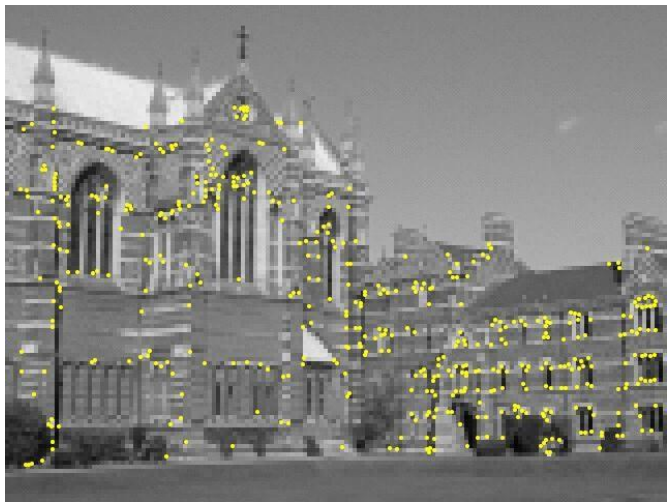
Scale Invariant Feature Transform (SIFT)

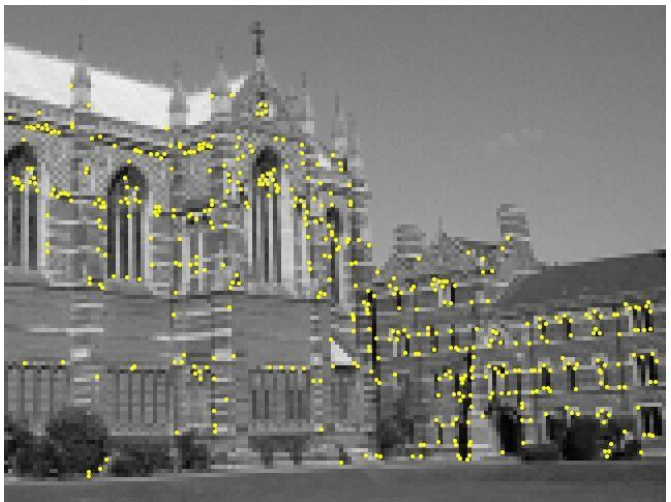
What is SIFT?

- It is a technique for **detecting** salient, stable feature points in an image.
- For every such point, it also provides a set of **"features"** that "characterize/describe" a small image region around the point.
- These features are invariant to rotation and scale.

Motivation for SIFT

- Image matching
  - Estimation of affine transformation/homography between images
  - Estimation of fundamental matrix in stereo
- Structure from motion, tracking, motion segmentation



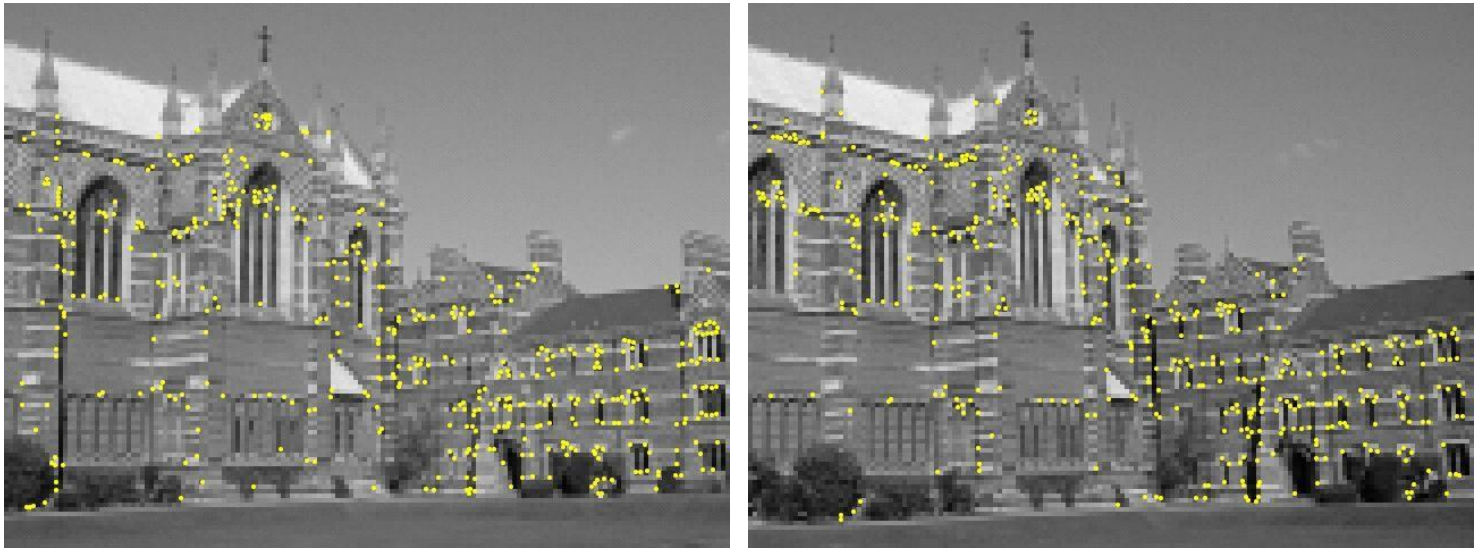


Motivation for SIFT

- All these applications need to
- (1) detect salient, stable points in two or more images, and (2) determine correspondences between them.
- To determine correspondences correctly, we need some features characterizing a salient point.
- These features must not change with:
  - Object position/pose
  - Scale
  - Illumination
  - Minor image artifacts/noise/blur

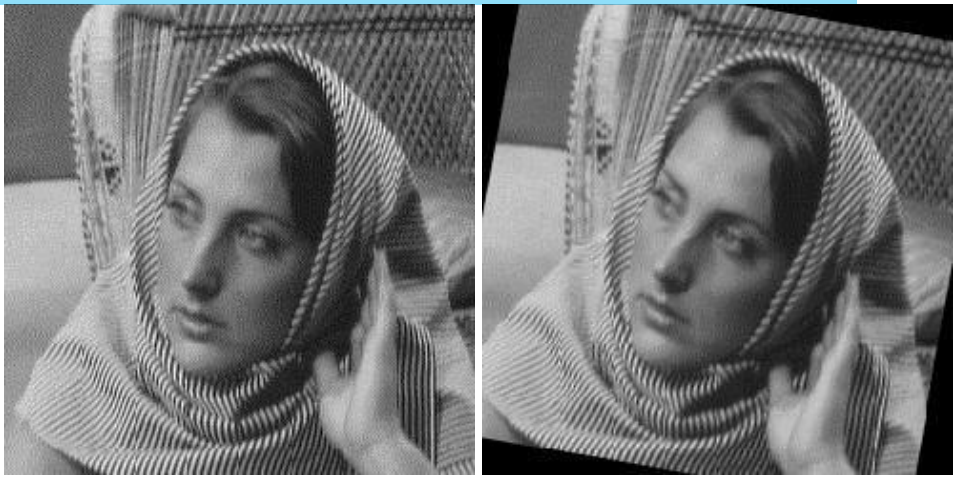
Motivation for SIFT

- Individual pixel color values are not an adequate feature to determine correspondences (why?).



Motivation for SIFT

- One could try matching patches around the salient feature points – but these patches will themselves change if there is change in object pose or illumination.
- So these patches will lead to several false matches/correspondences.



Motivation for SIFT

- SIFT provides features characterizing a salient point that remain invariant to changes in scale or rotation.

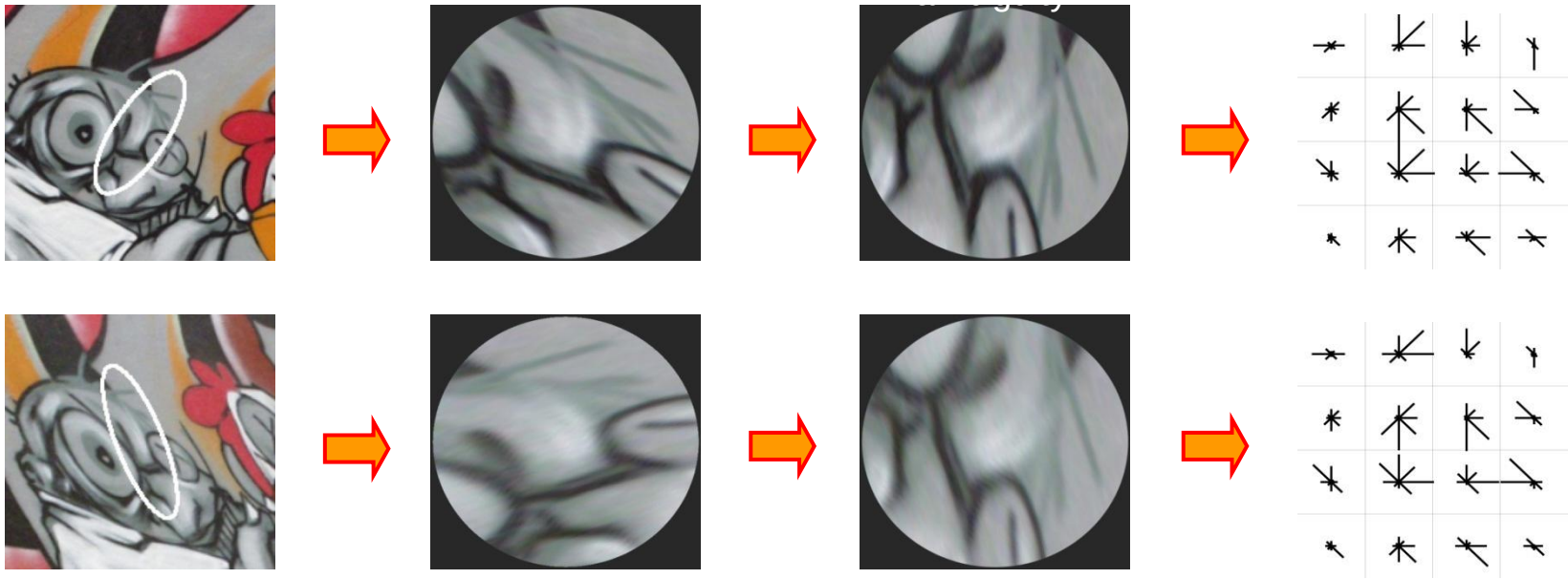


Image taken from slides by George Bebis (UNR).

Steps of SIFT algorithm

- Determine approximate location and scale of salient feature points (also called **keypoints**)
- Refine their location and scale
- Determine orientation(s) for each keypoint.
- Determine descriptors for each keypoint.

Step 1: Approximate keypoint location

- Look for intensity changes using the difference of Gaussians at two nearby scales:

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

Convolution operator: refers to the application of a filter (in this case Gaussian filter to an image)

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

Difference of Gaussians → "G"

Scale refers to the of the Gaussian.

0.0049	0.0092	0.0134	0.0152	0.0134	0.0092	0.0049
0.0092	0.0172	0.0250	0.0283	0.0250	0.0172	0.0092
0.0134	0.0250	0.0364	0.0412	0.0364	0.0250	0.0134
0.0152	0.0283	0.0412	**0.0467**	0.0412	0.0283	0.0152
0.0134	0.0250	0.0364	0.0412	0.0364	0.0250	0.0134
0.0092	0.0172	0.0250	0.0283	0.0250	0.0172	0.0092
0.0049	0.0092	0.0134	0.0152	0.0134	0.0092	0.0049

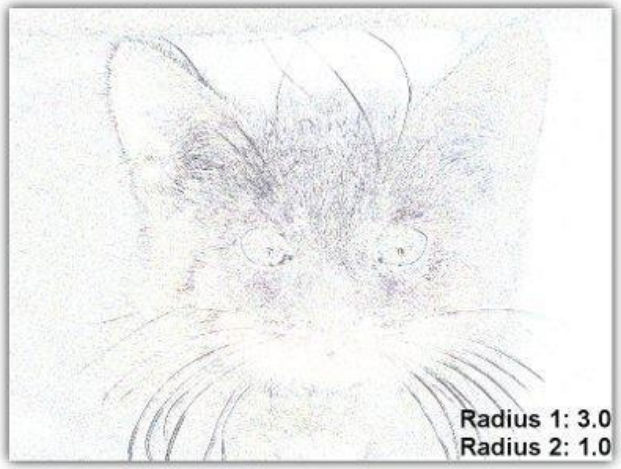
This is a 7 x 7 (truncated) Gaussian mask with mean zero and standard deviation $\sigma = 2$.

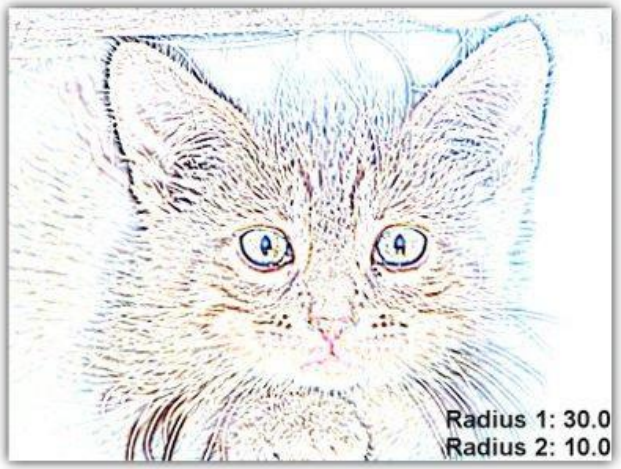
Convolution means the following (non-rigorously):

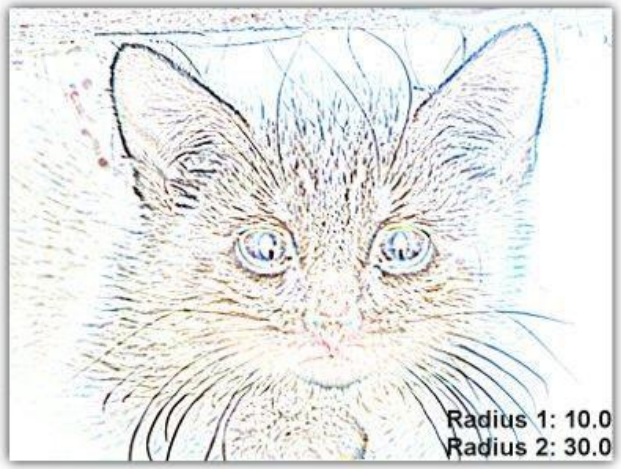
Let the original image be **A**. Let the new image be **B**. You move the mask all over the image **A**. Let us suppose the mask is centered at location (i,j) in image **A**. You compute the point-wise product between the mask entries and the corresponding entries in **A**. You store the sum of these products in **B**(i,j).

$$B(i,j) = \sum_{a=-r}^{r} \sum_{b=-r}^{r} A(i-a, j-b) M(a,b)$$

Whenever you are performing a filtering operation on image, the resultant image is obtained by ***convolving*** the original image with the filter, and is said to be the ***response*** to the filter. For further details, refer to Section 3.4 of the book on Digital Image Processing by Gonzalez, or see the animation at: http://en.wikipedia.org/wiki/Convolution





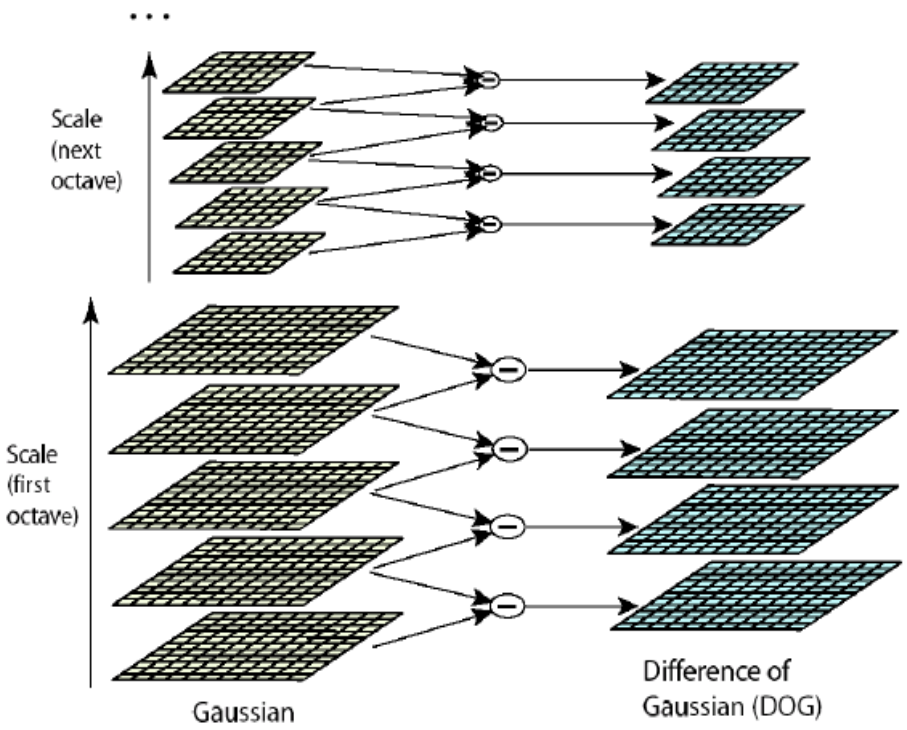


http://www.gimpbible.com/files/edge-detection-difference-of-gaussians/

This is an example of the DoG filter in gimp. In SIFT, however, the DoG is computed from Gaussians at nearby scales.

Step 1: Approximate keypoint location

Down-sampling



$$D(x, y, \sigma) = L(x, y, k\sigma) - L(x, y, \sigma)$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

Octave = doubling of σ_0. Within an octave, the adjacent scales differ by a constant factor k. If an octave contains s+1 images, then $k = 2^{(1/s)}$. The first image has scale σ_0, the second image has scale $k\sigma_0$, the third image has scale $k^2\sigma_0$, and the last image has scale $k^s\sigma_0$. Such a sequence of images convolved with Gaussians of increasing σ constitute a so-called **scale space**.

Scale = 0

Scale = 1

Scale = 4

Scale = 16

Scale = 64

Scale = 256

http://en.wikipedia.org/wiki/Scale_space

Step 1: Approximate keypoint location

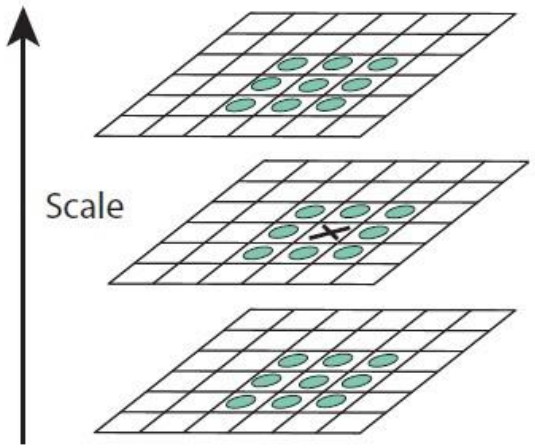


Image taken from D. Lowe, "Distinctive Image Features from Scale-Invariant Points", IJCV 2004

Figure 2: Maxima and minima of the difference-of-Gaussian images are detected by comparing a pixel (marked with X) to its 26 neighbors in 3x3 regions at the current and adjacent scales (marked with circles).

The keypoints are maxima or minima in the "scale-space pyramid", i.e. the stack of DoG images. Hereby, you get both the location as well as the scale of the keypoint.

Initial detection of keypoints

http://upload.wikimedia.org/wikipedia/commons/4/44/Sift_keypoints_filtering.jpg

Step 2: Refining keypoint location

- The keypoint location and scale is discrete – we can interpolate for greater accuracy.
- For this, we express the DoG function in a small 3D neighborhood around a keypoint (x_i,y_i,σ_i) by a second-order Taylor-series:

$$D(x,y,\sigma) = D(x_i,y_i,\sigma_i) + \left(\frac{\partial D(x,y,\sigma)}{\partial(x,y,\sigma)}\right)^T_{\substack{x=x_i,\\ y=y_i\\ \sigma=\sigma_i}} \Delta + \frac{1}{2}\Delta^T \left(\frac{\partial^2 D(x,y,\sigma)}{\partial(x,y,\sigma)^2}\right)_{\substack{x=x_i,\\ y=y_i\\ \sigma=\sigma_i}} \Delta;$$

$$\Delta = \begin{pmatrix} x - x_i \\ y - y_i \\ \sigma - \sigma_i \end{pmatrix}$$

Gradient vector evaluated digitally at the keypoint

3 x 3 Hessian matrix evaluated digitally at the keypoint

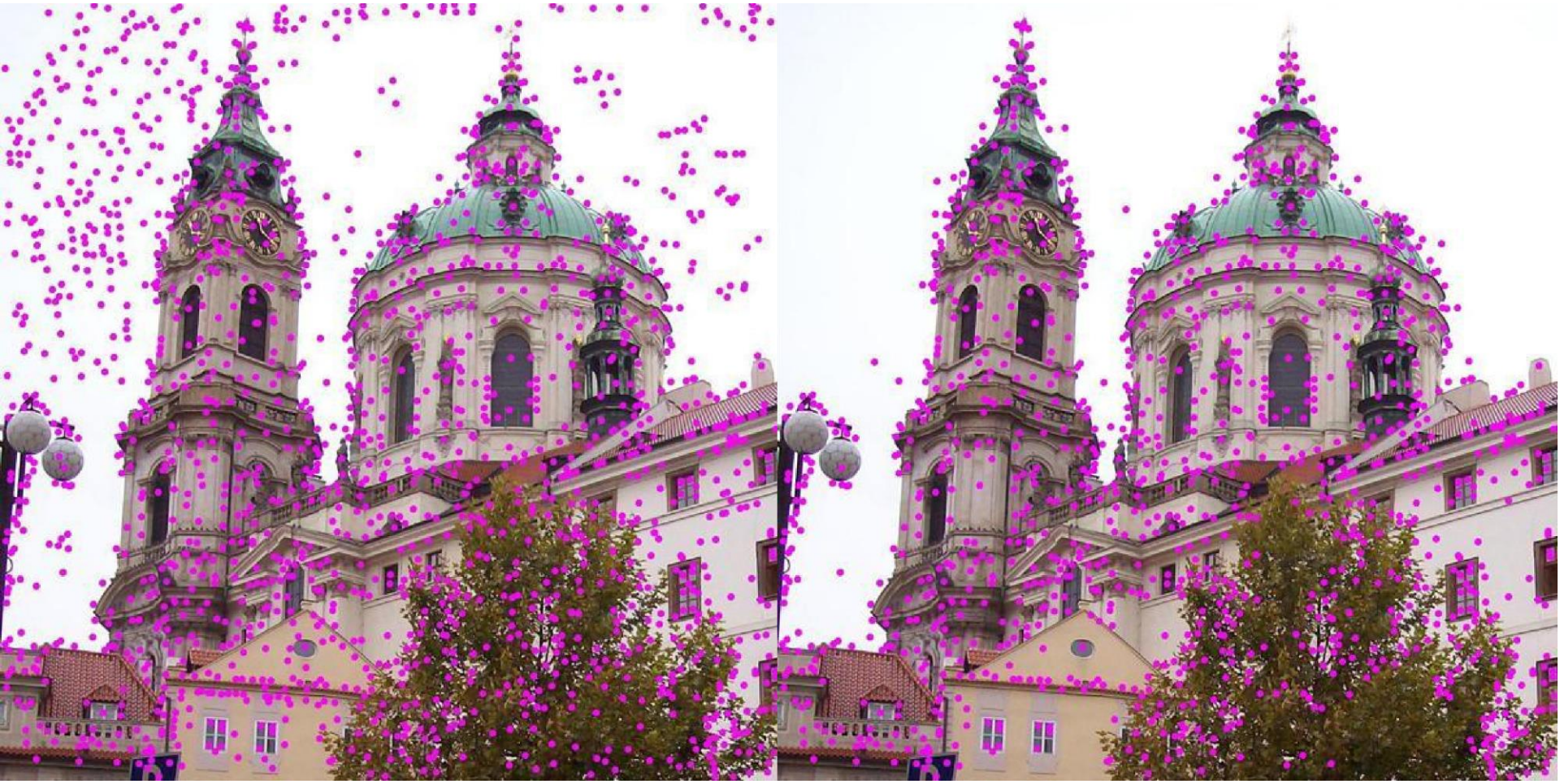
Step 2: Refining keypoint location

- To find an extremum of the DoG values in this neighborhood, set the derivative of D(.) to 0. This gives us:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{\sigma} \end{pmatrix} = -\left(\frac{\partial^2 D(x,y,\sigma)}{\partial (x,y,\sigma)^2} \right)^{-1}_{\substack{x=x_i, \\ y=y_i, \\ \sigma=\sigma_i}} \left(\frac{\partial D(x,y,\sigma)}{\partial (x,y,\sigma)} \right)_{\substack{x=x_i, \\ y=y_i, \\ \sigma=\sigma_i}}$$

$$\therefore D_{extremal} = D(x_i, y_i, \sigma_i) + \frac{1}{2} \left(\frac{\partial D(x,y,\sigma)}{\partial (x,y,\sigma)} \right)^{T}_{\substack{x=x_i, \\ y=y_i, \\ \sigma=\sigma_i}} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{\sigma} \end{pmatrix}$$

- The keypoint location is updated.
- All extrema with $|D_{extremal}| < 0.03$, are discarded as "weak extrema" or "low contrast points".



Removal of low-contrast keypoints

http://upload.wikimedia.org/wikipedia/commons/4/44/Sift_keypoints_filtering.jpg

Step 2: Refining keypoint location

- Some keypoints reside on edges, as edges always give a high response to a DoG filter.
- But edges should not be considered salient points (why?).
- So we discard points that lie on edges.
- In the case of KLT tracker, we saw how to detect points lying on salient edges using the structure tensor.

Step 2: Refining keypoint location

- The SIFT paper uses the 2nd derivative matrix (called the Hessian matrix):

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

- The eigenvalues of **H** give a lot of information about the local structure around the keypoint.
- In fact, the eigenvalues are the **maximal** and **minimal principal curvatures** of the surface

D(x,y), i.e. of the DoG function, at that point.

http://en.wikipedia.org/wiki/File:Minimal_surface_curvature_planes-en.svg

http://en.wikipedia.org/wiki/Principal_curvature

Step 2: Refining keypoint location

- An edge will have **high maximal** curvature, but very **low minimal** curvature.
- A keypoint which is a corner (not an edge) will have **high maximal and minimal** curvature.
- The following can be regarded as an edge-ness measure:

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

Should be less than a threshold (say 10).

For an edge, α >> β, leading to a large value of this measure.

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

$$\alpha = r\beta$$

Why this measure instead of r? To save computations – we need not compute eigenvalues!

- http://upload.wikimedia.org/wikipedia/commons/4/44/Sift_keypoints_filtering.jpg

Step 3: Assigning orientations

- Compute the gradient magnitudes and orientations in a small window around the keypoint – at the appropriate scale.

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$m(x, y) = \sqrt{(L(x+1, y) - L(x-1, y))^2 + (L(x, y+1) - L(x, y-1))^2}$$

$$\theta(x, y) = \tan^{-1}((L(x, y+1) - L(x, y-1))/(L(x+1, y) - L(x-1, y)))$$

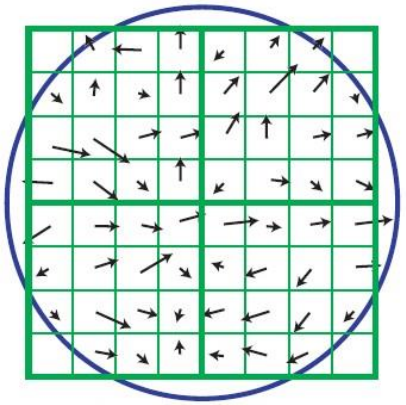
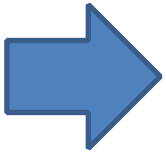
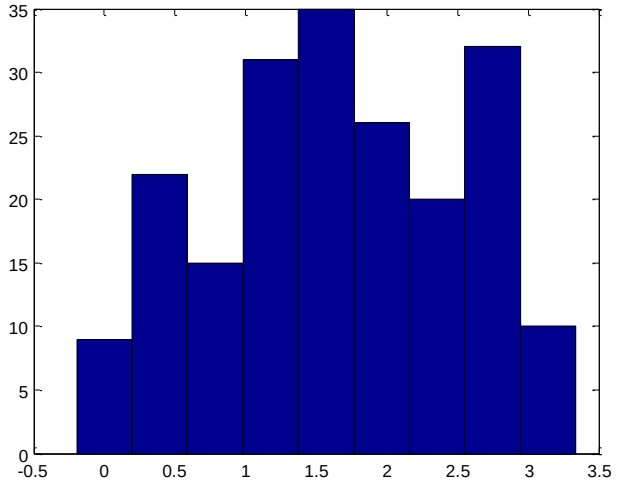


Image gradients





Histogram of gradient orientation – the bin-counts are weighted by gradient magnitudes and a Gaussian weighting function. Usually, 36 bins are chosen for the orientation.

Step 3: Assigning orientations

- Assign the **dominant orientation** as the orientation of the keypoint.
- In case of multiple peaks or histogram entries more than 0.8 x peak, create a **separate** descriptor for ***each*** orientation (they will all have the same scale and location).

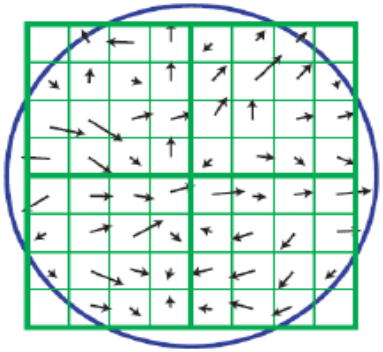
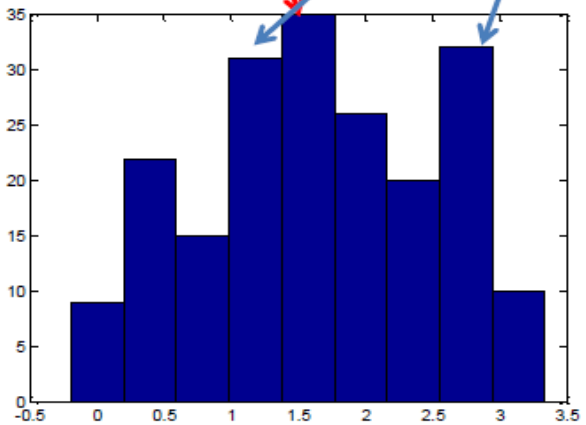


Image gradients



Histogram of gradient orientation – the bin-counts are weighted by gradient magnitudes and a Gaussian weighting function. Usually, 36 bins are chosen for the orientation.

Step 4: Descriptors for each keypoint

- Consider a small region around the keypoint. Divide it into *n* x *n* cells (usually *n* = 2). Each cell is of size 4 x 4.
- Build a **gradient orientation histogram** in each cell. Each histogram entry is weighted by the *gradient magnitude* and a *Gaussian weighting function* with σ = 0.5 times window width.
- **Sort** each gradient orientation histogram bearing in mind the dominant orientation of the keypoint (assigned in step 3).

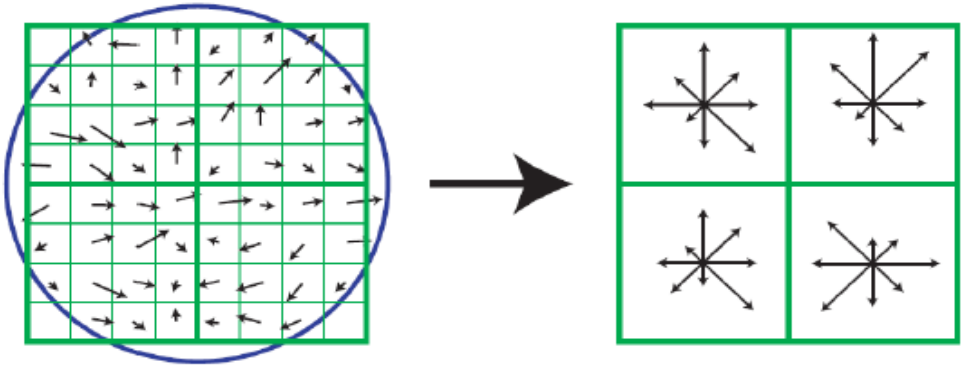


Image gradients

Keypoint descriptor

Image taken from D. Lowe, "Distinctive Image Features from Scale-Invariant Points", IJCV 2004

Step 4: Descriptors for each keypoint

- We now have a descriptor of size rn^2 if there are r bins in the orientation histogram.
- Typical case used in the SIFT paper: $r = 8$, $n = 4$, so length of each descriptor is 128.
- The descriptor is invariant to rotations due to the sorting.

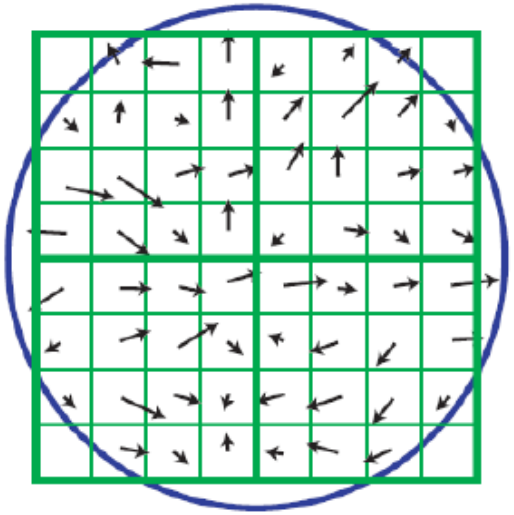
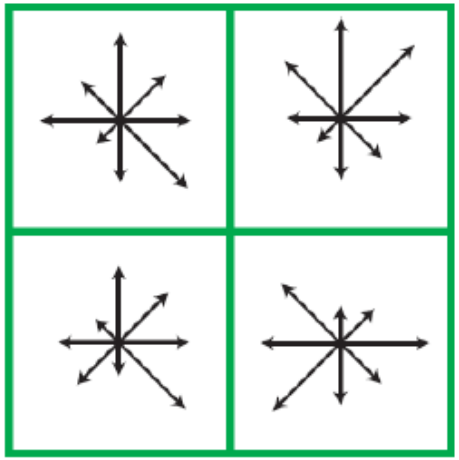


Image gradients

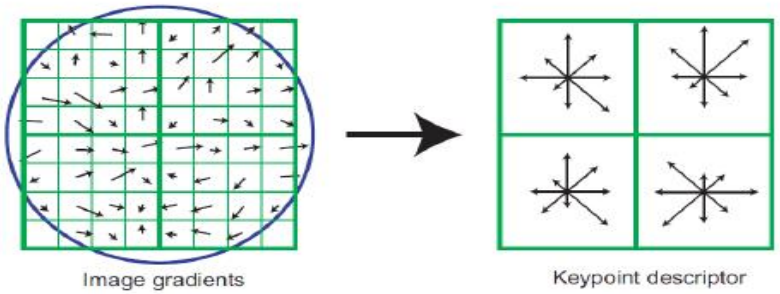


Keypoint descriptor

Image taken from D. Lowe, "Distinctive Image Features from Scale-Invariant Points", IJCV 2004

Step 4: Descriptors for each keypoint

- For scale-invariance, the size of the window should be adjusted as per scale of the keypoint. Larger scale = larger window.



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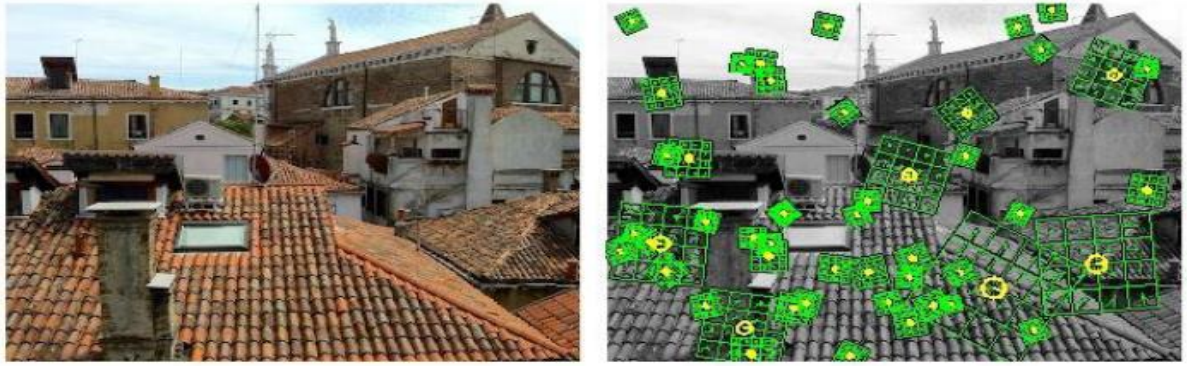


http://www.vlfeat.org/overview/sift.html

Step 4: Descriptors for each keypoint

- The SIFT descriptor (so far) is not illumination invariant –the histogram entries are weighted by gradient magnitude.
- Hence the descriptor vector is normalized to unit magnitude. This will normalize scalar multiplicative intensity changes.
- Scalar additive changes don't matter –gradients are invariant to constant offsets anyway.
- Not insensitive to non-linear illumination changes.

Step 1: Keypoint computations? (More details)

- Why are DoG filters used?
- The Gaussian filter (and its derivatives) is shown to be the only filter obeys all of the following:
  - ✓ Linearity
  - ✓ Shift-invariance
  - ✓ Structures at coarser scales are related to structures at finer scales in a consistent way (smoothing process does not produce new structures)
  - ✓ Rotational symmetry
  - ✓ Semi-group property: $G(x, y, \sigma_1) * G(x, y, \sigma_2) = G(x, y, \sigma_1 + \sigma_2)$
  - ✓ + some other properties

    http://en.wikipedia.org/wiki/Scale_space

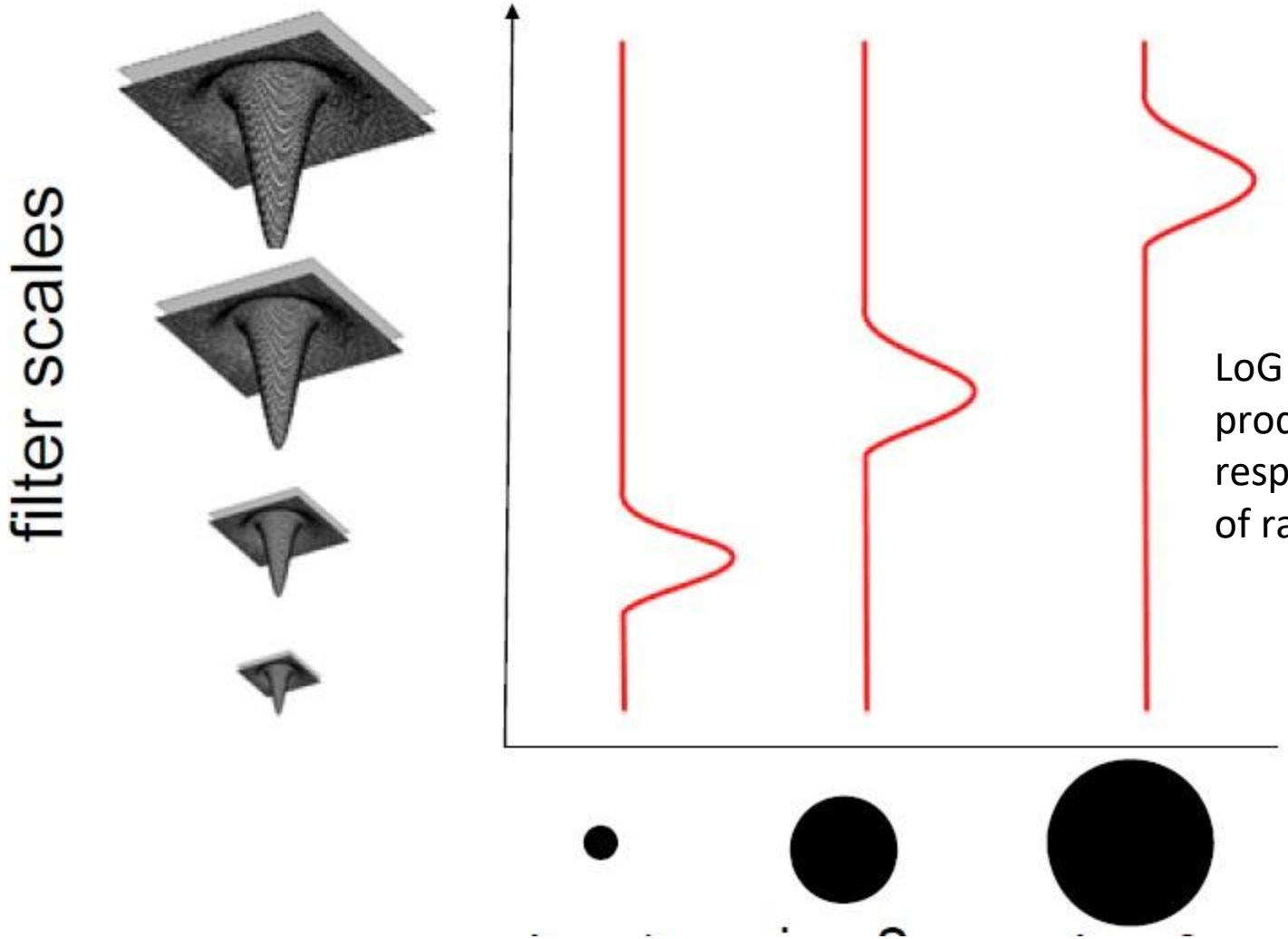
Step 1: Keypoint computations? (More details)

- Why **difference** of Gaussians?

✓ The DoG is an approximation to the scale multiplied Laplacian of Gaussian filter in image processing – a rotationally invariant filter.

✓ The DoG is a good model for how neurons in the retina extract image details to be sent to the brain for processing.

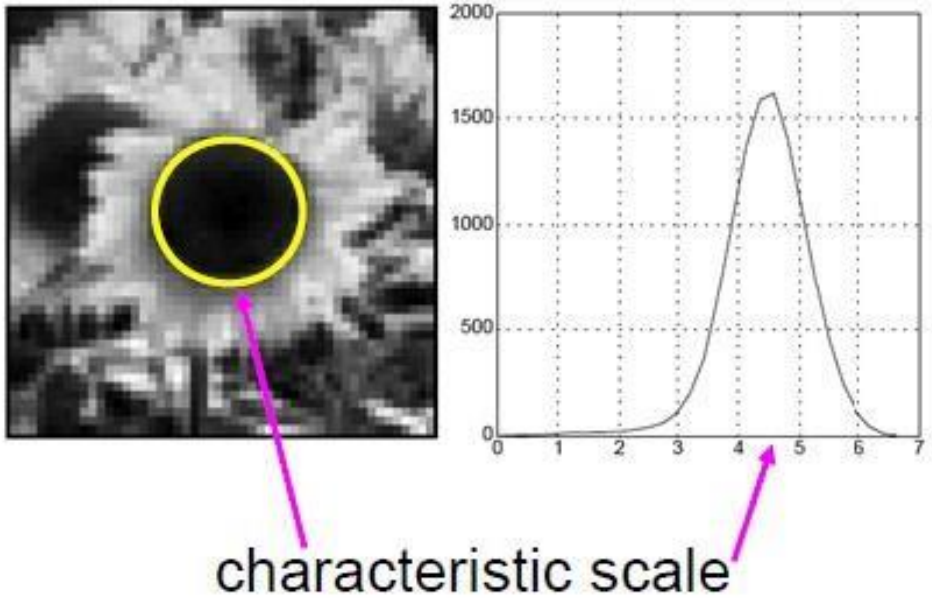
Laplacian-of-Gaussian = "blob" detector $\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$



LoG filter of scale σ produces strong responses for patterns of radius σ * sqrt(2).

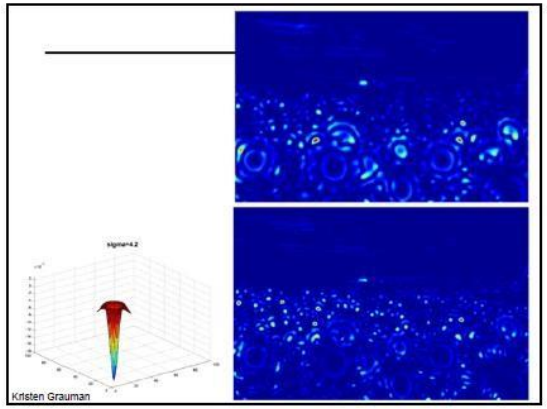
http://www.cs.utexas.edu/~grauman/courses/spring2011/slides/lecture14_localfeats.pdf

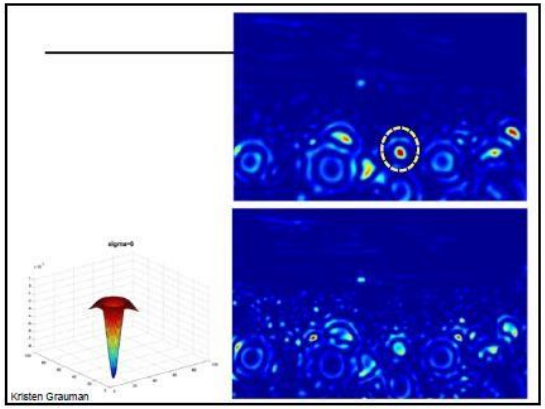
We define the *characteristic scale* as the scale that produces peak of Laplacian response

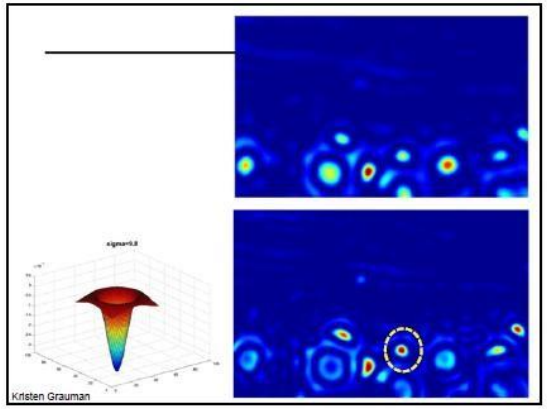


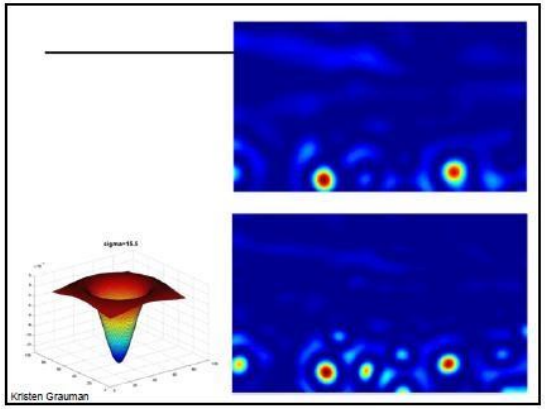
Keypoint = center of blob

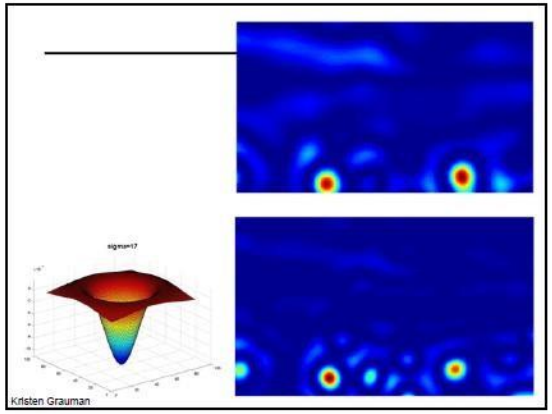
http://www.cs.utexas.edu/~grauman/courses/spring2011/slides/lecture14_localfeats.pdf











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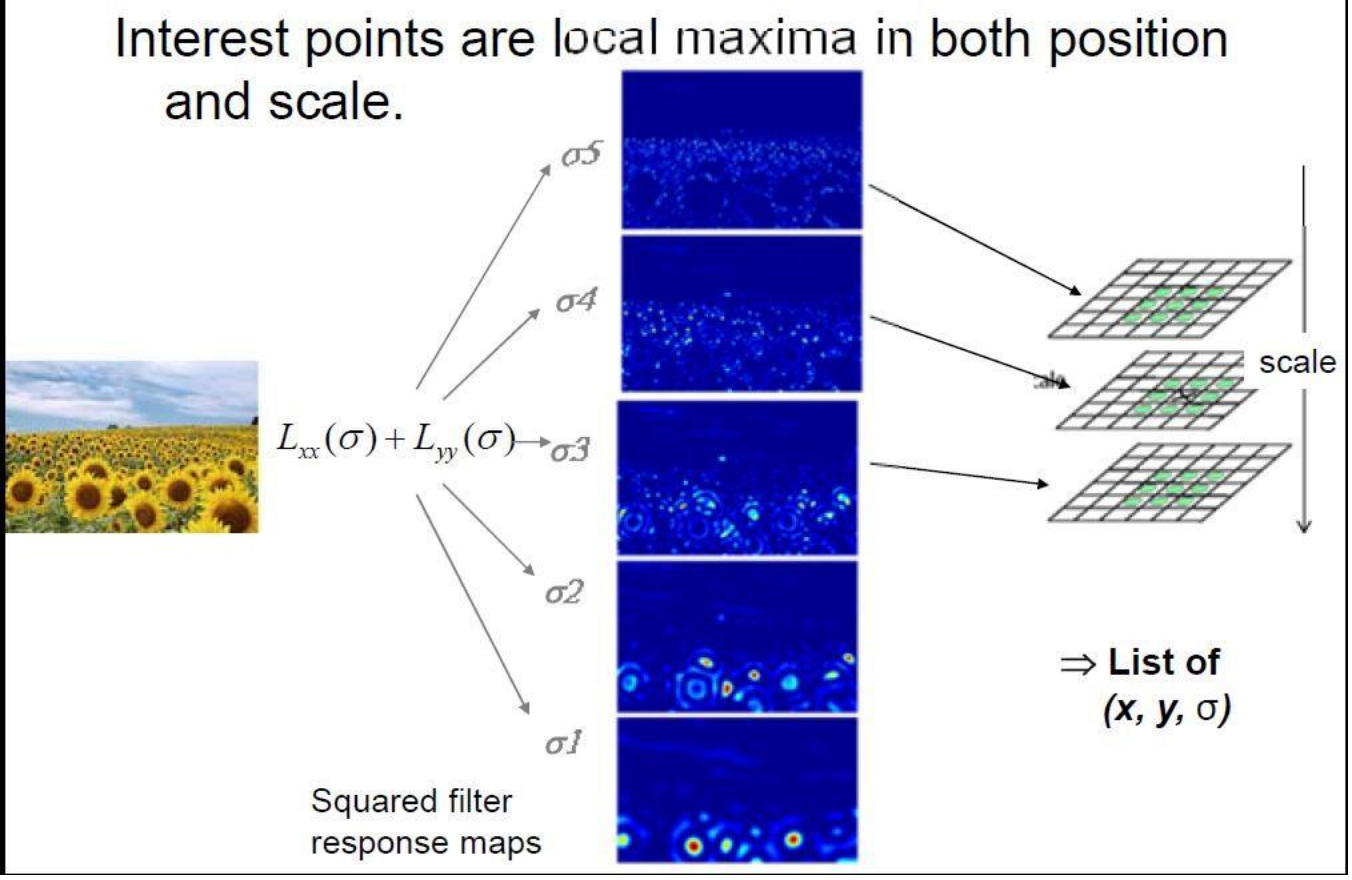
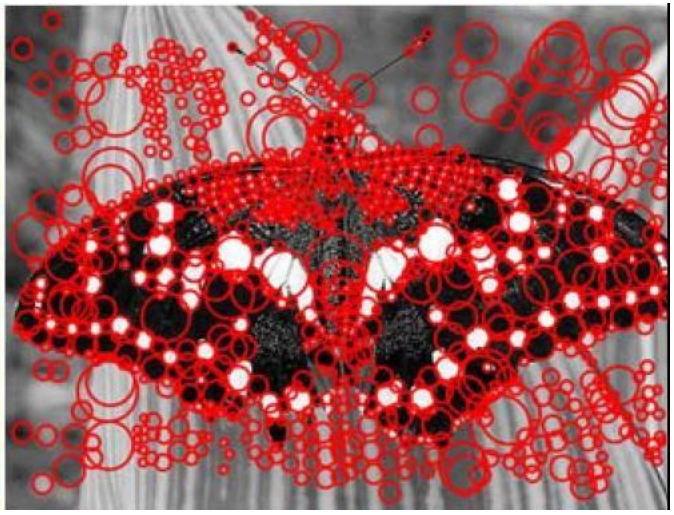


http://www.cs.utexas.edu/~grauman/courses/spring2011/slides/lecture14_localfeats.pdf



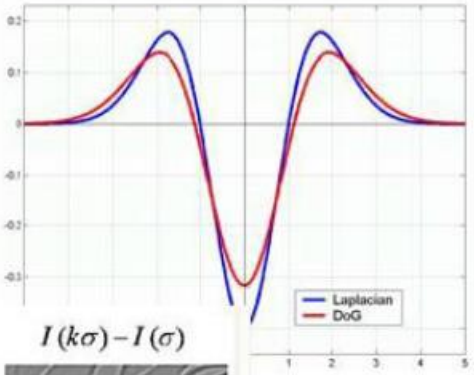
We can approximate the Laplacian with a difference of Gaussians; more efficient to implement.

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



http://www.cs.utexas.edu/~grauman/courses/spring2011/slides/lecture14_localfeats.pdf

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G. = G_{xx} + G_{yy}$$

Isotropic heat equation: running this equation on an image is equivalent to smoothing the image with a Gaussian.

$$\sigma \nabla^2 G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

Numerical approximation

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1)\sigma^2 \nabla^2 G.$$

Step 1: Keypoint computations?(More details)

- Why do we look for extrema of the DoG function?

✓Maxima of the DoG indicate dark points (blobs) on a bright background.

✓Minima of the DoG indicate bright points (blobs) on a dark background.

- Why do we look for extrema in a spatial as well as scale sense?

✓ It helps us pick the “scale” associated with the keypoint!

Step 1: Keypoint computations?

- How many scales per octave? Answer: 3 – it is empirically observed that this provides optimal **repeatability** under downsampling/upsampling/rotation of the image as well as image noise.

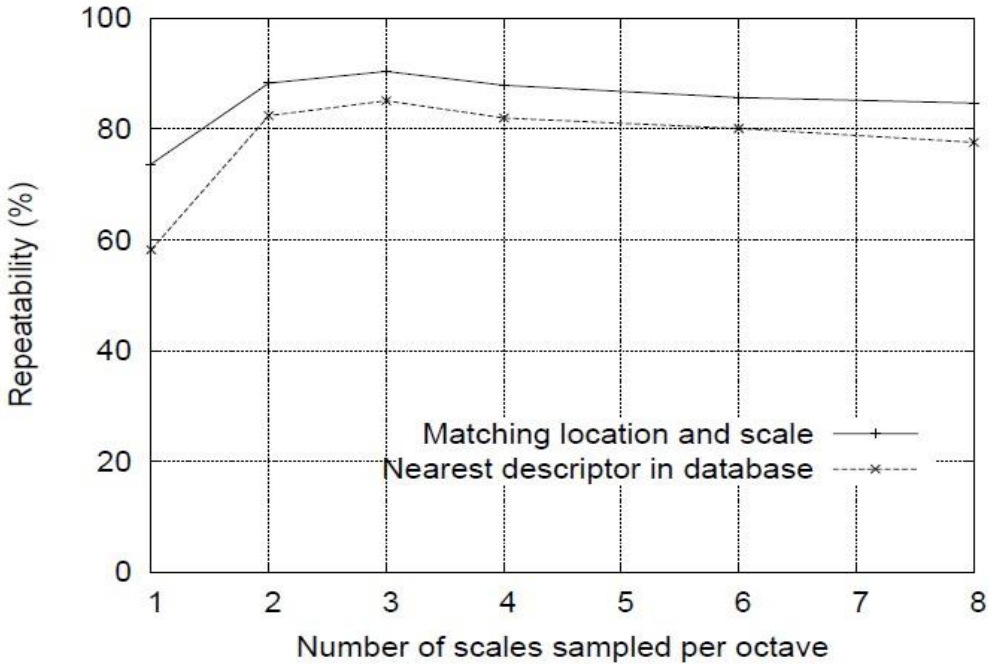


Image taken from D. Lowe, "Distinctive Image Features from Scale-Invariant Points", IJCV 2004

Step 1: Keypoint computations?

- Adding more scales per octave will increase the number of detected keypoints, but this does not improve the repeatability (in fact there is a small decrease) – so we settle for the computationally less expensive option.

Summary of SIFT descriptor properties

- Invariant to spatial rotation, translation, scale.
- Experimentally seen to be less sensitive to small spatial affine or perspective changes.
- Invariant to affine **illumination** changes.

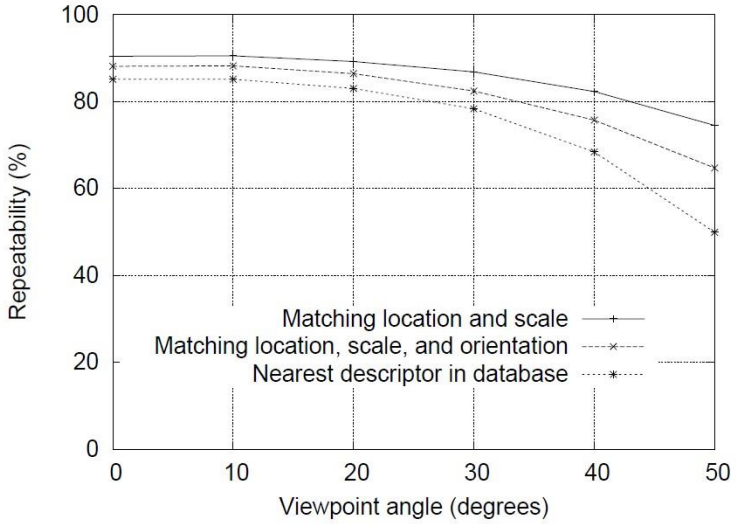


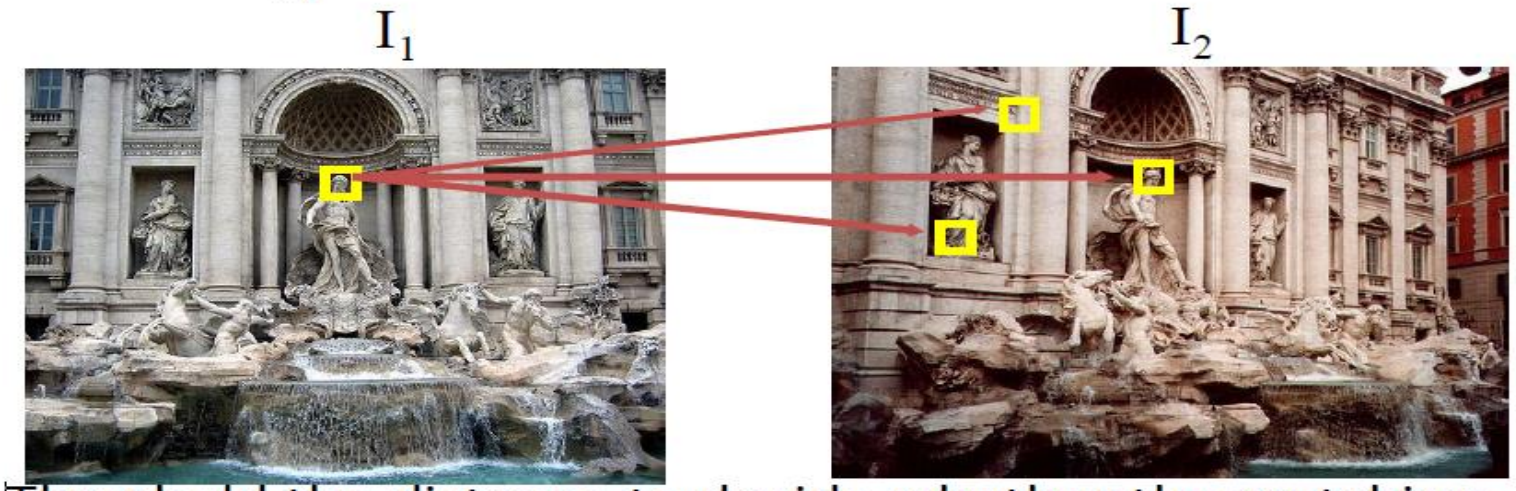
Figure 9: This graph shows the stability of detection for keypoint location, orientation, and final matching to a database as a function of affine distortion. The degree of affine distortion is expressed in terms of the equivalent viewpoint rotation in depth for a planar surface.

Image taken from D. Lowe, "Distinctive Image Features from Scale-Invariant Points", IJCV 2004

Application: Matching SIFT descriptors

- Given a keypoint descriptor in image 1, find its nearest neighbor in image 2. "Nearest" as defined typically by SSD:

$$d(a_1, a_2) = \sum_{i=1}^{rn^2} (a_1(i) - a_2(i))^2$$



- Threshold the distance to decide whether the matching pair was valid.

May lead to many false matches.

Application: Matching SIFT descriptors

- Consider the match between the keypoints to be valid if and only if the second nearest neighbor distance (SNND) in image 2 is **sufficiently larger** than the nearest neighbor distance (NND).
- Accept match as valid if SNND/NND > 0.8 (see next slide).

Image taken from D. Lowe, “Distinctive Image Features from Scale-Invariant Points”, IJCV 2004

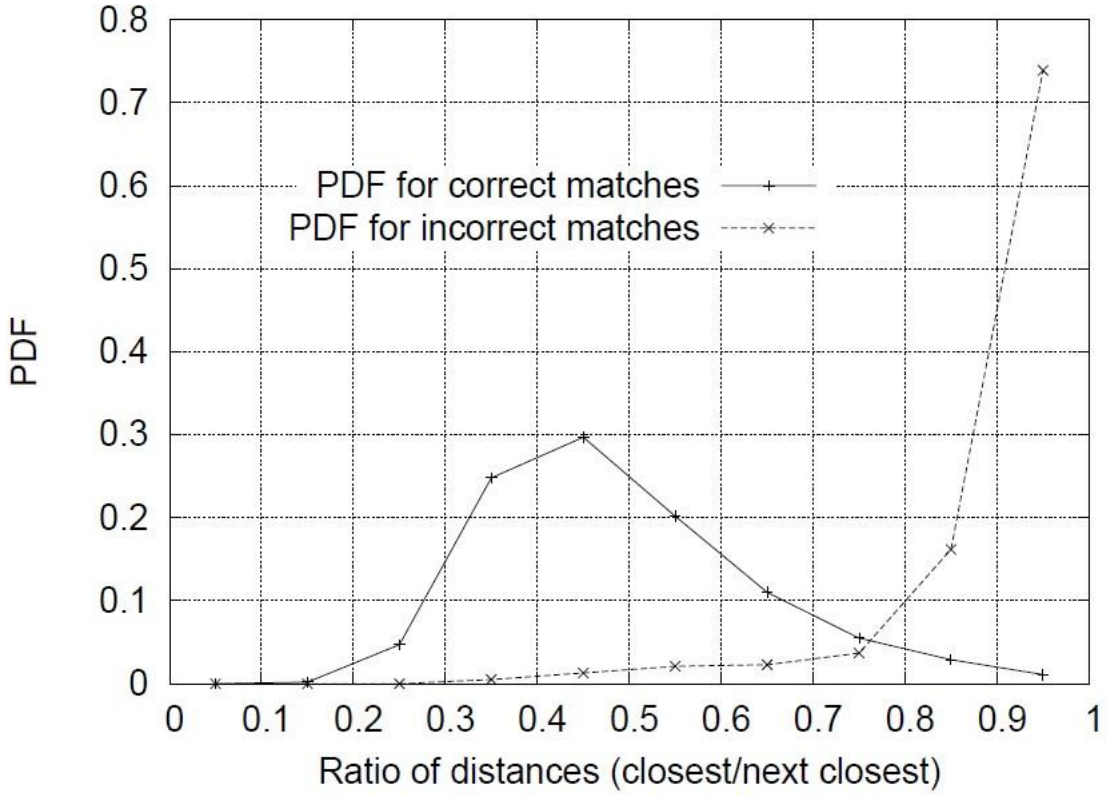


Figure 11: The probability that a match is correct can be determined by taking the ratio of distance from the closest neighbor to the distance of the second closest. Using a database of 40,000 keypoints, the solid line shows the PDF of this ratio for correct matches, while the dotted line is for matches that were incorrect.

Application: Object Recognition

- **Input (1):** A reference database of images of various objects. Each image is labeled by object name and object pose + scale.
- **Input (2):** Query images in which you locate one or more of these objects.

Application: Object Recognition

- Compute and store keypoints and their descriptors for each image in the reference database.

- Compute keypoint descriptors for the query image.

- For each keypoint, find the nearest matching descriptor in each image of the reference database subject to the SNND/NND constraint.

Application: Object Recognition

- Collect all keypoints in the query image that “vote for” a particular pose for object X in the reference database.
- “Vote for pose abc of object X” = “have a nearest neighbor in image of object X with pose θ (say)”

Application: Object Recognition

- Estimate an affine transformation between keypoint locations (x_i,y_i) from the query image and keypoint locations (u_i,v_i) for each candidate reference image.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- Verify the affine transformation: apply the transform to the keypoints and compute the difference between the transformed and target locations. Discard the point as an outlier if the difference between the orientations/scales/locations is too high.

Application: Object Recognition

- What is too high?

❑ Orientation difference > 20 degrees

❑ Scale difference more than 1.5

❑ Location difference > 0.2 * size of model

- Repeat the solution for affine transformation until no more points are thrown out.
- If number of points < 3, affine transformation cannot be estimated.

Figure 12: The training images for two objects are shown on the left. These can be recognized in a cluttered image with extensive occlusion, shown in the middle. The results of recognition are shown on the right. A parallelogram is drawn around each recognized object showing the boundaries of the original training image under the affine transformation solved for during recognition. Smaller squares indicate the keypoints that were used for recognition.

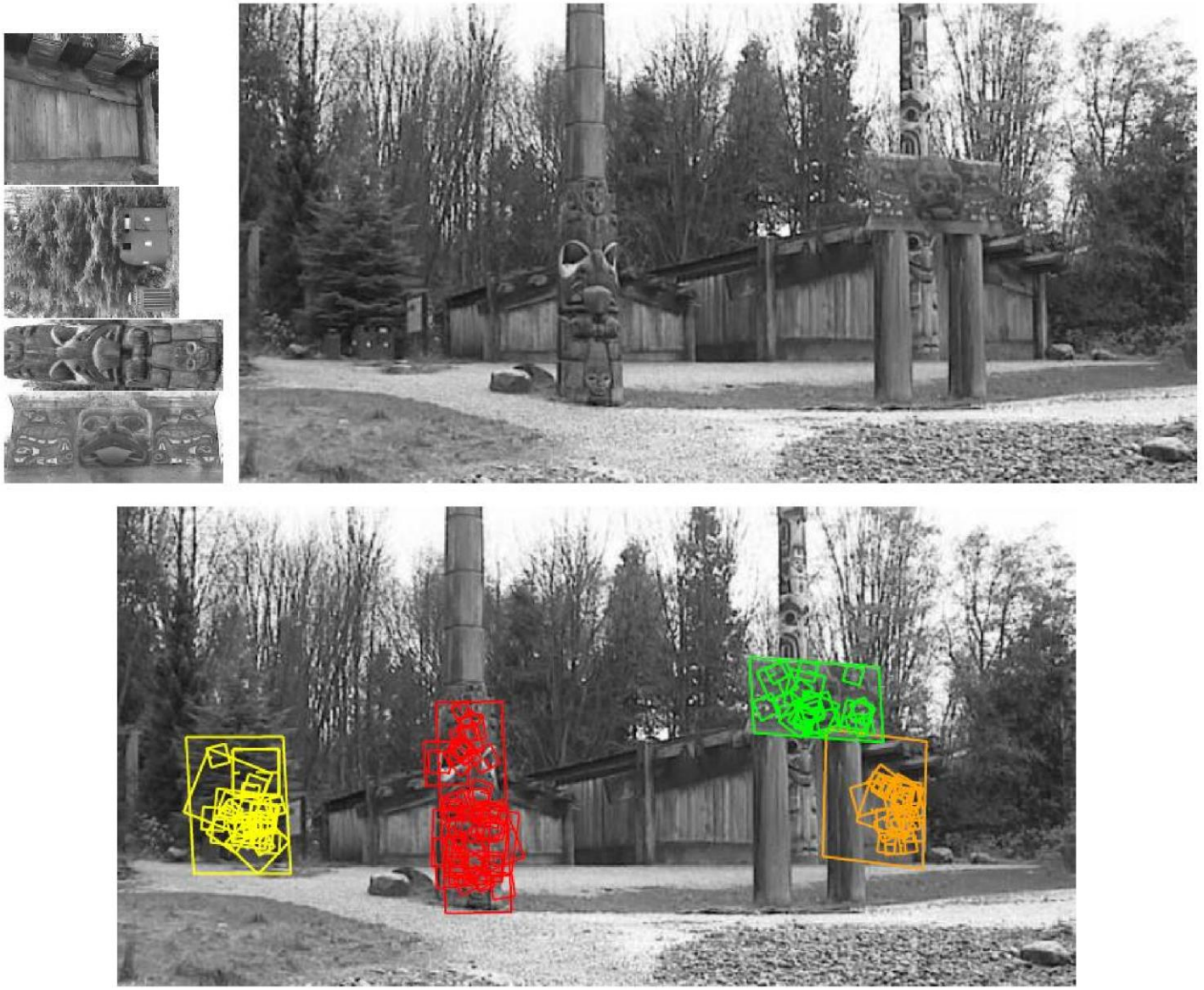


Figure 13: This example shows location recognition within a complex scene. The training images for locations are shown at the upper left and the 640x315 pixel test image taken from a different viewpoint is on the upper right. The recognized regions are shown on the lower image, with keypoints shown as squares and an outer parallelogram showing the boundaries of the training images under the affine transform used for recognition.

SIFT: ++ ☺

- Resistant to affine transformations of limited extent (works better for planar objects than full 3D objects).

- Resistant to a range of illumination changes

- Resistant to occlusions in object recognition, since SIFT descriptors are local.

SIFT: ☹

- Resistance to affine transformations is empirical – no hard-core theory provided.

- Several parameters in the algorithm: descriptor size, size of the region, various thresholds – theoretical treatment for their specification not clear.