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1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^{\mathrm{s}}, C_t, L_t^{\mathrm{s}}, I_t} U_t = \beta \mathrm{E}_t \left[U_{t+1}\right] + (1-\eta)^{-1} \left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t + p_t I_t = \pi_t^{\mathrm{C}} + \pi_t^{\mathrm{I}} + K_{t-1}^{\mathrm{s}} r_t + L_t^{\mathrm{s}} W_t \quad \left(\lambda_t^{\mathrm{CONSUMER}^1}\right) \tag{1.2}$$

$$K_t^{\mathrm{s}} = I_t + K_{t-1}^{\mathrm{s}} (1-\delta) \quad \left(\lambda_t^{\mathrm{CONSUMER}^2}\right) \tag{1.3}$$

1.2 Identities

$$Y_t = C_t + p_t I_t \tag{1.4}$$

1.3 First order conditions

$$-\lambda_t^{\mathrm{CONSUMER}^2} + \beta \left((1-\delta)\,\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^2}\right] + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}\right]\right) = 0 \quad (K_t^{\mathrm{s}}) \tag{1.5}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + \mu {C_t}^{-1+\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta} = 0 \quad (C_t) \tag{1.6}$$

$$\lambda_t^{\mathrm{CONSUMER}^1} W_t + (-1+\mu)\,{C_t}^{\mu}(1-L_t^{\mathrm{s}})^{-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta} = 0 \quad (L_t^{\mathrm{s}}) \tag{1.7}$$

$$\lambda_t^{\mathrm{CONSUMER}^2} - \lambda_t^{\mathrm{CONSUMER}^1} p_t = 0 \quad (I_t) \tag{1.8}$$

2 FIRM C

2.1 Optimisation problem

$$\max_{K_t^{\mathrm{C^d}}, L_t^{\mathrm{C^d}}, C_t^{\mathrm{s}}} \pi_t^{\mathrm{C}} = C_t^{\mathrm{s}} - L_t^{\mathrm{C^d}} W_t - r_t K_t^{\mathrm{C^d}} \tag{2.1}$$

s.t. :

$$C_t^{\mathrm{s}} = Z_t {K_t^{\mathrm{C^d}}}^{\alpha} {L_t^{\mathrm{C^d}}}^{1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM^{C^1}}}\right) \tag{2.2}$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\mathrm{FIRM^{C^1}}} Z_t K_t^{\mathrm{C^d}\,-1+\alpha} L_t^{\mathrm{C^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{C^d}}\right) \tag{2.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM^{C^1}}} Z_t \left(1-\alpha\right) K_t^{\mathrm{C^d}\,\alpha} L_t^{\mathrm{C^d}\,-\alpha} = 0 \quad \left(L_t^{\mathrm{C^d}}\right) \tag{2.4}$$

$$1 - \lambda_t^{\mathrm{FIRM^{C^1}}} = 0 \quad \left(C_t^{\mathrm{s}}\right) \tag{2.5}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\mathrm{C^d}\,-1+\alpha} L_t^{\mathrm{C^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{C^d}}\right) \tag{2.6}$$

$$-W_t + Z_t \left(1-\alpha\right) K_t^{\mathrm{C^d}\,\alpha} L_t^{\mathrm{C^d}\,-\alpha} = 0 \quad \left(L_t^{\mathrm{C^d}}\right) \tag{2.7}$$

3 FIRM I

3.1 Optimisation problem

$$\max_{K_t^{\mathrm{I^d}}, L_t^{\mathrm{I^d}}, I_t^{\mathrm{s}}} \pi_t^{\mathrm{I}} = I_t^{\mathrm{s}} - L_t^{\mathrm{I^d}} W_t - r_t K_t^{\mathrm{I^d}} \tag{3.1}$$

s.t. :

$$I_t^{\mathrm{s}} = Z_t K_t^{\mathrm{I^d}\,\sigma} L_t^{\mathrm{I^d}\,1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM^{I^1}}}\right) \tag{3.2}$$

3.2 First order conditions

$$-r_t + \sigma \lambda_t^{\mathrm{FIRM^{I^1}}} Z_t K_t^{\mathrm{I^d}\,-1+\sigma} L_t^{\mathrm{I^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{I^d}}\right) \tag{3.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM^{I^1}}} Z_t \left(1-\alpha\right) K_t^{\mathrm{I^d}\,\sigma} L_t^{\mathrm{I^d}\,-\alpha} = 0 \quad \left(L_t^{\mathrm{I^d}}\right) \tag{3.4}$$

$$1 - \lambda_t^{\mathrm{FIRM^{I^1}}} = 0 \quad \left(I_t^{\mathrm{s}}\right) \tag{3.5}$$

3.3 First order conditions after reduction

$$-r_t + \sigma Z_t K_t^{\mathrm{I^d}\,-1+\sigma} L_t^{\mathrm{I^d}\,1-\alpha} = 0 \quad \left(K_t^{\mathrm{I^d}}\right) \tag{3.6}$$

$$-W_t + Z_t \left(1-\alpha\right) K_t^{\mathrm{I^d}\,\sigma} L_t^{\mathrm{I^d}\,-\alpha} = 0 \quad \left(L_t^{\mathrm{I^d}}\right) \tag{3.7}$$

4 EQUILIBRIUM

4.1 Identities

$$K_t^{\mathrm{I^d}} + K_t^{\mathrm{C^d}} = K_{t-1}^{\mathrm{s}} \tag{4.1}$$

$$L_t^{\mathrm{I^d}} + L_t^{\mathrm{C^d}} = L_t^{\mathrm{s}} \tag{4.2}$$

$$C_t = C_t^{\mathrm{s}} \tag{4.3}$$

5 EXOG

5.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} \tag{5.1}$$

6 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t {K_t^{\mathrm{C^d}}}^{-1+\alpha} {L_t^{\mathrm{C^d}}}^{1-\alpha} = 0 \tag{6.1}$$

$$-r_t + \sigma Z_t \left(K_{t-1}^{\mathrm{s}} - K_t^{\mathrm{C^d}}\right)^{-1+\sigma} \left(L_t^{\mathrm{s}} - L_t^{\mathrm{C^d}}\right)^{1-\alpha} = 0 \tag{6.2}$$

$$-C_t + Z_t {K_t^{\mathrm{C^d}}}^{\alpha} {L_t^{\mathrm{C^d}}}^{1-\alpha} = 0 \tag{6.3}$$

$$-I_t^{\mathrm{s}} + Z_t \left(K_{t-1}^{\mathrm{s}} - K_t^{\mathrm{C^d}}\right)^{\sigma} \left(L_t^{\mathrm{s}} - L_t^{\mathrm{C^d}}\right)^{1-\alpha} = 0 \tag{6.4}$$

$$-W_t + Z_t \left(1-\alpha\right) {K_t^{\mathrm{C^d}}}^{\alpha} {L_t^{\mathrm{C^d}}}^{-\alpha} = 0 \tag{6.5}$$

$$-W_t + Z_t \left(1-\alpha\right) \left(K_{t-1}^{\mathrm{s}} - K_t^{\mathrm{C^d}}\right)^{\sigma} \left(L_t^{\mathrm{s}} - L_t^{\mathrm{C^d}}\right)^{-\alpha} = 0 \tag{6.6}$$

$$-Z_t + e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} = 0 \tag{6.7}$$

$$\beta \left(\mu \mathrm{E}_t \left[r_{t+1} {C_{t+1}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu} \left({C_{t+1}}^{\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right] + \mu \left(1-\delta\right) \mathrm{E}_t \left[p_{t+1} {C_{t+1}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu} \left({C_{t+1}}^{\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right]\right) - \mu p_t {C_t}^{-1+\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu} \left({C_t}^{\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu}\right. \tag{6.8}$$

$$(-1+\mu)\,{C_t}^{\mu}(1-L_t^{\mathrm{s}})^{-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_t {C_t}^{-1+\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{6.9}$$

$$C_t-Y_t+p_tI_t=0 \tag{6.10}$$

$$I_t-K_t^{\mathrm{s}}+K_{t-1}^{\mathrm{s}}\,(1-\delta)=0 \tag{6.11}$$

$$U_t-\beta\mathrm{E}_t\left[U_{t+1}\right]-(1-\eta)^{-1}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{1-\eta}=0 \tag{6.12}$$

$$I_t^{\mathrm{s}}+K_{t-1}^{\mathrm{s}}r_t-p_tI_t-r_tK_t^{\mathrm{C^d}}-r_t\left(K_{t-1}^{\mathrm{s}}-K_t^{\mathrm{C^d}}\right)+L_t^{\mathrm{s}}W_t-L_t^{\mathrm{C^d}}W_t-W_t\left(L_t^{\mathrm{s}}-L_t^{\mathrm{C^d}}\right)=0 \tag{6.13}$$

7 Steady state relationships (after reduction)

$$-r_{\mathrm{ss}}+\alpha Z_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{C^d}}}^{-1+\alpha}{L_{\mathrm{ss}}^{\mathrm{C^d}}}^{1-\alpha}=0 \tag{7.1}$$

$$-r_{\mathrm{ss}}+\sigma Z_{\mathrm{ss}}\left(K_{\mathrm{ss}}^{\mathrm{s}}-K_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{-1+\sigma}\left(L_{\mathrm{ss}}^{\mathrm{s}}-L_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{1-\alpha}=0 \tag{7.2}$$

$$-C_{\mathrm{ss}}+Z_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{C^d}}}^{\alpha}{L_{\mathrm{ss}}^{\mathrm{C^d}}}^{1-\alpha}=0 \tag{7.3}$$

$$-I_{\mathrm{ss}}^{\mathrm{s}}+Z_{\mathrm{ss}}\left(K_{\mathrm{ss}}^{\mathrm{s}}-K_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{\sigma}\left(L_{\mathrm{ss}}^{\mathrm{s}}-L_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{1-\alpha}=0 \tag{7.4}$$

$$-W_{\mathrm{ss}}+Z_{\mathrm{ss}}\,(1-\alpha)\,{K_{\mathrm{ss}}^{\mathrm{C^d}}}^{\alpha}{L_{\mathrm{ss}}^{\mathrm{C^d}}}^{-\alpha}=0 \tag{7.5}$$

$$-W_{\mathrm{ss}}+Z_{\mathrm{ss}}\,(1-\alpha)\left(K_{\mathrm{ss}}^{\mathrm{s}}-K_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{\sigma}\left(L_{\mathrm{ss}}^{\mathrm{s}}-L_{\mathrm{ss}}^{\mathrm{C^d}}\right)^{-\alpha}=0 \tag{7.6}$$

$$-Z_{\mathrm{ss}}+e^{\phi\log Z_{\mathrm{ss}}}=0 \tag{7.7}$$

$$\beta\left(\mu r_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu p_{\mathrm{ss}}\,(1-\delta)\,{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}\right)-\mu p_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{7.8}$$

$$(-1+\mu)\,{C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{7.9}$$

$$C_{\mathrm{ss}} - Y_{\mathrm{ss}} + p_{\mathrm{ss}} I_{\mathrm{ss}} = 0 \tag{7.10}$$

$$I_{\mathrm{ss}} - K^{\mathrm{s}}_{\mathrm{ss}} + K^{\mathrm{s}}_{\mathrm{ss}} (1 - \delta) = 0 \tag{7.11}$$

$$U_{\mathrm{ss}} - \beta U_{\mathrm{ss}} - (1 - \eta)^{-1} \left({C_{\mathrm{ss}}}^{\mu} (1 - L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu} \right)^{1-\eta} = 0 \tag{7.12}$$

$$I^{\mathrm{s}}_{\mathrm{ss}} - p_{\mathrm{ss}} I_{\mathrm{ss}} + r_{\mathrm{ss}} K^{\mathrm{s}}_{\mathrm{ss}} - r_{\mathrm{ss}} K^{\mathrm{C^d}}_{\mathrm{ss}} - r_{\mathrm{ss}} \left(K^{\mathrm{s}}_{\mathrm{ss}} - K^{\mathrm{C^d}}_{\mathrm{ss}} \right) + L^{\mathrm{s}}_{\mathrm{ss}} W_{\mathrm{ss}} - L^{\mathrm{C^d}}_{\mathrm{ss}} W_{\mathrm{ss}} - W_{\mathrm{ss}} \left(L^{\mathrm{s}}_{\mathrm{ss}} - L^{\mathrm{C^d}}_{\mathrm{ss}} \right) = 0 \tag{7.13}$$

8 Parameter settings

$$\alpha = 0.2 \tag{8.1}$$

$$\beta = 0.99 \tag{8.2}$$

$$\delta = 0.025 \tag{8.3}$$

$$\eta = 2 \tag{8.4}$$

$$\mu = 0.3 \tag{8.5}$$

$$\phi = 0.95 \tag{8.6}$$

$$\sigma = 0.4 \tag{8.7}$$

9 Steady-state values

	Steady-state value
p	1.5318
r	0.0538
C	0.3374
I	0.0439
I^{s}	0.0672
K^{s}	1.7551
$K^{\mathrm{C^d}}$	1.2551
L^{s}	0.2914
$L^{\mathrm{C^d}}$	0.243
U	-176.3002
W	1.111
Y	0.4046
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{array}{c} \\ K_t^{\mathrm{s}} \\ Z_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1}^{\mathrm{s}} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.9522 & -0.0054 \\ 0 & 0.95 \end{pmatrix} \end{array}$$

Matrix Q

$$\begin{array}{c} \\ K^{\mathrm{s}} \\ Z \end{array} \begin{array}{c} \begin{array}{c} \epsilon^{\mathrm{Z}} \end{array} \\ \begin{pmatrix} -0.0056 \\ 1 \end{pmatrix} \end{array}$$

Matrix R

$$\begin{array}{c} \\ p_t \\ r_t \\ C_t \\ I_t \\ I_t^{\mathrm{s}} \\ K_t^{\mathrm{C^d}} \\ L_t^{\mathrm{s}} \\ L_t^{\mathrm{C^d}} \\ U_t \\ W_t \\ Y_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1}^{\mathrm{s}} & Z_{t-1} \end{array} \\ \begin{pmatrix} -0.1506 & 1.1645 \\ -1.0646 & 0.95 \\ 0.3338 & 0.95 \\ -0.9139 & -0.2145 \\ -1.0646 & 0.95 \\ 1.3984 & 0 \\ -0.1646 & 0 \\ 0.0677 & 0 \\ 0.0257 & 0.0456 \\ 0.2661 & 0.95 \\ 0.1015 & 0.95 \end{pmatrix} \end{array}$$

Matrix S

$$\begin{array}{c} \\ p \\ r \\ C \\ I \\ I^{\mathrm{s}} \\ K^{\mathrm{C^d}} \\ L^{\mathrm{s}} \\ L^{\mathrm{C^d}} \\ U \\ W \\ Y \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \left(\begin{array}{c} 1.2258 \\ 1 \\ 1 \\ -0.2258 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0.048 \\ 1 \\ 1 \end{array} \right. \end{array}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
p	1.5318	0.113	0.0128	Y
r	0.0538	0.0922	0.0085	Y
C	0.3374	0.0922	0.0085	Y
I	0.0439	0.0208	0.0004	Y
I^{s}	0.0672	0.0922	0.0085	Y
K^{s}	1.7551	0.0018	0	Y
L^{s}	0.2914	0	0	Y
U	-176.3002	0.0044	0	Y
W	1.111	0.0922	0.0085	Y
Y	0.4046	0.0922	0.0085	Y

11.2 Correlation matrix

	p	r	C	I	I^{s}	K^{s}	U	W	Y
p	1	1	1	-0.997	1	-0.309	1	1	1
r		1	1	-0.995	1	-0.327	0.999	1	1
C			1	-0.997	1	-0.3	1	1	1
I				1	-0.995	0.229	-0.998	-0.997	-0.997
I^{s}					1	-0.327	0.999	1	1
K^{s}						1	-0.296	-0.301	-0.305
U							1	1	1
W								1	1
Y									1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
p_t	1.226	-0.019	0.107	0.269	0.469	0.712	1	0.714	0.473	0.273	0.112	-0.014
r_t	1.001	-0.028	0.098	0.26	0.463	0.708	1	0.72	0.482	0.285	0.125	-0.002
C_t	1	-0.014	0.112	0.273	0.472	0.714	1	0.712	0.469	0.268	0.107	-0.02
I_t	0.226	-0.023	-0.148	-0.305	-0.497	-0.727	-0.997	-0.688	-0.432	-0.223	-0.058	0.068
I_t^{s}	1.001	-0.028	0.098	0.26	0.463	0.708	1	0.72	0.482	0.285	0.125	-0.002
K_t^{s}	0.02	0.491	0.437	0.339	0.19	-0.022	-0.305	-0.493	-0.603	-0.651	-0.652	-0.616
U_t	0.048	-0.012	0.114	0.275	0.474	0.715	1	0.711	0.467	0.266	0.104	-0.022
W_t	1	-0.015	0.111	0.272	0.472	0.714	1	0.712	0.469	0.269	0.107	-0.019
Y_t	1	-0.017	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.017

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
p	0.713	0.471	0.271	0.11	-0.016
r	0.715	0.474	0.275	0.113	-0.013
C	0.713	0.47	0.27	0.109	-0.017
I	0.708	0.463	0.261	0.1	-0.025
I^{s}	0.715	0.474	0.275	0.113	-0.013
K^{s}	0.959	0.86	0.723	0.565	0.399
U	0.712	0.47	0.269	0.108	0
W	0.713	0.47	0.27	0.109	-0.017
Y	0.713	0.471	0.271	0.11	-0.017

12 Impulse response functions

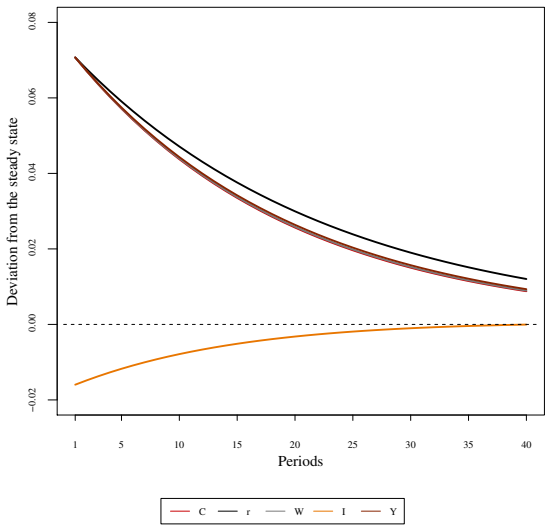

Figure 1: Impulse responses (C, r, W, I, Y) to ϵ^{Z} shock

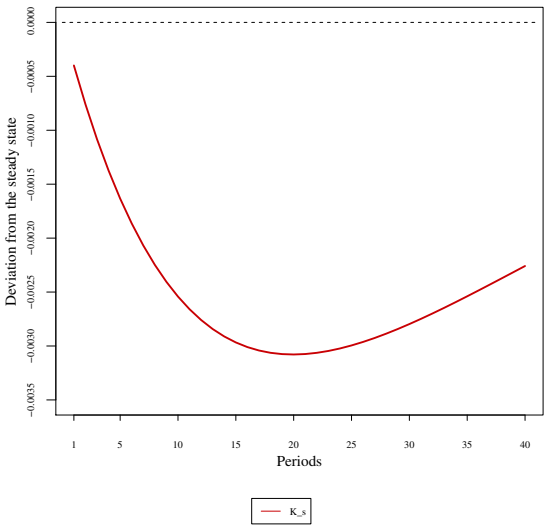

Figure 2: Impulse response (K^{s}) to ϵ^{Z} shock