

Designing a fast-charge urban electric bus services: an ILP model

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Abstract

Currently, there is serious political support for the decarbonization of transport locally, nationally and even internationally. Public transport operators are focusing on the use of electric buses as an opportunity to reduce greenhouse gas emissions and improve air quality. However, using electric buses, requires a functional infrastructure of urban charging points. Fast-charging can be made available thanks to the progress made on the major technological charging devices in recent years. In this study, we consider an optimization problem of the design of an infrastructure for a fast-charge city electric bus service. The decisions which have to be made include determining a **mixed fleet of conventional and electric buses**, points for electric chargers and power stations, quantities of charging plug devices, a distribution of electric buses between the routes, and matching chargers with power stations. **The objective is to maximize the route-weighted total passenger capacity of electric buses.** An Integer Linear Programming model has been developed to complement the existing non-linear model. The new model is efficient if the number of possible charging spots is small, which is natural and frequent in practice. Extensive computer experiments demonstrate that our approach delivers near-optimal solutions of the studied problem in ten minutes for real-world instances on a standard PC and it outperforms the earlier approach on every instance.

Keywords: electric bus; fast-charging; charging infrastructure; optimization; integer linear programming.

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1 Introduction

In 2000, exposure to fine particles suspended in the air caused an estimated 960,000 premature deaths, in the vast majority of cases as a result of cardio-pulmonary diseases. Without new air pollution control efforts, 3.1 million annual premature deaths could be expected in 2030, while the number of victims of lung cancer would quadruple according to Avere-France, a French association for the development of electric mobility [5]. The decarbonization of transport has become a major policy at local, national and international levels. The advent of electric buses is an opportunity that public transport operators are seizing to reduce greenhouse gas emissions and, at the same time, to improve air quality. In France, since 2015, the renewal of a public transport fleet must include a minimum proportion of low emission vehicles, according to the energy transition law for green growth. In addition, since January 1, 2020 service operators with a fleet of more than 20 buses and coaches, must, upon renewal, acquire or use 50% of low-emission buses and coaches. From January 1, 2025, the entire fleet must be comprised of low emission vehicles. In 2020, fleets of new electric buses in the EU increased by 18.4% from 1,448 units in 2019 to 1,714 units in 2020, taking 6.1% of the new bus market. According to ACEA (<https://www.acea.auto>), the market leaders in electric bus sales in 2020 are the Netherlands (446 units), Germany (388 units) and Poland (200 units). The use of urban electric buses requires the design of an operational infrastructure of urban charging points. Due to significant progress made with major technological charging devices in recent years, a fast-charging process can now be implemented.

Guschinsky et al. [7] have recently introduced a new optimization problem related to the design of a mixed fleet of conventional and electric buses and a complete charging infrastructure including power stations providing energy for chargers. This new problem has been denoted as OPT-FAST-CHAR. The reported results include proofs of strong NP-hardness for two special cases, a non-linear mathematical programming model and a local search method. In the present study, we explore a practical case of this problem where the number of feasible charging spots is small, in particular, it can be limited to terminal stops. While the previous problem formulation was non-linear and difficult to solve to optimality, the new model presented in this article is linear and efficiently solves practical cases as shown in a comparative numerical study. The model provided can be used by the municipality or urban transport agencies to improve the existing infrastructure and to optimize the facility costs in order to promote the introduction of electric buses in cities. Thus, the main theoretical contribution of this work is a collection of techniques to convert the

previous complex non-linear model into an efficient linear model while the numerical tests show that the proposed approach can be used by practitioners to obtain optimal solutions for real-case problem instances within acceptable time.

In the next section, our study is compared with the existing studies in the literature. In Section 3, a detailed description of the studied problem is given. In Section 4, the new Integer Linear Programming model is developed. The results of computer experiments are presented in Section 5. Final remarks are given in Section 6.

2 Literature review

Our study falls into the broader area of the network design, operations planning, scheduling and management of smart public transport systems. A recent survey of the developments in this area was provided by Kuo et al. [14]. They abbreviate main performance goals for public transport as SMART (Service, Mobility, Accessibility, Responsiveness and Technology). Our aim is to achieve these goals effectively for a specific yet sufficiently general city transport environment including fast charging electric buses. Kuo et al. [14] state that the current challenge for electric vehicles is their limited range and charging stations location, which make the design and planning of public transport systems a highly complex problem. We encountered this challenge in a case study (Guschinsky et al. [7]). Our current paper is a successful attempt to reduce complexity of the optimization model in [7] while keeping its practical adequacy.

A review of existing studies on electric bus planning and scheduling was recently presented by Perumal et al. [22]. It discusses strategic, tactical and operational decision levels. Since our study concerns the strategic level corresponding to the design of charging infrastructure for public electric transport, our literature review is focused on this area. This area was explored with *simulation*, *optimization* and a *combination of both techniques* in the literature.

The studies that mainly use the *simulation approach* provide useful insights on system performances, but cannot be used directly for designing a new infrastructure. Rogge et al. [25] presented a simulation model to evaluate the effect of fast-charging infrastructure utilization on the cost and energy characteristics of the entire urban transportation system. The study was based on a real bus network in Muenster, Germany. The authors showed the importance of focusing on the entire electric bus network for efficient planning, instead of individual trips. The impact of the load profile of a single charging station and that of the load profile of the entire network on the city electricity grid was also analyzed. This study did not include any optimization. Jefferies and

Göhlich [11] developed a simulation tool to evaluate energy, infrastructure and workforce resources for a given fleet of electric buses. These results were used to analyze the total cost of ownership. Further, this simulation tool was combined with a greedy algorithm in order to define the charging points. The approach developed was applied to a case study with 39 bus lines and 4748 passenger trips per day. This study was limited to a simplified charging infrastructure without considering different types of charging equipment. Raab et al. [24] examined complex charging strategies through a simulation framework. Further, *Mixed Integer Linear Programming (MILP)* models were developed in order to define and adjust the charging schedules themselves, without taking decisions on the charging infrastructure.

The *first optimization approaches* taking into account the charging infrastructure were developed for scheduling of charging operations within a given installation. The problem of charge scheduling for a given infrastructure has been studied by Adler and Mirchandani [1] who considered a multi-depot vehicle scheduling problem with a given set of charging stations and a charging capacity for the vehicles. A branch-and-price algorithm and a heuristic were proposed and tested on the case study of the Valley Metro bus network in Phoenix, Arizona. The study of van Kooten Niekerk et al. [26] was also dedicated to charge scheduling of electric buses for a given infrastructure. Two methods of obtaining approximate solutions were proposed. The methods were tested on a practical scheduling case in Leuven, Belgium. Wang et al. [27] considered electric bus recharging schedules with the aim of minimizing the total costs. The schedule generating method was implemented on a real case in Davis, California. Alvo et al. [2] studied the problem of dispatching electric and diesel buses and planning of charging the electric buses at a single public terminal equipped with given chargers. A Benders' type decomposition was used with a master sub-problem to determine bus itineraries and a satellite sub-problem to sequence charging events for given bus itineraries. Zhou et al. [34] investigated electric bus charging scheduling problem for a single public transport route with given trip tasks, fleet and chargers. They assumed a non-linear charging profile and battery degradation and a partial charging policy. The objective is to simultaneously determine a bus-to-trip assignment and charging schedule that minimizes the total cost. A mixed-integer nonlinear and non-convex programming model is built first, and then it is simplified to two approximating MILP models. A case study in Singapore was conducted. Zeng et al. [33] proposed an optimal electric bus charge scheduling model that employs peak-to-average power ratio, time-of-use electricity tariff, and battery wear to achieve a balanced space and time distribution of charging demand. The uncertainty of energy consumption in charging operations

was addressed by Liu et al. [16]. A robust model and a column generation algorithms were proposed in which the consumed power is optimized through proper allocation of power resources. A few studies were devoted to a combined problem of charger deployment, battery capacity and charging scheduling, see He et al. [8] and references therein. He et al. proposed a two-phase optimization framework with a rolling horizon charging strategy in the second phase.

A number of studies dealt with the battery selection, fleet composition and electric bus scheduling for a given charging network. Yıldırım and Yıldız [31] considered a given fast-charging and dynamic wireless power transfer environment. They studied optimal electric bus fleet composition and scheduling decisions with the aim to minimize the total procurement and operating costs of the buses. A column generation approach and a dynamic programming algorithm were proposed. An optimization model and a branch-and-price algorithm for electric fleet scheduling were presented by Zhang et al. [32]. They take into account battery degradation and non-linear charging profile. Wang et al. [28] proposed an optimization model for charger deployment and fleet scheduling in a given opportunity (fast) charging network. The model determines battery capacity, fleet size, and charger deployment to minimize the total annual costs. A case study was conducted in Oslo, Norway. An observation was made that the opportunity charging is cost-effective.

Several MILP models were presented in the literature for decisions about the design of charging infrastructure for electric buses, only including location decisions. Li et al. [15] proposed an *Integer Linear Programming (ILP)* model to consider only the location problem in order to optimize the bus fleet size, operational cost, passenger cost, and emissions generated by buses with different energy sources. A simplified model was suggested to find an approximate solution. The solution approaches were implemented for a case study in Hong Kong. Ma and Xie [19] studied an online vehicle-charger assignment model which was integrated with a fast-charging location model for dynamic electric bus ride sharing services. The problem was formulated as a bi-level optimization problem with: (1) an upper-level facility location problem to minimize total daily vehicle idle time and (2) a lower-level problem of dynamic ridesharing with dynamic vehicle-charging station assignment. The method was tested on a real case in Luxembourg.

Other authors considered purchasing problems without dealing with their location. Pelletier et al. [21] introduced an electric bus fleet transition problem dealing with investment decisions such as the number of buses per bus type and the number of chargers per charger type to be acquired. An et al. [4] studied the problem focused on the optimization of the number of batteries, chargers and their type of chargers as well as swapping robots needed for swapping operations. A MIP

model was developed, as well as a customized gradient algorithm, to handle the uncertainties in swapping demand caused by weather and traffic conditions. The authors tested their approaches on the case study in the east region of Melbourne, Australia.

Only few recent studies considered both location and equipment selection decisions within the design problem for the infrastructure of electric buses. Kunith et al. [13] formulated the problem of joint optimization of the charging infrastructure and the battery capacity as a capacitated set covering problem. A MILP model was proposed and tested for different scenarios of charging power and operating conditions. Xylia et al. (2017) [30] studied the design problem of charging infrastructure for urban electric buses. In their study, locations for charging station installation were limited to major public bus transport hubs connecting to the train and subway system. The model was tested on the case study of the city of Stockholm. Liu et al. [17] considered the installation of a series of wireless power transfer pads underneath the road used for charging electric buses while moving over these pads. A robust approach was developed to address the uncertainty of energy consumption and travel time. The proposed models were tested on the campus bus system of Utah State University, USA, which has four lines. He et al. [9] studied the design of the fast-charging infrastructure but without considering the traffic constraints. A MILP model was developed aiming at minimization of the overall cost of batteries, charging equipment and electricity. A case study was performed for the bus network in Salt Lake City, Utah. Iliopoulou and Kepaptsoglou [10] considered the integration of route planning and infrastructure design. A bi-level formulation was developed to select routes at the upper level and then to adapt the lower-level wireless charging infrastructures. A multi-objective particle swarm optimization algorithm embedded with a MIP model was developed to find the best trade-off between average travel time for passengers and the infrastructure cost. Alwesabi et al. [3] developed a MIP model to simultaneously select the optimal location of the dynamic wireless charging facilities and find the optimal battery sizes of electric buses for the system. The objective function included the inverter cost, the transmitter cable cost for all routes and the battery cost of electric buses on all routes. Finally, in our previous study, [7] a more complex infrastructure was considered with the constraints on the passenger load. A non-linear mathematical programming model and a local search method were developed for this new optimization problem.

As can be noted, the majority of existing studies propose MILP formulations for various practical problems related to the design and management of network of electric buses. Usually, the model is tested on a case study in one real network. [However, none of the previous studies,](#)

except for our previous paper [7], addresses the whole set of the following problem features: several public routes served by a mixture of conventional and fast-charging electric buses of several types, a number of feasible locations for capacitated charging stations and transformers, inter-bus intervals, total passenger capacity of all buses on the same route, limits on the capital and variable costs and value-based maximization criterion. The model in [7] is, unfortunately, non-linear and therefore, the solution time is too long for many practical instances. In the present study, we introduce several additional assumptions that allow to keep the completeness of the infrastructure but make it possible to develop an integer linear model. These assumptions are:

- Only few charges per ride are required for an electric bus. In the case study of [7], at most one charge was needed for any electric bus on its way between two terminals.
- Chargers are accessible from only one side of the road.
- The objective function does not only ensure the passenger flow, but also aims to reduce the number of conventional buses and non-used charging facilities. This assumption is not required to make the model linear but it helps to reduce the solution time.

The problem description and formulation are given in the next sessions.

3 Problem formulation

We consider a complex charging infrastructure of electric buses as shown in Fig. 1.

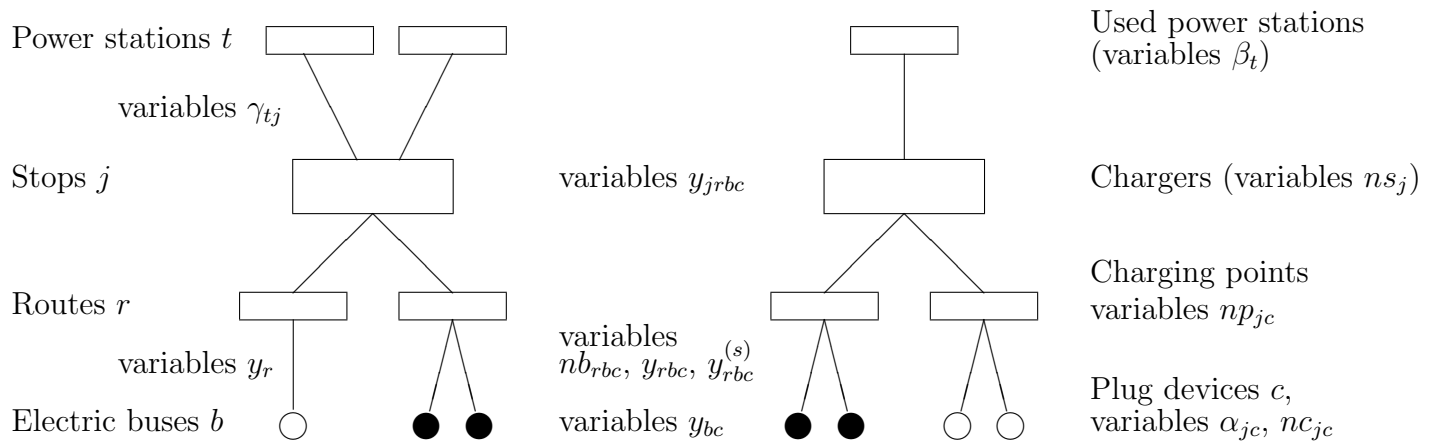


Figure 1: Hierarchy of charging infrastructure.

Due to fast technological evolution, a common urban bus system employs several electric bus types belonging to different generation models, each having their unique charging characteristics

and passenger capacities. The whole set of electric buses is supported by a charging infrastructure that provides the power recharge. The charging process is initialized by plugging an electric bus into a charging point via a charging plug. All charging points of the same bus stop are supplied by a common charging facility that is powered via geographically distributed electric power stations (transformers). Each power station can be linked to at most one charging facility, but every charging facility can be alimented by several power stations.

Passenger transfer system provides different routes r . Each route is characterized by a single depot and stops forming a unique *route cycle*. Each route is served by a number of electric and/or conventional (non-battery) buses. Each particular vehicle is associated with one specific route for the planning horizon. Each stop can be shared between several routes. If a charging facility is installed at one stop, it will be equipped with a number of charging points equal to the number of different routes concerned by power charging at this stop. This is a robust hypothesis that insures that if some delays in bus traffic occurs and all buses arrive at the same time to be charged at the stop, it will be possible to charge all of them in parallel.

Each electric bus starts fully charged from the depot at the beginning of its day, visits the stops on its route in the same repeated sequence (cycle), and arrives back at the depot at the end of its day. There are b different types of buses. Each bus of type b can be charged with c different technologies which require different charging power and thus differ by charging time. Each electric bus needs to be charged one or more times in each cycle. All depots of electric buses have a charging facility that can charge all the buses remaining there due to the long overnight rest time and fast charging time. Figure 2 shows two different charging technologies: slow on the left and fast on the right.

It is considered that an infrastructure for conventional and electric buses already exists. All the existing components are labeled as “old” (letter “o” is used in the names of variables related to “old” equipment). All new equipment to be introduced is labeled as “new”. All the old and new electric buses of the same type are assumed to be charged at the existing old charging points assigned to the electric buses of this type. There is a set of *obligatory charge* stops that includes all depots. A charging facility has to be installed at such a stop if it is visited by an electric bus. A full description of the input data for the considered optimization problem is given in Table 12 of Appendix 1.

The decisions to be made concern determining a fleet of new electric buses, points for charging facilities and power stations, quantities of charging plug devices, a distribution of new electric



Figure 2: Charging technologies

buses between the routes, and connecting charging facilities to power stations. The objective is to maximize the route-weighted total passenger capacity of new electric buses. The electric power supplied and consumed, and the capital and variable costs are the major limiting factors. The variable cost is the total annual operating, depreciation and energy cost of the fleet of electric buses and the charging equipment. The major constraints to be taken into account are:

- if conventional buses are replaced by electric ones, the total passenger capacity of the route must be maintained;
- number of charges required depends on the ranges to run by a bus and that number must be enough to maintain the bus charged throughout its route;
- traffic (inter-bus) intervals of all routes must be guaranteed;
- power characteristics of the charging facilities and power stations must be respected;
- limits on the capital and variable costs must be respected.

In the following section, we introduce a novel integer linear model for the described problem.

4 An ILP model

The main idea is to introduce feasible scenarios for locations of charging facilities for each route r , electric bus type $b \in B_r$ and charging type $c \in C_b$. Feasible scenario s , $s = 1, \dots, n_{rbc}$, is a sequence $S_{rbc}^{(s)} = (j_{rbc1}^{(s)}, j_{rbc2}^{(s)}, \dots, j_{rbc l_{rbc}^{(s)}}^{(s)})$ of $l_{rbc}^{(s)}$ stops of route r with charging points of type c for

b -type electric buses. It is assumed that any b -type electric bus is able to serve route r if it is (re)charged by type c at the stops of the sequence $S_{rbc}^{(s)}$, visiting them in this order. In general, the number of such scenarios can be large. However, if the number of feasible locations for charging facilities is small, then the number of such scenarios is small as well. For example, these locations could be limited to the terminal stops. The feasibility of a sequence $S_{rbc}^{(s)}$ is determined by the given electric bus type, route, charging type, charging times at the stops and distances between the stops according to the formula for driving ranges in Table 10 from [7]. The idea of feasible charging scenarios is similar to that used in the mathematical programming formulation of the cutting stock problem (Kantorovich and Zalgaller [12] and Gilmore and Gomory [6]).

If a route is already served by electric buses of type b , and then if new buses of the same type are assigned to the same route, these new buses will use the same charging type as that already used for this route. In this case, $n_{rbc} = 1$ for $c = co_b$, $r \in RO$ and $b \in BO$. Furthermore, the set of stops of one scenario cannot be a subset of stops of another scenario for the same triple (r, b, c) (in this case, the second scenario is redundant), and no scenario can include the same stop twice.

Fig. 3 illustrates an assignment of the electric buses to the charging points and plug devices.

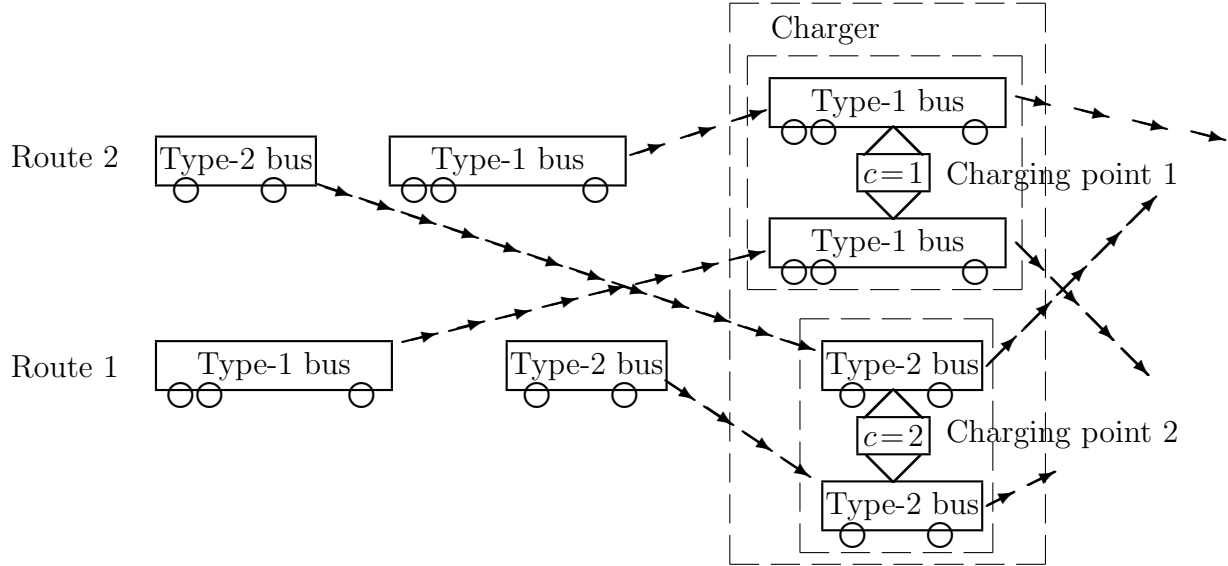


Figure 3: Assignment of e-buses of types $b = 1, 2$ to charging points of types $c = 1, 2$.

We introduce four groups of basic decision variables in our ILP formulation for the problem OFC2: 1) variables related to the quantity of new buses, 2) variables related to the assignment of electric buses for charging, 3) variables related to the charging equipment quantities, and 4) variables related to the allocation of the power stations.

1) Variables related to the quantity of new electric buses are


nb_{rbc} : number of new b -type electric buses assigned to route r and charged at a c -type charger;

y_{rbc} : binary indicator equal to 1 if and only if a new b -type electric bus is assigned to route r and charged at a c -type charger;

y_r : binary indicator equal to 1 if and only if at least one electric bus is assigned to route r .

2) Variables related to the assignment of electric buses for charging are

$y_{rbc}^{(s)}$: binary indicator equal to 1 if and only if b -type electric buses are charged by type c on route r according to scenario s ;

y_{bc} : binary indicator equal to 1 if and only if b -type electric buses are charged by type c on any route $r \in R \setminus RO$; 

y_{jrbc} : binary indicator equal to 1 if and only if b -type electric buses are charged by type c at a non-depot stop j of route $r \in R_{jc}$ according to some scenario.

3) Variables related to the charging equipment quantities are

ns_j : binary indicator equal to 1 if and only if a new charger is opened at stop j ;

α_{jc} : binary indicator equal to 1 if and only if a new charging plug device of type c is assigned to depot j ;

nc_{jc} : number of new plug devices of type c at stop j ;

np_{jc} : number of new charging points of type c at stop j .

4) Variables related to the allocation and links of power stations with the charging locations are

β_t : binary indicator equal to 1 if and only if an old or new power station is used at power station spot t ;

γ_{tj} : binary indicator equal to 1 if and only if there is an old or new link between power station spot t and stop or depot j .

We also introduce variables Z_r to represent contribution of route r to the passenger flow and variables nv_{rb} representing the number of non-battery b -type vehicles remaining on route r . Based on the input data, for each route r and electric bus type b , we calculate $dem_r^0 = dem_r - \sum_{b \in B_r} nob_{rb}cap_b$ (passenger capacity of route r to be satisfied by new electric buses and remaining non-battery vehicles), and $ub_{rb} = \left\lceil \frac{dem_r^0}{cap_b} \right\rceil$ (upper bound on the number of new b -type electric buses). The new linear objective function to be maximized is

$$\begin{aligned} \max \sum_{r \in R} \left(Z_r - \frac{\sum_{b \in V_r} nv_{rb}cap_b}{dem_r} \right) - \frac{\sum_{j \in N} \sum_{c \in C} nc_{jc}}{|N| \sum_{c \in C} uc_c} - \\ \frac{\sum_{j \in N} \sum_{c \in C} np_{jc}}{\sum_{j \in N} up_j} - \frac{\sum_{t \in T \setminus TO} \beta_t}{|T|} - \frac{\sum_{j \in N \setminus NO} \sum_{t \in T_j} \gamma_{tj}}{|N||T|} \end{aligned} \quad (1)$$

The following two constraints limit the overall capital cost and the overall variable cost, respectively:

$$\sum_{j \in N} csta_j ns_j + \sum_{c \in C} ccp_c np_{jc} + \sum_{r \in R} \sum_{b \in B_r} (cbus_b * \sum_{c \in C_b} nb_{rbc}) + \sum_{T \in T \setminus TO} ccps_t \beta_t + \sum_{t \in T \setminus TO} \sum_{j \in N \setminus NO} cl_{tj} \gamma_{tj} \leq cc, \quad (2)$$

$$\sum_{j \in N} (vcc_j ns_j + \sum_{c \in C} vcp_c np_{jc}) + \sum_{r \in R} \sum_{b \in B_r} vcb_{rb} \leq uoc. \quad (3)$$

Constraints (4) provide that the power demand of all charging plug devices at stop j is satisfied by the power stations linked with j :

$$\sum_{c \in C} (nc_{jc} + nod_{jc}) p_c \leq \sum_{t \in T_j} utp_t \beta_t, \quad j \in N. \quad (4)$$

Constraints (5) guarantee that any power station $t \in T$ is linked with at most one stop $j \in N$.

$$\beta_t = \sum_{j \in N} \gamma_{tj}, \quad t \in T. \quad (5)$$

Constraints (6) guarantee that the charging type is the same for all electric buses of the same type. They represent the implications $(y_{rb} = 1) \Rightarrow (y_{rbco_b} = 1)$, $r \in R$, $b \in BO \cap B_r$:

$$\sum_{c \in C_b} y_{rbc} - y_{rbco_b} \leq 0, \quad r \in R, \quad b \in BO \cap B_r. \quad (6)$$

Constraints (7-9) require b -type electric buses to be charged by a unique feasible charging type:

$$y_{bc} \leq \sum_{r \in R} y_{rbc}, \quad b \in B \setminus BO, \quad c \in C_b, \quad (7)$$

$$|R| y_{bc} \geq \sum_{r \in R} y_{rbc}, \quad b \in B \setminus BO, \quad c \in C_b, \quad (8)$$

$$\sum_{c \in C_b} y_{bc} \leq 1, \quad b \in B \setminus BO. \quad (9)$$

Constraints (10) introduce upper bound up_j on the total number of old and new charging points at stop j :

$$\sum_{c \in C} np_{jc} + \sum_{c \in C} nop_{jc} \leq up_j, \quad j \in N. \quad (10)$$

Constraints (11) require one c -type charging point to be opened at depot $d(r)$ of route r if this route is served by at least one electric bus charged by this type. Since $nop_{jc} = 1$ for $j \in D \cap NO$,

this requirement is caused by the implications $(\alpha_{jc} = 1) \Rightarrow (np_{jc} = 1)$, $j \in D \setminus NO$, which are equivalent to

$$\alpha_{jc} - np_{jc} \leq 0, \quad j \in D \setminus NO. \quad (11)$$

Constraints (12) state that the required passenger capacity of any route served by at least one electric bus is provided by the electric buses and the remaining non-battery vehicles.

$$\sum_{b \in B_r} (cap_b * \sum_{c \in C_b} nb_{rbc}) + \sum_{b \in V_r} cap_b nv_{rb} \geq dem_r^0 y_r, \quad r \in R. \quad (12)$$

Constraints (13) ensure that the number of non-battery vehicles does not increase.

$$nv_{rb} \leq nv_{rb}^0, \quad r \in R, b \in V_r. \quad (13)$$

Selecting a single charging point location scenario is modeled by the implications $(y_{rbc} = 1) \Rightarrow (\sum_{s=1}^{n_{rbc}} y_{rbc}^{(s)} = 1)$, $r \in R$, $b \in B_r$, $c \in C_b$, which are equivalent to constraints (14):

$$y_{rbc} - \sum_{s=1}^{n_{rbc}} y_{rbc}^{(s)} \leq 0, \quad r \in R, \quad b \in B_r, \quad c \in C_b. \quad (14)$$

The following constraints are implied by the definitions of the variables.

$$Z_r \leq \sum_{b \in B_r} cap_b \sum_{c \in C_b} nb_{rbc}, \quad r \in R, \quad (15)$$

$$Z_r \leq dem_r^0 y_r, \quad r \in R, \quad (16)$$

$$nb_{rbc} \geq y_{rbc}, \quad r \in R, \quad b \in B_r, \quad c \in C_b, \quad (17)$$

$$nb_{rbc} \leq ub_{rb} y_{rbc}, \quad r \in R, \quad b \in B_r, \quad c \in C_b, \quad (18)$$

$$y_{rb} = \sum_{c \in C_b} y_{rbc}, \quad r \in R, \quad b \in B_r, \quad (19)$$

$$ns_j \leq \sum_{c \in C} (np_{jc} + nop_{jc}), \quad j \in N \setminus NO, \quad (20)$$

$$up_j ns_j \geq \sum_{c \in C} (np_{jc} + nop_{jc}), \quad j \in N \setminus NO, \quad (21)$$

$$np_{jc} \geq \frac{nc_{jc} + nod_{jc}}{uc_c} - nop_{jc}, \quad j \in N \setminus D, \quad c \in C, \quad (22)$$

$$\sum_{c \in C} np_{jc} + \sum_{c \in C} nop_{jc} \leq up_j, \quad j \in N \setminus D, \quad (23)$$

$$y_r \leq \sum_{b \in B_r} \sum_{c \in C_b} y_{rbc}, \quad r \in R, \quad (24)$$

$$|B_r| y_r \geq \sum_{b \in B_r} \sum_{c \in C_b} y_{rbc}, \quad r \in R, \quad (25)$$

$$\alpha_{jc} \leq \sum_{r \in R_{jc}} y_r, \quad j \in D \setminus NO, \quad c \in C, \quad (26)$$

$$|R_{jc}| \alpha_{jc} \geq \sum_{r \in R_{jc}} y_r, \quad j \in D \setminus NO, \quad c \in C. \quad (27)$$

Traffic interval preservation is supported by the following relations.

$$L_r y_r \leq ut_r \sum_{b \in B_r} \left(\sum_{c \in C_b} nb_{rbc} + nob_{rb} \right) + ut_r \sum_{b \in V_r} nv_{rb}, \quad r \in R, \quad (28)$$

$$L_r y_r \geq lt_r \sum_{b \in B_r} \left(\sum_{c \in C_b} nb_{rbc} + nob_{rb} \right) + lt_r \sum_{b \in V_r} nv_{rb}, \quad r \in R. \quad (29)$$

According to Guschinsky et al. [7], the number nc_{jc} of new c -type plug devices at stop $j \in D \setminus NO$ satisfies implication $(\alpha_{jc} = 1) \Rightarrow (nc_{jc} = uc_c)$, which is transformed into

$$uc_c \alpha_{jc} - nc_{jc} \leq 0, \quad j \in D \setminus NO. \quad (30)$$

The number nc_{jc} , $j \in N \setminus D$, $c \in C$, is defined in [7] by the equation

$$nc_{jc} = \sum_{r \in R_{jc}} \min \left\{ \sum_{b \in B_{rc}} (nb_{rbc} + nob_{rb}), \max \left\{ noc_{jrc}^{(ct)}, \max \left\{ \frac{ct_{rjbc} y_{jrb}}{lt_r} \mid b \in B_{rc} \right\} \right\} \right\} - nod_{jc},$$

where $noc_{jrc}^{(ct)} = \frac{\max \{ ct_{rjbc} \mid b \in BO_{rc} \}}{lt_r}$. Linearization of this equation requires additional binary variables η_{jrc1} , η_{jrc2} , ξ_{jrc} , ξ_{jrb} , and the following variables:

nc_{jrc} : the number of new and old plug devices of type c used by electric buses on route r at stop j ;

$nc_{jrc}^{(b)}$: the number of new and old electric buses of type b on route r that charge by type c at stop j ;

$nc_{jrc}^{(ct)}$: the number of new and old plug devices of type c used by electric buses of route r at stop j .

With these variables and parameters $nc_{jrc}^{(\max)} = \lceil \frac{\max \{ ct_{jrb} \mid b \in B_{rc} \}}{lt_r} \rceil$ obtained from the input data, calculation of nc_{jc} is provided by the following linear constraints.

$$nc_{jc} = \sum_{r \in R_{jc}} nc_{jrc} - nod_{jc}, \quad j \in N \setminus D, c \in C, \quad (31)$$

$$nc_{jrc}^{(b)} = \sum_{b \in B_{rc}} nb_{rbc} + nob_{rb}, \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (32)$$

$$nc_{jrc} \leq nc_{jrc}^{(ct)}, \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (33)$$

$$nc_{jrc} \leq nc_{jrc}^{(b)}, \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (34)$$

$$nc_{jrc} \geq nc_{jrc}^{(ct)} - up_j uc_c (1 - \eta_{jrc1}), \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (35)$$

$$nc_{jrc} \geq nc_{jrc}^{(b)} - up_j uc_c (1 - \eta_{jrc2}), \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (36)$$

$$\eta_{jrc1} + \eta_{jrc2} = 1, \quad j \in N \setminus D, r \in R_{jc}, c \in C, \quad (37)$$

$$nc_{jrc}^{(ct)} \geq \frac{ct_{rjbc} y_{jrb}}{lt_r}, \quad j \in N \setminus D, r \in R_{jc}, c \in C, b \in B_{rc}, \quad (38)$$

$$nc_{jrc}^{(ct)} \leq \frac{ct_{rjbc}y_{jrb}}{lt_r} + nc_{jrc}^{(\max)}(1 - \xi_{jrb}), j \in N \setminus D, r \in R_{jc}, c \in C, b \in B_{rc}, \quad (39)$$

$$nc_{jrc}^{(ct)} \geq noc_{jrc}^{(ct)}, j \in N \setminus D, r \in R_{jc}, c \in C, b \in B_{rc}, \quad (40)$$

$$nc_{jrc}^{(ct)} \leq noc_{jrc}^{(ct)} + nc_{jrc}^{(\max)}(1 - \xi_{jrc}), j \in N \setminus D, r \in R_{jc}, c \in C, b \in B_{rc}, \quad (41)$$

$$\xi_{jrc} + \sum_{b \in B_{rc}} \xi_{jrb} = 1, j \in N \setminus D, r \in R_{jc}, c \in C. \quad (42)$$

Variables y_{jrb} are defined by the following equations involving additional binary variables $y_{jrb}^{(s)}$. The latter variable is equal to 1 if and only if b -type electric buses are charged by c -type at stop j of route r according to scenario s .

$$y_{jrb} = \sum_{s=1}^{n_{rbc}} y_{jrb}^{(s)}, r \in R, j \in \pi_r, b \in B_r, c \in C_b, \quad (43)$$

The following constraints ensure that a new b -type electric bus on route r is charged exactly at l_{rbc}^s stops of exactly one scenario $S_{rbc}^{(s)}$ for this route within one cycle.

$$\sum_{j \in S_{rbc}^{(s)}} y_{jrb}^{(s)} - l_{rbc}^{(s)} y_{rbc}^{(s)} = 0, r \in R, b \in B_r, s = 1, \dots, n_{rbc}, c \in C_b. \quad (44)$$

We also specify bounds on the variables in our ILP model.

$$\beta_t = 1, t \in TO, \quad (45)$$

$$\beta_t \in \{0, 1\}, t \in T \setminus TO, \quad (46)$$

$$\gamma_{tj} = 1, j \in NO, t \in TO_j, \quad (47)$$

$$\gamma_{tj} \in \{0, 1\}, t \in T \setminus TO, \quad (48)$$

$$Z_r \in [0, dem_r], r \in R, \quad (49)$$

$$nb_{rbc} \in [0, ub_{rb}], r \in R, b \in B_r, c \in C_b, \quad (50)$$

$$nv_{rb} \in [0, nv_{rb}^0], r \in R, b \in V_r, \quad (51)$$

$$y_r \in \{0, 1\}, r \in R, \quad (52)$$

$$nc_{jc} \in [0, up_j uc_c - nod_{jc}], j \in N \setminus D, c \in C, \quad (53)$$

$$np_{jc} \in [0, up_j - nop_{jc}], j \in N \setminus D, c \in C, \quad (54)$$

$$nc_{jc} \in [0, uc_c - nod_{jc}], j \in D \setminus NO, c \in C, \quad (55)$$

$$np_{jc} \in \{0, 1\}, j \in D \setminus NO, c \in C, \quad (56)$$

$$ns_j \in \{0, 1\}, j \in N \setminus NO, c \in C, \quad (57)$$

$$y_{rb} \in \{0, 1\}, r \in R, b \in B_r, \quad (58)$$

$$y_{rbc} \in \{0, 1\}, r \in R, b \in B_r, c \in C_b, \quad (59)$$

$$\alpha_{jc} \in \{0, 1\}, j \in D \setminus NO. \quad (60)$$

$$nc_{jrc}^{(ct)} \in [0, nc_{jrc}^{(\max)}], r \in R, j \in \pi_r, c \in C, \quad (61)$$

$$nc_{jrc}^{(b)} \in [0, \sum_{b \in B_r} (ub_{rb} + nob_{rb})], r \in R, j \in \pi_r, c \in C, \quad (62)$$

$$nc_{jrc} \in [0, \min\{up_j uc_c, nc_{jrc}^{(\max)}\}], r \in R, j \in \pi_r, c \in C, \quad (63)$$

$$y_{jrb}^{(s)} \in \{0, 1\}, r \in R, b \in B_r, j \in \pi_r, s = 1, \dots, n_{rbc}, c \in C_b, \quad (64)$$

$$y_{jrb}^{(s)} = 0, r \in R, b \in B_r, j \in \pi_r, j \notin S_{rbc}^{(s)}, s = 1, \dots, n_{rbc}, c \in C_b, \quad (65)$$

$$y_{rbc}^{(s)} \in \{0, 1\}, r \in R, b \in B_r, s = 1, \dots, n_{rbc}, c \in C_b. \quad (66)$$

The proposed ILP model aims to minimize the linear function given in (1), subject to the constraints (2)-(66). The variables $\beta_t, \gamma_{tj}, y_r, y_{rb}, y_{rbc}, \alpha_{jc}, y_{jrb}^{(s)}, y_{rbc}^{(s)}, ns_j$ are Boolean and the variables $Z_r, nb_{rbc}, nv_{rb}, nc_{jc}, np_{jc}, nc_{jrc}^{(ct)}, nc_{jrc}^{(b)}, nc_{jrc}$ are non-negative integers. In the next section, we evaluate this model for the problem OFC2 in computer experiments with an instance approximating a real case and random instances based on the real case.

5 Computer experiments

An instance that approximates a real case of the problem OFC2 in a large city is considered in Section 5.1. Random instances adapted from this case are considered in Section 5.2.

5.1 Real case based instance

Similar to Guschinsky et al. [7], the data used here is adapted from the real public transport system in a large city. We consider 26 routes, 23 non-depot charging stops, depot D_1 for routes 1-14 and depot D_2 for routes 15-26. All the non-depot stops are stops of some routes. Each charger can hold at most three charging points, and each charging point holds one plug device. The charging type c is the same for all charging points, and any electric bus can be charged at any charging point. The charging plug device power is $p_c = 260$ kW. Single charging points exist at depot D_1 and stops 1, 2, 12 and 14. Two charging points exist at stop 13. Any charging stop j can be linked with a given power station spot $t(j)$ (other links are too expensive). Existing (old) power station spots are given by the set $TO = \{t(j) \mid j = D_1, D_2, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$. The output power of any power station is 800 kW. Some of the old power stations were not used by electric buses in the past. The bus types b are described in Table 1. The distance is measured

in km and the costs are measured in euro.

Table 1: Parameters of electric bus types.

b (ID)	cap_b	d_b^{\max}	$ct_{rjbc}, \forall r, j, c$	c_{bus_b}	$vcb_{rb}, \forall r$
1 (E433)	153	15	6	500000	270000
2 (E420)	87	20	6	350000	180000
3 (E321)	85	40	10	400000	200000
4 (E490)	75	25	6	400000	170000
5 (321D)	90	15	40	300000	180000
6 (420D)	90	15	30	330000	200000

The other cost parameters are $ccp_c = 120000$, $vcp_c = 4500$, $ccc_j = 5000$, $vcc_j = 500$, $ccps_t = 200000$ and $cl_{tj} = 5000$. There are four types of non-battery vehicles identified with: M103, M105, T420 and T333. Their capacities are 100, 160, 115 and 170, respectively. The route parameters are given in Table 2. Each non-terminal stop is a pair of stops determined by the two directions of the respective route. The cycle time L_r of route r is obtained as the product of the traffic interval upper bound ut_r (past traffic interval of this route) and the number of all old vehicles operating on this route. The traffic interval lower bounds lt_r are determined by the experts.

The feasible charging scenarios are presented in Table 3. They are defined as follows. For each route r with terminal stops $T_1(r)$ and $T_2(r)$, we calculate distance I_0 between the depot and $T_1(r)$, distance I_1 between $T_1(r)$ and $T_2(r)$, and distance I_2 between $T_2(r)$ and $T_1(r)$ in the return direction. Then, define $n_{rbc} = 2$ if $d_b^{\max} \geq \max\{I_0 + I_1, I_1 + I_2, I_2 + I_0\}$, $n_{rbc} = 1$ if $d_b^{\max} \geq \max\{I_0, I_1, I_2\}$, and $n_{rbc} = 0$ if $d_b^{\max} < \max\{I_0, I_1, I_2\}$. For $n_{rbc} = 2$, there are two scenarios with one stop $T_1(r)$ or $T_2(r)$. For $n_{rbc} = 1$, the only scenario consists of both stops $T_1(r)$ and $T_2(r)$. For $n_{rbc} = 0$, there is no feasible charging scenario for an electric bus of type b . If $\sum_{b \in B_r} \sum_{c \in C_b} n_{rbc} = 0$, then no electric bus can serve the respective route.

The instances with ten pairs (cc, uoc) : $(10^7, 5 \cdot 10^6)$, $(1.5 \cdot 10^7, 7 \cdot 10^6)$, $(2 \cdot 10^7, 10^7)$, $(3 \cdot 10^7, 1.5 \cdot 10^7)$, $(4 \cdot 10^7, 2 \cdot 10^7)$, $(1.8 \cdot 10^7, 9 \cdot 10^6)$, $(2.2 \cdot 10^7, 1.1 \cdot 10^7)$, $(2.4 \cdot 10^7, 1.2 \cdot 10^7)$, $(2.6 \cdot 10^7, 1.3 \cdot 10^7)$, $(2.8 \cdot 10^7, 1.4 \cdot 10^7)$ were solved optimally with the academic version of CPLEX 20.1 within the limit of 5 minutes. The results are given in Table 4. For a given pair (cc, uoc) , u_1 and u_2 represent the first two digits of cc and uoc , respectively, $\#_b$ represents the number of new b -type electric buses.

In Table 5, we compare the results obtained with the MIP model proposed in this paper with heuristic HA presented in [7] where any stop was allowed for charging. It can be seen that heuristic HA provides a better solution for only one case out of 10 and for longer solution time. These results show that our MIP model is very well adapted to the practical problems with a low

Table 2: Route parameters. Terminals are labeled *.

$r(d(r))$	$[lt_r, ut_r]$	$nob_{rb},$ $b=E433$	$nv_{rb}^0,$ $b=M103$	$nv_{rb}^0,$ $b=M105$	$nv_{rb}^0,$ $b=T420$	$nv_{rb}^0,$ $b=T333$	$\pi_r = (j_1^{(r)}, \dots, j_{i(r)}^{(r)}, j_1^{(r)})$	Inter-stop distances
1(D_1)	[6,7]	4	3	2	0	0	(1*, 2*, 1*)	(3,9,9)
2(D_1)	[18,20]	0	0	2	0	0	(1*, 2, 3*, 2, 1*)	(3,9,4,4,9)
3(D_1)	[18,20]	0	1	2	0	0	(1*, 2, 4*, 2, 1*)	(3,9,6,6,9)
4(D_1)	[4,5]	0	0	0	2	5	(1*, 5*, 1*)	(3,5,5)
5(D_1)	[4,5]	0	0	0	4	6	(1*, 5, 6*, 5, 1*)	(3,5,4,4,5)
6(D_1)	[13,15]	0	5	0	0	0	(1*, 7*, 1*)	(3,11,11)
7(D_1)	[9,10]	0	0	0	2	5	(1*, 8, 9*, 8, 1*)	(3,5,4,4,5)
8(D_1)	[9,10]	0	0	0	3	6	(9*, 10, 11*, 10, 9*)	(2,5,13,13,5)
9(D_1)	[36,40]	0	0	2	0	0	(10*, 7*, 10*)	(3,9,9)
10(D_1)	[13,15]	0	0	0	2	2	(10*, 7*, 10*)	(3,9,9)
11(D_1)	[9,10]	6	0	0	2	0	(12*, 13*, 12*)	(2,13,13)
12(D_1)	[18,20]	0	1	2	0	0	(14*, 15*, 14*)	(4,8,8)
13(D_1)	[9,10]	4	0	0	2	0	(14*, 13, 14*)	(4,7,7)
14(D_1)	[9,10]	0	0	0	2	5	(15*, 14, 11*, 14, 15*)	(4,7,9,9,7)
15(D_2)	[18,20]	0	2	2	0	0	(14*, 11*, 14*)	(3,9,9)
16(D_2)	[18,20]	0	3	3	0	0	(14*, 16*, 14*)	(3,15,15)
17(D_2)	[9,10]	0	0	0	2	4	(10*, 16*, 10*)	(3,14,14)
18(D_2)	[13,15]	0	0	0	1	3	(14*, 16*, 14*)	(3,13,13)
19(D_2)	[9,10]	0	3	6	0	0	(14*, 17*, 14*)	(3,14,14)
20(D_2)	[6,7]	0	3	6	0	0	(18*, 19*, 18*)	(4,12,12)
21(D_2)	[18,20]	0	2	5	0	0	(18*, 2, 20*, 2, 18*)	(4,18,2,2,18)
22(D_2)	[9,10]	0	5	7	0	0	(14, 2, 21*, 2, 14, 13*, 14)	(3,9,2,2,9,7,7)
23(D_2)	[9,10]	0	0	0	3	0	(22*, 15*, 22*)	(2,6,6)
24(D_2)	[9,10]	0	2	5	0	0	(15*, 2, 23*, 2, 15*)	(6,10,6,6,10)
25(D_2)	[18,20]	0	1	2	0	0	(15*, 2, 23*, 2, 15*)	(6,10,7,7,10)
26(D_2)	[9,10]	0	0	0	6	8	(18*, 16*, 18*)	(6,21,21)

Table 3: Numbers of charging scenarios.

r	1	2	3	4	5	6	7	8	9	10	11	12	13
$n_{rbc}, b = E433$	1	1	1	2	1	1	1	0	1	1	1	1	2
$n_{rbc}, b = E420$	2	1	1	2	2	1	2	1	2	2	1	2	2
$n_{rbc}, b = E321$	2	2	2	2	2	2	2	2	2	2	2	2	2
$n_{rbc}, b = E490$	2	1	1	2	2	2	2	1	2	2	1	2	2
$n_{rbc}, b = 321D$	1	1	1	2	1	1	1	0	1	1	1	1	2
$n_{rbc}, b = 420D$	1	1	1	2	1	1	1	0	1	1	1	1	2
r	14	15	16	17	18	19	20	21	22	23	24	25	26
$n_{rbc}, b = E433$	1	1	1	1	1	1	1	0	0	2	0	0	0
$n_{rbc}, b = E420$	1	2	1	1	1	1	1	0	1	2	0	0	0
$n_{rbc}, b = E321$	2	2	2	2	2	2	2	2	2	2	2	2	1
$n_{rbc}, b = E490$	1	2	1	1	1	1	2	1	1	2	1	1	1
$n_{rbc}, b = 321D$	1	1	1	1	1	1	1	0	0	2	0	0	0
$n_{rbc}, b = 420D$	1	1	1	1	1	1	1	0	0	2	0	0	0

number of charging stops and outperforms the previously available method.

Table 4: Computer experiments for instances approximating the real-life case.

Characteristics \ (u_1, u_2)	(10,5)	(15,7)	(20,10)	(30,15)	(40,20)
Solution time, minutes	0.1	0.19	0.85	0.05	0.03
Number of new chargers	3	4	8	11	13
Number of new charging points	5	8	12	9	25
Numbers of new electric buses	$\#_{E433} = 17,$ $\#_{E420} = 1,$ $\#_{420D} = 1$	$\#_{E433} = 25,$ $\#_{420D} = 1$	$\#_{E433} = 36,$ $\#_{420D} = 1$	$\#_{E433} = 55$	$\#_{E433} = 54,$ $\#_{E420} = 18,$ $\#_{E321} = 1,$ $\#_{E490} = 4$
Number of new power stations	0	0	0	0	1
Number of new links	3	4	8	11	13
Objective value	2775.12	3911.86	5594.19	8360.1	9794.44
Capital cost	$9.81 \cdot 10^6$	$13.83 \cdot 10^6$	$19.85 \cdot 10^6$	$29.89 \cdot 10^6$	$38.63 \cdot 10^6$
Variable cost	$4.99 \cdot 10^6$	$6.99 \cdot 10^6$	$9.98 \cdot 10^6$	$14.94 \cdot 10^6$	$18.82 \cdot 10^6$
Characteristics \ (u_1, u_2)	(18,9)	(22,11)	(24,12)	(26,13)	(28,14)
Solution time, minutes	0.18	4.04	0.16	0.23	0.13
Number of new chargers	6	10	9	10	10
Number of new charging points	9	14	13	16	18
Numbers of new electric buses	$\#_{E433} = 33$	$\#_{E433} = 39,$ $\#_{420D} = 2$	$\#_{E433} = 44$	$\#_{E433} = 47,$ $\#_{420D} = 2$	$\#_{E433} = 50,$ $\#_{321D} = 2$
Number of new power stations	0	0	0	0	0
Number of new links	6	10	9	10	10
Objective value	5046.64	6142.04	6728.95	7276.54	7819.82
Capital cost	$17.64 \cdot 10^6$	$21.94 \cdot 10^6$	$23.65 \cdot 10^6$	$25.85 \cdot 10^6$	$27.96 \cdot 10^6$
Variable cost	$8.95 \cdot 10^6$	$11.0 \cdot 10^6$	$11.94 \cdot 10^6$	$12.97 \cdot 10^6$	$13.95 \cdot 10^6$

Table 5: Comparing MIP and heuristics.

(u_1, u_2) \ Characteristics	MIP objective	MIP time(min)	HA objective	HA time(min)
(10,5)	2775.12	0.1	2654.59	0.15
(15,7)	3911.86	0.19	3808.47	8.37
(20,10)	5594.19	0.85	5492.88	8.36
(30,15)	8360.1	0.05	8249.23	5.02
(40,20)	9794.44	0.03	10574.39	8.37
(18,9)	5046.64	0.18	4879.97	5.02
(22,11)	6142.04	4.04	6105.16	5.02
(24,12)	6728.95	0.16	6634.87	5.01
(26,13)	7276.54	0.23	7247.08	0.03
(28,14)	7819.82	0.13	7784.89	0.29

5.2 Random instances

Six groups of random instances of the problem OFC2 were generated. They are denoted as B1, MB1, B2, MB2, B3 and MB3. Group **B1** is associated with the base case in the previous subsection and the cost limits $cc = 2.2 \cdot 10^7$ and $uoc = 1.1 \cdot 10^7$ (the most difficult instance in the previous subsection), and group **MB1** is associated with the modified base case, which in addition to the base case, includes 28 new stops of 7 additional routes with the same additional depot D_3 . Each additional route r is served by three non-battery vehicles M103 and three non-battery vehicles M105 with parameters $ut_r = 10$ and parameters vcb_{rb} , $b = 1, \dots, 6$, from Table 1. Values of cc and uoc increased by 20% from the respective base case values.

Recall that in the base case, there is one charging type $c = 1$, any charging point holds exactly one plug device ($uc_c = 1$) and a charger can be linked with one power station. In the modified base case, there are charging types $c \in \{1, 2, 3\}$, any charging point can include one or two plug devices ($uc_c = 2$) and a charger can be linked with one or two power stations. The ILP problem for the base case includes 2732 variables and 2903 constraints and the ILP problem for the modified base case includes 13991 variables and 5253 constraints.

The random instances of group B1 (group MB1) keep the combinatorial structure of the base case (modified base case). The numerical characteristics of an instance of group B1 (group MB1), such as times, costs, distances, electric powers and passenger capacities randomly deviate $\pm 10\%$ from their base case (modified base case) values. For an instance of the group MB1, the parameters ct_{rjb2} are made dependent on j . In particular, ct_{rjb2} is equal to the same base case value if j is the depot or a terminal stop of route r . Otherwise, it is equal to the same base case value divided by two. Values ct_{rjbc} are made dependent on c such that $ct_{rjbc} = ct_{rjb2} \frac{p_2}{p_c}$ for $c = 1, 3$. Since distances between stops have changed for the modified base case, the feasible charging scenarios are re-calculated for this case.

Three instance families were generated for each group B1 and MB1. Each instance family of group B1 (group MB1) is characterized by the pair (n_1, n_2) (triple (n_1, n_2, n_3)), where n_i is the number of routes with depot D_i , which were selected randomly from the set of all routes of this depot to be included into the ground route set R , $i = 1, 2, 3$. For each instance family of any group, 50 random instances were obtained. Each instance was solved with CPLEX within the time limit of 5 minutes for the group B1 and of 10 minutes for the group MB1. The hardware used was a PC with Intel Core i5 2.3 GHz processor and 4 GB of RAM. Table 6 contains the following minimal, maximal and mean values for the 50 instances of the same family: running

times of CPLEX, ratios of total capital cost cc and total variable cost oc to the respective upper bounds cc and uoc , numbers of [binary and integer variables and constraints in the initial ILP models \(2\) - \(66\) before and after](#) the pre-solve stage of CPLEX, the optimality gap reported by CPLEX, and the number of optimal solutions found.

[The generation of a random instance of groups B2 and MB2 included two stages.](#) In the first stage, numerical parameters of stops, chargers, electric buses, non-battery vehicles and power stations were randomly generated to deviate $\pm 10\%$ from their values for the base case (group B2) or modified base case (group MB2). In the second stage, a set of nr routes was randomly generated. A route of this set was randomly selected from the set R in the respective basic case, and then its parameters (distances between stops, traffic interval) were randomly generated to deviate $\pm 10\%$ from the original parameters. Based on the new data, charging scenarios were re-calculated. [In contrast to groups B1 and MB1, for groups B2 and MB2, it was assumed that there is no old charging infrastructure.](#) Table 7 contains experimental results for groups B2 and MB2, and for each group, three families of 50 random instances, where each family is defined by $nr \in \{50, 75, 100\}$.

[The generation of a random instance of groups B3 and MB3 also included two stages.](#) In the first stage, a random transport network was generated. It was [defined by the triplet \$\(nd, nt, nr\)\$](#) where [\$nd\$ is the number of depot stops, \$nt\$ is the number of terminal stops, and \$nr\$ is the number of routes.](#) Each route consisted of three stops: one depot stop and two terminal stops. Abscissa and ordinate coordinates of each stop were randomly generated within the ranges $[0, 40]$ and $[0, 30]$, respectively. A Euclidean metric was used to calculate distances between the stops. [No old charging infrastructure was considered.](#) For group B3, at most one power station was allowed to be linked with each stop, and for group MB3, at most two power stations were allowed to be linked with each stop. For each generated route r , the sets B_r and V_r were randomly selected from the set $B = \{E433, E420, E321, E490, 321D, 420D\}$ and $V = \{M103, M105, T420, T333\}$, respectively. The traffic interval upper bound ut_r was randomly generated to deviate $\pm 10\%$ from the value $\frac{dst_r/30}{\sum_{b \in V_r} nvc_{rb}}$, where dst_r is the length of the cycle of route r , 30 is the average speed of any vehicle (30 km/h is a statistically average speed used in the project PLATON [23]), and nvc_{rb} is the number of b -type non-battery vehicles operating on route r , which is randomly generated from the set $\{1, 2, 3, 4\}$. The traffic interval lower bound lt_r was defined with deviation $\pm 10\%$ from ut_r . The generation of the other parameters for instances of groups B3 and MB3 was achieved with deviation $\pm 10\%$ from their values for the base case. Table 8 contains experimental results

Table 6: Groups B1 and MB1.

Group	B1			MB1		
Family (n_1, n_2, n_3)	(9,8,0)	(11,10,0)	(14,12,0)	(9,8,5)	(11,10,6)	(14,12,7)
Min time, sec	0.34	0.57	0.66	18.22	24.82	29.40
Max time, sec	45.19	44.08	300	359.8	600	600
Mean time, sec	4.38	7.22	18.53	95.26	145.5	301.4
Min cc/cc , %	83.44	88.93	88.22	95.36	91.99	93.83
Max cc/cc , %	99.99	100	100	99.98	100	99.99
Mean cc/cc , %	98.76	98.82	98.78	99.74	99.52	99.67
Min cc/uoc , %	77.73	81.19	76.83	755.04	74.45	74.04
Max cc/uoc , %	100	100	100	99.99	100	99.99
Mean cc/uoc , %	93.77	94.25	94.91	90.77	91.96	93.44
Min number of binary variables of the initial ILP	1239	1554	2024	7443	8803	11158
Max number of binary variables of the initial ILP	1472	1733	2061	16035	18965	22090
Mean number of binary variables of the initial ILP	1354	1656	2045	13533	17298	21234
Min number of integer variables of the initial ILP	405	499	628	1788	2205	2605
Max number of integer variables of the initial ILP	450	527	628	2022	2361	2704
Mean number of integer variables of the initial ILP	425	515	628	1903	2280	2700
Min number of constraints of the initial ILP	1791	2261	2925	9426	12073	14618
Max number of constraints of the initial ILP	2130	2469	2935	11261	13281	15310
Mean number of constraints of the initial ILP	1954	2385	2930	10373	12670	15274
Min number of binary variables of the transformed ILP	225	175	324	280	691	602
Max number of binary variables of the transformed ILP	278	318	377	969	1124	1311
Mean number of binary variables of the transformed ILP	250	300	369	850	1016	1244
Min number of integer variables of the transformed ILP	190	151	222	272	418	434
Max number of integer variables of the transformed ILP	218	260	314	757	895	1072
Mean number of integer variables of the transformed ILP	203	247	306	679	813	1004
Min number of constraints of the transformed ILP	628	461	768	756	1487	1396
Max number of constraints of the transformed ILP	705	850	1013	2357	2802	3340
Mean number of constraints of the transformed ILP	668	806	993	2144	2567	3156
Min gap, %	0	0	0	0	0	0
Max gap, %	0	0	0.05	0	0.32	0.67
Mean gap, %	0	0	0.001	0	0.02	0.075
Number of optimal solutions	50	50	49	50	47	39

Table 7: Groups B2 and MB2.

Group	B2			MB2		
Family (<i>nr</i>)	50	75	100	50	75	100
Min time, sec	4.53	6.86	9.16	83.15	127.8	484.6
Max time, sec	300	300	300	600	600	600
Mean time, sec	60.53	147.8	216.1	505.5	578.5	597.8
Min <i>cc/cc</i> , %	90.5	89.53	89.47	94.14	89.86	89.12
Max <i>cc/cc</i> , %	100	100	100	99.98	99.99	99.99
Mean <i>cc/cc</i> , %	99.12	99.06	98.94	99.46	98.46	97.72
Min <i>cc/uoc</i> , %	81.11	83.87	84.8	82.17	82.95	84.55
Max <i>oc/uoc</i> , %	100	100	100	100	99.99	99.99
Mean <i>oc/uoc</i> , %	97.63	96.91	97.08	96.24	96.56	97.0
Min number of binary variables of the initial ILP	3007	4633	6113	16879	44107	61508
Max number of binary variables of the initial ILP	4115	6096	7925	35553	53961	70777
Mean number of binary variables of the initial ILP	3388	5087	6800	32648	49399	65614
Min number of integer variables of the initial ILP	929	1366	1854	3667	5378	7085
Max number of integer variables of the initial ILP	1184	1738	2255	4110	6106	7967
Mean number of integer variables of the initial ILP	1034	1534	2037	3919	5739	7557
Min number of constraints of the initial ILP	4269	6506	8778	20774	30457	41566
Max number of constraints of the initial ILP	5892	8635	11133	24299	36839	48529
Mean number of constraints of the initial ILP	4850	7243	9653	22729	33898	44993
Min number of binary variables of the transformed ILP	292	699	948	835	686	2287
Max number of binary variables of the transformed ILP	649	925	1190	2173	3053	4024
Mean number of binary variables of the transformed ILP	559	832	1102	1780	2618	3541
Min number of integer variables of the transformed ILP	272	638	897	734	812	1803
Max number of integer variables of the transformed ILP	564	821	1065	1924	2754	3692
Mean number of integer variables of the transformed ILP	486	737	984	1598	2408	3319
Min number of constraints of the transformed ILP	768	1975	2814	2091	2036	5512
Max number of constraints of the transformed ILP	1782	2540	3303	5660	8068	10649
Mean number of constraints of the transformed ILP	1543	2319	3082	4689	6999	9589
Min gap, %	0.0	0.0	0.0	0.0	0.0	0.0
Max gap, %	0.33	0.62	0.58	2.52	5.86	12.38
Mean gap, %	0.01	0.07	0.15	0.35	1.11	1.65
Number of optimal solutions	48	37	21	18	3	1

for groups B3 and MB3, and for each group, three families of 50 random instances, where each family is defined by a triplet (nd, nt, nr) .

Only one instance of group B1 and one instance of group B3 were not solved optimally. The worst optimality gaps were 0.05%, 0.18%, 0.62%, 0.67%, 4.40% and 12.38% for the groups B1, B2, B3, MB1, MB3 and MB2, respectively. The hardest instance was in the group MB2. This group also contained the smallest number of optimally solved instances and an instance with the largest numbers of variables and constraints.

5.3 ILP-based heuristics

Based on the ILP model (1)-(66) we developed two heuristics that fix some variables of the model and then solve it. At each iteration, the first heuristics **HR** generates a set R_{HR} of routes and sets B_r for $r \in R_{HR}$ which satisfy constraints on the capital and operating costs by removing randomly chosen buses of routes. After generation of feasible sets we fix variables $y_r = 1$ for $r \in R_{HR}$ and $y_r = 0$ for $r \in R \setminus R_{HR}$ and solve the obtained problem. The second heuristics **HRBC** tries to generate sets R_{HRBC} of routes, sets B_r of buses for $r \in R_{HR}$, and charger types for each bus. After generation we fix variables y_r , y_{rb} , y_{rbc} and define lower and upper bounds on nb_{rbc} in the appropriate way and solve the corresponding problem. The maximal running time for heuristics **HR** and **HRBC** was set the same as for pure ILP models (up to 5 min for groups B1, B2 and B3 and up to 10 min for groups MB1, MB2 and MB3). The maximal available time for CPLEX for solving ILP subproblems was set up to 20 sec for groups B1, B2 and B3 and up to 30 sec for groups MB1, MB2 and MB3. In Tables 9-10, we present the results obtained using these heuristics for groups B1, MB1, B2, MB2, B3 and MB3. In the Tables **HR/ILP** (**HRBC/ILP**) correspond to ratios of the value of (1) obtained by **HR** (**HRBC**) and the value of (1) obtained by solving the ILP problem. Similarly, **HRBC/HR** is the ratio of the values of (1) obtained by **HRBC** and **HR**.

6 Concluding remarks

An optimal decision problem related to the design of infrastructure for a fast-charge urban electric bus system has been studied. The decisions to be made included determining a mixed fleet of conventional and electric buses, points for electric chargers and power stations, quantities of charging plug devices, a distribution of electric buses between the routes, and matching chargers with power stations. The objective is to maximize the route-weighted total passenger capacity of

Table 8: Groups B3 and MB3.

Group	B3			MB3		
Family (<i>nd, nt, nr</i>)	(3,25,50)	(4,40,75)	(5,50,100)	(3,25,50)	(4,40,75)	(5,50,100)
Min time, sec	4.53	6.86	9.16	83.15	127.8	484.6
Max time, sec	300	300	300	600	600	600
Mean time, sec	60.53	147.8	216.1	505.5	578.5	597.8
Min <i>cc/cc</i> , %	90.5	89.53	89.47	94.14	89.86	89.12
Max <i>cc/cc</i> , %	100	100	100	99.98	99.99	99.99
Mean <i>cc/cc</i> , %	99.12	99.06	98.94	99.46	98.46	97.72
Min <i>cc/uoc</i> , %	81.11	83.87	84.8	82.17	82.95	84.55
Max <i>oc/uoc</i> , %	100	100	100	100	99.99	99.99
Mean <i>oc/uoc</i> , %	97.63	96.91	97.08	96.24	96.56	97.0
Min number of binary variables of the initial ILP	3007	4633	6113	16879	44107	61508
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Mean number of binary variables of the initial ILP	3388	5087	6800	32648	49399	65614
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Max number of integer variables of the initial ILP	1184	1738	2255	4110	6106	7967
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Min number of constraints of the initial ILP	4269	6506	8778	20774	30457	41566
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Mean number of constraints of the initial ILP	4850	7243	9653	22729	33898	44993
Min number of binary variables of the transformed ILP	292	699	948	835	686	2287
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Mean number of binary variables of the transformed ILP	559	832	1102	1780	2618	3541
Min number of integer variables of the transformed ILP	272	638	897	734	812	1803
Max number of integer variables of the transformed ILP	564	821	1065	1924	2754	3692
Mean number of integer variables of the transformed ILP	486	737	984	1598	2408	3319
Min number of constraints of the transformed ILP	768	1975	2814	2091	2036	5512
Max number of constraints of the transformed ILP	1782	2540	3303	5660	8068	10649
Mean number of constraints of the transformed ILP	1543	2319	3082	4689	6999	9589
Min gap, %	0.0	0.0	0.0	0.0	0.0	0.0
Max gap, %	0.33	0.62	0.58	2.52	5.86	12.38
Mean gap, %	0.01	0.07	0.15	0.35	1.11	1.65
Number of optimal solutions	48	37	21	18	3	1

Table 9: Comparison of heuristics and ILP for groups B1 and MB1.

Group	B1			MB1		
Family (n_1, n_2, n_3)	(9,8,0)	(11,10,0)	(14,12,0)	(9,8,5)	(11,10,6)	(14,12,7)
Min HR/ILP, %	98.21	94.59	96.38	95.89	96.59	97.23
Max HR/ILP, %	99.91	99.89	99.85	100.47	101.58	101.09
Mean HR/ILP, %	99.74	99.6	99.56	99.4	99.31	99.08
Min HRBC/ILP, %	93.41	88.96	74.89	95.89	84.75	84.4
Max HRBC/ILP, %	99.82	99.18	97.48	100.47	98.09	97.5
Mean HRBC/ILP, %	97.65	95.73	92.01	99.4	91.39	90.72
Min HRBC/HR, %	93.63	91.19	75.73	78.57	85.17	85.65
Max HRBC/HR, %	100	99.47	97.87	98.74	98.25	100
Mean HRBC/HR, %	97.91	96.11	92.42	91.51	92.03	91.56

Table 10: Comparison of heuristics and ILP for groups B2 and MB2.

Group	B2			MB2		
Family nr	50	75	100	50	75	100
Min HR/ILP, %	56.34	51.9	49.18	79.85	68.97	28.76
Max HR/ILP, %	99.6	84.37	70	99.7	98.6	96.6
Mean HR/ILP, %	82.49	67.87	60.78	96.21	89.29	80.57
Min HRBC/ILP, %	44.44	40.77	36.43	67.54	57.44	40.32
Max HRBC/ILP, %	84.94	68.26	51.53	93.41	88.19	73.21
Mean HRBC/ILP, %	62.67	48.83	42.48	85.86	70.84	58.04
Min HRBC/HR, %	63.17	60.21	56.09	72.71	66.88	56.99
Max HRBC/HR, %	93.57	85.18	97.93	103.11	106.63	202.56
Mean HRBC/HR, %	76.26	72.25	70.2	89.27	79.42	73.94

Table 11: Comparison of heuristics and ILP for groups B3 and MB3.

Group	B3			MB3		
Family (nd, nt, nr)	(3,25,50)	(4,40,75)	(5,50,100)	(3,25,50)	(4,40,75)	(5,50,100)
Min HR/ILP, %	12.55	19.16	25.29	7.97	12.26	30.13
Max HR/ILP, %	88.66	90.46	75.88	93.54	99.6	97.8
Mean HR/ILP, %	51.32	56.44	51.84	53.63	70.33	73.28
Min HRBC/ILP, %	8.84	20.34	20.89	15.56	12.26	24.47
Max HRBC/ILP, %	80.18	81.64	67.13	98.07	94.78	93.59
Mean HRBC/ILP, %	47.69	50.62	45.9	65.3	68.04	68
Min HRBC/HR, %	60.31	68.36	58.5	86.09	81.69	51.46
Max HRBC/HR, %	183.97	113.77	106.1	134.08	115.81	115.09
Mean HRBC/HR, %	95.83	90.9	89.26	99.69	98.18	93.41

newly introduced electric buses, subject to the traffic interval, power demand and supply and cost constraints. We developed an original ILP model, which is efficient if the number of feasible charging spots is sufficiently small. **Extensive computer experiments demonstrate that our approach is able to deliver near-optimal solutions of the studied problem in ten minutes for practical-size instances on a standard PC.**

The proposed ILP model can be easily modified to address additional practical requirements such as whether any route must be served by non-battery vehicles only or by electric buses only.

In the future, several additional challenging assumptions can be considered. For example, it would be interesting to study a problem extension in which a combination of fast-charging and slow-charging technologies is used. Another assumption to explore is the case where the single-charge range of some electric buses is longer than the route length. Considering the state of charge level and the driving range after a charge as functions of the past driving conditions is practical, but it makes the proposed model much more uncertain and complicated.

Acknowledgment

This work was partially supported by the French Agency for Research under grant ANR-20-CE40-0021.

Appendix: Input parameters.

Table 12: Input parameters.

Name	Definition
ucc	capital cost upper limit
uoc	operating, depreciation & energy cost upper limit
N	feasible charging stop set
$NO, NO \subseteq N$	set of old charger stops
T	power station spot set
$TO, TO \subseteq T$	set of old power station spots
T_j	set of power station spots feasible for stop j
R	route set
D	depot set
RO	set of old electric bus routes
B	electric bus type set
BO	set of old electric bus types
V	non-battery vehicle type set
C	charging type set
d_b^{\max}	driving range of fully charged b -type electric bus

Continued on next page

Table 12: Input parameters. Continued from previous page

Notation	Description
C_b	feasible charging type set for b -type electric buses
co_b	Old Charging type of b -type electric bus
$cap_b, b \in V \cup B$	passenger capacity of b -type vehicle
$cbus_b$	b -type electric bus capital cost
V_r	route r set of non-battery vehicle types
nv_{rb}^0	number of b -type non-battery vehicles on route r
dem_r	past passenger capacity of all route r vehicles
B_{rc}	type set of c -type charging electric buses of route r
BO_{rc}	type set of c -type charging old electric buses of route r
B_r	electric bus type set of route r
BO_r	old electric bus type set of route r
L_r	cycle time of any vehicle on route r , time between two consecutive departures of the same vehicle on route r
ut_r	upper bound on traffic interval of route r (past traffic interval of route r)
lt_r	lower bound on traffic interval of route r
$d(r)$	route r depot
π_r	route r cycle
nob_{rbc}	number of old electric Buses of route r that are b -type and c -type charging
nob_{rb}	number of Old b -type electric Buses of route r
vcb_{rb}	variable Cost of b -type electric Bus on route r
nod_{jc}	number of Old c -type plug devices of stop j
$nos_j, nos_j \in \{0, 1\}$	Number of Old chargers of stop j
nop_{jc}	number of old c -type charging Points of stop j
up_j	upper limit on the number of charging points at stop j
ct_{rjbc}	charging Time of b -type electric bus at c -type charging point of non-depot stop j of route r
uc_c	upper limit on the number of plug devices of one c -type charging point
p_c	output power of one c -type plug device
ccp_c	capital cost of one c -type charging Point
vcp_c	variable cost of one c -type charging Point
ccc_j	capital cost of one charger at stop j
vcc_j	variable cost of one charger at stop j
utp_t	output power of a power station at spot $t \in T$
$ccps_t$	capital cost of a power station at $t \in T$
cl_{tj}	cost of linking power station spot t and stop j , $cl_{tj} = 0$ if $t \in TO$ and $j \in NO$
n_{rbc}	number of charging scenarios for route r , electric bus type b and charging type c
$S_{rbc}^{(s)}$	charging scenario s , sequence of intermediate stops of route r with charging points of type c for electric buses of type b , $S_{rbc}^{(s)} = (j_{rbc1}^{(s)}, j_{rbc2}^{(s)}, \dots, j_{rbcl_{rbc}^{(s)}}^{(s)})$
R_{jc}	set of routes with at least one occurrence of j in $S_{rbc}^{(s)}$ for all b and s

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