1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t, H_t} U_t = \beta E_t \left[U_{t+1} \right] + (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - persH_t)^\mu \right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \tag{1.2}$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \tag{1.3}$$

$$H_t = C_{t-1} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \tag{1.4}$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \tag{1.5}$$

$$-\lambda_t^{\text{CONSUMER}^1} + \beta E_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - persH_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - persH_t)^\mu \right)^{-\eta} = 0 \quad (C_t) \tag{1.6}$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu)(1 - L_t^s)^{-\mu} (C_t - persH_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - persH_t)^\mu \right)^{-\eta} = 0 \quad (L_t^s) \tag{1.7}$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \tag{1.8}$$

$$-\lambda_t^{\text{CONSUMER}^3} - \mu pers(1 - L_t^s)^{1-\mu} (C_t - persH_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - persH_t)^\mu \right)^{-\eta} = 0 \quad (H_t) \tag{1.9}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \tag{2.1}$$

s.t. :

$$Y_t = Z_t K_t^{d\,\alpha} L_t^{d\,1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \tag{2.2}$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\mathrm{FIRM}^1} Z_t K_t^{\mathrm{d}-1+\alpha} L_t^{\mathrm{d}1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1} Z_t \left(1-\alpha\right) K_t^{\mathrm{d}\alpha} L_t^{\mathrm{d}-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.4}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad \left(Y_t\right) \tag{2.5}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\mathrm{d}-1+\alpha} L_t^{\mathrm{d}1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.6}$$

$$-W_t + Z_t \left(1-\alpha\right) K_t^{\mathrm{d}\alpha} L_t^{\mathrm{d}-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.7}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{d}} = K_{t-1}^{\mathrm{s}} \tag{3.1}$$

$$L_t^{\mathrm{d}} = L_t^{\mathrm{s}} \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$C_{t-1} - H_t = 0 \tag{5.1}$$

$$-\lambda_t^{\mathrm{CONSUMER}^2} + \beta \left(\left(1-\delta\right) \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^2}\right] + \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^2} r_{t+1}\right] \right) = 0 \tag{5.2}$$

$$-r_t + \alpha Z_t K_{t-1}^{\mathrm{s}-1+\alpha} L_t^{\mathrm{s}1-\alpha} = 0 \tag{5.3}$$

$$-W_t + Z_t \left(1-\alpha\right) K_{t-1}^{\mathrm{s}\alpha} L_t^{\mathrm{s}-\alpha} = 0 \tag{5.4}$$

$$-Y_t + Z_t K_{t-1}^{s}{}^{\alpha} L_t^{s\,1-\alpha} = 0 \tag{5.5}$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \tag{5.6}$$

$$\lambda_t^{\mathrm{CONSUMER}^2} W_t + (-1 + \mu)(1 - L_t^s)^{-\mu}(C_t - pers H_t)^{\mu}\left((1 - L_t^s)^{1-\mu}(C_t - pers H_t)^{\mu}\right)^{-\eta} = 0 \tag{5.7}$$

$$-\lambda_t^{\mathrm{CONSUMER}^2} - \beta \mu pers \mathrm{E}_t\left[(1 - L_{t+1}^s)^{1-\mu}(C_{t+1} - pers H_{t+1})^{-1+\mu}\left((1 - L_{t+1}^s)^{1-\mu}(C_{t+1} - pers H_{t+1})^{\mu}\right)^{-\eta}\right] + \mu(1 - L_t^s)^{1-\mu}(C_t - pers H_t)^{-1+\mu}\left((1 - L_t^s)^{1-\mu}(C_t - pers H_t)^{\mu}\right)^{-\eta} = 0$$
$$\tag{5.8}$$

$$-C_t - I_t + Y_t = 0 \tag{5.9}$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \tag{5.10}$$

$$U_t - \beta \mathrm{E}_t[U_{t+1}] - (1 - \eta)^{-1}\left((1 - L_t^s)^{1-\mu}(C_t - pers H_t)^{\mu}\right)^{1-\eta} = 0 \tag{5.11}$$

3

6 Steady state relationships (after reduction)

$$-\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^2} + \beta\left(\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^2} r_{\mathrm{ss}} + \lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^2}(1 - \delta)\right) = 0 \tag{6.1}$$

$$-r_{\mathrm{ss}} + \alpha Z_{\mathrm{ss}} K_{\mathrm{ss}}^{s\,-1+\alpha} L_{\mathrm{ss}}^{s\,1-\alpha} = 0 \tag{6.2}$$

$$C_{\mathrm{ss}} - H_{\mathrm{ss}} = 0 \tag{6.3}$$

$$-W_{\mathrm{ss}} + Z_{\mathrm{ss}}(1 - \alpha) K_{\mathrm{ss}}^{s\,\alpha} L_{\mathrm{ss}}^{s\,-\alpha} = 0 \tag{6.4}$$

$$-Y_{\mathrm{ss}} + Z_{\mathrm{ss}} K_{\mathrm{ss}}^{s\,\alpha} L_{\mathrm{ss}}^{s\,1-\alpha} = 0 \tag{6.5}$$

$$-Z_{\mathrm{ss}} + e^{\phi \log Z_{\mathrm{ss}}} = 0 \tag{6.6}$$

$$\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^2} W_{\mathrm{ss}} + (-1 + \mu)(1 - L_{\mathrm{ss}}^s)^{-\mu}(C_{\mathrm{ss}} - pers H_{\mathrm{ss}})^{\mu}\left((1 - L_{\mathrm{ss}}^s)^{1-\mu}(C_{\mathrm{ss}} - pers H_{\mathrm{ss}})^{\mu}\right)^{-\eta} = 0 \tag{6.7}$$

$$-\lambda_{\text{ss}}^{\text{CONSUMER}^2} + \mu(1 - L_{\text{ss}}^{\text{s}})^{1-\mu}(C_{\text{ss}} - persH_{\text{ss}})^{-1+\mu}\left((1 - L_{\text{ss}}^{\text{s}})^{1-\mu}(C_{\text{ss}} - persH_{\text{ss}})^{\mu}\right)^{-\eta} - \beta\mu pers(1 - L_{\text{ss}}^{\text{s}})^{1-\mu}(C_{\text{ss}} - persH_{\text{ss}})^{-1+\mu}\left((1 - L_{\text{ss}}^{\text{s}})^{1-\mu}(C_{\text{ss}} - persH_{\text{ss}})^{\mu}\right)^{-\eta} = 0$$

$$\tag{6.8}$$

$$-C_{\text{ss}} - I_{\text{ss}} + Y_{\text{ss}} = 0 \tag{6.9}$$

$$I_{\text{ss}} - K_{\text{ss}}^{\text{s}} + K_{\text{ss}}^{\text{s}}(1 - \delta) = 0 \tag{6.10}$$

$$U_{\text{ss}} - \beta U_{\text{ss}} - (1 - \eta)^{-1}\left((1 - L_{\text{ss}}^{\text{s}})^{1-\mu}(C_{\text{ss}} - persH_{\text{ss}})^{\mu}\right)^{1-\eta} = 0 \tag{6.11}$$

7 Calibrating equations

$$-0.36 Y_{\text{ss}} + r_{\text{ss}} K_{\text{ss}}^{\text{s}} = 0 \tag{7.1}$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$

$$\delta = 0.025 \tag{8.2}$$

$$\eta = 2 \tag{8.3}$$

$$\mu = 0.3 \tag{8.4}$$

$$pers = 0.57 \tag{8.5}$$

$$\phi = 0.95 \tag{8.6}$$

9 Steady-state values

	Steady-state value
$\lambda^{\text{CONSUMER}^2}$	0.7116
r	0.0351
C	0.7494
H	0.7494
I	0.2584
K^{s}	10.3356
L^{s}	0.2721
U	-175.4236
W	2.3706
Y	1.0078
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$
\begin{array}{c}
 \\
C_t \\
K_t^{\text{s}} \\
Z_t
\end{array}
\begin{array}{ccc}
C_{t-1} & K_{t-1}^{\text{s}} & Z_{t-1} \\
\left(\begin{array}{ccc}
0.5544 & 0.0151 & 0.1764 \\
-0.5092 & 0.9817 & 1.1759 \\
0 & 0 & 0.95
\end{array} \right)
\end{array}
$$

Matrix Q

$$
\begin{array}{c}
 \\
C \\
K^{\text{s}} \\
Z
\end{array}
\begin{array}{c}
\epsilon^{\text{Z}} \\
\left(\begin{array}{c}
0.1857 \\
1.2377 \\
1
\end{array} \right)
\end{array}
$$

Matrix R

$$
\begin{array}{c}
 \\
\lambda_t^{\text{CONSUMER}^2} \\
r_t \\
H_t \\
I_t \\
L_t^{\text{s}} \\
U_t \\
W_t \\
Y_t
\end{array}
\begin{array}{ccc}
C_{t-1} & K_{t-1}^{\text{s}} & Z_{t-1} \\
\left(\begin{array}{ccc}
0.0599 & -0.0494 & -0.3592 \\
0.0016 & -0.0026 & 0.0471 \\
1 & 0 & 0 \\
-0.5092 & 0.0067 & 1.1759 \\
0.0191 & -0.0056 & 0.1666 \\
-0.9309 & 0.7188 & 11.4498 \\
-0.0598 & 0.1002 & 1.7296 \\
0.0452 & 0.0218 & 1.3522
\end{array} \right)
\end{array}
$$

Matrix S

$$
\begin{array}{c}
 \\
\lambda^{\text{CONSUMER}^2} \\
r \\
H \\
I \\
L^{\text{s}} \\
U \\
W \\
Y
\end{array}
\begin{array}{c}
\epsilon^{\text{Z}} \\
\left(\begin{array}{c}
-0.3781 \\
0.0496 \\
0 \\
1.2377 \\
0.1753 \\
12.0524 \\
1.8206 \\
1.4234
\end{array} \right)
\end{array}
$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.0046	0	N
C	0.7494	0.0333	0.0011	N
H	0.7494	0.0333	0.0011	N
I	0.2584	0.1077	0.0116	N
K^{s}	10.3356	0.3633	0.132	N
L^{s}	0.2721	0.0164	0.0003	N
U	-175.4236	1.1325	1.2825	N
W	2.3706	0.1719	0.0295	N
Y	1.0078	0.1325	0.0175	N
Z	1	0.0922	0.0085	N

11.2 Correlation matrix

	r	C	H	I	K^{s}	L^{s}	U	W	Y	Z
r	1	0.623	0.263	0.993	0.14	0.994	0.918	0.92	0.965	0.98
C		1	0.911	0.674	0.826	0.701	0.867	0.865	0.799	0.758
H			1	0.315	0.899	0.358	0.584	0.582	0.485	0.429
I				1	0.232	0.997	0.95	0.951	0.983	0.992
K^{s}					1	0.243	0.521	0.516	0.396	0.332
L^{s}						1	0.954	0.956	0.987	0.996
U							1	1	0.99	0.978
W								1	0.991	0.979
Y									1	0.998
Z										1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	0.035	0.126	0.245	0.388	0.556	0.749	0.965	0.613	0.331	0.109	-0.059	-0.182
C_t	0.251	-0.244	-0.131	0.025	0.228	0.485	0.799	0.867	0.804	0.677	0.527	0.375
H_t	0.251	-0.32	-0.244	-0.131	0.025	0.228	0.485	0.799	0.867	0.804	0.677	0.527
I_t	0.813	0.069	0.193	0.345	0.527	0.74	0.983	0.622	0.349	0.143	-0.011	-0.122
K_t^{s}	2.742	-0.459	-0.39	-0.278	-0.115	0.107	0.396	0.57	0.66	0.685	0.665	0.612
L_t^{s}	0.124	0.076	0.2	0.352	0.534	0.746	0.987	0.668	0.402	0.185	0.016	-0.113
U_t	8.55	-0.077	0.053	0.222	0.432	0.688	0.99	0.754	0.545	0.366	0.214	0.089
W_t	1.298	-0.073	0.057	0.226	0.436	0.69	0.991	0.755	0.546	0.365	0.211	0.085
Y_t	1	-0.005	0.124	0.286	0.486	0.724	1	0.724	0.486	0.286	0.124	-0.005
Z_t	0.696	0.028	0.155	0.314	0.507	0.735	0.998	0.698	0.446	0.24	0.076	-0.051

11.4 Autocorrelations

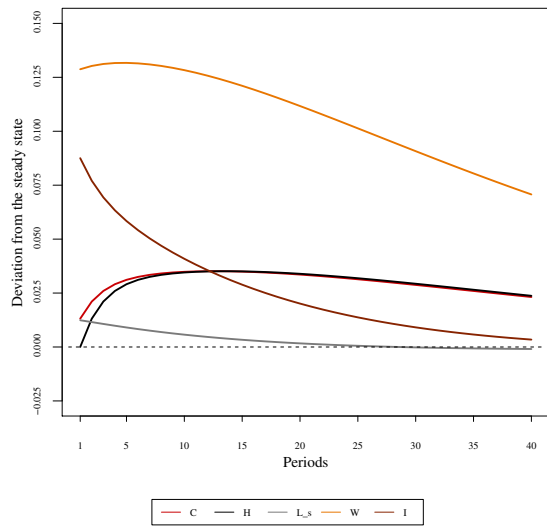|	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.707	0.462	0.261	0.101	0
C	0.911	0.738	0.537	0.337	0.156
H	0.911	0.738	0.537	0.337	0.156
I	0.667	0.413	0.217	0.069	-0.043
K^{s}	0.955	0.851	0.71	0.55	0.384
L^{s}	0.716	0.473	0.271	0.108	-0.02
U	0.737	0.509	0.316	0.155	0.025
W	0.74	0.512	0.317	0.155	0.024
Y	0.724	0.486	0.286	0.124	-0.005
Z	0.713	0.471	0.271	0.11	-0.016

12 Impulse response functions

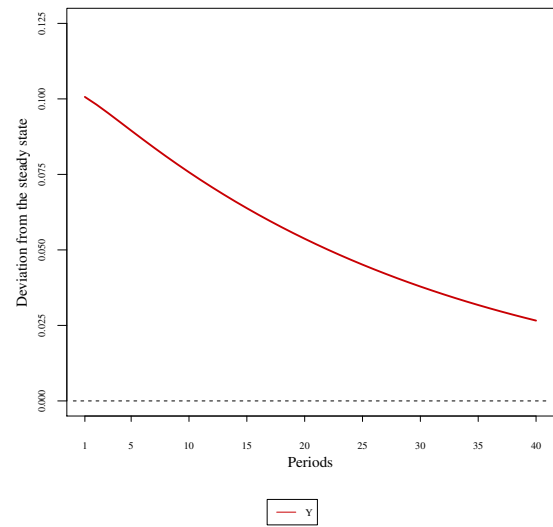Figure 1: Impulse responses (C, H, L^s, W, I) to ϵ^Z shock

Figure 2: Impulse response (Y) to ϵ^Z shock