

## 1 CONSUMER

### 1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left( \lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left( \lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.4)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} \left( (1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu = 0 \quad (L_t^s) \quad (1.6)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.7)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d1-\alpha} \quad \left( \lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

## 2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

## 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

# 3 EQUILIBRIUM

## 3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

# 4 EXOG

## 4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

## 5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}^{-1+\alpha}} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.1)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{-\alpha}} = 0 \quad (5.2)$$

$$-Y_t + Z_t K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.3)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$\beta \left( \mu \text{E}_t \left[ r_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] + \mu (1 - \delta) \text{E}_t \left[ C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] \right) - \mu C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left( C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^{\text{s}})^{-\mu} \left( C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left( C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$-C_t - I_t + Y_t = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.8)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.9)$$

## 6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.1)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.2)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.3)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (6.4)$$

$$\beta \left( \mu r_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) - \mu C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.5)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.7)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.8)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.9)$$

## 7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^s = 0 \quad (7.1)$$

## 8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$\phi = 0.95 \quad (8.5)$$

## 9 Steady-state values

	Steady-state values
$r$	0.0351
$C$	0.7422
$I$	0.2559
$K^s$	10.2368
$L^s$	0.2695
$U$	-136.2372
$W$	2.3706
$Y$	0.9981
$Z$	1

## 10 The solution of the perturbation

### 10.1 P

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ K^s & \begin{pmatrix} 0.9631 & 0.0962 \end{pmatrix} \\ Z & \begin{pmatrix} 0 & 0.95 \end{pmatrix} \end{matrix}$$

### 10.2 Q

$$\begin{matrix} & \epsilon^Z \\ K^s & \begin{pmatrix} 0.1012 \end{pmatrix} \\ Z & \begin{pmatrix} 1 \end{pmatrix} \end{matrix}$$

### 10.3 R

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ r & \begin{pmatrix} -0.7559 & 1.3521 \end{pmatrix} \\ C & \begin{pmatrix} 0.4919 & 0.4921 \end{pmatrix} \\ I & \begin{pmatrix} -0.4745 & 3.8461 \end{pmatrix} \\ L^s & \begin{pmatrix} -0.181 & 0.6282 \end{pmatrix} \\ U & \begin{pmatrix} -0.0418 & -0.0644 \end{pmatrix} \\ W & \begin{pmatrix} 0.4252 & 0.7238 \end{pmatrix} \\ Y & \begin{pmatrix} 0.2441 & 1.3521 \end{pmatrix} \end{matrix}$$

### 10.4 S

$$\begin{matrix} & \epsilon^Z \\ r & \begin{pmatrix} 1.4232 \end{pmatrix} \\ C & \begin{pmatrix} 0.518 \end{pmatrix} \\ I & \begin{pmatrix} 4.0485 \end{pmatrix} \\ L^s & \begin{pmatrix} 0.6613 \end{pmatrix} \\ U & \begin{pmatrix} -0.0678 \end{pmatrix} \\ W & \begin{pmatrix} 0.7619 \end{pmatrix} \\ Y & \begin{pmatrix} 1.4232 \end{pmatrix} \end{matrix}$$

## 11 Statistics of the model

### 11.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
$r$	0.0351	0.1893	0.0358	Y
$C$	0.7422	0.0711	0.0051	Y
$I$	0.2559	0.5284	0.2792	Y
$K^s$	10.2368	0.0469	0.0022	Y
$L^s$	0.2695	0.0867	0.0075	Y
$U$	-136.2372	0.009	0.0001	Y
$W$	2.3706	0.1011	0.0102	Y
$Y$	0.9981	0.1857	0.0345	Y
$Z$	1	0.1303	0.017	Y

### 11.2 Correlation matrix

	$r$	$C$	$I$	$K^s$	$L^s$	$U$	$W$	$Y$	$Z$
$r$	1	0.8683	0.9893	0.0841	0.9959	-0.9182	0.926	0.9689	0.9823
$C$	0.8683	1	0.9313	0.5674	0.9095	-0.9938	0.9913	0.964	0.9458
$I$	0.9893	0.9313	1	0.2285	0.9984	-0.9661	0.9711	0.9946	0.9991
$K^s$	0.0841	0.5674	0.2285	1	0.1737	-0.472	0.4541	0.3282	0.2693
$L^s$	0.9959	0.9095	0.9984	0.1737	1	-0.9502	0.9563	0.9873	0.9952
$U$	-0.9182	-0.9938	-0.9661	-0.472	-0.9502	1	-0.9998	-0.9877	-0.9761
$W$	0.926	0.9913	0.9711	0.4541	0.9563	-0.9998	1	0.9906	0.9803
$Y$	0.9689	0.964	0.9946	0.3282	0.9873	-0.9877	0.9906	1	0.9981
$Z$	0.9823	0.9458	0.9991	0.2693	0.9952	-0.9761	0.9803	0.9981	1

### 11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$
$r$	0.7098	0.4657	0.2647	0.1034
$C$	0.7596	0.5446	0.3568	0.1964
$I$	0.7109	0.4674	0.2668	0.1055
$K^s$	0.9597	0.8621	0.7271	0.5709
$L^s$	0.7092	0.4647	0.2636	0.1023
$U$	0.7389	0.5118	0.3185	0.1578
$W$	0.7356	0.5066	0.3125	0.1517
$Y$	0.7183	0.4791	0.2803	0.1192
$Z$	0.7133	0.4711	0.2711	0.1098

## 12 Statistics of the model

### 12.1 Moments relative to moments of the reference variable

	Steady-state value relative to $Y$	Std. dev. relative to $Y$	Variance relative to $Y$	Loglinear
$r$	0.0352	1.0193	1.0389	Y
$C$	0.7436	0.3827	0.1465	Y
$I$	0.2564	2.8453	8.0959	Y
$K^s$	10.2561	0.2526	0.0638	Y
$L^s$	0.27	0.4669	0.218	Y
$U$	-136.4937	0.0486	0.0024	Y
$W$	2.3751	0.5441	0.2961	Y
$Y$	1	1	1	Y
$Z$	1.0019	0.7018	0.4926	Y

## 12.2 Correlations with the reference variable

	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$
$r$	0.2344	0.3779	0.5477	0.7447	0.9689	0.6234	0.343	0.1215	-0.0479
$C$	-0.0128	0.1562	0.3722	0.64	0.964	0.7703	0.5919	0.4314	0.2902
$I$	0.1684	0.3233	0.5115	0.7348	0.9946	0.684	0.4257	0.2159	0.0501
$K^s$	-0.4149	-0.3135	-0.1616	0.0493	0.3282	0.5125	0.6196	0.6649	0.6624
$L^s$	0.1941	0.3451	0.5267	0.7405	0.9873	0.6625	0.3951	0.1802	0.0127
$U$	-0.0425	-0.2097	-0.4203	-0.6785	-0.9877	-0.7553	-0.5498	-0.3718	-0.221
$W$	0.0524	0.2191	0.4285	0.6846	0.9906	0.7516	0.5414	0.3605	0.2082
$Y$	0.1192	0.2803	0.4791	0.7183	1	0.7183	0.4791	0.2803	0.1192
$Z$	0.1486	0.3063	0.499	0.7291	0.9981	0.6988	0.448	0.2424	0.0783

## 13 Impulse response functions

### 13.1 Shock $\epsilon^Z$

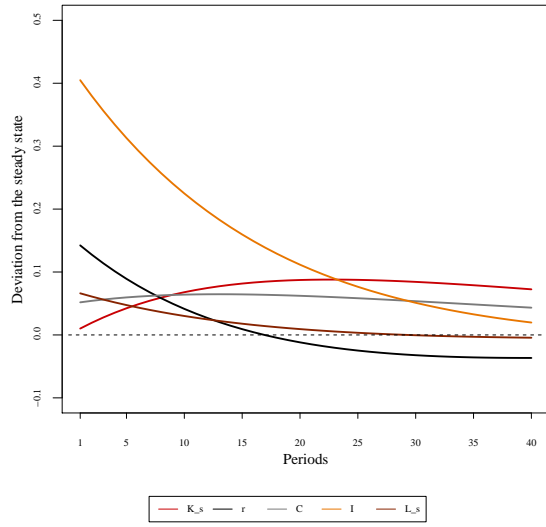


Figure 1: Impulse response function for  $\epsilon^Z$  shock

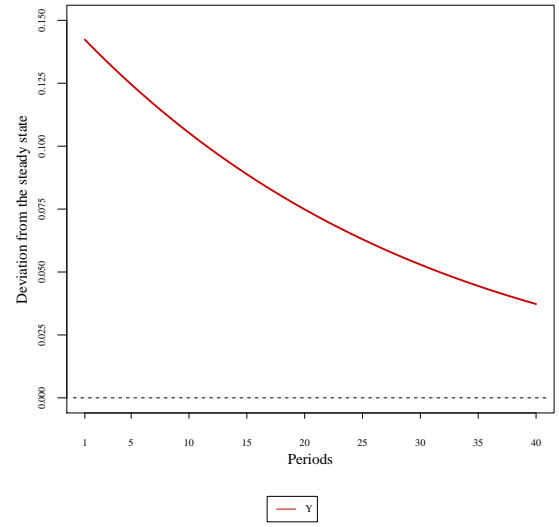


Figure 2: Impulse response function for  $\epsilon^Z$  shock