

## Macroeconomics LR/SR

Aggregation: micro → macro

GDP growth, unemployment, government policy

$$\text{Growth rate in year t} = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \quad \text{rule of 70}$$

Business Cycle: expansion, contraction

Inflation deflation hyperinflation (over 50% in a month)

fiscal policy, monetary policy

Summing individual economic variables to obtain economy wide totals

Positive analysis (objective) Normative analysis

## Chapter 2

National Income accounts: measuring current economic activity

- production, income, spending

GDP: market value of final goods and services newly produced within a nation during a fixed period of time

× Intermediate goods to avoid double counting (sum of value added)

Consumption goods, capital goods, Inventory investment

$\text{GDP} = \text{GNP} - \text{NFP}$  (Gross National Product - Net Factor payments from abroad)  
(domestic producer's goods will be included)

+ payments to domestically owned located abroad - payments to foreign factors located domestically

Not included in GDP

Depreciation = consumption of fixed capital

- Sales of used goods
- Intermediate goods and services
- sale of financial assets
- goods and services produced outside the region

### 3 Approaches:

Product Approach:  $GDP = \text{sum of the value of all goods and services in the economy}$   
- sum of value of all intermediate goods used in the production

Expenditure Approach:  $GDP = C + I + G + NX$

C: Consumer durables, non durable goods, Services

I: Business fixed investment (structures & equipment), Residential Investment, change in inventory

G: spending by the government on goods and services

X transfer payments, interest payments on the government debt

EX: Export - Import

After tax profits = Total revenue - Wages - Interest - Cost of intermediate inputs - taxes

National Income and Product Accounts (NIPA)

National income = compensation of employees + corporate profits + proprietor's income  
+ net interest + rental income of persons + taxes on production  
and imports + business current transfer payment + current surplus  
of government enterprises

GDP = Wage + Interest + Rent + Profit + Tax

GDP does not include: housework, informal sector, intangible capital / investment  
(software, R&D, artistic originals)

## GDP example:

### Producer

Revenue: 20 million  
 Wages: 5 million ✓  
 Interest on loan 0.5 million ✓  
 Taxes: 1.5 million  
 Gov.  
 Tax revenue: 0.5 million  
 Wages: 0.5 million ✓

### Restaurant

Revenue: 30 million  
 Cost of coconuts: 12 million  
 wages: 4 million ✓  
 Taxes: 3 million

### After tax profits

Coconut producer: 13 million ✓  
 Restaurant: 11 million ✓

### Consumers

Wage income: 14.5 million

Interest income: 0.5 million

Taxes: 1 million

Profits distributed to producers: 24 million

$$①: 30 + (20 - 12) + 5.5 = 43.5 \text{ million} / 20 + 18 + 0.5 = 43.5 \text{ million}$$

$$②: C + I + G + NX$$

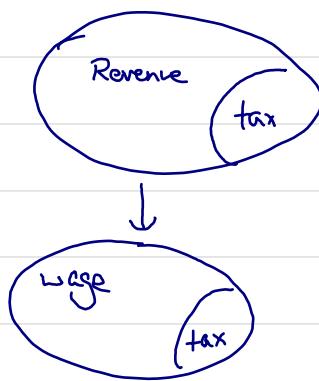
$$= 30 + 8 + 0 + 5.5 = 43.5 \text{ million}$$

③ Income approach (single count all the income)

$$= 13 + 11 + 5.5 + 5 + 4 + 0.5 + 4.5 = 43.5 \text{ million}$$

$$/ 24 + 14.5 + 0.5 - 1 + 5.5 = 43.5$$

$$\begin{array}{ccccccc} 20 & - & 5 & + & 30 & - & 12 \\ \underbrace{\quad}_{Pb+T} & \underbrace{\quad}_{Pb+T} & & & \underbrace{I} & \underbrace{g.v.r} & \underbrace{wage} \end{array}$$



$$\begin{aligned}
 \text{National Saving} &= \text{Private saving} + \text{Public Saving} \\
 &= Y - T + TR + INT (\text{interest payments on savings}) - C + \\
 &\quad T - TR - INT - G \\
 &= Y - C - G \quad \text{(interest payments on government debt)} \\
 &= I
 \end{aligned}$$

flow variables & stock variables  
 (continuous)      (discrete)

GDP deflator =  $\frac{\text{nominal}}{\text{real}} \times 100$  overall level of prices of goods and services included in GDP

$$\text{Inflation rate : } \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t}$$

$$CPI = \frac{\text{Cost of market basket in a given year}}{\text{Cost of market basket in base year}}$$

$$r + \pi = i \text{ (nominal)}$$

### Chapter 3

$$Y = Af(k, N)$$

- 1. upward sloping  $\frac{\partial Y}{\partial k} > 0$
- 2. diminishing marginal product  $\frac{\partial^2 Y}{\partial k^2} < 0$

Marginal product of capital

$$MPK = \frac{\partial Y}{\Delta K} = \frac{\partial Y}{\partial k}$$

$$MPN = \frac{\Delta Y}{\Delta N} = \frac{\partial Y}{\partial N} \text{ (same reasoning)}$$

Often consider A as exogenous (variables outside the model)

change in A: productivity shock (slope  $\uparrow \downarrow$  at each level of input)

## Cobb - Douglas Production Function

$$Y = AK^\alpha N^{1-\alpha} \quad 0 \leq \alpha \leq 1$$

$$\frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1} N^{1-\alpha} \geq 0$$

$$\frac{\partial^2 Y}{\partial K^2} = A\alpha(\alpha-1) \underbrace{K^{\alpha-2}}_{(-)} N^{1-\alpha} \leq 0$$

$$Y = AF(2K, 2N) = A(2K)^\alpha (2N)^{1-\alpha}$$

$$= 2A K^\alpha N^{1-\alpha}$$

$$= 2Y$$

$\uparrow$  both inputs by a factor of  $\beta$ ,  $Y \uparrow$  by a factor of  $\beta$

(constant returns to scale)

\* Constant factor shares:  $K$  and  $L$  as a share of output  $Y$

$$\text{Eg. labour share} = \frac{wN}{Y} = 1-\alpha$$

$$\begin{aligned} w &= \frac{\partial Y}{\partial N} = A(1-\alpha) K^\alpha N^{-\alpha} \\ &= (1-\alpha) \frac{AK^\alpha N^{1-\alpha}}{N} \\ &= (1-\alpha) \frac{Y}{N} \end{aligned}$$

$\Rightarrow$  total wage payment  $wN$  is a constant fraction  $(1-\alpha)$  of income  $Y$ .

Normally about 60% to 70%

Will there be constant returns to scale if  $a+b < 1$  (decreasing returns to scale)

$$AF(2K, 2N) = A(2K)^\alpha (2N)^b$$

$$= 2^{a+b} AK^\alpha N^b$$

$$= 2^{a+b} Y < 2Y$$

Demand:

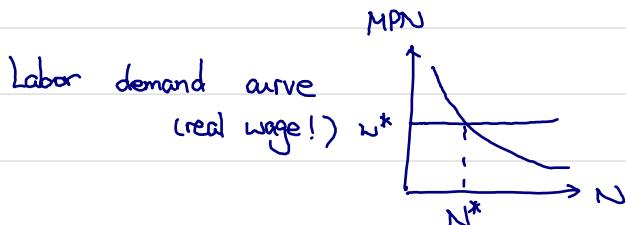
Firm's profit maximization problem

$$\max_N \text{Profit} = Y - wN - rk \quad (\text{rent} \times \text{capital}) \\ = Af(k, N) - wN - rk$$

$$\frac{\partial(Af(k, N) - wN - rk)}{\partial N} = MPN - w = 0 \\ \Rightarrow MPN = w$$

$w > MPN$  (profit decreases)

$w < MPN$  (profit can be higher)



Change in wage  $\Rightarrow$  a movement along the labor demand curve

Shift the labor demand curve: { supply shocks  
                                  ↑ K}

$$\text{Since } w = AK^{\alpha}(1-\alpha)N^{-\alpha}$$

$$N^{-\alpha} = \frac{w}{AK^{\alpha}(1-\alpha)} \\ N = \left(\frac{AK^{\alpha}(1-\alpha)}{w}\right)^{\frac{1}{\alpha}}$$

Supply of labor:

Utility maximization problem

$$\max_{C, N} U(C) - V(N) \quad \text{subject to } C = wN + T \\ \text{(disutility from working)} \qquad \qquad \qquad \text{wealth}$$

$$\frac{\partial U(C) - V(N)}{\partial N} = wU'(wN + T) - V'(N) = 0 \\ \Rightarrow U'(C)w = V'(N)$$

Substitution effect: keeping income level unchanged, effect of real wage change  
encourages work (one yr rise in the real wage)

Income effect: keeping real wage unchanged, effect of income change (lottery)  
chooses more consumption than work (leisure is normal good)

Labor supply curve relates quantity of labor supplied to current real wage  
 current real wage (otherwise income effect)



Shift the curve:  $\begin{cases} \uparrow \text{working age population} \\ \uparrow \text{labor force participation} \end{cases}$

E.g.  $\max_{C,N} U(C) - V(N)$  subject to  $C = w(1-\tau)N + T$   
 (income tax rate)

$$U(C) = \ln C \quad V(N) = \frac{1}{1+\frac{1}{\delta}} N^{1+\frac{1}{\delta}}$$

$$U'(C) = \frac{1}{C} w(1-\tau) \quad V'(N) = N^{\frac{1}{\delta}}$$

$$\Rightarrow \frac{1}{C} w(1-\tau) = N^{\frac{1}{\delta}}$$

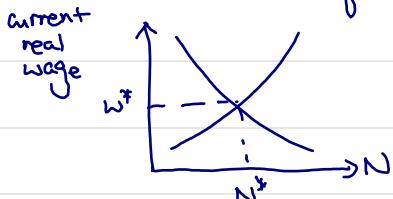
$$N = \left( \frac{w(1-\tau)}{C} \right)^{\delta}$$

Labor demand under Cobb-Douglas:  $w = (1-\alpha) \frac{F}{N}$

$$N = \left( \frac{(1-\alpha) \frac{F}{N} (1-\varepsilon)}{C} \right)^{\frac{1}{\alpha}}$$

$$N^* = \left( (1-\alpha) \frac{F}{C} (1-\varepsilon) \right)^{\frac{1}{1+\delta}}$$

Labor market equilibrium



Full employment Output  $\bar{Y} = AF(k, \bar{N})$

When labor market is in equilibrium

(affected by changes in full employment level  
 or production function)

Unemployment: Labor force = Employed + Unemployed

$$\text{Unemployment Rate} = \frac{\text{Unemployed}}{\text{Labor force}}$$

$$\text{Participation Rate} = \frac{\text{Labor force}}{\text{Adult Population}}$$

$$\text{Employment Ratio} = \frac{\text{Employed}}{\text{Adult Population}}$$

Unemployment spell: period of time an individual is continuously unemployed

Duration: length of unemployment spell

Frictional	Structural	Cyclical
matching process	Structural change	$u - \bar{u}$
ST	LT	Total - Natural rate of unemployment

Okun's Law:  $\frac{Y-Y}{Y} = 2(u-\bar{u})$

## Chapter 4. Consumption, Saving, and Investment

### Good and Capital markets

$$Y_{\text{expenditure}} = C + I + G \quad Y_{\text{production}} = AP(K, N)$$

When goods market is in equilibrium,

$$Y_{\text{production}} = Y_{\text{expenditure}}^{\text{desired}} = C^d + I^d + G$$

Unsophisticated Robinson: (consider current situation)

$$C^{\text{desired}} = \bar{C} + MPC \times Y(\text{income})$$

$$S^{\text{desired}} = Y - C^d = -\bar{C} + (1 - MPC)Y$$

Sophisticated Robinson: (consider foreign situation)

$$U(C_1, C_2) = U(C_1) + \beta U(C_2)$$

$\beta$  captures the impatience of consumers

Consumption-smoothing motive:

desire to have a relatively even pattern of consumption over time.

$$\max_{C_1, C_2, B} U(C_1) + \beta U(C_2) \quad \text{subject to} \quad C_1 + B = Y_1 \quad \text{and} \quad C_2 = Y_2 + (1+r)B \quad B: \text{saving / borrowing}$$

$$Y_1 - C_1 = \frac{C_2 - Y_2}{1+r}$$

$$Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} \quad (\text{PV of consumption})$$

$$C_2 = (1+r)Y_1 + Y_2 - C_1(1+r)$$

$$\Rightarrow U(C_1) + \beta U((1+r)Y_1 + Y_2 - C_1(1+r)) = 0$$

$$U'(C_1) + \beta U'(C_2) ((1+r)) = 0$$

$$\rightarrow U'(C_1) = (1+r)\beta U'(C_2)$$

Benefit of saving today  
and consuming  $tmr$

Benefit of consuming today

E.g.  $U(C) = \ln C$      $U'(C) = \frac{1}{C}$

$$U'(C_1) = (1+r)\beta U'(C_2)$$

$$\frac{1}{C_1} = (1+r)\beta \frac{1}{C_2}$$

$$\frac{C_2}{C_1} = (1+r)\beta$$

higher interest rate increases consumption growth

$\Rightarrow$  saves more and consumes more

ONLY apply to log utility

Substitution effect:  $r \uparrow$ , return to saving  $\uparrow$  leads to substitute current consumption for future consumption so that saving increases.

Income effect:  $r \uparrow$  target can be achieved with a smaller pool of saving  
saver: save less (increase in wealth)

borrower:  $\uparrow$  payment in the future, (save more instead of borrowing)

Consumption Euler equation:  $\frac{C_2}{C_1} = (1+r)\beta$  ①

Intertemporal budget constraint:  $C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$  ②

Substitute ① into ②:  $C_1 + \frac{C_1(1+r)\beta}{1+r} = Y_1 + \frac{Y_2}{1+r}$

$$C_1(1+\beta) = Y_1 + \frac{Y_2}{1+r}$$

$$C_1 = \frac{1}{1+\beta} \left( Y_1 + \frac{Y_2}{1+r} \right)$$

$$S_t = Y_t - C_t = Y_t - \frac{1}{1+\beta} Y_1 - \frac{Y_2}{(1+\beta)(1+r)} \\ = \frac{1}{1+\beta} \left( \beta Y_1 - \frac{Y_2}{1+r} \right)$$

Consumption can be always adjusted by saving / borrowing

People will spend money at a level consistent with their expected long term average income.

$$C_t = \frac{1}{\alpha} \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} Y_t$$

Fiscal policy :  $S^d = Y - C^d - G$  (directly affect desired national saving)

Lump-sum tax :  $G_1 + D = T_1$ ,  $G_2 = T_2 + (1+r)D$

$$\Leftrightarrow G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

Robinson's problem:  $G_1 + \frac{G}{1+r} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$

$$\max_{C_1, C_2, B} U(C_1) + \beta U(C_2)$$

In the case of  $U(C) = \ln C$

fraction of permanent income

$$C_1 = \frac{1}{1+\beta} (Y_1 - T_1 + \frac{Y_2 - T_2}{1+r})$$

$$C_1 = \frac{1}{1+\beta} (Y_1 - G_1 + \frac{Y_2 - G_2}{1+r})$$

$$(1+r)C_1 + C_2 = Y_1(1+r) - T_1(1+r) + Y_2 - T_2$$

$$C_2 = Y_1(1+r) - T_1(1+r) + Y_2 - T_2 - (1+r)C_1$$

$$\frac{1}{C_1} - \beta(1+r) \frac{1}{C_2} = 0$$

$$\frac{C_2}{C_1} = \beta(1+r)$$

$$\Rightarrow \Delta C_1 = -\frac{1}{1+\beta} \Delta G_1$$

$$S_1 = Y_1 - C_1 - G_1 = Y_1 - \frac{1}{1+\beta} (Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}) - G_1$$

$$\Rightarrow \Delta S_1 = (\frac{1}{1+\beta} - 1) \Delta G_1$$

$\uparrow G$ ,  $S, C \downarrow$

$$Y_1(1+r) - T_1(1+r) + Y_2 - T_2 - (1+r)C_1 = C_1 \beta (1+r)$$

$$Y_1(1+r) - T_1(1+r) + Y_2 - T_2 = C_1(1+r)(1+\beta)$$

$$C_1 = (Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}) \frac{1}{1+\beta}$$

Lump sum tax cut today, financed by higher future taxes (No change in gov. purchase)

$$\frac{G_1 + \frac{G_2}{1+r}}{\text{unchange}} = T_1 + \frac{T_2 \uparrow}{1+r} \quad (\text{Tax does not affect consumption decision})$$

Ricardian equivalence proposition: timing of taxes doesn't matter - no effect on your consumption decision

(may not hold, since we don't have information about the future)

Tax  $\uparrow$ ,

saving unchange if sophisticated

rises if unsophisticated

$$S^d = Y - C^d - G$$

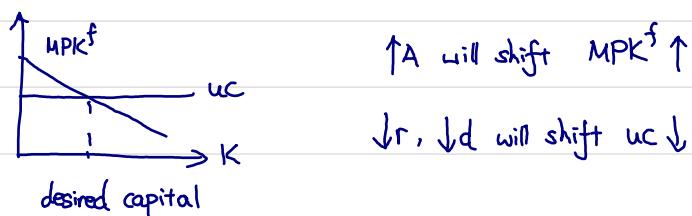
$\uparrow \downarrow$

Investment is more sensitive to the ups and downs of economy.

Return of additional investment :  $\underbrace{1-d}_{\text{undepreciated}} + \text{MPK}^f$  (investment becomes capital stock with a lag)

$$\text{MB} = \text{MPK}^f \quad \text{MC} = r+d \quad (\text{real interest cost and depreciation})$$

$$\Rightarrow \text{MPK}^f = r+d \quad (\text{user cost})$$



With taxes

$$(1-t)\text{MPK}^f = \text{uc}$$

$$\text{MPK}^f = \frac{\text{uc}}{1-t} = \frac{r+d}{1-t}$$

$\uparrow \uparrow$ : desired capital stock  $\downarrow$

Gross investment:  $I_t = K_{t+1} - K_t + dK_t$

1. New capital increases the gross investment
2. Current capital decreases the gross investment
3. depreciation,  $dK_t$

Net investment:  $K_{t+1} - K_t = I_t - dK_t$  (gross investment - depreciation)

$I_t = K_{t+1} - K_t + dK_t$  desired net increase, investment needed to

$\star = K_{t+1} - (1-d)K_t$  replace depreciated capital

Interest rate  $r$ : lending to firms is risky

Corporate debt yield = safe asset interest rate + spread (risk premium)

Tobin's  $g$  theory : Firms change investment in the same direction as the stock market

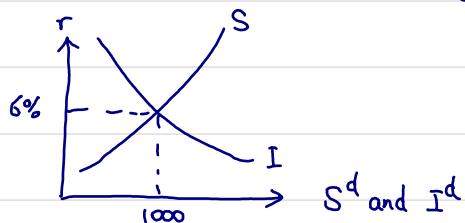
$$\text{Tobin's } g = \frac{\text{Capital's market value}}{\text{replacement cost}} = \frac{V}{P_k K}$$

$g < 1$ : don't invest     $g > 1$ : invest

### Goods Market Equilibrium

$$Y_{\text{production}} = Y^d_{\text{expenditure}} = C^d + I^d + G \text{ (exogenous given)}$$

$$\text{Since } S^d = Y - C^d - G \text{ and } Y^d = C^d + I^d + G \text{ and } Y = Y^d \\ \Rightarrow S^d = I^d \text{ (equilibrium)}$$



current output ↑  
 $\downarrow$  expected future output  
 $\downarrow$  wealth  
 $\downarrow$   $G$   
 $\uparrow$  taxes

} Saving shifts to the right

$r \uparrow$   
 $\uparrow T$  (tax rate)  
 $\downarrow MPK$

} Investment shifts to the left

$$\text{Gross Investment} = \text{Net investment} + \text{depreciation}$$

$$R_{t+1} - (1-d)R_t \text{ undepreciated}$$

### Chp 7 The Asset Market, Money, and Prices

Money: assets that are used and accepted as payment

1. Medium of exchange
2. unit of account
3. store of value

M1 monetary aggregate: currency and traveler's checks

checking accounts      bear interest

S-T securities, allow check-writing

M2 monetary aggregate: M1 + saving deposits + small time deposits + non-institutional MMMF

+ money - market deposit accounts (competitor to MMMFs)

passbook savings accounts

Monetary system: HKMA

Money supply: amount of money available in the economy

{ newly printed money  
sell financial assets  
open market operations

Money demand: portfolio allocation (expected return, risk, liquidity, time to maturity)

ER: increase of an asset in value per unit of time

interest rate, dividend yield + capital gains yield

Risk: degree of uncertainty in an asset's return (risk premium)

Liquidity: ease and quickness with which an asset can be traded  
money is liquid

Common assets: money (low return, low risk, high liquidity)

bonds (higher return, more risk, less liquidity)

stocks (more risky)

Ownership of a small business (risky and not liquid, very high return)

housing (capital gains, illiquid)

The sum of asset demands equals total wealth.

Affecting demand for Money:

Price level: higher the price level, more money you need for transactions

real income: more transactions you conduct, more liquidity you need. ( $\text{real GDP} \Rightarrow \uparrow C$ )  
money demand rises less than the rise in real income

interest rates:  $\uparrow$  interest rate,  $\downarrow$  demand for money

$$M^d = P \times L(Y, i)$$

↑ price level      ↑ real income      ↑ nominal interest rate on non monetary asset

$$= P \times L(Y, r + \pi e)$$

$$\Leftrightarrow \frac{M^d}{P} = L(Y, r + \pi e)$$

↑ real money demand

↑ Payment technologies : ↓ money demand

↑ Liquidity of alternative assets : ↓ money demand

Generally  $M^d = P \times L(Y, i, \text{wealth, risk, etc.})$   $i$  is the interest rate on nonmonetary assets

Elasticities of money demand:

income elasticity of demand :  $\frac{\% \Delta Q}{\% \Delta I} = \frac{2}{3}$

1% increase in income  $\Rightarrow 0.667\%$  increase in demand

interest elasticity of demand :  $\frac{\% \Delta Q}{\% \Delta r} = -\frac{1}{5}$

1% increase in nominal interest rate  $\Rightarrow 0.2\%$  decrease in demand

\* price elasticity of money demand is unitary

Velocity =  $\frac{\text{nominal GDP}}{\text{nominal money stock}} = \frac{PY}{M}$  how much money 'turns over' each period.

Quantity theory of money :  $M_t V_t = P_t Y_t$

$$\Rightarrow \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y}$$

$M_2$  velocity is closer to being a constant.

Demand for money :  $M^d = P \times L(Y, i)$ ,  $L(Y, i) = kY$

Assume money supply = money demand

$$\frac{M_t}{P_t} = kY_t$$

Assume constant velocity

$$\frac{M_t}{P_t} = \frac{1}{V} Y_t$$

$$\Rightarrow \frac{\Delta M}{M} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y} \quad (\text{neutrality of money})$$

## Asset Market Equilibrium

$m^d + nm^d = \text{total nominal wealth of an individual}$

$M^d + NM^d = \text{aggregate nominal wealth}$

$M + NM = \text{aggregate nominal wealth (supply of assets)}$

$$(M^d - M) + (NM^d - NM) = 0$$

Equilibrium condition:  $M^d = P \times L(Y, r + \pi^e)$

real money supply = real money demanded ( $M^d = M$ )

$\Rightarrow P = \frac{M}{L(Y, r + \pi^e)}$  (price level is the ratio of nominal money supply to real money demand)

## Chp 6 Long-Run Economic Growth

Growth accounting :  $Y = AF(K, L)$  which factor is important

$$Y = AK^\alpha N^{1-\alpha}$$

$$\frac{Y_t}{Y_{t-1}} = \frac{A_t K_t^\alpha N_t^{1-\alpha}}{A_{t-1} K_{t-1}^\alpha N_{t-1}^{1-\alpha}}$$

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln\left(\frac{A_t K_t^\alpha N_t^{1-\alpha}}{A_{t-1} K_{t-1}^\alpha N_{t-1}^{1-\alpha}}\right)$$

$$= \ln\left(\frac{A_t}{A_{t-1}}\right) + \alpha \ln\left(\frac{K_t}{K_{t-1}}\right) + (1-\alpha) \ln\left(\frac{N_t}{N_{t-1}}\right)$$

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln\left(1 + \frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\Delta Y}{Y}$$

$$\Rightarrow = \frac{A_t - A_{t-1}}{A_{t-1}} + \alpha \frac{K_t - K_{t-1}}{K_{t-1}} + (1-\alpha) \frac{N_t - N_{t-1}}{N_{t-1}}$$

$$\Rightarrow \frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta N}{N}$$

↓  
slow residual (TFP growth)

can estimate others from historical data

Assume  $\alpha = \frac{1}{3}$ , income per capita  $y = \frac{Y}{N}$ , capital per capita  $k = \frac{K}{N}$ .

$$y = \frac{Ak^{\frac{1}{3}}N^{\frac{2}{3}}}{N} = \frac{Ak^{\frac{1}{3}}N^{\frac{2}{3}}}{N^{\frac{1}{3}}N^{\frac{2}{3}}} = Ak^{\frac{1}{3}}$$

determined by TFP and capital per capita  
production function in per capita

## The Solow model

Population and work force  $N$  grows at rate  $n$ .

$$N_t = (1+n)N_{t-1}$$

Economy is closed and no government

$$Y_t = C_t + I_t$$

Assume constant returns to scale, diminishing marginal return to inputs, no A growth

$$\Rightarrow AF(\beta k, \beta N) = \beta AF(k, N) = \beta Y$$

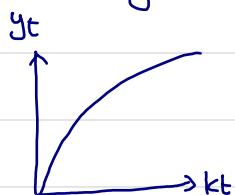
Capital accumulation :  $k_{t+1} = (1-d)k_t + \underbrace{I_t}_{\text{undepreciated}} + \underbrace{I_t}_{\text{new investment}}$

Saving decision.  $S_t = sY_t = I_t = Y_t - C_t$

$$y_t = \frac{Y_t}{N_t} \quad C_t = \frac{C_t}{N_t} \quad k_t = \frac{k_t}{N_t} \quad (\text{capital labor ratio})$$

$$y = \frac{AF(k, N)}{N} = AF\left(\frac{k}{N}, \frac{N}{N}\right) = A(k, 1) \equiv Af(k)$$

$$Y = AK^\alpha N^{1-\alpha} \Rightarrow y = \frac{AK^\alpha N^{1-\alpha}}{N} = A\left(\frac{k}{N}\right)^\alpha \left(\frac{N}{N}\right)^{1-\alpha} = Ak^\alpha$$



Solving:

$$k_{t+1} = (1-d)k_t + sY_t \Rightarrow k_{t+1} = (1-d)k_t + sAk_t^\alpha N_t^{1-\alpha}$$

$$\frac{k_{t+1}}{N_t} \frac{N_{t+1}}{N_t} = (1-d)\frac{k_t}{N_t} + sA\left(\frac{k_t}{N_t}\right)^\alpha (1)^{1-\alpha}$$

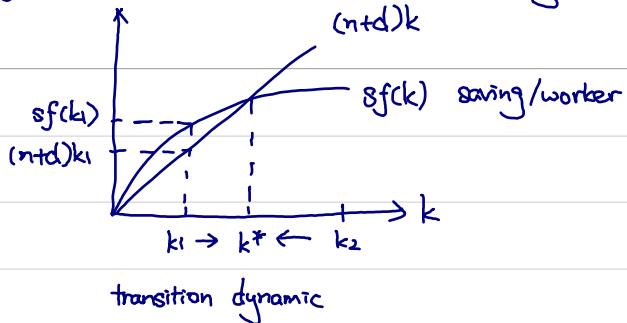
$$k_{t+1}(1+n) = (1-d)k_t + sAk_t^\alpha$$

Subtract  $(1+n)k_t$  from both sides

$$\cancel{(1+n)(k_{t+1} - k_t)} = sAk_t^\alpha - (n+d)k_t$$

steady state: point at which capital stops growing

saving and investment per worker



Steady state investment per worker

$$\begin{aligned} y^* &= Af(k^*) \\ c^* &= y^* - i^* \\ &= y^* - sy^* \\ &= Af(k^*) - (n+δ)k^* \end{aligned}$$

$$(1+n)\Delta k_{t+1} = sAk_t^\alpha - (n+δ)k_t$$

$$\Rightarrow 0 = sAk_t^\alpha - (n+δ)k_t$$

$$k^* = \left(\frac{sA}{n+δ}\right)^{\frac{1}{1-\alpha}} \quad y^* = A(k^*)^\alpha \quad c^* = (1-s)y^*$$

$$(1+n)k_{t+1} - (1+n)k_t = sAk_t^\alpha - (n+δ)k_t$$

$$(1+n)k_{t+1} = sAk_t^\alpha + (1-δ)k_t$$

$$k_{t+1} = \frac{sAk_t^\alpha + (1-δ)k_t}{1+n}$$

No productivity growth  $\Rightarrow$  all converge to constant steady state values

$\Rightarrow$  capital accumulation can't serve as an engine of long run growth

Rule of 70:  $1(1+g\%)^t = 2$

$$t = \frac{\ln 2}{\ln(1+g\%)}$$

$$\approx \frac{\ln 2}{\ln(1+g\%)}$$

$$\approx \frac{0.7}{g\%}$$

$$= \frac{70}{100g\%}$$

$$\ln\left(\frac{x_t}{x_{t-1}}\right) = \ln\left(1 + \frac{x_t - x_{t-1}}{x_{t-1}}\right) = \ln(1+g_t) \approx g_t$$

①: If  $z_t = x_t r_t$ ,  $g_t^z \approx g_t^x + g_t^r$

$$z_t = \frac{r_t}{x_t}, \quad g_t^z \approx g_t^r - g_t^x$$

$$z_t = x_t^\alpha, \quad g_t^z \approx \alpha g_t^x$$

$$\textcircled{1}: \ln\left(\frac{z_t}{z_{t-1}}\right) = \ln\left(\frac{x_t r_t}{x_{t-1} r_{t-1}}\right) = \ln\left(\frac{x_t}{x_{t-1}}\right) + \ln\left(\frac{r_t}{r_{t-1}}\right)$$

$$\Rightarrow g_t^z \approx g_t^x + g_t^r$$

## Exogenous model

### 1. Saving rate

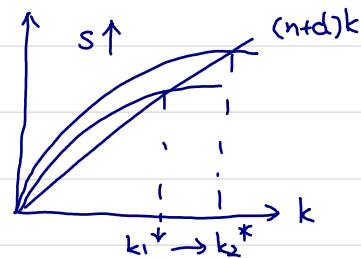
$$(1+n)\Delta k_{t+1} = SAk_t^\alpha - (n+d)k_t$$

$$\Delta k_{t+1} = 0 \quad (\text{steady state})$$

$$SAk_t^\alpha = (n+d)k_t$$

$$k^* = \left(\frac{SA}{n+d}\right)^{\frac{1}{1-\alpha}}$$

$I/N$



$$C_t = Y_t - i_t = (1-S)Ak_t^\alpha \quad (\text{lower consumption in the short run})$$

$$c^* = (1-S)A \left(\frac{SA}{n+d}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{\partial c}{\partial S} = -A\left(\frac{SA}{n+d}\right)^{\frac{1}{1-\alpha}} + (1-S)A\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{SA}{n+d}\right)^{\frac{\alpha}{1-\alpha}-1} = 0$$

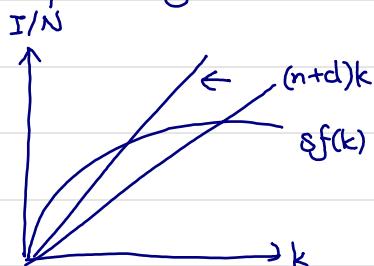
$$\Rightarrow \cancel{A\left(\frac{SA}{n+d}\right)^{\frac{1}{1-\alpha}}} = \frac{(1-S)A\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{SA}{n+d}\right)^{\frac{\alpha}{1-\alpha}}}{\cancel{\frac{SA}{n+d}}}\left(\frac{A}{n+d}\right)$$

$$\frac{1-\alpha}{\alpha} = \frac{1-S}{S}$$

$$S = \alpha$$

If the private market is efficient, gov shouldn't target a particular saving rate.

### 2. Population growth



doing so will raise consumption per capita

reduce total output and consumption

$$Y_t = Y_t N_t \Rightarrow g_Y = g_Y + g_N = g_Y + n \Rightarrow g_Y = n$$

### 3. Productivity growth (same as 1)

can always occur, raise output continuously (saving rate can't rise forever)

dominant factor

Countries below their steady state grow fast  
above ... slow

In the Solow model:

Strong convergence across OECD countries, US states, Japanese prefectures,  
European regions

Unconditional convergence: standards of living throughout the world become more or less  
the same.

Conditional convergence: Even if countries differ in their saving rates, population  
growth rates ... , they will converge to different steady  
state with different capital-labor ratios and different  
standards of living in the long run.

(standards of living will converge only within groups of countries having  
similar characteristics)

e.g. a low saving income country will catch up a richer country that also  
has a low saving rate, but it will never catch up a rich country that  
has a high saving rate.

No convergence: low income countries will never catch up.

Endogenous Growth Theory ( $A$  is explained endogenously in the model)

AK model:  $T = AK$  (constant MPK)

$F(k, k, n) = Ak^{\theta}(kn)^{1-\theta}$  :  $k \uparrow$ ,  $k \uparrow$  offsets decline in MPK from having more capital.

Human capital tends to increase in the same proportion as physical capital

Reasons some countries are so far behind the frontier?

### 1. Institutions as a barrier to rich

positive correlation between GDP per capita and Protection against risk of expropriation  
Corruption

### 2. Large difference in TFP

resource misallocation  $\Rightarrow$  TFP declines

giving more resources to low productivity firms

Policies to raise the rate of productivity growth

#### 1. Improving infrastructure

Empirical studies suggest a link between infrastructure and productivity  
but maybe reverse causation (high productivity  $\Rightarrow$  spends more)

#### 2. building human capital.

education and entrepreneurial skill

#### 3. encouraging research and development

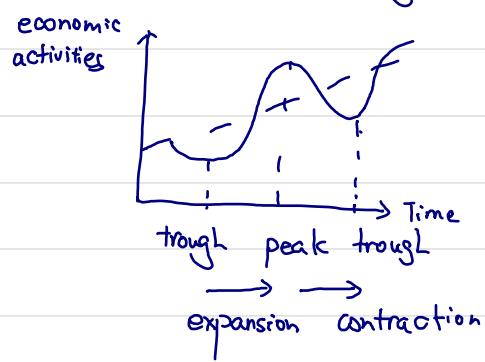
- gov support scientific research

- fund facilities

- provide grants to researchers

...

## Chp. 8 Business Cycle



leading : in advance

Coincident : at the same time

lagging : after

procyclical : same direction

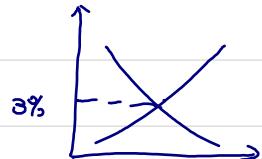
countercyclical : opposite

acyclical : no clear pattern

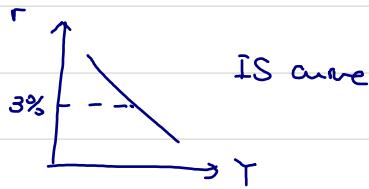
## Chapter 9 IS-LM / AD-AS

General equilibrium: Labor + Asset + Goods market equilibrium

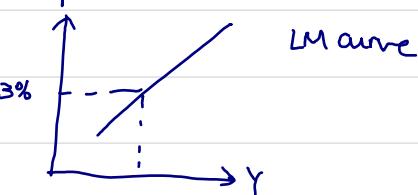
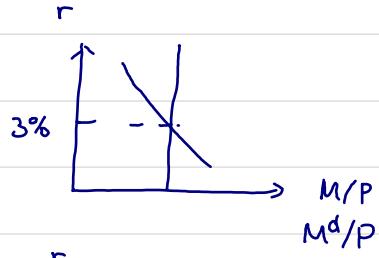
real interest rate,  $r$



Desired national saving,  $S^d$   
desired investment,  $I^d$



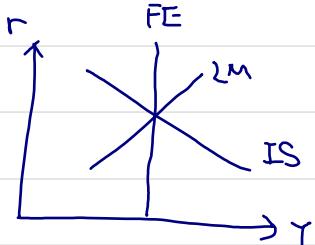
$r$



$M/P$   
 $M^d/P$



Combine

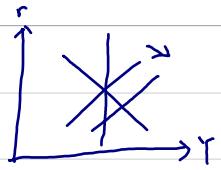


Shifter refer to Claudio's notes  
or book's table

Classical: prices are flexible and adjusts quickly

Keynesian: prices are sticky, equilibrium is at the intersection between IS and LM in the short run, labor market is not in equilibrium.

## Monetary expansion



SR: real interest rate  $\downarrow$

$\downarrow$  in real interest  $\Rightarrow$  consumption rises, investment increase temporarily

LR: no change in employment, output or the real interest rate

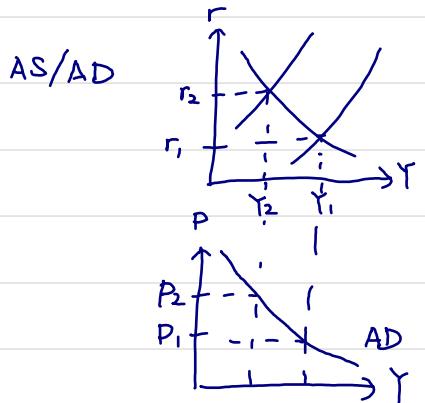
Keynesian economists think that the labor market does not function  $\Rightarrow$  gov. intervention

(1) Price adjustment and the self - correcting economy.

Labor input in the SR deviates from  $\bar{N}$ .

(2) Monetary neutrality

- change in the nominal money supply changes the price level proportionately
- has no effect on real variables

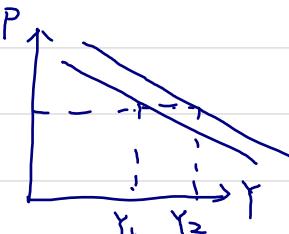
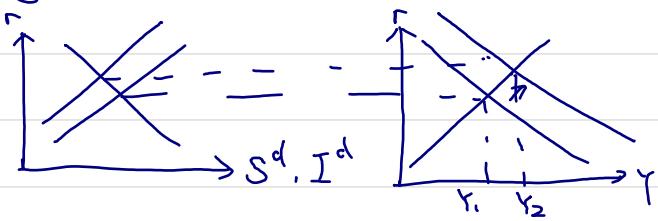


AD curve slopes downward because a higher price level is associated with lower real money supply, shifting the LM curve up, raising the real interest rate, and decreasing output demanded

Shifter: any factor that causes the intersection of the IS/LM curves

to shift to the left causes the AD curve to shift down and to the left.  
(right) (up) (right)

E.g.  $\uparrow$  Gov. purchase

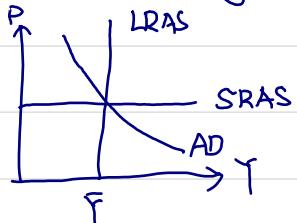


Another perspective

IS (and LM): Demand side  $Y^d = C^d + I^d + G + NX$

FE: Supply side  $Y = AK^\alpha N^{1-\alpha}$

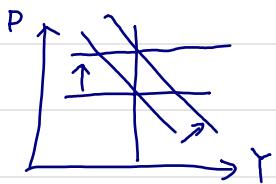
AS: SR, prices remain fixed, so firms supply whatever output is demanded  
LR, full employment output isn't affected by price level.



Factors leading to reduced prices shift SRAS down

Monetary neutrality in AD/AS

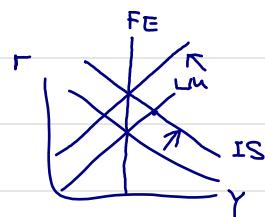
Suppose  $\uparrow$  money supply by 10%, AD  $\uparrow$ , price level  $\uparrow$  10%, SRAS  $\uparrow$



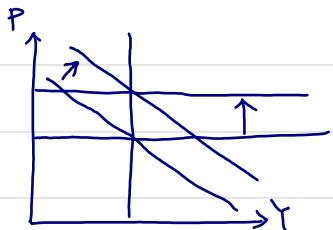
$\Rightarrow$  real money supply wouldn't change  
equal to real money demand

Fiscal policy: G & taxes

Government expenditure  $\uparrow$ , IS shifts to the right,  
 $r \uparrow$ ,  $y \uparrow$ , I goes down (user cost of capital  $\uparrow$ )



Classical economists think consumers are sophisticated



P goes up, C goes down

## Keynesian

Price level  $P$  does not adjust in the SR.

$C$  goes up in the SR

Employment increases in the SR, consumers demand more goods (has more money)

## Taxes / Transfer

$T \downarrow$  Consumer's disposable income  $\uparrow$

Assume gov. collects taxes in the future

Classical: Ricardian equivalence holds

do not stimulate the economy.

Keynesian:  $C = \bar{C} + MPC(Y - T)$

$T < 0$   $(Y - T) \uparrow$ ,  $C \uparrow$

## Monetary Policy / Fiscal policy

Classical: oppose attempts to dampen the cycle, price adjust quickly to restore money is neutral

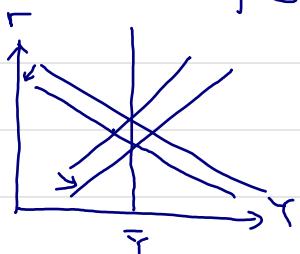
$\uparrow G \Rightarrow$  higher taxes,

lags in enacting the policy and implementing it,

Keynesian: favor gov. actions to stabilize the economy

· prices do not adjust, markets do not work in the SR.

· unemployed are hurt



recession.  $IS \downarrow$

monetary policy:  $\uparrow$  in  $M$ , LM shifts downward

fiscal policy:  $\uparrow G$ , IS shifts up.

$\uparrow G$ ,  $r \uparrow$ , user cost of capital  $\uparrow$ ,  $I \downarrow$  (crowding out)

financed by  $\uparrow T$ , if unsophisticated, reduce  $C$ .

## Chapter 14 Monetary Policy and the Fed reserve system

Central bank: issuing paper money and making monetary policy

Sum of reserve deposits (by banks) and currency (held by the nonbank public and by banks) is called the monetary base.

bank reserves = deposits      100% reserve banking  
reserve deposit ratio < 100% , fractional reserve banking

Open market operations: open market purchase ( $\uparrow$  monetary base)  
(prints money and uses it to buy assets in the market)  
open market sale: ( $\downarrow$  monetary base)  
(selling assets and collect money from the asset)

Interest rate when banks borrow reserves from each other is the fed fund rate.

Changing money supply: ① changing reserve requirements have a large impact  
on both the money supply and bank profits

$\uparrow$  reserve requirement  
 $\Rightarrow$  money supply  $\downarrow$   
 $\Rightarrow$  increasing the cost of credit

② discount window lending  
lending reserves to banks to meet depositors' demands

★ (discount rate)

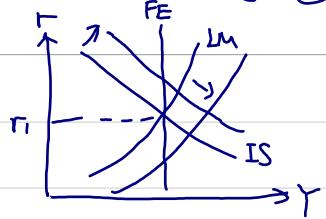
lender of last resort

a discount loan increases the monetary base

③ Interest rate on reserves

incentive of not spending resources

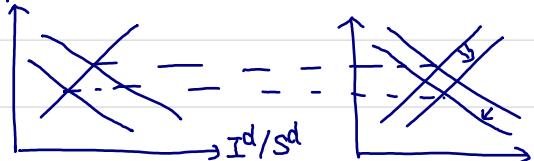
## Interest rate targeting ( $r_t$ )



If IS shock, increase in money supply  
 $LM \downarrow$

## Great Recession (2007/12 to 2009/6)

- Fall in house prices (decrease in wealth, IS curve to the left, recession)



Fed's response:  $\uparrow$  money supply,  $LM \uparrow$

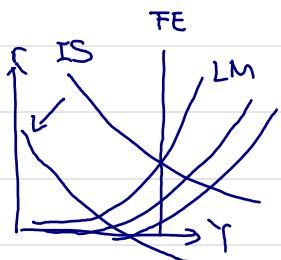
can't further lower  $r$ , reaches the zero lower bound

$\Rightarrow$  Increase in credit spreads,  $\downarrow$  investment,  $IS \downarrow$

Model the ZLB: people demand less money when nominal interest rate

on nonmonetary assets is higher

- Money demand curve is flat when the nominal interest rate is around zero.



Monetary policy has no use

$\Rightarrow$  Emergency lending (decline in credit spread)

$\uparrow$  investment and consumption (IS curve shifts to the right)

$\Rightarrow \uparrow G$  and tax cuts (IS shifts to the right)

## Unconventional monetary policy

### Forward Guidance

Announcing the path of future nominal interest rates

### Quantitative Easing (shifting IS back)

- keeps on supplying money by buying nonmonetary assets even at the ZLB.
- Targeted asset purchases: buy large quantities of non-traditional debt
- to push up bond prices and yields down and reduce credit spread.  
(higher the bond price, lower the interest rate)

Fiscal policy: three categories

- Government purchases (consumption and investment)
- Transfer payments / tax
- Net interest payments

Tax  
personal taxes  
contributions for social insurance  
taxes on production and imports  
Corporate taxes

$$\text{Deficit} = G + TR + INT - T \quad \text{primary deficit} = G + TR - T$$

Fiscal policy increases output by making workers worse off.

Even Keynesians admit that fiscal policy is difficult to use.

Fiscal space: (1) room in a government's budget that allows it to provide resources for a desired purpose

(2) simultaneously does not affect the sustainability of its financial position or the stability of the economy.