The Catastrophe Theory in Marketing

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Abstract

This project is the product of a summer's worth of marketing research on the Catastrophe Theory, primarily conducted by scientists Rene Thom and Christopher Zeeman in the late 1960s to early 1970s. The purpose of this study was to find an existing relationship between Thom's mathematical theory and current marketing strategy.

This research was initially set up in three parts, which I was later able to synthesize into one document. The first part consists of an academic essay, outlining Thom's seven elementary catastrophes and briefly outlining them, then goes into identifying advertising and/or marketing strategies and matching one or more of them to one of Thom's theories. The second part, which I put as examples and exhibits in the second essay, consists of finding real world examples of strategic marketing that would best exemplify these theories in a day to day setting.

As the research was conducted, there were several major things worth noting:

1. There has been very little research conducted on this topic, and its specificity made this little research very difficult to find.
2. Many of the articles that were found to be connected to marketing and these theories were only surrounding two major theories: fold and cusp. As this essay serves to cohesively summarize the ideas found through this research, many of the examples relatable to current marketing strategy only seemed seemed to fit the fold and cusp theories.
3. This project involved much more creative thinking than I initially expected, especially in terms of thinking about different marketing ideas with which I'm most familiar, and trying to decide which went best with which theories.

As a final note, please allow fair discretion through the duration of this research. I am unfamiliar with the higher level of physics and mathematics required to fully comprehend these theories. The ones written in these essays are summarized to a very basic level understanding of college mathematics, with basic analysis only on independent and dependent variables, producing more qualitative discussions than quantitative.

A Biography of René Thom

Born on September 2, 1923 in the small French village of Montbéliard, Doubs, René Thom became a mathematician most notably known for his work on the catastrophe theory. Thom came from a humble French family, and showed hints of academic promise from an early age. A shining star in school, Thom continued his studies at a university in Besançon, France (and later the University of Paris) where he received a degree in elementary mathematics; however, due to the outbreak of WWII, Thom had to suspend his career for some time, including a relocation to Switzerland. After waiting out the war and finishing his PhD, Thom moved to America in 1951. He moved back and forth between America and France, serving as a professor at multiple universities. It was here that he met revolutionary scientists including Albert Einstein, and even received the Fields Medal in 1958 for his work on the Thom transversality theorem. Over his life, Thom received five major honors, the last one posthumously: Speaker at International Congress (1958), Fields Medal (1958), Dutch Mathematical Society Brouwer Medal (1970), SIAM John von Neumann Lecture (1976), and the London Mathematical Society Lecture (1990). While René Thom is most famously known for his work regarding the catastrophe theory, he also worked on a number of other important theorems relevant to higher level science and mathematics.[1]

[1] Wikipedia. “René Thom.” *Wikipedia*, Wikimedia Foundation, 24 Aug. 2017.

The Catastrophe Theory

René Thom devoted much time in the 1960s to the development of the catastrophe theory, along with scientists Christopher Zeeman and Vladimir Arnold. This theory became crucial to the math and science fields, as it focuses on how “phenomena [is] characterized by sudden shifts in behavior arising from small changes in circumstances.” [2] There are seven different branches under the umbrella of the overall theory, and can be classified as elementary catastrophes involving one or two variables. The catastrophes involving a single active variable are the fold catastrophe, cusp catastrophe, swallowtail catastrophe, and butterfly catastrophe. The catastrophes involving two variables are the hyperbolic umbilic catastrophe, elliptic umbilic catastrophe, and parabolic umbilic catastrophe. Each catastrophe will be explored further later in this essay. Thom’s groundbreaking discovery forever changed modern industrial architecture, including bridge testing, ship stabilization, but could arguably be applied to human consumer behavior.

Before diving into the specifics of each catastrophe, it is crucial to understand some fundamental vocabulary words Thom uses throughout his book *Structural Stability and Morphogenesis*, which he wrote in 1972. Please see the glossary of key terms after the appendix. First and foremost, Thom realizes that this theory could be deemed as qualitative. On pages 4 through 6 of his book, Thom gives an example of an exception in which the theory could be qualitative, not completely quantitative, even though the theory tailors primarily to catastrophes under the realm of math and science.[3] As this research pertains to applying the

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[2] The Exploratorium. “Rene Thom and the Catastrophe Theory.” *Catastrophe Theory*, The Exploratorium, 1996.
[3] Thom, René. "Historicophilosophical Digression." *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*. By Vladimir Arnold and Christopher Zeeman. New York, NY: W.A. Benjamin, 1975. pp. 4-6. Print.

catastrophe theory in a marketing sphere, it is imperative to realize that the core of this research is to concisely and accurately show how this theory can be applied in a qualitative manner as well, specifically in marketing theory.

SINGLE ACTIVE VARIABLE CATASTROPHES

FOLD

Mathematically

Defined as $V = x^3 + ax$, the fold catastrophe gives a parabolic graph that opens to the side instead of opening upwards. While it isn't considered a quadratic function, it is in fact considered a conic function, with a single inflection point. According to Thom, at all values where a is negative, the graph holds the possibility of two extrema: stable and unstable.[4] When a = 0, the stable and unstable extrema meet at the bifurcation point on the graph, and "annihilate" at what could be considered "the tipping point" (Exhibit A).

Marketing

In marketing terms, the fold catastrophe is based on one independent variable (advertising) and one dependent variable (consumer behavior). From the theory, the bifurcation point represents a shift in consumer thinking or behavior. This is comparable to the "aha!" moment a consumer might have after a sudden realization about their consumption habits.[5] For example, shock advertising could be best used to describe the effects of the fold catastrophe. With this method, it typically only takes one, really strong and surprising advertisement to cause a consumer to shift their consumption habits.

[4] Wikipedia. "Catastrophe Theory, Fold Catastrophe." *Wikipedia*, Wikimedia Foundation, 24 Aug. 2017.
[5] Oliva, Terence A., and Alvin C. Burns. "Catastrophe Theory As a Model For Describing Consumer Behavior." *ACR North American Advances*, Association for Consumer Research, 1 Jan. 1978.

<u>CUSP</u>

Mathematically

Thom's second theory is the cusp catastrophe. Following the equation $V = x^4 + ax^2 + bx$, the cusp catastrophe could be best described as a phenomenon that occurs when a approaches the 'fold point,' or the buildup to a breaking point, further causing a catastrophe to occur (Exhibit B). Variables a and b represent the two independent variables. In order to qualify as a cusp catastrophe, the situation needs to possess five qualities: bimodality, sudden transactions, hysteresis, inaccessibility, and divergence (Glossary).

Marketing

The cusp catastrophe has two independent variables and one dependent variable, meaning it would take two externally unrelated factors to shift consumer behavior. This theory has been used in many ways, from tracking outer shell electron transfer in chemistry, to modeling real estate prices.[6] In business and marketing, it can be seen in customer adoption habits, and even a unique analysis of online retail competition.[7] In this sense, two independent variables will have to shift simultaneously in order to get the dependent outcome.

<u>SWALLOWTAIL</u>

Mathematically

The swallowtail catastrophe is Thom's third single active variable theory. Its equation is $V = x^5 + ax^3 + bx^2 + cx$, and has three independent variables with one dependent variable. It can be found

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[6] Xu, F (1990). "Application of catastrophe theory to the $\Delta G^{\neq}$ to $-\Delta G$ relationship in electron transfer reactions.". *Zeitschrift für Physikalische Chemie Neue Folge*. 166: 79–91.

Bełej, Mirosław; Kulesza, Sławomir (1 January 2012). "Modeling the Real Estate Prices in Olsztyn under Instability Conditions". *Folia Oeconomica Stetinensia*.

[7] Dou, Wenyu, and Sanjoy Ghose. "A Dynamic Nonlinear Model of Online Retail Competition Using Cusp Catastrophe Theory." *Journal of Business Research*, 7 Feb. 2006, pp. 839–846.

in three dimensional space, allowing for two minima and two maxima to all meet at a single value of x (Exhibit C).[8]

Marketing

Simply put, the swallowtail catastrophe is when there are three independent variables and one dependent variable, meaning it would take three externally unrelated factors to shift consumer behavior.

<u>BUTTERFLY</u>

Mathematically

The butterfly function is the last of Thom's single active variable catastrophe theories, and is made of both fold and cusp bifurcations. The equation is $V = x^6 + ax^4 + bx^3 + cx^2 + dx$. When a is greater than 0, three fold bifurcations and two cusp bifurcations come together and meet at the butterfly point. There was no sufficient evidence available to suggest a relationship between marketing and this theory.

TWO ACTIVE VARIABLE CATASTROPHES

Due to the intense level of higher mathematics and physics that come with two active variable catastrophes, this essay can only highlight the general background of two active variable catastrophes, and provide their respective equations.

Two active variable catastrophes are all umbilic, meaning they directly occur in the middle of three dimensional space. Thom's research suggests that these catastrophes can be seen in light reflections on spherical surfaces. These functions have only been seen in math and

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[8] Wikipedia. "Catastrophe Theory, Swallowtail Catastrophe." *Wikipedia*, Wikimedia Foundation, 24 Aug. 2017.

science, and there isn't enough research to prove that there can be a possible relationship between these catastrophes and marketing theory. Their respective mathematical equations are as follows:

HYPERBOLIC UMBILIC CATASTROPHE

$V = x^3 + y^3 + axy + bx + cy$ (Exhibit D)

Thom argued that this function could be used to model the breaking of a wave.[9]

ELLIPTIC UMBILIC CATASTROPHE

$V = (x^3/3) - xy^2 + a(x^2 + y^2) + bx + cy$ (Exhibit E)

Thom argued that this function could be used to model the foundation of hair strands, and things similar to it.[10]

PARABOLIC UMBILIC CATASTROPHE

$V = x^2y + y^4 + ax^2 + by^2 + cx + dy$ (Exhibit F)

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[9] Thom, Rene. "Historicophilosophical Digression." *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*. By Vladimir Arnold and Christopher Zeeman. New York, NY: W.A. Benjamin, 1975. Print.
[10] Ibid.

Appendix

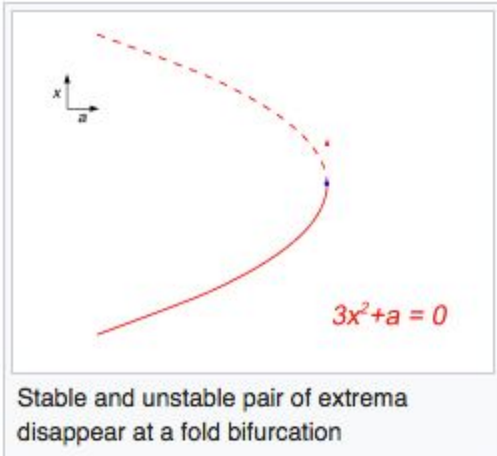
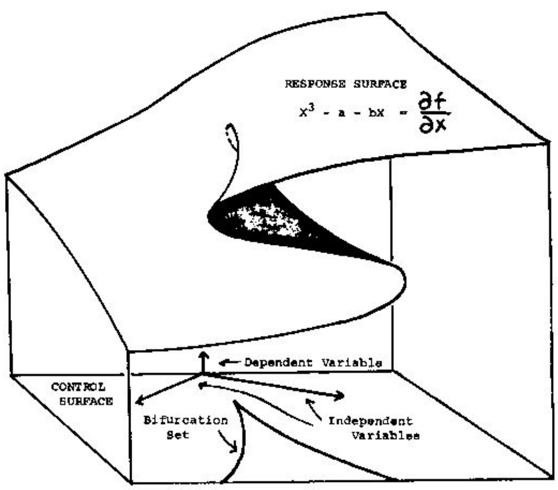


Exhibit A: Wikipedia. “René Thom.” *Wikipedia*, Wikimedia Foundation, 24 Aug. 2017.

FIGURE 2
A Cusp Catastrophe



(From Zeeman, 1976)

Exhibit B: Thom, René. "Historicophilosophical Digression." *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*. By Vladimir Arnold and Christopher Zeeman. New York, NY: W.A. Benjamin, 1975. pp. 4-6. Print.)

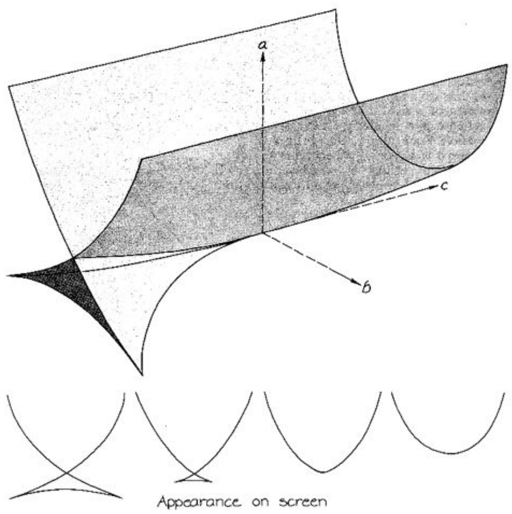


Exhibit C, Swallowtail

Walker, Jearl. " Caustics: Mathematical Curves Generated By Light Shined Through Rippled Plastic." *The Amateur Scientist*, The Amateur Scientist, 1983

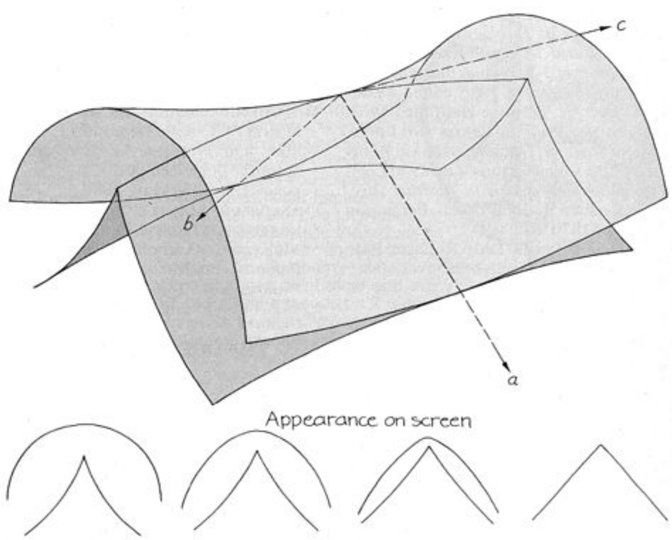


Exhibit D, Hyperbolic, Ibid.

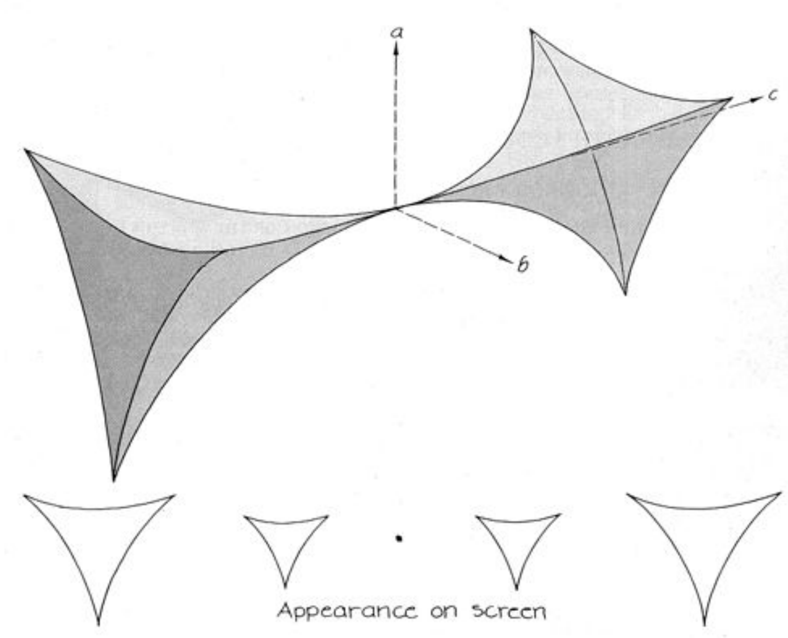


Exhibit E, Elliptical, Ibid.

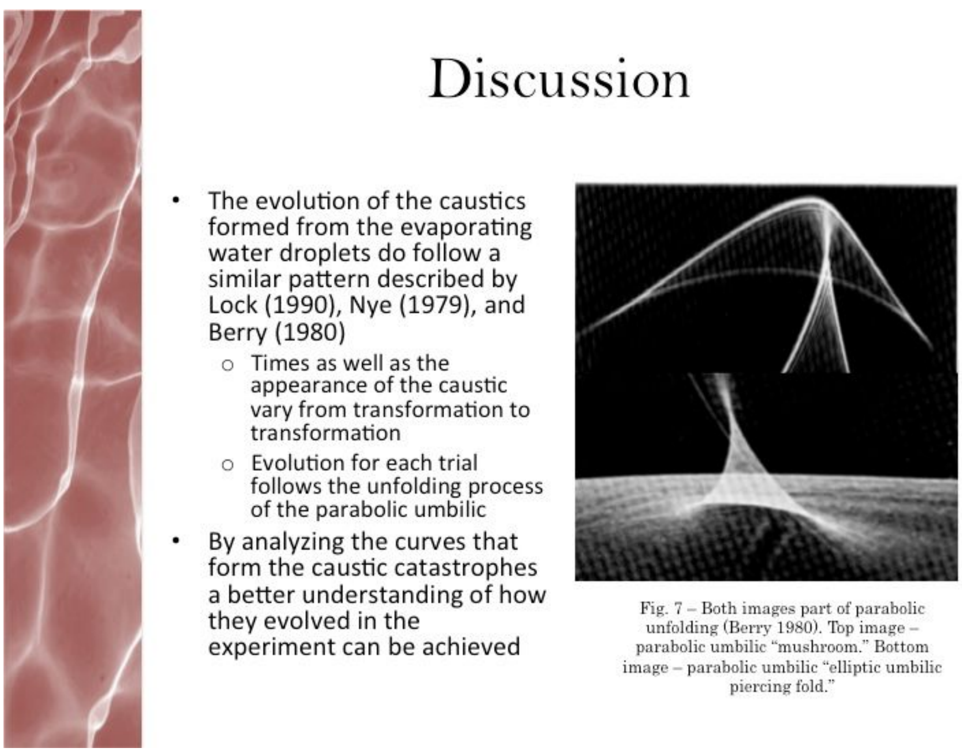


Exhibit F, Parabolic

Scibelli, Samantha. "Presentation 8.7.13." *Samantha Scibelli -- Laser Teaching Center Journal*, Stony Brook University.

Glossary

bifurcation: the division of something into two branches or parts

bimodality: created by the top and bottom surfaces of the fold; represents where the system tends to hold equal amounts of both control variables at the same time.[11]

catastrophe: a sudden change in state

stable equilibrium: a state which depends continuously on the parameters

parabola: plane curve generated by a point moving so that its distance from a fixed point is equal to its distance from a fixed line

hysteresis: a delayed reaction, in this case, in consumer or market behavior

butterfly effect: a property of chaotic systems (such as the atmosphere) by which small changes in initial conditions can lead to large-scale and unpredictable variation in the future state of the system

chaos theory: a branch of mathematics focused on the behavior of dynamical systems that are highly sensitive to initial conditions

domino effect: a cumulative effect produced when one event initiates a succession of similar events

inflection point: a point of a curve at which a change in the direction of curvature occurs

snowball effect: situation in which one action or event causes many other similar actions or events

sudden transitions: at the edges of the fold; as the system moves along the surface towards the pleat, at some point a small increase in the control variable will cause a shift in behavior

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[11]Oliva, Terence A., and Alvin C. Burns. “Catastrophe Theory As a Model For Describing Consumer Behavior.” *ACR North American Advances*, Association for Consumer Research, 1 Jan. 1978.

Marketing Examples

Fold:

1. Can be used to model a company's marketing strategy, and how these effects can cause a sudden realization among consumers about their consumption habits. This is most often seen in "shock advertising," an effective form of advertising that is meant to "deliberately, rather than inadvertently, startles and offends its audience by violating norms for social values and personal ideals."[1]
    a. Marketing example: Gary has been a smoker his whole life, and doesn't see the need to quit, even though he has a wife and young children. He sees a shock ad about smoking around children and it changes his whole perspective on this habit. (Exhibit A)
    b. Marketing example: Tamia loves Coca Cola and has been drinking it her whole life. She's relatively fit and goes to the gym often, but can't seem to shake that extra weight. She knows the nutrition facts for soda and knows that it isn't the healthiest, but what's 240 calories here or there? Well, Tamia sees this latest ad from the NYC Health Commission, and re-evaluates her decision after she does more research and sees that consuming one soda each day is equivalent to 35 lbs of sugar per year. (Exhibit B)
    c. Marketing example: Everyone loves shopping, but no one tends to think about all the child labor that goes into the production of clothes manufactured in third-world and underdeveloped countries. This advertisement frames materialistic consumption in the perspective of a child slave worker, triggering the consumer to think about the sides of the supply chain that is typically most forgotten about. (Exhibit C)

[1] Dahl, Darren W. et al. "Does it pay to shock? Reactions to Shocking and Nonshocking Advertising Content among University Students" Journal of Advertising Research 43 (2003): 268-280. Page 268, Retrieved August 9, 2017.

Cusp:

1. Can be used to model consumer behavior, in terms of consumer adoption and consumer complaining patterns
    a. Consumer adoption: consumers are satisfied with their current behaviors until some outside peer pressure forces them to conform new habits or reconsider their own habits.[2]
        i. Marketing example: Kevin has had his Blackberry since 2011. He's been loyal to this product for years, but finally switches to an iPhone after being the brunt of all the office jokes. All of his coworkers have smartphones (iPhones or Android devices).
    b. Online retailing data: "The Catastrophe Theory is uniquely positioned to capture the inherent nonlinearity and complexity in the e-commerce system."[3] E-commerce makes for a unique example, because of its more recently advanced business model that is making it a slow reaching leader in methods of consumer sales. A research study was performed by faculty from City University of Hong Kong and University of Wisconsin-Milwaukee, suggesting that lesser known brands might not always be losing when compared to their name brand counterparts.[4]
        i. Marketing example: Amazon (in 2006) vs. less established e-commerce organization. Both companies' website visit rate and competitive advantage would be the independent variables necessary to produce a solution highlighting a rival company's competitive dynamics.
    c. Zeeman's model of the stock market: Zeeman uses cusp to define the general patterns of the stock market, and what happens in both bear and bull economies. (Exhibit D)

As for the other five theories, there has not been sufficient research performed proving that there is even a relationship between marketing theory and Thom's other five elementary catastrophes. It is for this reason that there are no provided examples of how marketing

[2] Oliva, Terence A., and Alvin C. Burns. "Catastrophe Theory As a Model For Describing Consumer Behavior." *ACR North American Advances*, Association for Consumer Research, 1 Jan. 1978.
[3] Dou, Wenyu, and Sanjoy Ghose. "A Dynamic Nonlinear Model of Online Retail Competition Using Cusp Catastrophe Theory." *Journal of Business Research*, 7 Feb. 2006, pp. 839–846.
[4] Ibid.

strategies can relate to the swallowtail, butterfly, hyperbolic umbilic, elliptic umbilic, and parabolic umbilic theories. In fact, the further depth of discovering a relationship between these items might be better suited for a doctoral candidate or graduate student, who might have more familiarity with these higher level concepts.

Appendix

Exhibit A: "You Wouldn't Let Your Child Smoke." 2013. PETA.

Exhibit B: "Are You Pouring On the Pounds?" 2009. NYC Department of Health and Human Services.

Exhibit C: “Some Things Should Never Be For Sale.” 2009. World Vision.

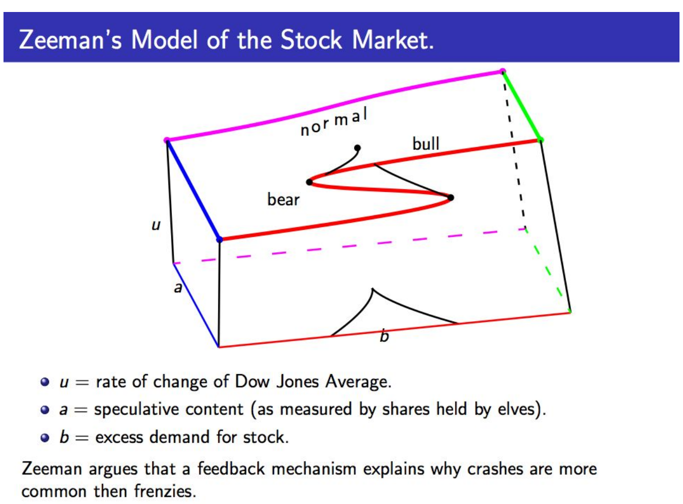

Exhibit D: JWR (UW Madison). Catastrophe Theory. Lecture. 19 February 2013.