

# Assignment - AI 41403

- Title: 8 puzzle using A\* Algorithm
- Problem definition: Solve 8-puzzle problem using A\* Algorithm. Assume any initial configuration & define goal configuration clearly.
- Learning Objectives:
  - learn informed search algorithms
  - learn A\* algorithm & its app<sup>n</sup>
  - Learn about 8 puzzle problem
  - To define perception, cognition, Action & Goal clearly.
- Outcomes: I will be able to design A\* algorithm to solve the 8 puzzle problem.
- S/W & H/W : OS 64 bit, Python 3, jupyter environment

Mathematical model:

$$S = \{s; e; X; Y; Fme; Ff; DD; NDD\}$$

$s$  = start state

$$\triangleright \{ \{1, 2, 3\}, \{0, 4, 6\}, \{7, 5, 8\} \}$$

$e$  = end state

$$\triangleright \{ \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 0\} \}$$

$$X = \{X1\}$$

$$\triangleright X1 = 5$$



$$Y = \{y_1\}$$

$$|y_1| = e$$

$$F_{me} = \{f_0\}$$

\*  $f_0$  - function to perform A\* searching

$$F_f = \{f_1, f_2, f_3, f_4, f_5\}$$

where

\*  $f_1$  - fun to find cost

\*  $f_2$  - fun to find states

\*  $f_3$  - fun to display grid

\*  $f_4$  - fun to generate next possible set of moves

\*  $f_5$  - fun to validate moves

DD - 3x3 grid of puzzle

NDD - No non deterministic data.

- Concept related theory.

8 - puzzle problem.

- 8 puzzle is a popular puzzle that consists of 8 tiles & one empty space

- The puzzle is divided into 3 rows & 3 columns

- The other 8 tiles have numbers 1 through 8 on it.

- The puzzle can be solved by moving the tiles one by one in the single empty space & thus achieving the goal state.



Start state.

$$s = \{ \{1, 2, 3\}, \{0, 4, 6\}, \{7, 5, 8\} \}$$

1	2	3
	4	6
7	5	8

- perception

- we visualize swapping the empty space with its neighbours
- The empty tile can have 9 possible locations
- $\exists x. (\text{move}(x) \wedge \text{valid location}(x))$
- The empty space can only move in four directions.
- The empty space cannot move diagonally and can take only one step at a time
- $\exists x (\text{move}(x) \rightarrow \sim \text{diagonal}(x))$

- cognition:

cost of A\* algorithm:  $f(n) = g(n) + h(n)$

-  $g(n)$  is the cost of the path traversed from the initial state to node  $n$ .

-  $h(n)$  is the estimated path-cost or the heuristic function cost from node  $n$  to goal node.

$g(n) = \text{path cost}$

- each step costs 1, so the path is no. of steps in the path.



For any  $k$ -puzzle's tile

$$\text{misplaced}(i) = \{ (x_1, y_1) \neq (x_2, y_2) \mid \forall x_1, y_1, x_2, y_2 \in \{0, 1, 2\} \}$$

where,

$(x_1, y_1)$  is the coordinate of the tile with  $i$  in current state

$(x_2, y_2)$  is the coordinate of the tile with  $i$  in goal state

- $x_1, y_1, x_2, y_2 \in \{0, 1, 2\}$
- $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $h(n) = \text{misplaced}(i)$  for  $0 \leq i \leq 8$
- Action =  $\{U, L, D, R\}$ 
  - action can be represented as set of 4
  - each action represents the interchanging of blank tile with the neighbour's one of the 4 directions
  - constraints
    - the neighbour should not be diagonally adjacent
    - the edge positions can interchange in only three directions
    - the corner in 2 positions

$$f_{\min} = \min(f_1, f_2, \dots, f_n)$$

- Goal state

1	2	3
4	5	6
7	8	

- conclusion: I have successfully designed a search algorithm for  $k$ -puzzle & defined perception cognition, Action & Goal for the same.