

## 1 CONSUMER

### 1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t - \psi K_{t-1}^s \left( -\delta + K_{t-1}^{s-1} I_t \right)^2 \quad (\lambda_t^c) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left( \lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

### 1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[ \lambda_{t+1}^c \left( r_{t+1} - \psi \left( -\delta + K_t^{s-1} I_{t+1} \right)^2 + 2\psi K_t^{s-1} I_{t+1} \left( -\delta + K_t^{s-1} I_{t+1} \right) \right) \right] \right) = 0 \quad (K_t^s) \quad (1.4)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^2} + \lambda_t^c \left( -1 - 2\psi \left( -\delta + K_{t-1}^{s-1} I_t \right) \right) = 0 \quad (I_t) \quad (1.7)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left( \lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$\pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad \left( \lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

## 2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^2} r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^2} W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^2} = 0 \quad (\pi_t) \quad (2.7)$$

## 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.8)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.9)$$

# 3 EQUILIBRIUM

## 3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

# 4 EXOG

## 4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

## 5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}-1+\alpha} L_t^{\text{s}1-\alpha} = 0 \quad (5.1)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}\alpha} L_t^{\text{s}-\alpha} = 0 \quad (5.2)$$

$$-Y_t + Z_t K_{t-1}^s L_t^{s^{1-\alpha}} = 0 \quad (5.3)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$\beta \left( \mu E_t \left[ \left( r_{t+1} - \psi \left( -\delta + K_t^{s^{-1}} I_{t+1} \right)^2 + 2\psi K_t^{s^{-1}} I_{t+1} \left( -\delta + K_t^{s^{-1}} I_{t+1} \right) \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] - \mu (1 - \delta) E_t \left[ \left( -1 - 2\psi \left( -\delta + K_t^{s^{-1}} I_{t+1} \right) \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] \right) \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.7)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.8)$$

$$-C_t - I_t + Y_t - \psi K_{t-1}^s \left( -\delta + K_{t-1}^{s^{-1}} I_t \right)^2 = 0 \quad (5.9)$$

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## 6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s^{-1}+\alpha} L_{ss}^{s^{1-\alpha}} = 0 \quad (6.1)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s^\alpha} L_{ss}^{s^{-\alpha}} = 0 \quad (6.2)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s^\alpha} L_{ss}^{s^{1-\alpha}} = 0 \quad (6.3)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (6.4)$$

$$\beta \left( \mu \left( r_{ss} - \psi \left( -\delta + I_{ss} K_{ss}^{s^{-1}} \right)^2 + 2\psi I_{ss} K_{ss}^{s^{-1}} \left( -\delta + I_{ss} K_{ss}^{s^{-1}} \right) \right) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} - \mu \left( -1 - 2\psi \left( -\delta + I_{ss} K_{ss}^{s^{-1}} \right) \right) (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) \quad (6.5)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.7)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.8)$$

$$-C_{ss} - I_{ss} + Y_{ss} - \psi K_{ss}^s \left( -\delta + I_{ss} K_{ss}^{s-1} \right)^2 = 0 \quad (6.9)$$

## 7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^s = 0 \quad (7.1)$$

## 8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$\phi = 0.95 \quad (8.5)$$

$$\psi = 0.8 \quad (8.6)$$

## 9 Steady-state values

	Steady-state values
$r$	0.0351
$C$	0.7422
$I$	0.2559
$K^s$	10.2368
$L^s$	0.2695
$U$	-136.2372
$W$	2.3706
$Y$	0.9981
$Z$	1

## 10 The solution of the perturbation

### 10.1 P

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ K^s & \begin{pmatrix} 0.9658 & 0.0863 \end{pmatrix} \\ Z & \begin{pmatrix} 0 & 0.95 \end{pmatrix} \end{matrix}$$

### 10.2 Q

$$\begin{matrix} & \epsilon^Z \\ K^s & \begin{pmatrix} 0.0908 \end{pmatrix} \\ Z & \begin{pmatrix} 1 \end{pmatrix} \end{matrix}$$

### 10.3 R

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ r & \begin{pmatrix} -0.7408 & 1.2972 \end{pmatrix} \\ C & \begin{pmatrix} 0.4748 & 0.5545 \end{pmatrix} \\ I & \begin{pmatrix} -0.3661 & 3.4511 \end{pmatrix} \\ L^s & \begin{pmatrix} -0.1575 & 0.5426 \end{pmatrix} \\ U & \begin{pmatrix} -0.0418 & -0.0644 \end{pmatrix} \\ W & \begin{pmatrix} 0.4167 & 0.7547 \end{pmatrix} \\ Y & \begin{pmatrix} 0.2592 & 1.2972 \end{pmatrix} \end{matrix}$$

### 10.4 S

$$\begin{matrix} & \epsilon^Z \\ r & \begin{pmatrix} 1.3655 \end{pmatrix} \\ C & \begin{pmatrix} 0.5837 \end{pmatrix} \\ I & \begin{pmatrix} 3.6328 \end{pmatrix} \\ L^s & \begin{pmatrix} 0.5711 \end{pmatrix} \\ U & \begin{pmatrix} -0.0678 \end{pmatrix} \\ W & \begin{pmatrix} 0.7944 \end{pmatrix} \\ Y & \begin{pmatrix} 1.3655 \end{pmatrix} \end{matrix}$$

## 11 Statistics of the model

### 11.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
$r$	0.0351	0.1814	0.0329	Y
$C$	0.7422	0.0783	0.0061	Y
$I$	0.2559	0.4741	0.2248	Y
$K^s$	10.2368	0.0422	0.0018	Y
$L^s$	0.2695	0.0749	0.0056	Y
$U$	-136.2372	0.009	0.0001	Y
$W$	2.3706	0.1047	0.011	Y
$Y$	0.9981	0.1781	0.0317	Y
$Z$	1	0.1303	0.017	Y

### 11.2 Correlation matrix

	$r$	$C$	$I$	$K^s$	$L^s$	$U$	$W$	$Y$	$Z$
$r$	1	0.9082	0.9901	0.0897	0.9965	-0.9321	0.9422	0.9726	0.9851
$C$	0.9082	1	0.9579	0.4983	0.9402	-0.9981	0.996	0.9806	0.9667
$I$	0.9901	0.9579	1	0.2284	0.9984	-0.9736	0.9798	0.9956	0.9995
$K^s$	0.0897	0.4983	0.2284	1	0.1733	-0.4445	0.4184	0.3187	0.2599
$L^s$	0.9965	0.9402	0.9984	0.1733	1	-0.9592	0.967	0.9887	0.9961
$U$	-0.9321	-0.9981	-0.9736	-0.4445	-0.9592	1	-0.9996	-0.9907	-0.9805
$W$	0.9422	0.996	0.9798	0.4184	0.967	-0.9996	1	0.9942	0.9858
$Y$	0.9726	0.9806	0.9956	0.3187	0.9887	-0.9907	0.9942	1	0.9981
$Z$	0.9851	0.9667	0.9995	0.2599	0.9961	-0.9805	0.9858	0.9981	1

### 11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
$r$	0.7103	0.4664	0.2655	0.1042	-0.0215
$C$	0.7446	0.5209	0.3292	0.1686	0.0376
$I$	0.7115	0.4684	0.2679	0.1066	-0.0193
$K^s$	0.9598	0.8626	0.7281	0.5723	0.4082
$L^s$	0.7098	0.4657	0.2647	0.1034	-0.0223
$U$	0.7346	0.505	0.3106	0.1498	0.0204
$W$	0.7304	0.4983	0.3028	0.1419	0.0131
$Y$	0.7179	0.4786	0.2798	0.1186	-0.0083
$Z$	0.7133	0.4711	0.2711	0.1098	-0.0163

## 12 Statistics of the model

### 12.1 Moments relative to moments of the reference variable

	Steady-state value relative to $Y$	Std. dev. relative to $Y$	Variance relative to $Y$	Loglinear
$r$	0.0352	1.0184	1.0372	Y
$C$	0.7436	0.4395	0.1931	Y
$I$	0.2564	2.6621	7.0869	Y
$K^s$	10.2561	0.2368	0.0561	Y
$L^s$	0.27	0.4205	0.1768	Y
$U$	-136.4937	0.0504	0.0025	Y
$W$	2.3751	0.5877	0.3453	Y
$Y$	1	1	1	Y
$Z$	1.0019	0.7319	0.5357	Y

## 12.2 Correlations with the reference variable

	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$r$	0.1089	0.228	0.3727	0.5446	0.7446	0.9726	0.6308	0.3527	0.1323	-0.0369	-0.1614
$C$	-0.1067	0.0213	0.1894	0.4025	0.665	0.9806	0.7609	0.5644	0.3923	0.2448	0.1212
$I$	0.039	0.1636	0.3192	0.5084	0.7335	0.9956	0.6875	0.4309	0.222	0.0566	-0.0702
$K^s$	-0.4795	-0.4216	-0.3213	-0.1704	0.0399	0.3187	0.5039	0.6124	0.6595	0.6589	0.6227
$L^s$	0.0671	0.1898	0.3414	0.5242	0.7397	0.9887	0.6664	0.4006	0.1865	0.0192	-0.1069
$U$	0.0765	-0.0517	-0.2183	-0.4279	-0.6842	-0.9907	-0.7507	-0.54	-0.3589	-0.2065	-0.0814
$W$	-0.0621	0.066	0.2318	0.4393	0.6925	0.9942	0.7449	0.5278	0.3426	0.1881	0.0624
$Y$	-0.0083	0.1186	0.2798	0.4786	0.7179	1	0.7179	0.4786	0.2798	0.1186	-0.0083
$Z$	0.0226	0.1481	0.3058	0.4986	0.7288	0.9981	0.6988	0.4479	0.2423	0.0782	-0.0488

## 13 Impulse response functions

### 13.1 Shock $\epsilon^Z$

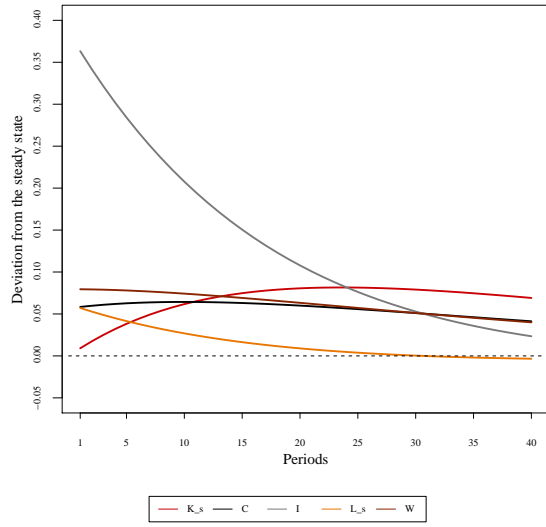


Figure 1: Impulse response function for  $\epsilon^Z$  shock

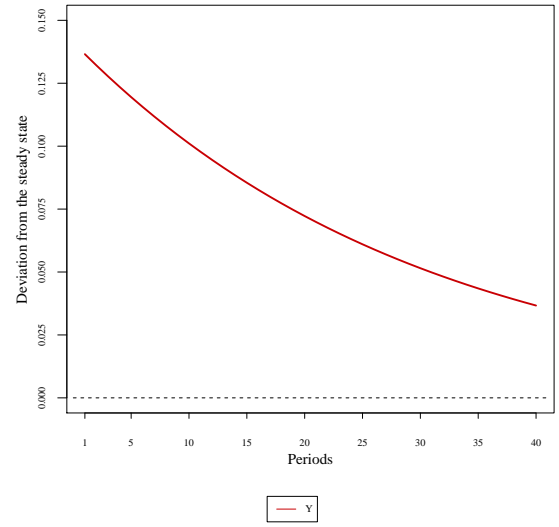


Figure 2: Impulse response function for  $\epsilon^Z$  shock