

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, I_t} U_t = \beta \left(E_t \left[U_{t+1}^{1-\theta^{EZ}} \right] \right)^{(1-\theta^{EZ})^{-1}} + (-1 + C_t^{1-\eta}) (1-\eta)^{-1} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1-\delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 Identities

$$L_t^s = 1 \quad (1.4)$$

1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^U} + \beta q_{t-1}^{\text{CONSUMER}^1-1+(1-\theta^{EZ})^{-1}} U_t^{-\theta^{EZ}} = 0 \quad (U_t) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^2} + E_t \left[\lambda_{t+1}^{\text{CONSUMER}^U} \left(\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} + \lambda_{t+1}^{\text{CONSUMER}^2} (1-\delta) \right) \right] = 0 \quad (K_t^s) \quad (1.6)$$

$$-\lambda_t^{\text{CONSUMER}^1} + C_t^{-\eta} = 0 \quad (C_t) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.6)$$

$$-W_t + Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^d = K_{t-1}^s \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$q_t^{\text{CONSUMER}^1} - E_t \left[U_{t+1}^{1-\theta^{\text{EZ}}} \right] = 0 \quad (5.1)$$

$$-r_t + \alpha Z_t 1^{1-\alpha} K_{t-1}^{s-1+\alpha} = 0 \quad (5.2)$$

$$-W_t + Z_t (1 - \alpha) 1^{-\alpha} K_{t-1}^{s\alpha} = 0 \quad (5.3)$$

$$-Y_t + Z_t 1^{1-\alpha} K_{t-1}^{s\alpha} = 0 \quad (5.4)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.5)$$

$$\beta q_t^{\text{CONSUMER}^1 - 1 + (1 - \theta^{\text{EZ}})^{-1}} E_t \left[(r_{t+1} C_{t+1}^{-\eta} + (1 - \delta) C_{t+1}^{-\eta}) U_{t+1}^{-\theta^{\text{EZ}}} \right] - C_t^{-\eta} = 0 \quad (5.6)$$

$$-C_t - I_t + Y_t = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.8)$$

$$U_t - \beta q_t^{\text{CONSUMER}^1 (1 - \theta^{\text{EZ}})^{-1}} - (-1 + C_t^{1-\eta}) (1 - \eta)^{-1} = 0 \quad (5.9)$$

6 Steady state relationships (after reduction)

$$q_{ss}^{\text{CONSUMER}^1} - U_{ss}^{1-\theta^{\text{EZ}}} = 0 \quad (6.1)$$

$$-r_{ss} + \alpha Z_{ss} 1^{1-\alpha} K_{ss}^{s-1+\alpha} = 0 \quad (6.2)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) 1^{-\alpha} K_{ss}^{s\alpha} = 0 \quad (6.3)$$

$$-Y_{ss} + Z_{ss} 1^{1-\alpha} K_{ss}^{s\alpha} = 0 \quad (6.4)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (6.5)$$

$$\beta (r_{ss} C_{ss}^{-\eta} + (1 - \delta) C_{ss}^{-\eta}) q_{ss}^{\text{CONSUMER}^1 - 1 + (1 - \theta^{\text{EZ}})^{-1}} U_{ss}^{-\theta^{\text{EZ}}} - C_{ss}^{-\eta} = 0 \quad (6.6)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.7)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.8)$$

$$U_{ss} - \beta q_{ss}^{\text{CONSUMER}^1} (1 - \theta^{\text{EZ}})^{-1} - (-1 + C_{ss}^{1-\eta}) (1 - \eta)^{-1} = 0 \quad (6.9)$$

7 Calibrating equations

$$-0.36Y_{ss} + r_{ss}K_{ss}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\phi = 0.95 \quad (8.4)$$

$$\theta^{\text{EZ}} = 0.05 \quad (8.5)$$

9 Steady-state values

	Steady-state values
q^{CONSUMER^1}	58.4346
r	0.0351
C	3.6213
I	1.4427
K^s	57.7077
U	72.3856
W	3.0384
Y	5.064
Z	1

10 The solution of the perturbation

10.1 P

$$\begin{matrix} K_{t-1}^s & Z_{t-1} \\ K^s & \begin{pmatrix} 0.9792 & 0.0632 \\ 0 & 0.95 \end{pmatrix} \\ Z & \end{matrix}$$

10.2 Q

$$\begin{matrix} \epsilon^Z \\ K^s & \begin{pmatrix} 0.0665 \\ 1 \end{pmatrix} \\ Z & \end{matrix}$$

10.3 R

$$\begin{matrix}
 & K_{t-1}^s & Z_{t-1} \\
 q^{\text{CONSUMER}^1} & \begin{pmatrix} 0.0571 & 0.0806 \\ -0.6 & 0.95 \\ 0.4918 & 0.3212 \\ 0.1696 & 2.5283 \\ 0.0614 & 0.0852 \\ 0.4 & 0.95 \\ 0.4 & 0.95 \end{pmatrix} \\
 r \\
 C \\
 I \\
 U \\
 W \\
 Y
 \end{matrix}$$

10.4 S

$$\begin{matrix}
 & \epsilon^Z \\
 q^{\text{CONSUMER}^1} & \begin{pmatrix} 0.0848 \\ 1 \\ 0.3381 \\ 2.6613 \\ 0.0897 \\ 1 \\ 1 \end{pmatrix} \\
 r \\
 C \\
 I \\
 U \\
 W \\
 Y
 \end{matrix}$$

11 Statistics of the model

11.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
q^{CONSUMER^1}	58.4346	0.1108	0.0123	Y
r	0.0351	1.3291	1.7664	Y
C	3.6213	0.4567	0.2085	Y
I	1.4427	3.4658	12.0115	Y
K^s	57.7077	0.3108	0.0966	Y
U	72.3856	0.1171	0.0137	Y
W	3.0384	1.301	1.6926	Y
Y	5.064	1.301	1.6926	Y
Z	1	1.3034	1.699	Y

11.2 Correlation matrix

	q^{CONSUMER^1}	r	C	I	K^s	U	W	Y	Z
q^{CONSUMER^1}	1	0.955	0.9839	0.9894	0.3667	1	0.9979	0.9979	0.9871
r	0.955	1	0.8866	0.9879	0.0743	0.9542	0.9723	0.9723	0.9902
C	0.9839	0.8866	1	0.9476	0.5272	0.9843	0.9701	0.9701	0.9426
I	0.9894	0.9879	0.9476	1	0.2279	0.9891	0.9968	0.9968	0.9999
K^s	0.3667	0.0743	0.5272	0.2279	1	0.3691	0.3053	0.3053	0.2131
U	1	0.9542	0.9843	0.9891	0.3691	1	0.9977	0.9977	0.9867
W	0.9979	0.9723	0.9701	0.9968	0.3053	0.9977	1	1	0.9954
Y	0.9979	0.9723	0.9701	0.9968	0.3053	0.9977	1	1	0.9954
Z	0.9871	0.9902	0.9426	0.9999	0.2131	0.9867	0.9954	0.9954	1

11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
q^{CONSUMER^1}	0.7256	0.4907	0.294	0.1331	0.0051
r	0.713	0.4707	0.2706	0.1094	-0.0168
C	0.753	0.5343	0.3451	0.1849	0.0529
I	0.7139	0.4721	0.2722	0.111	-0.0152
K^s	0.9605	0.8646	0.7317	0.5776	0.415
U	0.7259	0.4912	0.2946	0.1337	0.0057
W	0.7191	0.4804	0.2819	0.1208	-0.0062
Y	0.7191	0.4804	0.2819	0.1208	-0.0062
Z	0.7133	0.4711	0.2711	0.1098	-0.0163

12 Statistics of the model

12.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
q^{CONSUMER^1}	11.5392	0.0851	0.0072	Y
r	0.0069	1.0216	1.0436	Y
C	0.7151	0.351	0.1232	Y
I	0.2849	2.6639	7.0965	Y
K^s	11.3957	0.2389	0.0571	Y
U	14.2942	0.09	0.0081	Y
W	0.6	1	1	Y
Y	1	1	1	Y
Z	0.1975	1.0019	1.0038	Y

12.2 Correlations with the reference variable

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
q^{CONSUMER^1}	-0.0392	0.0887	0.2529	0.457	0.7045	0.9979	0.7361	0.5105	0.32	0.1628	0.0365
r	0.1127	0.2318	0.3763	0.5475	0.7461	0.9723	0.6325	0.3554	0.1353	-0.0341	-0.1591
C	-0.1292	-0.0014	0.1675	0.3827	0.6489	0.9701	0.7668	0.5819	0.4176	0.2745	0.1528
I	0.0346	0.1597	0.3161	0.5064	0.7329	0.9968	0.6939	0.4405	0.2334	0.0684	-0.0587
K^s	-0.4856	-0.429	-0.3301	-0.1807	0.0281	0.3053	0.4911	0.6017	0.6517	0.6545	0.6217
U	-0.0406	0.0874	0.2517	0.456	0.7038	0.9977	0.7368	0.5117	0.3215	0.1645	0.0382
W	-0.0062	0.1208	0.2819	0.4804	0.7191	1	0.7191	0.4804	0.2819	0.1208	-0.0062
Y	-0.0062	0.1208	0.2819	0.4804	0.7191	1	0.7191	0.4804	0.2819	0.1208	-0.0062
Z	0.0423	0.1669	0.3223	0.511	0.735	0.9954	0.6886	0.4327	0.224	0.0585	-0.0686

13 Impulse response functions

13.1 Shock ϵ^Z

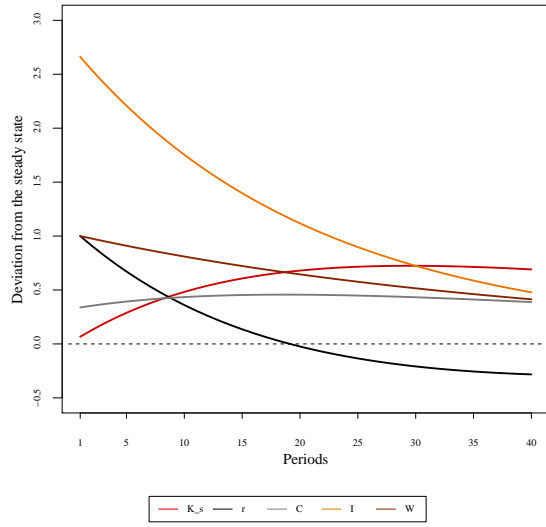


Figure 1: Impulse response function for ϵ^Z shock

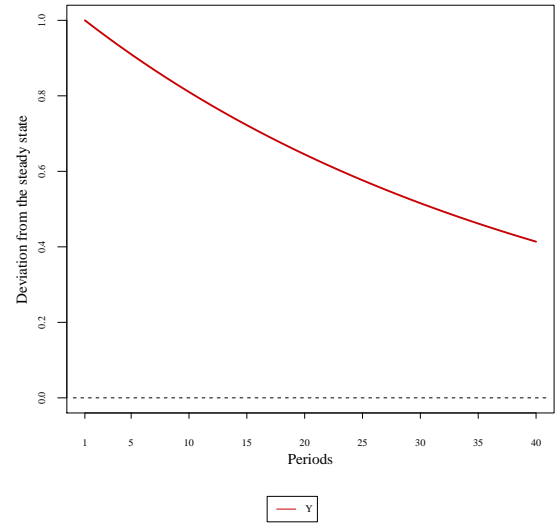


Figure 2: Impulse response function for ϵ^Z shock