

# 1 CONSUMER

## 1.1 Optimisation problem

$$\max_{C_t, N_t, a_t, L_t} U_t = \beta E_t [U_{t+1}] + \gamma^{-1} (C_t^\mu L_t^{1-\mu})^\gamma \quad (1.1)$$

s.t. :

$$C_t = \pi_t + N_t W_t \quad (\lambda_t^c) \quad (1.2)$$

$$L_t = 1 - \alpha N_t - \eta a_{t-1} (1 - \alpha) \quad (\lambda_t^{\text{CONSUMER}^2}) \quad (1.3)$$

$$a_t = N_t + a_{t-1} (1 - \eta) \quad (\lambda_t^{\text{CONSUMER}^3}) \quad (1.4)$$

## 1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.5)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} L_t^{1-\mu} (C_t^\mu L_t^{1-\mu})^{-1+\gamma} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^3} - \alpha \lambda_t^{\text{CONSUMER}^2} + \lambda_t^c W_t = 0 \quad (N_t) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^3} + E_t \left[ \lambda_{t+1}^U \left( \lambda_{t+1}^{\text{CONSUMER}^3} (1 - \eta) - \eta \lambda_{t+1}^{\text{CONSUMER}^2} (1 - \alpha) \right) \right] = 0 \quad (a_t) \quad (1.8)$$

$$-\lambda_t^{\text{CONSUMER}^2} + (1 - \mu) C_t^\mu L_t^{-\mu} (C_t^\mu L_t^{1-\mu})^{-1+\gamma} = 0 \quad (L_t) \quad (1.9)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t, N_t^d, Z_t, Y_t, S_t, X_t, \pi_t, S_t^{\text{lag}^1}, S_t^{\text{lag}^2}} \Pi_t = \pi_t + \lambda_t^c{}^{-1} \mathbb{E}_t [\lambda_{t+1}^U \lambda_{t+1}^c \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$Y_t = \left( \sigma Z_{t-1}^{-\nu} + \left( \Lambda_t K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-\nu^{-1}} \left( \lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$K_t = S_{t-3} + K_{t-1} (1 - \delta) \quad \left( \lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

$$X_t = \psi (S_{t-3} + S_{t-2} + S_{t-1} + S_t) \quad \left( \lambda_t^{\text{FIRM}^3} \right) \quad (2.4)$$

$$\pi_t = Z_{t-1} - X_t + Y_t - Z_t - N_t^d W_t \quad \left( \lambda_t^{\text{FIRM}^4} \right) \quad (2.5)$$

### 2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c{}^{-1} \lambda_t^U \lambda_t^c = 0 \quad (\Pi_t) \quad (2.6)$$

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$$-\lambda_t^{\text{FIRM}^2} + \mathbb{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^2} (1 - \delta) + \theta \lambda_{t+1}^{\text{FIRM}^1} \Lambda_{t+1} K_t^{-1+\theta} N_{t+1}^{\text{d}^{1-\theta}} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-1-\nu} \right) \right] = 0 \quad (K_t) \quad (2.7)$$

$$-\lambda_t^{\text{FIRM}^4} W_t + \lambda_t^{\text{FIRM}^1} \Lambda_t (1 - \theta) K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \left( \sigma Z_{t-1}^{-\nu} + \left( \Lambda_t K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_t K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \right)^{-1-\nu} = 0 \quad (N_t^d) \quad (2.8)$$

$$-\lambda_t^{\text{FIRM}^4} + \mathbb{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^4} + \sigma \lambda_{t+1}^{\text{FIRM}^1} Z_t^{-1-\nu} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \right) \right] = 0 \quad (Z_t) \quad (2.9)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^4} = 0 \quad (Y_t) \quad (2.10)$$

$$\psi \lambda_t^{\text{FIRM}^3} + \mathbb{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} + \psi \lambda_{t+1}^{\text{FIRM}^3} \right) \right] = 0 \quad (S_t) \quad (2.11)$$

$$-\lambda_t^{\text{FIRM}^3} - \lambda_t^{\text{FIRM}^4} = 0 \quad (X_t) \quad (2.12)$$

$$1 - \lambda_t^{\text{FIRM}^4} = 0 \quad (\pi_t) \quad (2.13)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} + \psi \lambda_{t+1}^{\text{FIRM}^3} \right) \right] = 0 \quad (S_t^{\text{lag}^1}) \quad (2.14)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^2} + \psi \lambda_{t+1}^{\text{FIRM}^3} \right) \right] = 0 \quad (S_t^{\text{lag}^2}) \quad (2.15)$$

### 2.3 First order conditions after reduction

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^{\text{c}}^{-1} \lambda_t^{\text{U}} \lambda_t^{\text{c}} = 0 \quad (\Pi_t) \quad (2.16)$$

$$-\lambda_t^{\text{FIRM}^2} + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( \lambda_{t+1}^{\text{FIRM}^2} (1 - \delta) + \theta \Lambda_{t+1} K_t^{-1+\theta} N_{t+1}^{\text{d}^{1-\theta}} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-1-\nu} \right) \right] = 0 \quad (K_t) \quad (2.17)$$

$$-W_t + \Lambda_t (1 - \theta) K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \left( \sigma Z_{t-1}^{-\nu} + \left( \Lambda_t K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_t K_{t-1}^\theta N_t^{\text{d}^{1-\theta}} \right)^{-1-\nu} = 0 \quad (N_t^{\text{d}}) \quad (2.18)$$

$$-1 + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( 1 + \sigma Z_t^{-1-\nu} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{\text{d}^{1-\theta}} \right)^{-\nu} \right)^{-1-\nu^{-1}} \right) \right] = 0 \quad (Z_t) \quad (2.19)$$

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$$-\psi + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( -\psi + \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} \right) \right] = 0 \quad (S_t) \quad (2.20)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( -\psi + \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} \right) \right] = 0 \quad (S_t^{\text{lag}^1}) \quad (2.21)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} + \text{E}_t \left[ \lambda_{t+1}^{\text{FIRM}^\Pi} \left( -\psi + \lambda_{t+1}^{\text{FIRM}^2} \right) \right] = 0 \quad (S_t^{\text{lag}^2}) \quad (2.22)$$

## 3 EQUILIBRIUM

### 3.1 Identities

$$N_t^{\text{d}} = N_t \quad (3.1)$$

## 4 EXOG

### 4.1 Identities

$$-1 + \Lambda_t = \epsilon_t^\Lambda + \phi^{\text{a}} (-1 + \Lambda_{t-1}) + \phi^{\text{b}} (-1 + \Lambda_{t-1}) \quad (4.1)$$

## 5 Equilibrium relationships (after reduction)

$$-1 + \beta C_t^{1-\mu} L_t^{-1+\mu} (C_t^\mu L_t^{1-\mu})^{1-\gamma} E_t \left[ \left( 1 + \sigma Z_t^{-1-\nu} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \right) C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.1)$$

$$-\psi + \beta C_t^{1-\mu} L_t^{-1+\mu} (C_t^\mu L_t^{1-\mu})^{1-\gamma} E_t \left[ \left( -\psi + \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} \right) C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.2)$$

$$S_{t-1} - S_t^{\text{lag}^1} = 0 \quad (5.3)$$

$$S_{t-1}^{\text{lag}^1} - S_t^{\text{lag}^2} = 0 \quad (5.4)$$

$$-\lambda_t^{\text{FIRM}^2} + \beta C_t^{1-\mu} L_t^{-1+\mu} (C_t^\mu L_t^{1-\mu})^{1-\gamma} E_t \left[ \left( \lambda_{t+1}^{\text{FIRM}^2} (1 - \delta) + \theta \Lambda_{t+1} K_t^{-1+\theta} N_{t+1}^{1-\theta} \left( \sigma Z_t^{-\nu} + \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_{t+1} K_t^\theta N_{t+1}^{1-\theta} \right)^{-1-\nu} \right) C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (L_{t+1}^\mu N_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.5)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} + \beta C_t^{1-\mu} L_t^{-1+\mu} (C_t^\mu L_t^{1-\mu})^{1-\gamma} E_t \left[ \left( -\psi + \lambda_{t+1}^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} \right) C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.6)$$

$$-\lambda_t^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} + \beta C_t^{1-\mu} L_t^{-1+\mu} (C_t^\mu L_t^{1-\mu})^{1-\gamma} E_t \left[ \left( -\psi + \lambda_{t+1}^{\text{FIRM}^2} \right) C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.7)$$

$$-W_t + \Lambda_t (1 - \theta) K_{t-1}^\theta N_t^{-\theta} \left( \sigma Z_{t-1}^{-\nu} + \left( \Lambda_t K_{t-1}^\theta N_t^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_t K_{t-1}^\theta N_t^{1-\theta} \right)^{-1-\nu} = 0 \quad (5.8)$$

$$-Y_t + \left( \sigma Z_{t-1}^{-\nu} + \left( \Lambda_t K_{t-1}^\theta N_t^{1-\theta} \right)^{-\nu} \right)^{-\nu^{-1}} = 0 \quad (5.9)$$

$$S_{t-1}^{\text{lag}^2} - K_t + K_{t-1} (1 - \delta) = 0 \quad (5.10)$$

$$-a_t + N_t + a_{t-1} (1 - \eta) = 0 \quad (5.11)$$

$$-\pi_t + \Pi_t - \beta (C_t^{-1+\mu})^{-1} (L_t^{1-\mu})^{-1} \left( (C_t^\mu L_t^{1-\mu})^{-1+\gamma} \right)^{-1} E_t \left[ \Pi_{t+1} C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] = 0 \quad (5.12)$$

$$\pi_t - C_t + N_t W_t = 0 \quad (5.13)$$

$$U_t - \beta E_t [U_{t+1}] - \gamma^{-1} (C_t^\mu L_t^{1-\mu})^\gamma = 0 \quad (5.14)$$

$$\beta \left( (1-\eta) \left( -\mu \mathbb{E}_t \left[ W_{t+1} C_{t+1}^{-1+\mu} L_{t+1}^{1-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] + \alpha (1-\mu) \mathbb{E}_t \left[ C_{t+1}^\mu L_{t+1}^{-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] \right) - \eta (1-\alpha) (1-\mu) \mathbb{E}_t \left[ C_{t+1}^\mu L_{t+1}^{-\mu} (C_{t+1}^\mu L_{t+1}^{1-\mu})^{-1+\gamma} \right] \right) \quad (5.15)$$

$$1 - L_t - \alpha N_t - \eta a_{t-1} (1 - \alpha) = 0 \quad (5.16)$$

$$-1 - \epsilon_t^\Lambda + \Lambda_t - \phi^a (-1 + \Lambda_{t-1}) - \phi^b (-1 + \Lambda_{t-1}) = 0 \quad (5.17)$$

$$Z_{t-1} - \pi_t + Y_t - Z_t - \psi \left( S_{t-1} + S_{t-1}^{\text{lag}^1} + S_{t-1}^{\text{lag}^2} + S_t \right) - N_t W_t = 0 \quad (5.18)$$

## 6 Steady state relationships (after reduction)

$$-1 + \beta \left( 1 + \sigma Z_{ss}^{-1-\nu} \left( \sigma Z_{ss}^{-\nu} + \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \right) 1 L_{ss}^{-1+\mu} L_{ss}^{1-\mu} (C_{ss}^\mu L_{ss}^{1-\mu})^{1-\gamma} (C_{ss}^\mu L_{ss}^{1-\mu})^{-1+\gamma} = 0 \quad (6.1)$$

$$-\psi + \beta \left( -\psi + \lambda_{ss}^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} L_{ss}^{-1+\mu} L_{ss}^{1-\mu} = 0 \quad (6.2)$$

$$S_{ss} - S_{ss}^{\text{lag}^1} = 0 \quad (6.3)$$

$$S_{ss}^{\text{lag}^1} - S_{ss}^{\text{lag}^2} = 0 \quad (6.4)$$

$$-\lambda_{ss}^{\text{FIRM}^2} + \beta \left( \lambda_{ss}^{\text{FIRM}^2} (1 - \delta) + \theta \Lambda_{ss} K_{ss}^{-1+\theta} N_{ss}^{1-\theta} \left( \sigma Z_{ss}^{-\nu} + \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-1-\nu} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} L_{ss}^{-1+\mu} L_{ss}^{1-\mu} = 0 \quad (6.5)$$

$$-\lambda_{ss}^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} + \beta \left( -\psi + \lambda_{ss}^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} L_{ss}^{-1+\mu} L_{ss}^{1-\mu} = 0 \quad (6.6)$$

$$-\lambda_{ss}^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} + \beta \left( -\psi + \lambda_{ss}^{\text{FIRM}^2} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} L_{ss}^{-1+\mu} L_{ss}^{1-\mu} = 0 \quad (6.7)$$

$$-W_{ss} + \Lambda_{ss} (1 - \theta) K_{ss}^\theta N_{ss}^{-\theta} \left( \sigma Z_{ss}^{-\nu} + \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-\nu} \right)^{-1-\nu^{-1}} \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-1-\nu} = 0 \quad (6.8)$$

$$-Y_{ss} + \left( \sigma Z_{ss}^{-\nu} + \left( \Lambda_{ss} K_{ss}^\theta N_{ss}^{1-\theta} \right)^{-\nu} \right)^{-\nu^{-1}} = 0 \quad (6.9)$$

$$-K_{ss} + S_{ss}^{\text{lag}^2} + K_{ss}(1 - \delta) = 0 \quad (6.10)$$

$$-a_{ss} + N_{ss} + a_{ss}(1 - \eta) = 0 \quad (6.11)$$

$$-\pi_{ss} + \Pi_{ss} - \beta \Pi_{ss} 1 L_{ss}^{-1+\mu} L_{ss}^{1-\mu} = 0 \quad (6.12)$$

$$\pi_{ss} - C_{ss} + N_{ss} W_{ss} = 0 \quad (6.13)$$

$$U_{ss} - \beta U_{ss} - \gamma^{-1} (C_{ss}^{\mu} L_{ss}^{1-\mu})^{\gamma} = 0 \quad (6.14)$$

$$\beta \left( (1 - \eta) \left( \alpha (1 - \mu) C_{ss}^{\mu} L_{ss}^{-\mu} (C_{ss}^{\mu} L_{ss}^{1-\mu})^{-1+\gamma} - \mu W_{ss} C_{ss}^{-1+\mu} L_{ss}^{1-\mu} (C_{ss}^{\mu} L_{ss}^{1-\mu})^{-1+\gamma} \right) - \eta (1 - \alpha) (1 - \mu) C_{ss}^{\mu} L_{ss}^{-\mu} (C_{ss}^{\mu} L_{ss}^{1-\mu})^{-1+\gamma} \right) - \alpha (1 - \mu) C_{ss}^{\mu} L_{ss}^{-\mu} (C_{ss}^{\mu} L_{ss}^{1-\mu})^{-1+\gamma} = 0 \quad (6.15)$$

$$1 - L_{ss} - \alpha N_{ss} - \eta a_{ss}(1 - \alpha) = 0 \quad (6.16)$$

$$-1 + \Lambda_{ss} - \phi^a(-1 + \Lambda_{ss}) - \phi^b(-1 + \Lambda_{ss}) = 0 \quad (6.17)$$

$$-\pi_{ss} + Y_{ss} - \psi \left( 2S_{ss} + S_{ss}^{\text{lag}^1} + S_{ss}^{\text{lag}^2} \right) - N_{ss} W_{ss} = 0 \quad (6.18)$$

## 7 Parameter settings

$$\alpha = 1 \quad (7.1)$$

$$\beta = 0.99 \quad (7.2)$$

$$\delta = 0.025 \quad (7.3)$$

$$\eta = 0.5 \quad (7.4)$$

$$\gamma = -1 \quad (7.5)$$

$$\mu = 0.34 \quad (7.6)$$

$$\nu = 3 \tag{7.7}$$

$$\phi^{\text{a}} = 0.906 \tag{7.8}$$

$$\phi^{\text{b}} = 0.088 \tag{7.9}$$

$$\psi = 0.25 \tag{7.10}$$

$$\sigma = 0.01 \tag{7.11}$$

$$\theta = 0.36 \tag{7.12}$$

## 8 Steady-state values

	Steady-state values
$a$	0.6064
$\pi$	0.1283
$C$	0.8261
$K$	11.0149
$L$	0.6968
$\Lambda$	1
$N$	0.3032
$\Pi$	12.8257
$S$	0.2754
$S^{\text{lag}^1}$	0.2754
$S^{\text{lag}^2}$	0.2754
$U$	-135.4461
$W$	2.3014
$Y$	1.1015
$Z$	1.0987

## 9 The solution of the perturbation

### 9.1 P

$$\begin{matrix} a \\ K \\ \Lambda \\ S \\ S^{\text{lag}^1} \\ S^{\text{lag}^2} \\ Z \end{matrix} \begin{pmatrix} a_{t-1} & K_{t-1} & \Lambda_{t-1} & S_{t-1} & S_{t-1}^{\text{lag}^1} & S_{t-1}^{\text{lag}^2} & Z_{t-1} \\ 0.5 & -0.0601 & 0.1549 & -0.0012 & -0.0024 & -0.0037 & -0.0086 \\ 0 & 0.975 & 0 & 0 & 0 & 0.025 & 0 \\ 0 & 0 & 0.994 & 0 & 0 & 0 & 0 \\ 0 & -8.077 & 6.257 & -1.0496 & -1.0055 & -0.8423 & 8.6102 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.453 & 0.2652 & 0.0002 & -0.0056 & -0.0193 & 0.4187 \end{pmatrix}$$

### 9.2 Q

$$\begin{matrix} a \\ K \\ \Lambda \\ S \\ S^{\text{lag}^1} \\ S^{\text{lag}^2} \\ Z \end{matrix} \begin{pmatrix} \epsilon^\Lambda \\ 0.1558 \\ 0 \\ 1 \\ 6.2947 \\ 0 \\ 0 \\ 0.2668 \end{pmatrix}$$

### 9.3 R

$$\begin{matrix} \lambda^{\text{FIRM}^2} \\ \lambda^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} \\ \lambda^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} \\ \pi \\ C \\ L \\ N \\ \Pi \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} a_{t-1} & K_{t-1} & \Lambda_{t-1} & S_{t-1} & S_{t-1}^{\text{lag}^1} & S_{t-1}^{\text{lag}^2} & Z_{t-1} \\ 0 & -0.0493 & 0.0617 & -0.0009 & -0.0019 & -0.0025 & 0.0167 \\ 0 & -0.0227 & 0.025 & -0.0016 & -0.0017 & -0.0015 & 0.0126 \\ 0 & -0.0376 & 0.044 & -0.0012 & -0.0024 & -0.0022 & 0.0165 \\ 0 & 1.3828 & -1.7195 & 0.0202 & 0.041 & 0.0647 & 0.1891 \\ 0 & 0.4442 & 0.6996 & 0.0019 & 0.0039 & 0.0062 & 0.0545 \\ 0 & 0.0523 & -0.1348 & 0.001 & 0.0021 & 0.0033 & 0.0075 \\ 0 & -0.1202 & 0.3098 & -0.0023 & -0.0048 & -0.0075 & -0.0172 \\ 0 & 0.8155 & 0.0939 & 0.0046 & 0.0091 & 0.014 & 0.0989 \\ 0 & -0.0465 & -0.2798 & -0.0003 & -0.0006 & -0.0009 & -0.0046 \\ 0 & 0.3919 & 0.8344 & 0.0009 & 0.0018 & 0.0029 & 0.047 \\ 0 & 0.2802 & 1.1803 & -0.0015 & -0.003 & -0.0047 & -0.0008 \end{pmatrix}$$



## 9.4 S

$$\begin{array}{c} \lambda^{\text{FIRM}^2} \\ \lambda^{\text{FIRM}^{\text{S}^{\text{lag}^1}}} \\ \lambda^{\text{FIRM}^{\text{S}^{\text{lag}^2}}} \\ \pi \\ C \\ L \\ N \\ \Pi \\ U \\ W \\ Y \end{array} \epsilon^{\Lambda} \begin{pmatrix} 0.0621 \\ 0.0252 \\ 0.0443 \\ -1.7299 \\ 0.7038 \\ -0.1356 \\ 0.3117 \\ 0.0945 \\ -0.2815 \\ 0.8395 \\ 1.1874 \end{pmatrix}$$

## 10 Statistics of the model

### 10.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
$C$	0.8261	0.2883	0.0831	Y
$K$	11.0149	0.093	0.0087	Y
$L$	0.6968	0.0533	0.0028	Y
$\Lambda$	1	0.4096	0.1678	Y
$N$	0.3032	0.1225	0.015	Y
$U$	-135.4461	0.1153	0.0133	Y
$W$	2.3014	0.3399	0.1155	Y
$Y$	1.1015	0.4723	0.2231	Y

### 10.2 Correlation matrix

	$C$	$K$	$L$	$\Lambda$	$N$	$U$	$W$	$Y$
$a$	0.9543	-0.1427	-0.9156	0.9519	0.9156	-0.9551	0.9532	0.9469
$\lambda^{\text{FIRM}^2}$	0.9535	-0.3445	-0.999	0.9851	0.999	-0.9786	0.9655	0.9808
$\lambda^{\text{FIRM}^{\text{S}^{\text{lag}^1}}}$	0.9189	-0.3123	-0.9813	0.9538	0.9813	-0.9461	0.9334	0.9536
$\lambda^{\text{FIRM}^{\text{S}^{\text{lag}^2}}}$	0.9415	-0.3346	-0.9959	0.9746	0.9959	-0.9675	0.9548	0.9726
$\pi$	-0.9228	0.4079	0.9932	-0.9663	-0.9932	0.9566	-0.9386	-0.9601
$C$	1	-0.0689	-0.9613	0.989	0.9613	-0.994	0.9991	0.9937
$K$	-0.0689	1	0.3125	-0.2068	-0.3125	0.171	-0.1075	-0.1669
$L$	-0.9613	0.3125	1	-0.9891	-1	0.9836	-0.9723	-0.9861
$\Lambda$	0.989	-0.2068	-0.9891	1	0.9891	-0.9992	0.9941	0.9983
$N$	0.9613	-0.3125	-1	0.9891	1	-0.9836	0.9723	0.9861
$\Pi$	0.5469	0.7454	-0.3036	0.4173	0.3036	-0.4523	0.5115	0.4537
$S$	0.6373	-0.1735	-0.7313	0.6684	0.7313	-0.66	0.6554	0.684
$S^{\text{lag}^1}$	0.5043	-0.2471	-0.5156	0.5098	0.5156	-0.5088	0.5086	0.5115
$S^{\text{lag}^2}$	0.3796	-0.2113	-0.316	0.3713	0.316	-0.3758	0.3715	0.3551
$U$	-0.994	0.171	0.9836	-0.9992	-0.9836	1	-0.9974	-0.9991
$W$	0.9991	-0.1075	-0.9723	0.9941	0.9723	-0.9974	1	0.9976
$Y$	0.9937	-0.1669	-0.9861	0.9983	0.9861	-0.9991	0.9976	1
$Z$	0.8794	0.2007	-0.766	0.8151	0.766	-0.833	0.8661	0.8414

### 10.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
$C$	0.7305	0.4987	0.3069	0.1534	0.0236
$K$	0.8674	0.72	0.6137	0.5297	0.3681
$L$	0.6809	0.4258	0.2416	0.1333	-0.0011
$\Lambda$	0.7212	0.4838	0.2859	0.1249	-0.0024
$N$	0.6809	0.4258	0.2416	0.1333	-0.0011
$U$	0.7238	0.4879	0.2907	0.1298	0.0022
$W$	0.721	0.4848	0.2936	0.1466	0.0162
$Y$	0.7051	0.4634	0.2752	0.1393	0.0069

## 11 Statistics of the model

### 11.1 Moments relative to moments of the reference variable

	Steady-state value relative to $Y$	Std. dev. relative to $Y$	Variance relative to $Y$	Loglinear
$C$	0.75	0.6104	0.3726	Y
$K$	10.0002	0.197	0.0388	Y
$L$	0.6326	0.1129	0.0127	Y
$\Lambda$	0.9079	0.8672	0.752	Y
$N$	0.2753	0.2594	0.0673	Y
$U$	-122.969	0.2441	0.0596	Y
$W$	2.0894	0.7196	0.5178	Y
$Y$	1	1	1	Y

### 11.2 Correlations with the reference variable

	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$C$	-0.0457	0.0881	0.2302	0.427	0.681	0.9937	0.7438	0.5265	0.3448	0.1984	0.0719
$K$	-0.4647	-0.472	-0.4503	-0.3907	-0.2912	-0.1669	0.018	0.2779	0.6192	0.6097	0.5685
$L$	-0.0857	-0.213	-0.3367	-0.5075	-0.7249	-0.9861	-0.6361	-0.3607	-0.1629	-0.0451	0.0902
$\Lambda$	0.0291	0.1602	0.295	0.4793	0.7138	0.9983	0.7105	0.4668	0.265	0.1019	-0.0259
$N$	0.0857	0.213	0.3367	0.5075	0.7249	0.9861	0.6361	0.3607	0.1629	0.0451	-0.0902
$U$	-0.0096	-0.1417	-0.2788	-0.4668	-0.7068	-0.9991	-0.7218	-0.4853	-0.2877	-0.1267	0.0009
$W$	-0.0253	0.1081	0.2481	0.4418	0.6914	0.9976	0.7308	0.5032	0.3181	0.1753	0.0468
$Y$	0.0069	0.1393	0.2752	0.4634	0.7051	1	0.7051	0.4634	0.2752	0.1393	0.0069

## 12 Impulse response functions

### 12.1 Shock $\epsilon^\Lambda$

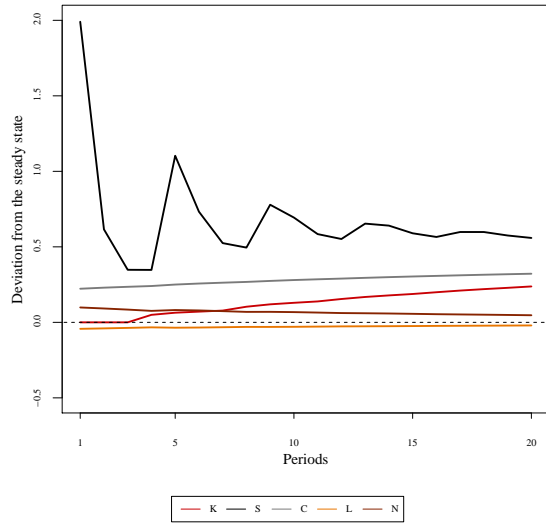


Figure 1: Impulse response function for  $\epsilon^\Lambda$  shock

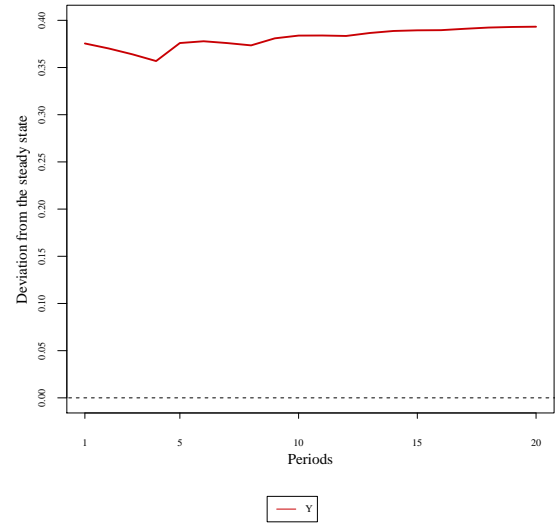


Figure 2: Impulse response function for  $\epsilon^\Lambda$  shock