1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L_t^{\mathrm{s}}} U_t = \beta \mathrm{E}_t\left[U_{t+1}\right] + (1-\eta)^{-1}\left(\left(1-L_t^{\mathrm{s}}\right)^{1-\mu}C_t{}^{\mu}\right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t = \pi_t + L_t^{\mathrm{s}}W_t \quad (\lambda_t^{\mathrm{c}}) \tag{1.2}$$

1.2 First order conditions

$$\beta - \lambda_t^{\mathrm{U}} = 0 \quad (U_t) \tag{1.3}$$

$$-\lambda_t^{\mathrm{c}} + \mu C_t{}^{-1+\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\left(\left(1-L_t^{\mathrm{s}}\right)^{1-\mu}C_t{}^{\mu}\right)^{-\eta} = 0 \quad (C_t) \tag{1.4}$$

$$\lambda_t^{\mathrm{c}}W_t + (-1+\mu)\left(1-L_t^{\mathrm{s}}\right)^{-\mu}\left(\left(1-L_t^{\mathrm{s}}\right)^{1-\mu}C_t{}^{\mu}\right)^{-\eta}C_t{}^{\mu} = 0 \quad (L_t^{\mathrm{s}}) \tag{1.5}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t, L_t^{\mathrm{d}}, Y_t, I_t, \pi_t, CapUt_t} \Pi_t = \pi_t + \lambda_t^{\mathrm{c}-1}\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{c}}\lambda_{t+1}^{\mathrm{U}}\Pi_{t+1}\right] \tag{2.1}$$

s.t. :

$$Y_t = L_t^{\mathrm{d}\,1-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CapUt_t)^{\alpha} \quad \left(\lambda_t^{\mathrm{FIRM}^1}\right) \tag{2.2}$$

$$K_t = I_t + K_{t-1}\left(1 - \delta CapUt_t{}^{\omega}\right) \quad \left(\lambda_t^{\mathrm{FIRM}^2}\right) \tag{2.3}$$

$$\pi_t = -I_t - L_t^{\mathrm{d}}W_t + P_tY_t \quad \left(\lambda_t^{\mathrm{FIRM}^3}\right) \tag{2.4}$$

2.2 First order conditions

$$-\lambda_t^{\mathrm{FIRM}^\Pi} + {\lambda_{t-1}^{\mathrm{c}}}^{-1}\lambda_t^{\mathrm{c}}\lambda_t^{\mathrm{U}} = 0 \quad (\Pi_t) \tag{2.5}$$

$$-\lambda_t^{\mathrm{FIRM}^2} + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{FIRM}^\Pi}\left(\lambda_{t+1}^{\mathrm{FIRM}^2}\left(1-\delta CapUt_{t+1}{}^\omega\right)+\alpha\lambda_{t+1}^{\mathrm{FIRM}^1}CapUt_{t+1}L_{t+1}^{\mathrm{d}}{}^{1-\alpha}Z_{t+1}{}^{1-\alpha}\left(K_t CapUt_{t+1}\right)^{-1+\alpha}\right)\right] = 0 \quad (K_t) \tag{2.6}$$

$$-\lambda_t^{\mathrm{FIRM}^3}W_t + \lambda_t^{\mathrm{FIRM}^1}\left(1-\alpha\right)L_t^{\mathrm{d}}{}^{-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CapUt_t)^\alpha = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.7}$$

$$-\lambda_t^{\mathrm{FIRM}^1} + \lambda_t^{\mathrm{FIRM}^3}P_t = 0 \quad (Y_t) \tag{2.8}$$

$$\lambda_t^{\mathrm{FIRM}^2} - \lambda_t^{\mathrm{FIRM}^3} = 0 \quad (I_t) \tag{2.9}$$

$$1 - \lambda_t^{\mathrm{FIRM}^3} = 0 \quad (\pi_t) \tag{2.10}$$

$$-\delta\omega K_{t-1}\lambda_t^{\mathrm{FIRM}^2}CapUt_t{}^{-1+\omega} + \alpha K_{t-1}\lambda_t^{\mathrm{FIRM}^1}L_t^{\mathrm{d}}{}^{1-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CapUt_t)^{-1+\alpha} = 0 \quad (CapUt_t) \tag{2.11}$$

2.3 First order conditions after reduction

$$-\lambda_t^{\mathrm{FIRM}^\Pi} + {\lambda_{t-1}^{\mathrm{c}}}^{-1}\lambda_t^{\mathrm{c}}\lambda_t^{\mathrm{U}} = 0 \quad (\Pi_t) \tag{2.12}$$

$$-1 + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{FIRM}^\Pi}\left(1-\delta CapUt_{t+1}{}^\omega+\alpha\lambda_{t+1}^{\mathrm{FIRM}^1}CapUt_{t+1}L_{t+1}^{\mathrm{d}}{}^{1-\alpha}Z_{t+1}{}^{1-\alpha}\left(K_t CapUt_{t+1}\right)^{-1+\alpha}\right)\right] = 0 \quad (K_t) \tag{2.13}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1}\left(1-\alpha\right)L_t^{\mathrm{d}}{}^{-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CapUt_t)^\alpha = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.14}$$

$$-\lambda_t^{\mathrm{FIRM}^1} + P_t = 0 \quad (Y_t) \tag{2.15}$$

$$-\delta\omega K_{t-1}CapUt_t{}^{-1+\omega} + \alpha K_{t-1}\lambda_t^{\mathrm{FIRM}^1}L_t^{\mathrm{d}}{}^{1-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CapUt_t)^{-1+\alpha} = 0 \quad (CapUt_t) \tag{2.16}$$

3 EQUILIBRIUM

3.1 Identities

$$P_t = 1 \tag{3.1}$$

$$L_t^{\mathrm{d}} = L_t^{\mathrm{s}} \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}}+\phi\log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$-1+\beta C_t^{1-\mu}(1-L_t^{\mathrm{s}})^{-1+\mu}\left(C_t^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{\eta}\mathrm{E}_t\left[\left(1-\delta CqUt_{t+1}{}^{\omega}+\alpha CqUt_{t+1}L_{t+1}^{\mathrm{s}}{}^{1-\alpha}Z_{t+1}{}^{1-\alpha}\left(K_t CqUt_{t+1}\right)^{-1+\alpha}\right)C_{t+1}{}^{-1+\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\left(C_{t+1}{}^{\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.1}$$

$$-W_t+(1-\alpha)\,L_t^{\mathrm{s}-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CqUt_t)^{\alpha}=0 \tag{5.2}$$

$$-Y_t+L_t^{\mathrm{s}1-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CqUt_t)^{\alpha}=0 \tag{5.3}$$

$$Z_t-e^{\epsilon_t^{\mathrm{z}}+\phi\log Z_{t-1}}=0 \tag{5.4}$$

$$-\delta\omega K_{t-1}CqUt_t{}^{-1+\omega}+\alpha K_{t-1}L_t^{\mathrm{s}1-\alpha}Z_t{}^{1-\alpha}(K_{t-1}CqUt_t)^{-1+\alpha}=0 \tag{5.5}$$

$$(-1+\mu)\,C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{-\mu}\left(C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_t C_t{}^{-1+\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\left(C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{5.6}$$

$$I_t-K_t+K_{t-1}\left(1-\delta CqUt_t{}^{\omega}\right)=0 \tag{5.7}$$

$$U_t-\beta\mathrm{E}_t\left[U_{t+1}\right]-(1-\eta)^{-1}\left(C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{1-\eta}=0 \tag{5.8}$$

$$-C_t+\Pi_t+L_t^{\mathrm{s}}W_t-\beta\left(C_t{}^{-1+\mu}\right)^{-1}\left((1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-1}\left(\left(C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}\right)^{-1}\mathrm{E}_t\left[\Pi_{t+1}C_{t+1}{}^{-1+\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\left(C_{t+1}{}^{\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.9}$$

$$-I_t-\Pi_t+Y_t-L_t^{\mathrm{s}}W_t+\beta\left(C_t{}^{-1+\mu}\right)^{-1}\left((1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-1}\left(\left(C_t{}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}\right)^{-1}\mathrm{E}_t\left[\Pi_{t+1}C_{t+1}{}^{-1+\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\left(C_{t+1}{}^{\mu}\left(1-L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.10}$$

6 Steady state relationships (after reduction)

$$-1+\beta\left(1-\delta CqUt_{\mathrm{ss}}{}^{\omega}+\alpha CqUt_{\mathrm{ss}}L_{\mathrm{ss}}^{\mathrm{s}}{}^{1-\alpha}Z_{\mathrm{ss}}{}^{1-\alpha}(CqUt_{\mathrm{ss}}K_{\mathrm{ss}})^{-1+\alpha}\right)C_{\mathrm{ss}}{}^{-1+\mu}C_{\mathrm{ss}}{}^{1-\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}=0 \tag{6.1}$$

$$-W_{\mathrm{ss}}+(1-\alpha)\,L_{\mathrm{ss}}^{\mathrm{s}}{}^{-\alpha}Z_{\mathrm{ss}}{}^{1-\alpha}(CqUt_{\mathrm{ss}}K_{\mathrm{ss}})^{\alpha}=0 \tag{6.2}$$

$$-Y_{\mathrm{ss}}+L_{\mathrm{ss}}^{\mathrm{s}}{}^{1-\alpha}Z_{\mathrm{ss}}{}^{1-\alpha}(CqUt_{\mathrm{ss}}K_{\mathrm{ss}})^{\alpha}=0 \tag{6.3}$$

$$Z_{\mathrm{ss}}-e^{\phi\log Z_{\mathrm{ss}}}=0 \tag{6.4}$$

$$-\delta\omega K_{\mathrm{ss}}CqUt_{\mathrm{ss}}{}^{-1+\omega}+\alpha K_{\mathrm{ss}}L_{\mathrm{ss}}^{\mathrm{s}}{}^{1-\alpha}Z_{\mathrm{ss}}{}^{1-\alpha}(CqUt_{\mathrm{ss}}K_{\mathrm{ss}})^{-1+\alpha}=0 \tag{6.5}$$

$$(-1+\mu)\,C_{\mathrm{ss}}{}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-\mu}\left(C_{\mathrm{ss}}{}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_{\mathrm{ss}}C_{\mathrm{ss}}{}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left(C_{\mathrm{ss}}{}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{6.6}$$

$$I_{\mathrm{ss}}-K_{\mathrm{ss}}+K_{\mathrm{ss}}\left(1-\delta CqUt_{\mathrm{ss}}{}^{\omega}\right)=0 \tag{6.7}$$

$$U_{\mathrm{ss}}-\beta U_{\mathrm{ss}}-(1-\eta)^{-1}\left(C_{\mathrm{ss}}{}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{1-\eta}=0 \tag{6.8}$$

$$-C_{\mathrm{ss}}+\Pi_{\mathrm{ss}}+L_{\mathrm{ss}}^{\mathrm{s}}W_{\mathrm{ss}}-\beta\Pi_{\mathrm{ss}}1(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}=0 \tag{6.9}$$

$$-I_{\mathrm{ss}}-\Pi_{\mathrm{ss}}+Y_{\mathrm{ss}}-L_{\mathrm{ss}}^{\mathrm{s}}W_{\mathrm{ss}}+\beta\Pi_{\mathrm{ss}}C_{\mathrm{ss}}{}^{-1+\mu}C_{\mathrm{ss}}{}^{1-\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}=0 \tag{6.10}$$

7 Parameter settings

$$\alpha = 0.36 \tag{7.1}$$

$$\beta = 0.99 \tag{7.2}$$

$$\delta = 0.025 \tag{7.3}$$

$$\eta = 2 \tag{7.4}$$

$$\mu = 0.3 \tag{7.5}$$

$$\omega = 1.45 \tag{7.6}$$

$$\phi = 0.95 \tag{7.7}$$

8 Steady-state values

	Steady-state values
C	0.7449
$CapUt$	0.9284
I	0.246
K	10.96
L^{s}	0.2673
Π	11.0707
U	-135.8123
W	2.3722
Y	0.9909
Z	1

9 The solution of the perturbation

9.1 P

$$
\begin{array}{c}
\quad\quad K_{t-1} \quad\quad Z_{t-1} \\
\begin{array}{c} K \\ Z \end{array}
\begin{pmatrix}
0.9758 & 0.0705 \\
0 & 0.95
\end{pmatrix}
\end{array}
$$

9.2 Q

$$
\begin{array}{c}
\quad\quad \epsilon^{\mathrm{Z}} \\
\begin{array}{c} K \\ Z \end{array}
\begin{pmatrix}
0.0742 \\
1
\end{pmatrix}
\end{array}
$$

9.3 R

$$
\begin{array}{c}
\quad\quad\quad K_{t-1} \quad\quad\quad Z_{t-1} \\
\begin{array}{c} C \\ CapUt \\ I \\ L^{\mathrm{s}} \\ \Pi \\ U \\ W \\ Y \end{array}
\begin{pmatrix}
0.2823 & 0.4185 \\
-0.74 & 1.0041 \\
-1.1491 & 4.5972 \\
-0.2604 & 0.7601 \\
0.9893 & 0.0146 \\
-0.0446 & -0.0408 \\
0.1873 & 0.6958 \\
-0.0731 & 1.456
\end{pmatrix}
\end{array}
$$

9.4 S

$$
\begin{array}{c}
\quad\quad \epsilon^{\mathrm{Z}} \\
\begin{array}{c} C \\ CapUt \\ I \\ L^{\mathrm{s}} \\ \Pi \\ U \\ W \\ Y \end{array}
\begin{pmatrix}
0.4405 \\
1.057 \\
4.8392 \\
0.8001 \\
0.0153 \\
-0.0429 \\
0.7325 \\
1.5326
\end{pmatrix}
\end{array}
$$

10 Statistics of the model

10.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
C	0.7449	0.0408	0.0017	Y
$CapUt$	0.9284	0.1001	0.01	Y
I	0.246	0.4485	0.2011	Y
K	10.96	0.0245	0.0006	Y
L^{s}	0.2673	0.0744	0.0055	Y
Π	11.0707	0.0242	0.0006	Y
U	-135.8123	0.004	0	Y
W	2.3722	0.0674	0.0045	Y
Y	0.9909	0.1414	0.02	Y
Z	1	0.0922	0.0085	Y

10.2 Correlation matrix

	C	$CapUt$	I	K	L^{s}	Π	U	W	Y	Z
C	1	0.9387	0.973	0.3867	0.9675	0.1719	-0.9947	0.9948	0.9834	0.9856
$CapUt$	0.9387	1	0.9929	0.0452	0.9954	-0.1781	-0.8984	0.9689	0.9857	0.9835
I	0.973	0.9929	1	0.1636	0.9997	-0.0599	-0.9442	0.9915	0.9987	0.998
K	0.3867	0.0452	0.1636	1	0.1408	0.975	-0.4794	0.2908	0.2128	0.2251
L^{s}	0.9675	0.9954	0.9997	0.1408	1	-0.0829	-0.9364	0.9882	0.9973	0.9963
Π	0.1719	-0.1781	-0.0599	0.975	-0.0829	1	-0.2722	0.0707	-0.0099	0.0027
U	-0.9947	-0.8984	-0.9442	-0.4794	-0.9364	-0.2722	1	-0.9791	-0.9595	-0.963
W	0.9948	0.9689	0.9915	0.2908	0.9882	0.0707	-0.9791	1	0.9967	0.9977
Y	0.9834	0.9857	0.9987	0.2128	0.9973	-0.0099	-0.9595	0.9967	1	0.9999
Z	0.9856	0.9835	0.998	0.2251	0.9963	0.0027	-0.963	0.9977	0.9999	1

10.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
C	0.7277	0.494	0.2979	0.137	0.0087
$CapUt$	0.7137	0.4719	0.272	0.1107	-0.0155
I	0.7115	0.4684	0.2679	0.1066	-0.0193
K	0.9603	0.8642	0.731	0.5765	0.4135
L^{s}	0.7114	0.4682	0.2676	0.1063	-0.0196
Π	0.9636	0.8695	0.7371	0.5827	0.4193
U	0.7427	0.518	0.3259	0.1654	0.0349
W	0.7174	0.4777	0.2787	0.1176	-0.0092
Y	0.7128	0.4703	0.2701	0.1089	-0.0172
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

11 Statistics of the model

11.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
C	0.7517	0.2886	0.0833	Y
$CapUt$	0.9369	0.7079	0.5011	Y
I	0.2483	3.1723	10.0637	Y
K	11.0607	0.1733	0.03	Y
L^{s}	0.2698	0.5261	0.2768	Y
Π	11.1724	0.1711	0.0293	Y
U	-137.0598	0.0286	0.0008	Y
W	2.394	0.4768	0.2274	Y
Y	1	1	1	Y
Z	1.0092	0.652	0.4251	Y

11.2   Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
C	-0.1167	0.01	0.1782	0.3932	0.66	0.9834	0.7485	0.5426	0.3656	0.2168	0.0945
$CapUt$	0.0755	0.1973	0.3474	0.5279	0.7405	0.9857	0.6585	0.3894	0.1736	0.0056	-0.1203
I	0.0103	0.1355	0.2939	0.4889	0.7231	0.9987	0.6988	0.4477	0.2423	0.0785	-0.0479
K	-0.5404	-0.4988	-0.413	-0.273	-0.0682	0.2128	0.4076	0.5307	0.5952	0.6134	0.5959
L^{s}	0.0229	0.1476	0.3046	0.497	0.7273	0.9973	0.6917	0.4369	0.2292	0.0645	-0.062
Π	-0.5491	-0.5352	-0.4842	-0.3865	-0.232	-0.0099	0.2549	0.436	0.5477	0.6029	0.6138
U	0.1712	0.046	-0.1236	-0.3437	-0.6205	-0.9595	-0.7577	-0.5754	-0.4142	-0.2746	-0.1563
W	-0.0614	0.0655	0.2305	0.438	0.6923	0.9967	0.7315	0.5043	0.3136	0.1572	0.0323
Y	-0.0172	0.1089	0.2701	0.4703	0.7128	1	0.7128	0.4703	0.2701	0.1089	-0.0172
Z	-0.0242	0.1021	0.264	0.4655	0.7099	0.9999	0.716	0.4759	0.2771	0.1165	-0.0095

12 Impulse response functions

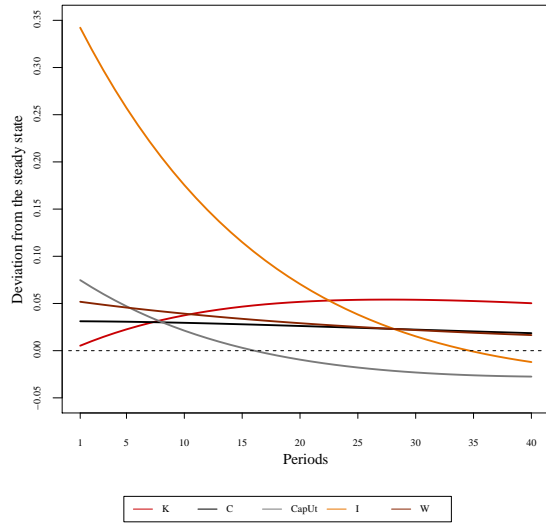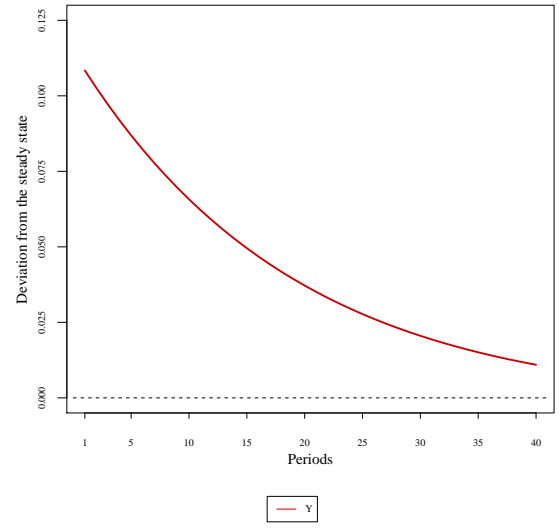## 12.1 Shock ϵ^Z

Figure 1: Impulse response function for ϵ^Z shock

Figure 2: Impulse response function for ϵ^Z shock