

## Index sets

$$COUNTRY = \{F, H\}$$

### 1 CONSUMER $c \in COUNTRY$

#### 1.1 Optimisation problem

$$\max_{K_t^{(c)}, C_t^{(c)}, H_t^{(c)}, I_t^{(c)}} U_t^{(c)} = \beta E_t \left[ U_{t+1}^{(c)} \right] + (1 - \eta)^{-1} \left( C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t^{(c)} + I_t^{(c)} + T_t^{(c)} = \pi_t^{(c)} + TR_t^{(c)} + K_{t-1}^{(c)} r_t^{(c)} + H_t^{(c)} W_t^{(c)} - \psi^{(c)} K_{t-1}^{(c)} \left( -\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)} \right)^2 \quad \left( \lambda_t^{c(c)} \right) \quad (1.2)$$

$$K_t^{(c)} = I_t^{(c)} + K_{t-1}^{(c)} \left( 1 - \delta^{(c)} \right) \quad \left( \lambda^{\text{CONSUMER}^2(c)}_t \right) \quad (1.3)$$

#### 1.2 First order conditions

$$-\lambda^{\text{CONSUMER}^2(c)}_t + \beta \left( \left( 1 - \delta^{(c)} \right) E_t \left[ \lambda^{\text{CONSUMER}^2(c)}_{t+1} \right] + E_t \left[ \lambda^{c(c)}_{t+1} \left( r_{t+1}^{(c)} - \psi^{(c)} \left( -\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right)^2 + 2\psi^{(c)} K_t^{(c)-1} I_{t+1}^{(c)} \left( -\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right) \right) \right] \right) = 0 \quad \left( K_t^{(c)} \right) \quad (1.4)$$

$$-\lambda^{c(c)}_t + \mu C_t^{(c)-1+\mu} (1 - H_t^{(c)})^{1-\mu} \left( C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{-\eta} = 0 \quad \left( C_t^{(c)} \right) \quad (1.5)$$

$$\lambda^{c(c)}_t W_t^{(c)} + (-1 + \mu) C_t^{(c)\mu} (1 - H_t^{(c)})^{-\mu} \left( C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{-\eta} = 0 \quad \left( H_t^{(c)} \right) \quad (1.6)$$

$$\lambda^{\text{CONSUMER}^2(c)}_t + \lambda^{c(c)}_t \left( -1 - 2\psi^{(c)} \left( -\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)} \right) \right) = 0 \quad \left( I_t^{(c)} \right) \quad (1.7)$$

## 2 FIRM $c \in COUNTRY$

### 2.1 Optimisation problem

$$\max_{K_t^{d(c)}, H_t^{d(c)}, Y_t^{(c)}, \pi_t^{(c)}} \Pi_t^{(c)} = \pi_t^{(c)} \quad (2.1)$$

s.t. :

$$Y_t^{(c)} = Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} \left( \lambda^{\text{FIRM}^1(c)} \right) \quad (2.2)$$

$$\pi_t^{(c)} = Y_t^{(c)} - H_t^{d(c)} W_t^{(c)} - r_t^{(c)} K_t^{d(c)} \left( \lambda^{\text{FIRM}^2(c)} \right) \quad (2.3)$$

### 2.2 First order conditions

$$-\lambda^{\text{FIRM}^2(c)} r_t^{(c)} + \alpha^{(c)} \lambda^{\text{FIRM}^1(c)} Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)-1+\alpha^{(c)}} = 0 \quad \left( K_t^{d(c)} \right) \quad (2.4)$$

$$-\lambda^{\text{FIRM}^2(c)} W_t^{(c)} + \lambda^{\text{FIRM}^1(c)} Z_t^{(c)} \left( 1 - \alpha^{(c)} \right) H_t^{d(c)-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad \left( H_t^{d(c)} \right) \quad (2.5)$$

$$-\lambda^{\text{FIRM}^1(c)} + \lambda^{\text{FIRM}^2(c)} = 0 \quad \left( Y_t^{(c)} \right) \quad (2.6)$$

$$1 - \lambda^{\text{FIRM}^2(c)} = 0 \quad \left( \pi_t^{(c)} \right) \quad (2.7)$$

### 2.3 First order conditions after reduction

$$-r_t^{(c)} + \alpha^{(c)} Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)-1+\alpha^{(c)}} = 0 \quad \left( K_t^{d(c)} \right) \quad (2.8)$$

$$-W_t^{(c)} + Z_t^{(c)} \left( 1 - \alpha^{(c)} \right) H_t^{d(c)-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad \left( H_t^{d(c)} \right) \quad (2.9)$$

## 3 EQUILIBRIUM

### 3.1 Identities

$$\sum_{c \in COUNTRY} TR_t^{(c)} = 0 \quad (3.1)$$

$$c \in COUNTRY: \quad K_t^{d(c)} = K_{t-1}^{(c)} \quad (3.2)$$

$$c \in COUNTRY: \quad H_t^{d\langle c \rangle} = H_t^{c\langle c \rangle} \quad (3.3)$$

$$c \in COUNTRY: \quad T_t^{c\langle c \rangle} = G_t^{d\langle c \rangle} \quad (3.4)$$

$$\lambda_t^{c\langle H \rangle} = \lambda_t^{c\langle F \rangle} \quad (3.5)$$

## 4 EXOG

### 4.1 Identities

$$c \in COUNTRY: \quad G_t^{d\langle c \rangle} = \epsilon_t^{G\langle c \rangle} + \phi^{G\langle c \rangle} G_{t-1}^{d\langle c \rangle} \quad (4.1)$$

$$c \in COUNTRY: \quad Z_t^{c\langle c \rangle} = e^{\epsilon_t^{Z\langle c \rangle} + \phi^{Z\langle c \rangle} \log Z_{t-1}^{c\langle c \rangle}} \quad (4.2)$$

## 5 Equilibrium relationships (before expansion and reduction)

$$-\lambda_t^{c\langle F \rangle} + \lambda_t^{c\langle H \rangle} = 0 \quad (5.1)$$

$$\sum_{c \in COUNTRY} TR_t^{c\langle c \rangle} = 0 \quad (5.2)$$

$$c \in COUNTRY: \quad -K_{t-1}^{c\langle c \rangle} + K_t^{d\langle c \rangle} = 0 \quad (5.3)$$

$$c \in COUNTRY: \quad -\lambda^{\text{CONSUMER}^2\langle c \rangle}_t + \beta \left( \left( 1 - \delta^{c\langle c \rangle} \right) E_t \left[ \lambda^{\text{CONSUMER}^2\langle c \rangle}_{t+1} \right] + E_t \left[ \lambda_{t+1}^{c\langle c \rangle} \left( r_{t+1}^{c\langle c \rangle} - \psi^{c\langle c \rangle} \left( -\delta^{c\langle c \rangle} + K_t^{c\langle c \rangle -1} I_{t+1}^{c\langle c \rangle} \right)^2 + 2\psi^{c\langle c \rangle} K_t^{c\langle c \rangle -1} I_{t+1}^{c\langle c \rangle} \left( -\delta^{c\langle c \rangle} + K_t^{c\langle c \rangle -1} I_{t+1}^{c\langle c \rangle} \right) \right] \right) \right] = 0 \quad (5.4)$$

$$c \in COUNTRY: \quad \lambda^{\text{CONSUMER}^2\langle c \rangle}_t + \lambda_t^{c\langle c \rangle} \left( -1 - 2\psi^{c\langle c \rangle} \left( -\delta^{c\langle c \rangle} + K_{t-1}^{c\langle c \rangle -1} I_t^{c\langle c \rangle} \right) \right) = 0 \quad (5.5)$$

$$c \in COUNTRY: \quad -\lambda_t^{c\langle c \rangle} + \mu C_t^{c\langle c \rangle -1+\mu} \left( 1 - H_t^{c\langle c \rangle} \right)^{1-\mu} \left( C_t^{c\langle c \rangle \mu} \left( 1 - H_t^{c\langle c \rangle} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$c \in COUNTRY: \quad -\pi_t^{c\langle c \rangle} + \Pi_t^{c\langle c \rangle} = 0 \quad (5.7)$$

$$c \in COUNTRY: \quad -r_t^{c\langle c \rangle} + \alpha^{c\langle c \rangle} Z_t^{c\langle c \rangle} H_t^{d\langle c \rangle} K_t^{d\langle c \rangle -1+\alpha^{c\langle c \rangle}} = 0 \quad (5.8)$$

$$c \in COUNTRY: \quad -G_t^{d\langle c \rangle} + T_t^{c\langle c \rangle} = 0 \quad (5.9)$$

$$c \in COUNTRY: \quad -H_t^{c\langle c \rangle} + H_t^{d\langle c \rangle} = 0 \quad (5.10)$$

$$c \in COUNTRY: \quad -W_t^{c\langle c \rangle} + Z_t^{c\langle c \rangle} \left( 1 - \alpha^{c\langle c \rangle} \right) H_t^{d\langle c \rangle -\alpha^{c\langle c \rangle}} K_t^{d\langle c \rangle \alpha^{c\langle c \rangle}} = 0 \quad (5.11)$$

$$c \in COUNTRY: \quad -Y_t^{(c)} + Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad (5.12)$$

$$c \in COUNTRY: \quad Z_t^{(c)} - e^{\epsilon^{Z_t^{(c)}} + \phi^{Z_t^{(c)}} \log Z_{t-1}^{(c)}} = 0 \quad (5.13)$$

$$c \in COUNTRY: \quad \lambda_t^{c(c)} W_t^{(c)} + (-1 + \mu) C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{-\mu} \left(C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (5.14)$$

$$c \in COUNTRY: \quad -\epsilon_t^{G^{(c)}} + G_t^{d(c)} - \phi^{G^{(c)}} G_{t-1}^{d(c)} = 0 \quad (5.15)$$

$$c \in COUNTRY: \quad I_t^{(c)} - K_t^{(c)} + K_{t-1}^{(c)} \left(1 - \delta^{(c)}\right) = 0 \quad (5.16)$$

$$c \in COUNTRY: \quad U_t^{(c)} - \beta E_t \left[U_{t+1}^{(c)}\right] - (1 - \eta)^{-1} \left(C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{1-\mu}\right)^{1-\eta} = 0 \quad (5.17)$$

$$c \in COUNTRY: \quad -\pi_t^{(c)} + Y_t^{(c)} - r_t^{(c)} K_t^{d(c)} - H_t^{d(c)} W_t^{(c)} = 0 \quad (5.18)$$

$$c \in COUNTRY: \quad \pi_t^{(c)} - C_t^{(c)} - I_t^{(c)} - T_t^{(c)} + TR_t^{(c)} + K_{t-1}^{(c)} r_t^{(c)} + H_t^{(c)} W_t^{(c)} - \psi^{(c)} K_{t-1}^{(c)} \left(-\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)}\right)^2 = 0 \quad (5.19)$$

## 6 Equilibrium relationships (after expansion and reduction)

$$-\lambda_t^{c(F)} + \lambda_t^{c(H)} = 0 \quad (6.1)$$

$$-\lambda_t^{c(F)} + \mu C_t^{(F)-1+\mu} \left(1 - H_t^{(F)}\right)^{1-\mu} \left(C_t^{(F)\mu} \left(1 - H_t^{(F)}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (6.2)$$

$$-\lambda_t^{c(H)} + \mu C_t^{(H)-1+\mu} \left(1 - H_t^{(H)}\right)^{1-\mu} \left(C_t^{(H)\mu} \left(1 - H_t^{(H)}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (6.3)$$

$$-r_t^{(F)} + \alpha^{(F)} Z_t^{(F)} K_{t-1}^{(F)-1+\alpha^{(F)}} H_t^{(F)1-\alpha^{(F)}} = 0 \quad (6.4)$$

$$-r_t^{(H)} + \alpha^{(H)} Z_t^{(H)} K_{t-1}^{(H)-1+\alpha^{(H)}} H_t^{(H)1-\alpha^{(H)}} = 0 \quad (6.5)$$

$$-W_t^{(F)} + Z_t^{(F)} \left(1 - \alpha^{(F)}\right) K_{t-1}^{(F)\alpha^{(F)}} H_t^{(F)-\alpha^{(F)}} = 0 \quad (6.6)$$

$$-W_t^{(H)} + Z_t^{(H)} \left(1 - \alpha^{(H)}\right) K_{t-1}^{(H)\alpha^{(H)}} H_t^{(H)-\alpha^{(H)}} = 0 \quad (6.7)$$

$$-Y_t^{(F)} + Z_t^{(F)} K_{t-1}^{(F)\alpha^{(F)}} H_t^{(F)1-\alpha^{(F)}} = 0 \quad (6.8)$$

$$-Y_t^{(H)} + Z_t^{(H)} K_{t-1}^{(H)\alpha^{(H)}} H_t^{(H)1-\alpha^{(H)}} = 0 \quad (6.9)$$

$$Z_t^{(F)} - e^{\epsilon Z_t^{(F)} + \phi Z^{(F)} \log Z_{t-1}^{(F)}} = 0 \quad (6.10)$$

$$Z_t^{(H)} - e^{\epsilon Z_t^{(H)} + \phi Z^{(H)} \log Z_{t-1}^{(H)}} = 0 \quad (6.11)$$

$$\beta \left( - \left( 1 - \delta^{(F)} \right) \text{E}_t \left[ \lambda_{t+1}^{c(F)} \left( -1 - 2\psi^{(F)} \left( -\delta^{(F)} + K_t^{(F)-1} I_{t+1}^{(F)} \right) \right) \right] + \text{E}_t \left[ \lambda_{t+1}^{c(F)} \left( r_{t+1}^{(F)} - \psi^{(F)} \left( -\delta^{(F)} + K_t^{(F)-1} I_{t+1}^{(F)} \right)^2 + 2\psi^{(F)} K_t^{(F)-1} I_{t+1}^{(F)} \left( -\delta^{(F)} + K_t^{(F)-1} I_{t+1}^{(F)} \right) \right) \right] \right) + \lambda_t^{c(F)} \left( -1 - 2\psi^{(F)} \left( -\delta^{(F)} + K_t^{(F)-1} I_{t+1}^{(F)} \right) \right) \right] \quad (6.12)$$

$$\beta \left( - \left( 1 - \delta^{(H)} \right) \text{E}_t \left[ \lambda_{t+1}^{c(H)} \left( -1 - 2\psi^{(H)} \left( -\delta^{(H)} + K_t^{(H)-1} I_{t+1}^{(H)} \right) \right) \right] + \text{E}_t \left[ \lambda_{t+1}^{c(H)} \left( r_{t+1}^{(H)} - \psi^{(H)} \left( -\delta^{(H)} + K_t^{(H)-1} I_{t+1}^{(H)} \right)^2 + 2\psi^{(H)} K_t^{(H)-1} I_{t+1}^{(H)} \left( -\delta^{(H)} + K_t^{(H)-1} I_{t+1}^{(H)} \right) \right) \right] \right) + \lambda_t^{c(H)} \left( -1 - 2\psi^{(H)} \left( -\delta^{(H)} + K_t^{(H)-1} I_{t+1}^{(H)} \right) \right) \right] \quad (6.13)$$

$$\lambda_t^{c(F)} W_t^{(F)} + (-1 + \mu) C_t^{(F)\mu} \left( 1 - H_t^{(F)} \right)^{-\mu} \left( C_t^{(F)\mu} \left( 1 - H_t^{(F)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (6.14)$$

$$\lambda_t^{c(H)} W_t^{(H)} + (-1 + \mu) C_t^{(H)\mu} \left( 1 - H_t^{(H)} \right)^{-\mu} \left( C_t^{(H)\mu} \left( 1 - H_t^{(H)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (6.15)$$

$$-\epsilon_t^{G(F)} + G_t^{d(F)} - \phi^{G(F)} G_{t-1}^{d(F)} = 0 \quad (6.16)$$

$$-\epsilon_t^{G(H)} + G_t^{d(H)} - \phi^{G(H)} G_{t-1}^{d(H)} = 0 \quad (6.17)$$

$$I_t^{(F)} - K_t^{(F)} + K_{t-1}^{(F)} \left( 1 - \delta^{(F)} \right) = 0 \quad (6.18)$$

$$I_t^{(H)} - K_t^{(H)} + K_{t-1}^{(H)} \left( 1 - \delta^{(H)} \right) = 0 \quad (6.19)$$

$$U_t^{(F)} - \beta \text{E}_t \left[ U_{t+1}^{(F)} \right] - (1 - \eta)^{-1} \left( C_t^{(F)\mu} \left( 1 - H_t^{(F)} \right)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.20)$$

$$U_t^{(H)} - \beta \text{E}_t \left[ U_{t+1}^{(H)} \right] - (1 - \eta)^{-1} \left( C_t^{(H)\mu} \left( 1 - H_t^{(H)} \right)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.21)$$

$$-C_t^{(F)} - G_t^{d(F)} - I_t^{(F)} - TR_t^{(H)} + Y_t^{(F)} - \psi^{(F)} K_{t-1}^{(F)} \left( -\delta^{(F)} + K_{t-1}^{(F)-1} I_t^{(F)} \right)^2 = 0 \quad (6.22)$$

$$-C_t^{(H)} - G_t^{d(H)} - I_t^{(H)} + TR_t^{(H)} + Y_t^{(H)} - \psi^{(H)} K_{t-1}^{(H)} \left( -\delta^{(H)} + K_{t-1}^{(H)-1} I_t^{(H)} \right)^2 = 0 \quad (6.23)$$

## 7 Steady state relationships (before expansion and reduction)

$$-\lambda_{ss}^{c(F)} + \lambda_{ss}^{c(H)} = 0 \quad (7.1)$$

$$\sum_{c \in COUNTRY} TR_{ss}^{(c)} = 0 \quad (7.2)$$

$$c \in COUNTRY: \quad -K_{ss}^{(c)} + K_{ss}^{d(c)} = 0 \quad (7.3)$$

$$c \in COUNTRY: \quad -\lambda_{ss}^{CONSUMER^2(c)} + \beta \left( \lambda_{ss}^{CONSUMER^2(c)} \left( 1 - \delta^{(c)} \right) + \lambda_{ss}^{c(c)} \left( r_{ss}^{(c)} - \psi^{(c)} \left( -\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right)^2 + 2\psi^{(c)} I_{ss}^{(c)} K_{ss}^{(c)-1} \left( -\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right) \right) \right) = 0 \quad (7.4)$$

$$c \in COUNTRY: \quad \lambda_{ss}^{CONSUMER^2(c)} + \lambda_{ss}^{c(c)} \left( -1 - 2\psi^{(c)} \left( -\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right) \right) = 0 \quad (7.5)$$

$$c \in COUNTRY: \quad -\lambda_{ss}^{c(c)} + \mu C_{ss}^{(c)-1+\mu} \left( 1 - H_{ss}^{(c)} \right)^{1-\mu} \left( C_{ss}^{(c)\mu} \left( 1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (7.6)$$

$$c \in COUNTRY: \quad -\pi_{ss}^{(c)} + \Pi_{ss}^{(c)} = 0 \quad (7.7)$$

$$c \in COUNTRY: \quad -r_{ss}^{(c)} + \alpha^{(c)} Z_{ss}^{(c)} H_{ss}^{d(c)1-\alpha^{(c)}} K_{ss}^{d(c)-1+\alpha^{(c)}} = 0 \quad (7.8)$$

$$c \in COUNTRY: \quad -G_{ss}^{d(c)} + T_{ss}^{(c)} = 0 \quad (7.9)$$

$$c \in COUNTRY: \quad -H_{ss}^{(c)} + H_{ss}^{d(c)} = 0 \quad (7.10)$$

$$c \in COUNTRY: \quad -W_{ss}^{(c)} + Z_{ss}^{(c)} \left( 1 - \alpha^{(c)} \right) H_{ss}^{d(c)-\alpha^{(c)}} K_{ss}^{d(c)\alpha^{(c)}} = 0 \quad (7.11)$$

$$c \in COUNTRY: \quad -Y_{ss}^{(c)} + Z_{ss}^{(c)} H_{ss}^{d(c)1-\alpha^{(c)}} K_{ss}^{d(c)\alpha^{(c)}} = 0 \quad (7.12)$$

$$c \in COUNTRY: \quad Z_{ss}^{(c)} - e^{\epsilon Z_{ss}^{(c)} + \phi Z_{ss}^{(c)} \log Z_{ss}^{(c)}} = 0 \quad (7.13)$$

$$c \in COUNTRY: \quad \lambda_{ss}^{c(c)} W_{ss}^{(c)} + (-1 + \mu) C_{ss}^{(c)\mu} \left( 1 - H_{ss}^{(c)} \right)^{-\mu} \left( C_{ss}^{(c)\mu} \left( 1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (7.14)$$

$$c \in COUNTRY: \quad -\epsilon_{ss}^{G(c)} + G_{ss}^{d(c)} - \phi^{G(c)} G_{ss}^{d(c)} = 0 \quad (7.15)$$

$$c \in COUNTRY: \quad I_{ss}^{(c)} - K_{ss}^{(c)} + K_{ss}^{(c)} \left( 1 - \delta^{(c)} \right) = 0 \quad (7.16)$$

$$c \in COUNTRY: \quad U_{ss}^{(c)} - \beta U_{ss}^{(c)} - (1 - \eta)^{-1} \left( C_{ss}^{(c)\mu} \left( 1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.17)$$

$$c \in COUNTRY: \quad -\pi_{ss}^{(c)} + Y_{ss}^{(c)} - r_{ss}^{(c)} K_{ss}^{d(c)} - H_{ss}^{d(c)} W_{ss}^{(c)} = 0 \quad (7.18)$$

$$c \in COUNTRY: \quad \pi_{ss}^{(c)} - C_{ss}^{(c)} - I_{ss}^{(c)} - T_{ss}^{(c)} + TR_{ss}^{(c)} + r_{ss}^{(c)} K_{ss}^{(c)} + H_{ss}^{(c)} W_{ss}^{(c)} - \psi^{(c)} K_{ss}^{(c)} \left( -\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right)^2 = 0 \quad (7.19)$$

## 8 Steady state relationships (after expansion and reduction)

$$-\lambda_{ss}^{c\langle F \rangle} + \lambda_{ss}^{c\langle H \rangle} = 0 \quad (8.1)$$

$$-\lambda_{ss}^{c\langle F \rangle} + \mu C_{ss}^{\langle F \rangle -1+\mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu} \left(C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.2)$$

$$-\lambda_{ss}^{c\langle H \rangle} + \mu C_{ss}^{\langle H \rangle -1+\mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu} \left(C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.3)$$

$$-r_{ss}^{\langle F \rangle} + \alpha^{\langle F \rangle} Z_{ss}^{\langle F \rangle} H_{ss}^{\langle F \rangle 1-\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle -1+\alpha^{\langle F \rangle}} = 0 \quad (8.4)$$

$$-r_{ss}^{\langle H \rangle} + \alpha^{\langle H \rangle} Z_{ss}^{\langle H \rangle} H_{ss}^{\langle H \rangle 1-\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle -1+\alpha^{\langle H \rangle}} = 0 \quad (8.5)$$

$$-W_{ss}^{\langle F \rangle} + Z_{ss}^{\langle F \rangle} \left(1 - \alpha^{\langle F \rangle}\right) H_{ss}^{\langle F \rangle -\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle \alpha^{\langle F \rangle}} = 0 \quad (8.6)$$

$$-W_{ss}^{\langle H \rangle} + Z_{ss}^{\langle H \rangle} \left(1 - \alpha^{\langle H \rangle}\right) H_{ss}^{\langle H \rangle -\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle \alpha^{\langle H \rangle}} = 0 \quad (8.7)$$

$$-Y_{ss}^{\langle F \rangle} + Z_{ss}^{\langle F \rangle} H_{ss}^{\langle F \rangle 1-\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle \alpha^{\langle F \rangle}} = 0 \quad (8.8)$$

$$-Y_{ss}^{\langle H \rangle} + Z_{ss}^{\langle H \rangle} H_{ss}^{\langle H \rangle 1-\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle \alpha^{\langle H \rangle}} = 0 \quad (8.9)$$

$$Z_{ss}^{\langle F \rangle} - e^{\phi^{Z\langle F \rangle} \log Z_{ss}^{\langle F \rangle}} = 0 \quad (8.10)$$

$$Z_{ss}^{\langle H \rangle} - e^{\phi^{Z\langle H \rangle} \log Z_{ss}^{\langle H \rangle}} = 0 \quad (8.11)$$

$$\beta \left( \lambda_{ss}^{c\langle F \rangle} \left( r_{ss}^{\langle F \rangle} - \psi^{\langle F \rangle} \left( -\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right)^2 + 2\psi^{\langle F \rangle} I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \left( -\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \right) - \lambda_{ss}^{c\langle F \rangle} \left( -1 - 2\psi^{\langle F \rangle} \left( -\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \right) \left( 1 - \delta^{\langle F \rangle} \right) \right) + \lambda_{ss}^{c\langle F \rangle} \left( -1 - 2\psi^{\langle F \rangle} \left( -\delta^{\langle F \rangle} + \right. \right. \quad (8.12)$$

$$\left. \lambda_{ss}^{c\langle H \rangle} \left( r_{ss}^{\langle H \rangle} - \psi^{\langle H \rangle} \left( -\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right)^2 + 2\psi^{\langle H \rangle} I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \left( -\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \right) - \lambda_{ss}^{c\langle H \rangle} \left( -1 - 2\psi^{\langle H \rangle} \left( -\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \right) \left( 1 - \delta^{\langle H \rangle} \right) \right) + \lambda_{ss}^{c\langle H \rangle} \left( -1 - 2\psi^{\langle H \rangle} \left( -\delta^{\langle H \rangle} + \right. \right) \quad (8.13)$$

$$\lambda_{ss}^{c\langle F \rangle} W_{ss}^{\langle F \rangle} + (-1 + \mu) C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{-\mu} \left(C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.14)$$

$$\lambda_{ss}^{c\langle H \rangle} W_{ss}^{\langle H \rangle} + (-1 + \mu) C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{-\mu} \left(C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.15)$$

$$G_{ss}^{d\langle F \rangle} - \phi^{G\langle F \rangle} G_{ss}^{d\langle F \rangle} = 0 \quad (8.16)$$

$$G_{ss}^{d\langle H \rangle} - \phi^{G\langle H \rangle} G_{ss}^{d\langle H \rangle} = 0 \quad (8.17)$$

$$I_{ss}^{(F)} - K_{ss}^{(F)} + K_{ss}^{(F)} \left(1 - \delta^{(F)}\right) = 0 \quad (8.18)$$

$$I_{ss}^{(H)} - K_{ss}^{(H)} + K_{ss}^{(H)} \left(1 - \delta^{(H)}\right) = 0 \quad (8.19)$$

$$U_{ss}^{(F)} - \beta U_{ss}^{(F)} - (1 - \eta)^{-1} \left( C_{ss}^{(F)\mu} \left(1 - H_{ss}^{(F)}\right)^{1-\mu} \right)^{1-\eta} = 0 \quad (8.20)$$

$$U_{ss}^{(H)} - \beta U_{ss}^{(H)} - (1 - \eta)^{-1} \left( C_{ss}^{(H)\mu} \left(1 - H_{ss}^{(H)}\right)^{1-\mu} \right)^{1-\eta} = 0 \quad (8.21)$$

$$-C_{ss}^{(F)} - G_{ss}^{d(F)} - I_{ss}^{(F)} - TR_{ss}^{(H)} + Y_{ss}^{(F)} - \psi^{(F)} K_{ss}^{(F)} \left( -\delta^{(F)} + I_{ss}^{(F)} K_{ss}^{(F)-1} \right)^2 = 0 \quad (8.22)$$

$$-C_{ss}^{(H)} - G_{ss}^{d(H)} - I_{ss}^{(H)} + TR_{ss}^{(H)} + Y_{ss}^{(H)} - \psi^{(H)} K_{ss}^{(H)} \left( -\delta^{(H)} + I_{ss}^{(H)} K_{ss}^{(H)-1} \right)^2 = 0 \quad (8.23)$$



## 9 Steady-state values

|                 | Steady-state values |
|-----------------|---------------------|
| $\lambda^{c^F}$ | 0.3934              |
| $\lambda^{c^H}$ | 0.3934              |
| $r^F$           | 0.0351              |
| $r^H$           | 0.0351              |
| $C^F$           | 0.9578              |
| $C^H$           | 0.9578              |
| $G^{d^F}$       | 0                   |
| $G^{d^H}$       | 0                   |
| $H^F$           | 0.2645              |
| $H^H$           | 0.2645              |
| $I^F$           | 0.3816              |
| $I^H$           | 0.3816              |
| $K^F$           | 15.2627             |
| $K^H$           | 15.2627             |
| $TR^H$          | 0                   |
| $U^F$           | -125.6048           |
| $U^H$           | -125.6048           |
| $W^F$           | 3.0384              |
| $W^H$           | 3.0384              |
| $Y^F$           | 1.3393              |
| $Y^H$           | 1.3393              |
| $Z^F$           | 1                   |
| $Z^H$           | 1                   |

## 10 The solution of the perturbation

### 10.1 P

$$\begin{matrix} G^{d^F} \\ G^{d^H} \\ K^F \\ K^H \\ Z^F \\ Z^H \end{matrix} \begin{pmatrix} G_{t-1}^{d^F} & G_{t-1}^{d^H} & K_{t-1}^F & K_{t-1}^H & Z_{t-1}^F & Z_{t-1}^H \\ 0.95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 \\ -0.1542 & -0.1542 & 0.9454 & 0.0244 & 2.2856 & -1.0704 \\ -0.1542 & -0.1542 & 0.0244 & 0.9454 & -1.0704 & 2.2856 \\ 0 & 0 & 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 \end{pmatrix}$$

### 10.2 Q

$$\begin{matrix} G^{d^F} \\ G^{d^H} \\ K^F \\ K^H \\ Z^F \\ Z^H \end{matrix} \begin{pmatrix} \epsilon^{Z^F} & \epsilon^{Z^H} & \epsilon^{G^F} & \epsilon^{G^H} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2.4059 & -1.1267 & -0.1623 & -0.1623 \\ -1.1267 & 2.4059 & -0.1623 & -0.1623 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

### 10.3 R

$$\begin{array}{l}
\lambda^{c^F} \\
\lambda^{c^H} \\
r^F \\
r^H \\
C^F \\
C^H \\
H^F \\
H^H \\
I^F \\
I^H \\
TR^H \\
U^F \\
U^H \\
W^F \\
W^H \\
Y^F \\
Y^H
\end{array}
\begin{pmatrix}
G_{t-1}^{d^F} & G_{t-1}^{d^H} & K_{t-1}^F & K_{t-1}^H & Z_{t-1}^F & Z_{t-1}^H \\
0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
0.0044 & 0.0044 & -0.0012 & -0.0004 & 0.0497 & -0.0046 \\
0.0044 & 0.0044 & -0.0004 & -0.0012 & -0.0046 & 0.0497 \\
-0.1525 & -0.1525 & 0.0187 & 0.0136 & 0.3448 & 0.1599 \\
-0.1525 & -0.1525 & 0.0136 & 0.0187 & 0.1599 & 0.3448 \\
0.0554 & 0.0554 & 0.0023 & -0.0049 & 0.2054 & -0.0581 \\
0.0554 & 0.0554 & -0.0049 & 0.0023 & -0.0581 & 0.2054 \\
-0.1542 & -0.1542 & -0.0296 & 0.0244 & 2.2856 & -1.0704 \\
-0.1542 & -0.1542 & 0.0244 & -0.0296 & -1.0704 & 2.2856 \\
-0.475 & 0.475 & 0.053 & -0.053 & -0.7338 & 0.7338 \\
-3.1408 & -3.1408 & 0.1608 & 0.2366 & 0.053 & 8.3603 \\
-3.1408 & -3.1408 & 0.2366 & 0.1608 & 8.3603 & 0.053 \\
-0.2547 & -0.2547 & 0.0689 & 0.0227 & 1.9424 & 0.2672 \\
-0.2547 & -0.2547 & 0.0227 & 0.0689 & 0.2672 & 1.9424 \\
0.1684 & 0.1684 & 0.0422 & -0.015 & 1.8966 & -0.1767 \\
0.1684 & 0.1684 & -0.015 & 0.0422 & -0.1767 & 1.8966
\end{pmatrix}$$

### 10.4 S

$$\begin{array}{l}
\lambda^{c^F} \\
\lambda^{c^H} \\
r^F \\
r^H \\
C^F \\
C^H \\
H^F \\
H^H \\
I^F \\
I^H \\
TR^H \\
U^F \\
U^H \\
W^F \\
W^H \\
Y^F \\
Y^H
\end{array}
\begin{pmatrix}
\epsilon^{Z^F} & \epsilon^{Z^H} & \epsilon^{G^F} & \epsilon^{G^H} \\
-0.1128 & -0.1128 & 0.1075 & 0.1075 \\
-0.1128 & -0.1128 & 0.1075 & 0.1075 \\
0.0523 & -0.0049 & 0.0046 & 0.0046 \\
-0.0049 & 0.0523 & 0.0046 & 0.0046 \\
0.3629 & 0.1683 & -0.1605 & -0.1605 \\
0.1683 & 0.3629 & -0.1605 & -0.1605 \\
0.2163 & -0.0612 & 0.0583 & 0.0583 \\
-0.0612 & 0.2163 & 0.0583 & 0.0583 \\
2.4059 & -1.1267 & -0.1623 & -0.1623 \\
-1.1267 & 2.4059 & -0.1623 & -0.1623 \\
-0.7724 & 0.7724 & -0.5 & 0.5 \\
0.0557 & 8.8003 & -3.3061 & -3.3061 \\
8.8003 & 0.0557 & -3.3061 & -3.3061 \\
2.0446 & 0.2812 & -0.2681 & -0.2681 \\
0.2812 & 2.0446 & -0.2681 & -0.2681 \\
1.9964 & -0.186 & 0.1773 & 0.1773 \\
-0.186 & 1.9964 & 0.1773 & 0.1773
\end{pmatrix}$$

## 11 Statistics of the model

### 11.1 Moments

|           | Steady-state value | Std. dev. | Variance | Loglinear |
|-----------|--------------------|-----------|----------|-----------|
| $r^H$     | 0.0351             | 0.0051    | 0        | N         |
| $C^H$     | 0.9578             | 0.034     | 0.0012   | N         |
| $G^{d^H}$ | 0                  | 0.0922    | 0.0085   | N         |
| $H^H$     | 0.2645             | 0.0249    | 0.0006   | N         |
| $I^H$     | 0.3816             | 0.2411    | 0.0581   | N         |
| $K^H$     | 15.2627            | 0.8242    | 0.6794   | N         |
| $TR^H$    | 0                  | 0.1586    | 0.0252   | N         |
| $U^H$     | -125.6048          | 0.7839    | 0.6144   | N         |
| $W^H$     | 3.0384             | 0.1864    | 0.0348   | N         |
| $Y^H$     | 1.3393             | 0.2022    | 0.0409   | N         |
| $Z^H$     | 1                  | 0.0922    | 0.0085   | N         |

## 11.2 Correlation matrix

|                | $r^H$   | $C^H$   | $G^{dH}$ | $H^H$   | $I^H$   | $K^H$   | $TR^H$  | $U^H$   | $W^H$   | $Y^H$   | $Z^H$   |
|----------------|---------|---------|----------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\lambda^{cF}$ | -0.226  | -0.8587 | 0.315    | -0.0583 | -0.2102 | -0.1567 | 0       | -0.6538 | -0.5296 | -0.2516 | -0.3535 |
| $\lambda^{cH}$ | -0.226  | -0.8587 | 0.315    | -0.0583 | -0.2102 | -0.1567 | 0       | -0.6538 | -0.5296 | -0.2516 | -0.3535 |
| $r^F$          | -0.0642 | 0.1313  | 0.0458   | -0.109  | -0.4976 | -0.1459 | -0.6228 | 0.7667  | 0.0158  | -0.0616 | -0.034  |
| $r^H$          | 1       | 0.6587  | 0.5543   | 0.9181  | 0.8885  | 0.1933  | 0.6228  | -0.2736 | 0.8886  | 0.9342  | 0.9831  |
| $C^F$          | 0.1313  | 0.6746  | -0.2132  | -0.072  | -0.0901 | -0.1031 | -0.2407 | 0.8996  | 0.351   | 0.0977  | 0.2227  |
| $C^H$          | 0.6587  | 1       | 0.06     | 0.5617  | 0.5915  | 0.393   | 0.2407  | 0.3136  | 0.8895  | 0.7121  | 0.769   |
| $G^{dF}$       | 0.0458  | -0.2132 | 0        | 0.0932  | -0.27   | -0.1198 | -0.493  | 0.1309  | -0.072  | 0.0289  | 0       |
| $G^{dH}$       | 0.5543  | 0.06    | 1        | 0.6253  | 0.3906  | 0.1507  | 0.493   | -0.3895 | 0.3801  | 0.5447  | 0.5     |
| $H^F$          | -0.109  | -0.072  | 0.0932   | -0.2342 | -0.5149 | -0.4537 | -0.4687 | 0.6968  | -0.1711 | -0.2157 | -0.1368 |
| $H^H$          | 0.9181  | 0.5617  | 0.6253   | 1       | 0.8128  | 0.5124  | 0.4687  | -0.4445 | 0.8777  | 0.9809  | 0.9273  |
| $I^F$          | -0.4976 | -0.0901 | -0.27    | -0.5149 | -0.8058 | -0.1844 | -0.8168 | 0.848   | -0.3366 | -0.4582 | -0.447  |
| $I^H$          | 0.8885  | 0.5915  | 0.3906   | 0.8128  | 1       | 0.2307  | 0.8168  | -0.5013 | 0.7916  | 0.829   | 0.8743  |
| $K^F$          | -0.1459 | -0.1031 | -0.1198  | -0.4537 | -0.1844 | -0.7884 | 0.1498  | 0.4338  | -0.3103 | -0.4093 | -0.2156 |
| $K^H$          | 0.1933  | 0.393   | 0.1507   | 0.5124  | 0.2307  | 1       | -0.1498 | -0.2587 | 0.5107  | 0.5274  | 0.3254  |
| $TR^H$         | 0.6228  | 0.2407  | 0.493    | 0.4687  | 0.8168  | -0.1498 | 1       | -0.534  | 0.3983  | 0.4544  | 0.5503  |
| $U^F$          | 0.7667  | 0.8996  | 0.1309   | 0.6968  | 0.848   | 0.4338  | 0.534   | -0.0945 | 0.906   | 0.8031  | 0.8435  |
| $U^H$          | -0.2736 | 0.3136  | -0.3895  | -0.4445 | -0.5013 | -0.2587 | -0.534  | 1       | -0.0639 | -0.3034 | -0.1974 |
| $W^F$          | 0.0158  | 0.351   | -0.072   | -0.1711 | -0.3366 | -0.3103 | -0.3983 | 0.906   | 0.1089  | -0.0625 | 0.0534  |
| $W^H$          | 0.8886  | 0.8895  | 0.3801   | 0.8777  | 0.7916  | 0.5107  | 0.3983  | -0.0639 | 1       | 0.9542  | 0.9576  |
| $Y^F$          | -0.0616 | 0.0977  | 0.0289   | -0.2157 | -0.4582 | -0.4093 | -0.4544 | 0.8031  | -0.0625 | -0.1601 | -0.0637 |
| $Y^H$          | 0.9342  | 0.7121  | 0.5447   | 0.9809  | 0.829   | 0.5274  | 0.4544  | -0.3034 | 0.9542  | 1       | 0.9679  |
| $Z^F$          | -0.034  | 0.2227  | 0        | -0.1368 | -0.447  | -0.2156 | -0.5503 | 0.8435  | 0.0534  | -0.0637 | 0       |
| $Z^H$          | 0.9831  | 0.769   | 0.5      | 0.9273  | 0.8743  | 0.3254  | 0.5503  | -0.1974 | 0.9576  | 0.9679  | 1       |

## 11.3 Autocorrelations

|          | $t-1$  | $t-2$  | $t-3$  | $t-4$  | $t-5$   |
|----------|--------|--------|--------|--------|---------|
| $r^H$    | 0.7036 | 0.4561 | 0.2537 | 0.0926 | -0.0318 |
| $C^H$    | 0.7497 | 0.5288 | 0.3382 | 0.1774 | 0.0454  |
| $G^{dH}$ | 0.7133 | 0.4711 | 0.2711 | 0.1098 | -0.0163 |
| $H^H$    | 0.7476 | 0.5249 | 0.3327 | 0.171  | 0.0384  |
| $I^H$    | 0.698  | 0.4473 | 0.2437 | 0.0826 | -0.0407 |
| $K^H$    | 0.9564 | 0.8523 | 0.7094 | 0.5455 | 0.3748  |
| $TR^H$   | 0.7199 | 0.4816 | 0.2831 | 0.1217 | -0.0057 |
| $U^H$    | 0.7308 | 0.4987 | 0.303  | 0.1419 | 0.0127  |
| $W^H$    | 0.748  | 0.5257 | 0.334  | 0.1726 | 0.0403  |
| $Y^H$    | 0.7475 | 0.5248 | 0.3327 | 0.171  | 0.0385  |
| $Z^H$    | 0.7133 | 0.4711 | 0.2711 | 0.1098 | -0.0163 |

## 11.4 Variance decomposition

|          | $\epsilon^{ZF}$ | $\epsilon^{ZH}$ | $\epsilon^{GF}$ | $\epsilon^{GH}$ |
|----------|-----------------|-----------------|-----------------|-----------------|
| $r^H$    | 0.0022          | 0.9869          | 0.0055          | 0.0055          |
| $C^H$    | 0.0587          | 0.6565          | 0.1424          | 0.1424          |
| $G^{dH}$ | 0               | 0.25            | 0               | 0.75            |
| $H^H$    | 0.0506          | 0.8796          | 0.0349          | 0.0349          |
| $I^H$    | 0.2139          | 0.7803          | 0.0029          | 0.0029          |
| $K^H$    | 0.2084          | 0.7853          | 0.0031          | 0.0031          |
| $TR^H$   | 0.4367          | 0.4367          | 0.0633          | 0.0633          |
| $U^H$    | 0.7325          | 0.0408          | 0.1133          | 0.1133          |
| $W^H$    | 0.0054          | 0.9676          | 0.0135          | 0.0135          |
| $Y^H$    | 0.0212          | 0.9689          | 0.005           | 0.005           |
| $Z^H$    | 0               | 1               | 0               | 0               |

## 12 Statistics of the model

### 12.1 Moments relative to moments of the reference variable

|                 | Steady-state value relative to $Y^H$ | Std. dev. relative to $Y^H$ | Variance relative to $Y^H$ | Loglinear |
|-----------------|--------------------------------------|-----------------------------|----------------------------|-----------|
| $\lambda^{c^F}$ | 0.2937                               | 0.0745                      | 0.0055                     | N         |
| $\lambda^{c^H}$ | 0.2937                               | 0.0745                      | 0.0055                     | N         |
| $r^F$           | 0.0262                               | 0.0251                      | 0.0006                     | N         |
| $r^H$           | 0.0262                               | 0.0251                      | 0.0006                     | N         |
| $C^F$           | 0.7151                               | 0.1683                      | 0.0283                     | N         |
| $C^H$           | 0.7151                               | 0.1683                      | 0.0283                     | N         |
| $G^{d^F}$       | 0                                    | 0.4558                      | 0.2077                     | N         |
| $G^{d^H}$       | 0                                    | 0.4558                      | 0.2077                     | N         |
| $H^F$           | 0.1975                               | 0.1233                      | 0.0152                     | N         |
| $H^H$           | 0.1975                               | 0.1233                      | 0.0152                     | N         |
| $I^F$           | 0.2849                               | 1.1922                      | 1.4214                     | N         |
| $I^H$           | 0.2849                               | 1.1922                      | 1.4214                     | N         |
| $K^F$           | 11.3957                              | 4.0759                      | 16.613                     | N         |
| $K^H$           | 11.3957                              | 4.0759                      | 16.613                     | N         |
| $TR^H$          | 0                                    | 0.7845                      | 0.6155                     | N         |
| $U^F$           | -93.7814                             | 3.8762                      | 15.0251                    | N         |
| $U^H$           | -93.7814                             | 3.8762                      | 15.0251                    | N         |
| $W^F$           | 2.2686                               | 0.9219                      | 0.8499                     | N         |
| $W^H$           | 2.2686                               | 0.9219                      | 0.8499                     | N         |
| $Y^F$           | 1                                    | 1                           | 1                          | N         |
| $Y^H$           | 1                                    | 1                           | 1                          | N         |
| $Z^F$           | 0.7466                               | 0.4558                      | 0.2077                     | N         |
| $Z^H$           | 0.7466                               | 0.4558                      | 0.2077                     | N         |

### 12.2 Correlations with the reference variable

|                 | $Y_{t-5}^H$ | $Y_{t-4}^H$ | $Y_{t-3}^H$ | $Y_{t-2}^H$ | $Y_{t-1}^H$ | $Y_t^H$ | $Y_{t+1}^H$ | $Y_{t+2}^H$ | $Y_{t+3}^H$ | $Y_{t+4}^H$ | $Y_{t+5}^H$ |
|-----------------|-------------|-------------|-------------|-------------|-------------|---------|-------------|-------------|-------------|-------------|-------------|
| $\lambda^{c^F}$ | 0.0871      | 0.052       | 0.0027      | -0.0629     | -0.147      | -0.2516 | -0.2295     | -0.2021     | -0.1717     | -0.1403     | -0.1092     |
| $\lambda^{c^H}$ | 0.0871      | 0.052       | 0.0027      | -0.0629     | -0.147      | -0.2516 | -0.2295     | -0.2021     | -0.1717     | -0.1403     | -0.1092     |
| $r^F$           | -0.0996     | -0.107      | -0.1091     | -0.1039     | -0.0889     | -0.0616 | 0.001       | 0.0457      | 0.0755      | 0.0931      | 0.1011      |
| $r^H$           | 0.1644      | 0.2797      | 0.415       | 0.5699      | 0.7436      | 0.9342  | 0.583       | 0.301       | 0.0807      | -0.0854     | -0.2047     |
| $C^F$           | -0.0881     | -0.0765     | -0.0547     | -0.0202     | 0.0296      | 0.0977  | 0.0894      | 0.0807      | 0.0719      | 0.063       | 0.0543      |
| $C^H$           | -0.0428     | 0.0529      | 0.1742      | 0.3236      | 0.5025      | 0.7121  | 0.5623      | 0.4245      | 0.3008      | 0.1923      | 0.0996      |
| $G^{d^F}$       | -0.0492     | -0.0463     | -0.0384     | -0.0243     | -0.0024     | 0.0289  | 0.0335      | 0.0357      | 0.0359      | 0.0346      | 0.0321      |
| $G^{d^H}$       | 0.0486      | 0.1179      | 0.2018      | 0.3009      | 0.4154      | 0.5447  | 0.3727      | 0.2295      | 0.1132      | 0.0212      | -0.0489     |
| $H^F$           | -0.0311     | -0.065      | -0.1021     | -0.1409     | -0.1796     | -0.2157 | -0.1962     | -0.1689     | -0.1372     | -0.1037     | -0.0706     |
| $H^H$           | 0.0572      | 0.1869      | 0.3437      | 0.5287      | 0.7415      | 0.9809  | 0.7249      | 0.5007      | 0.3086      | 0.1482      | 0.0177      |
| $I^F$           | -0.1696     | -0.2254     | -0.2844     | -0.3447     | -0.4037     | -0.4582 | -0.2427     | -0.0758     | 0.0488      | 0.1375      | 0.196       |
| $I^H$           | 0.1811      | 0.2834      | 0.4007      | 0.532       | 0.6757      | 0.829   | 0.5004      | 0.239       | 0.0372      | -0.1128     | -0.2185     |
| $K^F$           | 0.0822      | 0.0142      | -0.0694     | -0.1685     | -0.2824     | -0.4093 | -0.4701     | -0.4805     | -0.4542     | -0.4026     | -0.3353     |
| $K^H$           | -0.2716     | -0.1819     | -0.0602     | 0.097       | 0.2922      | 0.5274  | 0.6606      | 0.714       | 0.707       | 0.6563      | 0.576       |
| $TR^H$          | 0.2452      | 0.2926      | 0.3394      | 0.3838      | 0.4232      | 0.4544  | 0.1448      | -0.0813     | -0.2373     | -0.3358     | -0.3881     |
| $U^F$           | 0.0493      | 0.1549      | 0.2828      | 0.4337      | 0.6074      | 0.8031  | 0.583       | 0.3934      | 0.2336      | 0.1021      | -0.003      |
| $U^H$           | -0.1146     | -0.1545     | -0.1956     | -0.2359     | -0.2729     | -0.3034 | -0.189      | -0.0953     | -0.0206     | 0.0369      | 0.0793      |
| $W^F$           | -0.0682     | -0.0802     | -0.0881     | -0.0896     | -0.0821     | -0.0625 | -0.0566     | -0.0465     | -0.0341     | -0.0208     | -0.0075     |
| $W^H$           | 0.0068      | 0.1338      | 0.2908      | 0.4794      | 0.7006      | 0.9542  | 0.7261      | 0.5224      | 0.3447      | 0.1932      | 0.0675      |
| $Y^F$           | -0.0471     | -0.0732     | -0.0995     | -0.1244     | -0.1455     | -0.1601 | -0.1455     | -0.1244     | -0.0995     | -0.0732     | -0.0471     |
| $Y^H$           | 0.0385      | 0.171       | 0.3327      | 0.5248      | 0.7475      | 1       | 0.7475      | 0.5248      | 0.3327      | 0.171       | 0.0385      |
| $Z^F$           | -0.0898     | -0.0993     | -0.1037     | -0.101      | -0.0886     | -0.0637 | -0.022      | 0.0101      | 0.0338      | 0.0502      | 0.0605      |
| $Z^H$           | 0.106       | 0.2292      | 0.3768      | 0.5495      | 0.747       | 0.9679  | 0.6563      | 0.3977      | 0.1884      | 0.0235      | -0.1016     |

## 13 Impulse response functions

### 13.1 Shock $\epsilon^{Z^F}$

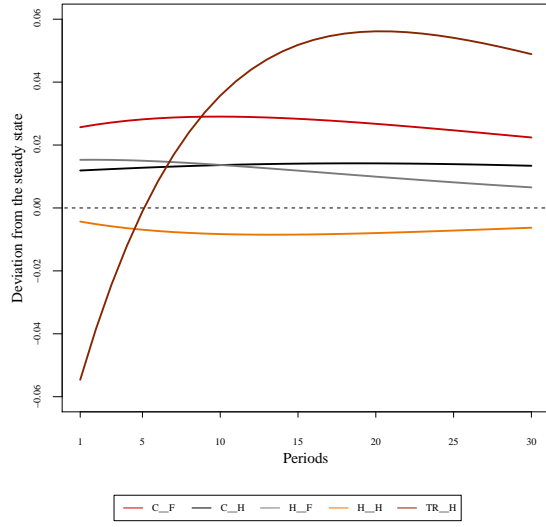


Figure 1: Impulse response function for  $\epsilon^{Z^F}$  shock

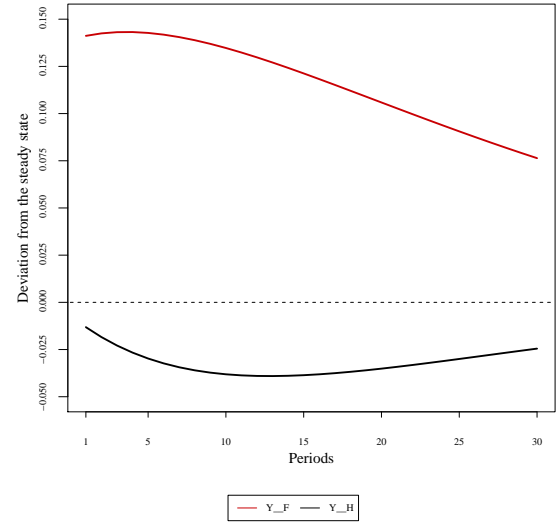


Figure 2: Impulse response function for  $\epsilon^{Z^F}$  shock

### 13.2 Shock $\epsilon^{Z^H}$

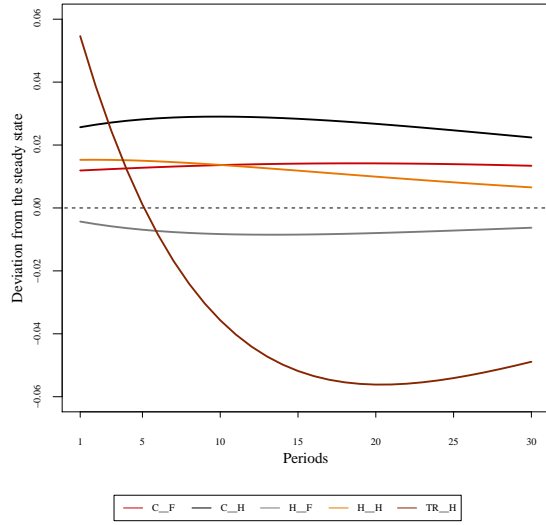


Figure 3: Impulse response function for  $\epsilon^{Z^H}$  shock

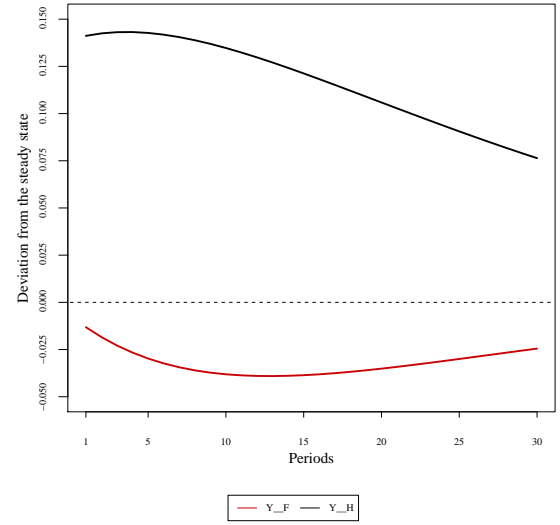


Figure 4: Impulse response function for  $\epsilon^{Z^H}$  shock