3. (1) $\Omega=\{THH, TTH, TTT, HTH, HHH, HTT, THT, HHT\}$

(2) $\Omega=\{H, TH, TTH, TTTH, \dots\}$

(3) $\Omega=\{BB, BW, BR, RB, RW, RR, WB, WR, WW\}$

(4) $\Omega=\{BW, BR, RB, RW, WB, WR\}$

4. (1) $\Omega=\{(w_i, w_j): i<j;\ i,j=1,2,3,4,5\}$ $\quad A=\{(w_i,w_j): i=1,2,3,\ j=4,5\}$

(2) $\Omega=\{(H,1),(H,2),\dots,(H,6),(T,1),\dots,(T,6)\}$ $\quad A=\{(H,2),(H,4),(H,6)\}$

(3) $\Omega=\{(H,T),(T,H),(T,T),(H,H)\}$ $\quad A=\{(H,H),(H,T)\}$ $\quad B=\{(H,H),(T,T)\}$ $\quad C=\{(H,H),(H,T),(T,H)\}$

7. (1) $\bar{A}_1A_2\bar{A}_3$ (2) $A_1A_2\bar{A}_3$ (3) $A_1+A_2+A_3$ (4) $A_1\bar{A}_2\bar{A}_3+\bar{A}_1A_2\bar{A}_3+\bar{A}_1\bar{A}_2A_3$

(5) $A_1A_2\bar{A}_3+A_1\bar{A}_2A_3+\bar{A}_1A_2A_3$ (6) $A_1A_2+A_2A_3+A_1A_3$ (7) $A_1A_2A_3$

(8) $\bar{A}_1\bar{A}_2A_3+\bar{A}_1A_2\bar{A}_3+A_1\bar{A}_2\bar{A}_3+\bar{A}_1\bar{A}_2\bar{A}_3$ (9) $\bar{A}_1\bar{A}_2\bar{A}_3+(4)$

2) (1) 至少一次没投中 (2) 前两次都不中 (3) 连续2次投中

8. $A=\{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),$
$(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)\}$

$B=\{(x,y): x\in(1,2,3,4,5,6) \text{ and } x=y\}$

$C=\{(1,1),(1,2),\dots,(1,6),(2,1),(2,2),\dots(2,6),(3,1),\dots,(3,6),(4,1),\dots(4,5),$
$(5,1),\dots,(5,4),(6,1),(6,2),(6,3)\}$

$B-A=A$ $\quad B\cap C=\{(1,1),(2,2),(3,3),(4,4)\}$ $\quad B+\bar{C}=B+\{(4,6),(5,6),(6,4),(6,5)\}$

9. $\bar{A}$ 三件都正 $\bar{B}$: 最多一件废 $A\cup B$: A $\quad AB$: B $\quad A\cup C=\Omega$ $\quad AC=\phi$ $\quad A-B$: 只有一废

10. (1) T (2) F (3) F (4) F (5) F (6) T (7) F 有complement 比较整个space

11. (1) $(A\cup B)-(A-B)$

$=B$

(2) $(A\cup B)\cap(A\cup\bar{B})$

$=(A\cup B)\cap A\cup(A\cup B)\cap\bar{B}$

$=((A\cap A)\cup(A\cap B))\cup((\bar{B}\cap A)\cup(\bar{B}\cap B))$

$= A\cup(A\cap\bar{B})=A$

(3) $(A-\bar{B})\cap(\overline{A\cup B})$

$= A\cap B\cap A^c\cap B^c$

$=\phi$

(4) $\overline{(\overline{AB}\cup C)(\overline{AC})}$

$=((A^c\cup B^c\cup C)\cap(A^c\cup C^c))^c$

$=(A^c\cup B^c\cup C)^c\cup(A^c\cup C^c)^c$

$=(A\cap B\cap C^c)\cup(A\cap C)$

$= A\cap(B\cap C^c)\cup A\cap C$

$= A\cap((B\cap C^c)\cup C)$

$= A\cap((B\cup C)\cap(C^c\cup C))$

$= A\cap(B\cup C)$

1-2

3. A为至少拿到一把能开门的 $P(A)=\frac{{}^3C_2+{}^3C_1{}^7C_1}{{}^{10}C_2}=\frac{8}{15}$

4. $P(A)=\frac{1}{10^5}$ 注意是否重复

7. $P(A)=\frac{{}^5C_1{}^3C_1{}^2C_1}{{}^{10}C_3}=0.25$

?5. $P(A)=\frac{3!\times3!\times2!}{10!}=\frac{1}{50400}$

6. $P(A)=\frac{{}^{N_1}C_k\,{}^{N-N_1}C_{n-k}}{{}^NC_n}$

8. $P(A)=\frac{{}^{12}C_2+{}^3C_2}{{}^{15}C_2}=\frac{23}{35}$

9. (1) $P(A)=\frac{n!}{N^n}$ (2) $P(B)=\frac{{}^NC_n\,n!}{N^n}$ 指定和恰有的区别

(3) $P(C)=\frac{{}^nC_m\,(N-1)^{n-m}}{N^n}$

10. $P(A)=\frac{9\times9\times8\times7\times6\times5\times4}{9\times10^6}=0.06048$

11. $P(A)=\frac{7!\times{}^8C_1\times3!}{10!}=\frac{1}{15}$

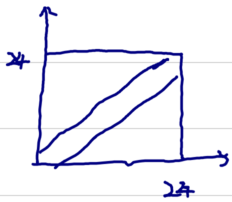
12. $P(A)=\frac{{}^8P_3\times5!\times12}{10!}=\frac{2}{15}$

13. $\frac{2}{5}$

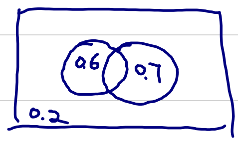
14. $P(A) = 1 - \frac{(1\times 24 - \frac{1}{2})\times 2}{24^2}$ $|x-y|\geq 1$

☆

$= 0.918$



15. $P(\text{都占}) = 0.6+0.7+0.2-1 = 0.5$



$P(1\text{都不占}) = 1-0.5 = 0.5$

16. $P(A) = \frac{{}^4C_1\,{}^{46}C_2 + {}^4C_2\,{}^{46}C_1 + {}^4C_3}{{}^{50}C_3} = 0.2155$

17. $(A^c \cap B^c)^c$ $P(A\cup B) = 0.8$

$= A\cup B$

18. $P(A+B) = P(A)+P(B)-P(AB)$ $P(AB) = 0.4$ $P(B-A) = 0.3$ $P(A-B) = 0.1$

$0.8 = 0.5+0.7-P(AB)$

19. $P(A^c\cap B^c)$ $P(A\cup B) = 0.85$

$= P(A\cup B)^c$

$= 0.15$

(1) $P(A\cap \bar{B}) = 0.5$

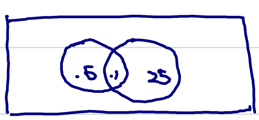
(2) $P(\bar{A}\cap B) = 0.25$

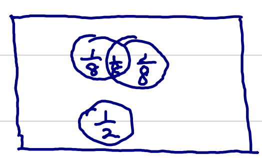
(3) $P(A\cup B) = 0.85$

$P(A\cup B) = P(A)+P(B)-P(AB)$

$0.85 = 0.6+P(B)-0.1$

$P(B) = 0.35$





20. $0.2 \leq \ln a \leq 1$

$e^{0.2} \leq a \leq e$

21. $P(A+B) = \frac{1}{4}+\frac{1}{4}-\frac{1}{8}$

$= \frac{3}{8}$

$P(\text{至少1个}) = \frac{3}{8}+\frac{1}{2} = \frac{7}{8}$

22. (1) $P(2\text{人同天}) = 1-P(\text{都不同})$

$= 1-\frac{365\times 364\times 363\times\cdots\times 356}{365^{10}}$

$= 1-\frac{{}^{365}P_{10}}{365^{10}}$

(2) $P(1\text{人10月1日}) = 1-P(\text{都不是})$

$= 1-\frac{364^{10}}{365^{10}}$

1-3

1. $P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$

考虑一女有7个Case

一女+至少一男6个Case

2. $P(A) = \frac{69}{98}$ $P(B) = \frac{95}{98}$ $P(AB) = \frac{67}{95}$ $P(B|A) = \frac{67}{69}$

$B(A|B) = \frac{67}{95}$ $P(\bar{A}|AB) = 0$

3. (1) $P(A) = \frac{1}{10} + \frac{9}{10} \times \frac{1}{9} + \frac{9}{10} \times \frac{8}{9} \times \frac{1}{8} = \frac{3}{10}$

(2) $P(A|\text{奇}) = \frac{1}{5} + \frac{4}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{3}{5}$

4. $P(15\text{岁}|10\text{岁}) = \frac{P(AB)}{P(10\text{岁})} = \frac{0.5}{0.8} = \frac{5}{8}$ 活到15岁本身就是P(AB)

5. A: Ⅰ有效 B: Ⅱ有效 $P(A) = 0.92$ $P(B) = 0.9$ $P(A|B) = 0.93$

(1) $P(AB) = P(A|B)P(B) = 0.93 \times 0.9 = 0.837$

(2) $P(B|A) = \frac{0.837}{0.92} = 0.910$

(3) $P(\text{至少1个}) = 0.92 + 0.9 - 0.837 = 0.983$

(4) $P(\quad) = 1 - 0.983 = 0.017$

6. (1) A_1: 第1次正, A_2: 第2次正

$P(A_1A_2) = P(A_1|A_2)P(A_2) = \frac{\frac{{}^8C_1 {}^7C_1}{90}}{\frac{{}^8C_1 {}^7C_1 + {}^2C_1 {}^8C_1}{10 \times 9}} \times \left(\frac{8 \times 7 + 16}{90}\right) = \frac{28}{45}$

(2) $P(\bar{A_1}\bar{A_2}) = \frac{{}^2C_1}{90} = \frac{1}{45}$

(3) $P(\bar{A_1}A_2) + P(A_1\bar{A_2}) = \frac{{}^2C_1 {}^8C_1 + {}^8C_1 {}^2C_1}{90} = \frac{16}{45}$

7. $P(B|A) = \frac{P(AB)}{P(A)}$

$0.8 = \frac{P(AB)}{0.5}$

$P(AB) = 0.4$

$P(\bar{A}\bar{B}) = 1 - 0.1 - 0.4 - 0.2 = 0.3$ 想不出来就画图

8. (1) $P(B|\bar{A}) = \dfrac{P(\bar{A}B)}{P(\bar{A})}$

$\dfrac{5}{6} = \dfrac{P(\bar{A}B)}{0.6}$

$P(\bar{A}B) = 0.5$

(2) $P(AB) = P(B) - P(\bar{A}B)$

$= 0.8 - 0.5$

$= 0.3$

(3) $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.3}{0.8} = 0.375$ (4) $P(A+B) = 0.4 + 0.8 - 0.3 = 0.9$

1-4

1. 设A_i表示为第i个车间的产品 $i=1,2,3$ B为废品

$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$

$= \frac{1}{2} \times 2\% + \frac{1}{4} \times 2\% + \frac{1}{4} \times 4\% = 0.025$

2. $P(2\text{次}3\text{新}) = \dfrac{{}^{6}C_3}{{}^{10}C_3} \times \dfrac{{}^{3}C_3}{{}^{10}C_3} + \dfrac{{}^{6}C_2\,{}^{4}C_1}{{}^{10}C_3} \times \dfrac{{}^{4}C_3}{{}^{10}C_3} + \dfrac{{}^{6}C_1\,{}^{4}C_2}{{}^{10}C_3} \times \dfrac{{}^{5}C_3}{{}^{10}C_3} + \dfrac{{}^{4}C_3}{{}^{10}C_3} \times \dfrac{{}^{6}C_3}{{}^{10}C_3}$

$= \dfrac{7}{144}$

3. 设A为甲取白，B为乙取白

$P(B) = P(A)P(B|A) + P(\bar{A})P(B|\bar{A})$

$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} = \frac{5}{12}$

4. $P(A|B) = \dfrac{P(A)P(B|A)}{\frac{5}{12}} = \dfrac{\frac{2}{3} \times \frac{1}{2}}{\frac{5}{12}} = \dfrac{4}{5}$

$P(\bar{A}|B) = \dfrac{\frac{1}{12}}{\frac{5}{12}} = \dfrac{1}{5}$

甲取白

5. $P(\text{迟到}) = 0.3 \times \frac{1}{4} + 0.2 \times \frac{1}{3} + 0.1 \times \frac{1}{12} = 0.15$

$P(\text{火}|\text{迟}) = \dfrac{0.3 \times \frac{1}{4}}{0.15} = 0.5$

6. $P(\text{色盲}) = \frac{22}{43} \times 5\% + \frac{21}{43} \times 0.25\% = \frac{461}{17200}$

$P(\text{男}|\text{色}) = \dfrac{\frac{22}{43} \times 5\%}{\frac{461}{17200}} = \dfrac{440}{461}$

7. (1) $P(\text{资}) = 1 - P(\text{2次不及格})$

$$= 1 - (1-p)(1-\frac{p}{2})$$

$$= 1 - (1 - \frac{p}{2} - p + \frac{p^2}{2})$$

$$= 1 - 1 + \frac{p}{2} + p - \frac{p^2}{2}$$

$$= \frac{3p - p^2}{2} = \frac{p}{2}(3-p)$$

(2) $P(1p|2p) = \dfrac{p^2}{\frac{3p^2+p}{2}}$ $= \dfrac{2p^2}{p^2+p} = \dfrac{2p}{p+1}$

$P(2p) = p \times p + (1-p)\frac{p}{2}$

$= p^2 + \frac{p}{2} - \frac{p^2}{2}$

$= \frac{p^2+p}{2}$

8. $P(\text{发}\cdot|\text{收}\cdot) = \dfrac{P(\text{发}\cdot\text{收}\cdot)}{P(\text{收}\cdot)} = \dfrac{0.6 \times 0.8}{0.6\times 0.8 + 0.4 \times 0.1} = 0.923$

1-5

1. (1) $P(A+B) = 0.3+0.4 = 0.7$ $P(AB)=0$ (2) $P(A+B) = 0.3+0.4-0.12 = 0.58$ $P(AB)=0.12$

2. $P(\bar{A})P(\bar{B}) = \frac{1}{9}$ $P(A)P(\bar{B}) = P(\bar{A})P(B)$ $\Rightarrow P(\bar{A}) = P(\bar{B}) = \frac{1}{3}$

$P(A)P(\bar{B}) = P(\bar{A})(1-P(\bar{B}))$ $P(A) = P(B) = \frac{2}{3}$

$(1-P(\bar{A}))P(\bar{B}) = P(\bar{A}) - \frac{1}{9}$

$P(\bar{B}) - \frac{1}{9} = P(\bar{A}) - \frac{1}{9}$

3. (1) $P(AB) = 0.42$ (2) $P(A\bar{B}) = 0.28$ (3) $P(\bar{A}B) = 0.18$ (4) $P(\bar{A}\bar{B}) = 0.12$ (5) $P(\ ) = 0.88$

4. $P(ABC) = 1 - 0.9 \times 0.95 \times 0.8 = 0.316$ 注意并不是3道都废才废

5. $P(\ ) = 1 - \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{3}{5}$

6. (1) $P(\text{2个}1) = {}^5C_2\,(0.2)^2(0.8)^3 = 0.2048$

(2) $P(\text{至少3个}) = {}^5C_3(0.2)^3(0.8)^2 + {}^5C_4(0.2)^4(0.8)^1 + {}^5C_5(0.2)^5 = 0.05792$

(3) $P(\text{最多2个}) = {}^5C_0(0.2)^0(0.8)^5 + {}^5C_1(0.2)(0.8)^4 + {}^5C_2(0.2)^2(0.8)^3 = 0.94208$

7. (1) $P(\text{1次3点}) = {}^6C_1\,(\frac{1}{6})(\frac{5}{6})^5 = 0.402$

(2) $P(\text{最多2次}) = {}^6C_0\,(\frac{5}{6})^6 + {}^6C_1(\frac{1}{6})(\frac{5}{6})^5 + {}^6C_2(\frac{1}{6})^2(\frac{5}{6})^4 = 0.938$

8. $P(\text{至少1次}) = 1 - P(\text{全部不成}) = 1 - {}^{20}C_0\,(0.8)^{20} = 0.9885$

9. $P(\text{成功}) = {}^7C_4\,(0.7)^4(0.3)^3 + {}^7C_5(0.7)^5(0.3)^2 + {}^7C_6(0.7)^6(0.3) + {}^7C_7(0.7)^7 = 0.874$

1. (1) $\frac{3!}{4!}=\frac{1}{4}$ (2) $\frac{{}^{3}C_1{}^{2}C_1+{}^{2}C_1{}^{3}C_1}{{}^{5}C_1{}^{4}C_1}=\frac{3}{5}$ (3) $\frac{3}{8}$ (4) $\frac{1}{4}$ (5) $\frac{1}{3}$ (6) $\frac{1}{5}$ (7) $\frac{1}{2}$ (8) 0.0395

2. (1) A, C (2) A, D (3) D (4) A, B, C, D (5) A, D (6) A, D (7) B (8) C (9) A (10) D

3. (1) $\frac{{}^{4}C_1{}^{13}C_5}{{}^{52}C_5}$ (2) $\frac{{}^{13}C_1{}^{4}C_3{}^{12}C_1{}^{4}C_2}{{}^{52}C_5}$ (3) $\frac{{}^{13}C_2{}^{4}C_2{}^{4}C_2{}^{44}C_1}{{}^{52}C_5}$ (4) $\frac{{}^{13}C_1{}^{4}C_4{}^{48}C_1}{{}^{52}C_5}$

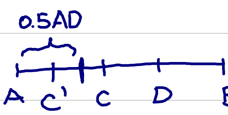
4. $\frac{1+2+3+4+5}{5\times5}=\frac{3}{5}$

5. (1) $\frac{{}^{N}P_n}{N^n}$ (2) $\frac{{}^{n}C_k(N-1)^{n-k}}{N^n}$

6. P(至少一双) = 1−P(一双没有) $= 1-\frac{{}^{5}C_4 2^4}{{}^{10}C_4}=\frac{13}{21}$

7. (1) P(有黑) $= 1-\frac{{}^{5}C_1{}^{5}C_1}{{}^{8}C_1{}^{8}C_1}=\frac{39}{64}$ (2) P(颜色不同) $=1-\frac{3\times3}{8\times8}-\frac{5\times5}{8\times8}=\frac{15}{32}$

8. P(乙胜 | 2胜一负) $= \frac{(\frac{1}{2})^3(\frac{1}{2})^2}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}=\frac{1}{4}$ 乙:250元 甲:750元

9.  or 

$P(\ )=\frac{1}{2}+\frac{1}{2}\times0.5=0.75$

10. ACB max: 0.6 $A+B=\Omega$ min: 0.3

11. P(k次黑) = 1 − P(摸白) $= 1-(1-\frac{1}{N})^{k-1}\frac{1}{N}$

12. A, B 独立 $\iff P(A|B)=P(A|\bar{B})$

$\Rightarrow$: Assume $P(AB)=P(A)P(B)$ $P(A|B)=\frac{P(AB)}{P(B)}$ $P(A|\bar{B})=\frac{P(A\bar{B})}{P(\bar{B})}$

$= P(A)$ $= P(A)$

$\Leftarrow$ Assume $P(A|B)=P(A|\bar{B})$

$$\frac{P(AB)}{P(B)}=\frac{P(A\bar{B})}{1-P(B)}$$

$$\frac{P(AB)}{P(B)}=\frac{P(A)-P(AB)}{1-P(B)}$$

$$P(AB)-P(B)P(AB)=P(A)P(B)-P(B)P(AB)$$

$$P(AB)=P(A)P(B)$$

13. P(正确) $=75\%+25\%(\frac{1}{4})=0.8125$

14. (1) P(事故) $=20\%\times0.05+50\%\times0.15+30\%\times0.3=0.175$

(2) P(谨 | 事故) $=\frac{0.2\times0.05}{0.175}=\frac{2}{35}$

15. $P(\text{合格}) = 80\% \times 0.8 + 18\% \times 0.2 + 2\% \times 0.1 = 0.678$

$P(\text{一等}|\text{没}) = \dfrac{80\% \times 0.2}{1-0.678} = 0.497$

16. $P(\text{击落}) = (P(AB\bar{C}) + P(A\bar{B}\bar{C}) + P(\bar{A}\bar{B}C)) \times 0.2 + (P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC)) \times 0.6$

$+ 0.4 \times 0.5 \times 0.7$

$= 0.458$

17. $P(\text{击落}) = (0.6 \times 0.4 \times 0.4) \times 3 \times 0.2 + 0.6^3 + (0.6 \times 0.6 \times 0.4) \times 3 \times 0.6$

$= 0.5328$

18. $P(\text{甲胜}|5\text{局}3\text{胜}) = 0.6^3 + 3 \times 0.6^3 \times 0.4 + {}^4C_2\,(0.6)^2(0.4)^2 \times 0.6 = 0.68256$ ✓

$P(\text{甲胜}|3\text{局}2\text{胜}) = 0.6^2 + 2 \times 0.6^2 \times 0.4 = 0.648$

19. $P(\text{至少一次}6) = 1 - \left(\frac{5}{6}\right)^4 = 0.5177$

$P(24\text{次至少一次双}6) = 1 - \left(\frac{35}{36}\right)^{24} = 0.4914$

20. 自然 0.25 非自然 0.75 至少4人服药治愈 $\longrightarrow$ 有效

(1) $P(\text{认为无效}) = {}^{10}C_0\,(0.35)^0(0.65)^{10} + {}^{10}C_1\,(0.35)(0.65)^9 + \cdots + {}^{10}C_3\,(0.35)^3(0.65)^7 = 0.514$

(2) $P(\text{认为有效}) = 1 - {}^{10}C_0\,(0.25)^0(0.75)^{10} - {}^{10}C_1\,(0.25)(0.75)^9 - \cdots - {}^{10}C_3\,(0.25)^3(0.75)^7 = 0.224$

2-1

1. (1) X表示取到的个数，取值为0,1,2,3 (2) $X \in \mathbb{Z}$ (3) $X = \begin{cases} 40 & \text{正} \\ 20 & \text{反} \end{cases}$ $Y = \begin{cases} 10 & \text{正} \\ 30 & \text{反} \end{cases}$

2. $X = \begin{cases} 5 & HH \\ 0 & \text{else} \\ -5 & TT \end{cases}$ $P\{X=0\} = \frac{1}{2}$ $P\{X \leq 1\} = \frac{3}{4}$ $P\{X > 5\} = 0$

3. $X = \begin{cases} 0 & <5 \\ 1 & =5 \\ 2 & >5 \end{cases}$ $P\{X=i\} = \frac{1}{10}$ $i = 0,1,\cdots,9$

2-2

1.(1) $\sum_{k=0}^{\infty} \frac{1}{2}\left(\frac{1}{3}\right)^k = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\left(\frac{1}{3}\right)^2 + \cdots$ No (2) $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$ Yes

$= \frac{1}{2} + \frac{1}{2}\left(\frac{\frac{1}{3}}{\frac{2}{3}}\right) = \frac{3}{4} \neq 1$

2. $\frac{2}{3}c + \frac{4}{9}c + \frac{8}{27}c = 1$

$c = \frac{27}{38}$

3.

x	3	4	5
P	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$

4. (1) $P\{X=1 \text{ or } X=2\} = \frac{1}{15}$ (2) $P\{\frac{1}{2}<X\leq\frac{5}{2}\} = \frac{1}{5}$

(3) $P\{1\leq X\leq 2\} = \frac{1}{5}$ (4) $P\{1<X\leq 2\} = \frac{2}{15}$

5.

X	0	1	2	3	4
P	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{3}{7}$	$\frac{4}{35}$	$\frac{1}{210}$

6.

X	0	1	2	3
P	$\frac{7}{10}$	$\frac{7}{30}$	$\frac{7}{120}$	$\frac{1}{120}$

7. $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\lambda c\int_{a}^{+\infty} \frac{1}{e^{\lambda x}}\,dx = 1$

$\lambda c\left[\frac{-1}{\lambda}\frac{1}{e^{\lambda x}}\right]_a^{\infty} = 1$

$c = e^{a\lambda}$

$P\{a-1<x\leq a+1\} = \int_a^{a+1} f(x)\,dx$

$= \lambda e^{a\lambda}\int_a^{a+1}\frac{1}{e^{\lambda x}}\,dx$

$= \lambda e^{a\lambda}\left[\frac{-1}{\lambda}\frac{1}{e^{\lambda x}}\right]_a^{a+1}$

$= -e^{a\lambda}\left(\frac{1}{e^{(a+1)\lambda}} - \frac{1}{e^{a\lambda}}\right)$

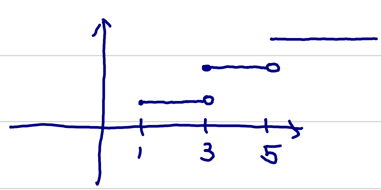
$= \frac{-e^{a\lambda}}{e^{(a+1)\lambda}} + 1$

$= e^{a\lambda-a\lambda-\lambda} + 1 = 1 - e^{-\lambda}$

8. Yes

9.

$$F(x) = \begin{cases} 0 & x<1 \\ 0.3 & 1\leq x<3 \\ 0.8 & 3\leq x<5 \\ 1 & 5\leq x \end{cases}$$



10. (1)

x	-1	1	3
P	0.4	0.4	0.2

★ (2) $P\{X<2 : X\neq 1\}$

$= \frac{0.4}{0.6}$

$= \frac{2}{3}$

11. (1) $\frac{1}{A}\int_0^1 x^{-\frac{1}{2}}dx = 1$

$\frac{1}{A}\left[2x^{\frac{1}{2}}\right]_0^1 = 1$

$A = 2$

(2) $F(x) = \begin{cases} 0 & x<0 \\ \sqrt{x} & 0\le x<1 \\ 0 & 1\le x \end{cases}$

$\int_{-\infty}^{x}\frac{1}{2}t^{\frac{1}{2}}dt$

$= \int_0^x \frac{1}{2}t^{-\frac{1}{2}}dt$

$= \sqrt{x}$

(3) $P\{x\le 0.5\} = \frac{\sqrt{2}}{2}$ $P\{x=0.5\}=0$ $P\{-1<x\le 0.5\} = F(\frac{1}{2})-F(-1) = \frac{\sqrt{2}}{2}$

12. (1) $\lim_{x\to 1^-}F(x)=1$

$\lim_{x\to 1^-} a+b\sin^{-1}x = 1$

$a+\frac{b\pi}{2} = 1$

$\lim_{x\to -1^+} a+b\sin^{-1}x = 0$

$a-\frac{b\pi}{2} = 0$

$\frac{b\pi}{2} = 1-\frac{b\pi}{2}$

$b\pi = 1$

$b = \frac{1}{\pi}$

$a = \frac{1}{2}$

(2) $F(x) = \frac{1}{2}+\frac{1}{\pi}\sin^{-1}x$

$f(x) = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}} & -1<x<1 \\ 0 & \text{else} \end{cases}$

13. (1) $P\{x<2\} = F(2) = \ln 2$ $P\{0<x\le 3\} = F(3)-F(0) = 1$ $P\{2<x<\frac{5}{2}\} = \ln\frac{5}{4}$

(2) $f(x) = \begin{cases} 0 & \text{else} \\ \frac{1}{x} & 1\le x\le e \end{cases}$

2-3

1.

x	0	1
P	$\frac{1}{3}$	$\frac{2}{3}$

$F(x) = \begin{cases} 0 & x<0 \\ \frac{1}{3} & 0\le x<1 \\ 1 & 1\le x \end{cases}$

2. $P\{X=k\} = (\frac{1}{4})^{k-1}(\frac{3}{4}) = \frac{3}{4^k}$ $k=1,2,\dots$ $P(\ ) = \frac{3}{4^2}+\frac{3}{4^4}+\cdots$

$= 3\left(\frac{\frac{1}{4^2}}{1-\frac{1}{4^2}}\right) = \frac{1}{5}$

3. (1) $P\{x=k\} = (0.98)^{k-1}(0.02)$ $k=1,2,\dots$

(2)

☆ $P\{x>m+n \mid x>m\} = \frac{P\{x>m+n\}}{P\{x>m\}} = \frac{P(\{x>m+n\}\cap\{x>m\})}{P(x>m)} = \frac{P\{x>m+n\}}{P\{x>m\}}$

$= \frac{\sum_{k=m+n+1}^{\infty}(1-p)^{k-1}p}{\sum_{k=m+1}^{\infty}(1-p)^{k-1}p} = \frac{\frac{p(1-p)^{m+n}}{p}}{\frac{p(1-p)^m}{p}} = \frac{p(1-p)^n}{p} = \sum_{k=n+1}^{\infty}(1-p)^{k-1}(p) = P\{x>n\}$

4. (1) $P\{X=3\} = {}^{10}C_3(0.7)^3(0.3)^7 = 0.009$

(2) $P\{X\geq 3\} = 1-P\{X=0\}-P\{X=1\}-P\{X=2\} = 1-{}^{10}C_0(0.7)^0(0.3)^{10}-{}^{10}C_1(0.7)(0.3)^9-{}^{10}C_2(0.7)^2(0.3)^8$

$= 0.9984$

5. 设废品数为 X，$X \sim B(20, 0.1)$，假设废品率为 n，$\frac{X}{20}=n$

$P\{n\leq 0.15\} = P\{X\leq 3\} = \sum_{k=0}^{3} {}^{20}C_k(0.1)^k(0.9)^{20-k} = 0.867$

6. $X\sim P(\lambda)$ $\quad P\{X=1\} = P\{X=2\}$ $\quad P\{X=k\} = \frac{2^k}{k!}e^{-2}$

$\frac{\lambda^k}{k!}e^{-\lambda} = \frac{\lambda^k}{k!}e^{-\lambda}$ $\quad \Rightarrow e^{-\lambda}(2\lambda-\lambda^2)=0$

$2\lambda e^{-\lambda} = \lambda^2 e^{-\lambda}$ $\quad \lambda=0$ or $\lambda=2$

$2\lambda e^{-\lambda}-\lambda^2e^{-\lambda}=0$

$P\{X=4\} = \frac{2^4}{4!}e^{-2} = 0.090224$

7. $X\sim B(500, 0.01)$ $\quad P\{X=k\} = {}^{500}C_k(0.01)^k(0.99)^{500-k}$

$np=5$ $\quad n>100$ $\quad X\sim P(5)$ $\quad P\{X=2\} = \frac{5^2}{2!}e^{-5} = 0.084224$

$P\{X\leq 2\} = P\{X=0\}+P\{X=1\}+P\{X=2\} = 0.124652$

8. $X\sim U[0,5]$ $\quad f(x) = \begin{cases} \frac{1}{5} & 0\leq x\leq 5 \\ 0 & \text{else}\end{cases}$ $\quad F(x) = \begin{cases} 0 & x<0 \\ \frac{x}{5} & 0\leq x<5 \\ 1 & x\geq 5\end{cases}$

$4x^2+4Xx+X+2=0$

$\Delta\geq 0$ $\quad P(\text{有实根}) = \frac{5-2}{5} = 0.6$

$(4X)^2-4(4)(X+2)\geq 0$

$16X^2-16(X+2)\geq 0$

$16X^2-16X-32\geq 0$

$X\geq 2$ or $X\leq -1$ (rej.)

9. $X \sim Exp\left(\frac{1}{1000}\right)$ $f(x) = \begin{cases} \frac{1}{1000} e^{-\frac{1}{1000}x} & x>0 \\ 0 & x\le 0 \end{cases}$

P{再用100 | 500小时没坏}

$$= \frac{\frac{1}{1000} e^{-\frac{1}{1000}\times 600}}{\frac{1}{1000} e^{-\frac{1}{1000}\times 500}}$$

$$= e^{-0.1}$$

10. $X \sim Exp\left(\frac{1}{3}\right)$ $f(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x} & x>0 \\ 0 & \text{else} \end{cases}$

(1) $P\{x \ge 3\} = 1 - P\{x<3\} = 1 - \left(1-e^{-\frac{1}{3}(3)}\right) = \frac{1}{e} = 0.3679$

(2) $P\{3 \le x \le 6\} = F(6) - F(3) = 1 - e^{-\frac{1}{3}(6)} - \left(1-e^{-\frac{1}{3}(3)}\right) = 0.2325$

11. $P(甲) = \frac{10\times 4}{60} = \frac{2}{3}$ $P(甲) = \frac{5+5+30}{60} = \frac{2}{3}$

12. $X \sim N(0,1)$ (1) $P\{0.02 < x < 2.33\} = \Phi_0(2.33) - \Phi_0(0.02) = 0.99^{2}0097 - 0.5080 = 0.482097$

$P\{-1.85 < x < 1.04\} = \Phi_0(1.04) - (1-\Phi_0(1.85)) = 0.8508 - 1 + 0.96784 = 0.8184$

$P\{x \le 0\} = 0.5$

(2) $X \sim N(4,9)$ $\mu=4$ $\sigma=3$ $P\{4<x<9.88\} = \Phi_0\left(\frac{9.88-4}{3}\right) - \Phi_0\left(\frac{4-4}{3}\right) = 0.475$

$P\{x>9.88\} = 1 - P\{x \le 9.88\} = 1 - \Phi_0\left(\frac{9.88-4}{3}\right) = 0.025$

$P\{|x|<9.88\} = P\{-9.88<x<9.88\} = \Phi_0\left(\frac{9.88-4}{3}\right) - \Phi_0\left(\frac{-9.88-4}{3}\right) = 0.974998172$

$P\{x = 9.88\} = 0$

13. $X \sim N(108,9)$ (1) $P\{101.1<x<117.6\} = \Phi_0\left(\frac{117.6-108}{3}\right) - \Phi_0\left(\frac{101.1-108}{3}\right) = 0.885929$

(2) $P\{x<a\} = 0.90$ $x_0 = 1.28$ $a = 3\times 1.28 + 108 = 111.84$

(3) $P\{|x-a|>a\} = 0.01$

$1 - P\{|x-a| \le a\} = 0.01$ $\Phi_0\left(\frac{2a-108}{3}\right) - \Phi_0\left(\frac{0-108}{3}\right) = 0.99$

$P\{|x-a| \le a\} = 0.99$ $\Phi_0\left(\frac{2a-108}{3}\right) = 0.99$

$P\{-a \le x-a \le a\} = 0.99$ $u_{0.99} = 2.33$

$P\{0 \le x \le 2a\} = 0.99$ $\frac{2a-108}{3} = 2.33 \Rightarrow a = 57.495$

14. $\varphi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $X \sim N(0, 1600)$

$P\{|X| \le 10\} = \Phi_0\left(\frac{10}{40}\right) - \Phi_0\left(\frac{-10}{40}\right)$

$= 0.5987 - 1 + 0.5987$

$= 0.1974$

$Y \sim B(3, 0.1974)$ $P\{Y \ge 1\} = 1 - {}^3C_0 (0.1974)^0 (0.8026)^3 = 0.483$

15. (1) $X \sim N(72, \sigma^2)$ $P\{X \ge 90\} = 0.04$

$1 - P\{X < 90\} = 0.04$

$P\{X < 90\} = 0.96$

$\Phi_0\left(\frac{90-72}{\sigma}\right) = 0.96$

$\frac{90-72}{\sigma} = 1.76$

$\sigma = 10.2272727\dot{3}$

$P\{X < 60\}$

$= \Phi_0\left(\frac{60-72}{10.227}\right)$

$= 1 - \Phi_0(1.17336)$

$= 0.121$

$= 12.1\%$

(2) $P\{65 < X < 80\} = \Phi_0\left(\frac{80-72}{10.227}\right) - \Phi_0\left(\frac{65-72}{10.227}\right) = 0.7823 - 1 + 0.7517 = 53.4\%$

16. $X \sim N(0.2, 0.05^2)$ $0.367 - 0.2 = 0.167$ $3\sigma = 0.15$ $x - \mu > 3\sigma$

$\Rightarrow P\{|x - \mu| < 3\sigma\}$ 可以认为异常

2-4

1.

Y	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Z	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

2. $X \sim B(3, 0.4)$

X	0	1	2	3
P	0.216	0.432	0.288	0.064

Y	0	-1	3
P	0.504	0.432	0.064

3. $X \sim U[0, 1]$

$F_Y(x) = P\{Y \le x\} = P\{e^X \le x\} = P\{X \le \ln x\} = F_X(\ln x)$

$f_Y(x) = f_X\left(\frac{1}{x}\right)$

$f_X(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$

$f_Y(x) = \begin{cases} \frac{1}{x} & 1 \le x \le e \\ 0 & \text{else} \end{cases}$

4. (1) $Y=2X+1$

$$F_Y(x)=P\{Y\le x\}=P\{2X+1\le x\}=P\{X\le \frac{x-1}{2}\}=F_X(\frac{x-1}{2})$$

$$f_Y(x)=\frac{1}{2}f_X(\frac{x-1}{2})$$

$$f_X(x)=\begin{cases}\frac{x}{2} & 0<x<2\\ 0 & \text{else}\end{cases} \qquad f_Y(x)=\begin{cases}\frac{x-1}{8} & 1\le x\le 5\\ 0 & \text{else}\end{cases}$$

(2) $Y=X^2$

$$F_Y(x)=P\{X^2\le x\}=P\{-\sqrt{x}\le X\le\sqrt{x}\}=F_X(\sqrt{x})-F_X(-\sqrt{x})$$

$$f_Y(x)=f_X(\sqrt{x})\frac{1}{2\sqrt{x}}-f_X(-\sqrt{x})(\frac{-1}{2\sqrt{x}})$$

$$f_Y(y)=\begin{cases}\frac{1}{4} & 0\le y\le 4\\ 0 & \text{else}\end{cases}$$

5. $X\sim N(0,1)$

$$F_Y(y)=P(Y\le y)=P(|X|\le y)=P(-y\le X\le y)$$

$$=\int_{-y}^{y}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$$

$$=\int_{-|y|}^{0}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt+\int_0^y\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$$

$$=\frac{1}{\sqrt{2\pi}}\left(2\int_0^y e^{-\frac{t^2}{2}}dt\right)$$

$$f_{|X|}(x)=\begin{cases}\frac{2}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} & x\geqslant 0\\ 0 & x<0\end{cases}$$

二

1. (1) $c^{-1}\frac{\lambda^k}{k!}=\frac{\lambda^k}{k!}e^{-\lambda}\Rightarrow c=e^{\lambda}$

(2) $\frac{\lambda^2}{2}e^{-\lambda}=2\lambda e^{-\lambda}$ $\qquad P\{x=3\}=\frac{4^3}{3!}e^{-4}$

$\lambda^2e^{-\lambda}=4\lambda e^{-\lambda}$ $\qquad =\frac{32}{3}e^{-4}$

$(\lambda^2-4\lambda)e^{-\lambda}=0$

$\lambda=4$

(3) $P\{X=-1\}=0.3$ $\qquad P\{x=1.5\}=0$ $\qquad$ 注意离散型的概率函数是分布列

☆ (4) $\lim\limits_{x\to\infty}A+B\tan^{-1}x=1$ $\qquad \lim\limits_{x\to-\infty}A+B\tan^{-1}x=0$ $\qquad B\pi=1$ $\qquad A=\frac{1}{2}$

$A+\frac{B\pi}{2}=1$ $\qquad A-\frac{B\pi}{2}=0$ $\qquad B=\frac{1}{\pi}$ $\qquad f(x)=\frac{1}{\pi(1+x^2)}$

(5)

Y	0	1
P	$\frac{1}{3}$	$\frac{2}{3}$

(6) $F_Y(x) = P\{2X \le x\} = P\{X \le \frac{x}{2}\} = F_X(\frac{x}{2})$ $\Rightarrow f_Y(x) = \frac{1}{\pi(1+\frac{x^2}{4})}(\frac{1}{2})$

$f_Y(x) = f_X(\frac{x}{2})(0.5)$ $= \frac{2}{\pi(4+x^2)}$

2.(1) $\sum_{k=1}^{\infty} b\lambda^k = 1$ C

$b\frac{\lambda}{1-\lambda} = 1$

$b\lambda = 1-\lambda$

$b\lambda + \lambda = 1$

$\lambda = \frac{1}{1+b}$

(2) $P\{X \le a\} = F(a)$

$P\{a < X \le b\} = P\{x \le b\} - P\{x \le a\}$

$= F(b) - F(A)$

A, C

(3) C (4) $\int_0^1 kx\,dx$ $\frac{k}{2} = 1$ A

$= \left[\frac{kx^2}{2}\right]_0^1$ $k=2$

$= \frac{k}{2}$

(5) B, D $F_Y(x) = P\{3X-1 \le x\} = P\{X \le \frac{x+1}{3}\} = F_X(\frac{x+1}{3})$

$f_Y(x) = \frac{1}{3} f_X(x)$ $f_Y(x) = \frac{1}{6}$

(6) C

(7) C (8) $X \sim N(2,4)$ $aX+b = \frac{X-2}{2}$

(9) $F(x) = aF_1(x) + bF_2(x)$ B

(10) $X \sim N(\mu, 25)$ $Y \sim N(\mu, 100)$ $P\{X \le \mu-5\} = \Phi_0(\frac{\mu-5-\mu}{5}) = 1-\Phi_0(1) = 0.758$

$P\{Y \ge \mu+10\} = 1 - P\{Y < \mu+10\} = 1 - \Phi_0(\frac{\mu+10-\mu}{10}) = 0.758$ A

3.

X	0	1	2	3
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

4.(1)

X	1	2	3	4
P	$\frac{7}{10}$	$\frac{7}{30}$	$\frac{7}{120}$	$\frac{1}{120}$

(2) $P\{X=k\} = (0.3)^{k-1}(0.7)$ $k=1,2,\cdots$

(3) 有限k次能解决的写分布列,
无限写函数(离散)

X	1	2	3	4
P	$\frac{7}{10}$	$\frac{6}{25}$	$\frac{27}{500}$	$\frac{3}{500}$

5. (1) $\int_{-\infty}^{\infty} Ae^{-|x|}dx=1$

$2\int_0^{\infty} Ae^{-x}dx=1$

$[-Ae^{-x}]_0^{\infty}=\frac{1}{2}$

$A=\frac{1}{2}$

(2) $P\{0<x<1\}$

$=\int_0^x \frac{1}{2}e^{-|t|}dt+\int_x^1 \frac{1}{2}e^{-|t|}dt$

$=[\frac{-1}{2}e^{-t}]_0^x+[\frac{-1}{2}e^{-t}]_x^1$

$=\frac{-1}{2}e^{-x}+\frac{1}{2}+\frac{-1}{2}e^{-1}+\frac{1}{2}e^{-x}$

$=\frac{1}{2}(1-\frac{1}{e})$

$P\{-1\le x\le 1\}=2\int_0^1 \frac{1}{2}e^{-|x|}dx$

$=1-\frac{1}{e}$

(3) $x\le 0$ $\int_{-\infty}^{x} f(t)dt$

$=\int_{-\infty}^{x}\frac{1}{2}e^{-|t|}dt$

$=\int_{-\infty}^{x}\frac{1}{2}e^{t}dt$

$=\frac{1}{2}e^{x}$

$x>0$ $\int_0^x f(t)dt+F(0)$

$=\int_0^x \frac{1}{2}e^{-t}dt+\frac{1}{2}$

$=\frac{-e^{-t}}{2}\Big|_0^x+\frac{1}{2}$

$=-\frac{e^{-x}}{2}+\frac{1}{2}+\frac{1}{2}$

$=1-\frac{1}{2e^{x}}$

6. $X\sim P(10)$ $P\{X\ge k\}=0.95$

$1-P\{x<k\}=0.95$

$P\{x<k\}=0.05$

$k=15$

注意 $X\sim P(\lambda)$ 也是离散型

7. $X\sim U[0,5]$ $P\{$至少2人不超过$\}=(\frac{2}{5})^2(\frac{3}{5})\times 3+(\frac{2}{5})^3$

$=0.096\times 3+\frac{8}{125}$

$=0.352$

8. $X\sim Exp(\frac{1}{5})$ $f(x)=\begin{cases}\lambda e^{-\lambda x} & x>0\\ 0 & else\end{cases}$

$P\{X>10\}=\int_{10}^{\infty}\frac{1}{5}e^{-\frac{1}{5}x}dx=[-e^{-\frac{1}{5}x}]_{10}^{\infty}=e^{-2}$

$Y\sim B(5,e^{-2})$

$P\{Y\ge 1\}=1-P\{Y=0\}=1-{}^5C_0(e^{-2})^0(1-e^{-2})^5$

$=1-(1-e^{-2})^5$

9. $X \sim N(170, 6^2)$ $\quad P\{X<k\} \geq 0.99$

$$\Phi_0\left(\frac{k-170}{6}\right) \geq 0.99$$

$$k = 2.33 \times 6 + 170$$

$$= 183.98$$

10. $P\{X<k\} = 1 - P\{X \geq k\} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\int_{-\infty}^{k} f(x)dx = \frac{1}{3}$$

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{x}{3} & 0 \leq x < 1 \\ \frac{1}{3} & 1 \leq x < 3 \\ \frac{1}{3} + \frac{2}{9}(x-3) & 3 \leq x < 6 \\ 1 & x \geq 6 \end{cases} \quad \Rightarrow 1 \leq k < 3$$

$$\int_0^x \frac{1}{3} dx = \frac{x}{3}$$

$$\int_{-\infty}^{x} f(t)dt$$

$$= \int_{-\infty}^{1} f(t)dt + \int_3^x f(t)dt$$

$$= \frac{1}{3} + \int_3^x \frac{2}{9} dt$$

$$= \frac{1}{3} + \frac{2}{9}(x-3)$$

11. $F_Y(x) = P\{Y \leq x\} = P\{\ln X \leq x\} = P\{X \leq e^x\} = F_X(e^x)$

$$f_X(x) = \begin{cases} \frac{2}{\lambda(1+x^2)} & x>0 \\ 0 & x \leq 0 \end{cases}$$

$$f_Y(x) = f_X(e^x)e^x$$

$$f_Y(x) = \begin{cases} \frac{2e^x}{\pi(1+e^{2x})} & x>1 \\ 0 & x \leq 1 \end{cases}$$

$$\int_0^\infty \frac{2}{\lambda(1+x^2)} dx = 1$$

$$\frac{2}{\lambda} \int_0^\infty \frac{1}{1+x^2} dx = 1$$

$$\frac{2}{\lambda} \left[\tan^{-1} x\right]_0^\infty = 1$$

$$\frac{2}{\lambda} \frac{\pi}{2} = 1$$

$$\lambda = \pi$$

12.(1) $P\{X\le 200\} = \Phi_0\left(\frac{200-220}{25}\right) = \Phi_0(-0.8) = 1-\Phi(0.8) = 0.2119$

$P\{200<X\le 240\} = \Phi_0\left(\frac{240-220}{25}\right) - \Phi_0\left(\frac{200-220}{25}\right) = 0.5762$

$P\{X>240\} = 1-P\{X\le 240\} = 0.2119$

$P(\text{损坏}) = 0.2119\times 0.1 + 0.5762\times 0.001 + 0.2119\times 0.2 = 0.0641462$

(2) $P(200\sim 240V \mid \text{损坏}) = \dfrac{0.5762\times 0.001}{0.0641462} = 0.008982605$

13. $X\sim Exp(2)$

$$f_X(x) = \begin{cases} 2e^{-2x} & x>0 \\ 0 & x\le 0 \end{cases}$$

$$F_Y(x) = P\{1-e^{-2X}\le x\} = P\left\{X\le \frac{-\ln(1-x)}{2}\right\} = F_X\left(\frac{-1}{2}\ln(1-x)\right)$$

$$= \int_0^{-\frac{1}{2}\ln(1-x)} 2e^{-2t}\,dt$$

$$= \left[-e^{-2t}\right]_0^{-\frac{1}{2}\ln(1-x)}$$

$$= -e^{-2(\frac{-1}{2})\ln(1-x)} + 1$$

$$= 1-e^{\ln(1-x)}$$

$$= x$$

$$f_Y(x) = \begin{cases} 1 & x>0 \\ 0 & x\le 0 \end{cases}$$

$\Rightarrow Y\sim U[0,1]$

14.(1) $X\sim U[-\frac{\pi}{2},\frac{\pi}{2}]$ $Y=\sin X$

$$f_X(x) = \begin{cases} \frac{1}{\pi} & -\frac{\pi}{2}\le x\le \frac{\pi}{2} \\ 0 & \text{else} \end{cases}$$

$F_Y(x) = P\{\sin X\le x\} = P\{X\le \sin^{-1}x\} = F_X(\sin^{-1}x)$

$f_Y(x) = f_X(\sin^{-1}x)\,\frac{1}{\sqrt{1-x^2}}$

$$f_Y(x) = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}} & -1\le x\le 1 \\ 0 & \text{else} \end{cases}$$

(2) $X\sim U[0,\pi]$

$$f_X(x) = \begin{cases} \frac{1}{\pi} & 0\le x\le \pi \\ 0 & \text{else} \end{cases}$$

$F_Y(x) = P\{0\le X\le \sin^{-1}x\} + P\{\pi-\sin^{-1}x\le X<\pi\}$

$= F_X(\sin^{-1}x) - F_X(0) + F_X(\pi) - F_X(\pi-\sin^{-1}x)$

$\rightarrow f_X(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}} + f_X(\pi-\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}$

$$f_Y(x) = \begin{cases} \frac{2}{\pi\sqrt{1-x^2}} & 0\le x\le 1 \\ 0 & \text{else} \end{cases}$$

3-1

2. (1) $F_X(x) = \lim_{y\to\infty} F(x,y) = 1-2^{-x} \qquad x\geq 0$

$F_Y(y) = \lim_{x\to\infty} F(x,y) = 1-2^{-y} \qquad y\geq 0$

(2) $P\{1<X\leq 2, 1<Y\leq 2\} = F(2,2) - F(1,2) - F(2,1) + F(1,1)$

$= 1-2^{-2}-2^{-2}+2^{-4} - (1-2^{-1}-2^{-2}+2^{-1-2}) - (1-2^{-2}-2^{-1}+2^{-2-1}) + 1-2^{-1}-2^{-1}+2^{-2}$

$= \frac{1}{16}$

3. $P\{-1<x<2, -1<y<2\}$

$= F(2,2) - F(-1,2) - F(2,-1) + F(-1,-1) \qquad \therefore$ No

$= 1-1-1+0$

$= -1 < 0$

4. $a=0.1 \quad c=0.6 \quad b=0.2 \quad f=0.4 \quad d=0.3 \quad e=0.3$

5.

X \ Y	1	2	3
1	0	$\frac{1}{6}$	$\frac{1}{12}$
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{12}$	$\frac{1}{6}$	0

6. (1)

X \ Y	0	1	2
0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$
1	$\frac{2}{9}$	$\frac{2}{9}$	0
2	$\frac{1}{9}$	0	0

(2)

X	0	1	2
P	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Y	0	1	2
P	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(3) $F_X(x) = \begin{cases} 0 & x<0 \\ \frac{4}{9} & 0\leq x<1 \\ \frac{8}{9} & 1\leq x<2 \\ 1 & 2\leq x \end{cases}$ $\qquad F_Y(y) = \begin{cases} 0 & y<0 \\ \frac{4}{9} & 0\leq y<1 \\ \frac{8}{9} & 1\leq y<2 \\ 1 & 2\leq y \end{cases}$

(4) $F(1, 1.2) = \frac{1}{9} + \frac{2}{9}\times 3 = \frac{7}{9}$

(5) $P\{X=Y\} = \frac{1}{3}$

7. $f(x,y) = \begin{cases} \frac{1}{16} & (x,y) \in G \\ 0 & \text{else} \end{cases}$

$$P\{X>3, Y>3\} = \int_3^4 \int_3^4 \frac{1}{16}\, dx\, dy$$
$$= \frac{1}{16}$$

8.(1) $F(x,y) = A(B + \tan^{-1}\frac{x}{2})(\frac{\pi}{2} + \tan^{-1} y)$

$$= A\left(\frac{B\pi}{2} + B\tan^{-1}y + \frac{\pi}{2}\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{2}\tan^{-1}y\right)$$
$$= \frac{AB\pi}{2} + AB\tan^{-1}y + \frac{A\pi}{2}\tan^{-1}\frac{x}{2} + A\tan^{-1}\frac{x}{2}\tan^{-1}y$$

$\lim\limits_{\substack{x\to\infty \\ y\to\infty}} F(x,y) = 1$ $\qquad \frac{AB\pi}{2} + AB\frac{\pi}{2} + \frac{A\pi}{2}\left(\frac{\pi}{2}\right) + A\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right) = 1$

$$AB\pi + A\frac{\pi^2}{2} = 1$$

$\lim\limits_{x\to-\infty} F(x,y) = 0$ $\qquad \frac{AB\pi}{2} + AB\tan^{-1}y - \frac{A\pi}{2}\left(\frac{\pi}{2}\right) - \frac{A\pi}{2}\tan^{-1}y = 0$

$$AB\left(\frac{\pi}{2} + \tan^{-1}y\right) - \frac{A\pi}{2}\left(\frac{\pi}{2} + \tan^{-1}y\right) = 0$$
$$\left(\frac{\pi}{2} + \tan^{-1}y\right)\left(AB - \frac{A\pi}{2}\right) = 0$$
$$A\left(B - \frac{\pi}{2}\right) = 0$$
$$B = \frac{\pi}{2}$$
$$A\frac{\pi^2}{2} + A\frac{\pi^2}{2} = 1$$
$$A = \frac{1}{\pi^2}$$

(2) $F(x,y) = \frac{1}{4} + \frac{1}{2\pi}\tan^{-1}y + \frac{1}{2\pi}\tan^{-1}\frac{x}{2} + \frac{1}{\pi^2}\tan^{-1}\frac{x}{2}\tan^{-1}y$

$$F_X(x) = \lim_{y\to\infty} F(x,y) = \frac{1}{4} + \frac{1}{2\pi}\left(\frac{\pi}{2}\right) + \frac{1}{2\pi}\tan^{-1}\frac{x}{2} + \left(\frac{1}{\pi^2}\right)\left(\frac{\pi}{2}\right)\tan^{-1}\frac{x}{2}$$
$$= \frac{1}{2} + \frac{1}{2\pi}\tan^{-1}\frac{x}{2} + \frac{1}{2\pi}\tan^{-1}\frac{x}{2}$$
$$= \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{x}{2}$$

$$F_Y(y) = \lim_{x\to\infty} F(x,y) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}y$$

(3) $$\frac{\partial F}{\partial y} = \frac{1}{2\pi(1+y^2)} + \frac{1}{\pi^2}\tan^{-1}\frac{x}{2}\left(\frac{1}{1+y^2}\right)$$
$$\frac{\partial^2 F}{\partial x \partial y} = \frac{1}{\pi^2}\left(\frac{2}{(4+x^2)(1+y^2)}\right)$$

$$\int_{-\infty}^{2}\int_{\frac{\sqrt{3}}{3}}^{\infty} \frac{2}{\pi^2}\left(\frac{1}{(4+x^2)(1+y^2)}\right) dy\, dx$$
$$= \int_{-\infty}^{2} \frac{2}{\pi^2(4+x^2)}\left[\tan^{-1}y\right]_{\frac{\sqrt{3}}{3}}^{\infty} dx$$
$$= \frac{2\pi}{3\pi^2}\int_{-\infty}^{2} \frac{1}{(4+x^2)}\, dx$$
$$= \frac{2}{3\pi}\ \frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_{-\infty}^{2}$$
$$= \frac{2}{3\pi}\left(\frac{1}{2}\right)\left(\frac{3\pi}{4}\right) = \frac{1}{4}$$

9. (1) $\int_0^\infty\int_0^\infty f(x,y)\,dxdy=1$

$c\int_0^\infty\int_0^\infty e^{-2x-2y}\,dxdy=1$

$c\int_0^\infty \left[\frac{e^{-2x-2y}}{-2}\right]_0^\infty dy=1$

$c\int_0^\infty \frac{e^{-2y}}{2}\,dy=1$

$c\left[\frac{e^{-2y}}{-4}\right]_0^\infty=1$

$c=4$

(2) $F(x,y)=\int_0^x\int_0^y 4e^{-2s-2t}\,dsdt$

$=4\int_0^x \left[\frac{e^{-2s-2t}}{-2}\right]_0^y dt$

$=4\int_0^x \left(\frac{e^{-2y-2t}}{-2}+\frac{e^{-2t}}{2}\right)dt$

$=2\int_0^x \left(e^{-2t}-e^{-2y-2t}\right)dt$

$=2\left(\left[\frac{-e^{-2t}}{-2}\right]_0^x+\left[\frac{e^{-2y-2t}}{-2}\right]_0^x\right)$

$=-\left[-e^{-2t}\right]_0^x-\left[-e^{-2y-2t}\right]_0^x$

$=-\left(-e^{-2x}+1\right)-\left(-e^{-2y-2x}+e^{-2y}\right)$

$=e^{-2x}-1+e^{-2y-2x}-e^{-2y}$

(3) $f_X(x)=\int_0^\infty f(x,y)\,dy$

$=\int_0^\infty 4e^{-2x-2y}\,dy$

$=4\int_0^\infty e^{-2x-2y}\,dy$

$=4\left[\frac{e^{-2x-2y}}{-2}\right]_0^\infty$

$=-2\left[e^{-2x-2y}\right]_0^\infty$

$=-2\left(0-e^{-2x}\right)$

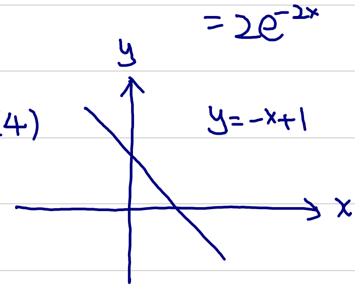
$=2e^{-2x}$

$f_Y(y)=\int_0^\infty f(x,y)\,dx$

$=4\int_0^\infty e^{-2x-2y}\,dx$

$=2e^{-2y}$

(4)



$\iint_G f(x,y)\,dxdy$

$=\int_0^1\int_0^{1-x} 4e^{-2x-2y}\,dydx$

$=-2\int_0^1 \left[e^{-2x-2y}\right]_0^{1-x} dx$

$=-2\int_0^1 \left(e^{-2x-2(1-x)}-e^{-2x}\right)dx$

$=-2\int_0^1 \left(e^{-2}-e^{-2x}\right)dx$

$=-2\left(e^{-2}+\left[\frac{e^{-2x}}{2}\right]_0^1\right)$

$=-2\left(e^{-2}+\frac{e^{-2}}{2}-\frac{1}{2}\right)=-2e^{-2}-e^{-2}+1=1-3e^{-2}$

10. $\varphi(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}e^{\frac{-1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2}-2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+\frac{(y-\mu_2)^2}{\sigma_2^2}\right)}$

$$f(x,y)=\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}(1+\sin x\sin y)=\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}-\sin x\sin y\left(\frac{x^2+y^2}{2}\right)$$

$$\Rightarrow \mu_1,\mu_2=0 \quad \rho=0$$

$$\varphi(x,y)=\frac{1}{2\pi}e^{\frac{-1}{2}\left(\frac{x^2}{\sigma_1^2}-\frac{2xy}{\sigma_1\sigma_2}+\frac{y^2}{\sigma_2^2}\right)} \qquad \sigma_1=\sigma_2=1$$

$$=\frac{1}{2\pi}e^{-\frac{1}{2}(x^2+y^2}$$

$$=\frac{1}{2\pi}e^{-\frac{1}{2}(x^2+y^2)}$$

不服从正态分布

3-2

1.(1)

X \ Y	1	2	3	4
1	$\frac{1}{4}$	0	0	0
2	$\frac{1}{8}$	$\frac{1}{8}$	0	0
3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

X	1	2	3	4
P	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Y	1	2	3	4
P	$\frac{25}{48}$	$\frac{13}{48}$	$\frac{7}{48}$	$\frac{1}{16}$

(2)

Y	1	2	3	4
$P\{Y=k\mid X=3\}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0

2.

X \ Y	1	2	3	4	5	6
1	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
2	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
3	0	0	$\frac{3}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	0	0	0	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
5	0	0	0	0	$\frac{5}{36}$	$\frac{1}{36}$
6	0	0	0	0	0	$\frac{6}{36}$

不独立

3.

X	1	2
P	$\frac{1}{3}$	$\frac{1}{3}+\alpha+\beta$

Y	1	2	3
P	$\frac{1}{2}$	$\frac{1}{9}+\alpha$	$\frac{1}{18}+\beta$

$$\frac{1}{3}\left(\frac{1}{9}+\alpha\right)=\frac{1}{9}$$
$$\frac{1}{27}+\frac{\alpha}{3}=\frac{1}{9}$$
$$\frac{\alpha}{3}=\frac{2}{27}$$
$$\alpha=\frac{2}{9}$$

$$\left(\frac{5}{9}+\beta\right)\left(\frac{1}{18}+\beta\right)=\beta$$
$$\frac{5}{162}+\frac{5}{9}\beta+\frac{1}{18}\beta+\beta^2=\beta$$
$$\beta^2-\frac{7}{18}\beta+\frac{5}{162}=0$$
$$\beta=\frac{1}{9} \text{ or } \beta=\frac{5}{18} \text{ (rej.)}$$

4.

X^2	0	1
P	$\frac{1}{4}$	$\frac{3}{4}$

Y^2	0	1
P	$\frac{1}{4}$	$\frac{3}{4}$

$P\{X^2=Y^2\}=1$

$P\{X^2=1,\ Y^2=1\}+P\{X^2=0,Y^2=0\}=1$

假设独立

$\frac{1}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{3}{4}=\frac{5}{8}\neq 1$

$\Rightarrow X^2, Y^2$ 不独立 $\Rightarrow X, Y$ 不独立

5.

X	-1	1
P	$\frac{1}{2}$	$\frac{1}{2}$

Y	-1	1
P	$\frac{1}{2}$	$\frac{1}{2}$

$$P\{X=Y\}=P\{X=-1,Y=-1\}+P\{X=1\}P\{Y=1\}$$
$$=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$$

6. (1) $f(y|x)=\dfrac{f(x,y)}{f_X(x)}$

$$f(x,y)=f_X(x)\cdot f(y|x)$$

(2) $\displaystyle\int_{-\infty}^{\infty} f_X(x)\cdot f(y|x)\,dx$

(3) $f(x|y)=\dfrac{f_X(x)f(y|x)}{\int_{-\infty}^{\infty} f_X(x)f(y|x)dx}$

7. (1) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy=1$

$$c\int_0^{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}}\sin(x+y)\,dx\,dy=1$$
$$c\int_0^{\frac{\pi}{4}}\left[-\cos(x+y)\right]_0^{\frac{\pi}{4}}dy=1$$
$$c\int_0^{\frac{\pi}{4}}\left(-\cos\left(\frac{\pi}{4}+y\right)+\cos y\right)dy=1$$

$$c\left[-\sin\left(\frac{\pi}{4}+y\right)+\sin y\right]_0^{\frac{\pi}{4}}=1$$
$$c\left(-\sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\sin\frac{\pi}{4}-\sin 0\right)=1$$
$$c\left(-1+\frac{\sqrt{2}}{2}\cdot 2\right)=1$$
$$c=\frac{1}{\sqrt{2}-1}$$
$$=\sqrt{2}+1$$

(2) $f(x,y)=\begin{cases}(\sqrt{2}+1)\sin(x+y) & 0\le x\le \frac{\pi}{4} \quad 0\le y\le \frac{\pi}{4}\\ 0 & \text{else}\end{cases}$

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy$$
$$=\int_0^{\frac{\pi}{4}}(\sqrt{2}+1)\sin(x+y)\,dy$$
$$=(\sqrt{2}+1)\left[-\cos(x+y)\right]_0^{\frac{\pi}{4}}$$
$$=(\sqrt{2}+1)\left(-\cos\left(\frac{\pi}{4}+x\right)+\cos x\right)$$
$$=(\sqrt{2}+1)\left(\cos x-\cos\left(\frac{\pi}{4}+x\right)\right)$$

$$f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\,dx$$
$$=(\sqrt{2}+1)\left(\cos y-\cos\left(\frac{\pi}{4}+y\right)\right)$$

(3) $$f_X(x)f_Y(y)=(\sqrt{2}+1)^2\left(\cos x-\cos\left(\frac{\pi}{4}+x\right)\right)\left(\cos y-\cos\left(\frac{\pi}{4}+y\right)\right)$$
$$=(\sqrt{2}+1)^2\left(-2\sin\frac{x+\frac{\pi}{4}+x}{2}\right)\left(-2\sin\frac{y+\frac{\pi}{4}+y}{2}\right)$$
$$=(\sqrt{2}+1)^2\left(-2\sin\left(x+\frac{\pi}{8}\right)\right)\left(-2\sin\left(y+\frac{\pi}{8}\right)\right)$$
$$=2(\sqrt{2}+1)^2\left(\cos\left(x+\frac{\pi}{8}+y+\frac{\pi}{8}\right)-\cos\left(x+\frac{\pi}{8}-y-\frac{\pi}{8}\right)\right)$$
$$=2(\sqrt{2}+1)^2\left(\cos\left(x+y+\frac{\pi}{4}\right)-\cos(x-y)\right)$$
$$\neq f(x,y)$$ 不独立

8.(1) $F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(u,v)\,dv\,du$

$= \int_0^x \int_0^y u e^{-u-v}\,dv\,du$

$= \int_0^x u e^{-u} \int_0^y e^{-v} dv\, du$

$= \int_0^x u e^{-u}\ [-e^{-v}]_0^y\, du$

$= \int_0^x u e^{-u} (1-e^{-y})\, du$

$= (1-e^{-y}) \int_0^x u e^{-u}\, du$

$= (1-e^{-y}) \int_0^x u\ d(-e^{-u})$

$= (1-e^{-y}) \left([-u e^{-u}]_0^x - \int_0^x (-e^{-u})\, du \right)$

$= (1-e^{-y}) \left(-x e^{-x} + \int_0^x e^{-u}\, du \right)$

$= (1-e^{-y}) \left(-x e^{-x} + [-e^{-u}]_0^x \right.$

$= (1-e^{-y}) (-x e^{-x} + 1 - e^{-x})$

$= (1-e^{-y}) (1-(x+1)e^{-x}) \qquad x>0,\ y>0$

(2) $P\{-1 \le X \le 1,\ 0 \le Y \le 2\} = F(1,2) - F(1,0) - F(-1,2) + F(-1,0)$

$= (1-e^{-2})(1-2e^{-1})$

* $F(-1,2)$ 不在这段函数定义域内

(3) $f_X(x) = \int_{-\infty}^{\infty} x e^{-(x+y)}\, dy$

$= x \int_0^{\infty} e^{-x-y}\, dy$

$= x\, [-e^{-x-y}]_0^{\infty}$

$= x e^{-x} \qquad x>0$

$f_X(x) \cdot f_Y(x) = x e^{-x-y} = f(x,y)$

$\Rightarrow$ 独立

$f_Y(y) = \int_{-\infty}^{\infty} x e^{-(x+y)}\, dx$

$= \int_{-\infty}^{\infty} \frac{x e^{-(x+y)}\, d(-e^{-(x+y)})}{e^{-(x+y)}}$

$= \int_{-\infty}^{\infty} x\ d(-e^{-(x+y)})$

$= [-x e^{-(x+y)}]_0^{\infty} - \int_0^{\infty} -e^{-(x+y)}\, dx$

$= \int_0^{\infty} e^{-(x+y)}\, dx$

$= [-e^{-x-y}]_0^{\infty}$

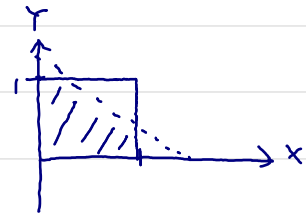
$= e^{-y} \qquad y>0$

9. $f_X(x) = \int_{-\infty}^{\infty} 8xy\,dy = [4xy^2]_0^1 = 4x$

$f_Y(y) = \int_{-\infty}^{\infty} 8xy\,dx = [4x^2y]_0^1 = 4y$

$f_X(x) \cdot f_Y(y) = 16xy \neq f(x,y) \Rightarrow$ 不独立

10. (1)



$x+y<1.2 \qquad 1=-x+1.2$

$y<-x+1.2 \qquad x=0.2$

$P\{X+Y<1.2\} = 1-\frac{0.8^2}{2} = 0.68$

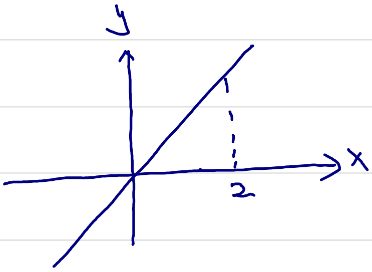
(2) $P\{XY<\frac{1}{4}\}$

$= \int_{\frac{1}{4}}^{1} \frac{1}{4x}dx+\frac{1}{4} = \frac{1}{4}\int_{\frac{1}{4}}^{1} x^{-1}dx+\frac{1}{4} = \frac{1}{4}[\ln x]_{\frac{1}{4}}^{1}+\frac{1}{4} = \frac{1}{4}(\ln 1-\ln\frac{1}{4})+\frac{1}{4} = \frac{-1}{4}\ln 4^{-1}+\frac{1}{4} = \frac{1}{2}\ln 2+\frac{1}{4}$

11. $X \sim U[0,2] \qquad Y \sim Exp(2)$

$f_X(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 2 \\ 0 & \text{else} \end{cases} \qquad f_Y(y) = \begin{cases} 2e^{-2y} & y>0 \\ 0 & y \le 0 \end{cases}$

$f(x,y) = \begin{cases} e^{-2y} & 0 \le x \le 2,\ y>0 \\ 0 & \text{else} \end{cases}$



$$P\{X>Y\} = \int_0^2 \int_0^x e^{-2y}\,dy\,dx$$
$$= \int_0^2 \left[\frac{e^{-2y}}{-2}\right]_0^x dx$$
$$= \int_0^2 \left(\frac{e^{-2x}}{-2}+\frac{1}{2}\right)dx$$
$$= \left[\frac{e^{-2x}}{4}+\frac{x}{2}\right]_0^2$$
$$= \frac{e^{-4}}{4}+1-\frac{1}{4}$$
$$= \frac{e^{-4}}{4}+\frac{3}{4}$$

3-3

1.

X\Y	0	1
0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{8}$	$\frac{1}{4}$
3	$\frac{1}{24}$	$\frac{1}{12}$

X+Y	0	1	2	3	4
P	$\frac{1}{6}$	$\frac{11}{24}$	$\frac{1}{4}$	$\frac{1}{24}$	$\frac{1}{12}$

2. (1) $Z = \sin(\pi X) + \cos(\pi Y)$

Z	-1	1
P	0.4	0.6

(2) $Z = XY$

Z	-6	-3	-2	0	2	4	-1
P	0.15	0.05	0.1	0.3	0.2	0.1	0.1

(3) $Z = \max(X, Y)$

Z	1	2
P	0.2	0.8

3.

$$f_X(x) = \begin{cases} 1 & -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & \text{else} \end{cases} \qquad f_Y(y) = \begin{cases} 1 & -\frac{1}{2} \le y \le \frac{1}{2} \\ 0 & \text{else} \end{cases}$$

$$F_Z(z) = P\{X+Y \le z\} = \iint_{x+y\le z} f(x,y)\,dx\,dy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{z-x} f(x,y)\,dy\,dx \qquad \text{Let } t = x+y$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{z} f(x, t-x)\,dt\,dx$$

$$= \int_{-\infty}^{z}\int_{-\infty}^{\infty} f(x, t-x)\,dx\,dt$$

$$f_Z(z) = \int_{-\infty}^{\infty} f(x, z-x)\,dx$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x)\, f_Y(z-x)\,dx$$

$$= \int_{-1}^{z} 1\,dx$$

$$= 1+z \qquad -1 \le z < 0$$

$$f_Z(z) = \int_{z}^{1} 1\,dx$$

$$= 1-z \qquad 0 \le z < 1$$

$$f_Z(z) = 0 \qquad \text{else}$$

4. $X \sim U[0,2]$ $\quad Y \sim U[0,1]$

$$f_X(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 2 \\ 0 & \text{else} \end{cases} \qquad f_Y(y) = \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 2 \\ 1 & x \ge 2 \end{cases} \qquad F_Y(y) = \begin{cases} 0 & y < 0 \\ y & 0 \le y < 1 \\ 1 & y \ge 1 \end{cases}$$

$$F_M(z) = \begin{cases} 0 & z < 0 \\ \frac{z^2}{2} & 0 \le z < 1 \\ \frac{z}{2} & 1 \le z < 2 \\ 1 & z \ge 2 \end{cases} \qquad f_M(z) = \begin{cases} z & 0 \le z < 1 \\ \frac{1}{2} & 1 \le z < 2 \\ 0 & \text{else} \end{cases}$$

$$F_N(z) = 1 - (1 - F_X(z))(1 - F_Y(z)) = \begin{cases} 0 & z < 0 \\ \frac{3z}{2} - \frac{z^2}{2} & 0 \le z < 1 \\ 1 & z \ge 1 \end{cases} \qquad f_N(z) = \begin{cases} \frac{3}{2} - z & 0 \le z < 1 \\ 0 & \text{else} \end{cases}$$

$$1 - \left(1 - \frac{z}{2}\right)(1 - z)$$
$$= 1 - \left(1 - z - \frac{z}{2} + \frac{z^2}{2}\right)$$
$$= z + \frac{z}{2} - \frac{z^2}{2}$$
$$= \frac{3z}{2} - \frac{z^2}{2}$$

5. $Z = X + 2Y$ $\qquad F(z) = P\{Z \le z\} = P\{X + 2Y \le z\}$ $\qquad z > 0$

$$= \iint_R 2e^{-(x+2y)}\,dx\,dy$$
$$= \int_0^{\frac{z}{2}} \int_0^{z-2y} 2e^{-(x+2y)}\,dx\,dy$$
$$= 2\int_0^{\frac{z}{2}} \left[-e^{-x-2y}\right]_0^{z-2y} dy$$
$$= 2\int_0^{\frac{z}{2}} \left(-e^{-z} + e^{-2y}\right) dy$$
$$= 2\left(-e^{-z}[y]_0^{\frac{z}{2}} + \left[\frac{e^{-2y}}{-2}\right]_0^{\frac{z}{2}}\right)$$
$$= -ze^{-z} - \left(e^{-z} - 1\right)$$
$$= -ze^{-z} - e^{-z} + 1$$

$$f(z) = -\left(e^{-z} - ze^{-z}\right) + e^{-z}$$
$$= ze^{-z} \qquad z > 0$$
$$f(z) = 0 \qquad \text{else}$$

三.

1.(1)

X \ Y	0	1
0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$

(2) $\Delta \geq 0$

$B^2 - 4C \geq 0$

$\frac{19}{36}$, $\frac{1}{18}$

(3) $P\{X_1X_2=0\}=1$

$\Rightarrow P\{X_1=-1, X_2=-1\}=0 \quad P\{X_1=-1, X_2=1\}=0 \quad P\{X_1=1, X_2=-1\}=0 \quad P\{X_1=1, X_2=1\}=0$

$P\{X_1=-1\} = P\{X_1=-1, X_2=-1\} + P\{X_1=-1, X_2=0\} + P\{X_1=-1, X_2=1\} = 0 + P\{X_1=-1, X_2=0\} + 0 = \frac{1}{4}$

$\Rightarrow P\{X_1=-1, X_2=0\} = \frac{1}{4}$

$P\{X_1=1, X_2=0\} = \frac{1}{4}$

$P\{X_2=0\} = P\{X_1=-1, X_2=0\} + P\{X_1=0, X_2=0\} + P\{X_1=1, X_2=0\} = \frac{1}{4} + P\{X_1=0, X_2=0\} + \frac{1}{4} = \frac{1}{2}$

$\Rightarrow P\{X_1=0, X_2=0\} = 0$

$P\{X_1=X_2\} = P\{X_1=-1, X_2=-1\} + P\{X_1=0, X_2=0\} + P\{X_1=1, X_2=1\} = 0$

4. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(6-x-y)\,dx\,dy = 1$

$\int_2^4 \int_0^2 k(6-x-y)\,dx\,dy = 1$

$\int_2^4 \int_0^2 (6k-kx-ky)\,dx\,dy = 1$

$\int_2^4 \left[6kx - \frac{k}{2}x^2 - kyx\right]_0^2 dy = 1$

$\int_2^4 (12k - 2k - 2ky)\,dy = 1$

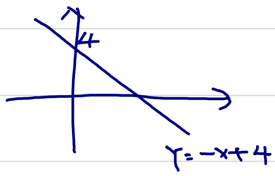
$\left[10ky - ky^2\right]_2^4 = 1$

$40k - 16k - 20k + 4k = 1$

$8k = 1$

$k = \frac{1}{8}$

$P\{X+Y \leq 4\}$



$= \int_0^2 \int_2^{4-x} \frac{1}{8}(6-x-y)\,dy\,dx$

$= \int_0^2 \int_2^{4-x} \left(\frac{3}{4} - \frac{1}{8}x - \frac{1}{8}y\right) dy\,dx$

$= \int_0^2 \left[\frac{3}{4}y - \frac{1}{8}xy - \frac{y^2}{16}\right]_2^{4-x} dx$

$= \int_0^2 \left(\frac{3}{4}(4-x) - \frac{1}{8}x(4-x) - \frac{(4-x)^2}{16} - \frac{3}{2} + \frac{1}{4}x + \frac{1}{4}\right) dx$

$= \int_0^2 \left(3 - \frac{3}{4}x - \frac{x}{2} + \frac{x^2}{8} - \frac{16-8x+x^2}{16} - \frac{5}{4} + \frac{1}{4}x\right) dx$

$= \int_0^2 \left(3 - \frac{3}{4}x - \frac{1}{2}x + \frac{1}{8}x^2 - 1 + \frac{1}{2}x - \frac{1}{16}x^2 - \frac{5}{4} + \frac{1}{4}x\right) dx$

$= \int_0^2 \left(\frac{1}{16}x^2 - \frac{1}{2}x + \frac{3}{4}\right) dx$

$= \left[\frac{x^3}{48} - \frac{x^2}{4} + \frac{3}{4}x\right]_0^2$

$= \frac{2}{3}$

(5) $X \sim N(2,3^2)$, $Y \sim N(-1,3^2)$ $\qquad \frac{1}{2}X+\frac{1}{3}Y \sim N\left(2(\frac{1}{2})+(-1)(\frac{1}{3}),\ \frac{1}{4}(3^2)+(\frac{1}{9})(3^2)\right)$

$$\sim N\left(\frac{2}{3},\frac{13}{4}\right)$$

※ 两个独立的正态分布 X,Y ，$Z=aX+bY$ 仍然服从正态分布

(6) $P\{X=0\} = P\{X=0,Y=0\} + P\{X=0,Y=1\} = 0.4+a$

$P\{X+Y=1\} = P\{X=0,Y=1\} + P\{X=1,Y=0\} = a+b$

$P\{X=0 \text{ and } X+Y=1\} = (0.4+a)(a+b)$ $\qquad 0.5+a+b=1$

$a = 0.4a+0.4b+a^2+ab$ $\qquad b=0.5-a$

$a = 0.4a+0.4(0.5-a)+a^2+a(0.5-a)$

$a = 0.4a+0.2-0.4a+a^2+0.5a-a^2$

$0.5a = 0.2$ $\qquad b=0.1$

$a = 0.4$

2. (1) $X+Y \sim N(1,2)$ $\qquad \Phi_0(\frac{z-1}{2})=0.5$ $\qquad$ B

$\frac{z-1}{2}=0$ $\quad z=1$

$X-Y \sim N(-1,2)$ $\qquad \Phi_0(\frac{z+1}{2})=0.5$ $\qquad \frac{z+1}{2}=0$ $\qquad z=-1$

(2) A

(3)

X\Y	-1	1
-1	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$

C

(4)

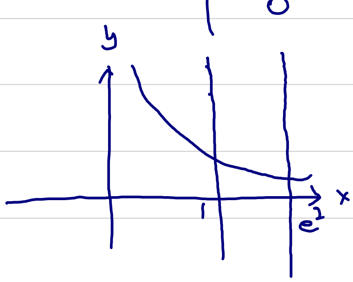
X\Y	0	1
0	$\frac{1}{9}$	$\frac{2}{9}$
1	$\frac{2}{9}$	$\frac{4}{9}$

$Z=\max(X,Y)$

Z	0	1
P	$\frac{1}{9}$	$\frac{8}{9}$

B

(5) $f(x,y) = \begin{cases} \frac{1}{S(G)} & (x,y)\in G \\ 0 & \text{else} \end{cases}$



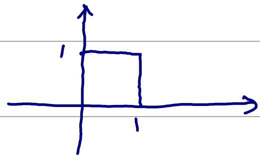
$$S(G) = \int_1^{e^2}\int_0^{\frac{1}{x}} 1 \, dy\,dx$$
$$= \int_1^{e^2} [y]_0^{\frac{1}{x}} \, dx$$
$$= \int_1^{e^2} \frac{1}{x}\,dx$$
$$= [\ln x]_1^{e^2}$$
$$= 2$$

$\Rightarrow f(x,y) = \begin{cases} \frac{1}{2} & (x,y)\in G \\ 0 & \text{else} \end{cases}$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$
$$= \int_0^{\frac{1}{x}} \frac{1}{2}\,dy$$
$$= \left[\frac{1}{2}y\right]_0^{\frac{1}{x}}$$
$$= \frac{1}{2x}$$

C

(6) A



$$f(x,y)=\begin{cases}1 & (x,y)\in D\\ 0 & \text{else}\end{cases}$$

$$f_X(x)=\int_0^1 1\,dy = 1 \qquad f_Y(x)=\int_0^1 1\,dx = 1 \qquad f(x,y)=f_X(x)\cdot f_Y(x)$$

$\Rightarrow$ 独立

3. (1)

Y	0	1	2
$P\{Y\mid X=1\}$	$\frac{4}{7}$	$\frac{3}{7}$	0

$P\{X=1\}=\frac{2}{9}+\frac{1}{6}=\frac{7}{18}$

(2)

X	0	1	2
$P\{X\mid Y=0\}$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

$P\{Y=0\}=\frac{5}{12}$

(3)

X	0	1	2
$P\{X\mid X+Y=1\}$	$\frac{3}{5}$	$\frac{2}{5}$	0

$P\{X+Y=1\}=\frac{1}{3}+\frac{2}{9}=\frac{5}{9}$

(4) $Z=X+Y$

Z	0	1	2	3	4
P	$\frac{1}{6}$	$\frac{5}{9}$	$\frac{5}{18}$	0	0

4.

$$f_X(x)=\begin{cases}\lambda^2 x e^{-\lambda x} & x>0\\ 0 & x\le 0\end{cases} \qquad f(y\mid x)=\begin{cases}\frac{1}{x} & y>0\\ 0 & y\le 0\end{cases}$$

$$f(y\mid x)=\frac{f(x,y)}{f_X(x)}$$

$$\frac{1}{x}=\frac{f(x,y)}{\lambda^2 x e^{-\lambda x}}$$

$$f(x,y)=\lambda^2 e^{-\lambda x}$$

$$f_Y(y)=\int_y^\infty \lambda^2 e^{-\lambda x}\,dx$$
$$=\lambda^2\int_y^\infty e^{-\lambda x}\,dx$$
$$=\frac{\lambda^2}{-\lambda}\left[e^{-\lambda x}\right]_y^\infty$$
$$=-\lambda-e^{-\lambda y}$$
$$=\lambda e^{-\lambda y} \quad y>0$$

5.(1) $f(x,y) = \begin{cases} 2 & (x,y)\in G \\ 0 & \text{else} \end{cases}$

$$S(G) = \int_0^1\int_0^x 1\,dy\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

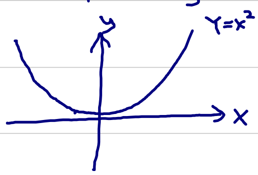
$$f_X(x) = \int_{-\infty}^{\infty} 2\,dy = \int_0^1 2\,dy = 2$$

$$f_Y(y) = \int_0^x 2\,dx = 2x$$

$f_X(x)\cdot f_Y(y) \neq f(x,y)$

$\Rightarrow$ 不独立

(2) $P\{Y > x^2\} = \int_0^1\int_{x^2}^x 2\,dy\,dx$



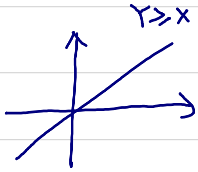
$$= \int_0^1 (2x - 2x^2)\,dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = \frac{1}{3}$$

6. $f(x,y) = \begin{cases} \frac{1}{2} & (x,y)\in G \\ 0 & \text{else} \end{cases}$

U\V	0	1
0	$\frac{1}{4}$	0
1	$\frac{1}{4}$	$\frac{1}{2}$

$-Y \le -X$

$Y > X$



$$\int_0^1\int_x^1 \frac{1}{2}\,dy\,dx = \int_0^1\left[\frac{1}{2}y\right]_x^1 dx = \int_0^1\left(\frac{1}{2} - \frac{1}{2}x\right)dx = \left[\frac{x}{2} - \frac{x^2}{4}\right]_0^1 = \frac{1}{4}$$

$$\int_0^1\int_0^{2y} \frac{1}{2}\,dx\,dy = \int_0^1 \left[\frac{x}{2}\right]_0^{2y} dy = \left[\frac{y^2}{2}\right]_0^1 = \frac{1}{2}$$

8. (1) $F(x,y) = \int_0^x \int_0^y 4uv \, dv \, du$ $\quad 0 \le x \le 1, \; 0 \le y \le 1$

$= \int_0^x \left[2uv^2\right]_0^y du$

$= \int_0^x 2uy^2 \, du$

$= \left[u^2y^2\right]_0^x$

$= x^2y^2$

When $0 \le x < 1, \; y \ge 1$

$F(x,y) = \int_0^x \int_0^1 4uv \, dv \, du$

$= \int_0^x \left[2uv^2\right]_0^1 du$

$= \int_0^x 2u \, du$

$= x^2$

When $0 \le y \le 1 \quad x \ge 0$

$F(x,y) = \int_0^y \int_0^1 4uv \, du \, dv$

$= \int_0^y \left[2u^2v\right]_0^1 dv$

$= y^2$

When $x > 1, \; y > 1$

$F(x,y) = \int_0^1 \int_0^1 4uv \, du \, dv$

$= \int_0^1 \left[2u^2v\right]_0^1 dv$

$= 1$

When $x < 0, \; y < 0 \quad F(x,y) = 0$

(2) $P\{0 \le X < \frac{1}{2}, \frac{1}{4} \le Y < 1\} = \int_0^{\frac{1}{2}} \int_{\frac{1}{4}}^1 4xy \, dy \, dx$

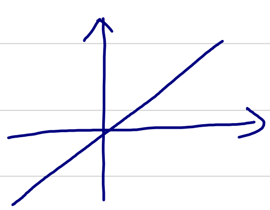
$= \int_0^{\frac{1}{2}} \left[2xy^2\right]_{\frac{1}{4}}^1 dx$

$= \int_0^{\frac{1}{2}} \frac{15}{8} x \, dx$

$= \left[\frac{15 x^2}{16}\right]_0^{\frac{1}{2}}$

$= \frac{15}{64}$

(3) $P\{X < Y\} = \int_0^1 \int_x^1 4xy \, dy \, dx$



$= \int_0^1 \left[2xy^2\right]_x^1 dx$

$= \int_0^1 (2x - 2x^3) \, dx$

$= \left[x^2 - \frac{x^4}{2}\right]_0^1$

$= \frac{1}{2}$

$-Y < -X$

$Y > X$

9. $X \sim B(n,p) \qquad Y \sim B(m,p)$

$P\{X=k\} = {}^{n}C_k\, p^k (1-p)^{n-k} \qquad P\{Y=k\} = {}^{m}C_k\, p^k (1-p)^{m-k}$

$Z = X+Y$

$$P\{Z=k\} = P\{X+Y=k\} = \sum_{k=0}^{i} P\{X=k\}P\{Y=i-k\}$$

$$= \sum_{k=0}^{i} {}^{n}C_k\, p^k (1-p)^{n-k}\; {}^{m}C_{i-k}\, p^{i-k} (1-p)^{m-i+k}$$

$$= \sum_{k=0}^{i} p^{i} (1-p)^{n+m-i}\; {}^{n}C_k\; {}^{m}C_{i-k}$$

Since $\sum_{k=0}^{i} {}^{n}C_k\, {}^{m}C_{i-k} = {}^{n+m}C_i$

$\Rightarrow \quad P\{X+Y=k\} = {}^{n+m}C_i\, p^{i} (1-p)^{n+m-i}$

$\Rightarrow \quad X+Y \sim B(n+m, p)$

10.

ξ	1	2	3
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

η	1	2	3
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\xi \backslash \eta$	1	2	3
1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
3	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

$X = \max(\xi, \eta)$

X	1	2	3
P	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{5}{9}$

$Y = \min(\xi, \eta)$

Y	1	2	3
P	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

$P\{\xi = \eta\} = \frac{1}{3}$

11. (1) $f(x,y) = \begin{cases} 2y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$

$0 < z-x < 1 \rightarrow z-1 < x < z$

(2) $Z = X+Y$

$0 \le z \le 2$

$$f_Z(z) = \int_{-\infty}^{\infty} f(x, z-x)\,dx$$

$$= \int_0^z 2(z-x)\,dx$$

$$= \left[2zx - x^2\right]_0^z$$

$$= 2z^2 - z^2$$

$$= z^2 \qquad 0 \le z \le 1$$

When $1 \le z < 2$

$$f_Z(z) = \int_{z-1}^{1} 2(z-x)\,dx$$

$$= \left[2zx - x^2\right]_{z-1}^{1}$$

$$= 2z - 1 - 2z(z-1) + (z-1)^2$$

$$= 2z - 1 - 2z^2 + 2z + z^2 - 2z + 1$$

$$= 2z - z^2$$

$f_Z(z) = 0 \qquad \text{else}$

7. $f(x_1, x_2) = \frac{1}{2\sqrt{3}\pi} e^{-\frac{1}{2}\left(\frac{(x_1-4)^2}{3} + (x_2-2)^2\right)}$

$\mu_1 = 4 \quad \mu_2 = 2 \quad \sigma_1^2 = 3 \quad \sigma_2^2 = 1 \quad \rho = 0$

$f_{X_1}(x) = \frac{1}{\sqrt{2\pi}\sqrt{3}} e^{\frac{-(x-4)^2}{6}} \quad -\infty < x < \infty$

$f_{X_2}(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(y-2)^2}{2}} \quad -\infty < y < \infty$

$X = \frac{X_1 + X_2}{2} \qquad Y = \frac{X_1 - X_2}{2}$

$\frac{1}{2}X_1 + \frac{1}{2}X_2 \sim N\left(\frac{1}{2}(4) + \frac{1}{2}(2),\ \left(\frac{1}{2}\right)^2(3) + \left(\frac{1}{2}\right)^2\right)$

$\sim N(3, 1)$

$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-3)^2}{2}}$

$\frac{1}{2}X - \frac{1}{2}X_2 \sim N\left(\frac{1}{2}(4) - \frac{1}{2}(2),\ \left(\frac{1}{2}\right)^2(3) + \left(-\frac{1}{2}\right)^2\right)$

$\sim N(1, 1)$

$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-1)^2}{2}}$

4-1

1. $EX = \sum_{k=1}^{n} \left|(-1)^k\left(\frac{2^k}{k}\right)\right|\left(\frac{1}{2^k}\right) = \sum_{k=1}^{n} |(-1)^k|\left(\frac{1}{k}\right) = \sum_{k=1}^{n}\left(\frac{1}{k}\right)$ diverge

2. $EX = \int_{-\infty}^{\infty} |x| f(x)\,dx$

$= \int_{-\infty}^{\infty} \frac{|x|}{\pi(1+x^2)}\,dx$

$= \left[\frac{x\ln(x^2+1)}{2|x|}\right]_{-\infty}^{\infty}$

$\Rightarrow$ diverge

3. $P\{X = k\} = \frac{k}{\frac{n(n+1)}{2}} \qquad EX = \sum_{k=1}^{n} \frac{2k^2}{n(n+1)} = \frac{2}{n(n+1)} \sum_{k=1}^{n} k^2$

$= \frac{2}{n(n+1)} \cdot \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n+1}{3}$

4. $EX = \sum_{n=1}^{\infty} \frac{n}{2^n}$

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + \cdots = S$$

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{2}{2^3} + \frac{1}{2^4} + \frac{3}{2^4} + \cdots = S$$

$$\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right) + \frac{1}{2}\left(\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots\right) = S$$

$$\frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{1}{2}S = S$$

$$1 = \frac{1}{2}S$$

$$S = 2$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n-1} \qquad |x|<1$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}$$

Let $x = \frac{1}{2}$ $\qquad \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n-1} = \frac{1}{(1-\frac{1}{2})^2}$

$$\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = 4$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$

5. $EX = \frac{1}{2}\times 2 + \frac{1}{3}\times 1 - \frac{1}{6}\times 1 = \frac{7}{6}$元

6. $P\{X \le 1\} = \int_0^1 \frac{1}{4}e^{-\frac{x}{4}}\,dx = \left[-\left(\frac{1}{4}\right)(4)e^{-\frac{x}{4}}\right]_0^1$

$$= 1 - e^{-\frac{1}{4}}$$

$$P\{x>1\} = \int_1^{\infty} \frac{1}{4}e^{-\frac{x}{4}}\,dx = \left[-\left(\frac{1}{4}\right)(4)e^{-\frac{x}{4}}\right]_1^{\infty}$$

$$= e^{-\frac{1}{4}}$$

$$EY = 100 - 300(1 - e^{-\frac{1}{4}})$$

$= 33.64$(元)

7. $EX = \int_0^{\infty} \frac{1}{\theta} x e^{-\frac{x}{\theta}}\,dx$ $\qquad d(-e^{-\frac{x}{\theta}}) = -\left(\frac{-1}{\theta}\right)e^{-\frac{x}{\theta}}\,dx$

$$= \frac{1}{\theta}\int_0^{\infty} x e^{-\frac{x}{\theta}}\,dx \qquad \frac{\theta\, d(-e^{-\frac{x}{\theta}})}{e^{-\frac{x}{\theta}}}\ dx$$

$$= \frac{1}{\theta}\int_0^{\infty} \theta x\, e^{-\frac{x}{\theta}} \frac{d(-e^{-\frac{x}{\theta}})}{e^{-\frac{x}{\theta}}}$$

$$= \int_0^{\infty} x\,d(-e^{-\frac{x}{\theta}})$$

$$= \left[-xe^{-\frac{x}{\theta}}\right]_0^{\infty} + \int_0^{\infty} e^{-\frac{x}{\theta}}\,dx$$

$$= \int_0^{\infty} e^{-\frac{x}{\theta}}\,dx = \left[-\theta e^{-\frac{x}{\theta}}\right]_0^{\infty} = \theta$$

8. $EX = 0.75$

$$\int_0^1 x\, kx^\alpha dx = 0.75 \qquad \int_0^1 kx^\alpha dx = 1$$

$$k\int_0^1 x^{\alpha+1} dx = 0.75 \qquad k\int_0^1 x^\alpha dx = 1$$

$$k\left[\frac{x^{\alpha+2}}{\alpha+2}\right]_0^1 = 0.75 \qquad k\left[\frac{x^{\alpha+1}}{\alpha+1}\right]_0^1 = 1$$

$$\frac{k}{\alpha+2} = 0.75 \qquad \frac{k}{\alpha+1} = 1$$

$$0.75(\alpha+2) = \alpha+1$$

$$0.75\alpha + 1.5 = \alpha + 1 \qquad \Rightarrow k=3$$

$$0.5 = 0.25\alpha$$

$$\alpha = 2$$

9.

x^2	1	0	1	9
P	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$

$\Rightarrow$

x^2	0	1	9
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$EX^2 = \frac{1}{2} + \frac{9}{4} = \frac{11}{4}$$

$2X-1$	-3	-1	1	5
P	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$

$$E(2X-1) = E(2X) - 1$$
$$= 2EX - 1 = 1$$

10.

$X+Y$	-1	0	1	2
P	$\frac{1}{4}$	0	$\frac{3}{4}$	0

$$E(X+Y) = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$$

XY	-1	0	1
P	0	1	0

$$E(XY) = 0$$

X	-1	0	1
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$EX = 0$

$EY = \frac{1}{2}$

$\Rightarrow E(X+Y) = \frac{1}{2}$

Y	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

$\max(X,Y)$	0	1
P	$\frac{1}{4}$	$\frac{3}{4}$

$$E(\max(X,Y)) = \frac{3}{4}$$

11. (1)

$$EY = 2\int_0^\infty x e^{-x} dx$$
$$= 2\int_0^\infty x\, d(-e^{-x})$$
$$= 2\left(\left[-xe^{-x}\right]_0^\infty + \int_0^\infty e^{-x} dx\right)$$
$$= 2\int_0^\infty e^{-x} dx$$
$$= 2\left[-e^{-x}\right]_0^\infty$$
$$= 2$$

(2)

$$EY = \int_0^\infty e^{-2x} e^{-x} dx$$
$$= \int_0^\infty e^{-3x} dx$$
$$= \left[\frac{e^{-3x}}{-3}\right]_0^\infty$$
$$= \frac{1}{3}$$

12. $f_X(x) = \int_0^1 2x\,dy = [2xy]_0^1 = 2x$

$EX = \int_0^1 2x^2\,dx = \frac{2}{3}$

$f_Y(y) = \int_0^1 2x\,dx = 1$

Since $f(x,y) = f_X(x)\cdot f_Y(y)$ $\Rightarrow$ X, Y 独立

$EY = \int_0^1 y\,dy = \frac{1}{2}$

$E(XY) = EX \cdot EY = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$

13. $E(X+Y) = EX + EY$

$EY = 6$

$$= \int_0^1 x^2 dx + \int_1^2 x(2-x)dx + \int_5^\infty y e^{-y+5} dy$$
$$= \frac{1}{3} + \int_1^2 (2x - x^2) dx + \int_5^\infty y\, d(-e^{-y+5})$$
$$= 1 + [-ye^{-y+5}]_5^\infty + \int_5^\infty e^{5-y} dy$$
$$= 1 + [-e^{5-y}]_5^\infty + 5$$
$$= 1 + 6$$
$$= 7$$

14. X为索赔

X	0	1
P	0.99	0.01

$EX = 500 \times 8000 - 40000 \times 0.01 \times 8000$
$= 800000$ 元

15. $X_i = \begin{cases} 0 & \text{不停} \\ 1 & \text{停} \end{cases}$ $i = 1, 2, \cdots, 10$

$P\{X_i = 0\} = (0.9)^{20}$

$P\{X_i = 1\} = 1 - (0.9)^{20}$

$EX = E(X_1 + X_2 + X_3 + \cdots + X_{10}) = 10(1 - (0.9)^{20}) = 8.784$ 次

16. $P\{X=1\} = \frac{{}^2C_1 \times 3}{16} = \frac{3}{8}$

$P\{Y=2 \cap X=1\} = \frac{2}{16}$

$EY = 1 \times \frac{\frac{2}{16}}{\frac{3}{8}} = \frac{1}{3}$

17. $f_Y(y) = \int_0^1 (x+y)\,dx = \left[\frac{x^2}{2} + yx\right]_0^1 = \frac{1}{2} + y$

$f(x|y) = \frac{x+y}{\frac{1}{2}+y}$

$E(X|Y=\frac{1}{2}) = \int_0^1 (x^2 + \frac{x}{2})\,dx = \left[\frac{x^3}{3} + \frac{x^2}{4}\right]_0^1 = \frac{7}{12}$

4-2

1. $EX=DX=\lambda$ $\quad E((x-1)(x-2))=1$

$DX=EX^2-(EX)^2=\lambda$ $\quad E(x^2-3x+2)=1$

$EX^2=\lambda+\lambda^2$ $\quad EX^2-3EX+2=1$

$\lambda+\lambda^2-3\lambda+2=1$

$\lambda=1$

2. $EX=-2$ $\quad E(x^2)=5$ $\quad D(1-3x)$

$DX=5-4$ $\quad =9DX$

$=1$ $\quad =9$

3. $DX=E(x-EX)^2$ $\quad E(x-C)^2$

$=E(x-EX+EX-C)^2$

$=E(x-EX)^2+(EX-C)^2 \geqslant DX$

4. $EX = 0.01\times0.1+0.02\times0.1+0.03\times0.2+0.04\times0.3+0.05\times0.2+0.06\times0.1) = 3.7\%$

收入 $=10$万 $\times(1+3.7\%)=10.37$万

$DX = (0.01-0.037)^2(0.1)+(0.02-0.037)^2\times0.1+(0.03-0.037)^2\times0.2+(0.04-0.037)^2\times0.3$
$+(0.05-0.037)^2(0.2)+(0.06-0.037)^2\times0.1 = 0.0201\%$

5. $EX=900\times0.1+1000\times0.8+1100\times0.1=1000$

$EY=950\times0.3+1000\times0.4+1050\times0.3=1000$

$DX=EX^2-(EX)^2=2000$

$DY=1500$ ✓

6. $E(2x+1)=2EX+1=1.6$ $\quad E(3X^2+5)$

$EX=-2\times0.1-1\times0.2+1\times0.3+2\times0.2=0.3$ $\quad =3E(x^2)+5$

$DX=E(x^2)-(EX)^2=1.7-(0.3)^2=1.61$ $\quad =10.1$

7. $EX = \int_0^1 x^2 dx + \int_1^2 x(2-x)dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \int_1^2 (2x - x^2)dx$

$= \frac{1}{3} + \left[x^2 - \frac{x^3}{3}\right]_1^2$

$= 1$

$EX^2 = \int_0^1 x^3 dx + \int_1^2 x^2(2-x)dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \int_1^2 (2x^2 - x^3)dx$

$= \frac{1}{4} + \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2$

$= \frac{7}{6}$

$DX = \frac{7}{6} - 1 = \frac{1}{6}$

8. $EX = \int_{-\infty}^{\mu} \frac{x}{2\lambda} e^{-\frac{\mu-x}{\lambda}} dx + \int_{\mu}^{\infty} \frac{x}{2\lambda} e^{-\frac{x-\mu}{\lambda}} dx$

$= \int_{-\infty}^{\mu} \frac{x}{2\lambda} e^{\frac{x-\mu}{\lambda}} dx + \int_{\mu}^{\infty} \frac{x}{2\lambda} e^{\frac{\mu-x}{\lambda}} dx$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2\lambda} \int_{-\infty}^{\mu} x e^{\frac{x}{\lambda}} dx + \frac{e^{\frac{\mu}{\lambda}}}{2\lambda} \int_{\mu}^{\infty} x e^{-\frac{x}{\lambda}} dx$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2} \int_{-\infty}^{\mu} x \, d(e^{\frac{x}{\lambda}}) + \frac{e^{\frac{\mu}{\lambda}}}{2} \int_{\mu}^{\infty} x \, d(-e^{-\frac{x}{\lambda}})$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\left[x e^{\frac{x}{\lambda}}\right]_{-\infty}^{\mu} - \int_{-\infty}^{\mu} e^{\frac{x}{\lambda}} dx\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\left[-x e^{-\frac{x}{\lambda}}\right]_{\mu}^{\infty} + \int_{\mu}^{\infty} e^{-\frac{x}{\lambda}} dx\right)$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\mu e^{\frac{\mu}{\lambda}} - \lambda\left[e^{\frac{x}{\lambda}}\right]_{-\infty}^{\mu}\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\mu e^{-\frac{\mu}{\lambda}} - \lambda\left[e^{-\frac{x}{\lambda}}\right]_{\mu}^{\infty}\right)$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\mu e^{\frac{\mu}{\lambda}} - \lambda e^{\frac{\mu}{\lambda}}\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\mu e^{-\frac{\mu}{\lambda}} + \lambda e^{-\frac{\mu}{\lambda}}\right)$

$= \frac{\mu e^{-\frac{\mu}{\lambda}+\frac{\mu}{\lambda}}}{2} - \frac{\lambda}{2} e^{-\frac{\mu}{\lambda}+\frac{\mu}{\lambda}} + \frac{\mu e^{\frac{\mu}{\lambda}+\frac{-\mu}{\lambda}}}{2} + \frac{\lambda e^{\frac{\mu}{\lambda}-\frac{\mu}{\lambda}}}{2}$

$= \frac{\mu}{2} - \frac{\lambda}{2} + \frac{\mu}{2} + \frac{\lambda}{2} = \mu$

$DX = \mu^2 + 2\lambda^2 - \mu^2$

$= 2\lambda^2$

$EX^2 = \int_{-\infty}^{\mu} \frac{x^2}{2\lambda} e^{\frac{x-\mu}{\lambda}} dx + \int_{\mu}^{\infty} \frac{x^2}{2\lambda} e^{\frac{\mu-x}{\lambda}} dx$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2\lambda} \int_{-\infty}^{\mu} x^2 e^{\frac{x}{\lambda}} dx + \frac{e^{\frac{\mu}{\lambda}}}{2\lambda} \int_{\mu}^{\infty} x^2 e^{-\frac{x}{\lambda}} dx$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2} \int_{-\infty}^{\mu} x^2 d(e^{\frac{x}{\lambda}}) + \frac{e^{\frac{\mu}{\lambda}}}{2} \int_{\mu}^{\infty} x^2 d(-e^{-\frac{x}{\lambda}})$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\left[x^2 e^{\frac{x}{\lambda}}\right]_{-\infty}^{\mu} - \int_{-\infty}^{\mu} 2x e^{\frac{x}{\lambda}} dx\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\left[-x^2 e^{-\frac{x}{\lambda}}\right]_{\mu}^{\infty} + \int_{\mu}^{\infty} 2x e^{-\frac{x}{\lambda}} dx\right)$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{\frac{\mu}{\lambda}} - 2\lambda \int_{-\infty}^{\mu} x \, d(e^{\frac{x}{\lambda}})\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{-\frac{\mu}{\lambda}} + 2\lambda \int_{\mu}^{\infty} x \, d(-e^{-\frac{x}{\lambda}})\right)$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{\frac{\mu}{\lambda}} - 2\lambda\left[x e^{\frac{x}{\lambda}}\right]_{-\infty}^{\mu} + 2\lambda \int_{-\infty}^{\mu} e^{\frac{x}{\lambda}} dx\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{-\frac{\mu}{\lambda}} + 2\lambda\left[-x e^{-\frac{x}{\lambda}}\right]_{\mu}^{\infty} + 2\lambda \int_{\mu}^{\infty} e^{-\frac{x}{\lambda}} dx\right)$

$= \frac{e^{-\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{\frac{\mu}{\lambda}} - 2\lambda\mu e^{\frac{\mu}{\lambda}} + 2\lambda^2 e^{\frac{\mu}{\lambda}}\right) + \frac{e^{\frac{\mu}{\lambda}}}{2}\left(\mu^2 e^{-\frac{\mu}{\lambda}} + 2\lambda\mu e^{-\frac{\mu}{\lambda}} + 2\lambda^2 e^{\frac{\mu}{\lambda}}\right)$

$= \frac{\mu^2}{2} - \lambda\mu + \lambda^2 + \frac{\mu^2}{2} + \lambda\mu + \lambda^2 = \mu^2 + 2\lambda^2$

9. $EX_i = i \quad DX_i = 5-i$

$$EY = E(2X_1 - X_2 + 3X_3 - \tfrac{1}{2}X_4)$$
$$= 2EX_1 - EX_2 + 3EX_3 - \tfrac{1}{2}EX_4$$
$$= 2 - 2 + 9 - 2$$
$$= 7$$

$$DY = D(2X_1 - X_2 + 3X_3 - \tfrac{1}{2}X_4)$$
$$= D(2X_1) - DX_2 + D(3X_3) - D(\tfrac{1}{2}X_4)$$
$$= 4DX_1 + DX_2 + 9DX_3 + \tfrac{1}{4}DX_4$$
$$= 4(4) + 3 + 9(2) + \tfrac{1}{4}$$
$$= 37.25$$

$$\int_5^\infty y^2 e^{5-y}\,dy$$
$$= e^5 \int_5^\infty y^2 e^{-y}\,dy$$
$$= e^5 \int_5^\infty y^2\, d(-e^{-y})$$
$$= e^5 \left[-y^2 e^{-y}\right]_5^\infty + 2e^5 \int_5^\infty y e^{-y}\,dy$$
$$= 25 + 2e^5 \int_5^\infty y \; d(-e^{-y})$$
$$= 25 + 2e^5 \left(\left[-ye^{-y}\right]_5^\infty + \int_5^\infty e^{-y}\,dy \right)$$
$$= 25 + 2e^5 \left(5e^{-5} + e^{-5} \right)$$
$$= 37$$

10. $E(X+Y)$

$$= EX + EY$$
$$= \int_0^1 2x^2\,dx + \int_5^\infty y e^{5-y}\,dy$$
$$= \left[\frac{2x^3}{3}\right]_0^1 + \int_5^\infty y\, d(-e^{5-y})$$
$$= \frac{2}{3} + \left[-ye^{5-y}\right]_5^\infty + \int_5^\infty e^{5-y}\,dy$$
$$= \frac{2}{3} + 5 + \left[-e^{5-y}\right]_5^\infty$$
$$= \frac{2}{3} + 5 + 1 = \frac{20}{3}$$

$E(3X-2Y)$

$$= 3EX - 2EY$$
$$= 3\int_0^1 2x^2\,dx - 2\int_5^\infty y e^{5-y}\,dy$$
$$= 2 - 2\left[-ye^{5-y}\right]_5^\infty - 2\int_5^\infty e^{5-y}\,dy$$
$$= 2 - 10 - 2$$
$$= -10$$

$E(XY)$

$$= EX\,EY$$
$$= \int_0^1 2x^2\,dx \quad \int_5^\infty y\, d(-e^{5-y})$$
$$= \frac{2}{3} \cdot 6$$
$$= 4$$

$D(X-Y)$

$$= DX + DY$$
$$= EX^2 - (EX)^2 + EY^2 - (EY)^2$$
$$= \int_0^1 2x^3\,dx - \frac{4}{9} + \int_5^\infty y^2 e^{5-y}\,dy - 36$$
$$= \frac{1}{2} - \frac{4}{9} + 37 - 36$$
$$= \frac{19}{18}$$

11. $E\bar{X}$

$= E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$

$= \frac{1}{n}\sum_{i=1}^{n} EX_i$

$= \frac{1}{n} n\mu$

$= \mu$

$D\bar{X}$

$= D\left(\frac{X_1}{n} + \frac{X_2}{n} + \cdots + \frac{X_n}{n}\right)$

$= \frac{1}{n^2} D(X_1 + X_2 + \cdots + X_n)$

$= \frac{1}{n^2} n\sigma^2$

$= \frac{1}{n}\sigma^2$

ES^2

$= \frac{1}{n-1} E\left(\sum_{i=1}^{n}(X_i-\bar{X})^2\right)$

$= \frac{1}{n-1}\left(E\left((X_1-\bar{X})^2 + (X_2-\bar{X})^2 + \cdots + (X_n-\bar{X})^2\right)\right)$

$= \frac{1}{n-1}\left(E\left(X_1^2 - 2X_1\bar{X} + \bar{X}^2 + \cdots + X_n^2 - 2X_n\bar{X} + \bar{X}^2\right)\right)$

$= \frac{1}{n-1}\left(E\left(X_1^2 + X_2^2 + \cdots + X_n^2 - 2\bar{X}(X_1+X_2+\cdots+X_n) + n\bar{X}^2\right)\right)$

$= \frac{1}{n-1}\left(E\left(\sum_{i=1}^{n} X_i^2 - 2\sum_{i=1}^{n} X_i\bar{X} + \sum_{i=1}^{n}(\bar{X})^2\right)\right)$

$= \frac{1}{n-1}\left(\sum_{i=1}^{n} EX_i^2 - 2\bar{X}E\sum_{i=1}^{n} X_i + E\sum_{i=1}^{n}(\bar{X})^2\right)$

$= \frac{1}{n-1}\left(\sum_{i=1}^{n} EX_i^2 - 2nE(\bar{X})^2 + nE(\bar{X})^2\right)$

$= \frac{1}{n-1}\left(\sum_{i=1}^{n} EX_i^2 - nE(\bar{X})^2\right)$

$DX = EX_i^2 - (EX_i)^2$

$EX_i^2 = \sigma^2 + \mu^2$

$E(\bar{X})^2 = D\bar{X} + (E(\bar{X}))^2$

$= \frac{\sigma^2}{n} + \mu^2$

$ES^2 = \frac{1}{n-1}\left((n\sigma^2 + n\mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right) = \sigma^2$

4-3

1. $X_i = \begin{cases} 0 & 不是 \\ 1 & 是 \end{cases}$ $i=1,2,3$ $P\{X_1=1\}=\frac{1}{4}$ $EX_1=\frac{1}{4}$

$EX_i = EX_1+EX_2+EX_3=\frac{3}{4}$

2. $X_i = \begin{cases} 0 & 出 \\ 1 & 不出 \end{cases}$ $i=1,2,3,4\cdots,n$ $P\{X_1=0\}=0.3\times0.2=0.06$ $P\{X_1=1\}=1-0.06=0.94$

$EX_i = nEX_1$ $DX_1=(1-0.06)^2\times0.06+(0-0.06)^2\times0.94$

$=0.06n$ $=0.0564$

$DX_i = 0.0564n$

3. $P(X=k)={}^{10}C_k(0.4)^k(0.6)^{10-k}$

$EX=10\times0.4=4$ $DX=npq=10\times0.4\times0.6=2.4$

$DX=EX^2-(EX)^2$

$2.4=EX^2-16$

$EX^2=18.4$

4. $EX=\sum_{k=1}^{\infty}\frac{k}{2^k}$ $P\{X=k\}=\frac{1}{2^k}$ $k=1,2,\cdots$

$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots=S$ $=(\frac{1}{2})^k$

$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{2}{2^3}+\cdots=S$ $=(\frac{1}{2})^{k-1}(\frac{1}{2})$

$(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots)+\frac{1}{2}(\frac{1}{2}+\frac{2}{2^2}+\cdots)=S$ $DX=\frac{1-\frac{1}{2}}{(\frac{1}{2})^2}=2$

$\frac{\frac{1}{2}}{1-\frac{1}{2}}+\frac{1}{2}S=S$

$S=2$

5.

$P\{X=k\}=(\frac{2}{7})^{k-1}(\frac{5}{7})$ $EX=\frac{1}{\frac{5}{7}}=\frac{7}{5}$ $DX=\frac{1-\frac{5}{7}}{(\frac{5}{7})^2}=\frac{14}{25}$

6. $X\sim P(\lambda)$ $E(X^2-5X+6)=2$ $DX=\lambda$ $EX=\lambda$

$EX^2-5EX=-4$ $DX=EX^2-(EX)^2$

$\lambda+\lambda^2-5\lambda=-4$ $\lambda=EX^2-\lambda^2$

$\lambda=2$ $EX^2=\lambda+\lambda^2$

7. $f(x)=\frac{1}{\sqrt{\pi}}e^{-x^2+2x-1}$

$=\frac{1}{\sqrt{\pi}}e^{-(x-1)^2}$ $\qquad \phi(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$X\sim N(1,\frac{1}{2})$ $\qquad \sigma^2=\frac{1}{2}$

$EX=1 \quad DX=\frac{1}{2}$

8. $X\sim U[0,1] \quad Y\sim[0,1]$ $\qquad E(X+Y)$

$=EX+EY$

$=\frac{1}{2}+\frac{1}{2}=1$

9. $E(X+Y)$ $\qquad X\sim Exp(2) \quad Y\sim Exp(4)$ $\qquad D(X+Y)$

$=EX+EY$ $\qquad =DX+DY$

$=\frac{1}{2}+\frac{1}{4}$ $\qquad =\frac{1}{4}+\frac{1}{16}$

$=\frac{3}{4}$ $\qquad =\frac{5}{16}$

4-4

1. $Cov(X,Y)=E(XY)-EXEY$

$f_X(x)=\int_{-\infty}^{\infty}6xy^2\,dy$ $\qquad f_Y(y)=\int_{-\infty}^{\infty}6xy^2\,dx$ $\qquad EXY=\int_0^1\int_0^1 6x^2y^3\,dxdy$

$=6x\int_0^1 y^2\,dy$ $\qquad =6y^2\int_0^1 x\,dx$ $\qquad =\int_0^1 2y^3\,dy$

$=2x$ $\qquad =3y^2$ $\qquad =\frac{1}{2}$

$EX=\int_0^1 2x^2\,dx$ $\qquad EY=\int_0^1 3y^3\,dy$ $\qquad Cov(X,Y)=\frac{1}{2}-\frac{1}{2}=0$

$=\frac{2}{3}$ $\qquad =\frac{3}{4}$

2. $f_X(x)=\int_0^{\infty}e^{-x-y}\,dy$ $\qquad f_Y(y)=\int_0^{\infty}e^{-x-y}\,dx$ $\qquad E(XY)=\int_0^{\infty}\int_0^{\infty}xye^{-x-y}\,dx\,dy$

$=\left[-e^{-x-y}\right]_0^{\infty}$ $\qquad =\left[-e^{-x-y}\right]_0^{\infty}$ $\qquad =\int_0^{\infty}y\int_0^{\infty}x\,d(-e^{-x-y})\,dy$

$=e^{-x}$ $\qquad =e^{-y}$ $\qquad =\int_0^{\infty}y\left(\left[-xe^{-x-y}\right]_0^{\infty}+\int_0^{\infty}e^{-x-y}\,dx\right)dy$

$EX=\int_0^{\infty}xe^{-x}\,dx$ $\qquad EY=1$ $\qquad =\int_0^{\infty}ye^{-y}\,dy$

$=\int_0^{\infty}x\,d(-e^{-x})$ $\qquad =\int_0^{\infty}y\,d(-e^{-y})$

$=\left[-xe^{-x}\right]_0^{\infty}+\int_0^{\infty}e^{-x}\,dx$ $\qquad =\left[-ye^{-y}\right]_0^{\infty}+\int_0^{\infty}e^{-y}\,dy$

$=\left[-e^{-x}\right]_0^{\infty}$ $\qquad Cov(X,Y)=0$ $\qquad =\left[-e^{-y}\right]_0^{\infty}$

$=1$ $\qquad \rho=0$ $\qquad =1$

3. $f_X(x)=\int_{-\infty}^{\infty}\frac{1}{\pi}dy$ $f_Y(x)=\int_{-\infty}^{\infty}\frac{1}{\pi}dx$

$=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\pi}dy$ $=\frac{2\sqrt{1-y^2}}{\pi}$

$=\frac{2\sqrt{1-x^2}}{\pi}$

$EX=\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\frac{2x\sqrt{1-x^2}}{\pi}dx$ $EY=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{2y\sqrt{1-y^2}}{\pi}dy$ $E(XY)=0$

$=0$ $=0$

$Cov(X,Y)=0$ $\rho=0$ 不独立

4.

XY	-2	-1	0	1	2
P	0.3	0	0.6	0.1	0

$E(XY)=-0.6+0.1=-0.5$

X	1	2
P	0.3	0.7

Y	-1	0	1
P	0.3	0.6	0.1

$E(2X-Y)=2EX-EY=2(0.3+1.4)-(-0.3+0.1)=3.6$

$Cov(X,Y)=-0.5-(1.7)(-0.2)=-0.16$ $EX^2=0.3+2.8=3.1$ $EY^2=0.4$

$\sqrt{DX}=\sqrt{3.1-(1.7)^2}$ $\sqrt{DY}=\sqrt{0.4-(-0.2)^2}$ $\rho=-0.5819$ 不独立

$=\sqrt{0.21}$ $=0.6$

5. $E(3X^2-2XY+Y^2-3)$ $DX=EX^2-(EX)^2$ $DY=EY^2-(EY)^2$

$=3E(X^2)-2E(XY)+E(Y^2)-3$ $25=EX^2-4$ $36=EY^2-16$

$=3\times29-2\times20+52-3$ $EX^2=29$ $EY^2=52$

$=96$ $0.4=\frac{Cov(X,Y)}{5\times6}$ $Cov(X,Y)=E(XY)-EX\,EY$

$D(X+Y)$ $Cov(X,Y)=12$ $12=E(XY)-8$

$=DX+DY+2Cov(X,Y)$ $E(XY)=20$

$=25+36+24$ $D(X-Y)=DX+DY-2Cov(X,Y)$

$=85$ $=25+36-24$

$=37$

$D(2X+3Y+5)=D(2X+3Y)=D(2X)+D(3Y)+2Cov(2X,3Y)$

$=4DX+9DY+12Cov(X,Y)=568$

6. $Y=aX+b$ $EY=aEX+b$ $DY=D(aX+b)=a^2DX$

$$Cov(X,Y)=E((X-EX)(Y-EY))=E((X-EX)(aX-aEX))$$
$$=E(aX^2-aXEX-aXEX+a(EX)^2)$$
$$=aE(X^2-2XEX+(EX)^2)$$
$$=aE((X-EX)^2)=aDX$$
$$\rho=\frac{aDX}{\sqrt{a^2DX}\sqrt{DX}}=\frac{a}{|a|}=\begin{cases}1 & a>0\\ -1 & a<0\end{cases}$$

10. $DX=9$ $DY=2$ $Cov(X,Y)=3$ $D(2X+Y)$

$$=4DX+DY+4Cov(X,Y)$$
$$=50$$

9. $f(x,y)=\frac{1}{3\sqrt{3}\pi}e^{-\frac{8}{15}\left(\frac{x^2}{3}+\frac{xy}{4\sqrt{3}}+\frac{y^2}{4}\right)}$

8. $DX=1$ $DY=4$ $Cov(X,Y)=1$ $\rho=\frac{1}{2}$

$DX_1=D(X-2Y)$ $DX_2=D(2X-Y)$

$=DX+4DY-4Cov(X,Y)$ $=4DX+DY-4Cov(X,Y)$

$=13$ $=4$

$$Cov(X_1,X_2)=E(XY)-EXEY$$
$$=E((X-2Y)(2X-Y))-E(X-2Y)E(2X-Y)$$
$$=E(2X^2-XY-4XY+2Y^2)-E(X-2Y)E(2X-Y)$$
$$=2EX^2-5EXY+2EY^2-(EX-2EY)(2EX-EY)$$
$$=2EX^2-5EXY+2EY^2-2(EX)^2+5EXEY-2(EY)^2$$
$$=2EX^2+2EY^2-2(EX)^2-2(EY)^2-5(EXY-EXEY)$$
$$=2(EX^2-(EX)^2)+2(EY^2-(EY)^2)-5Cov(X,Y)$$
$$=2DX+2DY-5Cov(X,Y)$$
$$=2+8-5=5$$
$$\rho=\frac{5}{2\sqrt{13}}$$

7. $DX_1 = D(aX+b)$ $\qquad DX_2 = D(cY+d)$

$= a^2 DX$ $\qquad = c^2 DY$

$$\begin{aligned}
Cov(X_1,Y_1) &= E(X_1Y_1) - EX_1EY_1 \\
&= E((aX+b)(cY+d)) - E(aX+b)E(cY+d) \\
&= E(acXY + adX + bcY + bd) - ((aEX+b)(cEY+d)) \\
&= acE(XY) + adEX + bcEY + bd - acEXEY - adEX - bcEY - bd \\
&= ac\, Cov(X,Y)
\end{aligned}$$

$$\begin{aligned}
\rho_{X_1,Y_1} &= \frac{ac\,Cov(X,Y)}{\sqrt{a^2DX}\sqrt{c^2DY}} \\
&= \frac{ac}{|ac|}\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} \\
&= \frac{ac}{|ac|}\rho
\end{aligned}$$

4-5.

1. $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{else} \end{cases}$

$$\mu_3 = \int_a^b \left(x - \frac{a+b}{2}\right)^3 \left(\frac{1}{b-a}\right) dx$$

Let $u = x - \frac{a+b}{2}$

$du = dx$

$$= \frac{1}{b-a}\int_{-\frac{b-a}{2}}^{\frac{b-a}{2}} u^3\, du$$

$= 0$

$$\begin{aligned}
V_k &= EX^k \\
&= \int_a^b \frac{x^k}{b-a} dx \\
&= \left[\frac{x^{k+1}}{(k+1)(b-a)}\right]_a^b \\
&= \frac{b^{k+1} - a^{k+1}}{(k+1)(b-a)} \\
&= \frac{1}{k+1}(b^k + b^{k-1}a + \cdots + ba^{k-1} + a^k)
\end{aligned}$$

2. $V_1 = EX$ $\qquad V_2 = EX^2$ $\qquad V_3 = EX^3$

$= \int_0^2 \frac{1}{2}x^2 dx$ $\qquad = \int_0^2 \frac{1}{2}x^3 dx$ $\qquad = \int_0^2 \frac{1}{2}x^4 dx$

$= \left[\frac{x^3}{6}\right]_0^2$ $\qquad = \left[\frac{x^4}{8}\right]_0^2$ $\qquad = \left[\frac{x^5}{10}\right]_0^2$

$= \frac{4}{3}$ $\qquad = 2$ $\qquad = 3.2$

$$\mu_1 = \int_0^2 (x-\tfrac{4}{3})(\tfrac{1}{2}x)\,dx$$
$$= \int_0^2 (\tfrac{1}{2}x^2 - \tfrac{2}{3}x)\,dx$$
$$= \left[\frac{x^3}{6} - \frac{x^2}{3}\right]_0^2$$
$$= 0$$

$$\mu_2 = \int_0^2 (x-\tfrac{4}{3})^2 (\tfrac{1}{2}x)\,dx$$
$$= \int_0^2 (x^2 - \tfrac{8}{3}x + \tfrac{16}{9})(\tfrac{1}{2}x)\,dx$$
$$= \int_0^2 (\tfrac{x^3}{2} - \tfrac{4}{3}x^2 + \tfrac{8}{9}x)\,dx$$
$$= \left[\frac{x^4}{8} - \frac{4x^3}{9} + \frac{4x^2}{9}\right]_0^2$$
$$= \frac{2}{9}$$

$$\mu_3 = \int_0^2 (x-\tfrac{4}{3})^3 (\tfrac{1}{2}x)\,dx$$
$$= \int_0^2 (\tfrac{x^3}{2} - \tfrac{4}{3}x^2 + \tfrac{8}{9}x)(x - \tfrac{4}{3})\,dx$$
$$= \int_0^2 (\tfrac{x^4}{2} - \tfrac{4}{3}x^3 + \tfrac{8}{9}x^2 - \tfrac{2}{3}x^3 + \tfrac{16}{9}x^2 - \tfrac{32}{27}x)\,dx$$
$$= \int_0^2 (\tfrac{x^4}{2} - 2x^3 + \tfrac{8}{3}x^2 - \tfrac{32}{27}x)\,dx$$
$$= \left[\frac{x^5}{10} - \frac{x^4}{2} + \frac{8x^3}{9} - \frac{16}{27}x^2\right]_0^2 = \frac{-8}{135}$$

3. $V_1 = EX \qquad V_2 = EX^2 \qquad V_3 = EX^3$

$$\mu_3 = E(x-EX)^3$$
$$= E\left((x^2 - 2X\,EX + (EX)^2)(x - EX)\right)$$
$$= E\left(x^3 - 2X^2 EX + X(EX)^2 - x^2 EX + 2x(EX)^2 - (EX)^3\right)$$
$$= EX^3 - 2EX^2 EX + (EX)^3 - EX^2 EX + 2(EX)^3 - (EX)^3$$
$$= V_3 - 2V_2V_1 + V_1^3 - V_2V_1 + 2V_1^3 - V_1^3$$
$$= V_3 - 3V_2V_1 + 2V_1^3$$

$X \sim Exp(\lambda)$ $\qquad v_1 = EX = \frac{1}{\lambda}$ $\qquad V_2 = EX^2$ $\qquad V_3 = EX^3$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x>0 \\ 0 & \text{else} \end{cases}$$

$$V_2 = \lambda \int_0^\infty x^2 e^{-\lambda x}\,dx = \frac{2}{\lambda^2}$$

$$V_3 = \lambda \int_0^\infty x^3 e^{-\lambda x}\,dx = \frac{6}{\lambda^3}$$

$$\mu_3 = \frac{6}{\lambda^3} - \frac{6}{\lambda^3} + \frac{2}{\lambda^3}$$
$$= \frac{2}{\lambda^3}$$

四

1.(1) $EX = 0.8\times2+0.2\times5=2.6$ $2.6\times3=7.8$元

(2) $np=6$ $npq=3.6$ $p=0.4$ $n=15$

(3) $DX=4$ $P(X=1)=\frac{2}{1}e^{-2}=2e^{-2}$

(4) $E(-2x+3)=-2EX+3=-2(4)+3=-5$ $D(-2X+3)=4DX=\frac{16}{3}$

(5) $DX=4$

(6) $X\sim Exp(1)$ $E(x+e^{-2X})=EX+E(e^{-2X})=1+\int_0^\infty e^{-3x}dx=1+\left[\frac{e^{-3x}}{-3}\right]_0^\infty=\frac{4}{3}$

(7) $D(3X-2Y)=9DX+4DY=36+8=44$

(8) $P\{X=0\}=0.4+a$ $P\{X+Y=1\}=a+b$ $P\{X=0\cap X+Y=1\}=a$ $b=0.1$

$0.4+a+0.1+b=1$

$a+b=0.5$

$(0.4+a)(a+b)=a$

$0.5(0.4+a)=a$

$0.2+0.5a=a$

$a=0.4$

XY	0	1
P	0.9	0.1

$E(XY)=0.1$

(9) $Cov(X^2,Y^2)=E(X^2Y^2)-EX^2EY^2=0.28-0.6\times0.5=-0.02$

X^2Y^2	0	1
P	0.72	0.28

(10) $\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}}=0.9$

$\frac{Cov(Y,X-0.4)}{\sqrt{DY}\sqrt{DZ}}$

$=0.9$

$DZ=D(X-0.4)$

$=DX$

$Cov(Y,X-0.4)$

$=E(XY-0.4Y)-EY(EX-0.4)$

$=E(XY)-0.4EY-EXEY+0.4EY$

$=Cov(X,Y)$

(11) $\frac{Cov(X,Y)}{4\times3}=0.2$

$Cov(X,Y)=2.4$

$D(X-Y)$

$=DX+DY-2Cov(X,Y)$

$=16+9-4.8$

$=20.2$

(1) $EX=\sum_{n=1}^{\infty}\frac{1}{n+1}$ D

(2) $E(2X+1)=2EX+1$ A,B

$=\int_0^{\infty}xe^{-\frac{1}{2}x}dx+1$

$=2\int_0^{\infty}x\,d(-e^{-\frac{1}{2}x})+1$

$=2\left[-xe^{-\frac{1}{2}x}\right]_0^{\infty}+2\int_0^{\infty}e^{-\frac{1}{2}x}dx+1$

$=5$

$EX=\frac{1}{2}\int_0^{\infty}xe^{-\frac{1}{2}x}dx$

$=\int_0^{\infty}x\,d(-e^{-\frac{1}{2}x})$

$=\left[-xe^{-\frac{1}{2}x}\right]_0^{\infty}+\int_0^{\infty}e^{-\frac{1}{2}x}dx$

$=\left[-2e^{-\frac{1}{2}x}\right]_0^{\infty}$

$=2$

$D(2X+1)$

$=4DX=16$

$EX^2=\frac{1}{2}\int_0^{\infty}x^2e^{-\frac{1}{2}x}dx$

$=\int_0^{\infty}x^2d(-e^{-\frac{1}{2}x})$

$=\left[-x^2e^{-\frac{1}{2}x}\right]_0^{\infty}+2\int_0^{\infty}xe^{-\frac{1}{2}x}dx$

$=4\int_0^{\infty}x\,d(-e^{-\frac{1}{2}x})$

$=4\left[-xe^{-\frac{1}{2}x}\right]_0^{\infty}+4\int_0^{\infty}e^{-\frac{1}{2}x}dx$

$=4\left[-2e^{-\frac{1}{2}x}\right]_0^{\infty}$

$=8$

(3) $\phi(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ A

(4) B, D

(5) A, B 不相关 $\Rightarrow \rho=0\Rightarrow\frac{Cov(X,Y)}{\sqrt{DXDY}}=0\Rightarrow EXY=EXEY$

(6) B $DX+DY+2Cov(X,Y)=DX+DY-2Cov(X,Y)\Rightarrow Cov(X,Y)=0\Rightarrow$ 不相关

(7) C

(8) B, D

(9) B $Cov(X_1,Y)=Cov(X_1,\frac{1}{n}\sum_{i=1}^{n}X_i)=\frac{1}{n}Cov(X_1,\sum_{i=1}^{n}X_i)=cov(X_1,\frac{1}{n}X_1)+Cov(X_1,\frac{1}{n}\sum_{n=2}^{n}X_i)$

$=\frac{\sigma^2}{n}$

(10) $Y=n-X$ $EY=n-EX$ $DY=DX$ $\rho=\frac{Cov(X,Y)}{DX}=\frac{E((X-EX)(Y-EY))}{DX}$ D

$=\frac{E((X-EX)(n-X-n+EX))}{DX}$

$=\frac{E((X-EX)(EX-X))}{DX}$

$=\frac{-DX}{DX}=-1$

(11) A, B, C, D

$\sum_{k=0}^{\infty}EX^2$

$=\sum_{k=1}^{n}\frac{k\lambda^k}{(k-1)!}e^{-\lambda}$

$=\sum_{k=1}^{\infty}\frac{(k-1)\lambda^k}{(k-1)!}e^{-\lambda}+\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!}e^{-\lambda}$

$=\lambda^2\sum_{k-2=0}^{\infty}\frac{\lambda^{k-2}}{(k-2)!}e^{-\lambda}+\lambda\sum_{k-1=0}^{\infty}\frac{\lambda^{k-1}}{(k-1)!}e^{-\lambda}$

$=\lambda^2+\lambda$

3. $P\{X=0\} = 0.9\times0.8\times0.7 = 0.504$ $\quad P\{X=1\} = 0.1\times0.8\times0.7 + 0.9\times0.2\times0.7 + 0.9\times0.8\times0.3 = 0.398$

$P\{X=2\} = 0.1\times0.2\times0.7 + 0.1\times0.3\times0.8 + 0.2\times0.3\times0.9 + 0.9\times0.2\times0.3 = 0.092$

$P\{X=3\} = 0.1\times0.2\times0.3 = 0.006$

$EX = 0.398 + 2\times0.092 + 3\times0.006 = 0.6$

$DX = 1\times0.398 + 4\times0.092 + 9\times0.006 = 0.46$

4. $EX = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x}{\pi}\cos^2 x\,dx$ odd function $\quad DX = \frac{\pi^2-6}{12}$

$= 0$

$EX^2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2x^2}{\pi}\cos^2 x\,dx$

$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}} x^2\cos^2 x\,dx$

$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} (x^2\cos 2x + x^2)\,dx$

$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx + \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} x^2\ d(\sin 2x) + \frac{\pi^2}{3}$

$= \frac{1}{\pi}\left[x^2\sin 2x\right]_0^{\frac{\pi}{2}} - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} 2x\sin 2x\,dx + \frac{\pi^2}{12}$

$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} x\ d(\cos 2x) + \frac{\pi^2}{3}$

$= \frac{1}{\pi}\left[x\cos 2x\right]_0^{\frac{\pi}{2}} - \frac{1}{\pi}\int \cos 2x\,dx + \frac{\pi^2}{12}$

$= \frac{-1}{2} - \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}\left(\frac{1}{\pi}\right) + \frac{\pi^2}{12}$

$= \frac{\pi^2-6}{12}$

5. $EX = \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + 1$ $\quad EX^2 = \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5} + 5$

$= 3$ $\quad = 11$

$E(X+2)^2 = E(x^2+4x+4)$ $\quad DX = 11-9$

$= EX^2 + 4EX + 4$ $\quad = 2$

$= 11 + 12 + 4$

$= 27$

6. $X \sim U[-\frac{1}{2}, \frac{1}{2}]$ $f(x) = \begin{cases} 1 & -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & \text{else} \end{cases}$

$$EY = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sin \pi x \, dx$$
$$= \left[\frac{-\cos \pi x}{\pi} \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$
$$= 0$$

7. (1) $\int_0^2 ax^2 dx + \int_2^4 (cx^2 + bx) dx = 2$

$$a\left[\frac{x^3}{3}\right]_0^2 + c\left[\frac{x^3}{3}\right]_2^4 + b\left[\frac{x^2}{2}\right]_2^4 = 2$$
$$\frac{8}{3}a - \frac{8}{3}c + \frac{64}{3}c + 8b - 2b = 2$$
$$\frac{8}{3}a + 6b + \frac{56}{3}c = 2$$

$$\int_0^2 ax\,dx + \int_2^4 (cx+b)\,dx = 1$$
$$a\left[\frac{x^2}{2}\right]_0^2 + \left[\frac{cx^2}{2}\right]_2^4 + \left[bx\right]_2^4 = 1$$
$$2a + 8c - 2c + 2b = 1$$
$$2a + 2b + 6c = 1$$

$$\int_1^2 ax\,dx + \int_2^3 (cx+b)\,dx = \frac{3}{4}$$
$$a\left[\frac{x^2}{2}\right]_1^2 + \left[\frac{cx^2}{2}\right]_2^3 + \left[bx\right]_2^3 = \frac{3}{4}$$
$$\frac{3}{2}a + \frac{9}{2}c - 2c + b = \frac{3}{4}$$
$$\frac{3}{2}a + b + \frac{5}{2}c = \frac{3}{4}$$

$a = \frac{1}{4}$ $b = 1$ $c = -\frac{1}{4}$

(2) $Ee^x = \frac{1}{4}\int_0^2 xe^x dx + \int_2^4 (-\frac{1}{4}xe^x + e^x) dx$

$$= \frac{e^2+1}{4} + \frac{e^4-3e^2}{4}$$
$$= \frac{e^4-2e^2+1}{4}$$
$$= \frac{(e^2-1)^2}{4}$$

$Ee^{2x} = \frac{1}{4}\int_0^2 xe^{2x} dx + \int_2^4 (-\frac{1}{4}xe^{2x} + e^{2x}) dx$

$$= \frac{3e^4+1}{16} + \frac{e^8-5e^4}{16}$$
$$= \frac{e^8-2e^4+1}{16}$$
$$= \frac{(e^4-1)^2}{16}$$

$$DX = Ee^{2x} - (Ee^x)^2$$
$$= \frac{(e^4-1)^2}{16} - \frac{(e^2-1)^4}{16}$$
$$= \frac{((e^2-1)(e^2+1))^2 - (e^2-1)^4}{16}$$
$$= \frac{(e^2-1)^2((e^2+1)^2 - (e^2-1)^2)}{16}$$
$$= \frac{(e^2-1)^2(e^4+2e^2+1-e^4+2e^2-1)}{16}$$
$$= \frac{e^2(e^2-1)^2}{4}$$

8. $\lambda>0 \Rightarrow e^{\lambda x}$ strictly increases

$P(X\geqslant a) = P(e^{\lambda X}\geqslant e^{\lambda a}) = P(Y\geqslant e^{\lambda a})$

$P(X\geqslant a)\leq e^{-\lambda a}EY \Leftrightarrow e^{\lambda a}P(Y\geqslant e^{\lambda a}) \leq EY$

We only have to prove that if $Y\geqslant 0$, $\forall b\geqslant 0$ $bP(Y\geqslant b)\leq EY$

$Y' = \begin{cases} b & Y\geqslant b \\ 0 & Y<b \end{cases} \Rightarrow Y\geqslant Y'$ Therefore $E(Y)\geqslant E(Y') = bP(Y'=b) = b(Y\geqslant b)$

9. $X\in[0,\infty)$ $P(X<x)\geqslant 1-\frac{EX}{x}$

$P(X<x)\geqslant \frac{x-EX}{x}$

$xP(X<x)\geqslant x-EX$

$EX\geqslant x-xP(X<x)$

$EX\geqslant x(1-P(X<x))$

$EX\geqslant xP(X\geqslant x)$

$xP(X\geqslant x)\leqslant EX$

$EX=\int_0^\infty x f(x)\,dx$

$xP(X\geqslant x) = \int_x^\infty x f(x)\,dx$

$\Rightarrow xP(X\geqslant x)\leqslant EX$

$\Rightarrow P(X<x)\geqslant 1-\frac{EX}{x}$

10. X Y independent DX, DY exist

$D(XY) = E(X^2Y^2) - E(XY)$

$= EX^2EY^2 - (EX)^2(EY)^2$

$= (DX+(EX)^2)(DY+(EY)^2) - (EX)^2(EY)^2$

$= DXDY + (EX)^2DY + (EY)^2DX + (EY)^2(EX)^2 - (EX)^2(EY)^2$

$= DXDY + (EX)^2DY + (EY)^2DX$

11. $X\sim U[1,2]$ $Y\sim U[1,2]$ $Z=\max\{X,Y\}$

when $1<Z<2$ $F(Z) = P\{Z\leqslant z\} = P\{\max\{X,Y\}\leqslant z\} = P(X\leqslant z, Y\leqslant z)$

$= P(X\leqslant z)P(Y\leq z) = \int_1^z dx \int_1^z dy$

$= (z-1)^2$

$f(z) = 2(z-1)$

$EZ = 2\int_1^2 (z^2-z)\,dz$

$= 2\left[\frac{z^3}{3}-\frac{z^2}{2}\right]_1^2$

$= \frac{5}{3}$

12. (1)

X	-1	0	1	2
P	0.1	0.2	0.3	0.4

Y	1	2
P	0.5	0.5

(2) 不独立

(3) $Cov(X,Y) = E(XY) - EXEY$

XY	-2	-1	0	1	2	4
P	0	0.1	0.2	0.2	0.2	0.3

$= -0.1+0.2+0.4+1.2-(\quad 1 \quad)(0.5+1)$

$= 1.7-1.5$

$= 0.2$

13. $Dra=16 \quad Drb=9 \quad Cov(ra,rb)=6$

(1) $\rho = \frac{6}{4\times 3} = \frac{1}{2}$

$rp = xra+(1-x)rb$

$= xra-xrb+rb$

(2) $Drp = D(xra-xrb+rb)$

$= D(xra+(1-x)rb)$

$= D(xra)+D((1-x)rb) + 2Cov(xra, (1-x)rb)$

$= x^2Dra + Drb+Dxrb - 2Cov(rb, xrb) + 2Cov(xra, (1-x)rb)$

$= x^2Dra + Drb + x^2Drb - 2xDrb + 2(x-x^2)Cov(ra, rb)$

$= 16x^2+9+9x^2-18x+12x-12x^2$

$= 13x^2-6x+9$

(3) $Drp = 13x^2-6x+9$

$= 13(x^2-\frac{6}{13}x)+9$

$= 13(x^2-\frac{6}{13}x+(\frac{6}{26})^2) - (\frac{6}{26})^2(13)+9$

$= 13(x-\frac{6}{26})^2 - 9$

$x=\frac{3}{13} \Rightarrow Drp \ \min.$

$Drp \le 9$

$13x^2-6x \le 0$

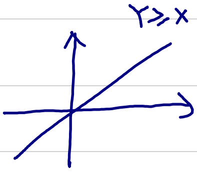
$\Rightarrow 0 \le x \le \frac{6}{13}$

19. $f(x,y) = \begin{cases} \frac{1}{2} & (x,y) \in G \\ 0 & \text{else} \end{cases}$

U\V	0	1
0	$\frac{1}{4}$	0
1	$\frac{1}{4}$	$\frac{1}{2}$

$-Y \le -X$

$Y \ge X$



$\int_0^1 \int_x^1 \frac{1}{2}\, dy\, dx$

$= \int_0^1 \left[\frac{1}{2}y\right]_x^1 dx$

$= \int_0^1 \left(\frac{1}{2} - \frac{1}{2}x\right) dx$

$= \left[\frac{x}{2} - \frac{x^2}{4}\right]_0^1$

$= \frac{1}{4}$

$\int_0^1 \int_0^{2y} \frac{1}{2}\, dx\, dy$

$= \int_0^1 \left[\frac{x}{2}\right]_0^{2y} dy$

$= \left[\frac{y^2}{2}\right]_0^1$

$= \frac{1}{2}$

U	0	1
P	$\frac{1}{4}$	$\frac{3}{4}$

V	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

UV	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

$Cov(U,V) = EUV - EUEV$

$= \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}$

$= \frac{1}{8}$

$DU = EU^2 - (EU)^2$

$= \frac{3}{4} - \left(\frac{3}{4}\right)^2$

$= \frac{3}{16}$

$DV = \frac{1}{2} - \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

$\rho = \dfrac{\frac{1}{8}}{\sqrt{\frac{1}{4}}\sqrt{\frac{3}{16}}}$

$= \frac{1}{\sqrt{3}}$

18.(1) $EZ = E\left(\frac{X}{3} + \frac{Y}{2}\right)$

$= \frac{1}{3}EX + \frac{1}{2}EY$

$= \frac{1}{3}$

$DZ = D\left(\frac{X}{3} + \frac{Y}{2}\right) + \frac{1}{3}Cov(X,Y)$

$= \frac{1}{9}DX + \frac{1}{4}DY - 2$

$= \frac{1}{9}(9) + \frac{1}{4}(16) - 2$

$= 3$

$-\frac{1}{2} = \frac{Cov(X,Y)}{3 \times 4}$

$Cov(X,Y) = -6$

$E(XY) - EXEY = -6$

$E(XY) = -6$

(2) $Cov(X,Z) = Cov\left(X, \frac{X}{3} + \frac{Y}{2}\right)$

$= \frac{1}{3}Cov(X,X) + \frac{1}{2}Cov(X,Y)$

$= \frac{1}{3}(9) - 3$

$= 0$

$\rho = 0$

15. (1) $Er = -0.03(0.015+0.025+0.06)+\cdots+0.07(0.015+0.025) = 2.755\%$

(2) $E(r \mid rf=1.5\%) = 2(-3\% \times 0.025 + \cdots + 7\% \times 0.025) = 3\%$

$P\{rf=1.5\%\} = 0.5$

14. 比例 $x, 1-x$ $\quad r_p = x r_a + (1-x) r_b$

$$Dr_p = x^2 Dr_a + (1-x)^2 Dr_b + 2x(1-x)\sqrt{Dr_a Dr_b}\,\rho_{ab}$$

$$= (x\sqrt{D_A} + \rho_{AB}(1-x)\sqrt{D_B})^2 + (1-\rho_{AB}^2)(1-x)^2 Dr_b$$

When $x=1$ $\quad Dr_p = Dr_a > 0$

When $x \neq 1$ $\quad |\rho_{AB}| \neq 1 \Rightarrow |\rho_{AB}| < 1$ and $Dr_b > 0$

$$Dr_p \geqslant (1-x)^2 Dr_b (1-\rho_{AB}^2) > 0$$

When $|\rho_{AB}| = 1$, $\rho_{AB} = 1$

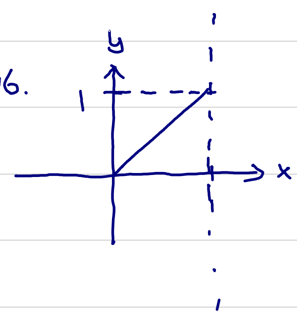
$$Dr_p = (x\sqrt{D_A} + (1-x)\sqrt{D_B})^2 = 0$$

$$\Rightarrow x = \frac{\sqrt{D_A}}{\sqrt{D_B}-\sqrt{D_A}}, \quad 1-x = \frac{-\sqrt{D_A}}{\sqrt{D_B}-\sqrt{D_A}}$$

When $\rho_{AB} = -1$ $\quad Dr_p = (x\sqrt{D_A} - (1-x)\sqrt{D_B})^2 = 0$

$$\Rightarrow x = \frac{\sqrt{D_B}}{\sqrt{D_A}+\sqrt{D_B}}, \quad 1-x = \frac{\sqrt{D_A}}{\sqrt{D_A}+\sqrt{D_B}}$$

16.



$x < y$

$-y < -x$

$y > x$

$$\text{Area} = \int_0^1 \int_x^1 dy\,dx = \int_0^1 (1-x)\,dx = \left[x - \frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

$$EX = \int_0^1 2x(1-x)\,dx = \int_0^1 2x - 2x^2\,dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$f(x,y) = \begin{cases} 2 & (x,y) \in D \\ 0 & \text{else} \end{cases}$$

$$EY = \int_0^1 2y^2\,dy = \frac{2}{3}$$

$$f_X(x) = \int_x^1 2\,dy = 2(1-x)$$

$$f_Y(x) = \int_0^y 2\,dx = 2y$$

$$EXY = \int_0^1 \int_x^1 2xy\,dy\,dx = \int_0^1 x[y^2]_x^1\,dx = \int_0^1 x(1-x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

$$Cov(X,Y) = \frac{1}{4} - \frac{2}{9} = \frac{1}{36}$$

$$EX^2 = \int_0^1 2x^2(1-x)\,dx = \int_0^1 2x^2 - 2x^3\,dx = \left[\frac{2x^3}{3} - \frac{x^4}{2}\right]_0^1 = \frac{1}{6}$$

$DX = \frac{1}{6} - (\frac{1}{3})^2 = \frac{1}{18}$

$EY^2 = \int_0^1 2y^3\,dy = [\frac{y^4}{2}]_0^1 = \frac{1}{2}$

$DY = \frac{1}{2} - (\frac{2}{3})^2 = \frac{1}{18}$

$\rho = \frac{\frac{1}{36}}{\sqrt{\frac{1}{18}}\sqrt{\frac{1}{18}}} = \frac{1}{2}$

17. $f(x,y) = \begin{cases} 8xy & 0 \le x \le y \le 1 \\ 0 & \text{else} \end{cases}$

$f_X(x) = \int_x^1 8xy\,dy \quad 0 \le x \le 1$
$= [4xy^2]_x^1$
$= 4x(1-x^2)$
$= 4x - 4x^3$

$f_Y(x) = \int_0^y 8xy\,dx \quad 0 \le y \le 1$
$= [4x^2y]_0^y$
$= 4y^3$

$E(X^2) = \int_0^1 (4x^3 - 4x^5)\,dx$
$= 1 - \frac{2}{3}$
$= \frac{1}{3}$

$EX = \int_0^1 (4x^2 - 4x^4)\,dx$
$= \frac{4}{3} - \frac{4}{5}$
$= \frac{8}{15}$

$Var(X) = \frac{1}{3} - (\frac{8}{15})^2 = \frac{11}{225}$

$E(Y) = \int_0^1 4y^4\,dy = \frac{4}{5}$

$E(Y^2) = \int_0^1 4y^5\,dy = \frac{2}{3}$

$Var(Y) = E(Y^2) - (E(Y))^2 = \frac{2}{75}$

$EXY = \int_0^1 \int_0^y 8x^2y^2\,dx\,dy$
$= \int_0^1 (\frac{8}{3}y^2)y^3\,dy$
$= \frac{8}{3} \times \frac{1}{6}$
$= \frac{4}{9}$

$Cov(X,Y) = EXY - EXEY$
$= \frac{4}{9} - \frac{8}{15} \times \frac{4}{5}$
$= \frac{4}{225}$

$\rho_{XY} = \frac{\frac{4}{225}}{\sqrt{\frac{11}{225} \times \frac{2}{75}}} = \frac{2\sqrt{66}}{33}$

5-1.

1. $P(|X-EX|\geqslant 2) \leqslant \frac{2}{4} = \frac{1}{2}$

2. $P(|\sum_{i=1}^{n} X_i| \geqslant n) = P(|X_1+X_2+\cdots+X_n| \geqslant n) \leqslant \frac{n}{n^2} = \frac{1}{n}$

3. $X \sim B(n,p)$ $\quad EX = np = 490$ $\quad DX = 4900 \times 0.1 \times 0.9 = 441$

$P(400 \leqslant X \leqslant 600) = P(400-490 \leqslant X-\mu \leqslant 600-490) = P(-90 \leqslant X-\mu \leqslant 110) \geqslant P(-90 \leqslant X-\mu \leqslant 90) = P(|X-\mu| \leqslant 90)$

$\geqslant 1 - \frac{441}{90^2}$

$= 0.9456$

4. $X \sim B(n,p)$ $\quad EX = 10000 \times 0.7 = 7000$ $\quad DX = 10000 \times 0.7 \times 0.3 = 2100$

$P(|X-7000| \leqslant 200) \geqslant 1 - \frac{2100}{200^2} = 0.9475$

5. $X \sim B(1000, 0.5)$ $\quad EX = 500$ $\quad DX = 250$

$P(|X-500| \leqslant 50) \geqslant 1 - \frac{250}{50^2} = 0.9$

6. $P\{|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n} EX_i| < \varepsilon\} \geqslant 1 - \frac{M}{n\varepsilon^2}$ $\quad D(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n^2}\sum_{i=1}^{n} DX_i \leqslant \frac{M}{n}$

$1 - \frac{D(\frac{1}{n}\sum_{i=1}^{n} X_i)}{n\varepsilon^2} \leqslant P\{|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n} EX_i| < \varepsilon\} \leqslant 1$

Since $\frac{1}{n^2} D(\sum_{i=1}^{n} X_i) \to 0 \quad (n \to \infty)$

$\Rightarrow \lim_{n\to\infty} \left(1 - \frac{D(\frac{1}{n}\sum_{i=1}^{n} X_i)}{n\varepsilon^2}\right) = 1$

By squeeze theorem

$\lim_{n\to\infty} P\{|\frac{1}{n}\sum_{i=1}^{n} X_i - \frac{1}{n}\sum_{i=1}^{n} EX_i| < \varepsilon\} = 1$

7. $EX = \sum_{k=1}^{\infty} \frac{1}{2^k}\left(\frac{2^k}{k^2}\right)$

$= \sum_{k=1}^{\infty} \frac{1}{k^2}$

$= 1 + \frac{1}{4} + \frac{1}{9} + \cdots$

Converges

$EX^2 = \sum_{k=1}^{\infty} \left(\frac{2^k}{k^2}\right)^2 \left(\frac{1}{2^k}\right)$

$= \sum_{k=1}^{\infty} \frac{2^k}{k^4}$

$\Rightarrow$ diverges

$\Rightarrow$ Var(X) does not exist

$\Rightarrow$ can apply weak law of large numbers

不适用切比雪夫

8 $f(x) = \frac{1}{a\pi} \frac{1}{1+\frac{x^2}{a^2}}$

$= \frac{1}{a\pi} \frac{a^2}{a^2+x^2}$

$= \frac{a}{\pi(a^2+x^2)}$

$EX = \int_{-\infty}^{\infty} \frac{ax}{\pi(a^2+x^2)} dx$

$= \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{x}{a^2+x^2} dx$

$= \frac{a}{\pi}\left[\ln|\sec\theta|\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

Since $\ln|\sec\frac{\pi}{2}| \to \infty$

$\Rightarrow$ EX does not exist.

Let $x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$\int \frac{x}{a^2+x^2} dx$

$= \int \frac{a^2\tan\theta\sec^2\theta}{a^2+a^2\tan^2\theta} d\theta$

$= \int \tan\theta\, d\theta$

$= \ln|\sec\theta| + C$

can't apply weak law

5-2

1. $\dfrac{\sum_{i=1}^{100} X_i - 100}{10\sqrt{0.01}} \overset{近似}{\sim} N(0,1)$ $\quad P(\sum_{i=1}^{100} X_i > 102) = 1 - P(\sum_{i=1}^{100} X_i \le 102)$

$= 1 - P\left(\dfrac{\sum_{i=1}^{100} X_i - 100}{1} \le \dfrac{102-100}{1}\right)$

$= 1 - \Phi_0(2)$

$= 0.02275$

2. $\dfrac{\sum_{i=1}^{100} X_i - 200}{10\sqrt{1.69}} \sim N(0,1)$ $\quad P(180 < \sum_{i=1}^{100} X_i < 200) = \Phi_0\left(\dfrac{200-200}{10\sqrt{1.69}}\right) - \Phi_0\left(\dfrac{180-200}{10\sqrt{1.69}}\right)$

$= 0.5 - 1 + \Phi_0(1.538461)$

$= 0.43822$

3. $\dfrac{\sum_{i=1}^{100} X_i - 900}{10\sqrt{0.6}} \sim N(0,1)$ $\quad EX = 10\times0.3 + 9\times0.4 + 8\times0.3 = 9$

$EX^2 = 100\times0.3 + 81\times0.4 + 64\times0.3 = 81.6$

$DX = 0.6$

$P(\sum_{i=1}^{100} X_i > 900) = 1 - \Phi_0\left(\dfrac{900-900}{10\sqrt{0.6}}\right) = 0.5$

4. $X \sim B(n, 0.1)$ $\quad np = 0.1n \quad npq = 0.09n$

$P(X > 10) = P\left(\dfrac{X - 0.1n}{\sqrt{0.09n}} > \dfrac{10-0.1n}{\sqrt{0.09n}}\right) = 1 - \Phi_0\left(\dfrac{10-0.1n}{\sqrt{0.09n}}\right) = 0.9$

$\Phi_0\left(\dfrac{10-0.1n}{\sqrt{0.09n}}\right) = 0.1$

$\Phi_0\left(\dfrac{0.1n-10}{\sqrt{0.09n}}\right) = 1 - \Phi_0\left(\dfrac{10-0.1n}{\sqrt{0.09n}}\right) = 0.9$

$\dfrac{0.1n-10}{\sqrt{0.09n}} = 1.29$

$0.1n - 10 = 1.29\sqrt{0.09}\sqrt{n}$

$n = 147$

5. $X \sim B(100, 0.9)$ $\quad np = 90 \quad npq = 9$

$P(X \geq 85) = P\left(\dfrac{X-90}{3} \geq \dfrac{85-90}{3}\right) = 1 - \Phi_0\left(-\dfrac{5}{3}\right) = 1 - 1 + \Phi_0\left(\dfrac{5}{3}\right) = 0.9525$

6. $X \sim B(100, 0.2)$ $\quad np = 20 \quad npq = 16$

$P(X \geq 30) = P\left(\dfrac{X-20}{4} \geq \dfrac{30-20}{4}\right) = 1 - \Phi_0(2.5) = 0.00621$

7. $X \sim B(500, 0.8)$ $np = 400$ $npq = 80$

$$P(2X \le a) = P(X \le \frac{a}{2}) \approx P\left(\frac{X-400}{\sqrt{80}} \le \frac{\frac{a}{2}-400}{\sqrt{80}}\right) \approx \Phi_0\left(\frac{\frac{a}{2}-400}{\sqrt{80}}\right) \geq 0.99$$

$$\frac{\frac{a}{2}-400}{\sqrt{80}} \geq 2.33$$

$$a \geq (2.33\sqrt{80}+400) 2 = 841.68 \text{ 千瓦}$$

8. (1) $X \sim B(10000, 0.006)$ $10000 \times 12 = 120000$元

$$P(1000X > 120000) = P(X > 120) = 1 - P(X \le 120) = \Phi_0\left(\frac{120-60}{7.72269}\right) = 0$$

(2) $P(120000 - 1000x \geq 60000)$

$$= P(-1000x \geq -60000) = P(x \le 60) = \Phi_0\left(\frac{60-60}{7.72269}\right) = 0.5$$

9. $X \sim B(n, p)$ $EX = np$ $DX = npq$

$$P\left(\left|\frac{X}{n} - p\right| < 0.01\right) \geq 0.95$$

$$P\left(-0.01 < \frac{X-np}{n} < 0.01\right) \geq 0.95$$

$$P(-0.01n + np < X < 0.01n + np) \geq 0.95$$

$$\Phi_0\left(\frac{0.01n+np-np}{\sqrt{npq}}\right) - \Phi_0\left(\frac{-0.01n+np-np}{\sqrt{npq}}\right) \geq 0.95$$

$$\Phi_0\left(\frac{0.01\sqrt{n}}{\sqrt{pq}}\right) - \Phi_0\left(\frac{-0.01\sqrt{n}}{\sqrt{pq}}\right) \geq 0.95$$

$$2\Phi_0\left(\frac{0.01\sqrt{n}}{\sqrt{pq}}\right) - 1 \geq 0.95$$

$$\Phi_0\left(\frac{0.01\sqrt{n}}{\sqrt{pq}}\right) \geq 0.975$$

$$\frac{0.01\sqrt{n}}{\sqrt{pq}} \geq 1.96$$

$$n \geq (196)^2 pq$$

When $0 \le p \le 1$ $p(1-p) \le \frac{1}{4}$

$$\Rightarrow n \geq (196)^2 \left(\frac{1}{4}\right) = 9604$$

$$p(1-p)$$
$$= -p^2 + p$$
$$= -(p^2 - p$$
$$= -\left(p^2 - p + \left(\frac{1}{2}\right)^2\right) + \left(\frac{P}{2}\right)^2$$
$$= -\left(p - \frac{1}{2}\right)^2 + \frac{1}{2}^2$$
$$= -\left(p - \frac{1}{2}\right)^2 + \frac{1}{4}$$

五

1.(1) $P(|X+Y|\geqslant 6)\leqslant \frac{D(X+Y)}{36}=\frac{1}{12}$ $Cov(X,Y)=-0.5\times1\times2=-1$

$D(X+Y)=DX+DY+2Cov(X,Y)=3$

(2) $P(|X-\mu|\geqslant 3\sigma)\leqslant \frac{\sigma^2}{9\sigma^2}=\frac{1}{9}$

(3) $X_i\sim Exp(\lambda)$ $\frac{1}{\lambda}$

(4) $X_i=\begin{cases}\frac{1}{\alpha} & x_i\in[0,\alpha]\\ 0 & else\end{cases}$ $EX_i=\frac{\alpha}{2}$ $DX=\frac{\alpha^2}{12}$ $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i\sim N(\frac{\alpha}{2},\frac{\alpha^2}{12n})$

(5) $P\{\sum_{i=1}^{100}X_i\geqslant 90\}=1-P\{\sum_{i=1}^{100}X_i<90\}=1-\Phi_0(\frac{90-100}{10\sqrt{2.5}})=1-1+\Phi_0(0.63)=0.7357$

2.(1) D

(2) $X=\frac{1}{9}\sum_{i=1}^{9}X_i$ $DX=D(\frac{1}{9}\sum_{i=1}^{9}X_i)=\frac{1}{81}(9)=\frac{1}{9}$ B, D

(3) A

(4) $f(x_n)=\begin{cases}\frac{1}{2n} & x\in[-n,n]\\ 0 & else\end{cases}$ $EX_n^2=\int_{-n}^{n}\frac{x^2}{2n}dx$ 方差无界，B, D

$=\left[\frac{x^3}{6n}\right]_{-n}^{n}$

$=\frac{n^2}{6}+\frac{n^2}{6}=\frac{n^2}{3}$

(5) $\lim_{n\to\infty}P\left(\frac{\sum_{i=1}^{n}X_i-n\mu}{\sqrt{n}\sigma}\leqslant x\right)=\Phi_0(x)$ B

$\sum_{i=1}^{n}X_i\sim N(n\mu,n\sigma^2)$ $\frac{1}{n}\sum_{i=1}^{n}X_i\sim N(\mu,\frac{\sigma^2}{n})$

(6) $X\sim B(100,0.2)$ $np=20$ $npq=16$ C, D

$P(10\leqslant X\leqslant 30)=P(X\leqslant 30)-P(X\leqslant 10)$

$=\Phi_0(\frac{30-20}{4})-\Phi_0(\frac{10-20}{4})$

$=2\Phi_0(2.5)-1$

(7) $X_i\sim P(\lambda)$ $Y=\frac{\sum_{i=1}^{n}X_i-n\lambda}{\sqrt{n\lambda}}\sim N(0,1)$ D

3 $EX_i=1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}=3.5$ $Y=10X$ $EY=35$

$EX_i^2=1\times\frac{1}{6}+4\times\frac{1}{6}+9\times\frac{1}{6}+16\times\frac{1}{6}+25\times\frac{1}{6}+36\times\frac{1}{6}=\frac{91}{6}$ $DY=10(\frac{35}{12})=\frac{175}{6}$

$DX=\frac{35}{12}$

$P(20<X<50)=P(20-35<X-\mu<50-35)=P(-15<X-\mu<15)=P(|X-\mu|<15)>1-\frac{\frac{175}{6}}{15^2}$

$=0.87$

4. $EX_k^2 = (-ka)^2\left(\frac{1}{2k^2}\right) + (ka)^2\left(\frac{1}{2k^2}\right)$

$= \frac{2k^2a^2}{2k^2}$ DX, EX exists

$= a^2$

$EX = (-ka)\left(\frac{1}{2k^2}\right) + (ka)\left(\frac{1}{2k^2}\right)$

$= 0$

5. $X \sim B(n, 0.75)$ $P(0.74n < nX < 0.76n) \geq 0.9$

$P\left(\frac{0.74n - 0.75n}{\sqrt{0.1875n}} < \frac{nX - 0.75n}{\sqrt{0.1875n}} < \frac{0.76n - 0.75n}{\sqrt{0.1875n}}\right) \geq 0.9$

$\Phi_0\left(\frac{0.01n}{\sqrt{0.1875n}}\right) - \Phi_0\left(\frac{-0.01n}{\sqrt{0.1875n}}\right) \geq 0.9$

$2\Phi_0\left(\frac{0.01n}{\sqrt{0.1875n}}\right) \geq 1.9$

$\frac{0.01n}{\sqrt{0.1875n}} \geq 1.65$

$n \geq 5104$

6. $X \sim B(n, 0.5)$ $P(0.4n < nX < 0.6n) \geq 0.9$ $EX = 0.5n$ $DX = 0.25n$

$P(|nX - 0.5n| < 0.1n) \geq 1 - \frac{0.25n}{(0.1n)^2} = 0.9$

$0.01n^2 - 0.25n = 0.9 \times 0.01n^2$

$n = 250$

$\Phi_0\left(\frac{0.6n - 0.5n}{\sqrt{0.25n}}\right) - \Phi_0\left(\frac{0.4n - 0.5n}{\sqrt{0.25n}}\right) \geq 0.9$

$2\Phi_0\left(\frac{0.1n}{\sqrt{0.25n}}\right) \geq 1.9$

$\frac{0.1n}{\sqrt{0.25n}} \geq 1.65$

$n \geq 68$

7. $X \sim B(100, 0.2)$ $P(14 \leq X \leq 30) = \Phi_0\left(\frac{30-20}{\sqrt{16}}\right) - \Phi_0\left(\frac{14-20}{4}\right)$

$= \Phi_0(2.5) - \Phi_0(-1.5)$

$= \Phi_0(2.5) - 1 + \Phi_0(1.5)$

$= 0.92698$

8. $X \sim B(1000, 0.6)$ $\quad P(X \leq N)$

$$\approx \Phi_0\left(\frac{N-600}{\sqrt{240}}\right) = 0.997$$

$$\frac{N-600}{\sqrt{240}} = 2.75$$

$$N = 643$$

9. $X \sim B(260, 0.04)$ $\quad P(X \leq N) \geq 0.95$

$$\Phi_0\left(\frac{N-10.4}{\sqrt{9.984}}\right) \geq 0.95$$

$$\frac{N-10.4}{\sqrt{9.984}} = 1.65$$

$$N = 16$$

10. $X_i \sim Exp(1)$ $\quad f(x_i) = \begin{cases} e^{-x} & x>0 \\ 0 & \text{else} \end{cases}$ $\quad EX_i = 30 \quad DX_i = 30$

$$P(T>35) = P\left(\sum_{i=1}^{30} X_i \geq 35\right) = P\left(\frac{\sum_{i=1}^{30} X_i - 30}{\sqrt{30}} \geq \frac{35-30}{\sqrt{30}}\right)$$

$$= 1 - \Phi_0(0.912)$$

$$= 0.1814$$

11. $EX = \frac{-1+1}{2} = 0$ $\quad DX = \frac{1}{3}$

$$P\left(\frac{1}{36}\sum_{i=1}^{36} X_i \leq \frac{1}{6}\right) = P\left(\sum_{i=1}^{36} X_i \leq 6\right)$$

$$= \Phi_0\left(\frac{6}{\sqrt{36}\sqrt{\frac{1}{3}}}\right)$$

$$= \Phi_0(1.73)$$

$$= 0.958$$

12. $X \sim P(0.05)$ $\quad P\left(\sum_{i=1}^{50} X_i > 3\right)$ $\quad E\sum_{i=1}^{50} X_i = 2.5 \quad D\sum_{i=1}^{50} X_i = 2.5$

$$= 1 - P\left(\sum_{i=1}^{50} X_i \leq 3\right)$$

$$= 1 - \Phi_0\left(\frac{3-2.5}{\sqrt{2.5}}\right)$$

$$= 1 - \Phi_0(0.316)$$

$$= 0.3745$$

13. $EX=0$ $DX=\frac{1500}{12}=125$

(1) $P(|\sum_{i=1}^{1500}x_i|>15) = P(\sum_{i=1}^{1500}x_i>15)+P(\sum_{i=1}^{1500}x_i<-15)$

$= 1-P(\sum_{i=1}^{1500}x_i\leq 15)+P(\sum_{i=1}^{1500}x_i<-15)$

$= 1-\Phi_0(\frac{15}{\sqrt{125}})-\Phi_0(\frac{15}{\sqrt{125}})+1$

$=0.18024$

(2) $P(|\sum_{i=1}^{n}x_i|<10)\leq 90\%$ $DX=\frac{n}{12}$

$P(-10<\sum_{i=1}^{n}x_i<10)\leq 90\%$

$\Phi_0(\frac{10}{\sqrt{\frac{n}{12}}})-\Phi_0(\frac{-10}{\sqrt{\frac{n}{12}}})\leq 0.9$

$2\Phi_0(\frac{10}{\sqrt{\frac{n}{12}}})\leq 1.9$

$\Phi_0(\frac{10\sqrt{12}}{\sqrt{n}})\leq 0.95$

$\frac{10\sqrt{12}}{\sqrt{n}}\leq 1.64$

$n\geq 446$

14. $X\sim B(n,0.01)$ $EX=0.01n$ $DX=0.0099n$

$P(n-X\geq 100)\geq 0.95$

$P(X\leq n-100)\geq 0.95$

$\Phi_0(\frac{n-100-0.01n}{\sqrt{0.0099n}})\geq 0.95$

$\frac{n-100-0.01n}{\sqrt{0.0099n}}\geq 1.64$

$n\geq 103$

16. 依辛钦大数定律 $\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{i=1}^{n}x_i-\mu|<\varepsilon\}=1$ $x_i\sim Exp(\lambda)$ $EX=\frac{1}{\lambda}$ $DX=\frac{1}{\lambda^2}$

$\lim_{n\to\infty}P(\frac{\sum_{i=1}^{n}x_i-n\mu}{\sqrt{n}\sigma}\leq x)=\Phi_0(x)$

$\Rightarrow \lim_{n\to\infty}P(\frac{\sum_{i=1}^{n}x_i-\frac{n}{\lambda}}{\sqrt{n}(\frac{1}{\lambda})}\leq x)=\Phi_0(x)$

$\Rightarrow \lim_{n\to\infty}P(\frac{1}{\sqrt{n}}\sum_{i=1}^{n}x_i-\frac{\sqrt{n}}{\lambda}\leq\frac{x}{\lambda})=\Phi_0(x)$

15. 相互独立 $\Rightarrow$ 两两不相关

由切比雪夫大数定律，

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}x_i-\frac{1}{n}\sum_{i=1}^{n}EX_i\right|<\varepsilon\right\}=1$$

$$\Rightarrow \lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{k=1}^{n}X_k-\frac{1}{n}\sum_{k=1}^{n}P_k\right|<\varepsilon\right\}=1$$

$X_k=\begin{cases}0\\1\end{cases}$

$\{X_k\}$ 由 k个 0-1 分布组成

$\Rightarrow EX_i=P_i$

6-1.

1. (1) 该地区 2011 年毕业的本专业本科生实习期满的月薪

(2) 30名毕业生，30

2. 独立同分布

3. 产品的不合格数，$X\sim B(m,p)$ 样本是抽出来的不合格数

$(X_1,X_2,\cdots,X_n)$ $X_i\sim B(m,p)$ $i=1,2,\cdots,n$

$$P(x_1,x_2,\ldots,x_n)=\prod_{i=1}^{n}{}^{m}C_{x_i}\,p^{x_i}(1-p)^{m-x_i}=\left(\prod_{i=1}^{n}{}^{m}C_{x_i}\right)p^{\sum_{i=1}^{n}x_i}(1-p)^{nm-\sum_{i=1}^{n}x_i},\quad x_i=0,1,\cdots,m$$

4. $X\sim Geo(p)$ $\quad p(x_1,x_2,x_3,\ldots,x_n)=\prod_{i=1}^{n}p(1-p)^{x_i-1}=p^n(1-p)^{\sum_{i=1}^{n}x_i-n}$, $x_i=1,2,\ldots$

5. 电容器寿命，抽出的n件 $X\sim Exp(\lambda)$

$$f(x_1,x_2,\ldots,x_n)=\begin{cases}\prod_{i=1}^{n}\lambda e^{-\lambda x_i}=\lambda^n e^{-\lambda\sum_{i=1}^{n}x_i}, & x_i>0\\ 0 & \text{else}\end{cases}$$

6. $$p(x_1,x_2,\cdots,x_m)=\prod_{i=1}^{m}{}^{n}C_{x_i}\,p^{x_i}(1-p)^{n-x_i}=\left(\prod_{i=1}^{m}{}^{n}C_{x_i}\right)p^{\sum_{i=1}^{m}x_i}(1-p)^{nm-\sum_{i=1}^{m}x_i}$$

$x_i=0,1,\cdots,n$ $\quad i=1,2,\cdots,m$

7. $X\sim U[\theta_1,\theta_2]$

$$P(x_1,x_2,\cdots,x_n)=\left(\frac{1}{\theta_2-\theta_1}\right)^n \quad \theta_1\le x_i\le\theta_2\ ;\ i=1,2,\cdots,n$$

8.

X	0	1
P	1-P	P

总体为打中与否

样本为由 0 或 1 组成的 N元数组

$x_i\sim B(n,p)$

$$P(x_1,x_2,\cdots,x_n)=\prod_{i=1}^{n}p^{x_i}(1-p)^{1-x_i}=p^{\sum_{i=1}^{n}x_i}(1-p)^{n-\sum_{i=1}^{n}x_i}$$

6-2

1\. (1) √ (2) √ (3) × (4) √ (5) √ (6) √

2\. (1)
$$\bar{Y} = \frac{Y_1+Y_2+\cdots+Y_n}{n} = \frac{x_1-\bar{x}+x_2-\bar{x}+\cdots+x_n-\bar{x}}{n} = \frac{x_1+\cdots+x_n-n\bar{x}}{n} = \bar{x}-\bar{x}=0$$

(2) (i)
$$\bar{Y}+a = \frac{x_1-a+x_2-a+\cdots+x_n-a}{n}+a = \frac{x_1+\cdots+x_n}{n}-\frac{an}{n}+a = \bar{x}$$

(ii)
$$\sum_{i=1}^{n}(Y_i-\bar{Y})^2 = (Y_1-\bar{Y})^2+(Y_2-\bar{Y})^2+\cdots+(Y_n-\bar{Y})^2$$
$$= (x_1-a-\bar{x}+a)^2+(x_2-a-\bar{x}+a)^2+\cdots+(x_n-a-\bar{x}+a)^2$$
$$= \sum_{i=1}^{n}(x_i-\bar{x})^2$$

3\. (1) $E\bar{X} = mp$ $\quad D\bar{X} = \frac{mp(1-p)}{n}$

(2) $E\bar{X} = \lambda$ $\quad D\bar{X} = \frac{\lambda}{n}$

(3) $E\bar{X} = \frac{1}{\lambda}$ $\quad D\bar{X} = \frac{1}{n\lambda^2}$

4\. $E\bar{X} = \frac{a+b}{2}$ $\quad D\bar{X} = \frac{(b-a)^2}{12n}$ $\quad ES^2 = \frac{(b-a)^2}{12}$

5\. $E\left(\sum_{i=1}^{100}(x_i-\bar{x})^2\right) = (n-1)\sigma^2 = 99\times10 = 990$

6\. $\bar{x} = 3$ $\quad S^2 = 3.78$ $\quad \sqrt{S^2} = 1.94$

7\. $\bar{x} = 3180$ $\quad S^2 = 112736.8$ $\quad \sqrt{S^2} = 335.763$ $\quad B_2 = \frac{1}{20}\sum_{i=1}^{20}(x_i-\bar{x})^2 = 107100$

8\.
$$ES_0^2 = E\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)$$
$$= \frac{1}{n}\sum_{i=1}^{n}E(x_i-\bar{x})^2$$
$$= \frac{1}{n}\sum_{i=1}^{n}E\left((x_i-\mu)^2-n(\bar{x}-\mu)^2\right)$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left(E(x_i-\mu)^2-nD\bar{x}\right)$$
$$= \frac{1}{n}\left(n\sigma^2-n\frac{\sigma^2}{n}\right) = \frac{n-1}{n}\sigma^2$$

6-3

1.(1) $\chi^2_{0.05}(8) = 15.5$ $\quad$ $\chi^2_{0.95}(8) = 2.73$

(2) $t_{0.05}(20) = 1.725$ $\quad$ $t_{0.95}(15) = -t_{0.05}(15) = -1.753$

(3) $F_{0.05}(12,20) = 2.28$ $\quad$ $F_{0.95}(12,20) = \frac{1}{F_{0.05}(20,12)} = \frac{1}{2.54}$ $\quad$ $F_{0.95}(20,12) = \frac{1}{2.28}$

2. $X \sim N(\mu, \sigma^2)$ $\quad$ $P(|\bar{X}-\mu|>4) = 0.02$

$$P((\bar{x}-\mu)>4) + P((\bar{x}-\mu)<-4) = 0.02$$

$$1-P((\bar{x}-\mu)\leq 4) + P((\bar{x}-\mu)<-4) = 0.02$$

$$1-\Phi_0\left(\frac{4}{\sigma}\sqrt{16}\right)+\Phi_0\left(\frac{-4}{\sigma}\sqrt{16}\right) = 0.02$$

$$1-\Phi_0\left(\frac{16}{\sigma}\right)+1-\Phi_0\left(\frac{16}{\sigma}\right) = 0.02$$

$$2-2\Phi_0\left(\frac{16}{\sigma}\right) = 0.02$$

$$\Phi_0\left(\frac{16}{\sigma}\right) = 0.99$$

$$\sigma = 6.87$$

3. $X \sim N(\mu, 1)$ $\quad$ $n=9$ $\quad$ $P(|\bar{x}-\mu| \leq 0.3)$

$$= 2\Phi_0\left(\frac{0.3}{1}\sqrt{9}\right) - 1$$

$$= 0.6318$$

4. $P\left(\sum_{i=1}^{6} Z_i^2 \leq b\right) = 0.95$

$$\sum_{i=1}^{6} Z_i^2 \sim \chi^2(6)$$

$$1-P\left(\sum_{i=1}^{6} Z_i^2 > b\right) = 0.95$$

$$P\left(\sum_{i=1}^{6} Z_i^2 > b\right) = 0.05$$

$$b = 12.6$$

5. $X \sim N(0, \sigma^2)$ $\quad$ $n=6$ $\quad$ $X_1+X_2+X_3 \sim N(0, 3\sigma^2)$

$$\frac{X_1+X_2+X_3-0}{\sqrt{3}\sigma} \sim N(0,1)$$

$$\frac{X_4+X_5+X_6}{\sqrt{3}\sigma} \sim N(0,1)$$

$$\left(\frac{X_1+X_2+X_3}{\sqrt{3}\sigma}\right)^2 + \left(\frac{X_4+X_5+X_6}{\sqrt{3}\sigma}\right)^2 \sim \chi^2(2)$$

$$cY = \frac{1}{3\sigma^2} Y \sim \chi^2(2)$$

$$\Rightarrow c = \frac{1}{3\sigma^2}$$

6. $X \sim N(0,3^2)$ $\quad Y \sim N(0,3^2)$

$$X_1+X_2+\cdots+X_9 \sim N(0,81)$$

$$\frac{X_1+\cdots+X_9}{9} \sim N(0,1)$$

$$\frac{Y_1}{3} \sim N(0,1)$$

$$\frac{Y_1^2+\cdots+Y_9^2}{9} \sim \chi^2(9)$$

$$\frac{\frac{X_1+\cdots+X_9}{9}}{\sqrt{\frac{Y_1^2+\cdots+Y_9^2}{9\times 9}}} \sim t(9)$$

$$\Rightarrow \frac{X_1+\cdots+X_9}{9} \times \frac{9}{\sqrt{Y_1^2+\cdots+Y_9^2}} \sim t(9)$$

$$\Rightarrow U \sim t(9)$$

7. $X \sim N(\mu,\sigma^2)$

(1) $P\{\frac{\sigma^2}{4} \leq \frac{1}{10}\sum_{i=1}^{10}(X_i-\mu)^2 \leq 2.3\sigma^2\}$

$= P\{2.5\sigma^2 \leq \sum_{i=1}^{10}(X_i-\mu)^2 \leq 23\sigma^2\}$

$= P\{2.5 \leq \frac{1}{\sigma^2}\sum_{i=1}^{10}(X_i-\mu)^2 \leq 23\}$

$= P\{\chi^2(10) \leq 23\} - P\{\chi^2(10) \leq 2.5\}$

$= 1-P\{\chi^2(10) > 23\} - 1 + P\{\chi^2(10) > 2.5\}$

$= 1-0.01-1+0.99$

$= 0.98$

(2) $P\{\frac{\sigma^2}{4} \leq \frac{1}{10}\sum_{i=1}^{10}(X_i-\bar{X})^2 \leq 2.3\sigma^2\}$

$= P\{2.5 \leq \frac{1}{\sigma^2}\sum_{i=1}^{10}(X_i-\bar{X})^2 \leq 23\}$

$= P\{\chi^2(9) \leq 23\} - P\{\chi^2(9) \leq 2.5\}$

$= 0.975-0.005$

$= 0.97$

8. $\bar{X} \sim N(\mu,\frac{\sigma^2}{n})$

$\bar{Y} \sim N(\mu,\frac{\sigma^2}{n})$

$\bar{X}-\bar{Y} \sim N(0,\frac{2}{n}\sigma^2)$

$P(|\bar{X}-\bar{Y}| > \sigma)$

$= P(\bar{X}-\bar{Y} < -\sigma) + P(\bar{X}-\bar{Y} > \sigma)$

$= \Phi_0\left(\frac{-\sigma}{\sqrt{\frac{2}{n}}\sigma}\right) + 1 - \Phi_0\left(\frac{\sigma}{\sqrt{\frac{2}{n}}\sigma}\right)$

$= 2-2\Phi_0(\sqrt{\frac{n}{2}})$

$2-2\Phi_0(\sqrt{\frac{n}{2}}) \approx 0.01$

$\Phi_0(\sqrt{\frac{n}{2}}) \approx 0.995$

$\sqrt{\frac{n}{2}} \approx 2.58$

$n = 14$

9. $X \sim N(50, 6^2)$ $Y \sim N(46, 4^2)$

(1) $\bar{X} \sim N(50, 3.6)$ $\bar{Y} \sim N(46, 2)$

$P\{0 < \bar{x} - \bar{Y} < 8\}$

$= P\left\{ \frac{0-50+46}{\sqrt{\frac{36}{10}+\frac{16}{8}}} < \frac{\bar{x}-\bar{Y}-50+46}{\sqrt{\frac{36}{10}+\frac{16}{8}}} < \frac{8-50+46}{\sqrt{\frac{36}{10}+\frac{16}{8}}} \right\}$

$= \Phi_0(1.69) - \Phi_0(-1.69)$

$= 2\Phi_0(1.69) - 1 = 0.909$

(2) $P\left\{\frac{S_1^2}{S_2^2} < 8.28\right\} = P\left\{ \frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2} < \frac{8.28 \times 16}{36} \right\} = 1 - P\left\{ \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \geqslant 3.68 \right\} = 0.95$

$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(9,7)$

10 $X \sim \chi^2(n)$ (1) $EX = E\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n} EX_i^2 = \sum_{i=1}^{n}(DX_i + (EX_i)^2) = n$

$X_i \sim N(0,1)$ (2) $DX = D\sum_{i=1}^{n} X_i^2 = nDX_i^2$ $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$EX^4 = \int_{-\infty}^{\infty} x^4 f(x)dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^4 e^{-\frac{x^2}{2}} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^3 \ d(-e^{-\frac{x^2}{2}})$

$= \frac{1}{\sqrt{2\pi}}\left(\left[-x^3 e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty} + 3\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \right)$

$= \frac{1}{\sqrt{2\pi}}(3\sqrt{2\pi})$

$= 3$

$(EX^2) = DX + (E(X))^2 = 1$ $DX_i^2 = 3 - 1 = 2$

$\Rightarrow DX = 2n$

11. $T \sim t(n)$

$\Rightarrow T = \frac{X}{\sqrt{\frac{Y}{n}}}$ $X \sim N(0,1)$ $Y \sim \chi^2(n)$

$T^2 = \frac{X^2}{\frac{Y}{n}}$ $\Rightarrow X^2 \sim \chi^2(1)$

$= \frac{X^2/1}{Y/n}$

Since $X^2 \sim \chi^2(1)$, $Y \sim \chi^2(n)$

By definition $T^2 \sim F(1,n)$

6-4

1. 样本取值.

0 0 0	0 0 2	1 1 0	1 0 2	0 2 2
0 0 1	0 2 0	0 1 1	2 0 1	2 0 2
0 1 0	2 0 0	0 1 2	1 2 0	2 2 0
1 0 0	1 0 1	0 2 1	2 1 0	

$P\{X_{(1)}=0\}=\frac{19}{27}$

$P\{X_{(2)}=0\}=\frac{7}{27}$

2. $F(x)=\begin{cases}0 & x<1\\ \frac{1}{3} & 1\le x<2\\ \frac{2}{3} & 2\le x<3\\ 1 & x\ge 3\end{cases}$

$F_{(1)}(x)=P\{X_{(1)}\le x\}=1-(1-F(x))^5$

$=\begin{cases}0 & x<1\\ \frac{211}{243} & 1\le x<2\\ \frac{242}{243} & 2\le x<3\\ 1 & x\ge 3\end{cases}$

$X_{(1)}$	1	2	3
P	$\frac{211}{243}$	$\frac{31}{243}$	$\frac{1}{243}$

$F_{(5)}(x)=(F(x))^5=\begin{cases}0 & x<1\\ \frac{1}{243} & 1\le x<2\\ \frac{32}{243} & 2\le x<3\\ 1 & x\ge 3\end{cases}$

$X_{(5)}$	1	2	3
P	$\frac{1}{243}$	$\frac{31}{243}$	$\frac{211}{243}$

3. $X\sim Exp(\lambda)$ $\quad F(x)=\begin{cases}1-e^{-\lambda x} & x>0\\ 0 & \text{else}\end{cases}$

$F_{X(1)}(x)=\begin{cases}1-e^{-n\lambda x} & x>0\\ 0 & \text{else}\end{cases}$ $\quad F_{X(n)}(x)=\begin{cases}(1-e^{-\lambda x})^n & x>0\\ 0 & \text{else}\end{cases}$

$P(X_{(1)}>a)=e^{-n\lambda a}$ $\quad P(X_{(n)}\le b)=(1-e^{-\lambda b})^n$

4. $F_{(6)}\,X=\begin{cases}0 & x<0.75\\ \frac{1}{6} & 0.75\le x<1.86\\ \frac{1}{3} & 1.86\le x<1.98\\ \frac{1}{2} & 1.98\le x<2.45\\ \frac{2}{3} & 2.45\le x<3.21\\ \frac{5}{6} & 3.21\le x<4.12\\ 1 & x\ge 4.12\end{cases}$

六

1.(1) 2015年毕业的统计学专业本科毕业生实习期满后的月薪情况

36名

(2) $E\bar{X}=\frac{a+b}{2}$ $D\bar{X}=\frac{(b-a)^2}{360}$ $ES^2=\frac{(b-a)^2}{12}$

(3) $P(1.5<\bar{x}<3.5)=P\left(\frac{1.5-2}{0.5}\sqrt{9}<\frac{\bar{x}-\mu}{0.5}\sqrt{9}<\frac{3.5-2}{0.5}\sqrt{9}\right)$

$=\Phi_0(9)-\Phi_0(-3)$

$=1-1+\Phi_0(3)=0.99865$

(4) $X\sim N(0,\sigma^2)$ $Y\sim N(0,\sigma^2)$

$\sqrt{m}\frac{x_1+\cdots+x_m}{m\sigma}\sim N(0,1)$ $\frac{Y}{\sigma}\sim N(0,1)$

$$\left(\frac{Y_1}{\sigma}\right)^2+\cdots+\left(\frac{Y_n}{\sigma}\right)^2\sim\chi^2(n)$$

$$\frac{Y_1^2+\cdots+Y_n^2}{\sigma^2}\sim\chi^2(n)$$

$$\frac{\frac{1}{\sqrt{m}}\frac{x_1+\cdots+x_m}{\sigma}}{\sqrt{\frac{Y_1^2+\cdots+Y_n^2}{\sigma^2 n}}}\sim t(n)$$

$$\frac{1}{\sqrt{m}}\frac{x_1+\cdots+x_m}{\sigma}\times\frac{\sqrt{\sigma^2 n}}{\sqrt{Y_1^2+\cdots+Y_n^2}}\sim t(n)$$

$$\frac{\sqrt{n}(x_1+\cdots+x_m)}{\sqrt{m}\sqrt{Y_1^2+\cdots+Y_n^2}}\sim t(n)$$

$$\Rightarrow\sqrt{\frac{n}{m}}=2\Rightarrow\frac{m}{n}=\frac{1}{4}$$

(5) $\frac{\bar{x}-a}{2}\sqrt{9}\sim N(0,1)$ $P\{|\bar{x}-a|<a\}=0.95$

$P\{-a<\bar{x}-a<a\}=0.95$

$P\left\{-\frac{3a}{2}<\frac{\bar{x}-a}{2}(3)<\frac{3a}{2}\right\}=0.95$

$2\Phi_0\left(\frac{3a}{2}\right)-1=0.95$

$\Phi_0\left(\frac{3a}{2}\right)=0.975$

$a=1.3067$

(6) $X \sim N(0,1)$

$$\frac{\sqrt{n-1}\,X_1}{\sqrt{\sum_{i=2}^{n} X_i^2}}$$

$X_1 \sim N(0,1)$

$$X_2^2+\cdots+X_n^2 \sim \chi^2(n-1)$$

$$\frac{X_1}{\sqrt{\frac{X_2^2+\cdots+X_n^2}{n-1}}} \sim t(n-1)$$

$$\frac{\sqrt{n-1}\,X_1}{\sqrt{\sum_{i=2}^{n} X_i^2}} \sim t(n-1)$$

(7) $X \sim N(\mu,4)$

$$E(\bar{X}-\mu)^2$$
$$=E(\bar{X}^2-2\mu\bar{X}+\mu^2)$$
$$=E\bar{X}^2-2\mu E\bar{X}+\mu^2$$
$$=E\bar{X}^2-2\mu^2+\mu^2$$
$$=E\bar{X}^2-\mu^2$$
$$=\frac{4}{n}$$

$D\bar{X}=E\bar{X}^2-(E\bar{X})^2$

$\frac{4}{n}=E\bar{X}^2-\mu^2$

$E\bar{X}^2=\frac{4}{n}+\mu^2$

$\frac{4}{n}\le 0.1$

$4\le 0.1n$

$n\geqslant 40$

(8) $P(X>a)=0.2$

$a=F_{0.2}(n,n)$

$F_{0.8}(n,n)=\frac{1}{a}$

$P(X>\frac{1}{a})=0.8$

(9) $X \sim N(\mu,4^2)$ $n=10$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(9)$

$P\{S^2>a\}=0.1$

$P\left\{\frac{(n-1)S^2}{\sigma^2} > \frac{a(n-1)}{\sigma^2}\right\}=0.1$

$P\left\{\frac{(n-1)S^2}{\sigma^2} > \frac{9a}{16}\right\}=0.1$

$\frac{9a}{16}=14.7$

$a=26.133$

(10) $F(x)=\frac{x}{2}$ $DF_n(x)=\frac{1}{n}\frac{x}{2}(1-\frac{x}{2})$

2.(1) A, D (2) $DX=EX^2-(EX)^2$

$=1$

$X \sim (-1,1)$ B

$\bar{X} \sim N(-1,\frac{1}{n})$

(3) C, B $\quad D\bar{X}=\frac{1}{n} \quad D(n\bar{X})=n^2(\frac{1}{n})=n \quad n\bar{X}\sim N(0,n)$

(4) $X\sim N(0,\sigma^2)$

$\frac{X}{\sigma}\sim N(0,1)$

$F(2,4)=\frac{\chi^2(2)/2}{\chi^2(4)/4}=\frac{2\chi^2(2)}{\chi^2(4)}$ A

(5) $X\sim N(\mu,\sigma^2) \quad E(X_1-X_2)=0 \quad D(X_1-X_2)=DX_1+DX_2=2\sigma^2$ C

$X_1-X_2\sim N(0,2\sigma^2)$

$X_3-X_4\sim N(0,2\sigma^2)$

$\frac{X_1-X_2}{\sqrt{2}\sigma}\sim N(0,1) \qquad \frac{X_3-X_4}{\sqrt{2}\sigma}\sim N(0,1)$

$\frac{\frac{X_1-X_2}{\sqrt{2}\sigma}}{\sqrt{\left(\frac{X_3-X_4}{\sqrt{2}\sigma}\right)^2}}\sim t(1) \qquad \left(\frac{X_3-X_4}{\sqrt{2}\sigma}\right)^2\sim\chi^2(1)$

$\frac{X_1-X_2}{\sqrt{(X_3-X_4)^2}}\sim t(1)$

(6) $X\sim t(n) \quad P\{X\le t_\alpha\}=1-\alpha \quad 0<\alpha<1$ D

$1-P\{X>t_\alpha\}=1-\alpha$

$P\{X>t_\alpha\}=\alpha$

$P(|X|>x)=b$

$P(X<-x)+P(X>x)=b$

$2P(X>x)=b$

$P(X>x)=\frac{b}{2}$

$x=t_{\frac{b}{2}}$

(7) $X\sim t(n) \qquad X^2=\frac{Z^2}{Y/n}$ C

$X=\frac{Z}{\sqrt{\frac{Y}{n}}}\sim t(n) \qquad \frac{1}{X^2}=\frac{Y/n}{Z^2}$

$Z\sim N(0,1) \qquad \frac{1}{X^2}\sim F(n,1)$

$Y\sim\chi^2(n)$

(8) C

(9) $X \sim N(0,\sigma^2)$ $\quad \bar{X} \sim N(0,\frac{\sigma^2}{n})$ $\quad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$\frac{\bar{X}}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$$

$$\frac{\sqrt{n}\bar{X}}{\sqrt{\sigma^2}} \sim N(0,1) \qquad \frac{\frac{\sqrt{n}\bar{X}}{\sqrt{\sigma^2}}}{\sqrt{\frac{S^2}{\sigma^2}}} \sim t(n-1)$$

A

(10) $\frac{\sqrt{n}\bar{X}}{\sqrt{\sigma^2}} \sim N(0,1)$ $\quad \frac{\frac{n\bar{X}^2}{\sigma^2}/1}{\frac{(n-1)S^2}{\sigma^2}/n-1} \sim F(1,n-1)$

$\frac{n\bar{X}^2}{\sigma^2} \sim \chi^2(1)$

$$\Rightarrow \frac{n\bar{X}^2}{\sigma^2} \times \frac{\sigma^2}{S^2} \sim F(1,n-1)$$

$$\Rightarrow \frac{n\bar{X}^2}{S} \sim F(1,n-1)$$

C

(11) $X \sim N(0,1)$ $\quad Y \sim N(0,1)$ $\quad X^2 \sim \chi^2(1)$ $\quad Y^2 \sim \chi^2(1)$ $\quad$ C

(12) A

3. $X \sim N(\mu,3)$ $\quad Y \sim N(\mu,3)$ $\quad \frac{\bar{X}-\mu}{\sqrt{3}}\sqrt{9} \sim N(0,1)$ $\quad \frac{\bar{Y}-\mu}{\sqrt{3}}\sqrt{27} \sim N(0,1)$

$$P\{|\bar{X}-\bar{Y}|>0.2\} = P(\bar{X}-\bar{Y}>0.2) + P(\bar{X}-\bar{Y}<-0.2)$$

$$= 1-P(\bar{X}-\bar{Y}<0.2) + P(\bar{X}-\bar{Y}<-0.2)$$

$$= 1-\Phi_0\left(\frac{0.2}{\sqrt{\frac{3}{9}+\frac{3}{27}}}\right) \quad 1-\Phi_0\left(\frac{0.2}{\sqrt{\frac{3}{9}+\frac{3}{27}}}\right)$$

$$= 2-2\Phi_0(0.3)$$

$$= 0.7642$$

4. $X \sim N(\mu,10)$ $\quad Y \sim N(\mu,21)$

$P\{S_X^2 \geqslant 2S_Y^2\}$ $\qquad \frac{S_X^2/10}{S_Y^2/21} \sim F(7,9)$

$$= P\left\{\frac{S_X^2}{S_Y^2} \geqslant 2\right\}$$

$$= P\left\{\frac{S_X^2/10}{S_Y^2/21} \geqslant \frac{2\times 21}{10}\right\}$$

$$= P\left\{\frac{S_X^2/10}{S_Y^2/21} \geqslant \frac{21}{5}\right\}$$

$$= P\left\{\frac{S_X^2/10}{S_Y^2/21} \geqslant 4.2\right\}$$

$$= 0.025$$

(5) $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

$= \left((x_1-\bar{x})^2 + (x_2-\bar{x})^2\right)$

$= \left(x_1^2 - 2x_1\bar{x} + \bar{x}^2 + x_2^2 - 2x_2\bar{x} + \bar{x}^2\right)$

$= \left(x_1^2 - (x_1+x_2)(x_1) + \frac{(x_1+x_2)^2}{2} - (x_1+x_2)(x_2) + x_2^2\right)$

$= \left(\cancel{x_1^2} - \cancel{x_1^2} - x_1x_2 + \frac{x_1^2}{2} + x_1x_2 + \frac{x_2^2}{2} - x_1x_2 - \cancel{x_2^2} + \cancel{x_2^2}\right)$

$= \left(\frac{x_1^2}{2} - x_1x_2 + \frac{x_2^2}{2}\right)$

$= \left(\frac{x_1}{\sqrt{2}} - \frac{x_2}{\sqrt{2}}\right)^2$

$= \left(\frac{x_1-x_2}{\sqrt{2}}\right)^2$

$= \frac{1}{2}(x_1-x_2)^2$

(6) $X_i \sim N(0, 2^2)$

$X = a(X_1-2X_2)^2 + b(3X_3-4X_4)^2$

Find a, b

$X \sim \chi^2(2)$

$\sqrt{a}(X_1-2X_2) \sim N(0,1)$ $\sqrt{b}(3X_3-4X_4) \sim N(0,1)$

$X_1-2X_2 \sim N(0,20)$

$3X_3-4X_4 \sim N(0,100)$

$D\sqrt{a}(X_1-2X_2) = 1$

$a = \frac{1}{20}$

$D\sqrt{b}(3X_3-4X_4) = 1$

$b = \frac{1}{100}$

(7) $X \sim F(n,n)$

$\frac{Y}{Z} \sim F(n,n)$

$Y \sim \chi^2(n)$

$Z \sim \chi^2(n)$

$P(X<1)$

$= P\left(\frac{Y}{Z} < 1\right)$

$= P(Y<Z)$

$= 0.5$

(8) $P(|\bar{x}-\mu| < 0\ 6)$

$\sqrt{S^2} = 1.79$

$\frac{\bar{x}-\mu}{S}\sqrt{n} \sim t(n-1)$

$T = \frac{\bar{x}-\mu}{1.79}\sqrt{16} \sim t(15)$

$= P(-0.6 < \bar{x}-\mu < 0.6)$

$= P\left(\frac{-0.6}{1.79}\sqrt{16} < \frac{\bar{x}-\mu}{1.79}\sqrt{16} < \frac{0.6}{1.79}\sqrt{16}\right)$

$= P(-1.341 < T < 1.341)$

$= 1 - 0.1 - 0.1$

$= 0.8$

9. $X_i \sim N(0,9)$ $\quad \frac{X_1^2}{9}+\frac{X_2^2}{9}+\cdots+\frac{X_{10}^2}{9} \sim \chi^2(10)$

$\frac{X_i}{3} \sim N(0,1)$ $\quad \frac{X_{11}^2+\cdots+X_{15}^2}{9} \sim \chi^2(5)$

$$\frac{\frac{X_1^2+\cdots+X_{10}^2}{9}\left(\frac{1}{10}\right)}{\frac{X_{11}^2+\cdots+X_{15}^2}{9}\left(\frac{1}{5}\right)} \sim F(10,5)$$

$Y \sim F(10,5)$

10. $F \sim F(n_1, n_2)$

$\frac{1}{F} \sim F(n_2, n_1)$

$F \sim F(n_1, n_2)$ $\quad \frac{1}{F} \sim F(n_2, n_1)$

$\underline{1-\alpha} = P(F > F_{1-\alpha}(n_1, n_2)) = P\left(\frac{1}{F} < \frac{1}{F_{1-\alpha}(n_1,n_2)}\right) \underline{= 1-P\left(\frac{1}{F} \geq \frac{1}{F_{1-\alpha}(n_1,n_2)}\right)}$

$\Rightarrow P\left(\frac{1}{F} > \underset{\text{上}\alpha\text{分位数}}{\underline{\frac{1}{F_{1-\alpha}(n_1,n_2)}}}\right) = \alpha$ $\quad F_\alpha(n_2, n_1) = \frac{1}{F_{1-\alpha}(n_1, n_2)}$

11. (1) $P\{|\bar{X}-\mu| \leq 0.1\} \geq 0.95$

$P\{-0.1 \leq \bar{X}-\mu \leq 0.1\} \geq 0.95$

$P\left\{\frac{-0.1\sqrt{n}}{2} \leq \frac{\bar{X}-\mu}{2}\sqrt{n} \leq \frac{0.1\sqrt{n}}{2}\right\} \geq 0.95$

$2\Phi_0\left(\frac{0.1\sqrt{n}}{2}\right) \geq 1.95$

$\Phi_0\left(\frac{0.1\sqrt{n}}{2}\right) \geq 0.975$

$0.1\sqrt{n} \geq 1.96 \times 2$

$n \geq 1537$

(2) $\bar{X} \sim N(\mu, \frac{4}{n})$ $\quad D\bar{X} = E\bar{X}^2 - (E\bar{X})^2$

$\bar{X}-\mu \sim N(0, \frac{4}{n})$ $\quad \frac{4}{n} = E\bar{X}^2 - \mu^2$

$E(|\bar{X}-\mu|^2) \leq 0.1$ $\quad E\bar{X}^2 = \frac{4}{n}+\mu^2$

$E(\bar{X}^2 - 2\mu\bar{X} + \mu^2) \leq 0.1$

$E\bar{X}^2 - 2\mu E\bar{X} + \mu^2 \leq 0.1$

$\frac{4}{n} + \mu^2 - 2\mu^2 + \mu^2 \leq 0.1$

$\frac{4}{n} \leq 0.1$

$n \geq 40$

12. $f_{X_{(5)}}(x) = \begin{cases} 30x(3x^2-2x^3)^4(1-x) & x \in (0,1) \\ 0 & \text{else} \end{cases}$

13. $P(X_{(16)} > 10) = 0.9370$ $\quad P(X_{(1)} > 5) = 0.3308$

15 $X_1+X_2+X_3 \sim \chi^2(3n)$

$X_4 \sim \chi^2(n)$

$$Y=\frac{(X_1+X_2+X_3)/3n}{X_4/n} \sim F(3n,n)$$

$$\Rightarrow \frac{X_1+X_2+X_3}{3X_4} \sim F(3n,n)$$

16. $X\sim N(\mu,\sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\Rightarrow \frac{15S^2}{\sigma^2} \sim \chi^2(15)$$

$$P\left\{\frac{S^2}{\sigma^2} \le 2.039\right\}$$

$$=P\left\{\frac{15S^2}{\sigma^2} \le 30.585\right\}$$

$$=1-P\left\{\frac{15S^2}{\sigma^2} > 30.585\right\}$$

$$=1-0.01$$

$$=0.99$$

17. $X\sim N(0,\sigma^2)$

$X_1 \sim N(0,\sigma^2)$

$X_2 \sim N(0,\sigma^2)$

$X_1+X_2 \sim N(0,2\sigma^2)$

$X_1-X_2 \sim N(0,2\sigma^2)$

$$\left(\frac{X_1+X_2}{\sqrt{2}\sigma}\right)^2 \sim \chi^2(1)$$

$$\left(\frac{X_1-X_2}{\sqrt{2}\sigma}\right)^2 \sim \chi^2(1)$$

$$\frac{\frac{(X_1+X_2)^2}{2\sigma^2}}{\frac{(X_1-X_2)^2}{2\sigma^2}} \sim F(1,1)$$

$E(X_1+X_2)=0$ $E(X_1-X_2)=0$

$Cov(X_1+X_2, X_1-X_2)$

$= E((X_1+X_2)(X_1-X_2)) - E(X_1+X_2)E(X_1-X_2)$

$= E(X_1^2-X_2^2)$ $DX=EX_1^2-E(X_1)^2$

$= EX_1^2 - EX_2^2$ $EX_1^2=\sigma^2$

$=0$ $EX_2^2=\sigma^2$

$\Rightarrow \rho=0$

$\Rightarrow$ X_1+X_2, X_1-X_2 are independent

18. $Y \sim \chi^2(3)$ $\quad X \sim N(0,4)$ $\quad X_i \sim N(0,4)$

$x_1+x_2 \sim N(0,8)$

$x_3+x_4+x_5 \sim N(0,12)$

$x_6+x_7+x_8+x_9 \sim N(0,16)$

$\left(\frac{x_1+x_2}{\sqrt{8}}\right)^2 \sim \chi^2(2)$ $\quad \left(\frac{x_3+x_4+x_5}{\sqrt{12}}\right)^2 \sim \chi^2(3)$ $\quad \left(\frac{x_6+x_7+x_8+x_9}{\sqrt{6}}\right)^2 \sim \chi^2(4)$

$a=\frac{1}{8}$ $\quad b=\frac{1}{12}$ $\quad c=\frac{1}{16}$

19. $ET = E\frac{1}{n}\sum_{i=1}^{n} x_i^2$

$= \frac{1}{n}\sum_{i=1}^{n} Ex_i^2$

$= \mu^2+\sigma^2$

$DX = EX^2-(EX)^2$

$\sigma^2 = EX^2-\mu^2$

$EX^2=\mu^2+\sigma^2$

20. $x \sim N(\mu,\sigma^2)$

$E(Y_1-Y_2) = E\left(\frac{1}{6}\sum_{i=1}^{6} x_i - \frac{x_7+x_8+x_9}{3}\right)$

$= \frac{1}{6}\sum_{i=1}^{6} Ex_i - \frac{1}{3}(3Ex_i)$

$= \mu-\mu$

$=0$

$\frac{(3-1)S^2}{\sigma^2} \sim \chi^2(2)$

$Y_1-Y_2 \sim N(0,\frac{1}{2}\sigma^2)$

$D(Y_1-Y_2) = D\left(\frac{1}{6}\sum_{i=1}^{6} x_i\right)+D\left(\frac{1}{3}(x_7+x_8+x_9)\right)$

$= \frac{1}{6}\sigma^2+\frac{1}{3}\sigma^2$

$= \frac{1}{2}\sigma^2$

$\frac{\sqrt{2}(Y_1-Y_2)}{\sigma} \sim N(0,1)$

$$\frac{\frac{\sqrt{2}(Y_1-Y_2)}{\sigma}}{\sqrt{\frac{S^2}{\sigma^2}}} \sim t(2)$$

$$\frac{\sqrt{2}(Y_1-Y_2)}{S} \sim t(2)$$

$\Rightarrow Z \sim t(2)$

21. $X \sim N(\mu_1, \sigma_1^2)$ $Y \sim N(\mu_2, \sigma_2^2)$ $m=8$ $n=10$

$$P\{\sigma_1^2 < \sigma_2^2\}$$
$$= P\left\{\frac{\sigma_1^2}{\sigma_2^2} < 1\right\}$$
$$= P\left\{\frac{\sigma_2^2}{\sigma_1^2} > 1\right\}$$
$$= P\left\{\frac{S_1^2\sigma_2^2}{\sigma_1^2 S_2^2} > \frac{8.75}{2.66}\right\}$$
$$= P\{F > 3.29\}$$
$$= 0.05$$

$$\frac{S_1^2\sigma_2^2}{S_2^2\sigma_1^2} \sim F(7,9)$$

14. $X \sim N(0,1)$ $x_i \sim N(0,1)$ $\bar{x} \sim N(0, \frac{1}{n})$ $D\bar{x} = \frac{1}{n}$

$$\sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$
$$= x_1^2 + \cdots + x_n^2 - n\bar{x}^2$$
$$= x_1^2 + \cdots + x_n^2 - \frac{(n\bar{x})^2}{n}$$
$$= (x_1^2 + \cdots + x_n^2) - \frac{1}{n}\left(\frac{n\bar{x}}{\sqrt{n}}\right)^2 n$$
$$= (x_1^2 + \cdots + x_n^2) - \left(\frac{n\bar{x}}{\sqrt{n}}\right)^2$$
$$= \chi^2(n-1)$$

$n\bar{x} \sim N(0, n)$ $Dn\bar{x} = n^2 D\bar{x} = n$

$$\frac{n\bar{x}}{\sqrt{n}} \sim N(0,1)$$
$$\left(\frac{n\bar{x}}{\sqrt{n}}\right)^2 \sim \chi^2(1)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$
$$\left(\frac{n\bar{x}}{\sqrt{n}}\right)^2 \sim \chi^2(1)$$
$$\frac{\left(\frac{n\bar{x}}{\sqrt{n}}\right)^2}{\frac{(n-1)S^2}{\sigma^2}/n-1} \sim F(1, n-1)$$
$$\frac{n\bar{x}^2}{S^2} \sim F(1, n-1)$$

7-1

1. $\bar{x} = \frac{1050 + \cdots + 1080}{8} = 1143.75 = \hat{\mu}$ $\qquad S^2 = \frac{1}{7}\sum_{i=1}^{8}(x_i - 1143.75)^2 = \hat{\sigma}^2 = 9226.785714 \qquad \hat{\sigma} = 96.056$

2. $\hat{\mu} = p \qquad p = \bar{x} \qquad P(x=1) = p \quad P(x=0) = 1-p$

$$L(p) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$$
$$= p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$$
$$\ln L(p) = \sum_{i=1}^{n} x_i \ln p + (n - \sum_{i=1}^{n} x_i)\ln(1-p)$$
$$\frac{d\ln L(p)}{dp} = \frac{\sum_{i=1}^{n} x_i}{p} - \frac{n - \sum_{i=1}^{n} x_i}{1-p} = 0$$
$$\Rightarrow (1-p)\sum_{i=1}^{n} x_i - p(n - \sum_{i=1}^{n} x_i) = 0$$
$$\sum_{i=1}^{n} x_i - p\sum_{i=1}^{n} x_i - np + p\sum_{i=1}^{n} x_i = 0$$
$$p(\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i - n) = -\sum_{i=1}^{n} x_i$$
$$p = \frac{1}{n}\sum_{i=1}^{n} x_i$$
$$= \bar{x}$$

3. $X \sim Geo(p) \qquad \mu = \frac{1}{p} \qquad p = \frac{1}{\mu} \qquad \hat{\mu} = \bar{x} \qquad \hat{p} = \frac{1}{\bar{x}}$

4. $X \sim B(n,p) \qquad \mu = np \qquad \hat{\mu} = \bar{x} \qquad \hat{p} = \frac{\bar{x}}{n}$

$$P(x=k) = {}^{n}C_{k}\, p^{k}(1-p)^{n-k}$$
$$L(p) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, p^{x_i}(1-p)^{n-x_i}$$
$$= \prod_{i=1}^{n} {}^{n}C_{x_i}\; p^{\sum_{i=1}^{n} x_i}(1-p)^{n^2 - \sum_{i=1}^{n} x_i}$$
$$\ln L(p) = \ln\left(\prod_{i=1}^{n} {}^{n}C_{x_i}\right) + \sum_{i=1}^{n} x_i \ln p + (n^2 - \sum_{i=1}^{n} x_i)\ln(1-p)$$
$$\frac{d(\ln L(p))}{dp} = \frac{\sum_{i=1}^{n} x_i}{p} - \frac{n^2 - \sum_{i=1}^{n} x_i}{1-p} = 0$$
$$(1-p)\sum_{i=1}^{n} x_i - n^2 p + p\sum_{i=1}^{n} x_i = 0$$
$$\sum_{i=1}^{n} x_i - p\sum_{i=1}^{n} x_i - n^2 p + p\sum_{i=1}^{n} x_i = 0$$
$$\hat{p} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i}{n}$$
$$= \frac{\bar{x}}{n}$$

6. $X \sim U[\theta-\frac{1}{2}, \theta+\frac{1}{2}]$ $\quad EX=\mu=\theta \quad \hat{\theta}=\overline{X}$

7. $X \sim N(0, \sigma^2)$

$$f(x;\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$$

$$L(\sigma^2)=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_i^2}{2\sigma^2}}$$

$$=(2\pi)^{-\frac{n}{2}}(\sigma^2)^{-\frac{n}{2}}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}x_i^2}$$

$$\ln L(\sigma^2)=-\frac{n}{2}\ln(2\pi)-\frac{n}{2}\ln(\sigma^2)-\frac{1}{2\sigma^2}\sum_{i=1}^{n}x_i^2$$

$$\frac{d\ln L(\sigma^2)}{d\sigma^2}=\frac{-n}{2\sigma^2}+\frac{1}{2\sigma^4}\sum_{i=1}^{n}x_i^2=0$$

$$\frac{-n\sigma^2+\sum_{i=1}^{n}x_i^2}{2\sigma^4}=0$$

$$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2=A_2$$

8. $X \sim P(\lambda)$

$$P\{X=k\}=\frac{\lambda^k}{k!}e^{-\lambda}$$

$$L(\lambda)=\prod_{i=1}^{n}\frac{\lambda^{x_i}}{x_i!}e^{-\lambda}=\frac{1}{\prod_{i=1}^{n}x_i!}\lambda^{\sum_{i=1}^{n}x_i}e^{-n\lambda}$$

$$\ln L=-\ln\left(\prod_{i=1}^{n}x_i!\right)+\left(\sum_{i=1}^{n}x_i\right)\ln\lambda-n\lambda$$

$$\frac{d\ln L}{d\lambda}=\frac{1}{\lambda}\sum_{i=1}^{n}x_i-n=0$$

$$\hat{\lambda}=\frac{1}{n}\sum_{i=1}^{n}x_i=\overline{x}$$

$$\overline{x}=1.90=\hat{\lambda}$$

5. $X \sim B(n,p)$ $\quad \mu_1=EX=np \quad V_2=\mathrm{Var}X=np(1-p)$

$$\begin{cases} np=\mu_1 \\ V_2=np(1-p) \end{cases} \qquad A_1=\overline{X}=\hat{\mu} \qquad B_2=S_0^2=\hat{V_2}$$

$$B_2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2$$

$$S^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2$$

$$\frac{n-1}{n}S^2=B_2$$

$$\Rightarrow \begin{cases} \hat{n}\hat{p}=\overline{X} \\ \frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{x})^2=\hat{n}\hat{p}(1-\hat{p}) \end{cases} \qquad 1-p=\frac{B_2}{\overline{x}} \qquad p=\frac{\overline{x}-B_2}{\overline{x}}$$

$$\hat{n}=\frac{\overline{x}}{\hat{p}}=\frac{\overline{x}^2}{\overline{x}-B_2}=\frac{\overline{x}^2}{\overline{x}-\frac{n-1}{n}S^2}$$

9. 设湖中有N条鱼，$P=\frac{1000}{N}$

$X\sim b(1,p)$ $\qquad \sum_{i=1}^{150}x_i=10$

$L(p)=\prod_{i=1}^{n}p^{x_i}(1-p)^{1-x_i}=p^{\sum_{i=1}^{n}x_i}(1-p)^{n-\sum_{i=1}^{n}x_i}$

$\ln L(p)=\sum_{i=1}^{n}x_i\ln p+(n-\sum_{i=1}^{n}x_i)\ln(1-p)$

$\frac{d\ln L(p)}{dp}=\sum_{i=1}^{n}x_i\frac{1}{p}-\frac{n-\sum_{i=1}^{n}x_i}{1-p}=0 \quad \Rightarrow p=\frac{1}{n}\sum_{i=1}^{n}x_i=\overline{X}$

$\hat{P}=\overline{X} \qquad N=\frac{1000}{P} \qquad \hat{N}=\frac{1000}{\overline{X}}$

$\Rightarrow \hat{N}=\frac{1000}{\frac{10}{150}}=15000$

7-2

1.(1)(i) $X_i\sim ?(\mu,\sigma^2)$ $\qquad EZ_1=EX_1=\mu$

(ii) $EZ_2=E\frac{1}{3}X_1+E\frac{2}{3}X_2=\frac{1}{3}\mu+\frac{2}{3}\mu=\mu$

(iii) $EZ_3=E\frac{1}{3}X_1+E\frac{1}{3}X_2+E\frac{1}{3}X_3=\mu$

(iv) $EZ_4=E\frac{1}{4}X_1+E\frac{1}{4}X_2+E\frac{1}{4}X_3+E\frac{1}{4}X_4=\mu$

(2) (i) $DZ_1=\sigma^2$ $\qquad$ (ii) $DZ_2=\frac{1}{9}\sigma^2+\frac{4}{9}\sigma^2=\frac{5}{9}\sigma^2$

(iii) $DZ_3=\frac{1}{9}\sigma^2+\frac{1}{9}\sigma^2+\frac{1}{9}\sigma^2=\frac{1}{3}\sigma^2$

(iv) $DZ_4=\frac{1}{4}\sigma^2$ ✓

2. (1) $X\sim U[0,\theta]$ $\qquad EX=\mu=\frac{\theta}{2}$ $\qquad Var(X)=\sigma^2=\frac{\theta^2}{12}$

$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$E\overline{X}=\frac{1}{n}\sum_{i=1}^{n}EX_i=\frac{1}{n}\sum_{i=1}^{n}\mu=\mu=\frac{\theta}{2}$

$E\hat{\theta}_1=E(2\overline{X})=2E\overline{X}=2(\frac{\theta}{2})=\theta$

(2) $F_{X_{(n)}}(x)=P(X_{(n)}\le x)=P(\max_{1\le i\le n}X_i\le x)=P(X_1\le x\cap\cdots\cap X_n\le x)=P(X_1\le x)\cdots P(X_n\le x)$

$$=\begin{cases}0 & x<0\\ (\frac{x}{\theta})^n=\frac{1}{\theta^n}x^n & 0\le x\le\theta\\ 1 & x>\theta\end{cases}$$

$$f_{X_{(n)}}(x)=\frac{d}{dx}F_{X_{(n)}}(x)=\begin{cases}\frac{n}{\theta^n}x^{n-1} & 0\le x\le\theta\\ 0 & \text{else}\end{cases}$$

$$E[X_{(n)}]=\int_0^\theta x\frac{n}{\theta^n}x^{n-1}dx=\frac{n}{\theta^n}\int_0^\theta x^n dx=\frac{n}{\theta^n}\left[\frac{x^{n+1}}{n+1}\right]_0^\theta=\frac{n}{\theta^n}\times\frac{\theta^{n+1}}{n+1}=\frac{n}{n+1}\theta$$

$E(\hat{\theta}_2) = E\left(\frac{n+1}{n}X_{(n)}\right) = \frac{n+1}{n}E(X_{(n)}) = \frac{n+1}{n}\times\frac{n}{n+1}\theta = \theta$

$Var(\hat{\theta}_1) = Var(2\bar{x}) = 4Var(\bar{x}) = 4Var\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right) = \frac{4}{n^2}Var\left(\sum_{i=1}^{n}x_i\right) = \frac{4}{n^2}\sum_{i=1}^{n}Var(x_i) = \frac{\theta^2}{3n}$

$E(X_{(n)}^2) = \int_0^\theta x^2\frac{n}{\theta^n}x^{n-1}dx = \frac{n}{\theta^n}\int_0^\theta x^{n+1}dx = \frac{n}{\theta^n}\left[\frac{x^{n+2}}{n+2}\right]_0^\theta = \frac{n}{\theta^n}\times\frac{\theta^{n+2}}{n+2} = \frac{n}{n+2}\theta^2$

$Var(X_{(n)}) = \left(\frac{n}{n+2}\theta^2\right) - \left(\frac{n}{n+1}\theta\right)^2 = \frac{n\theta^2}{n+2} - \frac{n^2\theta^2}{(n+1)^2} = \frac{n}{(n+1)^2(n+2)}\theta^2$

$Var(\hat{\theta}_2) = Var\left(\frac{n+1}{n}X_{(n)}\right) = \frac{(n+1)^2}{n^2}Var(X_{(n)}) = \frac{(n+1)^2}{n^2}\left(\frac{n}{(n+1)^2(n+2)}\right) = \frac{1}{n(n+2)}\theta^2$

$n\geq 1$, $n+2\geq 3$, $n(n+2)\geq 3n$, $\frac{1}{n(n+2)}\leq\frac{1}{3n}$, $\frac{1}{n(n+2)}\theta^2\leq\frac{\theta^2}{3n}$

$\Rightarrow Var(\hat{\theta}_2)\leq Var(\hat{\theta}_1)$

3. $X\sim N(\mu,\sigma^2)$

$E\hat{\sigma}^2 = EC\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2$

$= C\sum_{i=1}^{n-1}E(X_{i+1}-X_i)^2$

$= C\sum_{i=1}^{n-1}\left(D(x_{i+1}-x_i)+\left(E(x_{i+1}-x_i)\right)^2\right)$

$= C\sum_{i=1}^{n-1}(DX_{i+1}+Dx_i)$

$= C\ \ 2(n-1)\sigma^2$

$\Rightarrow C = \frac{1}{2(n-1)}$

4. $EX=\mu$ $DX=\sigma^2$

$E\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}E(x_i-\mu)^2$

$= \frac{1}{n}(n\sigma^2)$

$= \sigma^2$

5. $X\sim N(\mu,\sigma^2)$

$E\hat{\mu}^2 = E\bar{X}^2$

$= \mu^2+\frac{\sigma^2}{n}\neq\mu^2$

$\Rightarrow$ 不是无偏估计

$D\bar{X} = E\bar{X}^2-(E\bar{X})^2$

$\frac{\sigma^2}{n}+\mu^2 = E\bar{X}^2$

6. $E\hat{\theta}_1=\theta$ $E\hat{\theta}_2=\theta$ $D\hat{\theta}_1=\sigma_1^2$ $D\hat{\theta}_2=\sigma_2^2$

$E\hat{\theta} = E(c\hat{\theta}_1+(1-c)\hat{\theta}_2)$

$= cE\hat{\theta}_1+(1-c)E\hat{\theta}_2$

$= c\theta+(1-c)\theta$

$= \theta$

$D\hat{\theta} = D(c\hat{\theta}_1+(1-c)\hat{\theta}_2)$

$= c^2D\hat{\theta}_1+(1-c)^2D\hat{\theta}_2$

$= c^2\sigma_1^2+(1-c)^2\sigma_2^2$

$\frac{d(D\hat{\theta})}{dc} = 2c\sigma_1^2-2(1-c)\sigma_2^2 = 0$

$2c\sigma_1^2-2\sigma_2^2+2c\sigma_2^2=0$

$c = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$

7. $L(\theta)=\prod_{i=1}^{n} e^{-x_i+\theta}$

$$=\begin{cases} e^{-\sum_{i=1}^{n} x_i+n\theta} \\ 0 \end{cases} = \begin{cases} \dfrac{e^{n\theta}}{e^{\sum_{i=1}^{n} x_i}} & x_i>\theta \\ 0 & x_i<\theta \end{cases}$$

$\hat{\theta}=\min(x_1,x_2,\dots,x_n)$

$EX=\mu$

$$EX=\int_\theta^\infty x e^{-x+\theta}dx$$
$$=\int_\theta^\infty x\, d(-e^{-x+\theta})$$
$$=\left[-xe^{-x+\theta}\right]_\theta^\infty+\int_\theta^\infty e^{-x+\theta}dx$$
$$=-\theta+\left[-e^{-x+\theta}\right]_\theta^\infty$$
$$=1-\theta$$

不是

8. $\forall \varepsilon>0 \quad \lim_{n\to\infty} P(|\hat{\theta}-\theta|<\varepsilon)=1$

$\Rightarrow \lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} x_i^k - EX^k\right|<\varepsilon\right)=1$

EX^k 存在，$x_1,x_2,\cdots,x_n$ 独立同分布

$x_1^k,x_2^k,\cdots,x_n^k$ 也独立同分布

由辛钦大数定理

7-3

3. $X\sim N(\mu,25^2)$ $\quad n=9$

$\dfrac{\sqrt{9}(\bar{X}-\mu)}{25}\sim N(0,1)$

$\bar{x}=\dfrac{497+\cdots+511}{9}=509.3$

$1-\alpha=0.95$

$\alpha=0.05$

$P(U>u_{\frac{\alpha}{2}})=0.025$ $\qquad 1-P(U\leq u_{\frac{\alpha}{2}})=0.025$

$\Phi_0(u_{\frac{\alpha}{2}})=0.975$

$u_{\frac{\alpha}{2}}=1.96$

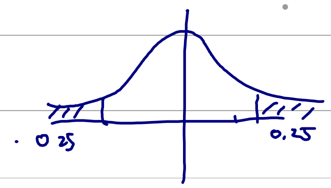
$P\left(-u_{\frac{\alpha}{2}}\leq \dfrac{\sqrt{n}(\bar{x}-\mu)}{\sigma}\leq u_{\frac{\alpha}{2}}\right)=1-\alpha$

$\bar{x}+\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}=525.6$

$CI=[493.0,525.6]$

$\bar{x}-\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}=493.0$

4. $X \sim N(\mu, \sigma^2)$ $\quad 1-\alpha = 0.05$



$$\bar{x} = \frac{68 + \cdots + 68}{10} = 70$$

$$T = \frac{\sqrt{n}(\bar{x}-\mu)}{S} \sim t(n-1) \qquad S = 2.49$$

$$P(T > t_{\frac{\alpha}{2}}) = 2.262$$

$$\left[\bar{x} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1),\ \bar{x} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1)\right]$$

$$= [\ 68.2\ ,\ 71.8\ ]$$

5. $X \sim N(\mu, \sigma^2)$ (1) $\sigma = 15$ $\quad \bar{x} = \frac{101 + \cdots + 144}{10} = 129.1$ kg

$$\sqrt{n}\frac{\bar{x}-\mu}{\sigma} \sim N(0,1) \qquad 1-\alpha = 0.05$$

$$u_{\frac{\alpha}{2}} = 1.96$$

$$\left[\frac{-1.96 \times 15}{\sqrt{10}} + 129.1\ ,\ \frac{1.96 \times 15}{\sqrt{10}} + 129.1\right]$$

$$= [\ 119.8\ ,\ 138.4\ ]$$

(2) $\sqrt{n}\frac{\bar{x}-\mu}{S} \sim t(n-1)$ $\quad S = 16.7$

$$P(T > t_{\frac{\alpha}{2}}) = 2.262$$

$$\left[\bar{x} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1),\ \bar{x} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1)\right]$$

$$= [\ 117.14\ ,\ 141.06\ ]$$

6. $X \sim N(\mu, \sigma^2)$

$$\sqrt{n}\frac{\bar{x}-\mu}{S} \sim t(n-1) \qquad P(T > t_{\frac{\alpha}{2}}) = 1.699$$

$$\bar{x} = 73.8\dot{3} \qquad S = 11.19446702$$

$$\left[\bar{x} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(29),\ \bar{x} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}(29)\right]$$

$$= [\ 70.36\ ,\ 77.3\ ]$$

7. $X \sim N(\mu, 1)$

$$CI = \left[\bar{X} - \frac{u_{\frac{\alpha}{2}}}{\sqrt{n}},\ \bar{X} + \frac{u_{\frac{\alpha}{2}}}{\sqrt{n}}\right] \leq 0.2$$

$$\frac{2u_{\frac{\alpha}{2}}}{\sqrt{n}} \leq 0.2 \qquad u_{\frac{\alpha}{2}} = 2.58$$

$$n \geq 665.64$$

$$n \geq 666$$

8. $X \sim N(0, \sigma^2)$ $\quad n=26 \quad \bar{X} = 0.2 \quad S=2 \quad 1-\alpha = 0.9$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \qquad \chi^2_{0.05}(25) = 37.652 \quad \chi^2_{0.95}(25) = 14.611$

$$P\left(\chi^2_{0.95}(25) \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{0.05}(25)\right) = 0.9$$

$$P\left(\chi^2_{0.95}(25) \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{0.05}(25)\right) = 0.9$$

$$CI = \left[\frac{(n-1)S^2}{\chi^2_{0.05}(25)},\ \frac{(n-1)S^2}{\chi^2_{0.95}(25)}\right]$$

$$= [2.65,\ 6.85]$$

9. $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \qquad \bar{X} = \frac{14.6 + \cdots + 15.1}{5} = 14.98 \quad S^2 = 0.057 \qquad \chi^2_{0.025}(4) = 11.1$

$\chi^2_{0.975}(4) = 0.484$

$$CI = \left[\frac{4\times 0.057}{11.1},\ \frac{4\times 0.057}{0.484}\right]$$

$$= [0.021,\ 0.479]$$

10. (1) $1-\alpha = 0.9 \quad \alpha = 0.1 \quad \frac{\alpha}{2} = 0.05 \qquad \bar{X} = 1147 \qquad t_{0.05}(9) = 1.833$

$\sqrt{n}\,\frac{\bar{X}-\mu}{S} \sim t(n-1) \qquad CI = \left[\bar{X} - \frac{S}{\sqrt{n}} t_{0.05}(9),\ \bar{X} + \frac{S}{\sqrt{n}} t_{0.05}(9)\right]$

$$= [1096.5,\ 1197.5]$$

(2) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \qquad \chi^2_{0.05}(9) = 16.919 \quad \chi^2_{0.95}(9) = 3.325$

$$CI = \left[\frac{(n-1)S^2}{\chi^2_{0.05}(9)},\ \frac{(n-1)S^2}{\chi^2_{0.95}(9)}\right]$$

$$= [4031.6,\ 20514.3]$$

11. $X \sim N(\mu, 2.18^2)$ $Y \sim N(\mu, 1.76^2)$ $n_X = 200$ $n_Y = 100$ $\bar{x} = 5.32$ $\bar{y} = 5.76$

$$U = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_X} + \frac{\sigma_2^2}{n_Y}}}$$

$$\sqrt{\frac{2.18^2}{200} + \frac{1.76^2}{100}} = 0.233961535$$

$1 - \alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$U_{\frac{\alpha}{2}} = 1.96$

$$CI = [5.32 - 5.76 - 1.96(0.233961535), 5.32 - 5.76 + 1.96(0.233961535)]$$

$$= [-0.899, 0.019]$$

12. $\bar{X} = 600$ $S_X^2 = 711.111$ $\bar{Y} = 570$ $S_Y^2 = 266.66667$

$1 - \alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$t_{0.025}(18) = 2.101$

$$T = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(18)$$

$$CI = [9.23, 50.77]$$

13. $S_X^2 = 0.5419$ $S_Y^2 = 0.6065$ $n_X = 10$ $n_Y = 10$

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(9,9)$$

$1 - \alpha = 0.9$

$\alpha = 0.1$

$\alpha = 0.05$

$F_{0.05}(9,9) = 3.18$

$$F_{0.95}(9,9) = \frac{1}{F_{0.05}(9,9)} = 0.314465408$$

$$CI = \left[\frac{S_1^2}{S_2^2} \frac{1}{F_{0.05}(9,9)}, \frac{S_1^2}{S_2^2}, \frac{1}{F_{0.95}(9,9)}\right]$$

$$= [0.28, 2.84]$$

七

1. (1) $\hat{\mu} = 34$ $\quad \hat{\sigma}_2 = 2178$

(2) $\mu = E\hat{\mu}$ $\qquad a = \frac{1}{2}$

$= \frac{1}{3}\mu + a\mu + \frac{1}{6}\mu$

(3) $L(\theta) = \prod_{i=1}^{7} \frac{1}{\theta} = \frac{1}{\theta^n}$

$\hat{\theta} = \max(x_1, \cdots, x_n)$

(4) $X \sim N(\mu, 0.9^2)$ $\qquad \sqrt{n}\frac{\overline{X}-\mu}{\sigma} \sim N(0,1)$

$1-\alpha = 0.99$

$\alpha = 0.01$ $\qquad [5 - \frac{0.9}{\sqrt{9}} 2.58 , \ 5 + \frac{0.9}{\sqrt{9}} 2.58]$

$\frac{\alpha}{2} = 0.005$ $\qquad = [4.226, 5.774]$

$u_{0.005} = 2.58$

2. (1) $\frac{1}{n}\sum_{i=1}^{5} x_i = \frac{2}{5} = EX = \hat{p}$ $\quad$ C

(2) $EX = \frac{\theta}{2}$ $\qquad \frac{1}{n}\sum_{i=1}^{5} x_i = \frac{10}{5} = 2$ $\qquad \hat{\theta} = 4$ $\quad$ D

(3) A, B, C, D

(4) $EX = mp$ $\qquad E\hat{p}_1 = \frac{EX_i}{m}$ $\qquad E\hat{p}_2 = \frac{E\overline{X}}{m}$ $\qquad E\hat{p}_3 = \frac{1}{2m}(EX_i + E\overline{X})$ $\qquad E\hat{p}_4 = \frac{1}{3m}(EX_i + 2E\overline{X})$

A, B, C, D $\qquad = p \qquad = p \qquad = p \qquad = p$

(5) D $\quad$ (6) A $\quad$ (7) C $\quad$ (8) C

3. $X \sim B(n,p)$ $\qquad \mu_1 = EX = np$ $\qquad \nu_2 = VarX = np(1-p)$

$\begin{cases} np = \mu_1 \\ \nu_2 = np(1-p) \end{cases}$ $\qquad A_1 = \overline{x} = \hat{\mu}$ $\qquad B_2 = S_0^2 = \hat{\nu}_2$ $\qquad B_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2$

$\Rightarrow \begin{cases} \hat{n}\hat{p} = \overline{x} \\ \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \hat{n}\hat{p}(1-p) \end{cases}$ $\qquad 1-p = \frac{B_2}{\overline{x}}$ $\qquad \frac{n-1}{n}S^2 = B_2$

$p = \frac{\overline{x} - B_2}{\overline{x}}$

$\hat{n} = \frac{\overline{x}}{\hat{p}} = \frac{\overline{x}^2}{\overline{x} - B_2} = \frac{\overline{x}^2}{\overline{x} - \frac{n-1}{n}S^2}$

4. $f(x;\theta) = \frac{2}{\theta} - \frac{2x}{\theta^2}$

$$EX = \int_0^\theta \left(\frac{2x}{\theta} - \frac{2x^2}{\theta^2}\right) dx$$

$$= \left[\frac{x^2}{\theta}\right]_0^\theta - \frac{2}{\theta^2}\left[\frac{x^3}{3}\right]_0^\theta$$

$$= \theta - \frac{2\theta}{3} = \frac{1}{3}\theta$$

$\hat{\mu} = \bar{x}$

$\hat{\theta} = 3\bar{x}$

5. $$EX = \int_\theta^\infty x e^{-x+\theta} dx$$

$$= \int_\theta^\infty x \, d(-e^{-x+\theta})$$

$$= \left[-xe^{-x+\theta}\right]_\theta^\infty + \int_\theta^\infty e^{-x+\theta} dx$$

$$= \theta - \left[e^{-x+\theta}\right]_\theta^\infty$$

$$= 1+\theta$$

$\bar{x} = 1+\theta$

$\hat{\theta} = \bar{x} - 1$

6 $$L(\theta) = \prod_{i=1}^n \theta x_i^{\theta-1} = \theta^n \left(\prod_{i=1}^n x_i\right)^{\theta-1}$$

$$\ln L(\theta) = n\ln\theta + (\theta-1)\ln\left(\prod_{i=1}^n x_i\right)$$

$$\frac{d\ln L(\theta)}{d\theta} = \frac{n}{\theta} + \ln\prod_{i=1}^n x_i = \frac{n}{\theta} + \sum_{i=1}^n \ln x_i = 0$$

$$\hat{\theta} = \frac{-n}{\sum_{i=1}^n \ln x_i}$$

7. (1) $X \sim U[\theta, \theta+1]$ $\quad EX = \frac{2\theta+1}{2} = \theta + \frac{1}{2} = \mu$ $\quad E\hat{X} = \theta + \frac{1}{2} \neq \theta$

(2) $\hat{\theta} = \bar{X} - \frac{1}{2}$ $\quad E\hat{\theta} = E\bar{X} - \frac{1}{2} = \theta + \frac{1}{2} - \frac{1}{2} = \theta$

8. $\bar{X} = 2.42$ $\quad \hat{\mu} = \lambda = 2.42$

$$P\{X=0\} = \frac{\lambda^0}{0!} e^{-2.42} = 0.089$$

$$L(\lambda) = \prod_{i=1}^n \frac{\lambda^{x_i}}{x_i!} e^{-\lambda} = \frac{1}{\prod_{i=1}^n x_i!} \lambda^{\sum_{i=1}^n x_i} e^{-n\lambda}$$

$$\ln L = -\ln\left(\prod_{i=1}^n x_i!\right) + \left(\sum_{i=1}^n x_i\right)\ln\lambda - n\lambda$$

$$\frac{d\ln L}{d\lambda} = \frac{1}{\lambda}\sum_{i=1}^n x_i - n = 0$$

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^n x_i = \bar{X}$$

9. (1) $\hat{\mu} = \bar{X} = 3140$ $\quad \hat{\sigma}^2 = B_2 = 178320$

(2) $\hat{\sigma}^2 = S^2 = 198133$

11. $X = \min\{X_1, X_2, \cdots, X_n\}$

$$F_X(x) = P(X \le x) = 1 - P(X > x) = 1 - P(X_1 > x)P(X_2 > x)\cdots P(X_n > x)$$
$$= 1 - [1 - P(X_1 \le x)]\cdots[1 - P(X_n \le x)]$$
$$= 1 - [1 - F_{X_1}(x)]\cdots[1 - F_{X_n}(x)]$$

$$F_{X_1}(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{\theta} & 0 < x \le \theta \\ 1 & x > \theta \end{cases} \qquad F_X(x) = \begin{cases} 0 & x \le 0 \\ 1 - (1 - \frac{x}{\theta})^n & 0 < x \le \theta \\ 1 & x > \theta \end{cases}$$

$$f(x) = \begin{cases} \frac{n}{\theta}(1 - \frac{x}{\theta})^{n-1} & 0 < x \le \theta \\ 0 & \text{else} \end{cases}$$

$$E[(n+1)X] = \int_{-\infty}^{+\infty} (n+1)\, x f(x)\, dx$$
$$= (n+1)\int_0^\theta x \frac{n}{\theta}(1 - \frac{x}{\theta})^{n-1} dx$$
$$= n(n+1)\int_0^\theta (\frac{x}{\theta} - 1 + 1)(1 - \frac{x}{\theta})^{n-1} dx$$
$$= n(n+1)\int_0^\theta \left(-(1 - \frac{x}{\theta})^n + (1 - \frac{x}{\theta})^{n-1}\right) dx$$
$$= n(n+1)\left[\theta \frac{(1 - \frac{x}{\theta})^{n+1}}{n+1} - \theta \frac{(1 - \frac{x}{\theta})^n}{n}\right]_0^\theta$$
$$= n(n+1)\theta\left(\frac{n+1-n}{n(n+1)}\right)$$
$$= \theta$$

$\Rightarrow \hat{\theta} = (n+1)\min(X_1, X_2, \cdots, X_n)$ 是 θ 的无偏估计量

12. $L(\theta) = \prod_{i=1}^{n} \frac{x_i}{\theta} e^{\frac{-x_i^2}{2\theta}}$

$= \frac{\prod_{i=1}^{n} x_i}{\theta^n} e^{\frac{-\sum_{i=1}^{n} x_i^2}{2\theta}}$

$\ln L(\theta) = \ln \prod_{i=1}^{n} x_i - \frac{\sum_{i=1}^{n} x_i^2}{2\theta} - n\ln\theta$

$\frac{d\ln L(\theta)}{d\theta} = \frac{\sum_{i=1}^{n} x_i^2}{2\theta^2} - \frac{n}{\theta} = 0$

$\frac{\sum_{i=1}^{n} x_i^2 - 2n\theta}{2\theta^2} = 0$

$\hat{\theta} = \frac{1}{2n}\sum_{i=1}^{n} x_i^2$

$E\hat{\theta} = \frac{1}{2n}\sum_{i=1}^{n} Ex_i^2 = \frac{1}{2n}(2n\theta) = \theta$

$EX^2 = \int_0^{\infty} \frac{x^3}{\theta} e^{-\frac{x^2}{2\theta}} dx$

$= 2\theta$

13. (1) $EX = \int_0^1 x\theta\, dx + \int_1^2 (x - x\theta)\, dx$

$= \left[\frac{x^2}{2}\right]_0^1 \theta + \left[\frac{x^2}{2} - \frac{x^2}{2}\theta\right]_1^2$

$= \frac{\theta}{2} + 2 - 2\theta - \frac{1}{2} + \frac{\theta}{2}$

$= 1.5 - \theta$

$\bar{x} = 1.5 - \theta$

$\hat{\theta} = 1.5 - \bar{x}$

(2) $L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$

$= \theta^N (1-\theta)^{n-N}$

$\ln L(\theta) = N\ln\theta + (n-N)\ln(1-\theta)$

$\frac{d\ln L(\theta)}{d\theta} = \frac{N}{\theta} - \frac{n-N}{1-\theta} = 0$

$N(1-\theta) - \theta(n-N) = 0$

$N - N\theta - n\theta + N\theta = 0$

$\hat{\theta} = \frac{N}{n}$

14. (1) $X \sim N(\mu, 0.01^2)$

$\bar{x} = 2.125$

$1-\alpha = 0.95 \quad \alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$u_{\frac{\alpha}{2}} = 1.96$

$\sqrt{n}\frac{\bar{X}-\mu}{\sigma} \sim N(0,1)$

$\left[2.125 - \frac{0.01}{\sqrt{16}} 1.96,\ 2.125 + \frac{0.01}{\sqrt{16}} 1.96\right]$

$= [2.1201, 2.1299]$

(2) $\frac{\bar{x}-\mu}{S}\sqrt{n} \sim t(n-1)$ $\quad t_{0.025}(15) = 2.131$

$$CI = \left[2.125 - \frac{0.01712}{\sqrt{16}}(2.131),\ 2.125 + \frac{0.01712}{\sqrt{16}}(2.131)\right]$$

$$= [2.116,\ 2.134]$$

15. $X \sim N(\mu, \sigma^2)$ $\quad n=12$ $\quad$ (1) $\mu = 2.7$ $\quad \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \sim \chi^2(n)$ $\quad \frac{\alpha}{2} = 0.05$

$$CI = \left[\frac{\sum_{i=1}^{12}(x_i-2.7)^2}{21.0},\ \frac{\sum_{i=1}^{12}(x_i-2.7)^2}{5.23}\right]$$

$\chi^2_{0.05}(12) = 21.0$

$\chi^2_{0.95}(12) = 5.23$

$\sum_{i=1}^{12}(x_i-2.7)^2$ $\quad = [0.193, 0.777]$

$= \sum_{i=1}^{12}(x_i^2 - 5.4x_i + 7.29)$

$= \sum_{i=1}^{12}x_i^2 - 5.4\sum_{i=1}^{12}x_i + 12\times 7.29$ $\quad \chi^2_{0.05}(11) = 19.7$

$= 4.06$ $\quad \chi^2_{0.95}(11) = 4.57$

(2) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$CI = \left[\frac{(n-1)S^2}{\chi^2_{0.05}(11)},\ \frac{(n-1)S^2}{\chi^2_{0.95}(11)}\right]$$

$S^2 = \frac{1}{n-1}\sum_{i=1}^{12}(x_i-\bar{x})^2$

$= \frac{4.06}{11} = 0.36909$

$$= [0.206,\ 0.886]$$

16.(1) $X \sim N(\mu, 2.8^2)$ $\quad n=10$ $\quad \bar{x} = 1500$ $\quad \frac{\alpha}{2} = 0.025$

$u_{0.975} = 1.96$

$\sqrt{10}\,\frac{\bar{x}-\mu}{2.8} \sim N(0,1)$

$$CI = \left[1500 - \frac{2.8}{\sqrt{10}}\times 1.96,\ 1500 + \frac{2.8}{\sqrt{10}}\times 1.96\right]$$

$$= [1498.265,\ 1501.735]$$

(2) CI's length $= \frac{2\sigma}{\sqrt{n}} \times 1.96 < 1$

$n > 121$

(3) $n = 100$ $\quad \bar{x}+1-\bar{x}+1 = 2$ $\quad 1-\alpha = 0.9996$

$\frac{2\times 2.8}{\sqrt{100}} \times u_\alpha = 2$

$u_\alpha = 3.5714$

17. (1) $f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2}}$

$EX = E(e^y) = \int_{-\infty}^{\infty} e^y \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2}} dy = e^{\frac{2\mu+1}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-(\mu+1))^2}{2}} dy = e^{\mu+\frac{1}{2}}$

(2) $\sqrt{n}\frac{\bar{Y}-\mu}{\sigma} \sim N(0,1)$ $\quad \bar{Y} = \frac{\ln 0.5 + \cdots + \ln 2}{4} = 0$

$CI = \left[-\frac{u_{\frac{\alpha}{2}}}{\sqrt{n}}, \frac{u_{\frac{\alpha}{2}}}{\sqrt{n}}\right] = [-0.98, 0.98]$

(3) $0.95 = P\{-0.98+\frac{1}{2} < \mu+\frac{1}{2} < 0.98+\frac{1}{2}\}$

$P\{e^{-0.48} < EX < e^{1.48}\} = 0.95$

$CI = [e^{-0.48}, e^{1.48}]$

18. $X \sim N(\mu_1, 9)$ $\quad Y \sim N(\mu_2, 16)$ $\qquad \alpha = 0.1$

$\frac{\alpha}{2} = 0.05$

$\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ $\qquad u_{\frac{\alpha}{2}} = 1.65$

$CI = \left[(95-90) - 1.65\sqrt{\frac{9}{25}+\frac{16}{30}}, (95-90)+1.65\sqrt{\frac{9}{25}+\frac{16}{30}}\right]$

$= [3.44, 6.56]$

19. $\bar{X} = 1.718$ $\quad S_X^2 = 0.00432$ $\quad \bar{Y} = 1.61$ $\quad S_Y^2 = 0.004$ $\qquad \frac{\alpha}{2} = 0.025$

$t_{0.025}(8) = 2.306$

$CI = \left[(1.718-1.61) - 2.306\sqrt{\frac{4\times0.00432+4\times0.004}{8}}\sqrt{\frac{1}{5}+\frac{1}{5}}, (1.718-1.61)+2.306\sqrt{\frac{4\times S_X^2+4\times S_Y^2}{8}}\sqrt{\frac{1}{5}+\frac{1}{5}}\right]$

$= [0.014, 0.202]$

20. $n_X = 6$ $\quad n_Y = 9$ $\qquad S_1^2 = 0.245$ $\quad S_2^2 = 0.357$ $\qquad F_{0.025}(5,8) = 4.82$

$F_{0.975}(5,8) = \frac{1}{F_{0.025}(8,5)} = \frac{1}{6.76}$

$CI = \left[\frac{0.245}{0.357}\frac{1}{4.82}, \frac{0.245}{0.357}\times 6.76\right]$

$= [0.142, 4.639]$

10.(1) $L(\theta)=\theta^{2n_1}(2\theta-2\theta^2)^{n_2}(1-\theta)^{2(n-n_1-n_2)}$

$\ln L(\theta)=2n_1\ln\theta+n_2\ln(2\theta-2\theta^2)+(2n-2n_1-2n_2)\ln(1-\theta)$

$\frac{d\ln L(\theta)}{d\theta}=\frac{2n_1}{\theta}+\frac{(2-4\theta)n_2}{2\theta(1-\theta)}-\frac{2n-2n_1-2n_2}{1-\theta}=0$

$4n_1-4n_1\theta+2n_2-4n_2\theta-4n\theta+4n_1\theta+4n_2\theta=0$

$\theta(-4n_1-4n_2-4n+4n_1+4n_2)=-4n_1-2n_2$

$\hat{\theta}=\frac{-4n_1-2n_2}{-4n}$

$=\frac{2n_1+n_2}{2n}$

(2) $EX=\overline{X}$

$\overline{X}=\theta^2+4\theta(1-\theta)+3(1-\theta)^2$

$=\theta^2+4\theta-4\theta^2+3-6\theta+3\theta^2$

$=3-2\theta$

$\overline{X}=3-2\theta$

$\hat{\theta}=\frac{3-\overline{X}}{2}$

(3) 极大似然：$n_1=3\quad n_2=2\quad n=6\qquad \hat{\theta}=\frac{2\times3+2}{2\times6}=\frac{2}{3}$

矩估计：$\hat{\theta}=\frac{3-\left(\frac{1+1+2+1+3+2}{6}\right)}{2}$

$=\frac{2}{3}$

21. n次独立 　　ξ_n A发生次数 　　n较大， $P(A)=p$

$$\xi_n \sim Bin(n,p)$$

$$E(\xi_n)=np$$

$$Var(\xi_n)=np(1-p)$$

$$\hat{p}=\frac{\xi_n}{n}$$

$$E(\hat{p})=E\left(\frac{\xi_n}{n}\right)=\frac{1}{n}E(\xi_n)=\frac{1}{n}\times np=p$$

$$Var(\hat{p})=Var\left(\frac{\xi_n}{n}\right)=\frac{1}{n^2}Var(\xi_n)=\frac{1}{n^2}\times np(1-p)=\frac{p(1-p)}{n}$$

$$\frac{\hat{p}-E(\hat{p})}{\sqrt{Var(\hat{p})}}=\frac{\frac{\xi_n}{n}-p}{\sqrt{\frac{p(1-p)}{n}}}\sim N(0,1)$$

$$P\left\{z_{1-\frac{\alpha}{2}}<\frac{\frac{\xi_n}{n}-p}{\sqrt{\frac{p(1-p)}{n}}}<z_{\frac{\alpha}{2}}\right\}=1-\alpha$$

$$P\left\{-z_{\frac{\alpha}{2}}<\frac{\frac{\xi_n}{n}-p}{\sqrt{\frac{p(1-p)}{n}}}<z_{\frac{\alpha}{2}}\right\}=1-\alpha$$

$$P\left\{\left|\frac{\frac{\xi_n}{n}-p}{\sqrt{\frac{p(1-p)}{n}}}\right|<z_{\frac{\alpha}{2}}\right\}=1-\alpha$$

$$\left|\frac{\xi_n}{n}-p\right|<z_{\frac{\alpha}{2}}\left|\sqrt{\frac{p(1-p)}{n}}\right|$$

$$\frac{\xi_n^2}{n^2}-2\frac{\xi_n}{n}p+p^2<z_{\frac{\alpha}{2}}\frac{p(1-p)}{n}$$

$$\frac{\xi_n^2}{n}-2\xi_n p+np^2<z_{\frac{\alpha}{2}}^2(p-p^2)$$

$$\frac{\xi_n^2}{n}-(2\xi_n+z_{\frac{\alpha}{2}}^2)p+(n+z_{\frac{\alpha}{2}}^2)p^2<0$$

$$(n+z_{\frac{\alpha}{2}}^2)p^2-(2\xi_n+z_{\frac{\alpha}{2}}^2)p<-\frac{\xi_n^2}{n}$$

$$p^2-\left(\frac{2\xi_n+z_{\frac{\alpha}{2}}^2}{n+z_{\frac{\alpha}{2}}^2}\right)p<-\frac{\xi_n^2}{n(n+z_{\frac{\alpha}{2}}^2)}$$

$$p^2-2\left(\frac{\xi_n+\frac{z_{\frac{\alpha}{2}}^2}{2}}{n+z_{\frac{\alpha}{2}}^2}\right)p+\left(\frac{\xi_n+\frac{z_{\frac{\alpha}{2}}^2}{2}}{n+z_{\frac{\alpha}{2}}^2}\right)^2<\left(\frac{\xi_n+\frac{z_{\frac{\alpha}{2}}^2}{2}}{n+z_{\frac{\alpha}{2}}^2}\right)^2-\frac{\xi_n^2}{n(n+z_{\frac{\alpha}{2}}^2)}$$

$$\left(p-\frac{\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}}{n+z^2_{\frac{\alpha}{2}}}\right)^2<\frac{n\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}\right)^2-\xi_n^2\left(n+z^2_{\frac{\alpha}{2}}\right)}{n\left(n+z^2_{\frac{\alpha}{2}}\right)^2}$$

$$=\frac{\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}\right)^2-\xi_n^2\left(1+\frac{z^2_{\frac{\alpha}{2}}}{n}\right)}{\left(n+z^2_{\frac{\alpha}{2}}\right)^2}$$

$$=\left(\xi_n^2+\xi_n z^2_{\frac{\alpha}{2}}+\frac{z^4_{\frac{\alpha}{2}}}{4}\right)-\xi_n^2-\frac{z^2_{\frac{\alpha}{2}}}{n}\xi_n^2$$

$$=\frac{\xi_n z^2_{\frac{\alpha}{2}}+\frac{z^4_{\frac{\alpha}{2}}}{4}-\frac{z^2_{\frac{\alpha}{2}}}{n}\xi_n^2}{\left(n+z^2_{\frac{\alpha}{2}}\right)^2}$$

$$=\frac{z^2_{\frac{\alpha}{2}}}{\left(n+z^2_{\frac{\alpha}{2}}\right)^2}\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{4}-\frac{1}{n}\xi_n^2\right)$$

$$\left|p-\frac{\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}}{n+z^2_{\frac{\alpha}{2}}}\right|<\frac{z_{\frac{\alpha}{2}}}{n+z^2_{\frac{\alpha}{2}}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}$$

$$-\frac{z_{\frac{\alpha}{2}}}{n+z^2_{\frac{\alpha}{2}}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}<p-\frac{\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}}{n+z^2_{\frac{\alpha}{2}}}<\frac{z_{\frac{\alpha}{2}}}{n+z^2_{\frac{\alpha}{2}}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}$$

$$\frac{\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}}{n+z^2_{\frac{\alpha}{2}}}-\frac{z_{\frac{\alpha}{2}}}{n+z^2_{\frac{\alpha}{2}}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}<p<\frac{\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}}{n+z^2_{\frac{\alpha}{2}}}+\frac{z_{\frac{\alpha}{2}}}{n+z^2_{\frac{\alpha}{2}}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}$$

$$\frac{1}{n+z^2_{\frac{\alpha}{2}}}\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}-z_{\frac{\alpha}{2}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}\right)<p$$

$$<\frac{1}{n+z^2_{\frac{\alpha}{2}}}\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}+z_{\frac{\alpha}{2}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}\right)$$

$$CI=\left[\frac{1}{n+z^2_{\frac{\alpha}{2}}}\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}-z_{\frac{\alpha}{2}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}\right),\ \frac{1}{n+z^2_{\frac{\alpha}{2}}}\left(\xi_n+\frac{z^2_{\frac{\alpha}{2}}}{2}+z_{\frac{\alpha}{2}}\sqrt{\frac{z^2_{\frac{\alpha}{2}}}{4}+\xi_n-\frac{1}{n}\xi_n^2}\right)\right]$$

8-1

5. $H_0: \mu = 32.0$ $H_1: \mu \neq 32.0$

6. $H_0: \mu = 10$ $H_1: \mu \neq 10$

7. $H_0: \mu > 30\%$ $H_1: \mu \leq 30\%$

8. $x_1, x_2, \cdots, x_{10} \sim b(1,p)$ $10\bar{x} \sim b(10,p)$

$\alpha = P(\bar{x} \geq 0.5 | H_0) = P(10\bar{x} \geq 5 | H_0) = \sum_{k=5}^{10} {}^{10}C_k \left(\frac{1}{5}\right)^k \left(\frac{4}{5}\right)^{10-k} = 0.0328$

$\beta = P(\bar{x} < 0.5 | H_1) = P(10\bar{x} < 5 | H_1) = \sum_{k=0}^{4} {}^{10}C_k \left(\frac{2}{5}\right)^k \left(\frac{3}{5}\right)^{10-k} = 0.6331$

8-2

1. $X \sim N(25, 0.02)$ $\mu_0 = 25$ $\bar{x} = 25.03$

$U = \frac{\bar{x} - \mu_0}{\sigma}\sqrt{n} \sim N(0,1)$ $\alpha = 0.05$ $U_{0.025} = 1.96$

$u = \frac{25.03 - 25}{\sqrt{0.02}}\sqrt{10} = 0.671 < 1.96$ ⇒ 接受H_0

2. $X \sim N(\mu, 30^2)$ $H_0: \mu \geq 2000$ $\alpha = 0.01$ $U_{0.005} = 2.58$

$u = \frac{1990 - 2000}{30}\sqrt{16} = -1.33 > -2.58$ ⇒ 接受

3. $X \sim N(\mu, \sigma^2)$ (1) $\alpha = 0.05$ $U_{0.05} = 1.65$ $\bar{x} = 100.104$

$u = \frac{100.104 - 100}{0.5}\sqrt{10} = 0.658 < 1.65$ ⇒ 接受

(2) $t = \frac{100.104 - 100}{0.476}\sqrt{10} = 0.691 < 1.833$ ⇒ 接受 $S = 0.476$ $t_{0.05}(9) = 1.833$

4. $X \sim N(\mu, \sigma^2)$ $\bar{x} = 0.52$ $\mu \leq 0.5\%$ $S = 0.02$ $t_{0.05}(4) = 2.132$

$t = \frac{0.52 - 0.5}{0.02}\sqrt{5} = 2.23 > 2.132$ 不接受

5. $\mu = 18$ $t_{0.025}(8) = 2.306$ $|t| = \frac{17.5 - 18}{0.7416}\sqrt{9} = |-2.02| < 2.306$

6. $t=\frac{70-66.5}{15}\times\sqrt{36}=1.4$ $t_{0.025}(35)=2.0301$ ⇒接受

7. $\mu\geqslant 49.1$ $\bar{x}=42.75$ $t_{0.05}(7)=1.895$ $t=\frac{42.75-49.1}{5.73}\sqrt{8}=-3.13<-1.895$ ⇒ No

8. $\sigma=10$ $n=17$ $S=11.8$ $\frac{(n-1)S^2}{\sigma_0^2}\sim\chi^2(16)$ $\chi^2_{0.025}(16)=28.8$

$\chi^2=\frac{16\times 11.8^2}{10^2}=22.2784<28.8$ $\chi^2_{0.975}(16)=6.91$ ✓

9. $\sigma=0.048$ (1) $\chi^2=\frac{4\times 0.00778}{0.048^2}=13.507$ $\chi^2_{0.025}(4)=11.1$ ×

(2) $\chi^2>11.1$ ×

10. $\mu_0=860$ $\sigma=220$ $H_0:\mu=860$ $H_1:\mu>860$

(1) $\frac{\bar{x}-\mu}{\sigma}\sqrt{n}\sim N(0,1)$ $U=\frac{1317.42-860}{220}\times\sqrt{31}=13.095$ $u_{0.025}=1.96$

⇒收入提高

(2) $H_0:\sigma^2=48400$ $H_1:\sigma^2>48400$

$\chi^2=\frac{30\times 215473}{48400}=133.56$ $\chi^2_{0.025}(30)=47$

⇒扩大了

8-3

1. $n_1=10$ $\bar{x}=1820$ $n_2=20$ $\bar{y}=1900$ $X,Y\sim N(\mu,\sigma^2)$ $\sigma_1=400$ $\sigma_2=380$

$H_0:\mu_1=\mu_2$ $H_1:\mu_1\neq\mu_2$ $u_{0.025}=1.96$

$$u=\frac{1820-1900}{\sqrt{\frac{400^2}{10}+\frac{380^2}{10}}}=-0.456<-1.96$$ ✓

2. $\mu_X = 12\%$ $\mu_Y = 12\%$ $H_0: \mu_X = \mu_Y$ $H_1: \mu_X \neq \mu_Y$

$\bar{X} = 12.02$ $S_X^2 = 0.907$ $\bar{Y} = 12.46$ $S_Y^2 = 0.973$ $t_{0.05}(8) = 1.860$

$$t = \frac{12.02 - 12.46}{\sqrt{\frac{4\times0.907 + 4\times0.973}{8}}\sqrt{\frac{1}{5}+\frac{1}{5}}} = |-0.718| < 1.86$$ 不比乙高

3. $\sigma_1^2 = \sigma_2^2 = \sigma$ $\bar{X} = 0.273$ $S_X^2 = 0.0281$ $\bar{Y} = 0.133$ $S_Y^2 = 0.00642$

$H_0: \mu_X \leq \mu_Y$ $H_1: \mu_X > \mu_Y$

$$t = \frac{0.273 - 0.133}{\sqrt{\frac{9\times0.0281 + 10\times0.00642}{19}}\sqrt{\frac{1}{10}+\frac{1}{11}}} = 2.480 > t_{0.025}(19) = 2.093$$ 有

4. (1) $\bar{X} = 0.141$ $S_X^2 = A$ $\bar{Y} = 0.1385$ $S_Y^2 = B$

✓ $f = \frac{A}{B} = 1.11$ $F_{0.025}(5,5) = 7.15$ $F_{0.975}(5,5) = 0.1399$

(2)

✓ $$t = \frac{0.141 - 0.1385}{\sqrt{\frac{5\times A + 5\times B}{10}}\sqrt{\frac{1}{6}+\frac{1}{6}}} = 1.583 < t_{0.025}(10) = 2.228$$

5. $S_X^2 = 100$ $S_Y^2 = 361$ $f = \frac{100}{361} = 0.277$ $F_{0.025}(15,15) = 2.86$

× $F_{0.975}(15,15) = 0.3497$

6. (1) $f = \frac{1.4}{4.38} = 0.3196 < F_{0.95}(11,9) = \frac{1}{F_{0.05}(9,11)} = 0.34$ ×

(2)

$F_{0.1}(11,9) = \frac{F_{0.1}(10,9) + F_{0.1}(12,9)}{2}$ $0.32 < 2.4$ ✓

≈ 2.4

7. $\bar{x}=1.422$ $S_x^2=0.01182$ $\bar{y}=1.516$ $S_y^2=0.01253$

$$t=\frac{1.422-1.516}{\sqrt{\frac{4\times0.01182+4\times0.01253}{8}}\sqrt{\frac{1}{5}+\frac{1}{5}}}=-1.347<-1.86 \quad \checkmark$$

$t_{0.05}(8)=1.86$

8-4

1. $X=\begin{cases}1 & 黄\\ 0 & 绿\end{cases}$ $H_0: P(X=1)=\frac{3}{4}$ $P(X=0)=\frac{1}{4}$

$r=0$

$A_1=(-\infty,\frac{1}{2}]$, $A_2=(\frac{1}{2},\infty)$ $\Rightarrow k=2$

$\chi^2_{0.05}(2-1)=\chi^2_{0.05}(1)=3.841$

$f_1=25$ $f_2=11$ $n=37$

$np_1=37\times\frac{3}{4}=27.75$ $np_2=9.25$

$\chi^2=\sum_{i=1}^{2}\frac{(f_i-np_i)^2}{np_i}=0.6036<3.841$

$\Rightarrow$ accept H_0

2. $n=100$ $H_0: P(X=k)=\frac{4^k}{k!}e^{-4}$ $k=0,1,2,\cdots$ $r=0$

$A_1=(-\infty,4]$ $A_2=(4,\infty)$

$\chi^2_{0.05}(1)=3.841$

$f_1=56$ $f_2=44$

$p_1=\sum_{k=0}^{4}\frac{4^k}{k!}e^{-4}=0.628838$ $p_2=\sum_{k=5}^{\infty}\frac{4^k}{k!}e^{-4}=0.370247$

$np_1=62.8838$ $np_2=37.0247$

$\chi^2=\sum_{i=1}^{2}\frac{(f_i-np_i)^2}{np_i}=2.0677<3.841$

$\Rightarrow$ accept H_0

3. $X \sim N(\mu, \sigma^2)$ $\quad \hat{\mu} = 1.406 \quad \hat{\sigma}^2 = 0.0482 \quad X \sim N(1.406, 0.002322)$

$f_1 = 12 \quad f_2 = 22 \quad f_3 = 23 \quad f_4 = 25 \quad f_5 = 18$

$P\{X \leq 1.355\} = \Phi_0\left(\frac{1.355-1.406}{0.0482}\right) = 1-\Phi_0(1.06) = 0.1446$

$P\{1.355 < X \leq 1.385\} = \Phi_0\left(\frac{1.385-1.406}{0.0482}\right) - \Phi_0(-1.06) = 0.1854$

$P\{1.385 < X \leq 1.415\} = \Phi_0\left(\frac{1.415-1.406}{0.0482}\right) - \Phi_0\left(\frac{1.385-1.406}{0.0482}\right) = 0.2453$

$P\{1.415 < X \leq 1.445\} = \Phi_0\left(\frac{1.445-1.406}{0.0482}\right) - \Phi_0\left(\frac{1.415-1.406}{0.0482}\right) = 0.2157$

$P\{X > 1.445\} = 1-\Phi_0\left(\frac{1.445-1.406}{0.0482}\right) = 0.209$

$np_1 = 14.46 \quad np_2 = 18.54 \quad np_3 = 24.53 \quad np_4 = 21.57 \quad np_5 = 20.9$

$\chi^2 = \sum_{i=1}^{5} \frac{(f_i - np_i)^2}{np_i} \sim \chi^2(2) \qquad \chi^2_{0.05}(2) = 5.99$

$\chi^2 = \frac{(12-14.46)^2}{14.46} + \cdots + \frac{(18-20.9)^2}{20.9}$

$= 2.11 < 5.99$

$\Rightarrow X \sim N(1.406, 0.002322)$

八

1.(1) 一

(2) $3|\bar{x}| \geq 1.96 \qquad \alpha = 0.05 \qquad 3|\bar{x}| \leq 1.96 \qquad P = 1-\Phi_0(1.04) - 1 + \Phi_0(4.959)$

$\sqrt{9}\left|\frac{\bar{x}-0}{1}\right| \geq 1.96 \qquad |\bar{x}| \leq 0.653 \qquad = 0.1492$

$-0.653 \leq \bar{x} \leq 0.653$

$-1.653 \leq \frac{\bar{x}-1}{1} \leq -0.347$

$-4.959 \leq \sqrt{9}\left(\frac{\bar{x}-1}{1}\right) \leq -1.041$

(3) $\sqrt{n}\,\frac{\bar{x}-\mu_0}{S}$

$= \sqrt{17}\left(\frac{23-21}{3.98}\right) \qquad t_{0.05}(16) = -1.746 \quad \checkmark$

$= 2.07$

(4) $X \sim N(\mu, \sigma^2)$ $H_0: \sigma^2 = \sigma_0^2$ $H_0: \sigma^2 \leq \sigma_0^2$ (5) 方差

(6) $H_0: p = 5\%$, 0.01

2. (1) A, C (2) B (3) A, D (4) B (5) B (6) A (7) C

3. (1) $\alpha_0 = \alpha$ (2) $\beta = P(\text{accept } H_0 | H_1 \text{ True}) = P(|\sqrt{n}(\bar{x}-\mu)| \leq u_{\frac{\alpha}{2}})$

$= P(-u_{\frac{\alpha}{2}} - \mu\sqrt{n} \leq \sqrt{n}\bar{x} - \mu\sqrt{n} \leq u_{\frac{\alpha}{2}} - \mu\sqrt{n})$

$= \Phi_0(u_{\frac{\alpha}{2}} - \sqrt{n}\mu) - \Phi_0(-u_{\frac{\alpha}{2}} - \sqrt{n}\mu)$

4. $u = \frac{\bar{x} - 4.55}{0.108}\sqrt{9} = -1.83$ $u_{0.025} = 1.96$ $-1.83 > -1.96$ ✓

5. $t = \frac{3.16 - 3.14}{0.3}\sqrt{20} = 0.298$ $t_{0.005}(19) = 1.729$ ✓

6. (1) $H_0: \mu < 50$ $\frac{\bar{x} - 50}{0.575}\sqrt{9} = -0.0579$ $u_{0.025} = 1.96$ ✓

(2) $H_0: \mu \geq 50$ ✓

(3) $H_0: \mu = 50$ ✓

7. $H_0: \mu \leq 0.5$ $t = \frac{0.52 - 0.5}{0.02}\sqrt{5} = 2.24$ $t_{0.05}(4) = 2.132$ × 显著超标

8. $\chi^2 = \frac{9 \times 75.74}{64} = 10.65$ $\chi^2_{0.05}(9) = 16.9$ ×

9. $H_0: \sigma^2 \leq 0.81$ $\chi^2 = \frac{18 \times 1.2^2}{0.81} \approx 32$ $\chi^2_{0.05}(18) = 28.9$ ✓

10. $H_0: \mu_X = \mu_Y$ $\frac{0.230 - 0.269}{\sqrt{\frac{8 \times 0.1337 + 7 \times 0.1736}{15}}\sqrt{\frac{1}{9} + \frac{1}{8}}} = -0.20565$

$t_{0.025}(15) = 2.131$ ✓

11. $H_0: \mu_X - \mu_Y \le 2$

$$T = \frac{5.25 - 1.5 - 2}{\sqrt{\frac{11 \times 0.931 + 11 \times 1}{22}} \sqrt{\frac{1}{12} + \frac{1}{12}}} = 4.3625 \qquad t_{0.05}(22) = 1.7171 \qquad \times$$

12. $\bar{X} = 15.0125 \quad S_X^2 = 0.0955 \quad \bar{Y} = 14.989 \quad S_Y^2 = 0.02611$

$$T = \frac{15.0125 - 14.989}{\sqrt{\frac{7 \times 0.0955 + 8 \times 0.02611}{15}} \sqrt{\frac{1}{8} + \frac{1}{9}}} = 0.199968 \qquad t_{0.025}(15) = 2.131 \quad \checkmark$$

13. $F = \frac{15.46}{9.66} = 1.600 \qquad F_{0.05}(13, 11) = \frac{2.79 + 2.72}{2} \approx 2.755 \quad \checkmark$

14. $F = \frac{1.82^2}{1.49^2} = 1.492 \qquad F_{0.005}(8, 10) = 6.12 \quad \checkmark$

15. $\bar{X} = 170.75 \quad S_X^2 = A \quad \bar{Y} = 169.5 \quad S_Y^2 = B \qquad H_0: \mu_X > \mu_Y \quad \checkmark$

$$t = \frac{170.75 - 169.5}{\sqrt{\frac{7 \times A + 7 \times B}{14}} \sqrt{\frac{1}{8} + \frac{1}{8}}} = 0.324 \qquad t_{0.025}(14) = -2.145$$

16. $X = \begin{cases} 1 & 正 \\ 0 & 反 \end{cases} \qquad A_1 = (-\infty, \frac{1}{2}], \quad A_2 = (\frac{1}{2}, \infty)$

$f_1 = 40 \qquad f_2 = 60$

$np_1 = 50 \qquad np_2 = 50$

$$\chi^2 = \sum_{i=1}^{2} \frac{(f_i - np_i)^2}{np_i} = 4 \qquad \chi^2_{0.05}(1) = 3.84 \qquad \times$$

17. $P\{X = k\} = {}^5C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{5-k} \qquad A_1 = (-\infty, 3.5] \quad A_2 = (3.5, \infty)$

$f_1 = 55 \qquad f_2 = 45$

$\sum_{k=0}^{3} {}^5C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{5-k} \qquad \frac{3}{16}$

$= \frac{13}{16}$

$np_1 = 81.25 \qquad 18.75$

$$\chi^2 = \sum_{i=1}^{2} \frac{(f_i - np_i)^2}{np_i} = 45.23 \qquad \chi^2_{0.05}(1) = 3.84 \qquad \times$$

18. $P\{X=k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ $k=0,1,\cdots$

$\hat{\mu} = \lambda = \overline{X} = 2$

$P\{X=k\} = \frac{2^k}{k!} e^{-2}$

$A_1 = (-\infty, 1.5]$ $A_2 = (1.5, 2.5]$ $A_3 = (2.5, \infty)$

$f_1 = 41$ $f_2 = 26$ $f_3 = 33$

$p_1 = \sum_{i=0}^{1} \frac{2^k}{k!} e^{-2} = 0.406006$ $p_2 = 0.270671$ $p_3 = 0.31879$

$np_1 = 40.6006$ $np_2 = 27.0671$ $np_3 = 31.879$

$$\chi^2 = \sum_{i=1}^{3} \frac{(f_i - np_i)^2}{np_i} = 0.08542$$

$\chi^2_{0.05}(1) = 3.84$

$\Rightarrow$ accept H_0: $X \sim P(2)$