

Index sets

$$IND = \{H, M\}$$

1 CONSUMER

1.1 Optimisation problem

$$\max_{\left(K_t^{(i)}\right)_{i \in IND}, \left(C_t^{(i)}\right)_{i \in IND}, \left(N_t^{(i)}\right)_{i \in IND}, \left(I_t^{(i)}\right)_{i \in IND}} U_t = \beta E_t [U_{t+1}] + \log \left(1 - \sum_{i \in IND} N_t^{(i)} \right) (1 - b) + b e^{-1} \log \left(a C_t^{(M)e} + (1 - a) C_t^{(H)e} \right) \quad (1.1)$$

s.t. :

$$C_t^{(M)} + \sum_{i \in IND} I_t^{(i)} = \pi_t + r_t K_{t-1}^{(M)} + W_t N_t^{(M)} \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$i \in IND: \quad K_t^{(i)} = I_t^{(i)} + K_{t-1}^{(i)} (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2 (i)} \right) \quad (1.3)$$

$$C_t^{(H)} = \Gamma Z_t^{(H)} K_{t-1}^{(H)\theta} N_t^{(H)1-\theta} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \quad (1.4)$$

1.2 Identities

$$K_t = \sum_{i \in IND} K_t^{(i)} \quad (1.5)$$

$$I_t = \sum_{i \in IND} I_t^{(i)} \quad (1.6)$$

$$N_t = \sum_{i \in IND} N_t^{(i)} \quad (1.7)$$

1.3 First order conditions

$$i \in IND: \quad -\lambda^{\text{CONSUMER}^2 (i)}_t + \beta \left(\delta^{(M,i)} E_t \left[\lambda^{\text{CONSUMER}^1}_{t+1} r_{t+1} \right] + (1 - \delta) E_t \left[\lambda^{\text{CONSUMER}^2 (i)}_{t+1} \right] + \delta^{(H,i)} \theta \Gamma K_t^{(H)-1+\theta} E_t \left[\lambda^{\text{CONSUMER}^3}_{t+1} Z_{t+1}^{(H)} N_{t+1}^{(H)1-\theta} \right] \right) = 0 \quad \left(K_t^{(i)} \right) \quad (1.8)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_t^{\text{CONSUMER}^3} + b e^{-1} \left(a C_t^{(M)e} + (1 - a) C_t^{(H)e} \right)^{-1} \left(\delta^{(M,i)} a e C_t^{(M)-1+e} + \delta^{(H,i)} e (1 - a) C_t^{(H)-1+e} \right) = 0 \quad \left(C_t^{(i)} \right) \quad (1.9)$$

$$i \in IND: \quad - (1-b) \left(1 - \sum_{i \in IND} N_t^{(i)} \right)^{-1} + \delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} W_t + \delta^{(H,i)} \Gamma \lambda_t^{\text{CONSUMER}^3} Z_t^{(H)} (1-\theta) K_{t-1}^{(H)\theta} N_t^{(H)-\theta} = 0 \quad \left(N_t^{(i)} \right) \quad (1.10)$$

$$i \in IND: \quad - \lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} I_t^{(i)} = 0 \quad \left(I_t^{(i)} \right) \quad (1.11)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\text{m}^d}, N_t^{\text{m}^d}, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$\pi_t = Y_t - N_t^{\text{m}^d} W_t - r_t K_t^{\text{m}^d} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$Y_t = \Gamma Z_t^{(M)} K_t^{\text{m}^d \alpha} N_t^{\text{m}^d 1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^1} r_t + \alpha \Gamma \lambda_t^{\text{FIRM}^2} Z_t^{(M)} K_t^{\text{m}^d - 1 + \alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad \left(K_t^{\text{m}^d} \right) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^1} W_t + \Gamma \lambda_t^{\text{FIRM}^2} Z_t^{(M)} (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d - \alpha} = 0 \quad \left(N_t^{\text{m}^d} \right) \quad (2.5)$$

$$\lambda_t^{\text{FIRM}^1} - \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (\pi_t) \quad (2.7)$$

2.3 First order conditions after reduction

$$-r_t + \alpha \Gamma Z_t^{(M)} K_t^{\text{m}^d - 1 + \alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad \left(K_t^{\text{m}^d} \right) \quad (2.8)$$

$$-W_t + \Gamma Z_t^{(M)} (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d - \alpha} = 0 \quad \left(N_t^{\text{m}^d} \right) \quad (2.9)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{m}^d} = K_{t-1}^{(M)} \quad (3.1)$$

$$N_t^{\text{m}^d} = N_t^{(M)} \quad (3.2)$$

4 EXOG

4.1 Identities

$$i \in IND: \quad Z_t^{(i)} = e^{\epsilon_t^{(i)} + \psi^{(i)} \log Z_{t-1}^{(i)}} \quad (4.1)$$

5 Equilibrium relationships (before expansion and reduction)

$$-\pi_t + \Pi_t = 0 \quad (5.1)$$

$$-r_t + \alpha \Gamma Z_t^{(M)} K_t^{m^d-1+\alpha} N_t^{m^d 1-\alpha} = 0 \quad (5.2)$$

$$I_t - \sum_{i \in IND} I_t^{(i)} = 0 \quad (5.3)$$

$$K_t - \sum_{i \in IND} K_t^{(i)} = 0 \quad (5.4)$$

$$K_t^{m^d} - K_{t-1}^{(M)} = 0 \quad (5.5)$$

$$N_t - \sum_{i \in IND} N_t^{(i)} = 0 \quad (5.6)$$

$$N_t^{m^d} - N_t^{(M)} = 0 \quad (5.7)$$

$$-W_t + \Gamma Z_t^{(M)} (1 - \alpha) K_t^{m^d \alpha} N_t^{m^d 1-\alpha} = 0 \quad (5.8)$$

$$-Y_t + \Gamma Z_t^{(M)} K_t^{m^d \alpha} N_t^{m^d 1-\alpha} = 0 \quad (5.9)$$

$$-C_t^{(H)} + \Gamma Z_t^{(H)} K_{t-1}^{(H) \theta} N_t^{(H) 1-\theta} = 0 \quad (5.10)$$

$$-\pi_t + Y_t - r_t K_t^{m^d} - N_t^{m^d} W_t = 0 \quad (5.11)$$

$$U_t - \beta E_t [U_{t+1}] - \log \left(1 - \sum_{i \in IND} N_t^{(i)} \right) (1 - b) - b e^{-1} \log \left(a C_t^{(M) e} + (1 - a) C_t^{(H) e} \right) = 0 \quad (5.12)$$

$$\pi_t - C_t^{(M)} + r_t K_{t-1}^{(M)} + W_t N_t^{(M)} - \sum_{i \in IND} I_t^{(i)} = 0 \quad (5.13)$$

$$i \in IND: \quad -\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2 (i)} = 0 \quad (5.14)$$

$$i \in IND: \quad -\lambda_t^{\text{CONSUMER}^2 (i)} + \beta \left(\delta^{(M, i)} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] + (1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2 (i)} \right] + \delta^{(H, i)} \theta \Gamma K_t^{(H) -1 + \theta} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} Z_{t+1}^{(H)} N_{t+1}^{(H) 1-\theta} \right] \right) = 0 \quad (5.15)$$

$$i \in IND: \quad Z_t^{(i)} - e^{\epsilon_t^{(i)} + \psi^{(i)} \log Z_{t-1}^{(i)}} = 0 \quad (5.16)$$

$$i \in IND: \quad I_t^{(i)} - K_t^{(i)} + K_{t-1}^{(i)} (1 - \delta) = 0 \quad (5.17)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_t^{\text{CONSUMER}^3} + b e^{-1} \left(a C_t^{(M)^e} + (1-a) C_t^{(H)^e} \right)^{-1} \left(\delta^{(M,i)} a e C_t^{(M)^{-1+e}} + \delta^{(H,i)} e (1-a) C_t^{(H)^{-1+e}} \right) = 0 \quad (5.18)$$

$$i \in IND: \quad -(1-b) \left(1 - \sum_{i \in IND} N_t^{(i)} \right)^{-1} + \delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} W_t + \delta^{(H,i)} \Gamma \lambda_t^{\text{CONSUMER}^3} Z_t^{(H)} (1-\theta) K_{t-1}^{(H)\theta} N_t^{(H)^{-\theta}} = 0 \quad (5.19)$$

6 Equilibrium relationships (after expansion and reduction)

$$-r_t + \alpha \Gamma Z_t^{(M)} K_{t-1}^{(M)^{-1+\alpha}} N_t^{(M)^{1-\alpha}} = 0 \quad (6.1)$$

$$-W_t + \Gamma Z_t^{(M)} (1-\alpha) K_{t-1}^{(M)\alpha} N_t^{(M)^{-\alpha}} = 0 \quad (6.2)$$

$$-Y_t + \Gamma Z_t^{(M)} K_{t-1}^{(M)\alpha} N_t^{(M)^{1-\alpha}} = 0 \quad (6.3)$$

$$-C_t^{(H)} + \Gamma Z_t^{(H)} N_t^{(H)^{1-\theta}} K_{t-1}^{(H)\theta} = 0 \quad (6.4)$$

$$Z_t^{(H)} - e^{\epsilon_t^{(H)} + \psi^{(H)} \log Z_{t-1}^{(H)}} = 0 \quad (6.5)$$

$$Z_t^{(M)} - e^{\epsilon_t^{(M)} + \psi^{(M)} \log Z_{t-1}^{(M)}} = 0 \quad (6.6)$$

$$\beta \left(ab E_t \left[r_{t+1} \left(a C_{t+1}^{(M)^e} + (1-a) C_{t+1}^{(H)^e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] + ab (1-\delta) E_t \left[\left(a C_{t+1}^{(M)^e} + (1-a) C_{t+1}^{(H)^e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] \right) - ab \left(a C_t^{(M)^e} + (1-a) C_t^{(H)^e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.7)$$

$$\beta \left(ab (1-\delta) E_t \left[\left(a C_{t+1}^{(M)^e} + (1-a) C_{t+1}^{(H)^e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] + b \theta \Gamma (1-a) K_t^{(H)^{-1+\theta}} E_t \left[Z_{t+1}^{(H)} \left(a C_{t+1}^{(M)^e} + (1-a) C_{t+1}^{(H)^e} \right)^{-1} C_{t+1}^{(H)^{-1+e}} N_{t+1}^{(H)^{1-\theta}} \right] \right) - ab \left(a C_t^{(M)^e} + (1-a) C_t^{(H)^e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.8)$$

$$-(1-b) \left(1 - N_t^{(H)} - N_t^{(M)} \right)^{-1} + ab W_t \left(a C_t^{(M)^e} + (1-a) C_t^{(H)^e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.9)$$

$$-(1-b) \left(1 - N_t^{(H)} - N_t^{(M)} \right)^{-1} + b \Gamma Z_t^{(H)} (1-a) (1-\theta) \left(a C_t^{(M)^e} + (1-a) C_t^{(H)^e} \right)^{-1} K_{t-1}^{(H)\theta} C_t^{(H)^{-1+e}} N_t^{(H)^{-\theta}} = 0 \quad (6.10)$$

$$I_t - I_t^{(H)} - I_t^{(M)} = 0 \quad (6.11)$$

$$K_t - K_t^{(H)} - K_t^{(M)} = 0 \quad (6.12)$$

$$N_t - N_t^{(H)} - N_t^{(M)} = 0 \quad (6.13)$$

$$I_t^{(H)} - K_t^{(H)} + K_{t-1}^{(H)} (1-\delta) = 0 \quad (6.14)$$

$$I_t^{(M)} - K_t^{(M)} + K_{t-1}^{(M)} (1 - \delta) = 0 \quad (6.15)$$

$$U_t - \beta E_t [U_{t+1}] - \log \left(1 - N_t^{(H)} - N_t^{(M)} \right) (1 - b) - be^{-1} \log \left(a C_t^{(M)^e} + (1 - a) C_t^{(H)^e} \right) = 0 \quad (6.16)$$

$$Y_t - C_t^{(M)} - I_t^{(H)} - I_t^{(M)} = 0 \quad (6.17)$$

7 Steady state relationships (before expansion and reduction)

$$-\pi_{ss} + \Pi_{ss} = 0 \quad (7.1)$$

$$-r_{ss} + \alpha \Gamma Z_{ss}^{(M)} K_{ss}^{m^d - 1 + \alpha} N_{ss}^{m^d 1 - \alpha} = 0 \quad (7.2)$$

$$I_{ss} - \sum_{i \in IND} I_{ss}^{(i)} = 0 \quad (7.3)$$

$$K_{ss} - \sum_{i \in IND} K_{ss}^{(i)} = 0 \quad (7.4)$$

$$K_{ss}^{m^d} - K_{ss}^{(M)} = 0 \quad (7.5)$$

$$N_{ss} - \sum_{i \in IND} N_{ss}^{(i)} = 0 \quad (7.6)$$

$$N_{ss}^{m^d} - N_{ss}^{(M)} = 0 \quad (7.7)$$

$$-W_{ss} + \Gamma Z_{ss}^{(M)} (1 - \alpha) K_{ss}^{m^d \alpha} N_{ss}^{m^d - \alpha} = 0 \quad (7.8)$$

$$-Y_{ss} + \Gamma Z_{ss}^{(M)} K_{ss}^{m^d \alpha} N_{ss}^{m^d 1 - \alpha} = 0 \quad (7.9)$$

$$-C_{ss}^{(H)} + \Gamma Z_{ss}^{(H)} K_{ss}^{(H) \theta} N_{ss}^{(H) 1 - \theta} = 0 \quad (7.10)$$

$$-\pi_{ss} + Y_{ss} - r_{ss} K_{ss}^{m^d} - N_{ss}^{m^d} W_{ss} = 0 \quad (7.11)$$

$$U_{ss} - \beta U_{ss} - \log \left(1 - \sum_{i \in IND} N_{ss}^{(i)} \right) (1 - b) - be^{-1} \log \left(a C_{ss}^{(M)^e} + (1 - a) C_{ss}^{(H)^e} \right) = 0 \quad (7.12)$$

$$\pi_{ss} - C_{ss}^{(M)} + r_{ss} K_{ss}^{(M)} + W_{ss} N_{ss}^{(M)} - \sum_{i \in IND} I_{ss}^{(i)} = 0 \quad (7.13)$$

$$i \in IND: \quad -\lambda_{ss}^{\text{CONSUMER}^1} + \lambda_{ss}^{\text{CONSUMER}^2 (i)} = 0 \quad (7.14)$$

$$i \in IND: \quad -\lambda_{ss}^{\text{CONSUMER}^2 (i)} + \beta \left(\lambda_{ss}^{\text{CONSUMER}^2 (i)} (1 - \delta) + \delta^{(M, i)} \lambda_{ss}^{\text{CONSUMER}^1} r_{ss} + \delta^{(H, i)} \theta \Gamma \lambda_{ss}^{\text{CONSUMER}^3} Z_{ss}^{(H)} K_{ss}^{(H) - 1 + \theta} N_{ss}^{(H) 1 - \theta} \right) = 0 \quad (7.15)$$

$$i \in IND: \quad Z_{ss}^{(i)} - e^{\epsilon_{ss}^{(i)} + \psi^{(i)} \log Z_{ss}^{(i)}} = 0 \quad (7.16)$$

$$i \in IND: \quad I_{ss}^{(i)} - K_{ss}^{(i)} + K_{ss}^{(i)} (1 - \delta) = 0 \quad (7.17)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_{ss}^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_{ss}^{\text{CONSUMER}^3} + be^{-1} \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} \left(\delta^{(M,i)} aeC_{ss}^{(M)-1+e} + \delta^{(H,i)} e(1-a) C_{ss}^{(H)-1+e} \right) = 0 \quad (7.18)$$

$$i \in IND: \quad -(1-b) \left(1 - \sum_{i \in IND} N_{ss}^{(i)} \right)^{-1} + \delta^{(M,i)} \lambda_{ss}^{\text{CONSUMER}^1} W_{ss} + \delta^{(H,i)} \Gamma \lambda_{ss}^{\text{CONSUMER}^3} Z_{ss}^{(H)} (1-\theta) K_{ss}^{(H)\theta} N_{ss}^{(H)-\theta} = 0 \quad (7.19)$$

8 Steady state relationships (after expansion and reduction)

$$-r_{ss} + \alpha \Gamma Z_{ss}^{(M)} K_{ss}^{(M)-1+\alpha} N_{ss}^{(M)1-\alpha} = 0 \quad (8.1)$$

$$-W_{ss} + \Gamma Z_{ss}^{(M)} (1-\alpha) K_{ss}^{(M)\alpha} N_{ss}^{(M)-\alpha} = 0 \quad (8.2)$$

$$-Y_{ss} + \Gamma Z_{ss}^{(M)} K_{ss}^{(M)\alpha} N_{ss}^{(M)1-\alpha} = 0 \quad (8.3)$$

$$-C_{ss}^{(H)} + \Gamma Z_{ss}^{(H)} K_{ss}^{(H)\theta} N_{ss}^{(H)1-\theta} = 0 \quad (8.4)$$

$$Z_{ss}^{(H)} - e^{\psi^{(H)} \log Z_{ss}^{(H)}} = 0 \quad (8.5)$$

$$Z_{ss}^{(M)} - e^{\psi^{(M)} \log Z_{ss}^{(M)}} = 0 \quad (8.6)$$

$$\beta \left(abr_{ss} \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} + ab(1-\delta) \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} \right) - ab \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.7)$$

$$\beta \left(ab(1-\delta) \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} + b\theta \Gamma Z_{ss}^{(H)} (1-a) \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(H)-1+e} K_{ss}^{(H)-1+\theta} N_{ss}^{(H)1-\theta} \right) - ab \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.8)$$

$$-(1-b) \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right)^{-1} + abW_{ss} \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.9)$$

$$-(1-b) \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right)^{-1} + b\Gamma Z_{ss}^{(H)} (1-a) (1-\theta) \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(H)-1+e} K_{ss}^{(H)\theta} N_{ss}^{(H)-\theta} = 0 \quad (8.10)$$

$$I_{ss} - I_{ss}^{(H)} - I_{ss}^{(M)} = 0 \quad (8.11)$$

$$K_{ss} - K_{ss}^{(H)} - K_{ss}^{(M)} = 0 \quad (8.12)$$

$$N_{ss} - N_{ss}^{(H)} - N_{ss}^{(M)} = 0 \quad (8.13)$$

$$I_{ss}^{(H)} - K_{ss}^{(H)} + K_{ss}^{(H)} (1-\delta) = 0 \quad (8.14)$$

$$I_{ss}^{(M)} - K_{ss}^{(M)} + K_{ss}^{(M)} (1-\delta) = 0 \quad (8.15)$$

$$U_{ss} - \beta U_{ss} - \log \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right) (1-b) - be^{-1} \log \left(aC_{ss}^{(M)e} + (1-a) C_{ss}^{(H)e} \right) = 0 \quad (8.16)$$

$$Y_{ss} - C_{ss}^{(M)} - I_{ss}^{(H)} - I_{ss}^{(M)} = 0 \quad (8.17)$$

9 Parameter settings

$$a = 0.337 \tag{9.1}$$

$$\alpha = 0.36 \tag{9.2}$$

$$b = 0.63 \tag{9.3}$$

$$\beta = 0.99 \tag{9.4}$$

$$\delta = 0.025 \tag{9.5}$$

$$e = 0.8 \tag{9.6}$$

$$\theta = 0.08 \tag{9.7}$$

$$\Gamma = 1 \tag{9.8}$$

$$\psi^{(\text{H})} = 0.95 \tag{9.9}$$

$$\psi^{(\text{M})} = 0.95 \tag{9.10}$$