = 0 \tag{5.7}$$

$$I_t - K_t^s + K_{t-1}^s\left(1-\delta\right) = 0 \tag{5.8}$$

$$U_t - \beta q_t^{\text{CONSUMER}^1\,\left(1-\theta^{\text{EZ}}\right)^{-1}} - \left(-1 + C_t^{1-\eta}\right)\left(1-\eta\right)^{-1} = 0 \tag{5.9}$$

$$-C_t - I_t + W_t + Y_t - L_t^d W_t = 0 \tag{5.10}$$

6 Steady state relationships

$$-1 + L_{\text{ss}}^d = 0 \tag{6.1}$$

$$q_{\text{ss}}^{\text{CONSUMER}^1} - U_{\text{ss}}^{1-\theta^{\text{EZ}}} = 0 \tag{6.2}$$

$$-r_{\text{ss}} + \alpha Z_{\text{ss}} K_{\text{ss}}^{s}{}^{-1+\alpha} L_{\text{ss}}^{d\,1-\alpha} = 0 \tag{6.3}$$

$$-W_{\text{ss}} + Z_{\text{ss}}\left(1-\alpha\right) K_{\text{ss}}^{s}{}^{\alpha} L_{\text{ss}}^{d\,-\alpha} = 0 \tag{6.4}$$

$$-Y_{\text{ss}} + Z_{\text{ss}} K_{\text{ss}}^{s}{}^{\alpha} L_{\text{ss}}^{d\,1-\alpha} = 0 \tag{6.5}$$

$$Z_{\text{ss}} - e^{\phi \log Z_{\text{ss}}} = 0 \tag{6.6}$$

$$\beta\left(r_{\text{ss}} C_{\text{ss}}^{-\eta} + \left(1-\delta\right) C_{\text{ss}}^{-\eta}\right) q_{\text{ss}}^{\text{CONSUMER}^1\,-1+\left(1-\theta^{\text{EZ}}\right)^{-1}} U_{\text{ss}}^{-\theta^{\text{EZ}}} - C_{\text{ss}}^{-\eta} = 0 \tag{6.7}$$

$$I_{\text{ss}} - K_{\text{ss}}^s + K_{\text{ss}}^s\left(1-\delta\right) = 0 \tag{6.8}$$

$$U_{\text{ss}} - \beta q_{\text{ss}}^{\text{CONSUMER}^1\,\left(1-\theta^{\text{EZ}}\right)^{-1}} - \left(-1 + C_{\text{ss}}^{1-\eta}\right)\left(1-\eta\right)^{-1} = 0 \tag{6.9}$$

$$-C_{\text{ss}} - I_{\text{ss}} + W_{\text{ss}} + Y_{\text{ss}} - L_{\text{ss}}^d W_{\text{ss}} = 0 \tag{6.10}$$

7 Calibrating equations

$$-0.36Y_{\text{ss}} + r_{\text{ss}}K_{\text{ss}}^s = 0 \tag{7.1}$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$
$$\delta = 0.025 \tag{8.2}$$
$$\eta = 2 \tag{8.3}$$
$$\phi = 0.95 \tag{8.4}$$
$$\theta^{\text{EZ}} = 0.05 \tag{8.5}$$

9 Steady state values

	Steady state values
q^{CONSUMER^1}	58.4346
r	0.0351
C	3.6213
I	1.4427
K^s	57.7077
L^d	1
U	72.3856
W	3.0384
Y	5.064
Z	1

10 The solution of the perturbation

10.1 P

$$
\begin{array}{c}
\begin{array}{cc} K_{t-1}^s & Z_{t-1} \end{array} \\
\begin{array}{c} K^s \\ Z \end{array}
\begin{pmatrix} 0.9792 & 0.0632 \\ 0 & 0.95 \end{pmatrix}
\end{array}
$$

10.2 Q

$$
\begin{array}{c}
\epsilon^Z \\
\begin{array}{c} K^s \\ Z \end{array}
\begin{pmatrix} 0.0665 \\ 1 \end{pmatrix}
\end{array}
$$

10.3 R

$$
\begin{array}{c}
\begin{array}{cc} K_{t-1}^s & Z_{t-1} \end{array} \\
\begin{array}{c} q^{\text{CONSUMER}^1} \\ r \\ C \\ I \\ L^d \\ U \\ W \\ Y \end{array}
\begin{pmatrix}
0.0571 & 0.0806 \\
-0.6 & 0.95 \\
0.4918 & 0.3212 \\
0.1696 & 2.5283 \\
0 & 0 \\
0.0614 & 0.0852 \\
0.4 & 0.95 \\
0.4 & 0.95
\end{pmatrix}
\end{array}
$$

10.4   S

$$
\begin{array}{c c}
 & \epsilon^Z \\
q^{\text{CONSUMER}^1} & \begin{pmatrix} 0.0848 \\ 1 \\ 0.3381 \\ 2.6613 \\ 0 \\ 0.0897 \\ 1 \\ 1 \end{pmatrix} \\
r & \\
C & \\
I & \\
L^d & \\
U & \\
W & \\
Y & \\
\end{array}
$$

11   Statistics of the model

11.1   Moments

	Steady state value	Std. dev.	Variance	Loglinear
q^{CONSUMER^1}	58.4346	0.1108	0.0123	Y
r	0.0351	1.3291	1.7664	Y
C	3.6213	0.4567	0.2085	Y
I	1.4427	3.4658	12.0115	Y
K^s	57.7077	0.3108	0.0966	Y
L^d	1	0	0	Y
U	72.3856	0.1171	0.0137	Y
W	3.0384	1.301	1.6926	Y
Y	5.064	1.301	1.6926	Y
Z	1	1.3034	1.699	Y

11.2   Correlation matrix

	q^{CONSUMER^1}	r	C	I	K^s	L^d	U	W	Y	Z
q^{CONSUMER^1}	1	0.955	0.9839	0.9894	0.3667	0	1	0.9979	0.9979	0.9871
r	0.955	1	0.8866	0.9879	0.0743	0	0.9542	0.9723	0.9723	0.9902
C	0.9839	0.8866	1	0.9476	0.5272	0	0.9843	0.9701	0.9701	0.9426
I	0.9894	0.9879	0.9476	1	0.2279	0	0.9891	0.9968	0.9968	0.9999
K^s	0.3667	0.0743	0.5272	0.2279	1	0	0.3691	0.3053	0.3053	0.2131
L^d	0	0	0	0	0	0	0	0	0	0
U	1	0.9542	0.9843	0.9891	0.3691	0	1	0.9977	0.9977	0.9867
W	0.9979	0.9723	0.9701	0.9968	0.3053	0	0.9977	1	1	0.9954
Y	0.9979	0.9723	0.9701	0.9968	0.3053	0	0.9977	1	1	0.9954
Z	0.9871	0.9902	0.9426	0.9999	0.2131	0	0.9867	0.9954	0.9954	1

11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
q^{CONSUMER^1}	0.7256	0.4907	0.294	0.1331	0.0051
r	0.713	0.4707	0.2706	0.1094	-0.0168
C	0.753	0.5343	0.3451	0.1849	0.0529
I	0.7139	0.4721	0.2722	0.111	-0.0152
K^s	0.9605	0.8646	0.7317	0.5776	0.415
L^d	NaN	NaN	NaN	NaN	NaN
U	0.7259	0.4912	0.2946	0.1337	0.0057
W	0.7191	0.4804	0.2819	0.1208	-0.0062
Y	0.7191	0.4804	0.2819	0.1208	-0.0062
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

12 Statistics of the model

12.1 Moments relative to moments of the reference variable

	Steady state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
q^{CONSUMER^1}	11.5392	0.0851	0.0072	Y
r	0.0069	1.0216	1.0436	Y
C	0.7151	0.351	0.1232	Y
I	0.2849	2.6639	7.0965	Y
K^s	11.3957	0.2389	0.0571	Y
L^d	0.1975	0	0	Y
U	14.2942	0.09	0.0081	Y
W	0.6	1	1	Y
Y	1	1	1	Y
Z	0.1975	1.0019	1.0038	Y

12.2 Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
q^{CONSUMER^1}	-0.0392	0.0887	0.2529	0.457	0.7045	0.9979	0.7361	0.5105	0.32	0.1628	0.0365
r	0.1127	0.2318	0.3763	0.5475	0.7461	0.9723	0.6325	0.3554	0.1353	-0.0341	-0.1591
C	-0.1292	-0.0014	0.1675	0.3827	0.6489	0.9701	0.7668	0.5819	0.4176	0.2745	0.1528
I	0.0346	0.1597	0.3161	0.5064	0.7329	0.9968	0.6939	0.4405	0.2334	0.0684	-0.0587
K^s	-0.4856	-0.429	-0.3301	-0.1807	0.0281	0.3053	0.4911	0.6017	0.6517	0.6545	0.6217
L^d	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
U	-0.0406	0.0874	0.2517	0.456	0.7038	0.9977	0.7368	0.5117	0.3215	0.1645	0.0382
W	-0.0062	0.1208	0.2819	0.4804	0.7191	1	0.7191	0.4804	0.2819	0.1208	-0.0062
Y	-0.0062	0.1208	0.2819	0.4804	0.7191	1	0.7191	0.4804	0.2819	0.1208	-0.0062
Z	0.0423	0.1669	0.3223	0.511	0.735	0.9954	0.6886	0.4327	0.224	0.0585	-0.0686

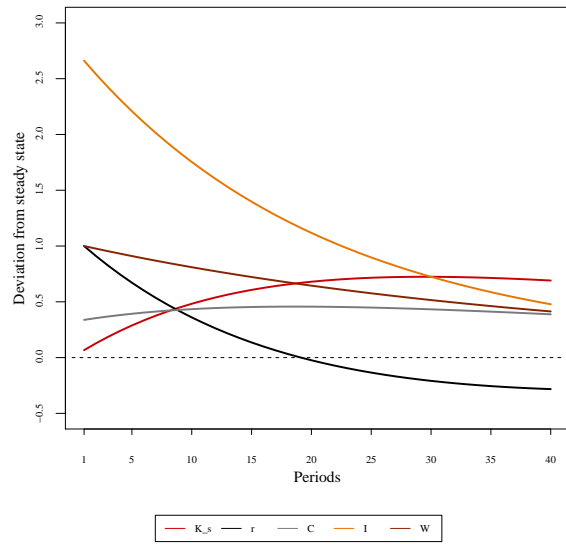# 13 Impulse response functions
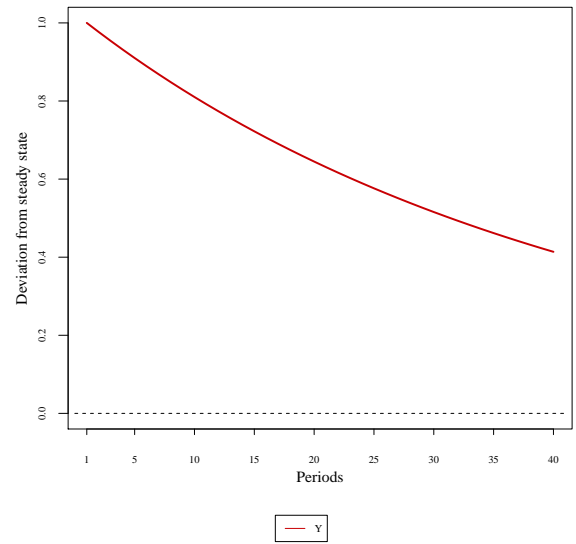
13.1 Shock ϵ^Z

Figure 1: Impulse response function for ϵ^Z shock

Figure 2: Impulse response function for ϵ^Z shock