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Model name: epstein_zin

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^{\mathrm{s}},C_t,I_t} U_t = \beta\left(\mathrm{E}_t\left[{U_{t+1}}^{1-\theta^{\mathrm{EZ}}}\right]\right)^{\left(1-\theta^{\mathrm{EZ}}\right)^{-1}} + \left(-1+{C_t}^{1-\eta}\right)(1-\eta)^{-1} \tag{1.1}$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^{\mathrm{s}} r_t + L_t^{\mathrm{s}} W_t \quad \left(\lambda_t^{\mathrm{CONSUMER}^1}\right) \tag{1.2}$$

$$K_t^{\mathrm{s}} = I_t + K_{t-1}^{\mathrm{s}}(1-\delta) \quad \left(\lambda_t^{\mathrm{CONSUMER}^2}\right) \tag{1.3}$$

1.2 Identities

$$L_t^{\mathrm{s}} = 1 \tag{1.4}$$

1.3 First order conditions

$$-\lambda_t^{\mathrm{CONSUMER}^{\mathrm{U}}} + \beta q_{t-1}^{{\mathrm{CONSUMER}^1}^{-1+\left(1-\theta^{\mathrm{EZ}}\right)^{-1}}} {U_t}^{-\theta^{\mathrm{EZ}}} = 0 \quad (U_t) \tag{1.5}$$

$$-\lambda_t^{\mathrm{CONSUMER}^2} + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^{\mathrm{U}}}\left(\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1} + \lambda_{t+1}^{\mathrm{CONSUMER}^2}(1-\delta)\right)\right] = 0 \quad (K_t^{\mathrm{s}}) \tag{1.6}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + {C_t}^{-\eta} = 0 \quad (C_t) \tag{1.7}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + \lambda_t^{\mathrm{CONSUMER}^2} = 0 \quad (I_t) \tag{1.8}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\mathrm{d}},L_t^{\mathrm{d}},Y_t} \pi_t = Y_t - L_t^{\mathrm{d}} W_t - r_t K_t^{\mathrm{d}} \tag{2.1}$$

s.t. :

$$Y_t = Z_t {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM}^1}\right) \tag{2.2}$$

2.2 First order conditions

$$-r_t + \alpha\lambda_t^{\mathrm{FIRM}^1} Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1} Z_t (1-\alpha) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.4}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad (Y_t) \tag{2.5}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.6}$$

$$-W_t + Z_t (1-\alpha) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.7}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{d}} = K_{t-1}^{\mathrm{s}} \tag{3.1}$$

$$L_t^{\mathrm{d}} = L_t^{\mathrm{s}} \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$q_t^{\mathrm{CONSUMER}^1} - \mathrm{E}_t\left[{U_{t+1}}^{1-\theta^{\mathrm{EZ}}}\right] = 0 \tag{5.1}$$

$$-r_t + \alpha Z_t 1^{1-\alpha} {K_{t-1}^{\mathrm{s}}}^{-1+\alpha} = 0 \tag{5.2}$$

$$-W_t + Z_t \left(1-\alpha\right) 1^{-\alpha} {K_{t-1}^{\mathrm{s}}}^{\alpha} = 0 \tag{5.3}$$

$$-Y_t + Z_t 1^{1-\alpha} {K_{t-1}^{\mathrm{s}}}^{\alpha} = 0 \tag{5.4}$$

$$-Z_t + e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} = 0 \tag{5.5}$$

$$\beta q_t^{\mathrm{CONSUMER}^{1\,-1+\left(1-\theta^{\mathrm{EZ}}\right)^{-1}}} \mathrm{E}_t\left[\left(r_{t+1}{C_{t+1}}^{-\eta} + \left(1-\delta\right){C_{t+1}}^{-\eta}\right){U_{t+1}}^{-\theta^{\mathrm{EZ}}}\right] - {C_t}^{-\eta} = 0 \tag{5.6}$$

$$-C_t - I_t + Y_t = 0 \tag{5.7}$$

$$I_t - K_t^{\mathrm{s}} + K_{t-1}^{\mathrm{s}}\left(1-\delta\right) = 0 \tag{5.8}$$

$$U_t - \beta q_t^{\mathrm{CONSUMER}^{1\left(1-\theta^{\mathrm{EZ}}\right)^{-1}}} - \left(-1 + {C_t}^{1-\eta}\right)\left(1-\eta\right)^{-1} = 0 \tag{5.9}$$

6 Steady state relationships (after reduction)

$$q_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} - {U_{\mathrm{ss}}}^{1-\theta^{\mathrm{EZ}}} = 0 \tag{6.1}$$

$$-r_{\mathrm{ss}} + \alpha Z_{\mathrm{ss}} 1^{1-\alpha} {K_{\mathrm{ss}}^{\mathrm{s}}}^{-1+\alpha} = 0 \tag{6.2}$$

$$-W_{\mathrm{ss}} + Z_{\mathrm{ss}}\left(1-\alpha\right) 1^{-\alpha} {K_{\mathrm{ss}}^{\mathrm{s}}}^{\alpha} = 0 \tag{6.3}$$

$$-Y_{\mathrm{ss}} + Z_{\mathrm{ss}} 1^{1-\alpha} {K_{\mathrm{ss}}^{\mathrm{s}}}^{\alpha} = 0 \tag{6.4}$$

$$-Z_{\mathrm{ss}} + e^{\phi \log Z_{\mathrm{ss}}} = 0 \tag{6.5}$$

$$\beta\left(r_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-\eta} + \left(1-\delta\right){C_{\mathrm{ss}}}^{-\eta}\right) q_{\mathrm{ss}}^{\mathrm{CONSUMER}^{1\,-1+\left(1-\theta^{\mathrm{EZ}}\right)^{-1}}} {U_{\mathrm{ss}}}^{-\theta^{\mathrm{EZ}}} - {C_{\mathrm{ss}}}^{-\eta} = 0 \tag{6.6}$$

$$-C_{\mathrm{ss}} - I_{\mathrm{ss}} + Y_{\mathrm{ss}} = 0 \tag{6.7}$$

$$I_{\mathrm{ss}} - K^{\mathrm{s}}_{\mathrm{ss}} + K^{\mathrm{s}}_{\mathrm{ss}} (1 - \delta) = 0 \tag{6.8}$$

$$U_{\mathrm{ss}} - \beta q^{\mathrm{CONSUMER}^1 \left(1-\theta^{\mathrm{EZ}}\right)^{-1}}_{\mathrm{ss}} - \left(-1 + {C_{\mathrm{ss}}}^{1-\eta}\right) (1 - \eta)^{-1} = 0 \tag{6.9}$$

7 Calibrating equations

$$-0.36 Y_{\mathrm{ss}} + r_{\mathrm{ss}} K^{\mathrm{s}}_{\mathrm{ss}} = 0 \tag{7.1}$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$

$$\delta = 0.025 \tag{8.2}$$

$$\eta = 2 \tag{8.3}$$

$$\phi = 0.95 \tag{8.4}$$

$$\theta^{\mathrm{EZ}} = 0.05 \tag{8.5}$$

9 Steady-state values

	Steady-state value
q^{CONSUMER^1}	58.4346
r	0.0351
C	3.6213
I	1.4427
K^{s}	57.7077
U	72.3856
W	3.0384
Y	5.064
Z	1

10 Model parameters

	Value
α	0.4
β	0.99
δ	0.025
η	2
ϕ	0.95
θ^{EZ}	0.05

11 The solution of the 1st order perturbation

Matrix P

$$\begin{array}{c} \\ K^{\mathrm{s}}_t \\ Z_t \end{array} \begin{array}{c} \begin{array}{cc} K^{\mathrm{s}}_{t-1} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.9792 & 0.0632 \\ 0 & 0.95 \end{pmatrix} \end{array}$$

Matrix Q

$$\begin{array}{c} \\ K^{\mathrm{s}} \\ Z \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 0.0665 \\ 1 \end{pmatrix} \end{array}$$

Matrix R

$$\begin{array}{l} \\ q_t^{\mathrm{CONSUMER}[1]} \\ r_t \\ C_t \\ I_t \\ U_t \\ W_t \\ Y_t \end{array} \begin{array}{c} \begin{array}{cc} K^{\mathrm{s}}_{t-1} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.0571 & 0.0806 \\ -0.6 & 0.95 \\ 0.4918 & 0.3212 \\ 0.1696 & 2.5283 \\ 0.0614 & 0.0852 \\ 0.4 & 0.95 \\ 0.4 & 0.95 \end{pmatrix} \end{array}$$

Matrix S

$$\begin{array}{l} \\ q^{\mathrm{CONSUMER}[1]} \\ r \\ C \\ I \\ U \\ W \\ Y \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 0.0848 \\ 1 \\ 0.3381 \\ 2.6613 \\ 0.0897 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
C	3.6213	0.4567	0.2085	Y
K^{s}	57.7077	0.3108	0.0966	Y
W	3.0384	1.301	1.6926	Y
I	1.4427	3.4658	12.0115	Y
r	0.0351	1.3291	1.7664	Y
Y	5.064	1.301	1.6926	Y

12.2 Correlation matrix

	r	C	I	K^{s}	W	Y
r	1	0.887	0.988	0.074	0.972	0.972
C		1	0.948	0.527	0.97	0.97
I			1	0.228	0.997	0.997
K^{s}				1	0.305	0.305
W					1	1
Y						1

12.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	1.022	0.113	0.232	0.376	0.547	0.746	0.972	0.632	0.355	0.135	-0.034	-0.159
C_t	0.351	-0.129	-0.001	0.167	0.383	0.649	0.97	0.767	0.582	0.418	0.275	0.153
I_t	2.664	0.035	0.16	0.316	0.506	0.733	0.997	0.694	0.441	0.233	0.068	-0.059
K^{s}_t	0.239	-0.486	-0.429	-0.33	-0.181	0.028	0.305	0.491	0.602	0.652	0.654	0.622
W_t	1	-0.006	0.121	0.282	0.48	0.719	1	0.719	0.48	0.282	0.121	-0.006
Y_t	1	-0.006	0.121	0.282	0.48	0.719	1	0.719	0.48	0.282	0.121	-0.006

12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.713	0.471	0.271	0.109	-0.017
C	0.753	0.534	0.345	0.185	0.053
I	0.714	0.472	0.272	0.111	-0.015
K^{s}	0.96	0.865	0.732	0.578	0.415
W	0.719	0.48	0.282	0.121	-0.006
Y	0.719	0.48	0.282	0.121	-0.006

13 Impulse response functions

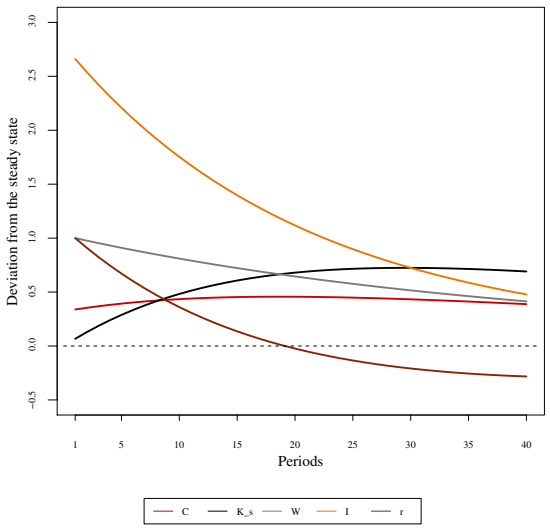

Figure 1: Impulse responses $(C, K^{\mathrm{s}}, W, I, r)$ to ϵ^{Z} shock

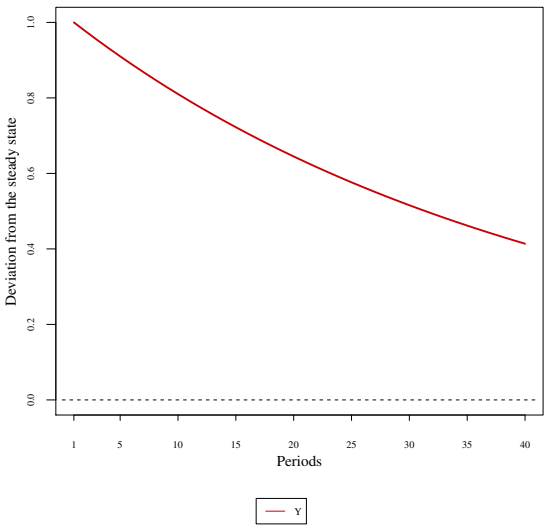

Figure 2: Impulse response (Y) to ϵ^{Z} shock