<u>CS 530 Midterm 1 Cheatsheet</u>

Complex Analysis

$z = a + ib = re^{i\theta} = r(cos(\theta) + isin(\theta))$
$\overline{z} = a - bi = re^{-i\theta}$
$|z| = \sqrt{a^2 + b^2}$
$\|\mathbf{z}\| = \sqrt{|z_1|^2 + \cdots + |z_n|^2}$

Fourier Transform

Fourier Basis for $\mathbb{C}^N$ is $\{F_0, F_1, \cdots, F_{N-1}\}$

$$F_m(n) = \frac{1}{N} e^{2\pi i \frac{mn}{N}}$$

Fourier basis is orthogonal, not orthonormal. Can be normalized the norm is $\|F_m\| = \frac{1}{\sqrt{N}}$

$$\hat{z}(m) = \sum_{n=0}^{N-1} z(n) e^{-2\pi i \frac{mn}{N}}$$

$$z(n) = \frac{1}{N} \sum_{m=0}^{N-1} \hat{z}(m) e^{2\pi i \frac{mn}{N}}$$

2D

$$\hat{z}(m_1, m_2) = \frac{1}{\sqrt{N_1 N_2}} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} z(n_1, n_2) e^{-2\pi i \frac{m_1 n_1}{N_1}} e^{-2\pi i \frac{m_2 n_2}{N_2}}$$

$$z(n_1, n_2) = \frac{1}{\sqrt{N_1 N_2}} \sum_{m_1=0}^{N_1-1} \sum_{m_2=0}^{N_2-1} \hat{z}(m_1, m_2) e^{2\pi i \frac{m_1 n_1}{N_1}} e^{2\pi i \frac{m_2 n_2}{N_2}}$$

Wavelets

Wavelet tools

$$(w * z)(m) = \sum_{n=0}^{N-1} w(n) z(m-n)$$

$(w * z)^\wedge(m) = \hat{w}(m)\hat{z}(m)$
$R_k z$ is called the translation of z by k.
$(R_k z)(n) = z(n-k)$
$(R_k z)^\wedge(m) = \hat{z}(m) e^{-2\pi i \frac{mk}{N}}$
$\langle z, R_k w \rangle = (z * \tilde{w})(k)$
$\langle z, R_k \tilde{w} \rangle = (z * w)(k)$
$\tilde{z}(n) = \overline{z(-n)}$
$(\tilde{z})^\wedge(m) = \overline{\hat{z}(m)}$
$z^*(n) = (-1)^n z(n)$
For the below, $N = 2M$
$(z^*)^\wedge(n) = \hat{z}(n + M)$
$|\hat{w}(n)|^2 + |\hat{w}(n+M)|^2 = 2$
$A(n)$ is the system matrix of u and v

$$A(n) = \frac{1}{\sqrt{2}} \begin{bmatrix} \hat{u}(n) & \hat{v}(n) \\ \hat{u}(n+M) & \hat{v}(n+M) \end{bmatrix}$$

Misc

$$\sum_{k=a}^{b} r^k = \frac{r^a - r^{b+1}}{1 - r}$$

Cauchy-Shwarz inequality $\overline{\langle u, v \rangle} \leq \|u\| \|v\|$
Unitary matrix $\iff \overline{U^T} U = I$
orthogonal $\iff \langle u, v \rangle = 0$
Parseval's formula

$$\langle z, w \rangle = \langle \hat{z}, \hat{w} \rangle$$

Plancherel's formula

$$\|z\|^2 = \|\hat{z}\|^2$$