

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t, H_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

$$H_t = C_{t-1} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \quad (1.4)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \beta E_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers} H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

$$-\lambda_t^{\text{CONSUMER}^3} - \mu \text{pers} (1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (H_t) \quad (1.9)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}^{-1}+\alpha} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}^{-1}+\alpha} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$C_{t-1} - H_t = 0 \quad (5.1)$$

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) \text{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + \text{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} r_{t+1} \right] \right) = 0 \quad (5.2)$$

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}^{-1}+\alpha} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.3)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{-\alpha}} = 0 \quad (5.4)$$

$$-Y_t + Z_t K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.5)$$

$$Z_t - e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.6)$$

$$\lambda_t^{\text{CONSUMER}^2} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers} H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.7)$$

$$-\lambda_t^{\text{CONSUMER}^2} - \beta \mu \text{pers} E_t \left[(1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^{-1+\mu} \left((1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^\mu \right)^{-\eta} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.8)$$

$$-C_t - I_t + Y_t = 0 \quad (5.9)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.10)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{1-\eta} = 0 \quad (5.11)$$

6 Steady state relationships (after reduction)

$$C_{ss} - H_{ss} = 0 \quad (6.1)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \beta \left(\lambda_{ss}^{\text{CONSUMER}^2} r_{ss} + \lambda_{ss}^{\text{CONSUMER}^2} (1 - \delta) \right) = 0 \quad (6.2)$$

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.3)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.4)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.5)$$

$$Z_{ss} - e^{\phi \log Z_{ss}} = 0 \quad (6.6)$$

$$\lambda_{ss}^{\text{CONSUMER}^2} W_{ss} + (-1 + \mu) (1 - L_{ss}^s)^{-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} = 0 \quad (6.7)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \mu (1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} - \beta \mu \text{pers} (1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} = 0 \quad (6.8)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.9)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.10)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{1-\eta} = 0 \quad (6.11)$$

7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$

$$\delta = 0.025 \tag{8.2}$$

$$\eta = 2 \tag{8.3}$$

$$\mu = 0.3 \tag{8.4}$$

$$pers = 0.57 \tag{8.5}$$

$$\phi = 0.95 \tag{8.6}$$

9 Steady-state values

	Steady-state values
$\lambda^{\text{CONSUMER}^2}$	0.7116
r	0.0351
C	0.7494
H	0.7494
I	0.2584
K^s	10.3356
L^s	0.2721
U	-175.4236
W	2.3706
Y	1.0078
Z	1

10 The solution of the perturbation

10.1 P

$$\begin{matrix} C \\ K^s \\ Z \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.5544 & 0.0151 & 0.1764 \\ -0.5092 & 0.9817 & 1.1759 \\ 0 & 0 & 0.95 \end{pmatrix}$$

10.2 Q

$$\begin{matrix} C \\ K^s \\ Z \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.1857 \\ 1.2377 \\ 1 \end{pmatrix}$$

10.3 R

$$\begin{matrix} \lambda^{\text{CONSUMER}^2} \\ r \\ H \\ I \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.0599 & -0.0494 & -0.3592 \\ 0.0016 & -0.0026 & 0.0471 \\ 1 & 0 & 0 \\ -0.5092 & 0.0067 & 1.1759 \\ 0.0191 & -0.0056 & 0.1666 \\ -0.9309 & 0.7188 & 11.4498 \\ -0.0598 & 0.1002 & 1.7296 \\ 0.0452 & 0.0218 & 1.3522 \end{pmatrix}$$

10.4 S

$$\begin{matrix} \lambda^{\text{CONSUMER}^2} \\ r \\ H \\ I \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} \epsilon^Z \\ -0.3781 \\ 0.0496 \\ 0 \\ 1.2377 \\ 0.1753 \\ 12.0524 \\ 1.8206 \\ 1.4234 \end{pmatrix}$$

11 Statistics of the model

11.1 Moments

	Steady-state value	Std. dev.	Variance	Loglinear
r	0.0351	0.0046	0	N
C	0.7494	0.0333	0.0011	N
H	0.7494	0.0333	0.0011	N
I	0.2584	0.1077	0.0116	N
K^s	10.3356	0.3633	0.132	N
L^s	0.2721	0.0164	0.0003	N
U	-175.4236	1.1325	1.2825	N
W	2.3706	0.1719	0.0295	N
Y	1.0078	0.1325	0.0175	N
Z	1	0.0922	0.0085	N

11.2 Correlation matrix

	r	C	H	I	K^s	L^s	U	W	Y	Z
$\lambda^{\text{CONSUMER}^2}$	-0.8032	-0.9421	-0.7242	-0.8546	-0.7018	-0.861	-0.9737	-0.9722	-0.9318	-0.9048
r	1	0.6233	0.2627	0.9934	0.1395	0.9944	0.9178	0.9204	0.9645	0.9804
C	0.6233	1	0.9106	0.6739	0.826	0.7011	0.8666	0.8652	0.7994	0.7579
H	0.2627	0.9106	1	0.3154	0.8989	0.358	0.5836	0.5818	0.4854	0.4287
I	0.9934	0.6739	0.3154	1	0.2315	0.9968	0.9495	0.9511	0.9826	0.9925
K^s	0.1395	0.826	0.8989	0.2315	1	0.2426	0.5211	0.5157	0.3959	0.3318
L^s	0.9944	0.7011	0.358	0.9968	0.2426	1	0.9542	0.9562	0.9869	0.9956
U	0.9178	0.8666	0.5836	0.9495	0.5211	0.9542	1	1	0.99	0.978
W	0.9204	0.8652	0.5818	0.9511	0.5157	0.9562	1	1	0.9909	0.9793
Y	0.9645	0.7994	0.4854	0.9826	0.3959	0.9869	0.99	0.9909	1	0.9976
Z	0.9804	0.7579	0.4287	0.9925	0.3318	0.9956	0.978	0.9793	0.9976	1

11.3 Autocorrelations

	$t-1$	$t-2$	$t-3$	$t-4$
r	0.7068	0.4619	0.2614	0.1009
C	0.9106	0.7381	0.5368	0.3373
H	0.9106	0.7381	0.5368	0.3373
I	0.6672	0.4127	0.2174	0.0686
K^s	0.9552	0.8509	0.7099	0.5498
L^s	0.7157	0.4728	0.2708	0.1076
U	0.7373	0.5093	0.3157	0.1551
W	0.7395	0.5115	0.3169	0.1551
Y	0.7236	0.4858	0.2864	0.1237
Z	0.7133	0.4711	0.2711	0.1098

12 Statistics of the model

12.1 Moments relative to moments of the reference variable

	Steady-state value relative to Y	Std. dev. relative to Y	Variance relative to Y	Loglinear
$\lambda^{\text{CONSUMER}^2}$	0.7061	0.2894	0.0838	N
r	0.0348	0.0351	0.0012	N
C	0.7436	0.2514	0.0632	N
H	0.7436	0.2514	0.0632	N
I	0.2564	0.8132	0.6612	N
K^s	10.2561	2.7424	7.521	N
L^s	0.27	0.124	0.0154	N
U	-174.0741	8.5496	73.0965	N
W	2.3524	1.2977	1.6841	N
Y	1	1	1	N
Z	0.9923	0.6958	0.4842	N

12.2 Correlations with the reference variable

	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}
$\lambda^{\text{CONSUMER}^2}$	0.0606	-0.1099	-0.3294	-0.6024	-0.9318	-0.7777	-0.6273	-0.4851	-0.3544
r	0.2446	0.3879	0.5562	0.7491	0.9645	0.6134	0.3309	0.1092	-0.0591
C	-0.1308	0.0247	0.2285	0.4854	0.7994	0.8668	0.8036	0.6775	0.5273
H	-0.2439	-0.1308	0.0247	0.2285	0.4854	0.7994	0.8668	0.8036	0.6775
I	0.1926	0.3445	0.5267	0.7398	0.9826	0.6219	0.3489	0.1427	-0.0109
K^s	-0.39	-0.2781	-0.115	0.1073	0.3959	0.5704	0.6596	0.6854	0.6651
L^s	0.2003	0.3524	0.5342	0.7461	0.9869	0.668	0.4016	0.1854	0.0157
U	0.0534	0.2219	0.4325	0.6881	0.99	0.7536	0.5453	0.3656	0.2138
W	0.0575	0.2257	0.4357	0.6904	0.9909	0.7554	0.5461	0.3647	0.2112
Y	0.1237	0.2864	0.4858	0.7236	1	0.7236	0.4858	0.2864	0.1237
Z	0.1554	0.3142	0.507	0.7349	0.9976	0.6975	0.4461	0.2401	0.0758

13 Impulse response functions

13.1 Shock ϵ^Z

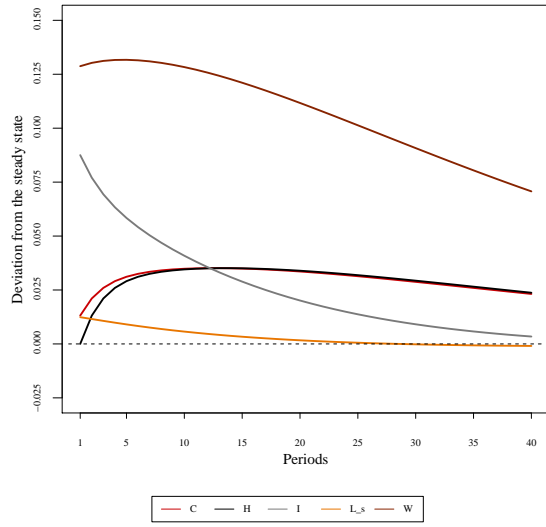


Figure 1: Impulse response function for ϵ^Z shock

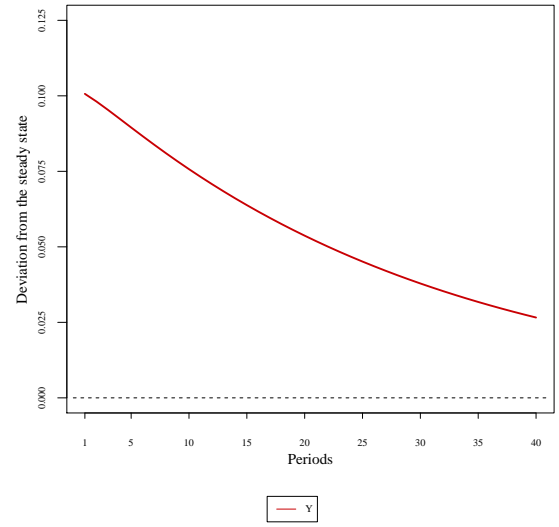


Figure 2: Impulse response function for ϵ^Z shock