Wireless Communication Project Report

Multi-Beam Power Allocation for mmWave Communications under Random Blockage

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Millimetre Wave - The price of high data rate

Communication using Millimetre wave is shaping the future. With newer technologies like 5G using millimetre wave to facilitate high data rates (in the order of Gbps), larger bandwidths and smaller component sizes the advantages are significant. But millimetre waves are also notoriously susceptible to blockage. When the direct line of sight is blocked, connectivity is notably impeded. However, when a direct line of sight is blocked, we can use a reflected path to maintain connectivity.

Finding the middle ground - Reflecting the Signal

When the direct line of sight is blocked, connectivity can be maintained by switching to an alternate reflected path. However, this switching operation is not the most efficient. The method requires detecting the blockage and then reconfiguring the transceiver to use the new path which incurs latency. In highly dynamic environments where path switching requirement is frequent, or for traffic with strict latency and reliability requirements, using both paths concurrently can be more beneficial.

The Paper proposes a better system

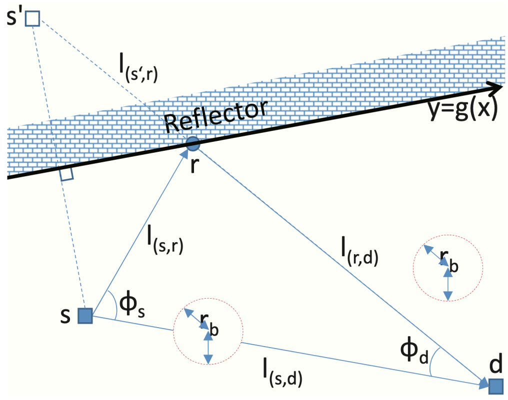
In this paper, the authors consider using multiple paths and dividing the transmission power over those paths, instead of path switching. They propose an algorithm to allocate power among the different mmWave communication paths to overcome link blockage under randomly distributed obstacles. Finally the performance of the proposed algorithm via simulation for various wireless environments.

System Model and design

We consider a mmWave network consisting of a pair of source node and destination node, a reflector, and random obstacles.

Obstacles are modelled as a disk with radius r_d and are assumed to be independently and randomly located. The distribution of the centres of obstacles follows a homogeneous Poisson distribution with density λ in two dimensional space, which is expressed as

$$\Pr\{k \text{ obstacles in in area } A\} = (\lambda A)^k \frac{e^{-\lambda A}}{k!}.$$

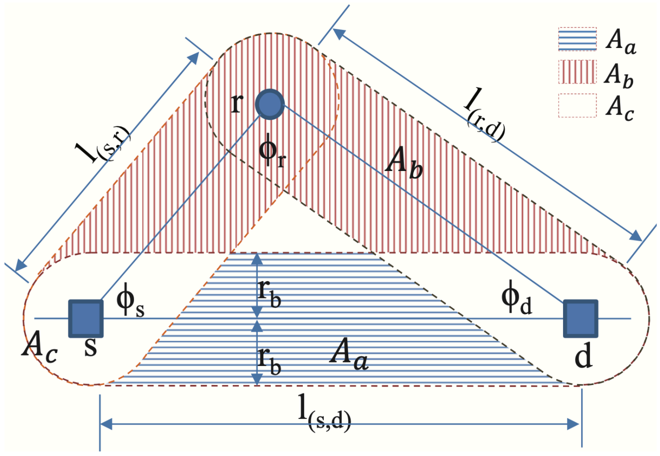


Signal-to-interference-and-noise ratio (SINR) - Notations

- γ is the signal-to-interference-and-noise ratio (SINR) at the receiver.
- Achievable link capacity is given by $C = B\log(\gamma+1)$; where B is the bandwidth
- SINR is defined as $\gamma = P_{rx}/(1+N)$, where P_{rx} is the receive power at the receiver, I is the interference power from other transmitters, and N is the ambient noise.

Analysis Link Capacity - Blockage Probability

Here we formulate the probability of the path that will be taken by the signal, by considering the corresponding the **area through which it passes**. Here **1** is the direct path and **2** is reflected path. So, there are three possibilities:



1) Signal takes only the direct path. The probability that only the direct path ($\rho_{1\bar{2}}$) is available is

$$\rho_{1\bar{2}} = \Pr(A_a + A_c)\Pr(\overline{A_b})$$
$$= e^{-\lambda(A_a+A_c)}\left(1 - e^{-\lambda A_b}\right)$$

2) Signal takes only the reflected path. The probability that only the reflected path ($\rho_{\bar{1}2}$) is available is

$$\rho_{\bar{1}2} = \left(1 - e^{-\lambda A_a}\right) e^{-\lambda(A_b+A_c)}$$

3) Signal takes both direct and reflected (optimum) path. The probability both the direct and reflected path is available is

$$\rho_{12} = e^{-\lambda(A_a+A_b+A_c)}$$

Calculated Areas and Calculations

For h = 6 units

$A_a = 11.35$

$A_b = 13.24$

$A_c = 0.96$

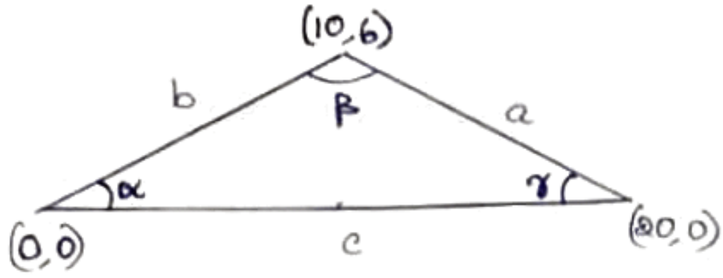
For h = 15 units

$A_a = 11.66$

$A_b = 21.24$

$A_c = 0.7$

(All areas in units^2)



Using Law of Cosines,

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\beta = \frac{b^2+a^2-c^2}{2ab}$$

$$\cos\gamma = \frac{c^2+a^2-b^2}{2ca}$$

$$a = \sqrt{(20-10)^2+(0-6)^2} = \sqrt{136}$$

$$b = \sqrt{(10-0)^2+(6-0)^2} = \sqrt{136}$$

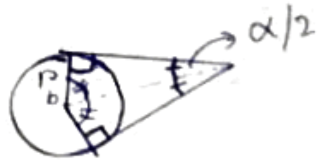
$$c = \sqrt{(20-0)^2+(0-0)^2} = 20$$

Substituting values of a,b,c in $\cos\alpha$, $\cos\beta$, $\cos\gamma$

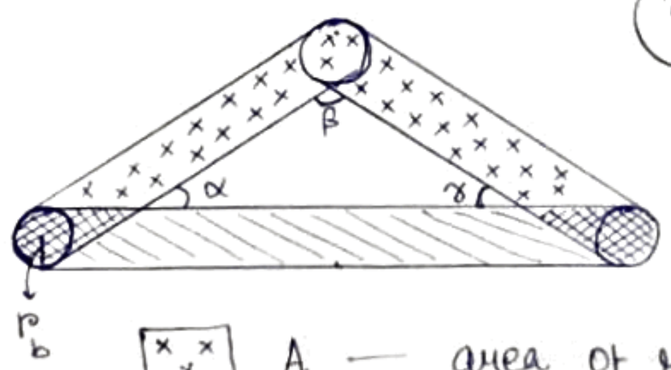
$$\cos\alpha = \frac{136+400-136}{2(\sqrt{136})20} = 0.857 \Rightarrow \alpha = \cos^{-1}(0.857) = 31^\circ$$

$$\cos\beta = \frac{136+136-400}{2(\sqrt{136})(\sqrt{136})} = -0.47 \Rightarrow \beta = \cos^{-1}(-0.47) = 118^\circ$$

$$\cos\gamma = \frac{400+136-136}{2(20)(\sqrt{136})} = 0.857 \Rightarrow \gamma = \cos^{-1}(0.857) = 31^\circ$$



$r_b = 0.3$



A_b — area of reflected path blockage

A_a — area of direct path blockage

A_c — area of reflected and direct path blockage

$$A_c = \left[\frac{1}{2}\left(r_b \tan\left(90-\frac{\alpha}{2}\right)\cdot r_b\right)\times 2 + \left(\frac{180+\alpha}{360}\right)\cdot \pi r_b^2\right]\times 2 \qquad (\because \alpha = \gamma)$$

$$= \left[\frac{1}{2}\cdot (0.3)^2 \tan\left(90-\frac{\alpha}{2}\right)\times 2 + \frac{180+\alpha}{360}\cdot \pi (0.3)^2\right]\times 2$$

$$= \left[\tan\left(90-\frac{\alpha}{2}\right)\times 0.09 + \frac{180+\alpha}{360}\times 0.09\pi\right]\times 2$$

$$= [0.32 + 0.16]\times 2 = 0.48\times 2 = \underline{\underline{0.96}}$$

$$A_b = \left[\frac{1}{2}\times 2r_b \times r_b \tan\left(90-\frac{\alpha}{2}\right) + \left(b - r_b - r_b\tan\left(90-\frac{\alpha}{2}\right)\right)\times 2r_b\right]\times 2$$

$$+ \left(\frac{1}{2}\cdot r_b \tan\left(90-\frac{\beta}{2}\right)\cdot r_b\right)\times 2 + \frac{180+\beta}{360}\times \pi r_b^2$$

$$(\because \alpha = \gamma,\ b = a)$$

$$= \left[\frac{1}{2}\times 0.6\times 0.3\tan\left(90-\frac{\alpha}{2}\right) + \left(11.66 - 0.3 - 0.3\tan\left(90-\frac{\alpha}{2}\right)\right)\times 0.6\right]\times 2$$

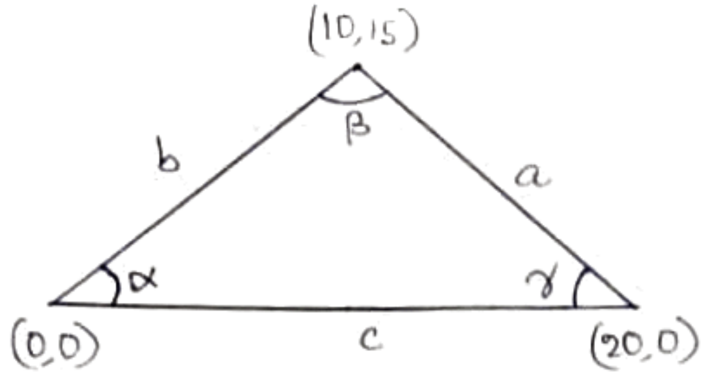
$$+ \left(\frac{1}{2}\times(0.3)^2\times \tan\left(90-\frac{\beta}{2}\right)\right)\times 2 + \frac{180+\beta}{360}\times 0.09\pi$$

$$= (0.32 + 6.16)\times 2 + 0.05 + 0.23 = \underline{\underline{13.24}}$$

$$A_a = \frac{1}{2}\times 2r_b \times \left(c - r_b\tan\left(90-\frac{\alpha}{2}\right)\right)\times 2 + c)$$

$$= \frac{1}{2}\times 0.6\times\left(20 - 0.6\tan\left(90-\frac{\alpha}{2}\right) + 20\right)$$

$$= 0.3\times 37.83 = \underline{\underline{11.35}}$$



Using Law of Cosines,

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\beta = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos\gamma = \frac{a^2+c^2-b^2}{2ac}$$

$$a = \sqrt{(20-10)^2+(0-15)^2} = \sqrt{325}$$

$$b = \sqrt{(10-0)^2+(15-0)^2} = \sqrt{325}$$

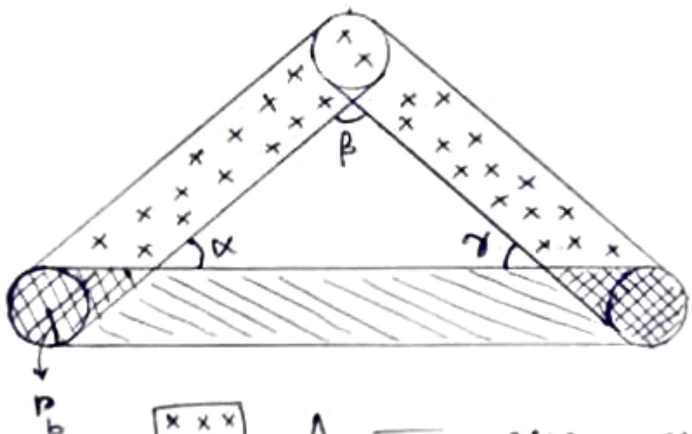
$$c = \sqrt{(20-0)^2+(0-0)^2} = 20$$

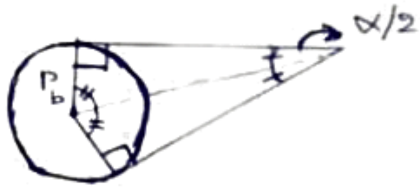
Substituting values of a, b, c in $\cos\alpha$, $\cos\beta$, $\cos\gamma$

$$\cos\alpha = \frac{325+400-325}{2(\sqrt{325})(20)} = 0.55 \Rightarrow \alpha = \cos^{-1}(0.55) = 56^\circ$$

$$\cos\beta = \frac{325+325-400}{2(\sqrt{325})(\sqrt{325})} = 0.38 \Rightarrow \beta = \cos^{-1}(0.38) = 68^\circ$$

$$\cos\gamma = \frac{325+400-325}{2(\sqrt{325})(20)} = 0.55 \Rightarrow \gamma = \cos^{-1}(0.55) = 56^\circ$$

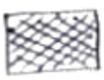




$r_b = 0.3$

 A_b — area of reflected path blockage

 A_a — area of direct path blockage

 A_c — area of reflected and direct path blockage

$$A_c = \left[\left(\frac{1}{2}\times r_b \tan\left(90-\frac{\alpha}{2}\right)\times r_b\right)\times 2 + \frac{180+\alpha}{360}\times \pi r_b^2\right]\times 2 \quad (\because \alpha=\gamma)$$

$$= \left[\frac{1}{2}\times 0.09 \tan\left(90-\frac{\alpha}{2}\right)\times 2 + \frac{180+\alpha}{360}\times 0.09\pi\right]\times 2$$

$$= (0.17+0.18)\times 2 = 0.7$$

$$A_b = \left[\frac{1}{2}\times 2r_b\times r_b\tan\left(90-\frac{\alpha}{2}\right) + \left(b - r_b - r_b\tan\left(90-\frac{\alpha}{2}\right)\right)\times 2r_b\right]\times 2$$

$$+ \left(\frac{1}{2}\times r_b\tan\left(90-\frac{\beta}{2}\right)\times r_b\right)\times 2 + \frac{180+\beta}{360}\times\pi r_b^2 \quad (\because \alpha=\gamma,\ b=a)$$

$$= 2\left[\frac{1}{2}\times 0.6\times 0.3\tan\left(90-\frac{\alpha}{2}\right) + \left(18.02-0.3-0.3\tan\left(90-\frac{\alpha}{2}\right)\right)\times 0.6\right]$$

$$+ 0.09\tan\left(90-\frac{\beta}{2}\right) + \frac{180+\beta}{360}\times 0.09\pi$$

$$= (0.17+10.29)\times 2 + 0.13+0.19 = \underline{\underline{21.24}}$$

$$A_a = \frac{1}{2}\times 2r_b\times\left(c - \tan\left(90-\frac{\alpha}{2}\right)r_b\times 2 + c\right)$$

$$= \frac{1}{2}\times 0.6\times\left(20-0.3\times 2\times\tan\left(90-\frac{\alpha}{2}\right)+20\right) = 0.3\times 38.87$$

$$= \underline{\underline{11.66}}$$

Impact of Blockage on Receiving Power

Formula of power at the receiver

$$P_{\mathrm{rx}} = KP_{\mathrm{tx}} l^{-\alpha}\beta$$
$$P_{\mathrm{rx}} = KP_{\mathrm{Tot}} p l^{-\alpha}\beta$$

K is a constant

P_{tx} is the transmit power for the antenna lobe serving that path

P_{Tot} is the total available power of the transmitter

p is the ratio of the path transmit power to the total power

l is the path length

α is the attenuation exponent

β is the reflection coefficient of a reflector

For the case of

Direct Path: The path length is $l_{(s,d)}$ in and the reflection coefficient is one

Reflected Path: In the case of a reflected path, the path length is $l_{(s,r)} + l_{(r,d)}$ and the coefficient is a value in the range of [0, 1]

Both direct and reflected path: When two beams are simultaneously activate at the transmitter and the receiver combines the signals received from the different antenna beams, the receiver power is

$$P_{\mathrm{rx}} = K_1 P_{\mathrm{Tot}} p_1 l_1^{-\alpha} + K_2 P_{\mathrm{Tot}} p_2 l_2^{-\alpha}\beta,$$

p1 (p2) is the power ratio allocated to the beam of the direct (reflected) path

l1 (l2) is the path length of the direct (reflected) path

Impact of Blockage on Capacity

There are four cases

1) When both paths are blocked, SINR is zero.
2) When only the direct path is available, SINR ($\gamma_{1\bar{2}}$). γ_1 is the SINR when all the power is allocated to the direct path.

$$\gamma_{1\bar{2}} = \frac{K_1 P_{\mathrm{Tot}} p_1 l^{-\alpha}}{I + N} = \gamma_1 p_1$$

3) For reflected path only, $P_{Tot}p_2$ is allocated to the beam, then SINR $\gamma_{\bar{1}2} = \gamma_2 p_2$.
4) When both the direct and reflected path is available, the SINR becomes γ_{12}

$$\gamma_{12} = \frac{K_1 P_{\mathrm{Tot}} p_1 l_1^{-\alpha} + K_2 P_{\mathrm{Tot}} p_2 l_2^{-\alpha}\beta}{I + N}$$
$$\gamma_{12} = \gamma_1 p_1 + \gamma_2 p_2,$$

<u>PROBLEM STATEMENT</u>: allocate power to antenna beams in order to maximise the expected capacity at node *d* under a power constraint, expressed as

$$\max_{(P_1,P_2)\in[0,1]^2} E[C(P_1,P_2)]$$
$$\text{subject to } P_1 + P_2 \leq P_{\text{Tot}}$$

where P_1, P_2, P_{Tot} are powers allocated to the direct path and the reflected path, and the total power, respectively.

Now using the SINR values for the four cases and the calculate blockage probabilities, we can have the expected capacity using the law of total expectation.

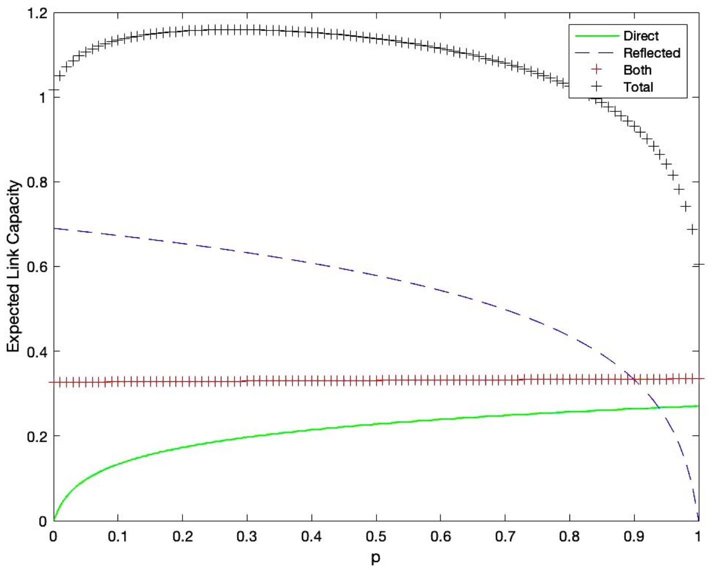
$$\begin{aligned}
&E[C]\\
&= E[C|L_{1\bar{2}}]\Pr(L_{1\bar{2}}) + E[C|L_{\bar{1}2}]\Pr(L_{\bar{1}2}) + E[C|L_{12}]\Pr(L_{12})\\
&= \rho_{1\bar{2}}\log\left(\gamma_{1\bar{2}}+1\right) + \rho_{\bar{1}2}\log\left(\gamma_{\bar{1}2}+1\right) + \rho_{12}\log\left(\gamma_{12}+1\right)\\
&= \rho_{1\bar{2}}\log\left(\gamma_1 p_1+1\right) + \rho_{\bar{1}2}\log\left(\gamma_2 p_2+1\right) + \rho_{12}\log\left(\gamma_1 p_1+\gamma_2 p_2+1\right)
\end{aligned}$$

$L_{1\bar{2}}$ is the event that only the direct path is available

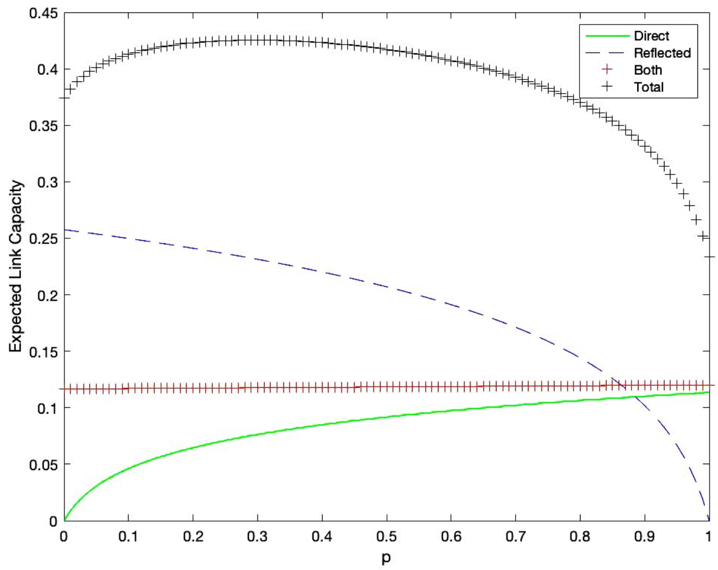
$L_{\bar{1}2}$ is the event that only the reflect path is available

L_{12} is the event that the direct and reflected path are simultaneously available

<u>RESULTS from running the code</u>



Horizontal reflector located at y = 6



Horizontal reflector located at y = 15

<u>Optimal Power Allocation Algorithm</u>

Since $p_1 + p_2 = 1$, we can express the above equation as a function of p_1 such that

$$
\begin{aligned}
f(p_1) &= \rho_{1\bar{2}} \log(\gamma_1 p_1 + 1) + \rho_{\bar{1}2} \log(\gamma_2(1-p_1)+1) + \rho_{12} \log(\gamma_1 p_1 + \gamma_2(1-p_1)+1) \\
&= \rho_{1\bar{2}} \log(\gamma_1 p_1 + 1) + \rho_{\bar{1}2} \log(-\gamma_2 p_1 + \gamma_2 + 1) + \rho_{12} \log((\gamma_1 - \gamma_2)p_1 + \gamma_2 + 1)\,.
\end{aligned}
$$

Since the log function is strictly concave and the non-negative weighted sum of concave functions is also concave, $f(\cdot)$ is also strictly concave with respect to p_1. Hence, our problem is a convex optimization problem to find optimal power of direct path p^*_1.

$$
\begin{aligned}
p_1^* &= \underset{p_1}{\operatorname{argmax}} f(p_1) \\
&\text{subject to } 0 \le p_1 \le 1
\end{aligned}
\qquad \text{(eq.a)}
$$

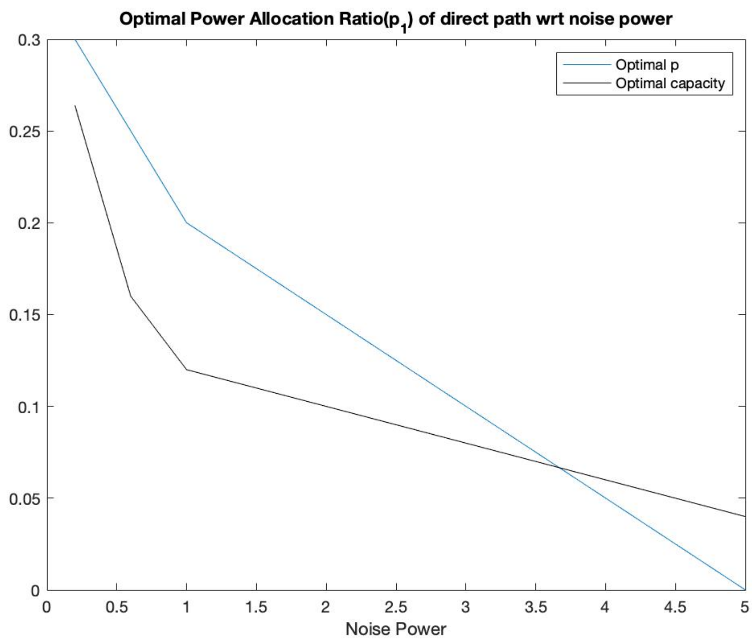
The optimization problem is a standard convex optimization problem with an inequality constraint. We first find a solution $\acute{p}_1$ to satisfy the first order necessary condition, which is expressed as

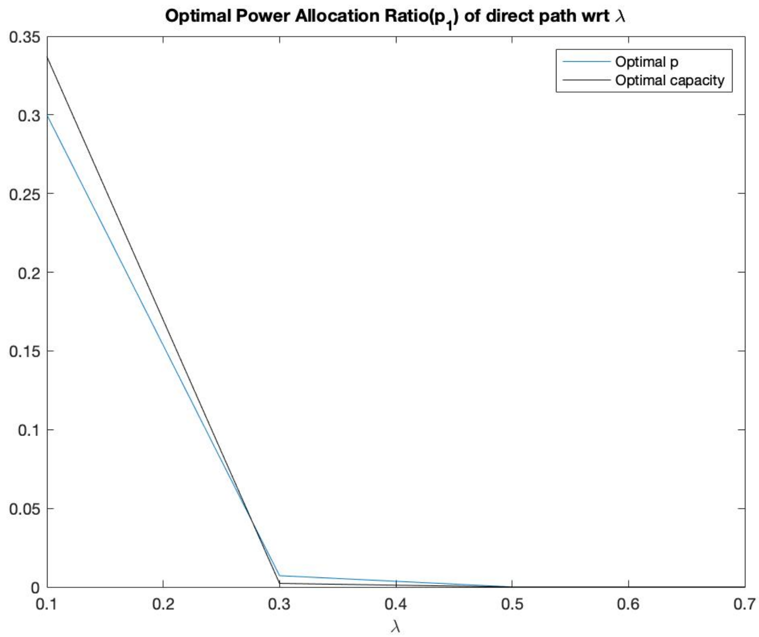
$$f'(\hat{p}_1) = 0,$$

where $\dot{f}(\cdot)$ is the derivative of $f(\cdot)$

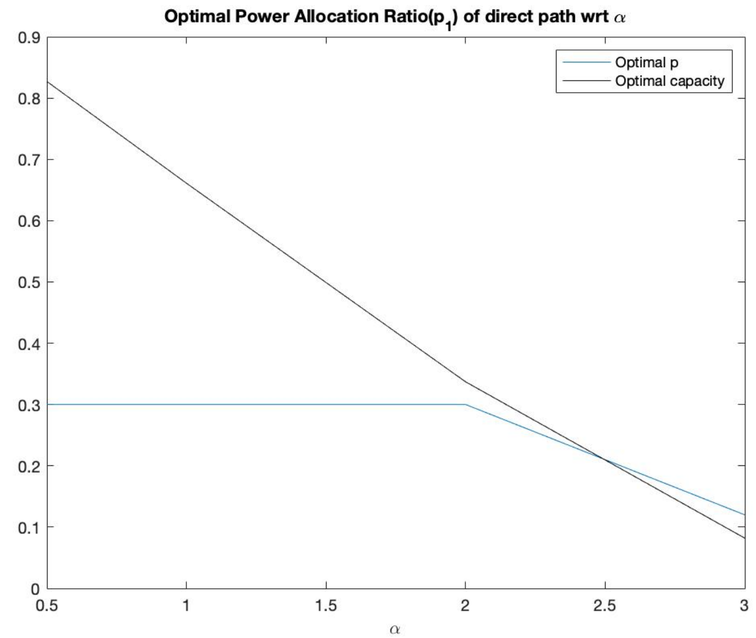
If the solution of the above equation is in (0,1) the solution becomes the solution of (a). If it is less than or equal to zero, the solution is zero, which means that all power is allocated to the reflected path. If it is greater than or equal to one, the solution is one, which means that all power is allocated to the direct path.

Results from the code - optimization

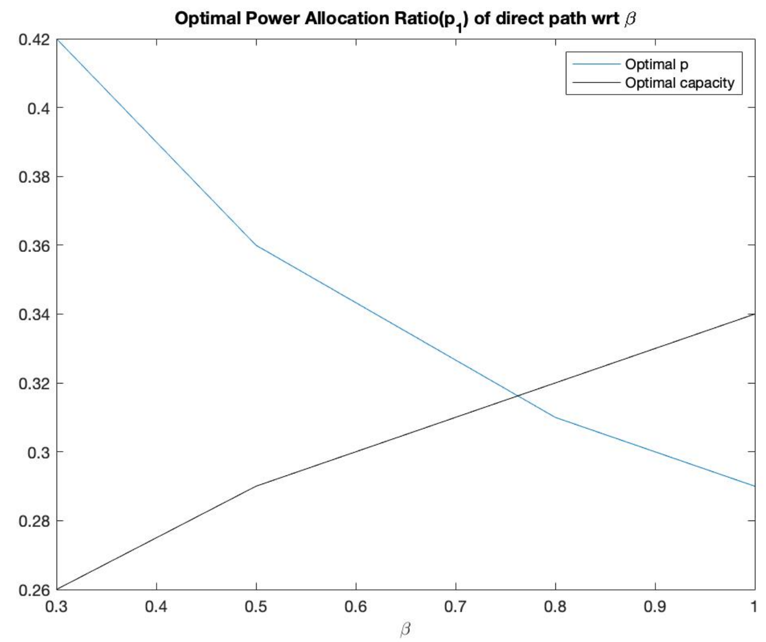




Optimal Power Allocation Ratio(p_1) of direct path wrt λ
Optimal p
Optimal capacity
λ



Optimal Power Allocation Ratio(p_1) of direct path wrt α
Optimal p
Optimal capacity
α



Optimal Power Allocation Ratio(p_1) of direct path wrt β
Optimal p
Optimal capacity
β