

Equations for Pipelining:**Speed Up:**

$$S_n = \frac{\text{Execution time}_{old}}{\text{Execution time}_{new}} = \frac{nt_n}{(k+n-1)t_p}$$

Where n is the number of tasks, t_n is the time to complete each task on a non-pipeline unit, k is the number of stages for each task, and t_p is the time period to complete a k-stage pipeline.

Problem 1:

Consider a machine where each task passes by 4 stages in order to be executed, each of length 20 ns, If 100 tasks to be executed:

- On a non-pipelined machine, what is the execution time for the tasks on this machine?
- On a 4-stage pipeline machine, what is the execution time for the tasks on this machine?
- What is the speed up gained from pipelining?

Solution:

- Total execution time in a non-pipelined machine = $20 \times 4 \times 100 = 8000$ ns
- Total execution time in a pipelined machine = $(k + n - 1) \times t_p = (4 + 99) \times 20$
- = 2060 ns
- Speedup = $8000 / 2060 = 3.88$

Problem 2:

Consider a machine where each instruction passes by 5 stages, the 5 stages needs 10, 8, 10, 10, 7 ns respectively. If a pipeline is applied to the machine, it adds a latch of 1 ns between each stage and the other. If 50 tasks to be executed:

- On a non-pipelined machine, what is the execution time?
- On a 5-stage pipeline machine with a latch, what is the execution time?
- What is the speed up gained from pipelining?

Solution:

- a) Total execution time in a non-pipelined machine = $(10+8+10+10+7) \times 50 = 2250$ ns
- b) Total execution time in a pipelined machine = $(k + n - 1) \times t_p = (5 + 49) \times 11 = 594$ ns
- c) Speedup = $2250 / 594 = 3.78$

Problem 3:

Consider a non pipelined machine with 6 stages of lengths 50 ns, 50 ns, 60 ns, 60 ns, 50 ns, and 50 ns.

- a) What is the clock cycle time on this machine?
- b) How much time does it take to execute 100 instructions?

Suppose we introduce pipelining on this machine. Assume that when introducing pipelining, the clock adds 5ns latch to each execution stage.

- c) What is the clock cycle time on this machine?
- d) How much time does it take to execute 100 instructions?
- e) What is the speedup obtained from pipelining for 100 instructions?

Solution:

- a) The clock cycle time for this machine = $50+50+60+60+50+50 = 320$ ns
- b) Time to execute 100 instructions = $100 \times 320 = 32000$ ns

Remember that in the pipelined implementation, the length of the pipe stages must all be the same, i.e., the speed of the slowest stage plus overhead. With 5ns latch it comes to:

$$\begin{aligned} \text{The length of pipelined stage} &= \text{MAX}(\text{lengths of non-pipelined stages}) + \text{latch} \\ &= 60 + 5 = 65 \text{ ns} \end{aligned}$$

- c) The clock cycle time for this machine = 65 ns
- d) Time to execute 100 instructions = $(6+99) \times 65 = 6825$ ns
- e) Speedup for 100 instructions = $32000 / 6825 = 4.69$

Problem 4:

Consider the following snippet of code:

```
FOR i = 1 TO 100 DO
```

```
{  
    A[i] = (B[i]xC[i]) + D[i]  
}
```

Assume that each operation, multiplication and addition, requires 10 ns to complete. Consider a pipelined unit that could break this computation into two stages, the first stage performs the multiplication and the second stage performs the addition.

- What is the total execution time for the code before pipeline?
- What is the total execution time for the code after pipeline if no latch is added?
- What is the total execution time for the code after pipeline if a latch of 2ns is added between stages?
- Calculate the speedup gained from pipeline with latch.

Solution

- Total execution time for the code before pipeline = $(10+10) \times 100 = 2000$ ns
- Total execution time for the code after pipeline without latch = $(2+99) \times 10 = 1010$ ns
- Total execution time for the code after pipeline with latch = $(2+99) \times 12 = 1212$ ns
- Speedup = $2000/1212 = 1.65$

Problem 5:

Assume that the individual stages of on a machine takes the following execution time:

IF	ID	EX	MEM	WB
300ps	400ps	350ps	500ps	100ps

Assume that when pipelining, each pipeline stage costs 20ps extra between pipeline stages.

- What is the clock cycle time in a pipelined and non-pipelined processor?
- If you could split one of the pipeline stages into 2 equal halves, which one would you choose? What is the new cycle time?

Solution

- a) Clock cycle time in a non-pipelined = $300 + 400 + 350 + 500 + 100 = 1650$ ps
 Clock cycle time in a pipelined = $500 + 20 = 520$ ps
- b) Splitting the longest stage is the only way to reduce the cycle time. After splitting it, the new cycle time is based on the new longest stage.
 Old longest stage is MEM.
 New Clock cycle time = 420ps

Problem 6:

Consider a machine where each instruction passes by 4 stages, fetch (IF), decode(ID), execute(EX), and write back (WB).

- a) Fill table 1 to show 3 instructions execution on a non-pipelined machine.
 b) Fill table 2 to show 3 instructions execution on a pipelined machine.

Table 1

Inst1															
Instr2															
Instr3															

Table 2

Inst1															
Instr2															
Instr3															

Solution:

Table 1

Inst1	IF	ID	EX	WB											
Instr2					IF	ID	EX	WB							
Instr3									IF	ID	EX	WB			

Table 2

Inst1	IF	ID	EX	WB											
Instr2		IF	ID	EX	WB										
Instr3			IF	ID	EX	WB									

Problem 7:

Draw a space time diagram for a six segment pipeline showing the time it takes to process eight tasks. Determine the number of clock cycles that it takes to process 200 tasks in a six-segment pipeline.

Solution:

Segment/Cycles	1	2	3	4	5	6	7	8	9					
S1	T1	T2	T3	T4	T5	T6	T7	T8						
S2		T1	T2	T3	T4	T5	T6	T7	T8					
S3			T1	T2	T3	T4	T5	T6	T7	T8				
S4				T1	T2	T3	T4	T5	T6	T7	T8			
S5					T1	T2	T3	T4	T5	T6	T7	T8		
S6						T1	T2	T3	T4	T5	T6	T7	T8	

Number of clock cycles = $k + n - 1 = 6 + 200 - 1 = 205$ clock cycles

Problem 8:

A non-pipeline system takes 50 ns to process a task. The same task can be processed in a six-segment pipeline with a clock cycle of 10 ns. Determine the speedup ratio of the pipeline for 100 tasks. What is the maximum speed up that can be achieved?

Solution:

Total execution time in a non-pipelined machine = $nt_n = 100 \times 50 = 5000$ ns

Total execution time in a pipelined machine = $(k + n - 1) \times t_p = (6 + 99) \times 10 = 1050$ ns

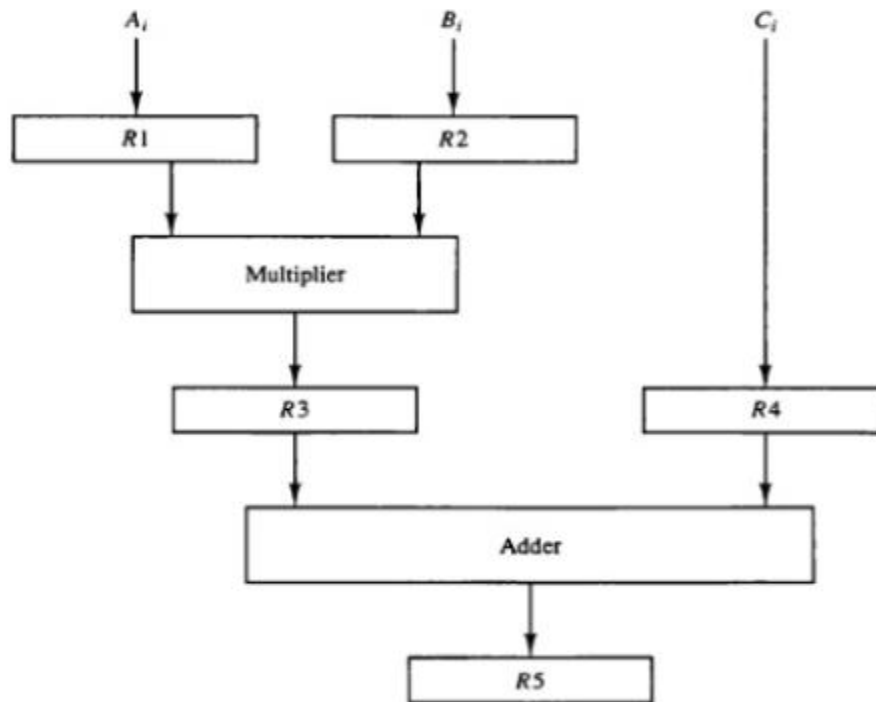
Speedup = $5000 / 1050 = 4.76$

Maximum Speedup achieved = $t_n / t_p = 50 / 10 = 5$

Problem 9:

The pipeline of the below figure has the following propagation times: 40 ns for the operands to be read from memory into registers R1 and R2, 45 ns for the signal to propagate through the multiplier, 5 ns for the transfer into R3 and R4, and 15 ns to add the two numbers into R5.

- What is the minimum clock cycle time that can be used?
- A non-pipeline system can perform the same operation by removing R3 and R4. How long will it take to multiply and add the operands without using the pipeline?
- Calculate the speedup of the pipeline for 100 tasks.
- What is the maximum speed up that can be achieved?



Solution:

- Minimum clock cycle time = $t_p = 45$ ns.
- Time on non-pipelined machine if no R3 or R4 = $40 + 45 + 15 = 100$ ns.
- Total execution time in a non-pipelined machine = $nt_n = 100 \times 105 = 10500$ ns
Total execution time in a pipelined machine = $(k + n - 1) \times t_p = (4 + 99) \times 45 = 4635$ ns
Speedup = $10500 / 4635 = 2.26$
- Maximum Speedup = $t_n / t_p = 105 / 45 = 2.33$