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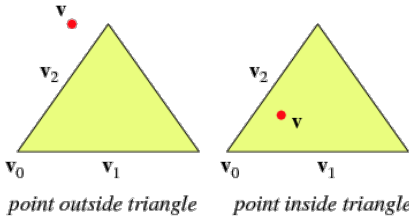
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Created, developed, and  
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Triangle Interior

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The interior of the triangle is the set of all points inside a triangle, i.e., the set of all points in the convex hull of the triangle's vertices.

The simplest way to determine if a point lies inside a triangle is to check the number of points in the convex hull of the vertices of the triangle adjoined with the point in question. If the hull has three points, the point lies in the triangle's interior; if it is four, it lies outside the triangle.

To determine if a given point  $\mathbf{v}$  lies in the interior of a given triangle, consider an individual vertex, denoted  $\mathbf{v}_0$ , and let  $\mathbf{v}_1$  and  $\mathbf{v}_2$  be the vectors from  $\mathbf{v}_0$  to the other two vertices. Expressing the vector from  $\mathbf{v}_0$  to  $\mathbf{v}$  in terms of  $\mathbf{v}_1$  and  $\mathbf{v}_2$  then gives

$$\mathbf{v} = \mathbf{v}_0 + a \mathbf{v}_1 + b \mathbf{v}_2, \tag{1}$$

where  $a$  and  $b$  are constants. Solving for  $a$  and  $b$  gives

$$a = \frac{\det(\mathbf{v} \mathbf{v}_2) - \det(\mathbf{v}_0 \mathbf{v}_2)}{\det(\mathbf{v}_1 \mathbf{v}_2)} \tag{2}$$

$$b = -\frac{\det(\mathbf{v} \mathbf{v}_1) - \det(\mathbf{v}_0 \mathbf{v}_1)}{\det(\mathbf{v}_1 \mathbf{v}_2)}, \tag{3}$$

where

$$\det(\mathbf{u} \mathbf{v}) = \mathbf{u} \times \mathbf{v} = u_x v_y - u_y v_x \tag{4}$$

is the determinant of the matrix formed from the column vectors  $\mathbf{u}$  and  $\mathbf{v}$ . Then the point  $\mathbf{v}$  lies in the interior of the triangle if  $a, b > 0$  and  $a + b < 1$ .

If the convex hull of the triangle vertices plus the point  $\mathbf{v}_0$  is bounded by four points, the point  $\mathbf{v}_0$  lies outside the triangle. However, if it contains three points, the point  $\mathbf{v}_0$  may lie either in the interior or in the exterior.

SEE ALSO:  
Convex Hull, Triangle, Triangle Point Picking

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