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Fleet and charging infrastructure decisions for fast-charging city electric bus service

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ABSTRACT

Decision aspects concerning the introduction of fast-charging city electric buses are studied in this paper. The main studied problem consists of determining a fleet of electric buses and their charging infrastructure such that a social-ecological value is maximized. In the most representative time period, all the electric buses should be available to drive, the required inter-bus interval should be maintained, the output power of any charging station and transformer should not be exceeded, and the total capital cost and the total annual operating, depreciation and energy cost should not exceed certain maximum thresholds. The total passenger demand satisfied by the electric buses can be considered as the value to be maximized. A secondary problem consists of finding a passenger load balanced schedule of the vehicles on the same route. Mathematical models for these two problems are proposed. A randomized heuristic algorithm combined with the Particle Swarm Optimization is developed for the main problem, and a known polynomial time algorithm is adapted for the secondary problem. A case study for the city of Minsk (Belarus) and computer experiments with random instances are provided. The proposed approach delivered solutions with values deviating at most 12% on average and 24% in the worst case from the upper bounds obtained as optima of a relaxed problem.

1. Introduction

We see four major interrelated reasons for replacing conventional vehicles with the electric ones: (i) the global warming and, as a consequence, natural disasters because of the combustion of hydrocarbons, (ii) the carcinogenic effect of the internal combustion engines exhaust, (iii) the energy efficiency of the electric vehicles, and (iv) their economic efficiency in the long-term perspective because of the easier and cheaper maintenance. In this article, a city electric bus system planning problem is studied. Such problems are more complicated for electric vehicles than for the conventional ones because of the much smaller driving range and, as a consequence, more frequent recharging.

We call an electric bus an *e-bus* and a fleet of e-buses an *e-fleet*. An e-bus is equipped with a battery which requires recharging to be operational. A *fast-charging* technology is considered, according to which (re)charging duration is usually 3–30 min and the single-charge e-bus range is 10–30 km (Andersson, 2017; Kunith et al., 2017; He et al., 2019, project ZeEUS). Most of the input numerical parameters of the studied problem are uncertain. We assume that their deterministic values are given by the decision makers. These values can be nominal values, statistically average values, worst-case values, or values provided by the experts.

A set of city routes and fleets of conventional and electric vehicles of different types serving these routes are considered. It is assumed that any vehicle serves a single route. An e-bus type is characterized by a set of charging times at charging stations of different types at different stops, energy consumption for the same route over the year, capital, operating, depreciation and energy costs over the year, passenger capacity, and other parameters defined in Table 9 of Appendix A.

Each route is characterized by a unique *route cycle*, which consists of a depot and a sequence of intermediate (en route) and terminal stops visited cyclically by the e-buses and conventional vehicles assigned to this route. It is assumed that any e-bus charges once in the depot at a night time and at least once in each route cycle. Due to the long standing time of the e-buses in the depot, one appropriate charging station with a single charging point equipped with the maximum number of connectors is sufficient to charge e-buses of the same type in the depot. These assumptions are valid for the e-buses with the fast-charging batteries. Both the charging time and the charging range after a charge are assumed to depend on the initial *State of Charge (SOC)* level, e-bus type, route, charging station type and stop (longer time for passenger loading/unloading can imply longer charging time).

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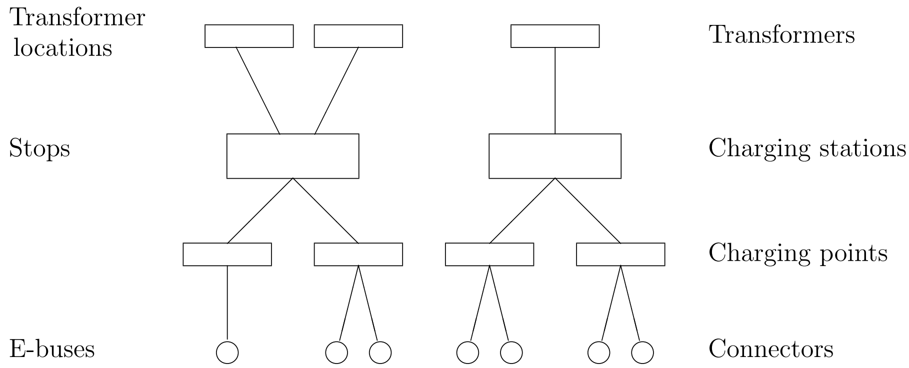

Fig. 1. Hierarchy of charging infrastructure.

Two decision problems are introduced and solved. The primary problem is to determine an e-fleet and its assignment to the routes, a fleet of the remaining conventional vehicles, places for charging stations and transformer substations (transformers), assignment of charging stations to the transformers, and quantities of charging points and connectors (see Fig. 1 for an illustration of these concepts). The total social-ecological value has to be maximized. Additionally, in a *decisive time period*, the passenger demand of each route must be satisfied, all the e-buses must be available to drive, the required traffic interval is maintained by the e-buses and the remaining conventional vehicles, the output power of any station and transformer is not exceeded. Finally, the total capital cost and the total annual operating, depreciation and energy cost of the e-buses and the charging infrastructure cannot exceed maximum thresholds previously set. We denote this problem as Opt-Fast-Char.

The decisive time period is a time period such that the traffic (inter-bus) intervals and electric power supply from the city grid are stable. The decisions made for this period with respect to the e-bus fleet and charging infrastructure ensure their feasible operation in any other time period. With some degree of uncertainty, the decisive time period can be characterized by the highest SOC loss of the e-buses while driving over the same route segments, and the smallest traffic intervals. The passenger demand of a route is determined as the past total passenger capacity of all vehicles operating on this route in the decisive time period.

We assume that the vehicle schedule is determined by the vehicle sequence and the traffic interval rather than a list of the fixed departure times from the stops, which is motivated by the observed real-life cases. The total passenger demand satisfied by the e-buses is considered as the value to be maximized. The problem Opt-Fast-Char can be solved repeatedly for several successive planning periods (e.g. years). Decisions made in the past periods can be used as a part of the input for a future period.

The secondary problem consists of distributing uniformly the departures of vehicles with different passenger capacities assigned to the same route. The corresponding schedules are called *balanced*. For example, if there are two buses, denoted as L, with large passenger capacity, and two buses, denoted as S, with small passenger capacity, then the schedule determined by the sequence (L, S, L, S) is balanced and (L, L, S, S) is not. The balanced schedule problem is solved for each route separately, provided that the set of vehicles serving this route is determined, for example, via solving the primary problem Opt-Fast-Char.

Our studies are motivated and supported by the project “Planning process and tool for step-by-step conversion of the conventional or mixed bus fleet to a 100% electric bus fleet” (PLATON) (PLATON, 2020). It was the PLATON consortium decision that commercial software should not be used in the tool to be developed. Therefore, the problem Opt-Fast-Char is not formulated as a mathematical programming problem (MPP) to be further solved by a commercial software. We made an effort to formulate Opt-Fast-Char as an MPP and solve it by a free software, but realized that an extremely complicated formulation with a huge number of variables and constraints cannot be avoided, even if some of the constraints are relaxed or aggregated, because of the complex combinatorial structure of the problem. Several instances of the relaxed MPP problem in Section 3.2, which is much simpler than Opt-Fast-Char, were not solved by an academic version of ILOG CPLEX to proven optimality in two hours, and we forecast that the corresponding instances of Opt-Fast-Char will demand years of computations. We have chosen a randomized heuristic and the PSO as solution techniques for Opt-Fast-Char because these methods work well with the mathematical objects of different types and levels of formalization, which are needed for the appropriate modelling of Opt-Fast-Char. Furthermore, the real-life problem motivating our studies changes over time. As examples we consider its different versions described in Kovalyov et al. (2020) as well as in the current paper. The changes of the constraints or the objective function are sufficiently easily addressed in both the randomized heuristic and the PSO. With regard to the choice of optimal or suboptimal solutions, achieving optimality is not important for problems with uncertain or imprecise input data and sensitive to changes in input data, such as problems that admit more than one optimal solution, including the problem Opt-Fast-Char. Any little change of an input number associated with the distinct parts of two optimal solutions can make one of them suboptimal.

A state-of-the-art in the field of optimal decision making for planning fast-charging e-bus systems is given in the next section. Differences of the problem Opt-Fast-Char from the earlier studied problems are demonstrated. The problem Opt-Fast-Char is considered in Section 3. Its mathematical model is presented in Section 3.1. A relaxed problem for Opt-Fast-Char is formulated in Section 3.2. An optimal solution of the relaxed problem is used to evaluate the quality of heuristic solutions of Opt-Fast-Char. In Section 3.3, problem Opt-Fast-Char is proved computationally difficult, namely, *NP-hard in the strong sense* for two important special cases. A *randomized heuristic* algorithm combined with the *Particle Swarm Optimization (PSO)* is developed for Opt-Fast-Char in Section 3.4. The balanced schedule problem is formulated and solved in Section 4. Section 5 is devoted to the computer experiments. A real-life case in the city of Minsk (Belarus) is considered in Section 5.1. Section 5.2 presents results for random instances based on the real-life case. The last section contains concluding remarks.

2. State-of-the-art

A comprehensive review of the e-bus system design problems, their mathematical models and solution techniques, based on 69 recent publications, is provided by Jefferies and Göhlich (2020). We shall concentrate on the most relevant literature in which optimal decisions with regard to the e-bus fleet and fast-charging infrastructure are considered.

Rogge et al. (2015) provide a simulation study of the effect of deployment of fast-charging infrastructure on the cost and energy consumption of a public transport system. In the studied case, each e-bus serves several routes, charging stations are located at the terminal stops, and simultaneous charging of several e-buses at the same charging station is possible with the only restriction on the available power. A schedule of the e-buses can be fixed or adjusted for the required recharging times. A trade-off between the required battery capacity and the charging power is discussed, as well as the impact of the charging load on the electric grid.

Kunith et al. (2017) propose a *Mixed Integer Linear Programming (MILP)* model to determine locations and numbers of fast-charging stations and transformers, and battery types of e-buses such that the total cost of the selected equipment is minimized. No charging station can be shared by different routes and the bus timetable is assumed to be fixed. The SOC loss of any battery is assumed to be dependent on the operational conditions and a battery can be charged to any feasible level at a charging station. The energy consumption of an e-bus is an input parameter and it is determined via simulation.

Wang et al. (2017) develop a MILP model to decide about the number of charging stations, charging connectors and recharging schedules for e-buses to replace all conventional vehicles of a public transit network with the aim of minimizing the total annual costs. The specific assumptions are: the e-bus timetables are fixed, the e-buses charge at a single depot and two transit centres, all the e-buses are homogeneous and have the same driving range, the charge consumed is proportional to the driving distance, the energy recharged is proportional to the charging duration, all the charging stations are homogeneous fast-charging, and the recharging duration is location dependent and fixed.

The decisions in Liu et al. (2018) concern the types and the numbers of the fast-charging stations to be installed at the depots and terminals, and the battery types of the e-buses. Each e-bus serves a single cyclic route with a single depot and a single terminal. Different routes can share the same charging station. Each e-bus has to be fully charged each time when it visits the terminal. The objective is to minimize the total equipment cost. A MILP model is proposed which implements a robust optimization approach to address the energy consumption uncertainty of the e-buses.

Vepsäläinen et al. (2018) consider a single route with two terminals, which is served by e-buses driving with a given traffic interval. Three charging infrastructure scenarios are considered: one charging station, two charging stations located at the same terminal, and two charging stations located at different terminals. The costs and the utilization rates of the e-buses are analysed depending of the traffic intervals.

Liu and Wei (2018) consider a replacement of the conventional vehicles by the e-buses without changing the vehicle tasks. The e-buses and the charging stations are assumed to be identical and the energy issues are not considered. Upper bounds on the number of e-buses that can charge simultaneously at the same depot station and at the same intermediate station are given. A MILP model is presented to determine the number of e-buses and the number of charging stations at each location such that the total equipment cost is minimized.

Li et al. (2019) propose a time–space–energy indexed MILP formulation for a vehicle scheduling problem with multiple depots and a mixed fleet of e-buses and conventional vehicles of different types. Each vehicle can be assigned a task that is a sequence of passenger transfer trips, and a trip is a sequence of arcs between two terminals of the same route in a time–space–energy network. Each arc in this network is associated with the passenger demand between the origin and the destination (which are the space characteristic of the arc) in a time interval (which is the time characteristic of the arc) and the SOC losses of different vehicle types (which are the energy characteristic of the arc). Locations of the charging stations are fixed and they are characterized by the upper bounds on the number of simultaneously charged e-buses. The decision concerns opening or not charging stations and assigning e-buses to the trips and the charging stations over time so that the passenger demand and the charging constraints are satisfied and the total operator cost, the passenger demand dissatisfaction cost, and the cost of emissions is minimized.

Less complex problems than in Li et al. (2019) are studied by Wen et al. (2016) and Adler and Mirchandani (2017). The main differences are that any number of vehicles can be recharged simultaneously at the same charging station and the objective is to minimize the total travelling cost of all the vehicles. MILP formulations are proposed. An adaptive large neighbourhood search heuristic and a branch-and-price algorithm are developed in Wen et al. (2016) and Adler and Mirchandani (2017), respectively. van Kooten Niekerk et al. (2017) extend these two models by the fixed costs and the energy capacities of the charging stations, and describe column generation methods.

He et al. (2019) address a deployment of the fast-charging stations equipped with the energy storage devices for an e-bus system. Each route is cyclic with a single terminal (base) station. The same terminal or bus stop can belong to several routes. A non-shared route-owned charging station can be opened at a terminal and a shared charging station can be opened at a bus stop. Scheduling of the e-buses on the routes and their scheduling at the charging stations are not considered. The problem is to determine the charging station locations, the types of the energy storage devices used by the stations and the battery capacities of the e-buses such that the total system cost including the energy cost is minimized. A MILP model is proposed.

Raab et al. (2019) examine three recharging scenarios at an e-bus depot. E-buses of three types are considered. Each e-bus has a fixed timetable and an associated energy demand. All the e-buses are recharged at the same depot. Scenario 1 assumes that slow recharging is only possible after the required e-bus technical service. Scenario 2 assumes that fast recharging during the service is possible. Scenario 3 permits performing e-bus service and slow recharging in any order and this order can be decided. The cost of the energy supplied from the city grid is time dependent. For each scenario, an optimal recharging timetable is determined to minimize the sum of the operating and energy costs.

Jefferies and Göhlich (2020) provide a simulation tool for an e-bus system, in which e-buses of different types perform their multi-route passenger transfer tasks and recharge their batteries at a depot and terminals. A Genetic Algorithm is proposed to decide about the terminals for recharging such that the system cost is minimized. A heuristic algorithm is used to schedule e-bus charging processes at the charging points. The algorithm employs fixed e-bus timetables and a list of charging points as the input. The total cost of ownership is determined for the optimized system.

None of the above works consider transformer assignment decisions. The problem in Kovalyov et al. (2020) differs from Opt-Fast-Char in that the charging infrastructure is simple: each charging station consists of a single charging point with a single connector. Furthermore, it is assumed that the SOC level and the driving range of any e-bus are maximal after each charge. The objective function is the ratio of the total value and the total cost, and the bus schedule is not decided in Kovalyov et al. (2020). The solution method in Kovalyov et al. (2020) is a simplified version of the method in Section 3.4.

Characteristics of Opt-Fast-Char and the earlier studied optimal decision problems dealing with the fast-charging e-bus technology are given in Table 1, which demonstrates the differences between the problems. There, symbols D, T and I denote charging station locations at the depots, terminals and intermediate stops, respectively. The location decision includes decisions on the places for the charging stations, transformers and links between them. The equipment decision includes decisions on the types and quantities of the e-buses and charging stations and their distribution between the routes.

Decision problems for the other types of electric vehicles such as private cars, trucks and long-distance buses are studied by Alonso et al. (2014), Wen et al. (2014), Yu et al. (2016), Hiermann et al. (2016),

Table 1
Problem characteristics.

Reference	Daily bus task	Fleet	Fixed timetable or fixed traffic intervals	Charging station locations	Charging station capacities	Time of charge	SOC level at departure	Location decision	Equipment decision
This paper	Single-route	Electric & conventional	Traffic intervals	D,T,I	# of e-buses & power	Stop, e-bus, route & station dependent	Initial SOC level, Stop, e-bus, route & Station dependent	Yes	Yes
Vepsäläinen et al. (2018)	Single-route	Electric	Traffic intervals	T	Calculated	Calculated	Calculated	No	Yes
Liu et al. (2018)	Single-route	Electric	Timetable	D,T	Power	Stop dependent	Variable	Yes	Yes
He et al. (2019)	Single-route	Electric	Irrelevant	T,I	No	0	Irrelevant	Yes	Yes
Li et al. (2019)	Multi-route	Electric & conventional	Timetable	D,T,I	# of e-buses	Distance, speed & load dependent	Variable	Yes	No
Wang et al. (2017)	Multi-route	Electric	Timetable	D,T	# of e-buses	Stop dependent	Variable	Yes	Yes
Liu and Wei (2018)	Multi-route	Electric	Timetable	D,T,I	# of e-buses	0	Maximal	Yes	No
Kunith et al. (2017)	Multi-route	Electric	Timetable	D,T,I	# of e-buses	Variable	Variable	Yes	Yes
Wen et al. (2016)	Multi-route	Electric	Timetable	D,T,I	# of e-buses	Distance dependent	Variable	No	No
Raab et al. (2019)	Multi-route	Electric	Timetable	D	Power	Simulated	Maximal	No	No
Rogge et al. (2015)	Multi-route	Electric	Timetable	T	Power	Simulated	Simulated	No	No
van Kooten Niekerk et al. (2017)	Multi-route	Electric	Timetable	D,T,I	No	Distance dependent	Variable	No	No
Adler and Mirchandani (2017)	Multi-route	Electric	Timetable	D,T,I	No	0	Maximal	No	No
Jefferies and Göhlich (2020)	Multi-route	Electric	Timetable	D,T	No	Simulated	Simulated	Yes	No

Desaulniers et al. (2016), Wielinski et al. (2017), Bruglieri et al. (2017), Pelletier et al. (2016, 2017, 2018), Froger et al. (2017a,b), Xylia et al. (2017a,b), Mohamed et al. (2017), Leou and Hung (2017), Teoh et al. (2016), Zhang et al. (2018), Rogge et al. (2018), Hosseini and Sarder (2019), Wang et al. (2018a,b), Pelletier et al. (2019), Liu et al. (2019), Yao et al. (2019), Lin et al. (2019), Gampa et al. (2020) and Rinaldi et al. (2020). Various aspects of the introduction of electric vehicles into the city life are studied within the international projects Electric Mobility Europe initiative and projects therein, EIT Urban Mobility, ELECTRIFIC, ELIPTIC, MAtchUP, Mega-E, mySMARTLife, REPLICATE, Ruggedised, Triangulum, ZeEUS and Ultra-E.

3. Problem Opt-Fast-Char

There is the following hierarchy of the charging infrastructure. At the lower level, there are *charging connectors*, which are directly connected to the e-buses and work in parallel. Charging connectors of the same type output the same power. Charging connectors of the same stop and type are grouped into the *charging points* of this stop and type, and each charging point accepts as many connectors as permitted by its *number of connectors capacity*, or equivalently, as it is permitted by its output power. Charging points of the same stop are united into a single *charging station* of this stop, which accepts a limited number of charging points due to the space capacity of the stop. Any e-bus of any route arriving to a stop can be assigned to any charging point of an appropriate type with a free connector. If an e-bus is not charged at a stop, then it does not block any charging point at this stop. Finally, each charging station is linked with a number of transformer locations, and each such a location accommodates at most one transformer. The total power of connectors of the same charging station is limited by the total output power of the transformers linked with this station. Each transformer is linked with at most one charging station. See Fig. 1 for an illustration.

Some e-buses and elements of the charging infrastructure can already be in operation. We call them *"old"* and we call *"new"* e-buses and infrastructure elements to be decided. All the e-buses of the same type charge at all stops of their route where a charging station of an appropriate type is opened. A unique charging type is used for all the e-buses of the same type. If there are old e-buses of a certain type, then the old and new e-buses of this type charge at the charging points of the old type, else a unique charging type has to be selected from the set of the appropriate charging types. The charging time and the SOC level after a charge depend on the charging station type, the e-bus type and the stop.

There is a set of *obligatory charge* stops, which includes depots of all routes, and which can include some other stops, for example, terminal stations. A charging station must be opened at the obligatory charge stop if at least one e-bus goes via this stop. In any depot, due to the long non-operational time of the e-buses, a single charging point equipped with the maximum number of connectors is sufficient for charging all the e-buses of the same type. It is assumed that the e-buses of different routes can arrive simultaneously to the same stop, and therefore, the number of routes of these e-buses is a lower bound for the number of charging connectors of the same type that they need at the same stop. Fig. 2 illustrates an assignment of the e-buses to the charging points and connectors.

3.1. Mathematical model

The input data for the problem Opt-Fast-Char are described in Table 9 of Appendix A. Denote by X a solution of the problem Opt-Fast-Char. We describe it by the variables given in Table 10 of Appendix A. Problem Opt-Fast-Char can be formulated as follows.

Problem Opt-Fast-Char : $\max_X f(X)$, subject to

$$cc(X) \le ucc, \tag{1}$$

$$oc(X) \le uoc, \tag{2}$$

$$\sum_{c\in C}(nc_{jc}(X) + noc_{jc})p_c \le \sum_{t\in T_j(X)\cup TO_j} utp_t,\ \ j \in N, \tag{3}$$

$$c_b(X) = co_b,\ \ b \in BO, \tag{4}$$

$$c_b(X) \in C_b,\ \ b \in B\setminus BO, \tag{5}$$

$$nop_j + np_j(X) \le up_j,\ \ j \in N, \tag{6}$$

$$nop_{d(r)c} + np_{d(r)c}(X) = 1,\ \ c \in CO_{d(r)} \cup C_{d(r)}(X),\ \ r \in RO \cup R(X), \tag{7}$$

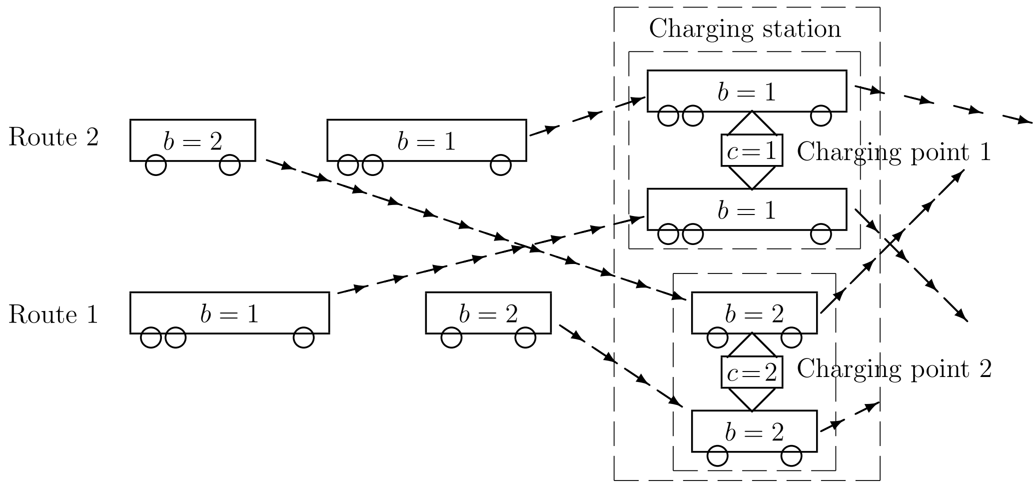

Fig. 2. Assignment of e-buses of types $b = 1, 2$ to charging points of types $c = 1, 2$.

$$nos_j + ns_j(X) = 1,\ j \in NM \cap (NO \cup N(X)), \tag{8}$$

$$\sum_{b\in BO_r} nbo_{rb}cap_b + \sum_{b\in B_r(X)} nb_{rb}(X)cap_b + \sum_{b\in V_r(X)} nv_{rb}(X)cap_b$$
$$\geq dem_r,\ r \in R(X), \tag{9}$$

$$nv_{rb}(X) \leq nv^0_{rb},\ r \in R(X),\ b \in B_r(X), \tag{10}$$

$$d^0_{ribc}(X) \geq d_{rij},\ (i,j) \in S_{rc}(X),\ b \in B_r(X),\ r \in R(X),\ c = c_b(X). \tag{11}$$

The objective function is

$$f(X) = \sum_{r\in R(X)} \left(f_r(Z_r(X)) - \frac{\sum_{b\in V_r(X)} nv_{rb}(X)cap_b}{dem_r} \right),$$

where $R(X)$ is a subset of the routes selected for the introduction of the new e-buses, $Z_r(X) = \min\{dem_r, \sum_{b\in B_r(X)} nb_{rb}(X)cap_b\}$ is the passenger demand of route r to be satisfied by the new e-buses according to X, dem_r is the passenger demand of route r (total passenger capacity of all old vehicles on this route), and $\sum_{b\in B_r(X)} nb_{rb}(X)cap_b$ and $\sum_{b\in V_r(X)} nv_{rb}(X)cap_b$ are the total capacities of the new e-buses and the remaining conventional vehicles, respectively, on route r. The functions f_r can be defined such that $f_r(Z_r(X)) = w_r Z_r(X)$, where $w_r > 0$ is a priority coefficient of route r. The minus total capacity of the remaining conventional vehicles divided by dem_r is added to the objective function in order to minimize this capacity with a negligible effect on the maximization of the total value $\sum_{r\in R(X)} f_r(Z_r(X))$.

Constraints (1) and (2) bound the total capital cost $cc(X)$ and the total operating, depreciation and energy cost $oc(X)$ from above, see Table 10 in Appendix A for the definition of these costs. It is assumed that there is no capital, operating and depreciation cost associated with the numbers of charging connectors as soon they do not change the numbers of the charging points. The required numbers of the charging connectors are determined based on the traffic intervals ut_r, charging times ct_{rjbc} and the numbers of new e-buses $nb_{rbc}(X)$ required to satisfy the demands dem_r, see Table 10. Constraints (3) ensure that the total power demand of old (sum of noc_{jc}) and new (sum of $nc_{jc}(X)$) charging connectors of all types at stop j does not exceed the total output power (sum of utp_t) of all transformers linked with this stop. Constraints (4) and (5) define a unique charging type $c_b(X)$ for the e-buses of each type b. Constraints (6) limit the total number of old (nop_j) and new ($np_j(X)$) charging points of all types at stop j by the upper bound up_j, which is related to the available space at stop j. The total number of the old and new charging points of a certain type is obtained by rounding up the total number of connectors of this type divided by the respective number of connectors capacity, see Table 10 for the formula. Constraints (7) state that exactly one old or new charging point of each type c used by the e-buses of route r must be opened at the depot $d(r)$ of this route if this route is served by at least one e-bus. Constraints (8) guarantee that exactly one old or new charging station is opened at each obligatory charge stop visited by at least one e-bus. Constraints (9) require that the passenger demand on route r served by at least one e-bus is satisfied by the e-buses and the remaining conventional vehicles of this route. The number of the remaining conventional vehicles is limited by their original number nv^0_{rb} in the constraints (10). Constraints (11) guarantee that any new e-bus of type b can feasibly run on route r to which it is assigned, provided that the appropriate charging stations are opened at the stops of this route.

Note that the solution in which no new e-bus or new charging infrastructure element is used is feasible for the problem Opt-Fast-Char and it has zero value. Furthermore, an optimal solution of Opt-Fast-Char is a *non-dominated (efficient, Pareto-optimal)* solution of a tri-criteria problem of maximizing $f(X)$ and minimizing $cc(X)$ and $oc(X)$. A solution is a non-dominated solution if there is no other solution which is no worse in all the criteria values and strictly better in one of the criteria values than the non-dominated solution.

3.2. Relaxed problem

If the feasible domain of a maximization problem is extended by relaxing some of the constraints, then the optimal solution value of this *relaxed* problem is an upper bound on the optimal value of the original problem. This upper bound can be used to evaluate the quality of an approximate solution of the original problem. We will use such an upper bound to evaluate the quality of the heuristic solutions of the real-life and random instances of the problem Opt-Fast-Char in Section 5.

We formulate the following relaxed problem, denoted as Relax, for the problem Opt-Fast-Char. Firstly, assume that the charging points opened at the obligatory charge stops of the set NM_r are sufficient to feasibly serve all the e-buses of the same route r. This assumption is obviously a relaxation. Furthermore, we omit the electrical power consumption constraints and assume that the capital, operating and depreciation costs of the charging stations, charging points, transformers and connections between them are minimal over all types and locations.

Recall that Z_r is the total passenger demand satisfied by the new e-buses of route r. Variables Z_r are variables in the relaxed problem. Besides, introduce variables x_{rb} to represent the number of the new e-buses of type b assigned to route r and variables μ_{rb} to represent the number of the conventional vehicles of type b left on route r, which is served by at least one new e-bus. Variables x_{rb} and μ_{rb} are analogues of the variables $nb_{rb}(X)$ and $nv_{rb}(X)$, respectively.

Introduce 0–1 variables y_r such that $y_r = 1$ if and only if there is at least one new e-bus on route r, that is, if $x_{rb} > 0$ for at least one $b \in B$.

Denote by nop_j and noc_j the number of the old charging points and the number of their connectors at the stop $j \in NM \backslash D$, respectively: $nop_j = \sum_{c \in C} nop_{jc}$, $noc_j = \sum_{c \in C} noc_{jc}$. Denote by $nc_{\max} = \max_{c \in C}\{uc_c\}$ the maximum connector capacity over all charging types.

According to the definition of the number of new charging connectors of type c at stop $j \in NM \backslash D$, $nc_{jc}(X)$ in Table 10, the number of such new connectors of all types should be at least $Y_j := \max\{0, \sum_{r \in RM_j} k_{rj} y_r - noc_j\}$, where $k_{rj} \in \{1, 2\}$ is the number of times of visiting stop j by the e-buses of route r, and the number Y_j^p of new charging points required at stop j should be at least $\frac{Y_j + noc_j}{uc_{\max}} - nop_j$. Values Y_j and Y_j^p are variables in Relax. Introduce 0–1 indicator variable $Y_j^s = 1$ if $Y_j^p > 0$ and $Y_j^s = 0$ if $Y_j^p = 0$, $j \in NM$.

Problem Relax : $\max\limits_{Z,x,y} \sum\limits_{r \in R} \left(f_r(Z_r) - \frac{\sum_{b \in B_r} cap_b \mu_{rb}}{dem_r} \right)$, subject to

$$\sum_{j \in NM} \left((1 - nos_j)(csta_j + cl_j^{\min}) Y_j^s + cpoi_{\min} Y_j^p \right) + \sum_{r \in R, b \in B} cbus_b x_{rb} \le ucc,$$

$$\sum_{j \in NM} \left((1 - nos_j) osta_j Y_j^s + opoi_{\min} Y_j^p \right) + \sum_{r \in R, b \in B} obus_{rb} x_{rb} \le uoc,$$

$$\frac{Y_j^p}{up_j} \le Y_j^s \le Y_j^p, \ j \in NM,$$

$$\sum_{r \in RM_j} k_{rj} y_r - noc_j \le Y_j \le up_j uc_{\max}, \ j \in NM \backslash D,$$

$$\frac{Y_j + noc_j}{uc_{\max}} \le Y_j^p + nop_j \le up_j, \ j \in NM \backslash D,$$

$$Y_j^p \ge (1 - nos_j) y_r, \ j \in D, r \in RM_j,$$

$$Z_r \le \sum_{b \in B} cap_b x_{rb}, \ r \in R,$$

$$Z_r \le dem_r, \ r \in R,$$

$$\sum_{b \in BO_r} nbo_{rb} cap_b + \sum_{b \in B_r} cap_b x_{rb} + \sum_{b \in NV_r} cap_b \mu_{rb} \ge dem_r y_r, \ r \in R,$$

$$\mu_{rb} \le nv_{rb}^0 y_r, \ r \in R, \ b \in B,$$

$$\frac{1}{dem_r + 1} Z_r \le y_r \le Z_r, \ r \in R,$$

$$Z_r, \ x_{rb}, \ \mu_{rb}, \ Y_j, \ Y_j^p \text{ are non-negative integers}, \ Y_j^s \in \{0, 1\},$$
$$y_r \in \{0, 1\}, \ b \in B, \ r \in R, \ j \in NM.$$

If $f_r(Z_r)$ is a linear function of Z_r, then this problem is an *Integer Linear Programming* problem, which can be solved by a standard software.

3.3. Computational complexity

We show in Theorems 1 and 2 that sharing charging stations by the routes (their e-buses) or sharing transformers by the charging stations makes the problem Opt-Fast-Char computationally difficult.

Theorem 1. *Problem* Opt-Fast-Char *is NP-hard in the strong sense even if there is a single charging station type, a single e-bus type, a single transformer with an unlimited power and eligible for linking with any stop, and all the costs are equal to zero.*

Proof. We will use a pseudo-polynomial reduction from the NP-complete problem Exact Cover by 3-sets (X3C), see Garey and Johnson (1979).

Exact Cover by 3-Sets (X3C): Given a family $S = \{S_1, \dots, S_l\}$ of 3-element subsets of the set $K = \{1, \dots, 3k\} = \cup_{i=1}^{l} S_i$, does S contain an exact cover of K, i.e., a subfamily $Y \subseteq S$, $|Y| = k$, such that each $j \in K$ belongs to exactly one 3-element set in Y?

Given an instance of X3C, we construct an instance of the problem Opt-Fast-Char, in which there are l routes. Route r goes via stops of the set S_r, has the total passenger demand $dem_r = 1$, the traffic interval $ut_r = 1$ and the value function $f_r(Z) = Z$, $r = 1, \dots, l$. All the stops of each route are obligatory charge stops, there is a single charging type c^* and the number of the charging points and connectors at any stop cannot exceed 1. There is a single e-bus type b^* with the passenger capacity $cap_{b^*} = 1$ and the charging time $ct_{rjb^*c^*} = 1$, $j \in K$, $r = 1, \dots, l$. No old e-bus or charging infrastructure element is in use. We will show that there exists a feasible solution X for this instance with the value $f(X) \ge k$ if and only if the original instance of X3C has a solution.

"Only if". Assume that there exists a feasible solution X of the constructed instance of the problem Opt-Fast-Char with the value $f(X) \ge k$. Since $dem_r = 1$, $r = 1, \dots, k$, there exists a feasible solution with $f(X) \ge k$, in which the e-buses are used on exactly k routes, with one e-bus on each such a route. Furthermore, no two e-buses (of different routes) visit the same stop, because otherwise this stop will require two charging connectors. Therefore, exactly k e-buses, each of a different route, are used, no stop of these routes belongs to two of these routes, and the sets of stops of these routes represent an exact cover of the set K, being a solution of the original instance of X3C.

"If". Let Y be a solution of the instance of X3C. Construct a solution X of the instance of Opt-Fast-Char, in which a charging station with a single charging point and a single connector is opened at each stop of the set K, and, if $S_r \in Y$, then a single e-bus is assigned to route r. This solution is feasible and $f(X) = k$, as it is required for the part "if". ■

Theorem 2. *Problem* Opt-Fast-Char *is NP-hard in the strong sense even if there is a single route and all the costs are equal to zero.*

Proof. A pseudo-polynomial reduction from the NP-complete in the strong sense problem 3-Partition (Garey and Johnson, 1979) is used.

3-Partition: Given $3k + 1$ positive integer numbers $h_1, \dots, h_{3k}$ and H satisfying $\sum_{i=1}^{3k} h_i = kH$ and $H/4 < h_i < H/2$, $i = 1, \dots, 3k$, does there exist a partition of the set $\{1, \dots, 3k\}$ into subsets $X_1, \dots, X_k$ such that $\sum_{i \in X_j} h_i = H$ for $j = 1, \dots, k$?

Given an instance of 3-Partition, we construct an instance of the problem Opt-Fast-Char, in which no old e-bus or charging infrastructure element is in use and there is a single route r^* with the total passenger demand $dem_{r^*} = 1$, and the value function $f_{r^*}(Z) = Z$. Furthermore, there are k stops j with the upper bound on the number of charging points $up_j = 1$, one e-bus type b^* with the passenger capacity $cap_{b^*} = 1$, one charging point type c^* with the number of connectors capacity $uc_{c^*} = 1$, and the output power of one connector $p_{c^*} = H$. Any e-bus of the type b^* must be charged at every stop. Finally, there are $3k$ transformer locations t with the transformer output power $utp_t = h_t$, $t = 1, \dots, 3k$. We will show that there exists a feasible solution X for this instance with the value $f(X) \ge 1$ if and only if the original instance of 3-Partition has a solution.

"Only if". Assume that there exists a feasible solution X of the constructed instance of the problem Opt-Fast-Char with the value $f(X) \ge 1$. Then, there exists a feasible solution with the value $f(X) \ge 1$, in which a single e-bus of the type b^* is assigned to the route r^* and it is charged at every stop. Consider the set of transformer locations $T_j(X)$ linked with the stop j, $j = 1, \dots, k$. For feasibility, the relations $\sum_{t \in T_j(X)} utp_t = \sum_{t \in T_j(X)} h_t \ge H$ must be satisfied for $j = 1, \dots, k$. Since $\sum_{t=1}^{3k} h_t = kH$, the above relations are satisfied only if $\sum_{t \in T_j(X)} h_t = H$ for $j = 1, \dots, k$. Hence, the sets $X_j = T_j(X)$, $j = 1, \dots, k$, constitute a solution of the instance of 3-Partition.

"If". Let $X_1, \dots, X_k$ be a solution of the instance of 3-Partition. Construct a solution X of the instance of Opt-Fast-Char, in which a single e-bus of the type b^* is charged at each stop, and the set of transformers linked with the stop j is $T_j(X) = X_j$, $j = 1, \dots, k$. This solution is feasible, $\sum_{t \in T_j(X)} utp_t = H$, $j = 1, \dots, k$, and $f(X) = 1$, as it is required for the part "if". ■

3.4. Randomized heuristic

Metaheuristics such as Simulated Annealing, Tabu Search, Genetic Algorithm, Ant Colony Optimization, Particle Swarm Optimization (PSO) and other local search methods are the most popular techniques for solving practical NP-hard problems. We have chosen the PSO for the problem Opt-Fast-Char, because it works well for the problems with a complex solution structure such as the solution structure of the problem Opt-Fast-Char, it is easily adapted to the problem changes which happen to the real-life cases of Opt-Fast-Char, it is easy for implementation because it operates with only three parameters of the candidate solution, and it convergence rate is often faster than that of the other metaheuristic methods (Sengupta et al., 2019).

Denote by $\mathcal{X}$ the set of all feasible solutions of the problem Opt-Fast-Char. Our heuristic RH combines a randomized choice of feasible or infeasible partial solutions with the PSO technique. A set of feasible complete solutions $\mathcal{Q} \in \mathcal{X}$ is constructed, which we expect to contain solutions close to the optimal solution. Steps of the heuristic RH are performed sequentially, unless it is stated differently.

Randomized heuristic RH.

Step 1. (Initializing) Define $\mathcal{Q} = \{Q_0\}$, where Q_0 is a feasible solution obtained from the potentially infeasible solution of the corresponding problem Relax by iteratively removing new e-buses and charging infrastructure elements required for them. Note that a solution with no new e-bus or charging infrastructure element is feasible for Opt-Fast-Char. In Steps 2–4, a partial solution Q of Opt-Fast-Char is generated. It is extended to a feasible or infeasible complete solution in Step 5. A feasible solution is added to the set $\mathcal{Q}$.

Step 2. (Generating sets of routes, new e-bus types, remaining conventional vehicle types and new charging types) Generate a random set $R(Q) \subseteq R$ of routes served by at least one new e-bus, a random set $B_r(Q) \subseteq B$ of new e-bus types and a random set $V_r(Q) \subseteq V_r$ of the remaining conventional vehicle types to serve route $r \in R(Q)$, and a unique charging type $c_b(Q) \in C_b$ for each e-bus type $b \in \cup_{r\in R(Q)} B_r(Q)$. A higher route selection probability can be given to the routes with the old charging stations that have larger free connector capacity at their stops. For a given $r \in R(Q)$, a higher selection probability for the pair (e-bus type b, charging type c_b) can be given to the pairs (b, c_b) such that the maximum charging time $\max_{j\in r}\{ct_{rjbc_b}\}$ is minimized, and a higher selection probability for the conventional vehicle type can be given to the types with the smaller passenger capacities. Recall that if there are old e-buses of a type b, then the old and new e-buses of this type are charged at the points of the old charging type co_b.

Step 3. (Verifying driving feasibility and extending set of charging stations) Initialize set $N(Q)$ of charging stops to comprise obligatory charge stops of the routes from $R(Q)$. For all routes $r \in R(Q)$ and e-bus types $b \in B_r(Q)$, recursively calculate driving ranges $d^0_{rjbc_b(Q)}(Q)$ starting from the depot ($j = d(r)$), where $d^0_{rjbc_b(Q)}(Q) = d_b^{(\max)}$. If $d^0_{ribc_b(Q)}(Q) \geq d_{rij}$, $(i,j) \in S_{rc_b(Q)}(Q)$, $b \in B_r(Q)$, $r \in R(Q)$, then all the e-buses can feasibly drive. Pass to Step 4. If $d^0_{ribc_b(Q)}(Q) < d_{rij}$ for a segment $(i,j) \in S_{rc_b(Q)}(Q)$, then assign charging type $c_b(Q)$ to a certain stop of route r between stops i and j. A preference is given to an intersection stop of several routes from $R(Q)$ and to the closest stop to the equidistant point between i and j. Repeat Step 3.

Step 4. (Generating sets $T_j(Q)$ of transformer locations) For each stop $j \in N(Q)\backslash NO$, generate a random set $T_j(Q)$ of new transformer locations linked with this stop. A preference is given to the sets of minimum cardinality, including the case $T_j(Q) = \emptyset$ if $TO_j \neq \emptyset$. For the transformer locations with $T_j(Q) \neq \emptyset$, a preference is given to the sets minimizing the total linking and transformer building cost.

Step 5. (Extending Q to a complete solution) Solve the following problem to determine numbers $nb_{rb}(Q)$ and $nv_{rb}(Q)$ of new e-buses and remaining conventional vehicles, and, as a result, numbers $nc_{jc}(Q)$ and $np_{jc}(Q)$ of the new charging connectors and points.

Problem Opt-Fast-Char(Q) : $\max f(Q)$, subject to

$$cc(Q) \leq ucc, \tag{12}$$

$$oc(Q) \leq uoc, \tag{13}$$

$$\sum_{c\in C}(nc_{jc}(Q) + noc_{jc})p_c \leq \sum_{t\in T_j(Q)\cup TO_j} utp_t,\ j \in N, \tag{14}$$

$$nop_j + np_j(Q) \leq up_j,\ j \in N, \tag{15}$$

$$nv_{rb}(Q) \leq nv^0_{rb},\ r \in R(Q),\ b \in B_r(Q), \tag{16}$$

$$\sum_{b\in BO_r} nbo_{rb}cap_b + \sum_{b\in B_r(Q)\cup BO_r} nb_{rb}(Q)cap_b + \sum_{b\in V_r(Q)} nv_{rb}(Q)cap_b \geq dem_r,\ r \in R(Q). \tag{17}$$

All the functions of Q are the same functions of X defined in Table 10 of Appendix A. Note that $cc(Q)$ includes capital costs of the new charging stations, transformers and transformer links, and $oc(Q)$ includes operating costs of the new charging stations, charging points and transformers which are determined in Steps 3–4. A Particle Swarm Optimization (PSO) technique is used to solve the problem Opt-Fast-Char(Q), see Clerc (2010), Kennedy and Eberhart (1995) and Pedersen and Chipperfield (2010). It is described in Appendix B.

Let (Q, Q^0) denote a solution of Opt-Fast-Char(Q) delivered by the PSO algorithm. Thus, (Q, Q^0) is a complete solution of the original problem Opt-Fast-Char. If (Q, Q^0) is feasible, then extend the set of the candidate solutions, $\mathcal{Q} := \mathcal{Q} \cup \{(Q, Q^0)\}$. If the total computational time permits, then perform Step 2, else perform Step 6.

Step 6. Output solution Q^* such that $f(Q^*) = \max_{Q\in\mathcal{Q}} f(Q)$. If there are solutions $Q \in \mathcal{Q}$ with values $f(Q)$ slightly smaller than $f(Q^*)$ but with much smaller costs $cc(Q)$ and $oc(Q)$, then they can be returned as well. ■

4. Balanced schedule problem

In this section, we collectively call new and old e-buses and conventional vehicles as buses. The same problem, denoted as BS (Balanced Schedule), is solved for each route $r \in R(Q^*)$, where Q^* is a solution of Opt-Fast-Char. The input of the problem BS for any route r consists of the numbers $h_b := nb_{rb}(Q^*) + nbo_{rb}$ of the e-buses of type b and the numbers $h_{b'} = nv_{rb'}(Q^*)$ of the conventional vehicles of type b' for b and b' from the set $\{1, \ldots, n\}$, where $n := |B_r(Q^*) \cup BO_r \cup VC_r(Q^*)|$. All the buses assigned to the route r are assumed to depart with the same traffic interval ut_r.

The objective of the problem BS is to distribute buses of the same type assigned to the same route as uniform as possible over all the buses serving this route, which will determine a balanced bus schedule. It is assumed that buses of different types have different passenger capacities. Therefore, a balanced schedule ensures a uniform allocation of bus passenger capacities in a single cycle bus sequence on the same route.

Denote $H = \sum_{b=1}^{n} h_b$ and $s_b = h_b/H$, $b = 1, \ldots, n$. The value of H is equal to the total number of buses of the same route. According to the balanced schedule objective, the number of buses of type b among the first k buses must be kept as close to $s_b k$ as possible for $b = 1, \dots, n$. Introduce non-negative integer variables x_{bk} representing the number of the buses of type b among the first k buses, $b = 1, \dots, n$, $k = 1, \dots, H$. Denote by x matrix with the entries x_{bk}. The problem BS admits the following two formulations.

Problem BS-Sum : $\min_x \sum_{k=1}^{H} \sum_{b=1}^{n} (x_{bk} - s_b k)^2$, subject to

$$\sum_{b=1}^{n} x_{bk} = k, \ k = 1, \dots, H, \tag{18}$$

$$0 \le x_{bk} - x_{b(k-1)}, \ b = 1, \dots, n, \ k = 1, \dots, H, \tag{19}$$

$$x_{b0} = 0, \ b = 1, \dots, n, \tag{20}$$

$$x_{bH} = h_b, \ b = 1, \dots, n, \tag{21}$$

$$x_{bk} \in Z_0, \ b = 1, \dots, n, \ k = 1, \dots, H. \tag{22}$$

Problem BS-Max : $\min_x \max_{1 \le k \le H, 1 \le b \le n} |x_{bk} - s_b k|$, subject to (18)–(22).

Kubiak and Sethi (1991) reduce the problem BS-Sum to an assignment problem which can be solved in $O(H^3)$ time. Denote by M^* the optimal objective value of BS-Max and denote $h_{\max} = \max_{1 \le b \le n}\{h_b\}$. Steiner and Yeomans (1993) prove that $\frac{H - h_{\max}}{H} \le M^* \le 1$ and reduce the problem BS-Max to a single machine scheduling problem solvable in $O(H \log H)$ time. Thus, both problems are not NP-hard in the strong sense. The relation $M^* \le 1$ means that an optimal solution of BS-Max is such that the total number of buses departing in the first k traffic intervals never deviates from the desired number ks_b by more than one for any vehicle type. Kovalyov et al. (2001) provide extensive computer experiments with both models BS-Sum and BS-Max. Brauner and Crama (2004) strengthen the earlier results by demonstrating that $M^* \le \frac{H-1}{H}$ and that BS-Max can be solved in $O(g(n) \log H)$ time, where $g(n)$ is a function of n.

We employ formulation BS-Max for our purposes because it is adequate and easy for implementation. We use the $O(H \log H)$ time algorithm of Steiner and Yeomans (1993), denoted as SY, to solve this problem. The algorithm can be described as follows. Consider the following auxiliary decision problem.

Problem BS-Max(M) : $\max_{1 \le k \le H, 1 \le b \le n} |x_{bk} - s_b k| \le M$,

subject to (18)–(22),

where M satisfies $\frac{H - h_{\max}}{H} \le M \le \frac{H-1}{H}$. This problem is equivalent to the scheduling problem $1|r_j, \bar{d}_j, p_j = 1|\cdot$, in which there is a single machine (route), unit-time ($p_j = 1$) jobs (buses with a unit-time traffic interval) of a set N, each job j has a release date r_j and a deadline $\bar{d}_j$, no two jobs can be processed concurrently, and each job j has to be processed between r_j and $\bar{d}_j$. For any instance of BS-Max(M), an equivalent instance of $1|r_j, \bar{d}_j, p_j = 1|\cdot$ is constructed by setting $N = \{(b, i) \mid b = 1, \dots, n, \ i = 1, \dots, h_b\}$, $r_{(b,i)} = \lceil \frac{i-H}{s_b} \rceil$ and $\bar{d}_{(b,i)} = \lfloor \frac{i-1+M}{s_b} + 1 \rfloor$, $(b, i) \in N$, where (b, i) is the bus of type b numbered ith. The problem $1|r_j, \bar{d}_j, p_j = 1|\cdot$ is solved by applying the following rule formulated by Horn (1974): In any time interval k, $k = 1, \dots, H$, schedule an available unassigned job j ($r_j \le k \le \bar{d}_j$) with the smallest deadline. If no job is available for $k \in \{1, \dots, H\}$, then $1|r_j, \bar{d}_j, p_j = 1|\cdot$, and hence, the corresponding instance of BS-Max(M), has no solution.

Algorithm SY.

Step 1. (Initialization) Set $L = \frac{H - h_{\max}}{H}$ and $U = \frac{H-1}{H}$. We have $L \le M^* \le U$. Solve the problem BS-Max(L). If a feasible solution of this problem is found, then it is optimal for BS-Max, stop. Assume that it is not found. Solve the problem BS-Max(U). Observe that $|M_1 - M_2| \ge \frac{1}{H^2}$ for any two distinct objective function values M_1 and M_2 of BS-Max.

Step 2. (Bisection search) If $U - L < \frac{1}{H^2}$, then stop: a feasible solution for the problem BS-Max(U) is optimal for BS-Max. Else, solve the problem BS-Max($(L+U)/2$). If a feasible solution of this problem is found, then re-set $U := (L+U)/2$ and repeat Step 2. Else, re-set $L := (L+U)/2$ and repeat Step 2. ■

The number of iterations of the bisection search in Step 2 is $O(\log V)$.

Remark. Algorithm SY constructs a balanced schedule for a single bus cycle. A balanced schedule for $K \ge 2$ cycles in the decisive time period can be constructed by solving a modification of the problem BS in which the number of vehicles of each type is increased K times.

Table 2
E-bus types $b \in B$.

b	Name	Capacity cap_b	Range of one charge	Char.time ct_{rjbc}, $\forall r, j, c$	Cap.cost $cbus_b$	Oper., depr. & ener.cost $obus_{rb}$, $\forall r$
1	E433	153	15	6	500 000	270 000
2	E420	87	20	6	350 000	180 000
3	E321	85	40	10	400 000	200 000
4	E490	75	25	6	400 000	170 000
5	321D	90	15	40	300 000	180 000
6	420D	90	15	30	330 000	200 000

5. Computational experiments

Computational experiments are provided for a real-life case (Section 5.1) and for random instances based on the real-life case (Section 5.2).

5.1. Real-life case

A set of public transport routes in the city of Minsk (Belarus) is considered. Super-capacitor fast-charging technology is used in the batteries of all the e-buses. There are 26 routes, 23 intermediate stops for charging, e-bus depot D_1 for routes 1–14 and e-bus depot D_2 for routes 15–26. All the intermediate stops are terminal stops of some routes, and the intermediate stops are visited by the e-buses in both direct and backward directions of their routes. The set NM_r of obligatory charge stops of each route r consists of the depot and the terminal stops of this route.

Each stop j can accommodate up to $up_j = 3$ charging points and each charging point includes a single connector. New and old charging points are of the same type c and they can charge e-buses of any type. The power of one charging connector is $p_c = 260$ kW. One old charging point is opened at the depot D_1 and at each of the stops 1, 2, 12, 13 and 14. Stop or depot j can only be linked with a unique transformer location $t(j)$ (any other link has an unacceptable cost). Transformer locations with the old transformers are from the set $TO = \{t(j) \mid j = D_1, D_2, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$. The output power of any transformer is 800 kW. The e-bus types and their characteristics are given in Table 2.

The other costs are $cpoi_c = 120\,000$ (charging point capital cost), $opoi_c = 4500$ (charging point operating and depreciation cost), $csta_j = 5000$ (charging station capital cost), $osta_j = 500$ (charging station operating and depreciation cost), $ctb_t = 200\,000$ (transformer capital cost) and $cl_{tj} = 5000$ (cost of linking transformer location $t(j)$ with stop j). There are four types of conventional vehicles: diesel buses M103 and M105 and trolleybuses T420 and T333 with passenger capacities 100, 160, 115 and 170, respectively. The value function is defined as $f_r(Z_r) = Z_r$ for any route. The other route characteristics are given in Table 3.

Solution. The running time of the algorithm RH, the total number of iterations of Steps 2–5 of this algorithm and the number of consecutive iterations of these steps were limited to two hours, two millions and five hundred thousands, respectively. The following pairs of upper bounds

Table 3
Routes. Terminal stops are marked with *.

r	ut_r	nbo_{rb}, $b = E433$	nv^0_{rb}, $b = M103$	nv^0_{rb}, $b = M105$	nv^0_{rb}, $b = T420$	nv^0_{rb}, $b = T333$	$\pi_r = (d(r), j_1^{(r)}, \dots, j_{i(r)}^{(r)}, j_1^{(r)})$	Inter-stop distance
1	7	4	3	2	0	0	$(D_1, 1^*, 2^*, 1^*)$	(3,9,9)
2	20	0	0	2	0	0	$(D_1, 1^*, 2, 3^*, 2, 1^*)$	(3,9,4,4,9)
3	20	0	1	2	0	0	$(D_1, 1^*, 2, 4^*, 2, 1^*)$	(3,9,6,6,9)
4	5	0	0	0	2	5	$(D_1, 1^*, 5^*, 1^*)$	(3,5,5)
5	5	0	0	0	4	6	$(D_1, 1^*, 5, 6^*, 5, 1^*)$	(3,5,4,4,5)
6	15	0	5	0	0	0	$(D_1, 1^*, 7^*, 1^*)$	(3,11,11)
7	10	0	0	0	2	5	$(D_1, 1^*, 8, 9^*, 8, 1^*)$	(3,5,4,4,5)
8	10	0	0	0	3	6	$(D_1, 9^*, 10, 11^*, 10, 9^*)$	(2,5,13,13,5)
9	40	0	0	2	0	0	$(D_1, 10^*, 7^*, 10^*)$	(3,9,9)
10	15	0	0	0	2	2	$(D_1, 10^*, 7^*, 10^*)$	(3,9,9)
11	10	6	0	0	2	0	$(D_1, 12^*, 13^*, 12^*)$	(2,13,13)
12	20	0	1	2	0	0	$(D_1, 14^*, 15^*, 14^*)$	(4,8,8)
13	15	4	0	0	2	0	$(D_1, 14^*, 13, 14^*)$	(4,7,7)
14	10	0	0	0	2	5	$(D_1, 15^*, 14, 11^*, 14, 15^*)$	(4,7,9,9,7)
15	20	0	2	2	0	0	$(D_2, 14^*, 11^*, 14^*)$	(3,9,9)
16	20	0	3	3	0	0	$(D_2, 14^*, 16^*, 14^*)$	(3,15,15)
17	10	0	0	0	2	4	$(D_2, 10^*, 16^*, 10^*)$	(3,14,14)
18	15	0	0	0	1	3	$(D_2, 14^*, 16^*, 14^*)$	(3,13,13)
19	10	0	3	6	0	0	$(D_2, 14^*, 17^*, 14^*)$	(3,14,14)
20	7	0	3	6	0	0	$(D_2, 18^*, 19^*, 18^*)$	(4,12,12)
21	20	0	2	5	0	0	$(D_2, 18^*, 2, 20^*, 2, 18^*)$	(4,18,2,2,18)
22	10	0	5	7	0	0	$(D_2, 14, 2, 21^*, 2, 14, 13^*, 14)$	(3,9,2,2,9,7,7)
23	10	0	0	0	3	0	$(D_2, 22^*, 15^*, 22^*)$	(2,6,6)
24	10	0	2	5	0	0	$(D_2, 15^*, 2, 23^*, 2, 15^*)$	(6,10,6,6,10)
25	20	0	1	2	0	0	$(D_2, 15^*, 2, 23^*, 2, 15^*)$	(6,10,7,7,10)
26	10	0	0	0	6	8	$(D_2, 18^*, 16^*, 18^*)$	(6,21,21)

Table 4
$(ucc, uoc) = (10^7, 5 \cdot 10^6)$. Balanced schedules.

Route	Departure sequence
20	(E433,E433,E433,E433,M103,E433,E433,E433,E433)
22	(E433,E433,E433,E433,E433,M103,E433,E433,E433,E433,E433)

Table 5
$(ucc, uoc) = (1.5 \cdot 10^7, 7 \cdot 10^6)$. Balanced schedules.

Route	Departure sequence
1	(E433,E433,E433,E433,M103,E433,E433,E433,E433)
2	(321D,M105,321D)
8	(E433,E433,E433,T333,E433,E433,T420,E433,E433,E433)
10	(E433,T420,T333,E433)
11	(E433,E433,E433,E433,T420,E433,E433,E433,E433)
13	(E433,E433,T420,E433,E433,E433)
20	(E433,E433,E433,E433,M103,E433,E433,E433,E433)

Table 6
$(ucc, uoc) = (2 \cdot 10^7, 10^7)$. Balanced schedules.

Route	Departure sequence
1	(E433,E433,E433,E433,M103,E433,E433,E433,E433)
5	(E433,E433,E433,E433,T420,E433,E433,E433,E433,E433)
16	(E433,E433,M103,E433,E433,E433)
19	(E433,E433,E433,E433,M103,E433,E433,E433,E433)
22	(E433,E433,E433,E433,E433,M103,E433,E433,E433,E433,E433)

on the costs were considered: $(ucc, uoc) \in \{(10^7, 5 \cdot 10^6), (1.5 \cdot 10^7, 7 \cdot 10^6), (2 \cdot 10^7, 10^6)\}$. Denote solution delivered by RH for the pair (ucc, uoc) as $Q_{RH}^{u_1,u_2}$, where u_1 and u_2 are the first two digits in ucc and uoc, respectively. For a given solution, denote by $\#_b$ the number of new e-buses of type b.

For $(ucc, uoc) = (10^7, 5 \cdot 10^6)$, the value and cost characteristics of the solution are $f(Q_{RH}^{10,5}) = 2753.86$, $cc(Q_{RH}^{10,5}) = 9.72 \cdot 10^6$ and $oc(Q_{RH}^{10,5}) = 4.88 \cdot 10^6$. New e-buses are assigned to the following routes. Route 20: $\#_{E433} = 8$. Route 22: $\#_{E433} = 10$. A new charging station is opened at the depot D_2 and each of the stops 18, 19, 21. A single new charging point is opened at the depot D_2 and each of the stops 18, 19, 21. A new transformer is installed at the location $t(21)$. The single cycle balanced schedules for the routes affected by the decision are given in Table 4.

For $(ucc, uoc) = (1.5 \cdot 10^7, 7 \cdot 10^6)$, the value and cost characteristics of the solution are $f(Q_{RH}^{15,7}) = 3849.55$, $cc(Q_{RH}^{15,7}) = 1.43 \cdot 10^7$ and $oc(Q_{RH}^{15,7}) = 6.9 \cdot 10^6$. New e-buses are assigned to the following routes. Route 1: $\#_{E433} = 4$. Route 2: $\#_{321D} = 2$. Route 8: $\#_{E433} = 8$. Route 10: $\#_{E433} = 2$. Route 11: $\#_{E433} = 1$. Route 13: $\#_{E433} = 1$. Route 20: $\#_{E433} = 8$. A new charging station is opened at the depot D_2 and each of the stops 3, 7, 9, 10, 11, 18, 19. A single new charging point is opened at the depots D_2 and each of the stops 7, 9, 10, 11, 13, 18, 19. Two new charging points are opened at each of the stops 1, 3. A new transformer is installed at the location $t(3)$. The single cycle balanced schedules are given in Table 5.

For $(ucc, uoc) = (2 \cdot 10^7, 10^7)$, the value and cost characteristics of the solution are $f(Q_{RH}^{20,10}) = 5507.49$, $cc(Q_{RH}^{20,10}) = 1.93 \cdot 10^7$ and $oc(Q_{RH}^{20,10}) = 9.76 \cdot 10^6$. New e-buses are assigned to the following routes. Route 1: $\#_{E433} = 4$. Route 5: $\#_{E433} = 9$. Route 16: $\#_{E433} = 5$. Route 19: $\#_{E433} = 8$. Route 22: $\#_{E433} = 10$. A new charging station is opened at the depot D_2 and each of the stops 6, 16, 17, 21. A single new charging point is opened at the depots D_2 and each of the stops 14, 16, 17, 21. Two new charging points are opened at each of the stops 1, 6. A new transformer is installed at the location $t(21)$. The single cycle balanced schedules are given in Table 6.

In order to evaluate the quality of the solutions delivered by the randomized heuristic RH, we formulated the Relax problem (Section 3.2) for the Minsk case of the problem Opt-Fast-Char and solved it by the academic version of the specialized software IBM ILOG CPLEX Optimization Studio. Similar to $Q_{RH}^{u_1,u_2}$, denote by $Q_{Relax}^{u_1,u_2}$ the optimal solution of the Relax problem for the Minsk case with the upper bounds ucc and uoc on the total costs, where u_1 and u_2 are the first two digits in ucc and uoc, respectively. Denote by $Q_0^{u_1,u_2}$ feasible solution of the Minsk case obtained from $Q_{Relax}^{u_1,u_2}$ by iteratively removing new e-buses and required charging infrastructure elements. The obtained results are: $f(Q_{Relax}^{10,5}) = 2780.92$, $f(Q_0^{10,5}) = 2600.55$, $f(Q_{Relax}^{15,7}) = 3914.81$, $f(Q_0^{15,7}) = 3824.75$, $f(Q_{Relax}^{20,10}) = 5597.55$ and $f(Q_0^{20,10}) = 5507.49$. Calculate $f(Q_{RH}^{10,5})/f(Q_{Relax}^{10,5}) = 0.99$, $f(Q_0^{10,5})/f(Q_{Relax}^{10,5}) = 0.94$, $f(Q_{RH}^{15,7})/f(Q_{Relax}^{15,7}) = 0.98$, $f(Q_0^{15,7})/f(Q_{Relax}^{15,7}) = 0.98$ and $f(Q_{RH}^{20,10})/f(Q_{Relax}^{20,10}) = 0.98$, $f(Q_0^{20,10})/f(Q_{Relax}^{20,10}) = 0.98$.

We see that the total value of the solution delivered by the randomized heuristic for the Minsk case is within 2% of the upper bound.

Furthermore, the total capital cost and the total operating, depreciation and energy cost are within 5% and 2% of their upper bounds, respectively. Several other instances of the problems Opt-Fast-Char and Relax were solved, which are fixed modifications of the Minsk case instances. Some instances of Relax were not solved to optimality in two hours, which is a justification for selecting the metaheuristic algorithm RH to solve the much more complicated problem Opt-Fast-Char.

5.2. Experiments with random instances

Two groups of random instances of the problem Opt-Fast-Char are generated. Each group is associated with a basic instance of Opt-Fast-Char. For group 1, the basic instance is the Minsk case with the cost upper bounds $ucc = 2.4 \cdot 10^7$ and $uoc = 1.2 \cdot 10^7$, and for group 2, it is the following *Modified Minsk case*.

In the Modified Minsk case, an extra stop is added in the middle between any two consecutive stops of each route of the Minsk case if the distance between these two stops is at least 7. It can be seen from Table 3 that the number of new stops is 28. We number them $24, 25, \dots, 51$. The new stops are partitioned into 7 new routes numbered $27, 28, \dots, 33$ and each of them consists of 4 consecutively indexed new stops to be visited in the increasing order. Terminal stops of a new route are the stops with the smallest and largest indices of this route. The new routes have the same new depot denoted as D_3. The set NM of obligatory charge stops consists of the depots D_1, D_2, D_3 and the old 23 stops. Recall that an obligatory charge stop of an old route is not necessarily an obligatory charge stop of another old route. Each new stop belongs to the old route by which it is defined and a new route. The distances between any two consecutive stops of a new route, including D_3, are randomly selected from the interval $[2, 10]$. Each new route r is served by 3 diesel buses M103 and 3 diesel buses M105 with the same traffic interval $ut_r = 10$, and it is associated with the e-bus operating, depreciation and energy costs $obus_{rb}$ given in Table 2. The cost upper bounds ucc and uoc are increased by 20% from their Minsk case values.

Further on, there are three charging types, $C = \{1, 2, 3\}$, and a charging point of any type c includes up to $uc_c = 2$ connectors. The powers of the charging connectors are $p_1 = 200$, $p_2 = 250$ and $p_3 = 300$, and the charging point costs are $cpoi_2 = 120\,000$, $opoi_2 = 4500$ and $cpoi_c = cpoi_2 \frac{p_c}{p_2}$, $opoi_c = opoi_2 \frac{p_c}{p_2}$ for $c = 1, 3$. The set of the e-bus types does not change, and any charging type is feasible for any e-bus type. There is no old e-bus or old charging infrastructure element. For each stop j, there are two feasible transformer locations $t_1(j)$ and $t_2(j)$. Transformer at location $t_1(j)$ is assigned the output power $utp_{t_1(j)} = 500$ and the capital cost $ctb_{t_1(j)} = 160\,000$, and transformer at location $t_2(j)$ is assigned the output power $utp_{t_2(j)} = 800$ and the capital cost $ctb_{t_2(j)} = 200\,000$ for any j. The other input data are the same as in the Minsk case.

This modification increases the search space of the problem Opt-Fast-Char for the Minsk case about $2^{70} \cdot 2^{28} \cdot 3^{54} \geq 2^{152}$ times, where 2^{70} is the number of combinations of 10 vehicle types (6 e-bus types and 4 conventional vehicle types) for each of the 7 new routes, 2^{29} is the number of decisions of opening or not a charging point at each of the 28 new stops and the depot D_3, and 3^{54} is the number of decisions of whether to link each of the 51 stops and 3 depots with the first or the second transformer location ($t_1(j)$ or $t_2(j)$), or with both these locations.

For any instance of group 1 (resp., group 2), the combinatorial structure of the Minsk case (resp., Modified Minsk case) is kept, but all the numerical parameters (times, costs, distances, electrical powers, passenger capacities) randomly deviate ±10% from their Minsk case (resp., Modified Minsk case) values. For group 2, the charging times ct_{rjb2} are made dependent of j such that ct_{rjb2} coincides with the same Minsk case value (depending solely of b) if j is the depot or a terminal stop of route r, else it is half of the Minsk case value. Values ct_{rjbc} are made dependent of c such that $ct_{rjbc} = ct_{rjb2} \frac{p_2}{p_c}$ for $c = 1, 3$.

Four series of instances are considered for each group. Each series of group 1 (resp., group 2) is characterized by the pair (n_1, n_2) (resp., triple (n_1, n_2, n_3)), where n_i is the number of routes of depot D_i selected randomly from the set of all routes of this depot and included into the set R, $i = 1, 2, 3$. For each series of any group, 20 instances of the problem Opt-Fast-Char are generated. Each instance is solved by the algorithm RH, and the corresponding Relax problem is solved by CPLEX with the time limit of two hours (7200 s) for both solution procedures. The experiments were run on a PC with Intel Core i5 2.3 GHz processor and 4 GB of RAM under OS Windows. Tables 7 and 8 contain the following information for the 20 instances of the same series: number $\#^{feas}_{Relax}$ of instances of Relax which are feasible for the original problem, mean and maximal running times of CPLEX for Relax, and mean and minimal values of $\frac{f(Q_{RH})}{f(Q_{Relax})}$, $\frac{f(Q_0)}{f(Q_{Relax})}$, $\frac{f(Q_0)}{f(Q_{RH})}$, $\frac{cc(Q_{RH})}{ucc}$, $\frac{oc(Q_{RH})}{uoc}$, $\frac{cc(Q_{Relax})}{ucc}$, $\frac{oc(Q_{Relax})}{uoc}$, $\frac{cc(Q_0)}{ucc}$ and $\frac{oc(Q_0)}{uoc}$, where the solutions Q_{RH}, Q_{Relax} and Q_0 are defined in the same way as for the basic Minsk case.

Table 7
Group 1 of random instances.

Series (n_1, n_2)		(3,2)	(5,4)	(9,8)	(14,12)
$\#^{feas}_{Relax}$		0	3	0	0
Time CPLEX, sec	mean	0.11	1.0	51.2	96.3
	max	0.31	4.41	898	999
$\frac{f(Q_{RH})}{f(Q_{Relax})}$	mean	0.97	0.96	0.96	0.93
	min	0.88	0.9	0.91	0.85
$\frac{f(Q_0)}{f(Q_{Relax})}$	mean	0.76	0.82	0.8	0.73
	min	0.18	0.24	0.35	0.09
$\frac{f(Q_0)}{f(Q_{RH})}$	mean	0.78	0.86	0.83	0.78
	min	0.21	0.25	0.37	0.11
$\frac{cc(Q_{RH})}{ucc}$	mean	0.98	0.98	0.97	0.99
	min	0.89	0.94	0.93	0.96
$\frac{oc(Q_{RH})}{uoc}$	mean	0.93	0.95	0.96	0.95
	min	0.73	0.87	0.86	0.87
$\frac{cc(Q_{Relax})}{ucc}$	mean	0.97	0.98	0.98	0.99
	min	0.89	0.91	0.9	0.91
$\frac{oc(Q_{Relax})}{uoc}$	mean	0.97	0.98	0.99	0.99
	min	0.81	0.94	0.92	0.92
$\frac{cc(Q_0)}{ucc}$	mean	0.81	0.85	0.81	0.76
	min	0.28	0.27	0.39	0.15
$\frac{oc(Q_0)}{uoc}$	mean	0.73	0.8	0.79	0.72
	min	0.2	0.23	0.33	0.1

Table 8
Group 2 of random instances.

Series (n_1, n_2, n_3)		(3,2,2)	(5,4,4)	(9,8,6)	(14,12,7)
$\#^{feas}_{Relax}$		1	0	0	0
Time CPLEX, sec	mean	0.08	0.65	20.2	403
	max	0.234	6.53	152	7200
$\frac{f(Q_{RH})}{f(Q_{Relax})}$	mean	0.93	0.92	0.89	0.88
	min	0.84	0.8	0.77	0.76
$\frac{f(Q_0)}{f(Q_{Relax})}$	mean	0.76	0.76	0.75	0.7
	min	0.33	0.43	0.17	0.21
$\frac{f(Q_0)}{f(Q_{RH})}$	mean	0.82	0.82	0.83	0.79
	min	0.36	0.5	0.18	0.26
$\frac{cc(Q_{RH})}{ucc}$	mean	0.96	0.98	0.98	0.97
	min	0.9	0.93	0.91	0.89
$\frac{oc(Q_{RH})}{uoc}$	mean	0.84	0.88	0.88	0.88
	min	0.66	0.73	0.78	0.75
$\frac{cc(Q_{Relax})}{ucc}$	mean	0.98	0.99	0.99	0.99
	min	0.95	0.96	0.96	0.95
$\frac{oc(Q_{Relax})}{uoc}$	mean	0.9	0.96	0.98	0.98
	min	0.69	0.81	0.88	0.85
$\frac{cc(Q_0)}{ucc}$	mean	0.92	0.86	0.85	0.79
	min	0.65	0.52	0.36	0.4
$\frac{oc(Q_0)}{uoc}$	mean	0.69	0.73	0.72	0.68
	min	0.34	0.42	0.18	0.2

For 3 instances of the series (5,4) and one instance of the series (3,2,2) the same optimal solutions were obtained by solving both problems Relax and Opt-Fast-Char. One instance of Relax for the series (14,12,7) was not solved optimally by CPLEX in two hours. The final relative error gap was 0.08%.

In the experiments with the random instances of group 1, mean deviations of the values delivered by RH from the upper bound on the optimal value of Opt-Fast-Char and the cost upper bounds ucc and uoc are within 7%, 3% and 7%, respectively, and the same worst values are within 15%, 11% and 27%, respectively. The costs of the solutions of Relax, which are not necessarily feasible for Opt-Fast-Char, are often closer to the upper bounds ucc and occ than the solutions of Opt-Fast-Char. The tighter satisfaction of these constraints can be related to the violation of the other constraints of Opt-Fast-Char which are not present in Relax.

For the experiments with the random instances of group 2, mean deviations of the values delivered by RH from the upper bound on the optimal value of Opt-Fast-Char and the cost upper bounds ucc and uoc are within 12%, 4% and 16%, respectively, and the same worst values are within 24%, 11% and 34%, respectively. The deviation of about 20% from the optimum is acceptable for a heuristic solution, taking into account difficulty and uncertainty of the problem Opt-Fast-Char. Similar quality of the heuristic solutions for difficult problems is often reported, see, for example, Puka et al. (2021) for the flow shop scheduling problems and Tarhan and Oguz (2021) for an order acceptance and scheduling problem.

It can be observed that the average solution time of RH is an increasing function with values 0.11, 1.0, 51.2 and 96.2, and the solution quality is a decreasing function with values 0.97, 0.96, 0.96 and 0.93 over the instance sizes (3,2), (5,4), (9,8) and (14,12) of group 1. Similarly, the average solution time is an increasing function with values 0.08, 0.65, 20.2 and 403, and the solution quality is a decreasing function with values 0.93, 0.92, 0.89 and 0.88 over the instance sizes (3,2,2), (5,4,4), (9,8,6) and (14,12,7) of group 2. A similar dependence is observed for the worst solution time and the worst solution quality, with a slight violation for the worst solution quality in group 1 (sequence 0.88, 0.9, 0.91, 0.85 is not decreasing).

The proposed models, solution approaches and software represent a decision tool which is competitive to the currently applied human expertise and the approach of electrifying city routes one by one. A justification for this statement is that, in general, a combination of the optimal solutions of sub-problems of a global problem does not give an optimal solution of the global problem. Consider a simple example of two routes intersecting at an intermediate stop. If the routes are electrified one by one, then the only feasible decision is to build four charging stations, one at each of the four terminals. However, if both routes are electrified together, then one station at the intersection stop can replace two stations at the terminals of both routes close to this stop, which saves the cost of one charging station and, perhaps, transformer connection costs. This decision is quite evident, but optimal or near-optimal decision can be difficult to obtain if more routes are considered and there is a choice for the e-bus types and charging infrastructure elements. The proposed tool can be employed to obtain solutions for different input data, for example, different budget values, or route sets, or e-bus types, in order to have several decision options. Preferences of the decision makers with regard to the routes to be electrified can also be taken into account by selecting appropriate value functions $f_r(Z_r)$ or preference coefficients w_r. By doing this, for example, routes in the city centre can be prioritized, or routes in the green zones.

6. Conclusions

A complex decision problem is studied, which is to plan a transfer of the conventional public transport service into a fast-charging e-bus service for a given set of routes. It is decomposed into two problems. The primary problem consists of determining an e-fleet, places for charging stations and transformers, an assignment of e-buses to the routes, charging points to the charging stations, charging stations to the transformers, and quantities of charging connectors at the charging points such that the total value is maximized. In the most representative time period, all the electric buses should be available to drive, the required inter-bus interval should be maintained, the output power of any charging station and transformer should not be exceeded, and the total capital cost and the total annual operating, depreciation and energy cost should not exceed certain maximum thresholds. A secondary problem consists of finding a balanced schedule of all the vehicles on the same route.

Appropriate mathematical problems are introduced for the primary and secondary problems, respectively. The primary problem is proved NP-hard in the strong sense for two special cases and a metaheuristic algorithm is developed for it, which combines random search with the Particle Swarm Optimization. The secondary problem is solved by an efficient polynomial time algorithm. The proposed models and methods are applied for a real-life case in the city of Minsk (Belarus). The value of the obtained solution is within 2% from the optimum. Extensive computer experiments with randomly generated large scale instances demonstrated that the objective function values of the heuristic solutions deviate at most 12% on average and 24% in the worst case from of the optima.

If minimizing the number of charging points is a higher priority than the vehicle schedule balancing, then the former priority can be addressed by employing vehicle sequences in which e-buses and conventional vehicles are uniformly distributed over all vehicles serving the same route. Let $k_r = \sum_{b\in B} nb_{rb}(X)$ be the number of e-buses (of all types) on a route r. If the number of conventional vehicles left on this route, $\sum_{b\in V} nv_{rb}(X)$, is at least $i_r(k_r - 1)$ for $i_r \in \{1, 2, \dots\}$, then, by placing i_r conventional vehicles between any two consecutive e-buses, the traffic interval between these e-buses can be made equal to $ut_r := (i_r + 1)ut_r$, and this interval can be used in the formula for the number of charging connectors $nc_{jc}(X)$ (see Table 10) to make it and the number of charging points smaller.

CRediT authorship contribution statement

Nikolai Guschinsky: Elaboration and rectification of models and algorithms, Computer coding, Computer experiments. **Mikhail Y. Kovalyov:** Discussions and developments of models, Algorithms, Proofs, Computer experiments, Writing. **Boris Rozin:** Discussions of models, Verification of theoretical results. **Nadia Brauner:** Discussions of the models and algorithms, Verification of theoretical results.

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Appendix A. Input data and variables

See Tables 9 and 10.

Table 9
Input data.

Notation	Description
ucc	Upper bound on total capital cost
uoc	Upper bound on total annual operating, depreciation and energy cost
T	Set of transformer locations
TO	Set of old transformer locations, $TO \subseteq T$
N	Set of stops for charging
NO	Set of stops with an old charging station opened, $NO \subseteq N$
R	Set of routes
NM_r	Set of obligatory charge stops of route r, at each of which a charging station must be opened if an e-bus goes via this stop
NM	Set of all obligatory charge stops, $NM = \cup_{r\in R} NM_r$
RM_j	Set of routes for each of which j is an obligatory charge stop, $RM_j = \{r \in R \mid j \in NM_r\}$
D	Set of depot stops, $D \subseteq \cup_{r\in R} NM_r$
RO	Set of routes served by at least one old e-bus
RO_j	Set of routes with old e-buses charging at stop j
B	Set of e-bus types
V	Set of conventional vehicle types
C	Set of charging types
C_b	Set of feasible charging types for b-type e-buses
co_b	Charging type of old b-type e-buses
cap_b	Passenger capacity of vehicle of type $b \in V \cup B$
$cbus_b$	Capital cost of one b-type e-bus
V_r	Set of conventional vehicle types on route r
nv^0_{rb}	Number of b-type conventional vehicles on route r
dem_r	Passenger demand of route r, which is equal to the total past passenger capacity of all old vehicles on this route
$f_r(Z_r)$	Value function of route r. Z_r is passenger demand of route r satisfied by new e-buses. For example, $f_r(Z_r) = w_r Z_r$, where $w_r > 0$ is a preference coefficient
B_c	Set of e-bus types that are feasible for charging type c
BO_{rc}	Set of e-bus types of old e-buses that charge at c-type points on route r
BO_r	Set of e-bus types of old e-buses on route r
pm_r	$\min_{b\in V_r}\{cap_b\}$
ut_r	Traffic interval of route r
$d(r)$	Depot of route r
$\pi_r = (d(r), j^{(r)}_1, \dots, j^{(r)}_{i(r)}, j^{(r)}_1)$	Sequence of stops of route r. E-bus goes from $d(r)$ to $j^{(r)}_1$, then visits $j^{(r)}_1, \dots, j^{(r)}_{i(r)}$ cyclically in this order and goes back from $j^{(r)}_1$ to $d(r)$
k_{jr}	Number of times of visiting stop j by e-buses of route r in the same cycle, $k_{jr} \in \{1,2\}$, and $k_{jr} = 2$ if e-buses visit j in both directions
nbo_{rbc}	Number of old b-type e-buses that charge at c-type points on route r
nbo_{rb}	Number of old b-type e-buses on route r
$obus_{rb}$	Annual operating, depreciation and energy cost of b-type e-bus on route r
noc_{jc}	Number of old c-type connectors at stop j
nos_j	Number of old charging stations at stop j, $nos_j \in \{0,1\}$
nop_{jc}	Number of old c-type charging points at stop j
up_j	Upper bound on the number of charging points at stop j
$ct^{(\max)}_{bc}$	Charging time of b-type e-bus at c-type charging point to maximum SOC level
ct_{rjbc}	Charging time of b-type e-bus of route r at c-type charging point at stop j, $ct_{rjbc} = ct^{(\max)}_{bc}$ if j is the depot $d(r)$ of route r
uc_c	Upper bound on the number of connectors of one c-type charging point, exact number of connectors of the only c-type charging point at any depot
d_{rij}	Distance between stops i and j of route r
$d^{(max)}_b$	Driving range of a fully charged b-type e-bus
p_c	Output power of one c-type connector
$cpoi_c$	Capital cost of one c-type charging point
$cpoi_{\min}$	$\min\{cpoi_c \mid c \in C\}$
$opoi_c$	Annual operating and depreciation cost of one c-type charging point
$opoi_{\min}$	$\min\{opoi_c \mid c \in C\}$
$csta_j$	Capital cost of one charging station at stop j
$osta_j$	Annual operating and depreciation cost of one charging station at stop j
utp_t	Output power of a transformer at location $t \in T$
ctb_t	Capital cost of a transformer at $t \in T$
cl_{tj}	Cost of linking transformer location t and stop j
$cl^{\min}_j$	$\min\left\{\min\{cl_{tj} \mid t \in TO\}, \min\{cl_{tj} + ctb_t \mid t \in T\backslash TO\}\right\}$

Appendix B. PSO for Opt-Fast-Char(Q)

Each solution of the problem Opt-Fast-Char(Q) is considered as a *particle* in a *swarm*. There are several consecutive applications of the PSO algorithm, and each application consists of several iterations. Each application starts with a group of random particles (Q, Q_i), $i = 1, \dots, k$, where Q is a partial solution determined in Steps 1–4 of the algorithm RH and Q_i is an extension of Q to a complete solution of Opt-Fast-Char. Each particle is associated with its *position*, *velocity* and *value*. Velocity determines the increment of the position when passing from one iteration of the same application of PSO algorithm to the next.

Define parameters related to the position range:

$$pos^0_{rb} = \begin{cases} \lceil dem_r / cap_b \rceil - nbo_{rb} & \text{if } b \in B_r(Q) \cup BO_r, \\ nv^0_{rb} & \text{if } b \in V_r(Q). \end{cases}$$

Table 10
Variables.

Notation	Description
$R(X)$	Set of routes with at least one new e-bus
$R_j(X)$	Set of routes with at least one new e-bus charging at j
$C_j(X)$	Set of new charging types at stop j
$N(X)$	Set of stops with at least one new connector
$S_{rc}(X)$	Set of segments (i,j) of route r with c-type charging points opened at consecutive stops i and j of this route
$d^0_{rjbc}(X)$	Driving range of b-type e-bus on route r charged at c-type point of stop j, $d^0_{rjbc}(X) = \min\{d^{(\max)}_b, d^0_{ribc}(X) - d_{rij} + d^{(\max)}_b \frac{ct_{rjbc}}{ct^{(\max)}_{bc}}\}$[a], $(i,j) \in S_{rc}(X)$, $d^0_{rjbc}(X) = d^{(\max)}_b$ if j is the depot of route r
$B_{rc}(X)$	Set of e-bus types of new e-buses charging at points of type c on route r
$B_r(X)$	Set of e-bus types of new e-buses on route r, $B_r(X) = \cup_{c\in C} B_{rc}(X)$
$nb_{rbc}(X)$	Number of new b-type e-buses assigned to route r and charging type c
$nb_{rb}(X)$	Number of new b-type e-buses assigned to route r, $nb_{rb}(X) = \sum_{c\in C} nb_{rbc}(X)$
$V_r(X)$	Set of types of conventional vehicles left on route r
$nv_{rb}(X)$	Number of b-type conventional vehicles left on route $r \in R(X)$
$Z_r(X)$	Passenger demand of route r satisfied by new e-buses, $Z_r(X) = \min\{dem_r, \sum_{b\in B_r(X)} cap_b nb_{rb}(X)\}$
$f(X)$	Total value, $\sum_{r\in R(X)}\left(f_r(Z_r(X)) - \frac{\sum_{b\in V_r(X)} nv_{rb}(X) cap_b}{dem_r}\right)$
$nc_{jc}(X)$[b]	Number of new charging connectors of type $c \in C_j(X)$ at stop j, $nc_{jc}(X) = \begin{cases} \sum\limits_{r\in R_j(X)\cup RO_j} k_{jr} \min\left\{ \sum\limits_{b\in B_{rc}(X)\cup BO_{rc}} (nb_{rbc}(X) + nbo_{rbc}), \left\lceil \frac{\max_{b\in B_{rc}(X)\cup BO_{rc}}\{ct_{rjbc}\}}{ut_r} \right\rceil \right\} - noc_{jc} & \text{if } j \notin D, \\ uc_c & \text{if } j \in D \end{cases}$
$np_{jc}(X)$	Number of new charging points of type $c \in C_j(X)$ at stop j, $np_{jc}(X) = \begin{cases} \left(\left\lceil \frac{noc_{jc} + nc_{jc}(X)}{uc_c} \right\rceil - nop_{jc}\right) & \text{if } j \notin D, \\ 1 & \text{if } j \in D \end{cases}$
$np_j(X)$	Number of new charging points at stop j, $np_j(X) = \sum_{c\in C} np_{jc}(X)$
$ns_j(X)$	Number 0 or 1 of new charging stations at stop j, $ns_j(X) = \begin{cases} 0 & \text{if } nos_j = 1 \\ 1 & \text{if } nos_j = 0 \text{ and } np_{jc}(X) \geq 1 \text{ for a } c \in C \end{cases}$
$T(X)$	Set of transformer locations linked with at least one new charging station
$T_j(X)$	New transformer locations linked with stop j
$cc(X)$	Total capital cost, the sum of charging station, charging point, e-bus, transformer connection and transformer building costs, $cc(X) = \sum\limits_{j\in N}\left(csta_j ns_j(X) + \sum\limits_{c\in C_j(X)} cpoi_c np_{jc}(X)\right) + \sum\limits_{r\in R}\sum\limits_{b\in B_r(X)} cbus_b nb_{rb}(X) + \sum\limits_{j\in N(X)}\sum\limits_{t\in T_j(X)} cl_{tj} + \sum_{t\in T(X)\setminus TO} ctb_t$
$oc(X)$	Total annual operating, depreciation and energy cost, the sum of charging station, charging point and e-bus costs, $oc(X) = \sum\limits_{j\in N}\left(osta_j ns_j(X) + \sum\limits_{c\in C_j(X)} opoi_c np_{jc}(X)\right) + \sum\limits_{r\in R}\sum_{b\in B_r(X)} obus_{rb} nb_{rb}(X)$

[a]This formula is derived from the linear relation between the SOC decrease and the distance between two charges (Ma and Xie, 2021) and the linear relation between the SOC increase during charging and the charging duration (Ng et al., 2009).
[b]This value is an upper bound on the required number of new c-type charging connectors at stop j, which is attained if e-buses of the same route with the longest charging time arrive consecutively within their entire charging time interval, and e-buses of different routes are charged simultaneously.

In our algorithm, position of each particle (Q, Q_i) is determined by a random integer number set

$$POS(Q,Q_i) = \{pos_{rb}(Q,Q_i) \mid r \in R(Q),\ b \in B_r(Q) \cup BO_r \cup V_r(Q),\ 0 \leq pos_{rb}(Q,Q_i) \leq pos^0_{rb}\},$$

where if $b \in B_r(Q) \cup BO_r$, then $pos_{rb}(Q,Q_i) = nb_{rb}(Q,Q_i)$ is the number of new e-buses, and if $b \in V_r(Q)$, then $pos_{rb}(Q,Q_i) = nv_{rb}(Q,Q_i)$ is the number of the remaining conventional vehicles. Velocity of a particle (Q,Q_i) is determined as a random integer number set

$$VEL(Q,Q_i) = \{vel_{rb}(Q,Q_i) \mid r \in R(Q),\ b \in B_r(Q) \cup BO_r \cup V_r(Q),\ -pos^0_{rb} \leq vel_{rb}(Q,Q_i) \leq pos^0_{rb}\}.$$

Value of a particle (Q,Q_i) is determined by the objective function value $f(Q,Q_i)$, which is set to -1 for infeasible solutions. PSO operates with the *best known positions* of the k particles and the swarm as a whole over all the past iterations of the same application. The best known position of a particle (Q,Q_i) is a set denoted as

$$BEP(Q,Q_i) = \{bep_{rb}(Q,Q_i) \mid r \in R(Q),\ b \in B_r(Q) \cup BO_r \cup V_r(Q)\},$$

where $BEP(Q,Q_i) = POS(Q,Q_i)$ in the first iteration. The content of $BEP(Q,Q_i)$ in any other iteration is described below. The best known position of the swarm is the position of its "best particle" that has the largest objective function value $f(Q,Q_{i^*})$ and is denoted as (Q,Q_{i^*}). The best known position of the swarm is a set denoted as

$$BES = \{bes_{rb} \mid r \in R(Q),\ b \in B_r(Q) \cup BO_r \cup V_r(Q)\},$$

where $bes_{rb} = bep_{rb}(Q,Q_{i^*})$.

In each iteration of one application of the PSO algorithm, all k particles of the swarm are updated, their best known positions are re-calculated and the best particle is determined. An application stops if a given upper bound on the number of iterations without an improvement of the largest objective function value is exceeded. In this case, a new application launches, and the applications are repeated until an upper bound on the solution time is not exceeded. In each iteration, each particle (Q,Q_i), sets $BEP(Q,Q_i)$, $POS(Q,Q_i)$, $i \in \{1,\dots,k\}$, and BES, and the best solution (Q,Q_{i^*}) are updated according to the following steps (a)–(e).

(a) Select standard random PSO control parameters α, β, ω, ϕ and ξ from $(0,1)$.

(b) For each pair (r,b), $r \in R(Q)$, $b \in B_r(Q) \cup BO_r \cup V_r(Q)$, update velocity such that

$$vel_{rb}(Q,Q_i) := \omega \cdot vel_{rb}(Q,Q_i) + \phi\alpha(bep_{rb}(Q,Q_i) - pos_{rb}(Q,Q_i)) + \xi\beta(bes_{rb} - pos_{rb}(Q,Q_i)).$$

(c) Select standard random PSO control parameters $\lambda_{rb} \in (0,1)$, $r \in R(Q)$, $b \in B_r(Q) \cup BO_r \cup V_r(Q)$. According to the PSO methodology, if $\frac{\omega}{\omega+\phi\alpha+\xi\beta} \leq \lambda_{rb} < \frac{\omega+\phi\alpha}{\omega+\phi\alpha+\xi\beta}$, then update $pos_{rb}(Q,Q_i) := pos_{rb}(Q,Q_i) + \lfloor \omega \cdot vel_{rb}(Q,Q_i) \rfloor$. If $\lambda_{rb} \geq \frac{\omega+\phi\alpha}{\omega+\phi\alpha+\xi\beta}$, then update $pos_{rb}(Q,Q_i) := pos_{rb}(Q,Q_i) + \lceil \omega \cdot vel_{rb}(Q,Q_i) \rceil$. Recall that $nb_{rb}(Q,Q_i) = pos_{rb}(Q,Q_i)$ for $b \in B_r(Q) \cup BO_r$ and $nv_{rb}(Q,Q_i) = pos_{rb}(Q,Q_i)$ for $b \in V_r(Q)$.

(d) Calculate the number of connectors $nc_{jc_b(Q)}(Q,Q_i)$, numbers of charging points $np_{jc_b(Q)}(Q,Q_i)$ and $np_j(Q,Q_i)$, $b \in B_r(Q) \cup BO_r$, $r \in R(Q)$, $j \in N(Q)$, and costs $cc(Q,Q_i)$ and $oc(Q,Q_i)$ according to their formulas in Table 10 of Appendix A. If the constraints (12)–(17) are satisfied, then the complete solution (Q,Q_i) is feasible, else it is not feasible. Calculate new value $f(Q,Q_i)$ of the particle (Q,Q_i). Update sets $BEP(Q,Q_i)$, $POS(Q,Q_i)$, BES and the best solution (Q,Q_{i^*}). ■

References

Adler, J.D., Mirchandani, P.B., 2017. The vehicle scheduling problem for fleets with alternative-fuel vehicles. Transp. Sci. 51, 441–456.

Alonso, M., Amaris, H., Germain, J.G., Galan, J.M., 2014. Optimal charging scheduling of electric vehicles in smart grids by heuristic algorithm. Energies 7, 2449–2475.

Andersson, M., 2017. Energy Storage Solutions for Electric Bus Fast Charging Stations: Cost Optimization of Grid Connection and Grid Reinforcements (Master Thesis). Uppsala University, Finland.

Brauner, N., Crama, Y., 2004. The maximum deviation just-in-time scheduling problem. Discrete Appl. Math. 134 (1–3), 25–50.

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., Suraci, S., 2017. A three-phase matheuristic for the time-effective electric vehicle routing problem with partial recharges. Electron. Notes Discrete Math. 58, 95–102.

Clerc, M., 2010. Particle Swarm Optimization. John Wiley & Sons.

Desaulniers, G., Errico, F., Irnich, S., Schneider, M., 2016. Exact algorithms for electric vehicle-routing problems with time windows. Oper. Res. 64 (6), 1388–1405.

Froger, A., Mendoza, J., Jabali, O., Laporte, G., 2017a. New Formulations for the Electric Vehicle Routing Problem with Nonlinear Charging Functions. CIRRELT-2017-30, CIRRELT, Montreal.

Froger, A., Mendoza, J., Jabali, O., Laporte, G., 2017b. A Matheuristic for the Electric Vehicle Routing Problem with Capacitated Charging Stations. CIRRELT-2017-31, CIRRELT, Montreal.

Gampa, S.R., Jasthi, K., Goli, P., Das, D., Bansal, R.C., 2020. Grasshopper optimization algorithm based two stage fuzzy multiobjective approach for optimum sizing and placement of distributed generations, shunt capacitors and electric vehicle charging stations. J. Energy Storage 27, 101117.

Garey, M.R., Johnson, D.S., 1979. Computers and Intractability: A Guide to the Theory of NP-Completeness. Freeman, San Francisco.

He, Y., Song, Z., Liu, Z., 2019. Fast-charging station deployment for battery electric bus systems considering electricity demand charges. Sustainable Cities Soc. 48, 101530.

Hiermann, G., Puchinger, G., Ropke, S., Hartl, R.F., 2016. The electric fleet size and mix vehicle routing problem with time windows and recharging stations. European J. Oper. Res. 252, 995–1018.

Horn, W.A., 1974. Some simple scheduling algorithms. Nav. Res. Logist. Q. 21, 177–185.

Hosseini, S., Sarder, M.D., 2019. Development of a Bayesian network model for optimal site selection of electric vehicle charging station. Int. J. Electr. Power Energy Syst. 105, 110–122.

Jefferies, D., Göhlich, D., 2020. A comprehensive TCO evaluation method for electric bus systems based on discrete-event simulation including bus Scheduling and charging infrastructure optimisation. World Electr. Veh. J. 11, 56. http://dx.doi.org/10.3390/wevj11030056.

Kennedy, J., Eberhart, R., 1995. Particle swarm optimization. In: Proceedings of IEEE International Conference on Neural Networks, IV. pp. 1942–1948.

Kovalyov, M.Y., Kubiak, W., Yeomans, J.S., 2001. A computational analysis of balanced JIT optimization algorithms. INFOR Inf. Syst. Oper. Res. 39 (3), 299–316.

Kovalyov, M.Y., Rozin, B.M., Guschinsky, N.N., 2020. Mathematical model and random search algorithm for the optimal planning problem of replacing traditional public transport with electric. Autom. Remote Control 81, 803–818.

Kubiak, W., Sethi, S., 1991. A note on Level schedules for mixed-model assembly lines in Just-in-Time production systems. Manage. Sci. 37 (1), 121–122.

Kunith, A., Mendelevitch, R., Göehlich, D., 2017. Electrification of a city bus network - An optimization model for cost-effective placing of charging infrastructure and battery sizing of fast-charging electric bus systems. Int. J. Sustain. Transp. 11 (10), 707–720.

Leou, R.-C., Hung, J.-J., 2017. Optimal charging schedule planning and economic analysis for electric bus charging stations. Energies 10, 483.

Li, L., Lo, H.K., Xiao, F., 2019. Mixed bus fleet scheduling under range and refueling constraints. Transp. Res. Part Emerg. Technol. 104, 443–462, 2019.

Lin, Y., Zhang, K., Shen, Z.-J.M., Yee, B., Miao, L., 2019. Multistage large-scale charging station planning for electric buses considering transportation network and power grid. Transp. Res. C 107, 423–443.

Liu, Z., Song, Z., He, Y., 2018. Planning of fast-charging stations for a battery electric bus system under energy consumption uncertainty. Transp. Res. Rec. http://dx.doi.org/10.1177/0361198118772953.

Liu, X.C., Wei, R., 2018. Strategic planning and design for electric bus systems. In: Mountain-Plains Consortium. report MPC 18-355, North Dakota State University.

Liu, Y., Yao, E., Li, S., 2019. Energy consumption optimization model of multi-type bus operating organization based on time-space network. Appl. Sci. 9, 3352.

Ma, T.-Y., Xie, S., 2021. Optimal fast charging station locations for electric ridesharing with vehicle-charging station assignment. Transp. Res. D 90, 102682.

Mohamed, M., Farag, H., El-Taweel, N., Ferguson, M., 2017. Simulation of electric buses on a full transit network: Operational feasibility and grid impact analysis. Electr. Power Syst. Res. 142, 163–175.

Ng, K.S., Moo, C.-S., Chen, Y.-P., Hsieh, Y.-C., 2009. Enhanced coulomb counting method for estimating state-of-charge and state-of-health of lithium-ion batteries. Appl. Energy 86, 1506–1511.

Pedersen, M.E.H., Chipperfield, A.J., 2010. Simplifying particle swarm optimization. Appl. Soft Comput. 10, 618–628.

Pelletier, S., Jabali, O., Laporte, G., 2016. 50th anniversary invited article. Goods distribution with electric vehicles: Review and research perspectives. Transp. Sci. 50 (1), 3–22.

Pelletier, S., Jabali, O., Laporte, G., 2018. Charge scheduling for electric freight vehicle. Transp. Res. B 115, 246–269.

Pelletier, S., Jabali, O., Laporte, G., Veneroni, M., 2017. Battery degradation and behaviour for electric vehicles: Review and numerical analyses of several models. Transp. Res. B 103, 158–187.

Pelletier, S., Jabali, O., Mendoza, J.E., Laporte, G., 2019. The electric bus fleet transition problem. Transp. Res. C 109, 174–193.

Project PLATON, 2020. https://www.electricmobilityeurope.eu/projects/, http://platon.publictransport.info/.

Puka, R., Duda, J., Stawowy, A., Skalna, I., 2021. N-NEH+ algorithm for solving permutation flow shop problems. Comput. Oper. Res. 132, 105296.

Raab, A.F., Lauth, E., Strunz, K., Göhlich, D., 2019. Implementation schemes for electric bus fleets at depots with optimized energy procurements in virtual power plant operations. World Electr. Veh. J. 10 (5), http://dx.doi.org/10.3390/wevj10010005.

Rinaldi, M., Picarelli, E., D'Ariano, A., Viti, F., 2020. Mixed-fleet single-terminal bus scheduling problem: Modelling, solution scheme and potential applications. Omega 96, 102070. http://dx.doi.org/10.1016/j.omega.2019.05.006.

Rogge, M., van der Hurk, E., Larsen, A., Sauer, D.U., 2018. Electric bus fleet size and mix problem with optimization of charging infrastructure. Appl. Energy 211, 282–295.

Rogge, M., Wollny, S., Sauer, D.U., 2015. Fast charging battery buses for the electrification of urban public transport - a feasibility study focusing on charging infrastructure and energy storage requirements. Energies 8 (5), 4587–4606.

Sengupta, S., Basak, S., Peters II, R.A., 2019. Particle Swarm Optimization: A survey of historical and recent developments with hybridization perspectives. Mach. Learn. Knowl. Extr. 1 (1), 157–191.

Steiner, G., Yeomans, S., 1993. Level schedules for mixed-model, Just-In-Time assembly processes. Manage. Sci. 39 (6), 728–735.

Tarhan, I., Oguz, C., 2021. Generalized order acceptance and scheduling problem with batch delivery: models and metaheuristics. Comput. Oper. Res. Accepted.

Teoh, T., Kunze, O., Teo, C.-C., 2016. Scenario-based electric bus operation: A case study of Putrajaya, Malaysia. Int. J. Transp. Sci. Technol. 7, 10–25.

van Kooten Niekerk, M.E., van den Akker, J.M., Hoogeveen, J.A., 2017. Scheduling electric vehicles. Public Transp. 9, 155–176.

Vepsäläinen, J., Baldi, F., Lajunen, A., Kivekäs, K., Tammi, K., 2018. Cost–benefit analysis of electric bus fleet with various operation intervals. In: 21st International Conference on Intelligent Transportation Systems (ITSC). Maui, HI, pp. 1522–1527. http://dx.doi.org/10.1109/ITSC.2018.8569583.

Wang, Y., Bi, J., Guan, W., Zhao, X., 2018a. Optimising route choices for the travelling and charging of battery electric vehicles by considering multiple objectives. Transp. Res. D 64, 246–261.

Wang, Y., Huang, Y., Xu, J., Barclay, N., 2017. Optimal recharging scheduling for urban electric buses: A case study in davis. Transp. Res. E 100, 115–132.

Wang, Y.-W., Lin, C.-C., Lee, T.-J., 2018b. Electric vehicle tour planning. Transp. Res. D 63, 121–136.

Wen, M., Laporte, G., Madsen, O.B.G., Norrelund, A.V., Olsen, A., 2014. Locating replenishment stations for electric vehicles: application to danish traffic data. J. Oper. Res. Soc. 65 (10), 1555–1561.

Wen, M., Linde, E., Ropke, S., Mirchandani, P., Larsen, A., 2016. An adaptive large neighborhood search heuristic for the electric vehicle scheduling problem. Comput. Oper. Res. 76, 73–83.

Wielinski, G., Trépanier, M., Morency, C., 2017. Electric and hybrid car use in a free-floating car sharing system. Int. J. Sustain. Transp. 11 (3), 161–169.

Xylia, M., Leduc, S., Patrizio, P., Kraxner, F., Silveira, S., 2017a. Locating charging infrastructure for electric buses in Stockholm. Transp. Res. C 78, 183–200.

Xylia, M., Leduc, S., Patrizio, P., Silveira, S., Kraxner, F., 2017b. Developing a dynamic optimization model for electric bus charging infrastructure. Transp. Res. Procedia 27, 776–783.

Yao, F., Wang, J., Wen, F., Tseng, C.-L., Zhao, X., Wang, Q., 2019. An integrated planning strategy for a power network and the charging infrastructure of electric vehicles for power system resilience enhancement. Energies 12, 3918.

Yu, Z., Chen, S., Tong, L., 2016. An intelligent energy management system for large-scale charging of electric vehicles. CSEE J. Power Energy Syst. 2 (1), 47–53.

Zhang, H., Sheppard, C.J.R., Lipman, T.E., Moura, S.J., 2018. Joint fleet sizing and charging system planning for autonomous electric vehicles. arXiv:1811.00234v1 [math.OC].