

Index sets

$$HH = \{1, 2\}$$

$$SEC = \{A, B, C\}$$

1 CONSUMER $h \in HH$

1.1 Optimisation problem

$$\max_{(D^{(s,h)})_{s \in SEC}} U^{(h)} = \left(\sum_{s \in SEC} \alpha^{(s,h)} D^{(s,h)} \omega^{-1} (-1+\omega) \right)^{\omega(-1+\omega)^{-1}} \quad (1.1)$$

s.t. :

$$INC^{(h)} + \Pi^{(h)} = \sum_{s \in SEC} p^{(s)} D^{(s,h)} \quad \left(\lambda^{\text{CONSUMER}^1(h)} \right) \quad (1.2)$$

1.2 Identities

$$INC^{(h)} = L^{(h)} + p^k K^{(h)} \quad (1.3)$$

$$K^{(h)} = k s^{\text{data}(h)} \quad (1.4)$$

$$L^{(h)} = l s^{\text{data}(h)} \quad (1.5)$$

1.3 First order conditions

$$s \in SEC: \quad \lambda^{\text{CONSUMER}^1_t(h)} p_t^{(s)} + \alpha^{(s,h)} D_t^{(s,h)} \omega^{-1} (-1+\omega) \left(\sum_{s \in SEC} \alpha^{(s,h)} D_t^{(s,h)} \omega^{-1} (-1+\omega) \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad \left(D_t^{(s,h)} \right) \quad (1.6)$$

2 FIRM $s \in SEC$

2.1 Optimisation problem

$$\max_{Y^{(s)}, K^{(s)}, L^{(s)}, (X^{(si, s)})_{si \in SEC}} \pi^{(s)} = -L^{(s)} - p^k K^{(s)} + p^{(s)} Y^{(s)} - \sum_{si \in SEC} p^{(si)} X^{(si, s)} \quad (2.1)$$

s.t. :

$$Y^{(s)} = \gamma^{(s)} K^{(s) \beta^k(s)} L^{(s) \beta^l(s)} \left(\prod_{si \in SEC} X^{(si, s) \beta^x(s)} \right) \left(\lambda^{\text{FIRM}^1(s)} \right) \quad (2.2)$$

2.2 First order conditions

$$-\lambda^{\text{FIRM}^1(s)} + p^{(s)} = 0 \quad \left(Y^{(s)} \right) \quad (2.3)$$

$$-p^k + \beta^k(s) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} K^{(s) -1 + \beta^k(s)} L^{(s) \beta^l(s)} \left(\prod_{si \in SEC} X^{(si, s) \beta^x(s)} \right) = 0 \quad \left(K^{(s)} \right) \quad (2.4)$$

$$-1 + \beta^l(s) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} K^{(s) \beta^k(s)} L^{(s) -1 + \beta^l(s)} \left(\prod_{si \in SEC} X^{(si, s) \beta^x(s)} \right) = 0 \quad \left(L^{(s)} \right) \quad (2.5)$$

$$si \in SEC: \quad -p_t^{(si)} + \beta^x(s) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} X_t^{(si, s) -1} K_t^{(s) \beta^k(s)} L_t^{(s) \beta^l(s)} \left(\prod_{si' \in SEC} X_t^{(si', s) \beta^x(s)} \right) = 0 \quad \left(X_t^{(si, s)} \right) \quad (2.6)$$

2.3 First order conditions after reduction

$$-p^k + \beta^k(s) \gamma^{(s)} p^{(s)} K^{(s) -1 + \beta^k(s)} L^{(s) \beta^l(s)} \left(\prod_{si \in SEC} X^{(si, s) \beta^x(s)} \right) = 0 \quad \left(K^{(s)} \right) \quad (2.7)$$

$$-1 + \beta^l(s) \gamma^{(s)} p^{(s)} K^{(s) \beta^k(s)} L^{(s) -1 + \beta^l(s)} \left(\prod_{si \in SEC} X^{(si, s) \beta^x(s)} \right) = 0 \quad \left(L^{(s)} \right) \quad (2.8)$$

$$si \in SEC: \quad -p_t^{(si)} + \beta^x(s) \gamma^{(s)} p_t^{(s)} X_t^{(si, s) -1} K_t^{(s) \beta^k(s)} L_t^{(s) \beta^l(s)} \left(\prod_{si' \in SEC} X_t^{(si', s) \beta^x(s)} \right) = 0 \quad \left(\left(X_t^{(si, s)} \right)_{si \in SEC} \right) \quad (2.9)$$

3 EQUILIBRIUM

3.1 Identities

$$\sum_{h \in HH} K^{\langle h \rangle} = \sum_{s \in SEC} K^{\langle s \rangle} \quad (3.1)$$

$$s \in SEC: \quad p_t^{\langle s \rangle} = 1 \quad (3.2)$$

$$h \in HH: \quad \Pi_t^{\langle h \rangle} = \sum_{s \in SEC} \pi^h \pi_t^{\langle s \rangle} \quad (3.3)$$

4 Equilibrium relationships (before expansion and reduction)

$$\sum_{h \in HH} K^{\langle h \rangle} - \sum_{s \in SEC} K^{\langle s \rangle} = 0 \quad (4.1)$$

$$h \in HH: \quad -ks^{\text{data}\langle h \rangle} + K^{\langle h \rangle} = 0 \quad (4.2)$$

$$h \in HH: \quad -ls^{\text{data}\langle h \rangle} + L^{\langle h \rangle} = 0 \quad (4.3)$$

$$h \in HH: \quad \Pi^{\langle h \rangle} - \sum_{s \in SEC} \pi^h \pi^{\langle s \rangle} = 0 \quad (4.4)$$

$$h \in HH: \quad U^{\langle h \rangle} - \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (4.5)$$

$$h \in HH: \quad -INC^{\langle h \rangle} - \Pi^{\langle h \rangle} + \sum_{s \in SEC} p^{\langle s \rangle} D^{\langle s, h \rangle} = 0 \quad (4.6)$$

$$h \in HH: \quad INC^{\langle h \rangle} - L^{\langle h \rangle} - p^k K^{\langle h \rangle} = 0 \quad (4.7)$$

$$h \in HH: \quad s \in SEC: \quad \lambda^{\text{CONSUMER}^1 \langle h \rangle} p^{\langle s \rangle} + \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle -1+\omega^{-1}(-1+\omega)} \left(\sum_{s \in SEC} \alpha^{\langle s, h \rangle} D^{\langle s, h \rangle \omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (4.8)$$

$$s \in SEC: \quad -1 + p^{\langle s \rangle} = 0 \quad (4.9)$$

$$s \in SEC: \quad -1 + \beta^{l\langle s \rangle} \gamma^{\langle s \rangle} p^{\langle s \rangle} K^{\langle s \rangle \beta^{k\langle s \rangle}} L^{\langle s \rangle -1 + \beta^{l\langle s \rangle}} \left(\prod_{\mathbf{s}\dot{\mathbf{i}} \in SEC} X^{\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle \beta^{x\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle}} \right) = 0 \quad (4.10)$$

$$s \in SEC: \quad -p^k + \beta^{k\langle s \rangle} \gamma^{\langle s \rangle} p^{\langle s \rangle} K^{\langle s \rangle -1 + \beta^{k\langle s \rangle}} L^{\langle s \rangle \beta^{l\langle s \rangle}} \left(\prod_{\mathbf{s}\dot{\mathbf{i}} \in SEC} X^{\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle \beta^{x\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle}} \right) = 0 \quad (4.11)$$

$$s \in SEC: \quad -Y^{\langle s \rangle} + \gamma^{\langle s \rangle} K^{\langle s \rangle \beta^{k\langle s \rangle}} L^{\langle s \rangle \beta^{l\langle s \rangle}} \left(\prod_{\mathbf{s}\dot{\mathbf{i}} \in SEC} X^{\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle \beta^{x\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle}} \right) = 0 \quad (4.12)$$

$$s \in SEC: \quad \pi^{\langle s \rangle} + L^{\langle s \rangle} + p^k K^{\langle s \rangle} - p^{\langle s \rangle} Y^{\langle s \rangle} + \sum_{\mathbf{s}\dot{\mathbf{i}} \in SEC} p^{\langle \mathbf{s}\dot{\mathbf{i}} \rangle} X^{\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle} = 0 \quad (4.13)$$

$$s \in SEC: \quad \mathbf{s}\dot{\mathbf{i}} \in SEC: \quad -p^{\langle \mathbf{s}\dot{\mathbf{i}} \rangle} + \beta^{x\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle} \gamma^{\langle s \rangle} p^{\langle s \rangle} X^{\langle \mathbf{s}\dot{\mathbf{i}}, s \rangle -1} K^{\langle s \rangle \beta^{k\langle s \rangle}} L^{\langle s \rangle \beta^{l\langle s \rangle}} \left(\prod_{\mathbf{s}\dot{\mathbf{i}}' \in SEC} X^{\langle \mathbf{s}\dot{\mathbf{i}}', s \rangle \beta^{x\langle \mathbf{s}\dot{\mathbf{i}}', s \rangle}} \right) = 0 \quad (4.14)$$

5 Equilibrium relationships (after expansion and reduction)

$$-1 + p^{\langle A \rangle} = 0 \quad (5.1)$$

$$-1 + p^{\langle B \rangle} = 0 \quad (5.2)$$

$$-1 + p^{\langle C \rangle} = 0 \quad (5.3)$$

$$-1 + \beta^{l\langle A \rangle} \gamma^{\langle A \rangle} p^{\langle A \rangle} K^{\langle A \rangle \beta^{k\langle A \rangle}} L^{\langle A \rangle -1 + \beta^{l\langle A \rangle}} X^{\langle B, A \rangle \beta^{x\langle B, A \rangle}} X^{\langle C, A \rangle \beta^{x\langle C, A \rangle}} X^{\langle A, A \rangle \beta^{x\langle A, A \rangle}} = 0 \quad (5.4)$$

$$-1 + \beta^{l\langle B \rangle} \gamma^{\langle B \rangle} p^{\langle B \rangle} K^{\langle B \rangle \beta^{k\langle B \rangle}} L^{\langle B \rangle -1 + \beta^{l\langle B \rangle}} X^{\langle B, B \rangle \beta^{x\langle B, B \rangle}} X^{\langle C, B \rangle \beta^{x\langle C, B \rangle}} X^{\langle A, B \rangle \beta^{x\langle A, B \rangle}} = 0 \quad (5.5)$$

$$-1 + \beta^{l\langle C \rangle} \gamma^{\langle C \rangle} p^{\langle C \rangle} K^{\langle C \rangle \beta^{k\langle C \rangle}} L^{\langle C \rangle -1 + \beta^{l\langle C \rangle}} X^{\langle B, C \rangle \beta^{x\langle B, C \rangle}} X^{\langle C, C \rangle \beta^{x\langle C, C \rangle}} X^{\langle A, C \rangle \beta^{x\langle A, C \rangle}} = 0 \quad (5.6)$$

$$-k_{\mathbf{s}}^{\text{data}\langle 1 \rangle} + K^{\langle 1 \rangle} = 0 \quad (5.7)$$

$$-k_{\mathbf{s}}^{\text{data}\langle 2 \rangle} + K^{\langle 2 \rangle} = 0 \quad (5.8)$$

$$-l_s^{\text{data}\langle 1 \rangle} + L^{\langle 1 \rangle} = 0 \quad (5.9)$$

$$-l^{\text{data}(2)} + L^{(2)} = 0 \quad (5.10)$$

$$-p^k + \beta^{k(A)} \gamma^{(A)} p^{(A)} K^{(A)-1 + \beta^{k(A)}} L^{(A)\beta^{l(A)}} X^{(A,A)\beta^{x(A,A)}} X^{(B,A)\beta^{x(B,A)}} X^{(C,A)\beta^{x(C,A)}} = 0 \quad (5.11)$$

$$-p^k + \beta^{k(B)} \gamma^{(B)} p^{(B)} K^{(B)-1 + \beta^{k(B)}} L^{(B)\beta^{1(B)}} X^{(A,B)\beta^{x(A,B)}} X^{(B,B)\beta^{x(B,B)}} X^{(C,B)\beta^{x(C,B)}} = 0 \quad (5.12)$$

$$-p^k + \beta^{k\langle C \rangle} \gamma^{\langle C \rangle} p^{\langle C \rangle} K^{\langle C \rangle - 1 + \beta^{k\langle C \rangle}} L^{\langle C \rangle \beta^{1\langle C \rangle}} X^{\langle A, C \rangle \beta^{\mathbf{x}\langle A, C \rangle}} X^{\langle B, C \rangle \beta^{\mathbf{x}\langle B, C \rangle}} X^{\langle C, C \rangle \beta^{\mathbf{x}\langle C, C \rangle}} = 0 \quad (5.13)$$

$$-p^{\langle A \rangle} + \beta^{\mathbf{x} \langle A, A \rangle} \gamma^{\langle A \rangle} p^{\langle A \rangle} X^{\langle A, A \rangle - 1} K^{\langle A \rangle \beta^{\mathbf{k} \langle A \rangle}} L^{\langle A \rangle \beta^{\mathbf{l} \langle A \rangle}} X^{\langle A, A \rangle \beta^{\mathbf{x} \langle A, A \rangle}} X^{\langle B, A \rangle \beta^{\mathbf{x} \langle B, A \rangle}} X^{\langle C, A \rangle \beta^{\mathbf{x} \langle C, A \rangle}} = 0 \quad (5.14)$$

$$-p^{(A)} + \beta^{\mathbf{x}(A,B)} \gamma^{(B)} p^{(B)} X^{(A,B)-1} K^{(B)} \beta^{\mathbf{k}(B)} L^{(B)} \beta^{\mathbf{l}(B)} X^{(A,B)} \beta^{\mathbf{x}(A,B)} X^{(B,B)} \beta^{\mathbf{x}(B,B)} X^{(C,B)} \beta^{\mathbf{x}(C,B)} = 0 \quad (5.15)$$

$$-p^{(A)} + \beta^{x(A,C)} \gamma^{(C)} p^{(C)} X^{(A,C)-1} K^{(C)\beta^{k(C)}} L^{(C)\beta^{l(C)}} X^{(A,C)\beta^{x(A,C)}} X^{(B,C)\beta^{x(B,C)}} X^{(C,C)\beta^{x(C,C)}} = 0 \quad (5.16)$$

$$-p^{(B)} + \beta^{x(B,A)} \gamma^{(A)} p^{(A)} X^{(B,A)-1} K^{(A)\beta^{k(A)}} L^{(A)\beta^{l(A)}} X^{(A,A)\beta^{x(A,A)}} X^{(B,A)\beta^{x(B,A)}} X^{(C,A)\beta^{x(C,A)}} = 0 \quad (5.17)$$

$$-p^{(B)} + \beta^{x(B,B)} \gamma^{(B)} p^{(B)} X^{(B,B)-1} K^{(B)} \beta^{k(B)} L^{(B)} \beta^{l(B)} X^{(A,B)} \beta^{x(A,B)} X^{(B,B)} \beta^{x(B,B)} X^{(C,B)} \beta^{x(C,B)} = 0 \quad (5.18)$$

$$-p^{(B)} + \beta^{x(B,C)} \gamma^{(C)} p^{(C)} X^{(B,C)-1} K^{(C)\beta^{k(C)}} L^{(C)\beta^{l(C)}} X^{(A,C)\beta^{x(A,C)}} X^{(B,C)\beta^{x(B,C)}} X^{(C,C)\beta^{x(C,C)}} = 0 \quad (5.19)$$

$$-p^{(C)} + \beta^{x(C,A)} \gamma^{(A)} p^{(A)} X^{(C,A)-1} K^{(A)} \beta^{k(A)} L^{(A)} \beta^{l(A)} X^{(A,A)} \beta^{x(A,A)} X^{(B,A)} \beta^{x(B,A)} X^{(C,A)} \beta^{x(C,A)} = 0 \quad (5.20)$$

$$-p^{(C)} + \beta^{x(C,B)} \gamma^{(B)} p^{(B)} X^{(C,B)-1} K^{(B)\beta^{k(B)}} L^{(B)\beta^{l(B)}} X^{(A,B)\beta^{x(A,B)}} X^{(B,B)\beta^{x(B,B)}} X^{(C,B)\beta^{x(C,B)}} = 0 \quad (5.21)$$

$$-p^{(C)} + \beta^{\mathbf{x}(C,C)} \gamma^{(C)} p^{(C)} X^{(C,C)-1} K^{(C)\beta^{k(C)}} L^{(C)\beta^{l(C)}} X^{(A,C)\beta^{\mathbf{x}(A,C)}} X^{(B,C)\beta^{\mathbf{x}(B,C)}} X^{(C,C)\beta^{\mathbf{x}(C,C)}} = 0 \quad (5.22)$$

$$U^{(1)} - \left(\alpha^{\langle A,1 \rangle} D^{\langle A,1 \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle B,1 \rangle} D^{\langle B,1 \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle C,1 \rangle} D^{\langle C,1 \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.23)$$

$$U^{(2)} - \left(\alpha^{\langle A,2 \rangle} D^{\langle A,2 \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle B,2 \rangle} D^{\langle B,2 \rangle \omega^{-1}(-1+\omega)} + \alpha^{\langle C,2 \rangle} D^{\langle C,2 \rangle \omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.24)$$

$$-Y^{\langle A \rangle} + \gamma^{\langle A \rangle} K^{\langle A \rangle \beta^k \langle A \rangle} L^{\langle A \rangle \beta^l \langle A \rangle} X^{\langle A, A \rangle \beta^x \langle A, A \rangle} X^{\langle B, A \rangle \beta^x \langle B, A \rangle} X^{\langle C, A \rangle \beta^x \langle C, A \rangle} = 0 \quad (5.25)$$

$$-Y^{\langle B \rangle} + \gamma^{\langle B \rangle} K^{\langle B \rangle \beta^k \langle B \rangle} L^{\langle B \rangle \beta^l \langle B \rangle} X^{\langle A, B \rangle \beta^x \langle A, B \rangle} X^{\langle B, B \rangle \beta^x \langle B, B \rangle} X^{\langle C, B \rangle \beta^x \langle C, B \rangle} = 0 \quad (5.26)$$

$$-Y^{(C)} + \gamma^{(C)} K^{(C)\beta^k(C)} L^{(C)\beta^l(C)} X^{(A,C)\beta^x(A,C)} X^{(B,C)\beta^x(B,C)} X^{(C,C)\beta^x(C,C)} = 0 \quad (5.27)$$

$$\lambda^{\text{CONSUMER}^1 \langle 1 \rangle} p^{\langle A \rangle} + \alpha^{\langle A, 1 \rangle} D^{\langle A, 1 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 1 \rangle} D^{\langle A, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 1 \rangle} D^{\langle B, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 1 \rangle} D^{\langle C, 1 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.28)$$

$$\lambda^{\text{CONSUMER}^1 \langle 1 \rangle} p^{\langle B \rangle} + \alpha^{\langle B, 1 \rangle} D^{\langle B, 1 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 1 \rangle} D^{\langle A, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 1 \rangle} D^{\langle B, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 1 \rangle} D^{\langle C, 1 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.29)$$

$$\lambda^{\text{CONSUMER}^1 \langle 1 \rangle} p^{\langle C \rangle} + \alpha^{\langle C, 1 \rangle} D^{\langle C, 1 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 1 \rangle} D^{\langle A, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 1 \rangle} D^{\langle B, 1 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 1 \rangle} D^{\langle C, 1 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.30)$$

$$\lambda^{\text{CONSUMER}^{1\langle 2 \rangle}} p^{\langle A \rangle} + \alpha^{\langle A, 2 \rangle} D^{\langle A, 2 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 2 \rangle} D^{\langle A, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 2 \rangle} D^{\langle B, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 2 \rangle} D^{\langle C, 2 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.31)$$

$$\lambda^{\text{CONSUMER}^1 \langle 2 \rangle} p^{\langle B \rangle} + \alpha^{\langle B, 2 \rangle} D^{\langle B, 2 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 2 \rangle} D^{\langle A, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 2 \rangle} D^{\langle B, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 2 \rangle} D^{\langle C, 2 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.32)$$

$$\lambda^{\text{CONSUMER}^{1\langle 2 \rangle}} p^{\langle C \rangle} + \alpha^{\langle C, 2 \rangle} D^{\langle C, 2 \rangle - 1 + \omega^{-1}(-1 + \omega)} \left(\alpha^{\langle A, 2 \rangle} D^{\langle A, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle B, 2 \rangle} D^{\langle B, 2 \rangle \omega^{-1}(-1 + \omega)} + \alpha^{\langle C, 2 \rangle} D^{\langle C, 2 \rangle \omega^{-1}(-1 + \omega)} \right)^{-1 + \omega(-1 + \omega)^{-1}} = 0 \quad (5.33)$$

$$INC^{(1)} - L^{(1)} - p^k K^{(1)} = 0 \quad (5.34)$$

$$INC^{(2)} - L^{(2)} - p^k K^{(2)} = 0 \quad (5.35)$$

$$\Pi^{(1)} - \pi^{h(1)} \pi^{(A)} - \pi^{h(1)} \pi^{(B)} - \pi^{h(1)} \pi^{(C)} = 0 \quad (5.36)$$

$$\Pi^{\langle 2 \rangle} - \pi^{\text{h}\langle 2 \rangle} \pi^{\langle \text{A} \rangle} - \pi^{\text{h}\langle 2 \rangle} \pi^{\langle \text{B} \rangle} - \pi^{\text{h}\langle 2 \rangle} \pi^{\langle \text{C} \rangle} = 0 \quad (5.37)$$

$$-INC^{\langle 1 \rangle} - \Pi^{\langle 1 \rangle} + p^{\langle \text{A} \rangle} D^{\langle \text{A},1 \rangle} + p^{\langle \text{B} \rangle} D^{\langle \text{B},1 \rangle} + p^{\langle \text{C} \rangle} D^{\langle \text{C},1 \rangle} = 0 \quad (5.38)$$

$$-INC^{\langle 2 \rangle} - \Pi^{\langle 2 \rangle} + p^{\langle \text{A} \rangle} D^{\langle \text{A},2 \rangle} + p^{\langle \text{B} \rangle} D^{\langle \text{B},2 \rangle} + p^{\langle \text{C} \rangle} D^{\langle \text{C},2 \rangle} = 0 \quad (5.39)$$

$$K^{\langle 1 \rangle} + K^{\langle 2 \rangle} - K^{\langle \text{A} \rangle} - K^{\langle \text{B} \rangle} - K^{\langle \text{C} \rangle} = 0 \quad (5.40)$$

$$\pi^{\langle \text{A} \rangle} + L^{\langle \text{A} \rangle} + p^{\text{k}} K^{\langle \text{A} \rangle} + p^{\langle \text{A} \rangle} X^{\langle \text{A},\text{A} \rangle} - p^{\langle \text{A} \rangle} Y^{\langle \text{A} \rangle} + p^{\langle \text{B} \rangle} X^{\langle \text{B},\text{A} \rangle} + p^{\langle \text{C} \rangle} X^{\langle \text{C},\text{A} \rangle} = 0 \quad (5.41)$$

$$\pi^{\langle \text{B} \rangle} + L^{\langle \text{B} \rangle} + p^{\text{k}} K^{\langle \text{B} \rangle} + p^{\langle \text{A} \rangle} X^{\langle \text{A},\text{B} \rangle} + p^{\langle \text{B} \rangle} X^{\langle \text{B},\text{B} \rangle} - p^{\langle \text{B} \rangle} Y^{\langle \text{B} \rangle} + p^{\langle \text{C} \rangle} X^{\langle \text{C},\text{B} \rangle} = 0 \quad (5.42)$$

$$\pi^{\langle \text{C} \rangle} + L^{\langle \text{C} \rangle} + p^{\text{k}} K^{\langle \text{C} \rangle} + p^{\langle \text{A} \rangle} X^{\langle \text{A},\text{C} \rangle} + p^{\langle \text{B} \rangle} X^{\langle \text{B},\text{C} \rangle} + p^{\langle \text{C} \rangle} X^{\langle \text{C},\text{C} \rangle} - p^{\langle \text{C} \rangle} Y^{\langle \text{C} \rangle} = 0 \quad (5.43)$$

6 Calibrating equations

$$-d^{\text{data}\langle \text{B},1 \rangle} + D^{\langle \text{B},1 \rangle} = 0 \quad (6.1)$$

$$-d^{\text{data}\langle \text{B},2 \rangle} + D^{\langle \text{B},2 \rangle} = 0 \quad (6.2)$$

$$-d^{\text{data}\langle \text{C},1 \rangle} + D^{\langle \text{C},1 \rangle} = 0 \quad (6.3)$$

$$-d^{\text{data}\langle \text{C},2 \rangle} + D^{\langle \text{C},2 \rangle} = 0 \quad (6.4)$$

$$-l^{\text{data}\langle \text{A} \rangle} + L^{\langle \text{A} \rangle} = 0 \quad (6.5)$$

$$-l^{\text{data}\langle \text{B} \rangle} + L^{\langle \text{B} \rangle} = 0 \quad (6.6)$$

$$-l^{\text{data}\langle \text{C} \rangle} + L^{\langle \text{C} \rangle} = 0 \quad (6.7)$$

$$-x^{\text{data}\langle \text{A},\text{A} \rangle} + X^{\langle \text{A},\text{A} \rangle} = 0 \quad (6.8)$$

$$-x^{\text{data}\langle A,B \rangle} + X^{\langle A,B \rangle} = 0 \quad (6.9)$$

$$-x^{\text{data}\langle A,C \rangle} + X^{\langle A,C \rangle} = 0 \quad (6.10)$$

$$-x^{\text{data}\langle B,A \rangle} + X^{\langle B,A \rangle} = 0 \quad (6.11)$$

$$-x^{\text{data}\langle B,B \rangle} + X^{\langle B,B \rangle} = 0 \quad (6.12)$$

$$-x^{\text{data}\langle B,C \rangle} + X^{\langle B,C \rangle} = 0 \quad (6.13)$$

$$-x^{\text{data}\langle C,A \rangle} + X^{\langle C,A \rangle} = 0 \quad (6.14)$$

$$-x^{\text{data}\langle C,B \rangle} + X^{\langle C,B \rangle} = 0 \quad (6.15)$$

$$-x^{\text{data}\langle C,C \rangle} + X^{\langle C,C \rangle} = 0 \quad (6.16)$$

$$-y^{\text{data}\langle A \rangle} + Y^{\langle A \rangle} = 0 \quad (6.17)$$

$$-y^{\text{data}\langle B \rangle} + Y^{\langle B \rangle} = 0 \quad (6.18)$$

$$-y^{\text{data}\langle C \rangle} + Y^{\langle C \rangle} = 0 \quad (6.19)$$

$$-1 + \pi^{\text{h}\langle 1 \rangle} + \pi^{\text{h}\langle 2 \rangle} = 0 \quad (6.20)$$

$$-1 + \alpha^{\langle A,1 \rangle \omega} + \alpha^{\langle B,1 \rangle \omega} + \alpha^{\langle C,1 \rangle \omega} = 0 \quad (6.21)$$

$$-1 + \alpha^{\langle A,2 \rangle \omega} + \alpha^{\langle B,2 \rangle \omega} + \alpha^{\langle C,2 \rangle \omega} = 0 \quad (6.22)$$

$$-1 + \beta^{\text{k}\langle A \rangle} + \beta^{\text{l}\langle A \rangle} + \beta^{\text{x}\langle A,A \rangle} + \beta^{\text{x}\langle B,A \rangle} + \beta^{\text{x}\langle C,A \rangle} = 0 \quad (6.23)$$

$$-1 + \beta^{\text{k}\langle B \rangle} + \beta^{\text{l}\langle B \rangle} + \beta^{\text{x}\langle A,B \rangle} + \beta^{\text{x}\langle B,B \rangle} + \beta^{\text{x}\langle C,B \rangle} = 0 \quad (6.24)$$

$$-1 + \beta^{\text{k}\langle C \rangle} + \beta^{\text{l}\langle C \rangle} + \beta^{\text{x}\langle A,C \rangle} + \beta^{\text{x}\langle B,C \rangle} + \beta^{\text{x}\langle C,C \rangle} = 0 \quad (6.25)$$

7 Equilibrium values

	Equilibrium values
p^k	1
$\lambda^{\text{CONSUMER}^{11}}$	-1
$\lambda^{\text{CONSUMER}^{12}}$	-1
p^A	1
p^B	1
p^C	1
π^A	0
π^B	0
π^C	0
D^{A^1}	52.94
D^{A^2}	64.45
D^{B^1}	11.7
D^{B^2}	30.79
D^{C^1}	18.6
D^{C^2}	43.6
INC^1	83.24
INC^2	138.84
K^1	65.07
K^2	68.77
K^A	38.1
K^B	35.01
K^C	60.73
L^1	18.17
L^2	70.07
L^A	9.44
L^B	31.6
L^C	47.2
Π^1	0
Π^2	0
U^1	83.24
U^2	138.84
X^{A^A}	68.4
X^{A^B}	131.01
X^{A^C}	28.28
X^{B^A}	111.91
X^{B^B}	92.3
X^{B^C}	86.92
X^{C^A}	117.23
X^{C^B}	43.7
X^{C^C}	111.65
Y^A	345.08
Y^B	333.62
Y^C	334.78

8 Parameters of the model

	Parameters
ω	2
α^{A^1}	0.7975
α^{A^2}	0.6813
α^{B^1}	0.3749
α^{B^2}	0.4709
α^{C^1}	0.4727
α^{C^2}	0.5604
β^{k^A}	0.1104
β^{k^B}	0.1049
β^{k^C}	0.1814
β^{l^A}	0.0274
β^{l^B}	0.0947
β^{l^C}	0.141
$\beta^{x^{AA}}$	0.1982
$\beta^{x^{AB}}$	0.3927
$\beta^{x^{AC}}$	0.0845
$\beta^{x^{BA}}$	0.3243
$\beta^{x^{BB}}$	0.2767
$\beta^{x^{BC}}$	0.2596
$\beta^{x^{CA}}$	0.3397
$\beta^{x^{CB}}$	0.131
$\beta^{x^{CC}}$	0.3335
$d^{\text{data}^{B^1}}$	11.7
$d^{\text{data}^{B^2}}$	30.79
$d^{\text{data}^{C^1}}$	18.6
$d^{\text{data}^{C^2}}$	43.6
γ^A	4.0329
γ^B	4.2572
γ^C	4.5311
ks^{data^1}	65.07
ks^{data^2}	68.77
l^{data^A}	9.44
l^{data^B}	31.6
l^{data^C}	47.2
ls^{data^1}	18.17
ls^{data^2}	70.07
π^{h^1}	0.5
π^{h^2}	0.5
$x^{\text{data}^{AA}}$	68.4
$x^{\text{data}^{AB}}$	131.01
$x^{\text{data}^{AC}}$	28.28
$x^{\text{data}^{BA}}$	111.91
$x^{\text{data}^{BB}}$	92.3
$x^{\text{data}^{BC}}$	86.92
$x^{\text{data}^{CA}}$	117.23
$x^{\text{data}^{CB}}$	43.7
$x^{\text{data}^{CC}}$	111.65
y^{data^A}	345.08
y^{data^B}	333.62
y^{data^C}	334.78