Stochastic models of the income and wealth distribution

Income/wealth follows the stochastic process:

$$\frac{dX_t}{X_t} = \mu(X_t)\,dt + \sigma_t(X_t)\,dX_t$$

Increasing mean at the top, quasi-constant variance at the top

In[1]:= $\mu[x\_] = -c1 + \frac{c2\ x^2}{1 + c3\ x^2};$

In[2]:= $\sigma[x\_] = \sqrt{\frac{c4 + x^2}{x^2}};$

In[3]:= $\xi[x\_] = \text{Simplify}\left[2 - \frac{2\,\mu[x]}{\sigma[x]^2} + \frac{x}{\sigma[x]^2}\,D\left[\sigma[x]^2, x\right]\right]$

Out[3]= $\frac{2\ x^2\ \left(1 + c1 - c2\ x^2 + c3\ x^2 + c1\ c3\ x^2\right)}{\left(c4 + x^2\right)\ \left(1 + c3\ x^2\right)}$

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In[4]:= CoefficientList[Numerator[Together[ξ[x] / x]], x] // InputForm
Out[4]//InputForm= {0, 2 + 2*c1, 0, -2*c2 + 2*c3 + 2*c1*c3}

In[5]:= CoefficientList[Denominator[Together[ξ[x] / x]], x] // InputForm
Out[5]//InputForm= {c4, 0, 1 + c3*c4, 0, c3}
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Constant mean, increasing variance at the top

In[6]:= $\mu[x\_] = -1;$

In[7]:= $\sigma[x\_] = \sqrt{\frac{c1 + c2\ x^2}{x^2} + \frac{c3\ x^2}{1 + c4\ x^2}};$

In[8]:= $\xi[x\_] = \text{Simplify}\left[2 - \frac{2\,\mu[x]}{\sigma[x]^2} + \frac{x}{\sigma[x]^2}\,D\left[\sigma[x]^2, x\right]\right]$

Out[8]= $\frac{2\ x^2\ \left(\left(1 + c4\ x^2\right)^2 + c2\ \left(1 + c4\ x^2\right)^2 + c3\ x^2\ \left(2 + c4\ x^2\right)\right)}{\left(1 + c4\ x^2\right)\ \left(c1 + c2\ x^2 + c1\ c4\ x^2 + c3\ x^4 + c2\ c4\ x^4\right)}$

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In[9]:= CoefficientList[Numerator[Together[ξ[x] / x]], x] // InputForm
Out[9]//InputForm= {0, 2 + 2*c2, 0, 4*c3 + 4*c4 + 4*c2*c4, 0, 2*c3*c4 + 2*c4^2 + 2*c2*c4^2}
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In[10]:=
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CoefficientList[Denominator[Together[ξ[x] / x]], x] // InputForm
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Out[10]//InputForm=
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{c1, 0, c2 + 2*c1*c4, 0, c3 + 2*c2*c4 + c1*c4^2, 0, c3*c4 + c2*c4^2}
```