

## Index sets

$$\begin{aligned} agents &= \{A, B\} \\ goods &= \{1, 2, 3\} \end{aligned}$$

## 1 AGENTS $a \in agents$

### 1.1 Optimisation problem

$$\max_{(C^{(a,g)})_{g \in goods}} U^{(a)} = \prod_{g \in goods} C^{(a,g)} \alpha^{(a,g)} \quad (1.1)$$

s.t. :

$$\sum_{g \in goods} p^{(g)} C^{(a,g)} = \sum_{g \in goods} e^{(a,g)} p^{(g)} \quad \left( \lambda^{\text{AGENTS}^1(a)} \right) \quad (1.2)$$

### 1.2 Identities

$$g \in goods: \quad e_t^{(a,g)} = e^{\text{calibr}(a,g)} \quad (1.3)$$

### 1.3 First order conditions

$$g \in goods: \quad -\lambda^{\text{AGENTS}^1(a)}_t p_t^{(g)} + \alpha^{(a,g)} C_t^{(a,g)}{}^{-1} \left( \prod_{g' \in goods} C_t^{(a,g')} \alpha^{(a,g')} \right) = 0 \quad (C_t^{(a,g)}) \quad (1.4)$$

## 2 EQUILIBRIUM

### 2.1 Identities

$$p^{(1)} = 1 \quad (2.1)$$

$$g \in goods \setminus \{1\}: \quad \sum_{a \in agents} C_t^{(a,g)} = \sum_{a \in agents} e_t^{(a,g)} \quad (2.2)$$

## 3 Equilibrium relationships (before expansion and reduction)

$$-1 + p^{(1)} = 0 \quad (3.1)$$

$$a \in agents: \quad U^{(a)} - \prod_{g \in goods} C^{(a,g)} \alpha^{(a,g)} = 0 \quad (3.2)$$

$$a \in agents: \quad \sum_{g \in goods} e^{(a,g)} p^{(g)} - \sum_{g \in goods} p^{(g)} C^{(a,g)} = 0 \quad (3.3)$$

$$a \in agents: \quad g \in goods: \quad -e^{\text{calibr}(a,g)} + e^{(a,g)} = 0 \quad (3.4)$$

$$a \in agents: \quad g \in goods: \quad -\lambda^{\text{AGENTS}^1(a)} p^{(g)} + \alpha^{(a,g)} C^{(a,g)}{}^{-1} \left( \prod_{g' \in goods} C^{(a,g')} \alpha^{(a,g')} \right) = 0 \quad (3.5)$$

$$g \in goods \setminus \{1\}: \quad -\sum_{a \in agents} e^{(a,g)} + \sum_{a \in agents} C^{(a,g)} = 0 \quad (3.6)$$

## 4 Equilibrium relationships (after expansion and reduction)

$$-1 + p^{(1)} = 0 \quad (4.1)$$

$$-e^{\text{calibr}\langle A,1 \rangle} + e^{\langle A,1 \rangle} = 0 \quad (4.2)$$

$$-e^{\text{calibr}\langle A,2 \rangle} + e^{\langle A,2 \rangle} = 0 \quad (4.3)$$

$$-e^{\text{calibr}\langle A,3 \rangle} + e^{\langle A,3 \rangle} = 0 \quad (4.4)$$

$$-e^{\text{calibr}\langle B,1 \rangle} + e^{\langle B,1 \rangle} = 0 \quad (4.5)$$

$$-e^{\text{calibr}\langle B,2 \rangle} + e^{\langle B,2 \rangle} = 0 \quad (4.6)$$

$$-e^{\text{calibr}\langle B,3 \rangle} + e^{\langle B,3 \rangle} = 0 \quad (4.7)$$

$$U^{\langle A \rangle} - C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.8)$$

$$U^{\langle B \rangle} - C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.9)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{\langle 1 \rangle} + \alpha^{\langle A,1 \rangle} C^{\langle A,1 \rangle -1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.10)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{\langle 2 \rangle} + \alpha^{\langle A,2 \rangle} C^{\langle A,2 \rangle -1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.11)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{\langle 3 \rangle} + \alpha^{\langle A,3 \rangle} C^{\langle A,3 \rangle -1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.12)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{\langle 1 \rangle} + \alpha^{\langle B,1 \rangle} C^{\langle B,1 \rangle -1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.13)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{\langle 2 \rangle} + \alpha^{\langle B,2 \rangle} C^{\langle B,2 \rangle -1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.14)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{\langle 3 \rangle} + \alpha^{\langle B,3 \rangle} C^{\langle B,3 \rangle -1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.15)$$

$$-e^{\langle A,2 \rangle} - e^{\langle B,2 \rangle} + C^{\langle A,2 \rangle} + C^{\langle B,2 \rangle} = 0 \quad (4.16)$$

$$-e^{\langle A,3 \rangle} - e^{\langle B,3 \rangle} + C^{\langle A,3 \rangle} + C^{\langle B,3 \rangle} = 0 \quad (4.17)$$

$$e^{\langle A,1 \rangle} p^{\langle 1 \rangle} + e^{\langle A,2 \rangle} p^{\langle 2 \rangle} + e^{\langle A,3 \rangle} p^{\langle 3 \rangle} - p^{\langle 1 \rangle} C^{\langle A,1 \rangle} - p^{\langle 2 \rangle} C^{\langle A,2 \rangle} - p^{\langle 3 \rangle} C^{\langle A,3 \rangle} = 0 \quad (4.18)$$

$$e^{\langle B,1 \rangle} p^{\langle 1 \rangle} + e^{\langle B,2 \rangle} p^{\langle 2 \rangle} + e^{\langle B,3 \rangle} p^{\langle 3 \rangle} - p^{\langle 1 \rangle} C^{\langle B,1 \rangle} - p^{\langle 2 \rangle} C^{\langle B,2 \rangle} - p^{\langle 3 \rangle} C^{\langle B,3 \rangle} = 0 \quad (4.19)$$

## 5 Equilibrium values

	Equilibrium values
$e^{A^1}$	3
$e^{A^2}$	2
$e^{A^3}$	1
$\lambda^{\text{AGENTS}^1 A}$	0.2674
$C^{A^1}$	2.2667
$C^{A^2}$	1.7
$C^{A^3}$	2.2667
$U^A$	2.0203

## 6 Equilibrium values

	Equilibrium values
$e^{B^1}$	1
$e^{B^2}$	1
$e^{B^3}$	3
$\lambda^{\text{AGENTS}^1 B}$	0.2674
$C^{B^1}$	1.7333
$C^{B^2}$	1.3
$C^{B^3}$	1.7333
$U^B$	1.5449