

1 CONSUMER

1.1 Optimization problem

$$\max_{C_t, L_t^s} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t = \pi_t + L_t^s W_t \quad (\lambda_t^c) \quad (1.2)$$

1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.3)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.4)$$

$$\lambda_t^c W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu = 0 \quad (L_t^s) \quad (1.5)$$

2 FIRM

2.1 Optimization problem

$$\max_{K_t, L_t^d, Y_t, I_t, \pi_t, CqU_t} \Pi_t = \pi_t + \lambda_t^{c-1} E_t [\lambda_{t+1}^c \lambda_{t+1}^U \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$Y_t = L_t^{d^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha \quad (\lambda_t^{\text{FIRM}^1}) \quad (2.2)$$

$$K_t = I_t + K_{t-1} (1 - \delta CqU_t)^\omega \quad (\lambda_t^{\text{FIRM}^2}) \quad (2.3)$$

$$\pi_t = -I_t - L_t^d W_t + P_t Y_t \quad (\lambda_t^{\text{FIRM}^3}) \quad (2.4)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^{c-1} \lambda_t^c \lambda_t^U = 0 \quad (\Pi_t) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^2} + E_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(\lambda_{t+1}^{\text{FIRM}^2} (1 - \delta CqU_{t+1})^\omega + \alpha \lambda_{t+1}^{\text{FIRM}^1} CqU_{t+1} L_{t+1}^{d^{1-\alpha}} Z_{t+1}^{1-\alpha} (K_t CqU_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.6)$$

$$-\lambda_t^{\text{FIRM}^3} W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{d^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (L_t^d) \quad (2.7)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^3} P_t = 0 \quad (Y_t) \quad (2.8)$$

$$\lambda_t^{\text{FIRM}^2} - \lambda_t^{\text{FIRM}^3} = 0 \quad (I_t) \quad (2.9)$$

$$1 - \lambda_t^{\text{FIRM}^3} = 0 \quad (\pi_t) \quad (2.10)$$

$$-\delta \omega K_{t-1} \lambda_t^{\text{FIRM}^2} CqU_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{d^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} CqU_t)^{-1+\alpha} = 0 \quad (CqU_t) \quad (2.11)$$

2.3 First order conditions after reduction

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c{}^{-1} \lambda_t^c \lambda_t^U = 0 \quad (\Pi_t) \quad (2.12)$$

$$-1 + E_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(1 - \delta CqU_{t+1}^\omega + \alpha \lambda_{t+1}^{\text{FIRM}^1} CqU_{t+1} L_{t+1}^d{}^{1-\alpha} Z_{t+1}^{1-\alpha} (K_t CqU_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.13)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{d-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (L_t^d) \quad (2.14)$$

$$-\lambda_t^{\text{FIRM}^1} + P_t = 0 \quad (Y_t) \quad (2.15)$$

$$-\delta \omega K_{t-1} CqU_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{d1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^{-1+\alpha} = 0 \quad (CqU_t) \quad (2.16)$$

3 EQUILIBRIUM

3.1 Identities

$$P_t = 1 \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships

$$-1 + \lambda_t^{c-1} E_t \left[\lambda_{t+1}^c \lambda_{t+1}^U \left(1 - \delta CqU_{t+1}^\omega + \alpha CqU_{t+1} L_{t+1}^s{}^{1-\alpha} Z_{t+1}^{1-\alpha} (K_t CqU_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (5.1)$$

$$\beta - \lambda_t^U = 0 \quad (5.2)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} = 0 \quad (5.3)$$

$$-W_t + (1 - \alpha) L_t^{s-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (5.4)$$

$$-Y_t + L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha = 0 \quad (5.5)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.6)$$

$$\lambda_t^c W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{-\eta} C_t^\mu = 0 \quad (5.7)$$

$$-\delta \omega K_{t-1} CqU_t^{-1+\omega} + \alpha K_{t-1} L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^{-1+\alpha} = 0 \quad (5.8)$$

$$I_t - K_t + K_{t-1} (1 - \delta CqU_t^\omega) = 0 \quad (5.9)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} C_t^\mu \right)^{1-\eta} = 0 \quad (5.10)$$

$$-C_t + \Pi_t - \lambda_t^{c-1} E_t [\lambda_{t+1}^c \lambda_{t+1}^U \Pi_{t+1}] + L_t^s W_t = 0 \quad (5.11)$$

$$-I_t - \Pi_t + Y_t + \lambda_t^{c-1} E_t [\lambda_{t+1}^c \lambda_{t+1}^U \Pi_{t+1}] - L_t^s W_t = 0 \quad (5.12)$$

6 Steady state relationships

$$-1 + \lambda_{ss}^U \left(1 - \delta CqUt_{ss}^\omega + \alpha CqUt_{ss} L_{ss}^s{}^{1-\alpha} Z_{ss}^{1-\alpha} (CqUt_{ss} K_{ss})^{-1+\alpha} \right) = 0 \quad (6.1)$$

$$\beta - \lambda_{ss}^U = 0 \quad (6.2)$$

$$-\lambda_{ss}^c + \mu C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.3)$$

$$-W_{ss} + (1 - \alpha) L_{ss}^s{}^{-\alpha} Z_{ss}^{1-\alpha} (CqUt_{ss} K_{ss})^\alpha = 0 \quad (6.4)$$

$$-Y_{ss} + L_{ss}^s{}^{1-\alpha} Z_{ss}^{1-\alpha} (CqUt_{ss} K_{ss})^\alpha = 0 \quad (6.5)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (6.6)$$

$$\lambda_{ss}^c W_{ss} + (-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.7)$$

$$-\delta \omega K_{ss} CqUt_{ss}^{-1+\omega} + \alpha K_{ss} L_{ss}^s{}^{1-\alpha} Z_{ss}^{1-\alpha} (CqUt_{ss} K_{ss})^{-1+\alpha} = 0 \quad (6.8)$$

$$I_{ss} - K_{ss} + K_{ss} (1 - \delta CqUt_{ss}^\omega) = 0 \quad (6.9)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.10)$$

$$-C_{ss} + \Pi_{ss} - \lambda_{ss}^U \Pi_{ss} + L_{ss}^s W_{ss} = 0 \quad (6.11)$$

$$-I_{ss} - \Pi_{ss} + Y_{ss} + \lambda_{ss}^U \Pi_{ss} - L_{ss}^s W_{ss} = 0 \quad (6.12)$$

7 Parameter settings

$$\alpha = 0.36 \quad (7.1)$$

$$\beta = 0.99 \quad (7.2)$$

$$\delta = 0.025 \quad (7.3)$$

$$\eta = 2 \quad (7.4)$$

$$\mu = 0.3 \quad (7.5)$$

$$\omega = 1.45 \quad (7.6)$$

$$\phi = 0.95 \quad (7.7)$$

8 Steady state values

| | Steady state values |
|-------------|---------------------|
| λ^c | 0.547 |
| λ^U | 0.99 |
| C | 0.7449 |
| $CqUt$ | 0.9284 |
| I | 0.246 |
| K | 10.96 |
| L^s | 0.2673 |
| Π | 11.0707 |
| U | -135.8123 |
| W | 2.3722 |
| Y | 0.9909 |
| Z | 1 |

9 The solution of the perturbation

9.1 P

$$\begin{array}{c} K_{t-1} \quad Z_{t-1} \\ K \left(\begin{array}{cc} 0.9758 & 0.0705 \\ 0 & 0.95 \end{array} \right) \\ Z \end{array}$$

9.2 Q

$$\begin{array}{c} \epsilon^Z \\ K \left(\begin{array}{c} 0.0742 \\ 1 \end{array} \right) \\ Z \end{array}$$

9.3 R

$$\begin{array}{c} K_{t-1} \quad Z_{t-1} \\ \lambda^c \\ \lambda^U \\ C \\ CqUt \\ I \\ L^s \\ \Pi \\ U \\ W \\ Y \end{array} \left(\begin{array}{cc} -0.4335 & -0.3499 \\ 0 & 0 \\ 0.2823 & 0.4185 \\ -0.74 & 1.0041 \\ -1.1491 & 4.5972 \\ -0.2604 & 0.7601 \\ 0.9893 & 0.0146 \\ -0.0446 & -0.0408 \\ 0.1873 & 0.6958 \\ -0.0731 & 1.456 \end{array} \right)$$

9.4 S

$$\begin{array}{c} \epsilon^Z \\ \lambda^c \\ \lambda^U \\ C \\ CqUt \\ I \\ L^s \\ \Pi \\ U \\ W \\ Y \end{array} \left(\begin{array}{c} -0.3683 \\ 0 \\ 0.4405 \\ 1.057 \\ 4.8392 \\ 0.8001 \\ 0.0153 \\ -0.0429 \\ 0.7325 \\ 1.5326 \end{array} \right)$$

10 Statistics of the model

10.1 Moments

| | Steady state value | Std. dev. | Variance | Loglinear |
|-------------|--------------------|-----------|----------|-----------|
| λ^c | 0.547 | 0.035 | 0.0012 | Y |
| λ^U | 0.99 | 0 | 0 | Y |
| C | 0.7449 | 0.0408 | 0.0017 | Y |
| $CqUt$ | 0.9284 | 0.1001 | 0.01 | Y |
| I | 0.246 | 0.4485 | 0.2011 | Y |
| K | 10.96 | 0.0245 | 0.0006 | Y |
| L^s | 0.2673 | 0.0744 | 0.0055 | Y |
| Π | 11.0707 | 0.0242 | 0.0006 | Y |
| U | -135.8123 | 0.004 | 0 | Y |
| W | 2.3722 | 0.0674 | 0.0045 | Y |
| Y | 0.9909 | 0.1414 | 0.02 | Y |
| Z | 1 | 0.0922 | 0.0085 | Y |

10.2 Correlation matrix

| | λ^c | λ^U | C | $CqUt$ | I | K | L^s | Π | U | W | Y | Z |
|-------------|-------------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--------|
| λ^c | 1 | 0 | -0.9905 | -0.8825 | -0.9322 | -0.5097 | -0.9236 | -0.3055 | 0.9994 | -0.9714 | -0.9491 | -0.953 |
| λ^U | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | -0.9905 | 0 | 1 | 0.9387 | 0.973 | 0.3867 | 0.9675 | 0.1719 | -0.9947 | 0.9948 | 0.9834 | 0.9856 |
| $CqUt$ | -0.8825 | 0 | 0.9387 | 1 | 0.9929 | 0.0452 | 0.9954 | -0.1781 | -0.8984 | 0.9689 | 0.9857 | 0.9835 |
| I | -0.9322 | 0 | 0.973 | 0.9929 | 1 | 0.1636 | 0.9997 | -0.0599 | -0.9442 | 0.9915 | 0.9987 | 0.998 |
| K | -0.5097 | 0 | 0.3867 | 0.0452 | 0.1636 | 1 | 0.1408 | 0.975 | -0.4794 | 0.2908 | 0.2128 | 0.2251 |
| L^s | -0.9236 | 0 | 0.9675 | 0.9954 | 0.9997 | 0.1408 | 1 | -0.0829 | -0.9364 | 0.9882 | 0.9973 | 0.9963 |
| Π | -0.3055 | 0 | 0.1719 | -0.1781 | -0.0599 | 0.975 | -0.0829 | 1 | -0.2722 | 0.0707 | -0.0099 | 0.0027 |
| U | 0.9994 | 0 | -0.9947 | -0.8984 | -0.9442 | -0.4794 | -0.9364 | -0.2722 | 1 | -0.9791 | -0.9595 | -0.963 |
| W | -0.9714 | 0 | 0.9948 | 0.9689 | 0.9915 | 0.2908 | 0.9882 | 0.0707 | -0.9791 | 1 | 0.9967 | 0.9977 |
| Y | -0.9491 | 0 | 0.9834 | 0.9857 | 0.9987 | 0.2128 | 0.9973 | -0.0099 | -0.9595 | 0.9967 | 1 | 0.9999 |
| Z | -0.953 | 0 | 0.9856 | 0.9835 | 0.998 | 0.2251 | 0.9963 | 0.0027 | -0.963 | 0.9977 | 0.9999 | 1 |

10.3 Autocorrelations

| | $t-1$ | $t-2$ | $t-3$ | $t-4$ | $t-5$ |
|-------------|--------|--------|--------|--------|---------|
| λ^c | 0.7487 | 0.5276 | 0.3371 | 0.1768 | 0.0454 |
| λ^U | NaN | NaN | NaN | NaN | NaN |
| C | 0.7277 | 0.494 | 0.2979 | 0.137 | 0.0087 |
| $CqUt$ | 0.7137 | 0.4719 | 0.272 | 0.1107 | -0.0155 |
| I | 0.7115 | 0.4684 | 0.2679 | 0.1066 | -0.0193 |
| K | 0.9603 | 0.8642 | 0.731 | 0.5765 | 0.4135 |
| L^s | 0.7114 | 0.4682 | 0.2676 | 0.1063 | -0.0196 |
| Π | 0.9636 | 0.8695 | 0.7371 | 0.5827 | 0.4193 |
| U | 0.7427 | 0.518 | 0.3259 | 0.1654 | 0.0349 |
| W | 0.7174 | 0.4777 | 0.2787 | 0.1176 | -0.0092 |
| Y | 0.7128 | 0.4703 | 0.2701 | 0.1089 | -0.0172 |
| Z | 0.7133 | 0.4711 | 0.2711 | 0.1098 | -0.0163 |

11 Statistics of the model

11.1 Moments relative to moments of the reference variable

| | Steady state value relative to Y | Std. dev. relative to Y | Variance relative to Y | Loglinear |
|-------------|------------------------------------|---------------------------|--------------------------|-----------|
| λ^c | 0.552 | 0.2476 | 0.0613 | Y |
| λ^U | 0.9991 | 0 | 0 | Y |
| C | 0.7517 | 0.2886 | 0.0833 | Y |
| $CapUt$ | 0.9369 | 0.7079 | 0.5011 | Y |
| I | 0.2483 | 3.1723 | 10.0637 | Y |
| K | 11.0607 | 0.1733 | 0.03 | Y |
| L^s | 0.2698 | 0.5261 | 0.2768 | Y |
| Π | 11.1724 | 0.1711 | 0.0293 | Y |
| U | -137.0598 | 0.0286 | 0.0008 | Y |
| W | 2.394 | 0.4768 | 0.2274 | Y |
| Y | 1 | 1 | 1 | Y |
| Z | 1.0092 | 0.652 | 0.4251 | Y |

11.2 Correlations with the reference variable

| | Y_{t-5} | Y_{t-4} | Y_{t-3} | Y_{t-2} | Y_{t-1} | Y_t | Y_{t+1} | Y_{t+2} | Y_{t+3} | Y_{t+4} | Y_{t+5} |
|-------------|-----------|-----------|-----------|-----------|-----------|---------|-----------|-----------|-----------|-----------|-----------|
| λ^c | 0.1893 | 0.0649 | -0.1047 | -0.3261 | -0.6056 | -0.9491 | -0.759 | -0.5852 | -0.4297 | -0.2936 | -0.1769 |
| λ^U | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| C | -0.1167 | 0.01 | 0.1782 | 0.3932 | 0.66 | 0.9834 | 0.7485 | 0.5426 | 0.3656 | 0.2168 | 0.0945 |
| $CapUt$ | 0.0755 | 0.1973 | 0.3474 | 0.5279 | 0.7405 | 0.9857 | 0.6585 | 0.3894 | 0.1736 | 0.0056 | -0.1203 |
| I | 0.0103 | 0.1355 | 0.2939 | 0.4889 | 0.7231 | 0.9987 | 0.6988 | 0.4477 | 0.2423 | 0.0785 | -0.0479 |
| K | -0.5404 | -0.4988 | -0.413 | -0.273 | -0.0682 | 0.2128 | 0.4076 | 0.5307 | 0.5952 | 0.6134 | 0.5959 |
| L^s | 0.0229 | 0.1476 | 0.3046 | 0.497 | 0.7273 | 0.9973 | 0.6917 | 0.4369 | 0.2292 | 0.0645 | -0.062 |
| Π | -0.5491 | -0.5352 | -0.4842 | -0.3865 | -0.232 | -0.0099 | 0.2549 | 0.436 | 0.5477 | 0.6029 | 0.6138 |
| U | 0.1712 | 0.046 | -0.1236 | -0.3437 | -0.6205 | -0.9595 | -0.7577 | -0.5754 | -0.4142 | -0.2746 | -0.1563 |
| W | -0.0614 | 0.0655 | 0.2305 | 0.438 | 0.6923 | 0.9967 | 0.7315 | 0.5043 | 0.3136 | 0.1572 | 0.0323 |
| Y | -0.0172 | 0.1089 | 0.2701 | 0.4703 | 0.7128 | 1 | 0.7128 | 0.4703 | 0.2701 | 0.1089 | -0.0172 |
| Z | -0.0242 | 0.1021 | 0.264 | 0.4655 | 0.7099 | 0.9999 | 0.716 | 0.4759 | 0.2771 | 0.1165 | -0.0095 |

12 Impulse response functions

12.1 Shock ϵ^Z

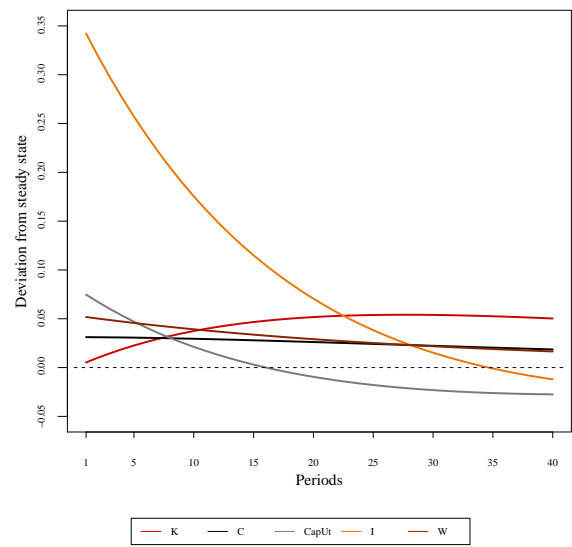


Figure 1: Impulse response function for ϵ^Z shock

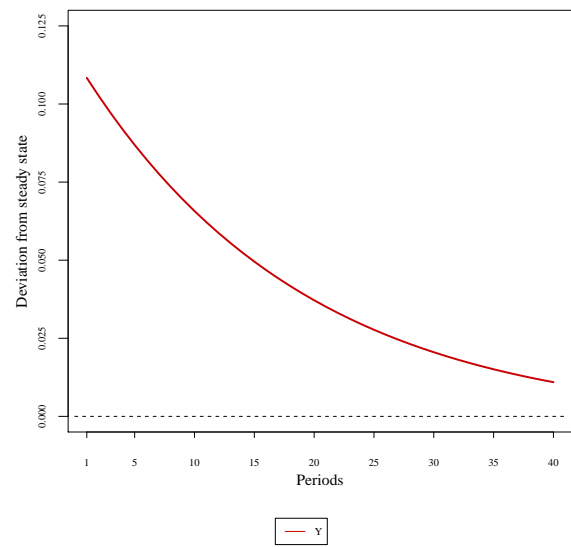


Figure 2: Impulse response function for ϵ^Z shock